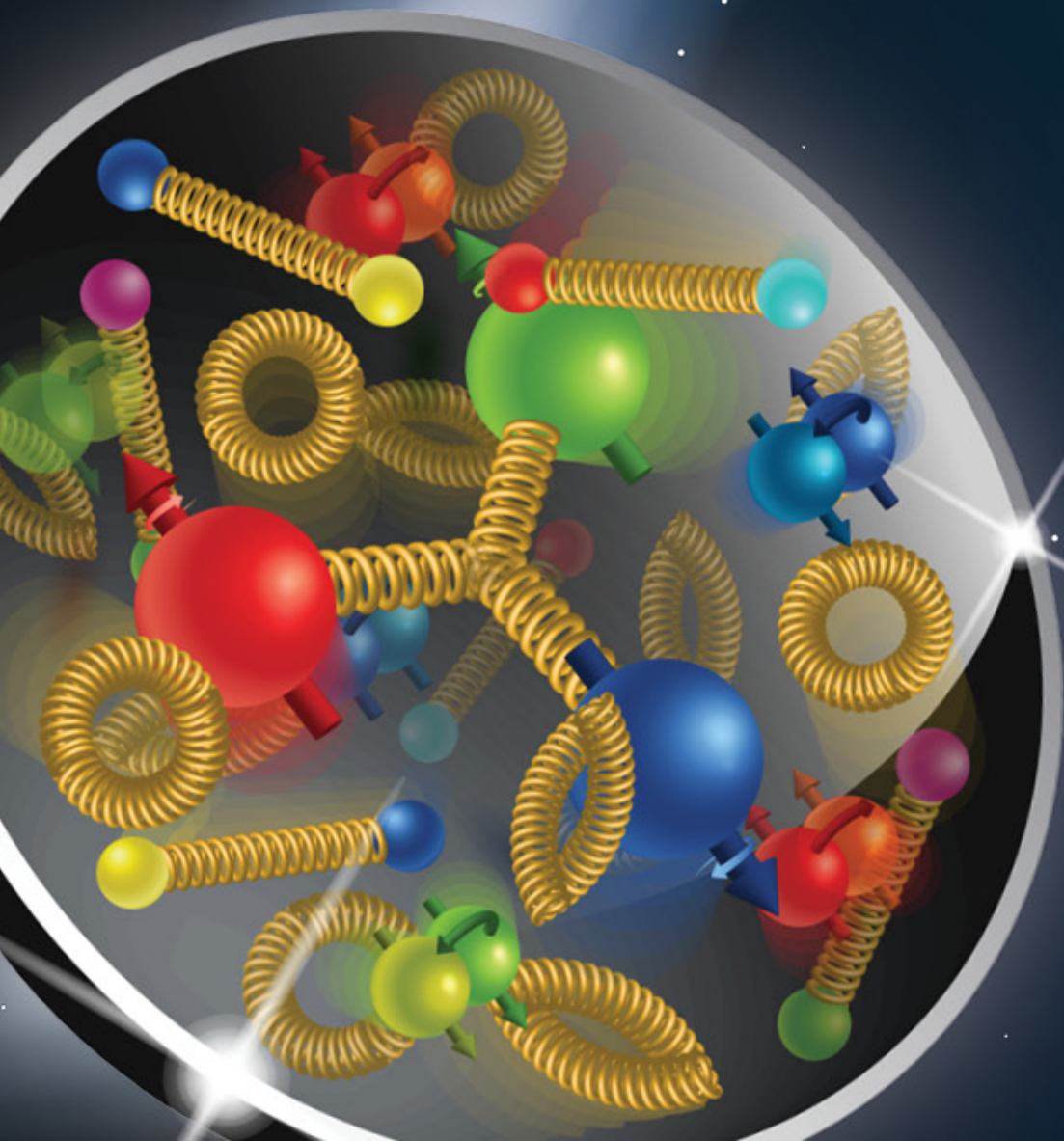


# SPIN

# at the EIC



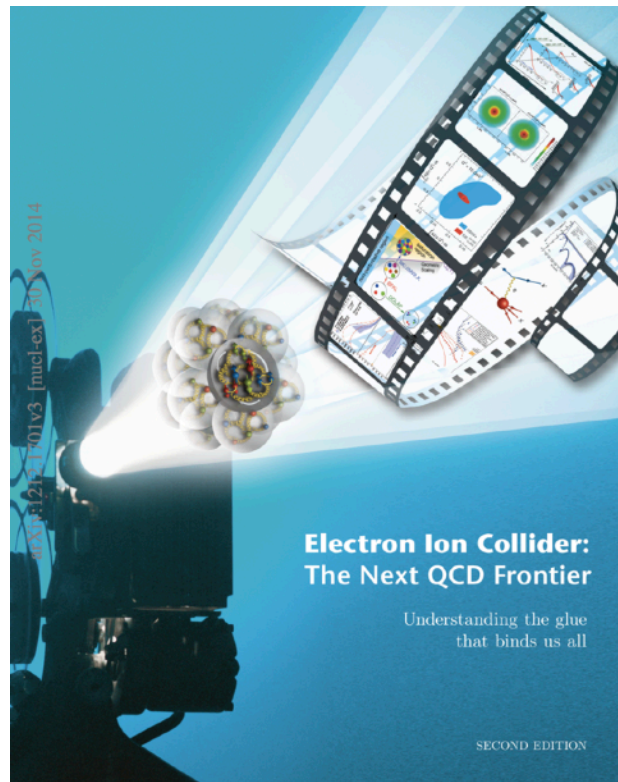
Alexei Prokudin

Pennsylvania State University Berks and Jefferson Lab

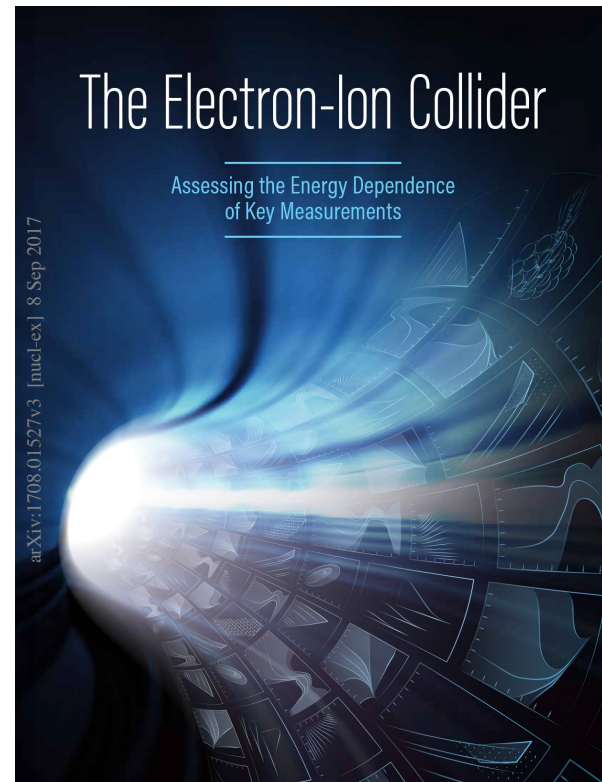


# THE ELECTRON-ION COLLIDER: RELEVANT DOCUMENTS

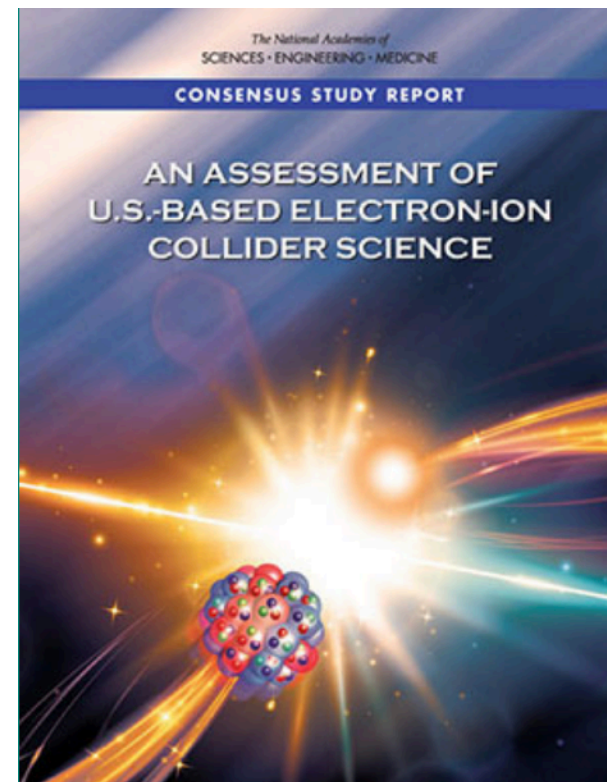
.....



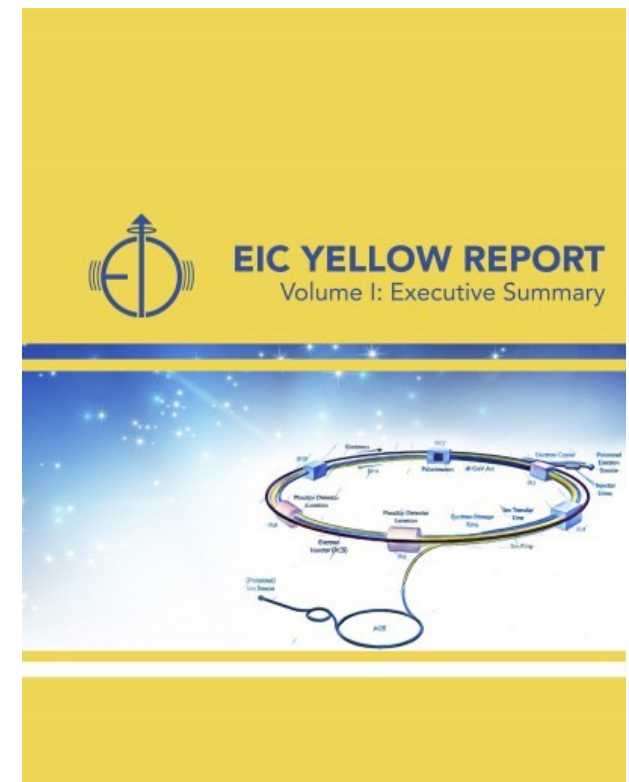
White Paper (2012)  
Accardi et al,  
arXiv:1212:1701



BNL Report (2017)  
Aschenauer et al,  
arXiv:1708.01527



NAS Study (2018)



EIC Yellow Report (2021)  
arXiv:2103.05419

Yellow Paper (2016)  
Accardi et al, Eur. Phys. J. A  
(2016) 52: 268

# WHY SPIN?

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Spin is a fundamental degree of freedom originated from the space-time symmetry

Spin plays a critical role in determining the basic structure of fundamental interactions

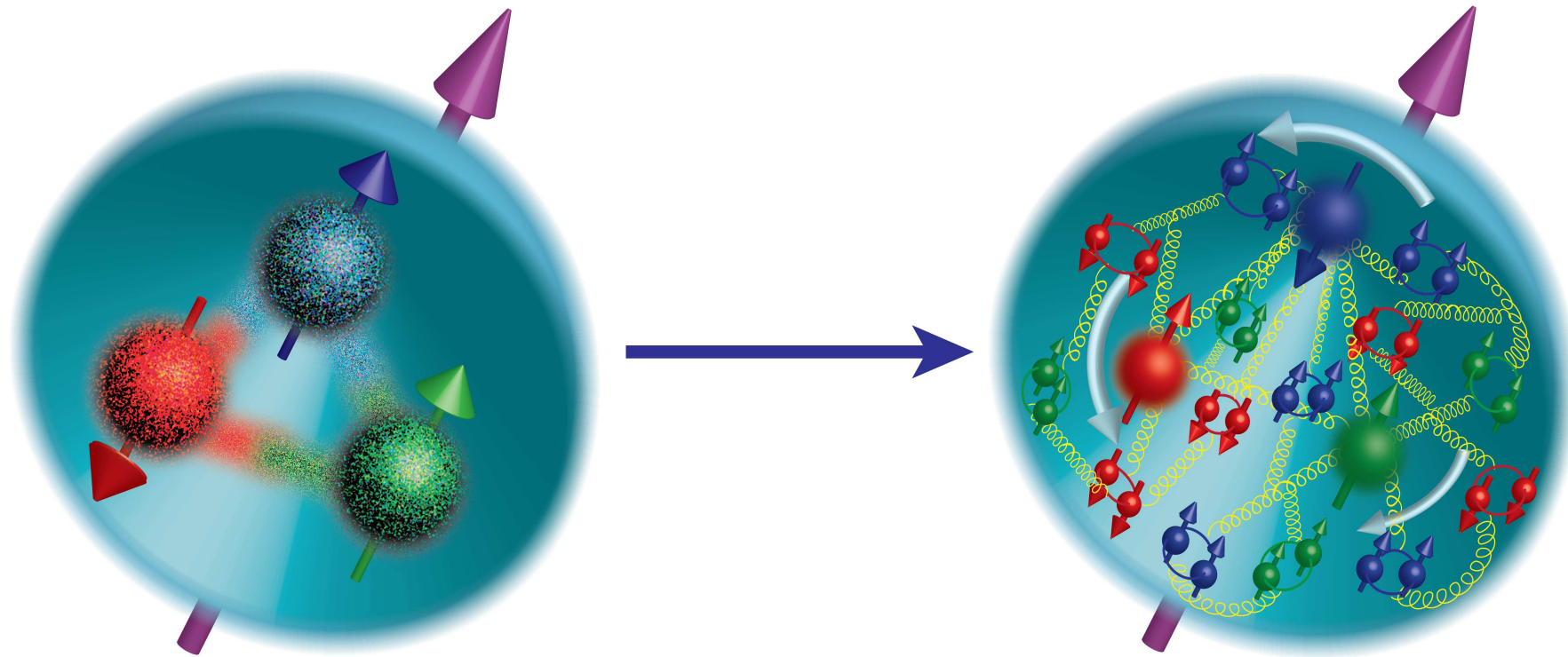


Spin provides a unique opportunity to probe the inner structure of a composite system (such as the proton) and hence to test our ability to understand the working of non-perturbative QCD

Test of a theory is not complete without a full exploration of spin-dependent decays and scatterings

# EVOLUTION OF OUR UNDERSTANDING OF THE SPIN STRUCTURE

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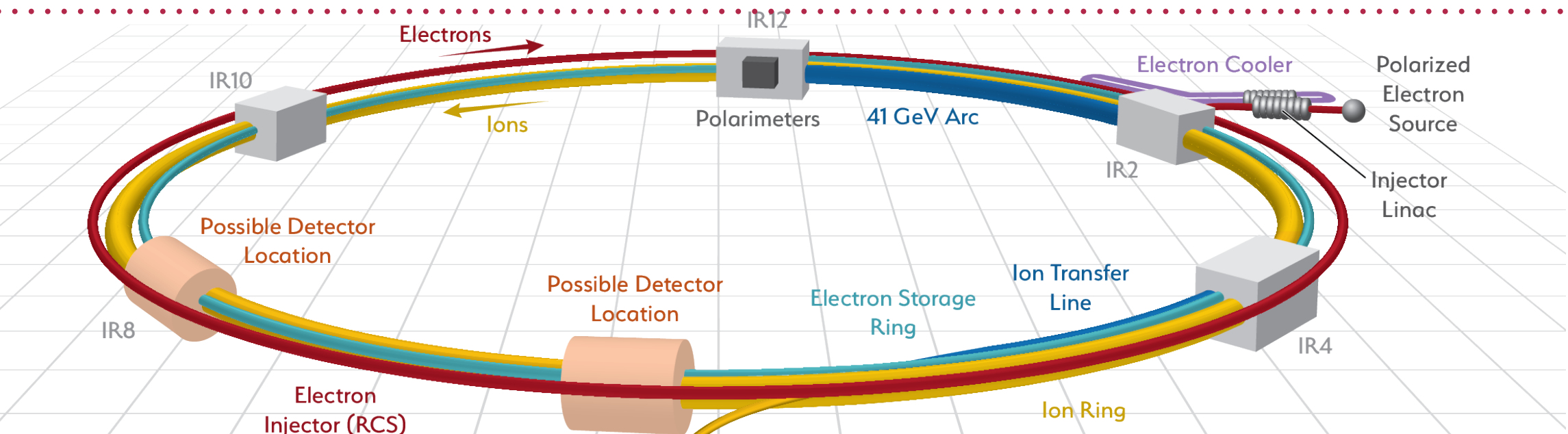


1980' - the spin of the nucleon  
is due to the valence quarks

Modern concept: valence quarks, sea quarks,  
and gluons together with orbital angular  
momentum are contributing



# THE ELECTRON-ION COLLIDER @ BNL



- High luminosity: ( $\sim 10^{33} - 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ ) ( $\sim 1000$  times that of HERA)
- **Variable** CM energy: **20 — 100 GeV** upgradable to **140 GeV**
- Highly polarized  **$\sim 70\%$**  electron and  **$\sim 70\%$**  nucleon beams
- Ion beams from deuterons to heavy nuclei such as gold, lead, or uranium
- Possibility of more than one interaction region (none of the major facilities operates with one detector only - important for discovery potential)

White Paper (2012)  
Accardi et al, arXiv:1212:1701



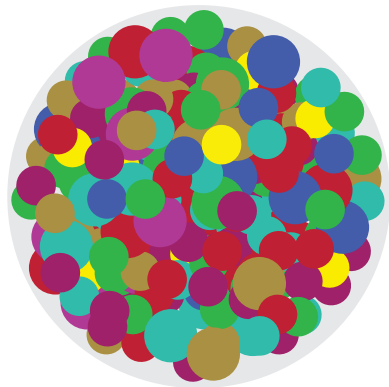
**WHAT DO WE STUDY?**



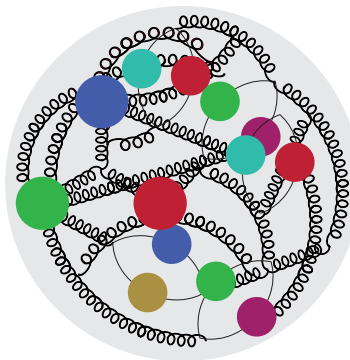
# THE ELECTRON-ION COLLIDER: KINEMATICS

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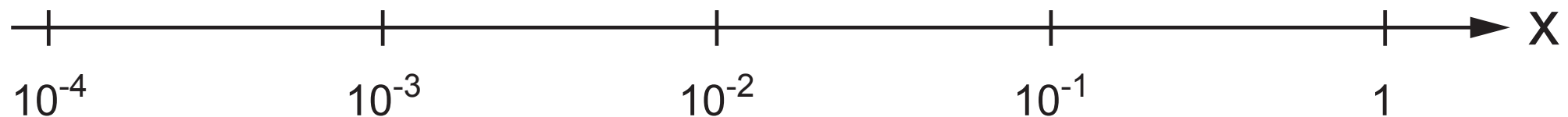
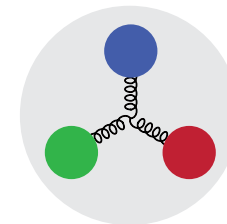
Non-Linear Dynamics  
Regime



Radiation Dominated  
Regime

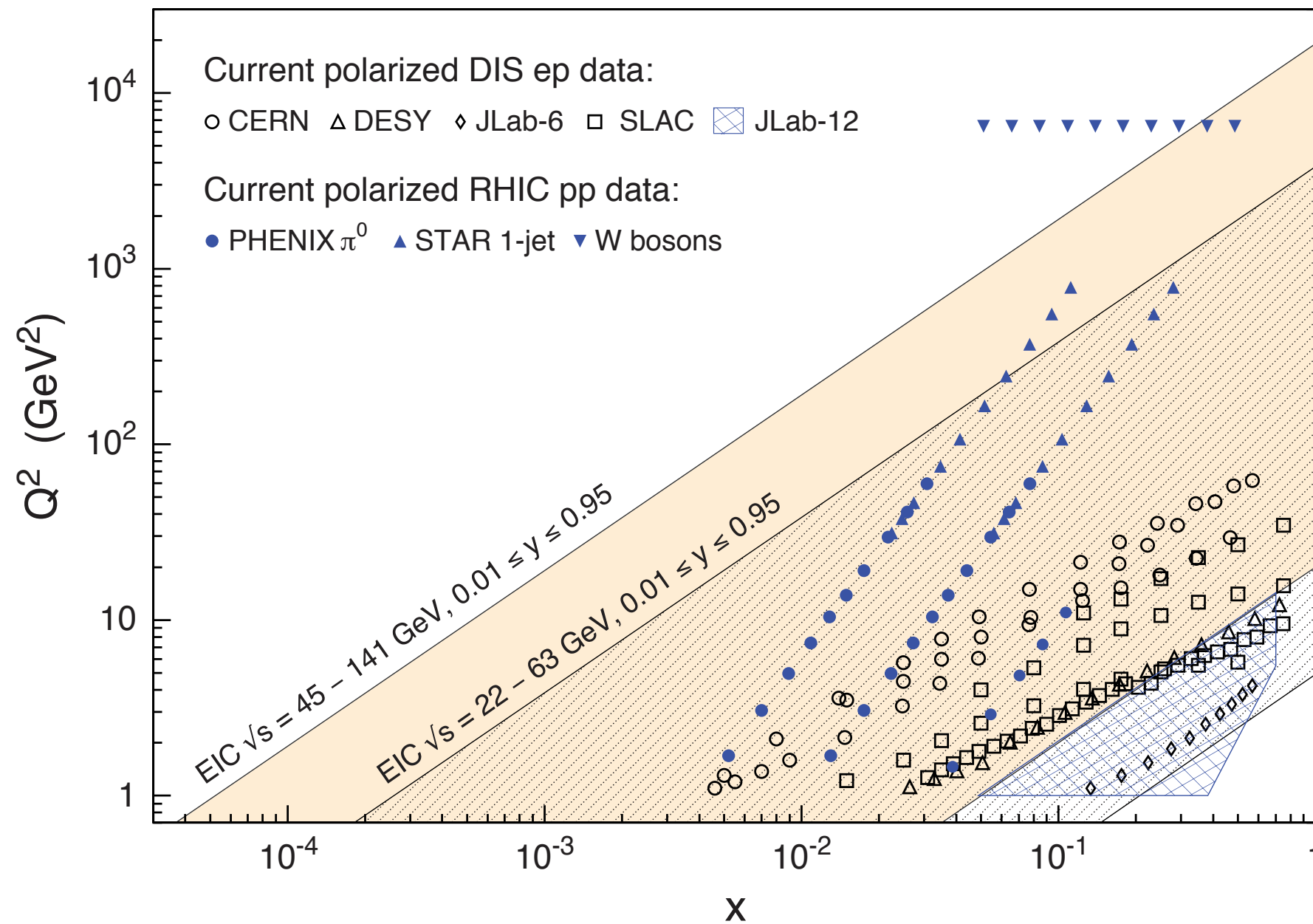


Valence Quark  
Regime



BNL Report (2017)  
Aschenauer at el, arXiv:1708.01527

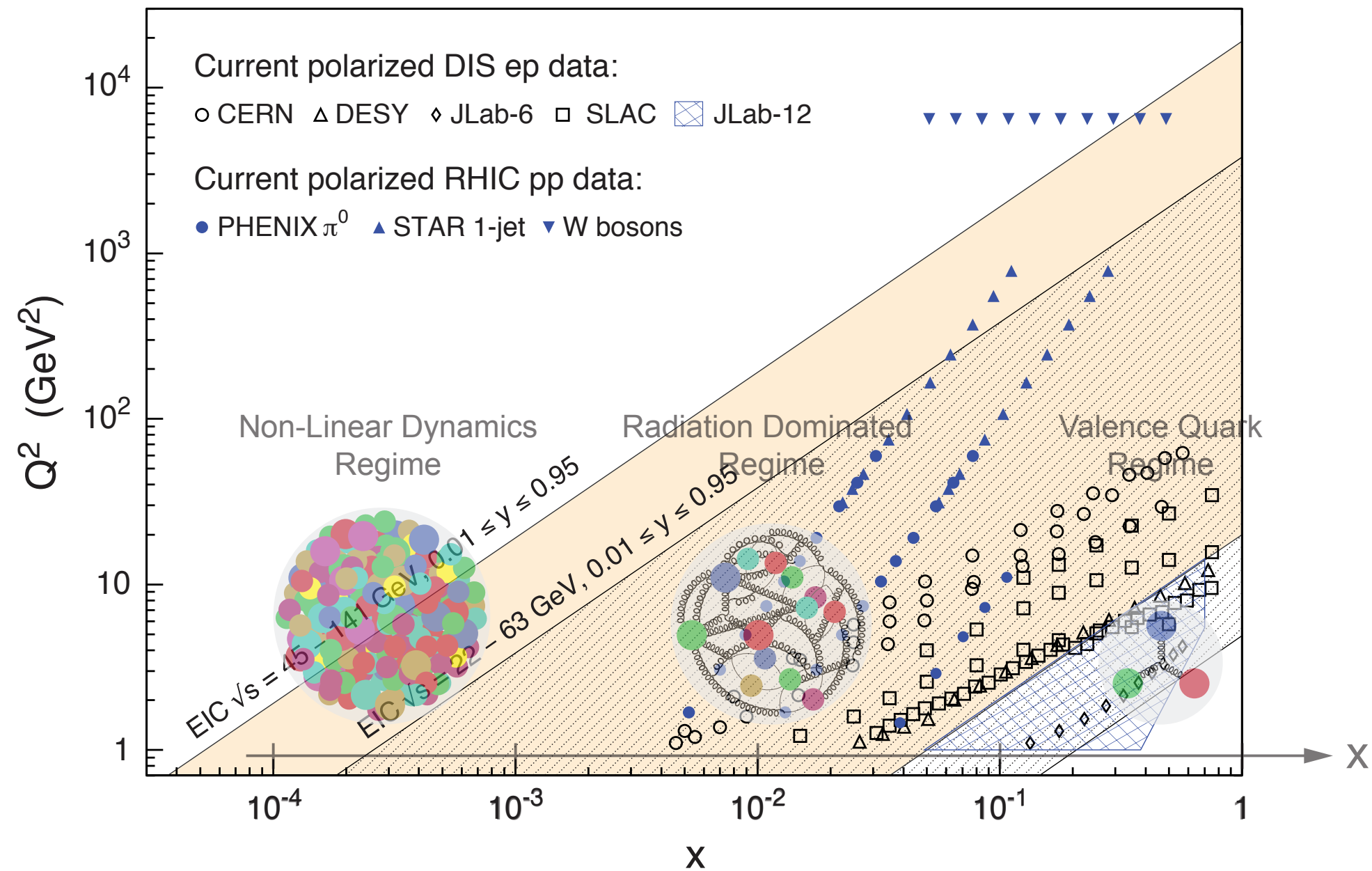
# THE ELECTRON-ION COLLIDER: KINEMATICS



BNL Report (2017)  
Aschenauer et al, arXiv:1708.01527



# THE ELECTRON-ION COLLIDER: KINEMATICS






BNL Report (2017)  
Aschenauer et al, arXiv:1708.01527

# COLLINEAR SPIN STRUCTURE

---

Spin-1/2 nucleon can be described by three collinear parton distribution functions (pdf)

<div><div>N</div><div>q</div></div>	U	L	T
U			
L			
T			

unpolarized pdf

helicity pdf

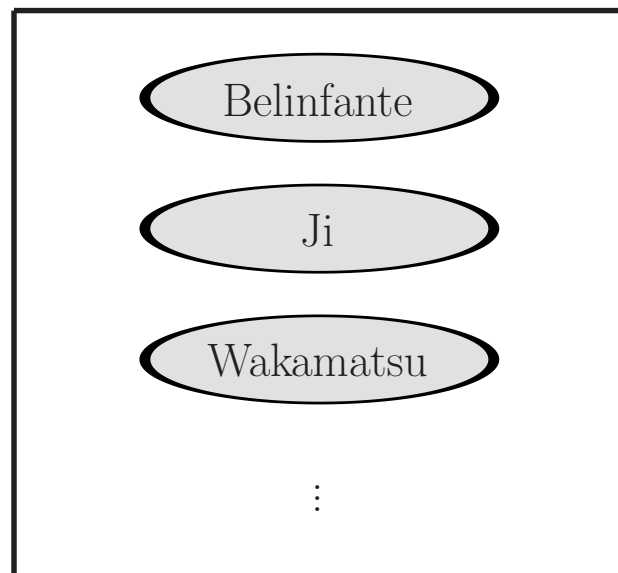
transversity pdf



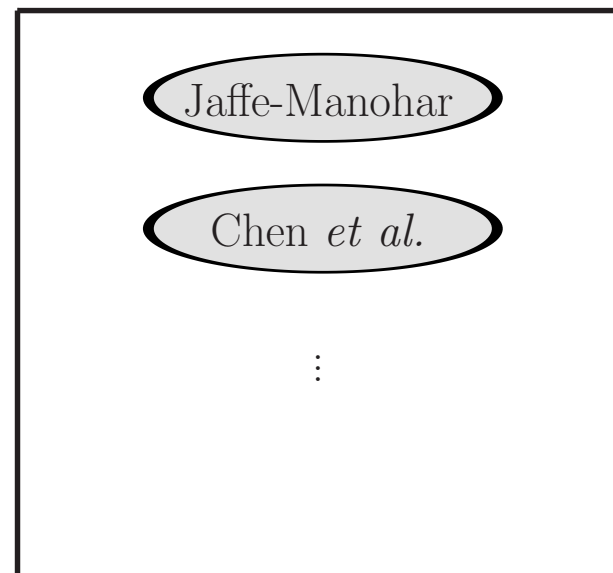
# SPIN DECOMPOSITION

- The nucleon is a composite system. The spin is carried by its constituents: quarks, anti-quarks and gluons and the angular momentum generated by their motion.
- The nucleon at rest has spin  $1/2$ , however its decomposition in terms of spin and orbital contributions associated with quarks and gluons is not unique.
- There are two types of decompositions of the proton spin operator: kinetic (also known as mechanical) and canonical. These two types differ by how the OAM operator is split into the quark and gluon contributions. They share the same quark spin operator.

Kinetic family



Canonical family



R. L. Jaffe and A. Manohar, Nucl. Phys. B337, 509 (1990)  
 S. Bashinsky and R. L. Jaffe, Nucl. Phys. B 536 (1998)  
 X. Ji, Phys. Rev. Lett. 78 (1997)  
 X. -S. Chen, X. -F. Lu, W. -M. Sun, F. Wang and T. Goldman, Phys. Rev. Lett. 100 (2008)  
 M. Wakamatsu, Phys. Rev. D 83 (2011)  
 Y. Hatta, Phys. Rev. D 84 (2011)  
 E. Leader and C. Lorce, Phys.Rept. 541, 163 (2014)  
 C. Lorcé and B. Pasquini, JHEP 09 (2013)  
 C. Lorcé and B. Pasquini, Phys. Rev. D 84, (2011)  
 C. Lorcé, B. Pasquini, X. Xiong and F. Yuan, Phys. Rev. D 85, (2012)  
 L. Adhikari and M. Burkard, Phys.Rev.D 94 (2016)

- Kinetic family is related to Generalized Parton Distributions, while canonical in light cone gauge is related to collinear helicity distribution functions

# LONGITUDINAL SPIN

When the proton or the neutron are polarized, quarks and gluons are polarized as well. Helicity distribution functions: number of quarks/gluons with spin parallel to the nucleon momentum minus the number of quarks/gluons with the spin opposite to the nucleon momentum

$$\Delta f(x, Q^2) = g_1(x, Q^2) \equiv f^+(x, Q^2) - f^-(x, Q^2)$$

The relevant spin decomposition is by Jaffe and Manohar

R. L. Jaffe and A. Manohar, Nucl. Phys. B337, 509 (1990)

$$\frac{1}{2} = S_q + \mathcal{L}_q + S_g + \mathcal{L}_g$$

Related to measured observables:

Quark spin contribution

$$S_q = \frac{1}{2} \int_0^1 \Delta \Sigma(x, Q^2) dx \equiv \frac{1}{2} \int_0^1 (\Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s})(x, Q^2) dx$$

Difficult to measure in experiment:

Gluon spin contribution

$$S_g = \int_0^1 \Delta g(x, Q^2) dx$$

$$\mathcal{L}_q + \mathcal{L}_g$$

quark and gluon orbital angular momenta (OAM)  
via twist-3 GPDs, Wigner functions

D.V. Kiptily, M.V. Polyakov, Eur. Phys. J. C 37 (2004)

A. Courtoy, G. R. Goldstein, J. O. Gonzalez Hernandez, S. Liuti, A. Rajan, PLB 731 (2014)

Y. Hatta, Phys. Lett. B 708 (2012);

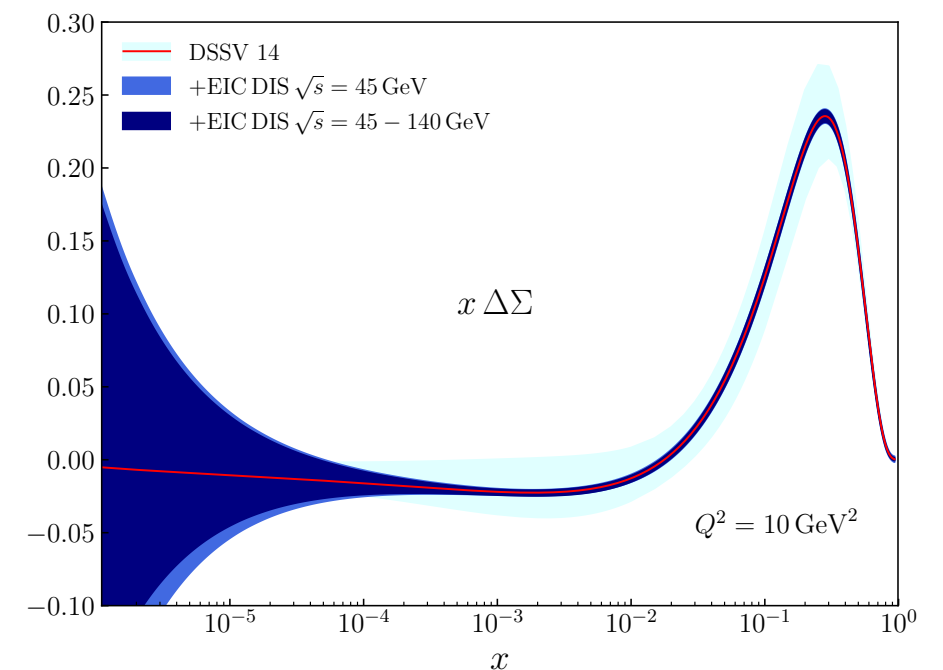
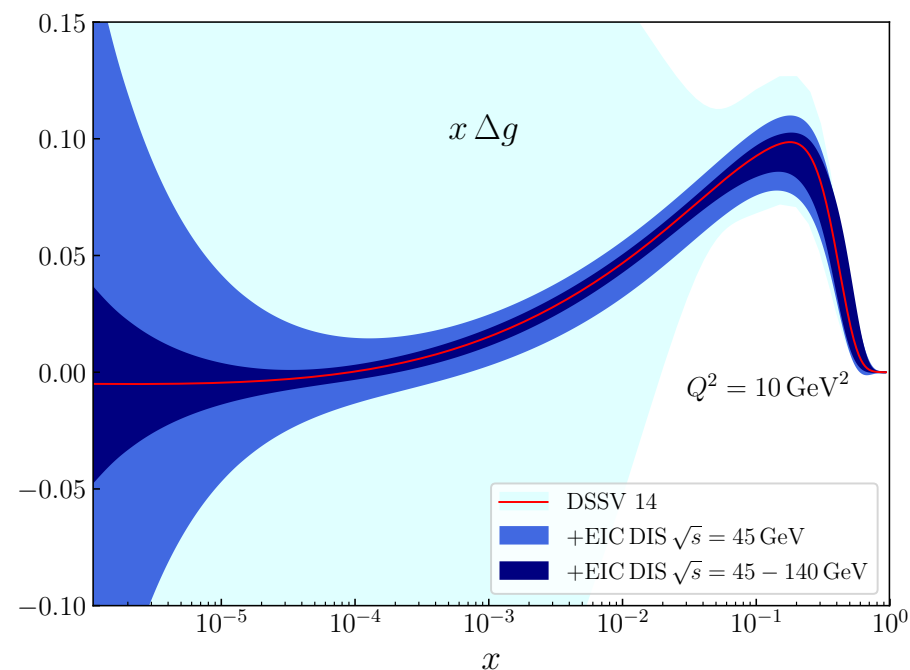
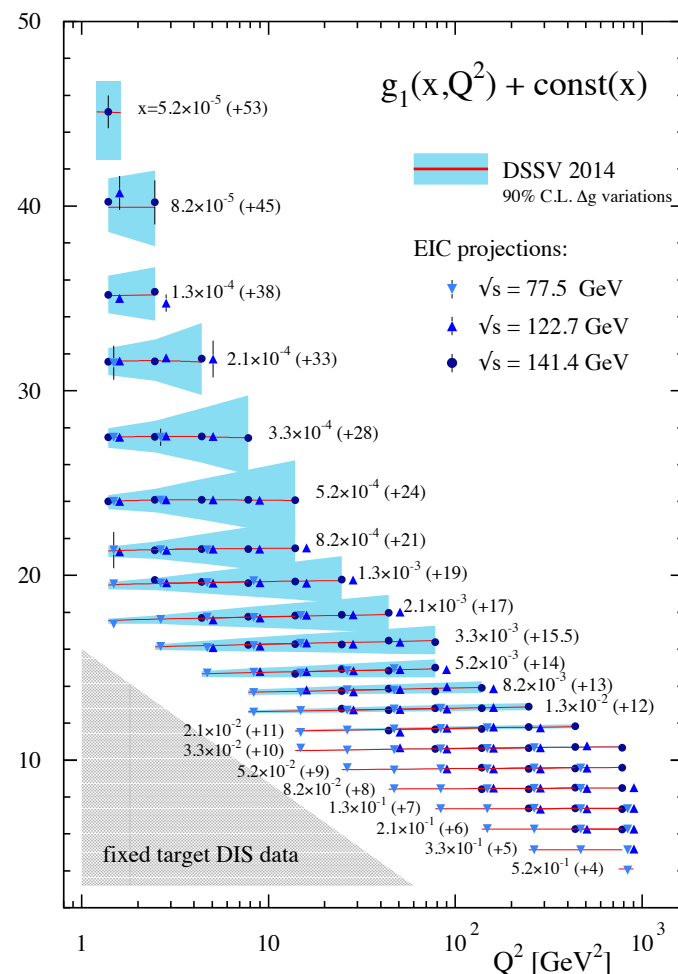
Y. Hatta, S. Yoshida, J. High Energy Phys. 1210 (2012)



# LONGITUDINAL SPIN

- Global QCD analyses are performed to extract helicity pdfs:  
 DSSV: D. de Florian, R. Sassot, M. Stratmann and W. Vogelsang, Phys. Rev. Lett. 113 (2014)  
 NNPDFpol: E. R. Nocera, R. D. Ball, S. Forte, G. Ridolfi, J. Rojo, Nucl. Phys. B 887 (2014)  
 JAM: J. J. Ethier, N. Sato, W. Melnitchouk, Phys. Rev. Lett. 119 (13) (2017)
- At present around 25 % of the spin is attributed to quarks and anti-quarks.
- The evidence for non-zero gluon contribution, around 30 % , is mainly due to RHIC spin program E. R. Nocera, Impact of Recent RHIC Data on Helicity-Dependent Parton Distribution Functions (2017). arXiv:1702.05077.

## The impact of the EIC on determination of the quark and gluon contributions

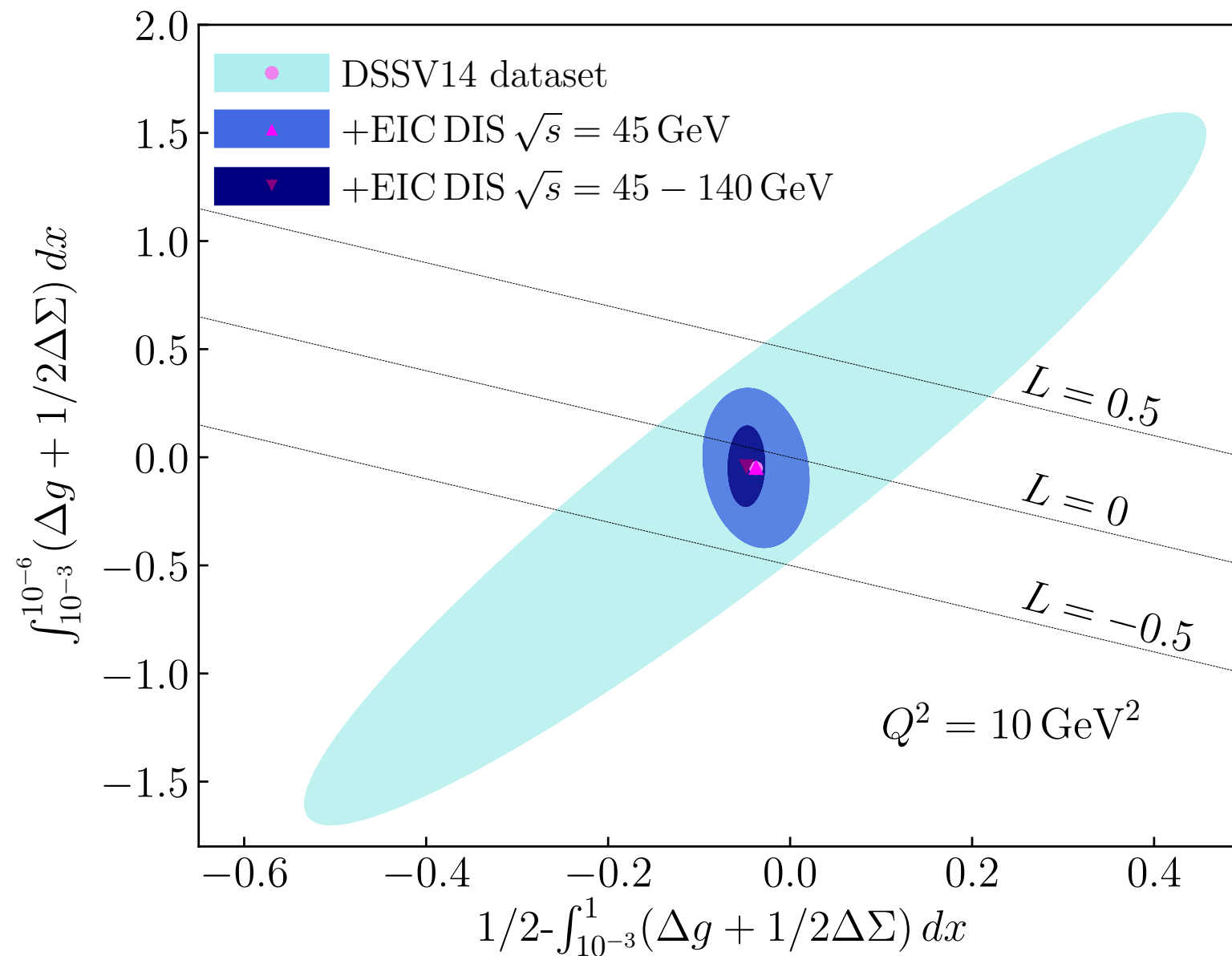


Yellow Report (2021) arXiv:2103.05419

Aschenauer , Sassot, Stratmann, Phys.Rev.D 92 (2015)

# LONGITUDINAL SPIN

Studies of the room left for potential OAM contributions at the EIC

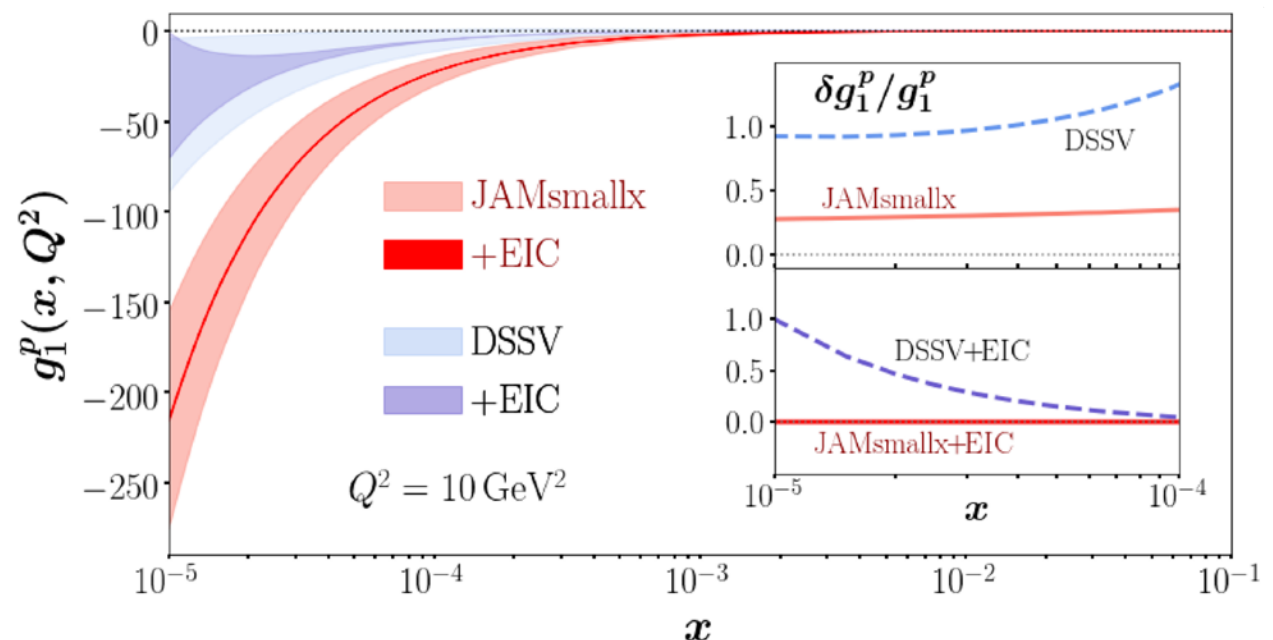
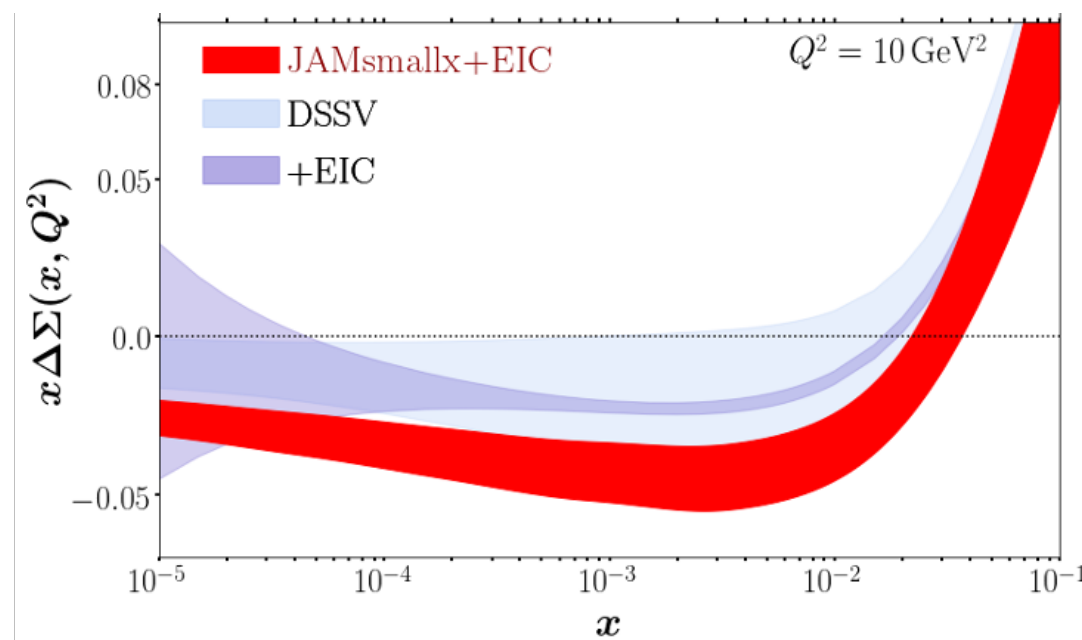


Yellow Report (2021) arXiv:2103.05419

# SMALL-x SPIN CHALLENGE

- Can we constrain theoretically the amount of proton spin and OAM coming from small x?
  - Existing and future experiments probe the helicity distributions and OAM down to some min x
- $$S_g = \int_0^1 \Delta g(x, Q^2) dx \quad S_q = \frac{1}{2} \int_0^1 \Delta \Sigma(x, Q^2) dx$$
- If we want to predict helicity PDFs at small x, we need a different evolution equation similar to BK/JIMWLK evolving in x starting from some value of  $x \approx 10^{-3}$ . Pitonyak, Sievert, Kovchegov (15), (18)
  - Potentially negative 10-20% of the proton spin may be carried by small-x quarks helicity (JAMsmallx, preliminary)

JAMsmallx: Adamiak, Melnitchouk, Pitonyak, Sato, Sievert, Kovchegov (2102.06159)



Better constraint at small-x achieved compared to the traditional approaches

$$g_1(x, Q^2) = \frac{1}{2} \sum_f e_f^2 [\Delta q_f + \Delta q_{\bar{f}}]$$

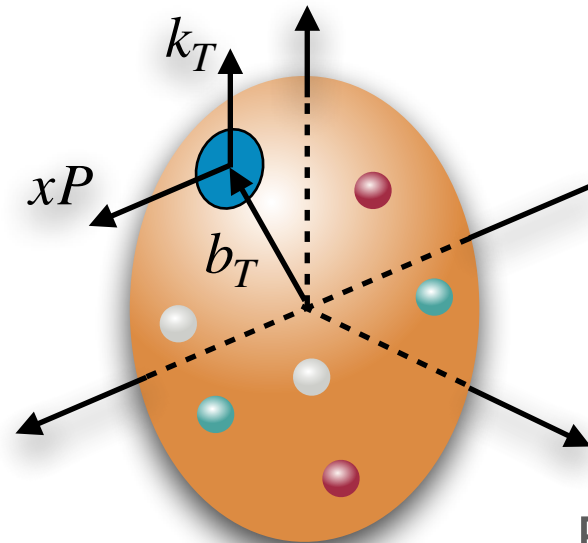


**BEYOND THE COLLINEAR PICTURE**



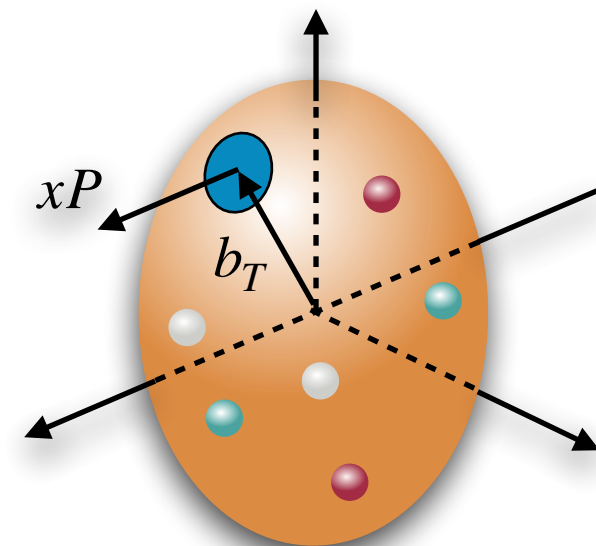
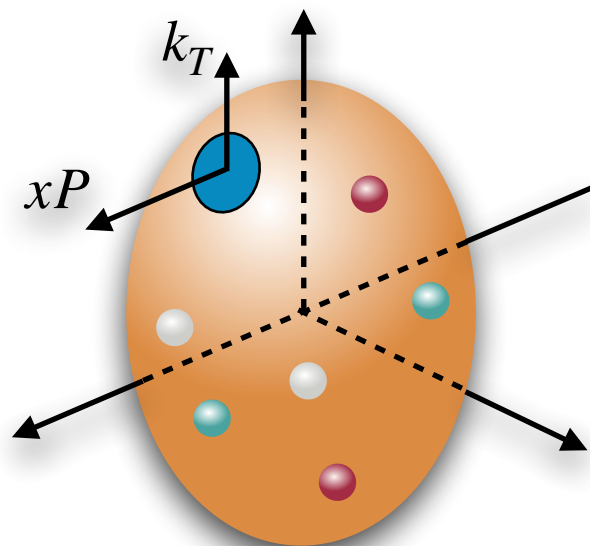
# WIGNER DISTRIBUTIONS AND THREE DIMENSIONAL STRUCTURE

Wigner distributions (Fourier transform of Generalized Transverse Momentum Distributions)



Fourier transform  
of Generalized Parton Distributions  
(GPDs)

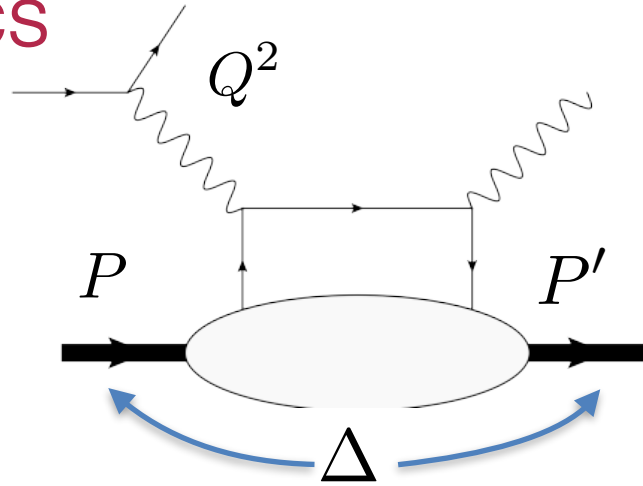
Transverse Momentum Dependent Distributions  
(TMDs)



see, e.g., C. Lorcé, B. Pasquini, M. Vanderhaeghen, JHEP 1105 (11)

# GPD

## DVCS



Ji (1997)  
Radyushkin (1997)

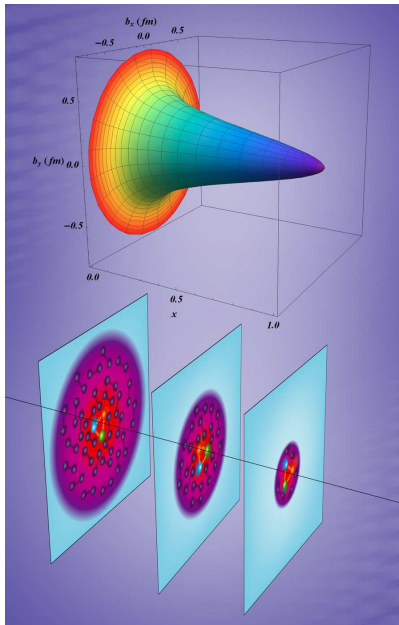
$Q^2$  ensures hard scale, pointlike interaction

$\Delta = P' - P$  momentum transfer can be varied independently

## Connection to 3D structure

Burkardt (2000)  
Burkardt (2003)

$$\rho(x, \vec{b}_\perp) = \int \frac{d^2 \vec{\Delta}_\perp}{(2\pi)^2} e^{-i \vec{\Delta}_\perp \cdot \vec{b}_\perp} H_q(x, \xi = 0, t = -\vec{\Delta}_\perp^2)$$

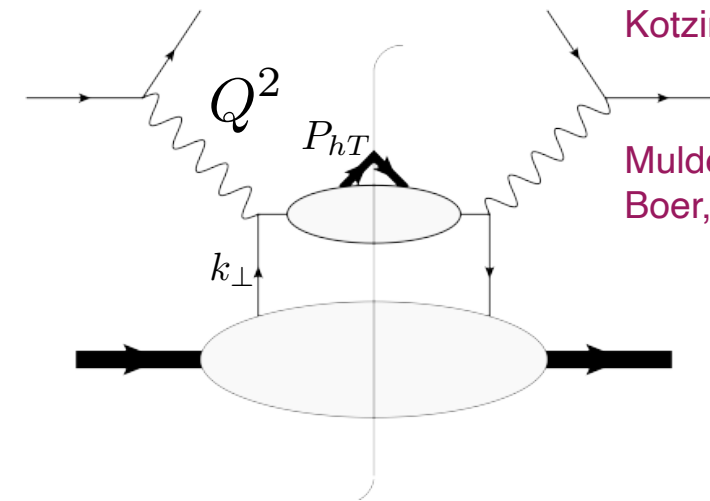


Drell-Yan frame  $\Delta^+ = 0$

Dupré, Guidal, Niccolai,  
Vanderhaeghen,  
arXiv:1704.07330

# TMD

## SIDIS



Kotzinian (1995)

Mulders, Tangerman (1995)  
Boer, Mulders (1998)

$Q^2$  ensures hard scale, pointlike interaction

$P_{hT}$  final hadron transverse momentum can be varied independently

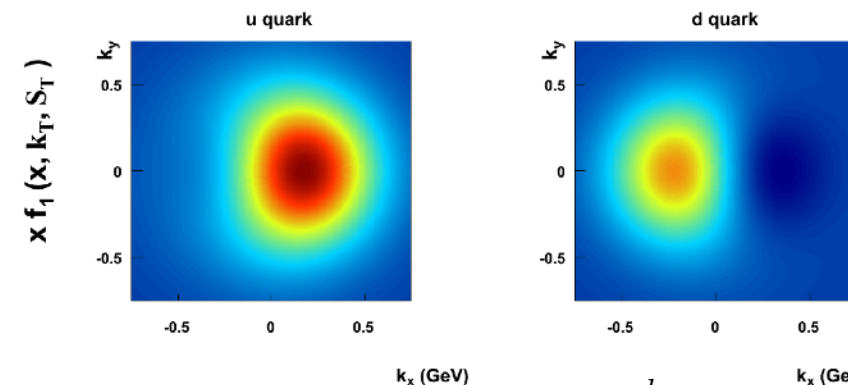
## Connection to 3D structure

Ji, Ma, Yuan (2004)  
Collins (2011)

$$f(x, \vec{k}_T) = \int \frac{d^2 \vec{b}_T}{(2\pi)^2} e^{i \vec{k}_T \cdot \vec{b}_T} \tilde{f}(x, \vec{b}_T)$$

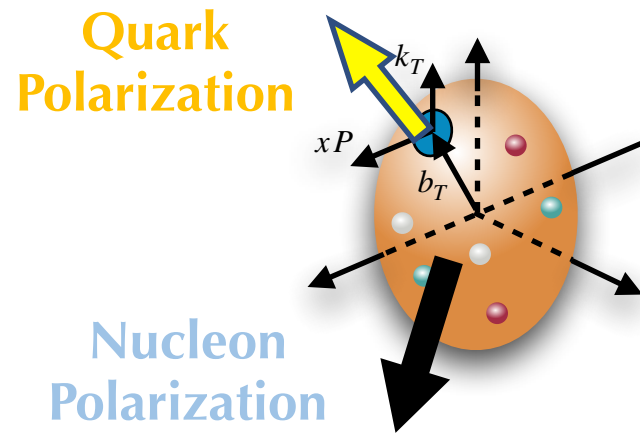
$\vec{b}_T$  is the transverse separation of parton fields in configuration space

White Paper (2012) Accardi et al, arXiv:1212:1701



$$\rho_{1;q \leftarrow h^\uparrow}(x, \vec{k}_T, S_T, \mu) = f_{1;q \leftarrow h}(x, k_T; \mu, \mu^2) - \frac{k_{Tx}}{M} f_{1T;q \leftarrow h}^\perp(x, k_T; \mu, \mu^2)$$

# Our understanding of the hadron evolves: SPIN



Nucleon emerges as a strongly interacting, relativistic bound state of quarks and gluons

TMDs

GPDs

		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x, k_T^2)$ <i>Unpolarized</i>		$h_1^\perp(x, k_T^2)$ <i>Boer-Mulders</i>
	L		$g_1(x, k_T^2)$ <i>Helicity</i>	$h_{1L}^\perp(x, k_T^2)$ <i>Kozinian-Mulders, "worm" gear</i>
	T	$f_{1T}^\perp(x, k_T^2)$ <i>Sivers</i>	$g_{1T}(x, k_T^2)$ <i>Kozinian-Mulders, "worm" gear</i>	$h_1(x, k_T^2)$ <i>Transversity</i> $h_{1T}^\perp(x, k_T^2)$ <i>Pretzelosity</i>

		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$H$		$\mathcal{E}_T$
	L		$\tilde{H}$	
	T	$E$		$H_T, \tilde{H}_T$

Many TMDs and GPDs cannot exist without OAM.

Examples: TMD Sivers function  $f_{1T}^\perp$  and GPD  $E$

# LONGITUDINAL SPIN

Studies of DVCS process were highly motivated by Ji decomposition

$$\frac{1}{2} = S_q + L_q + J_g$$

X. Ji, Phys. Rev. Lett. 78 (1997) 610

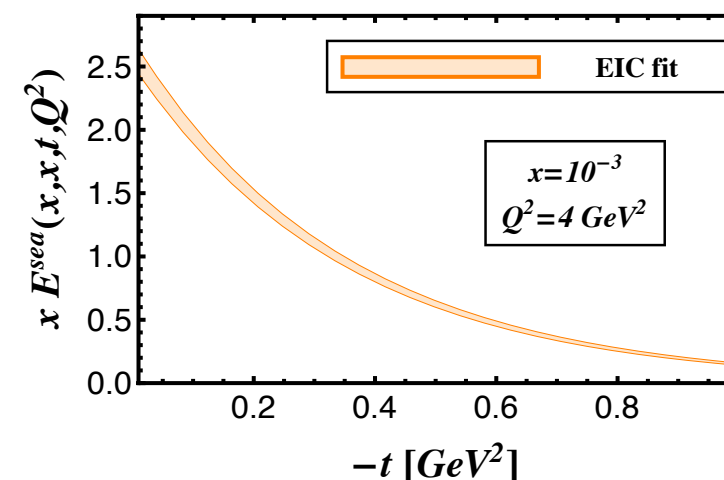
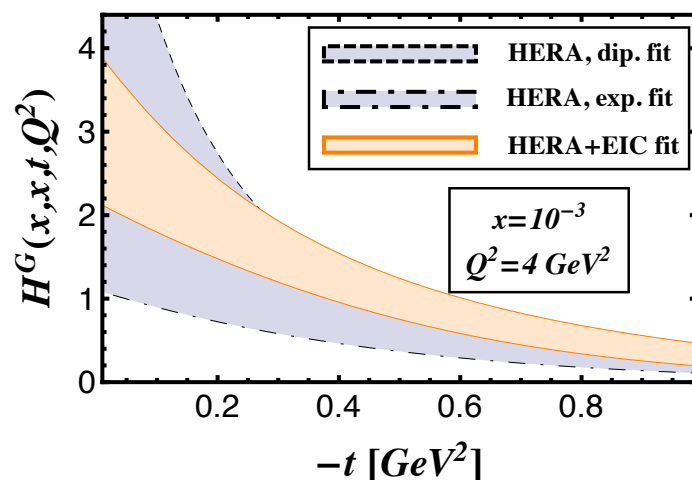
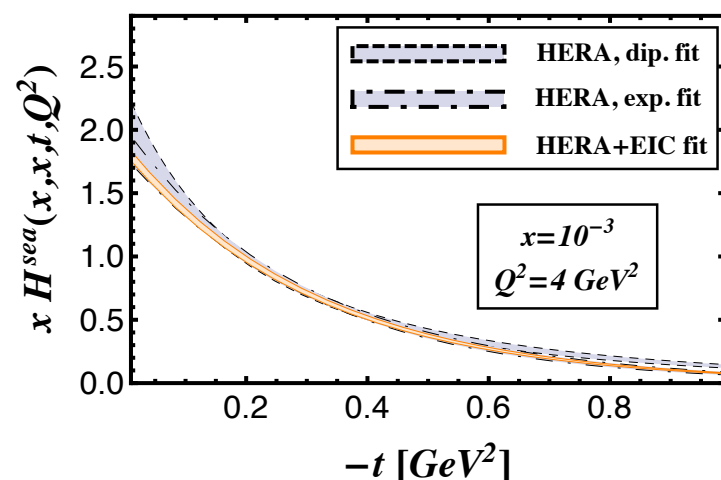
Related to twist-2 GPDs:

$$J_q \equiv S_q + L_q = \frac{1}{2} \int_0^1 \Delta \Sigma(x, Q^2) dx + L_q = \frac{1}{2} \int_{-1}^1 dx x (H^q(x, \xi = 0, t = 0) + E^q(x, \xi = 0, t = 0))$$

$$J_g = \frac{1}{2} \int_{-1}^1 dx x (H^g(x, \xi = 0, t = 0) + E^g(x, \xi = 0, t = 0))$$

These quantities can be computed also in lattice simulations

The impact of the EIC on determination of the sea-quark and gluon contributions



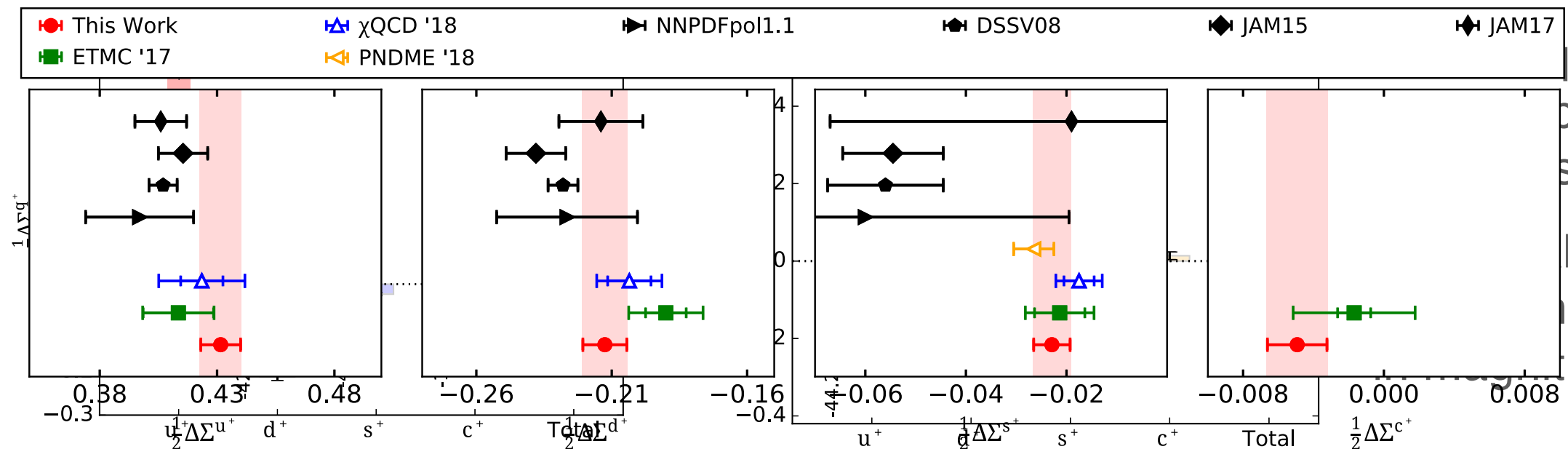
E. Aschenauer, S. Fazio, K. Kumericki, D. Mueller, JHEP 09 (2013) 093



# LONGITUDINAL SPIN AND LATTICE QCD

- Many methods are explored by lattice QCD to calculate the spin and OAM, subtraction, direct computations  
See e.g. review of Keh-Fei Liu and Cédric Lorcé, Eur.Phys.J.A 52 (2016)

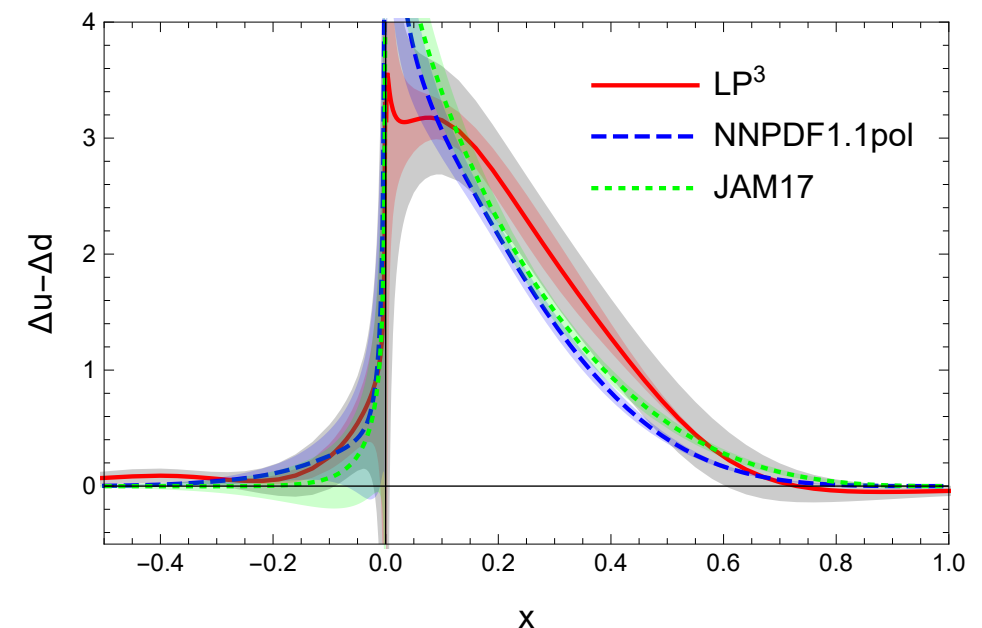
- Spin contributions can be computed at physical pion mass. An example from ETMC Collaboration C. Alexandrou et al, Phys.Rev.D 101 (2020)



quarks  
opposite  
S.  
Ms are  
ately equal  
ude

- Lattice QCD computes also the shape of pdfs, GPDs, TMDs using various approaches. An example from LP<sup>3</sup> Collaboration

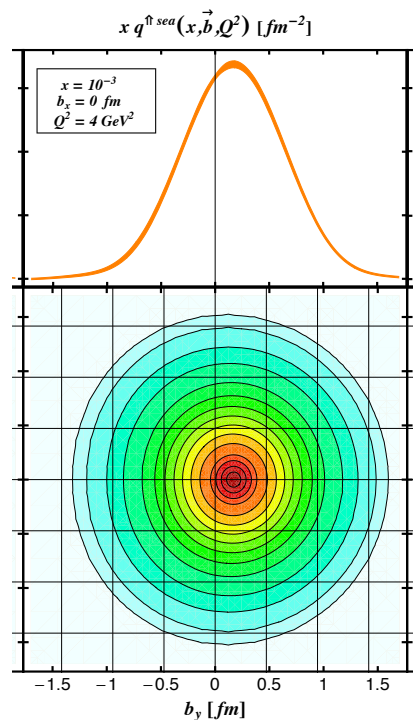
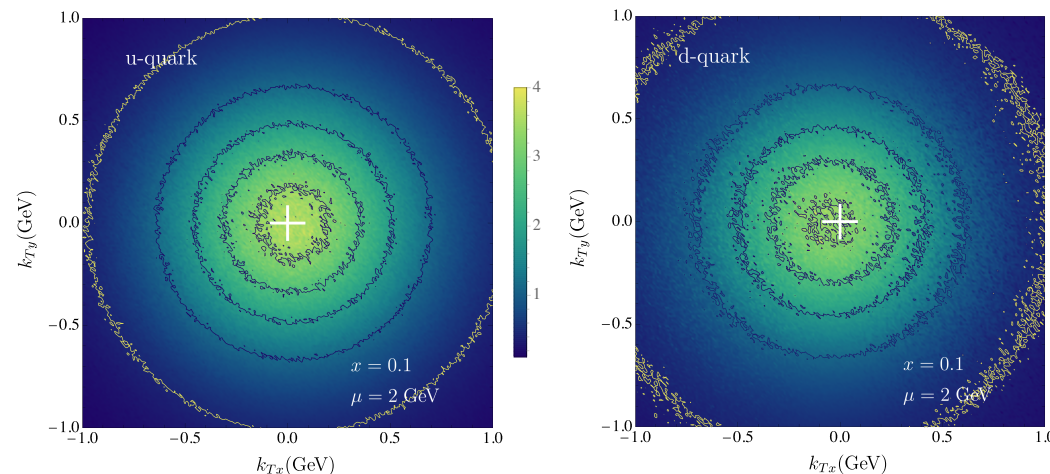
H. -W. Lin et al Phys.Rev.Lett. 121 (2018)



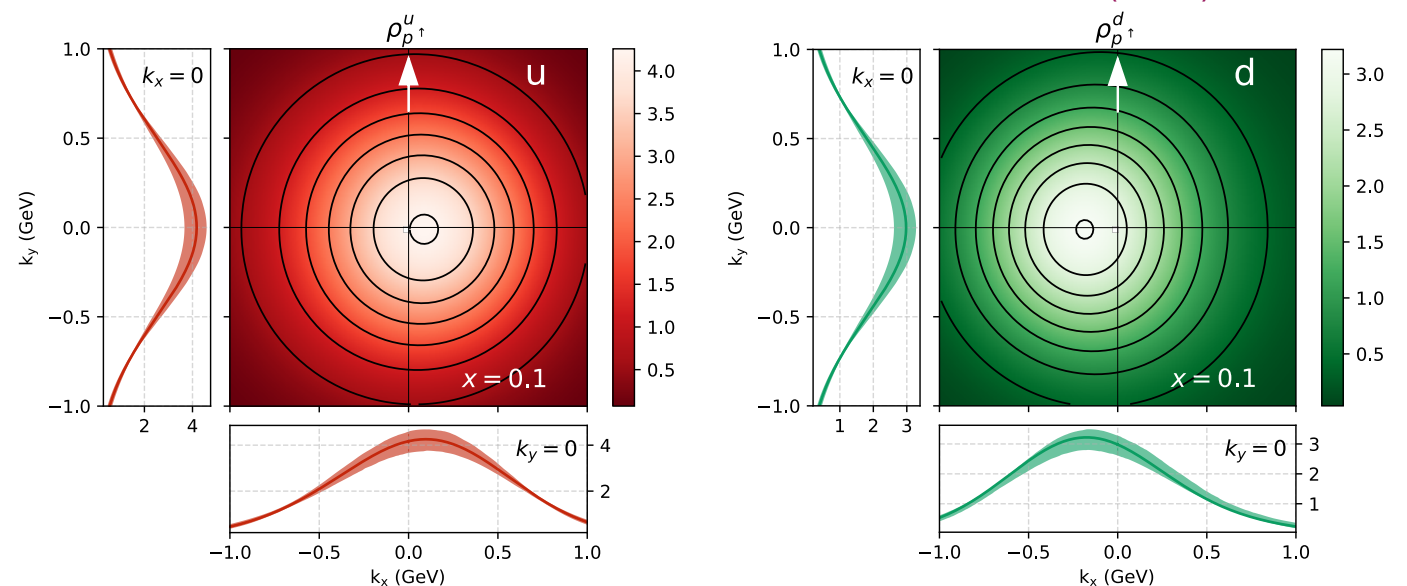
# NUCLEON TOMOGRAPHY – THE FINAL GOAL OF THE EIC

$$\rho_{1;q \leftarrow h^\uparrow}(x, \mathbf{k}_T, \mathbf{S}_T, \mu) = f_{1;q \leftarrow h}(x, k_T; \mu, \mu^2) - \frac{k_{Tx}}{M} f_{1T;q \leftarrow h}^\perp(x, k_T; \mu, \mu^2)$$

M. Bury, A. Prokudin, A. Vladimirov, Phys.Rev.Lett. 126 (2021)



A. Bacchetta, F. Delcarro, C. Pisano, M. Radici (2020)

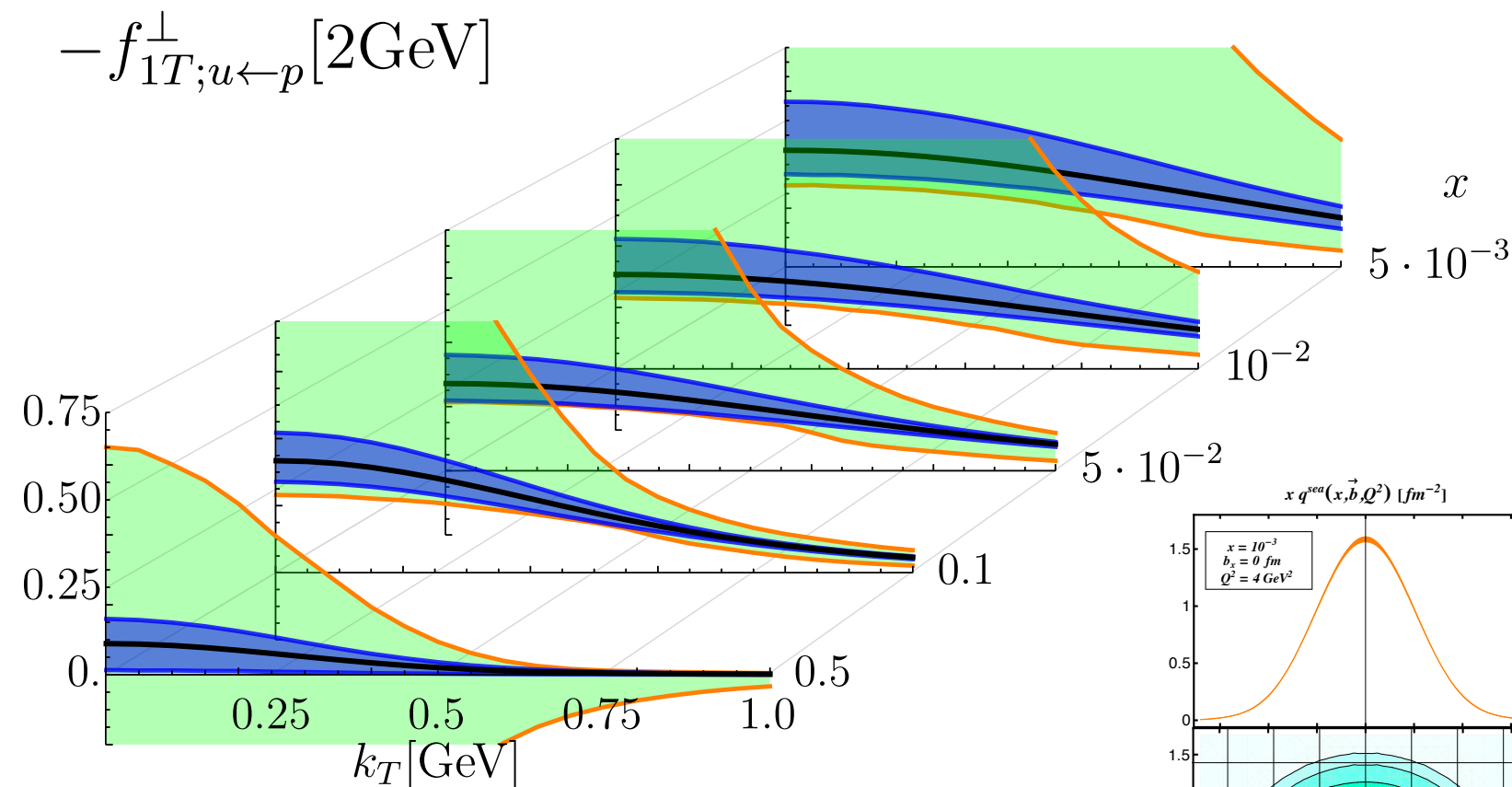


$$q^\uparrow(x, \vec{b}, Q^2) = q(x, \vec{b}, Q^2) - \frac{1}{2M_p} \frac{\partial}{\partial b_y} E(x, \vec{b}, Q^2)$$

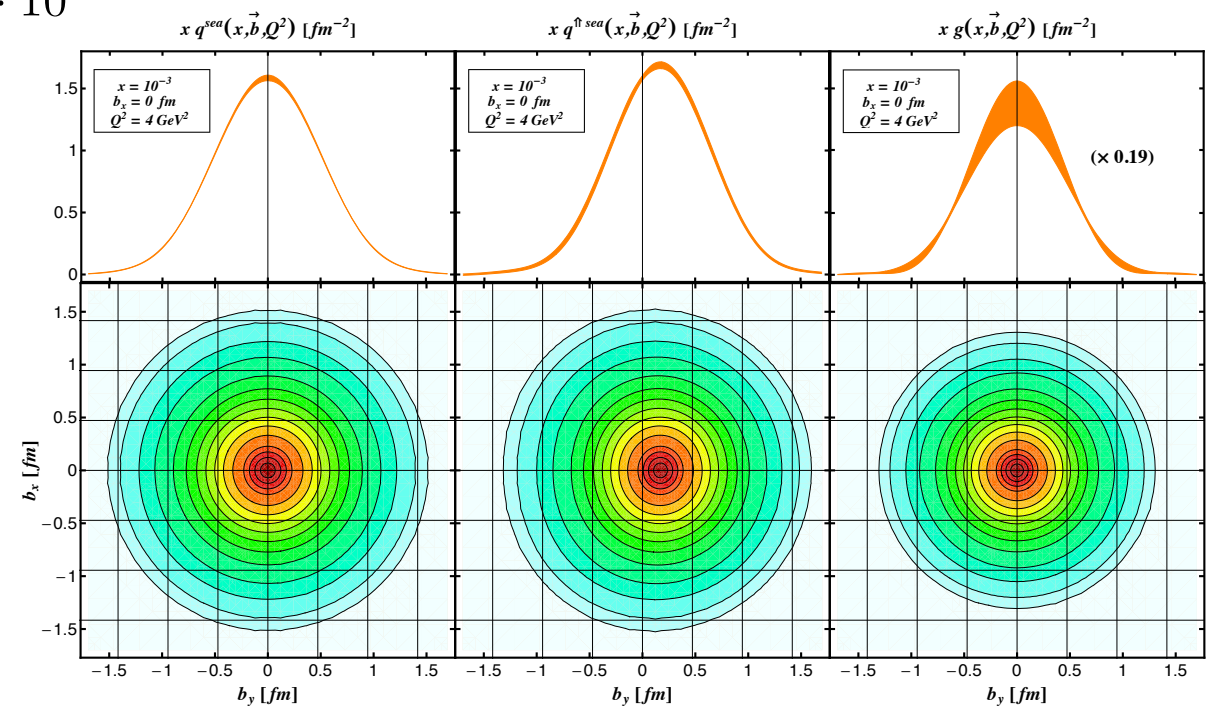
E. Aschenauer, S. Fazio, K. Kumericki, D. Mueller, JHEP 09 (2013) 093

- The shift in the transverse plane is generated by the Sivers function and GPD  $E$  that cannot exist without OAM
- The opposite signs of the shift is consistent with lattice QCD findings on the opposite signs of the OAM for u and d quarks

# THE SIVERS FUNCTION AND GPD E AT THE EIC



A. Vladimirov for EIC Yellow Report (2021) arXiv:2103.05419



$x = 0.001$  and  $Q^2 = 4 \text{ GeV}^2$

E. Aschenauer, S. Fazio, K. Kumericki, D. Mueller, JHEP 09 (2013) 093

► The impact of the EIC is very substantial

# TRANSVERSE SPIN

- The situation is more subtle due to the fact that boost and rotations do not commute
- Two sum rules are proposed

E. Leader, PRD 85 (2012), canonical

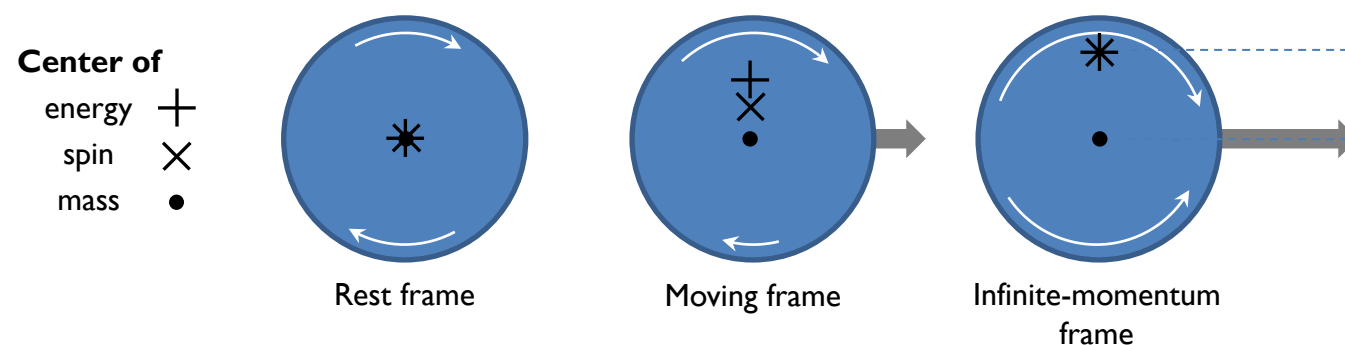
$$\langle J_a^T \rangle = \frac{1}{2} \left[ \int_{-1}^1 dx x H^a(x, \xi = 0, t = 0) + \frac{p_0}{M} \int_{-1}^1 dx x E^a(x, \xi = 0, t = 0) \right]$$

X. Ji and F. Yuan, PLB 810 (2020), covariant

$$\langle J_a^T \rangle = \frac{p_0}{2M} \left[ \int_{-1}^1 dx x H^a(x, \xi = 0, t = 0) + \int_{-1}^1 dx x E^a(x, \xi = 0, t = 0) \right]$$

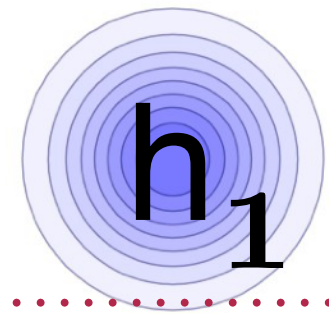
- Both turn out to be correct and correspond to different definitions of the relativistic center of the system: Leader - spin, Ji-Yuan - mass

C. Lorcé, EPJC 81 (2021)





# TRANSVERSITY

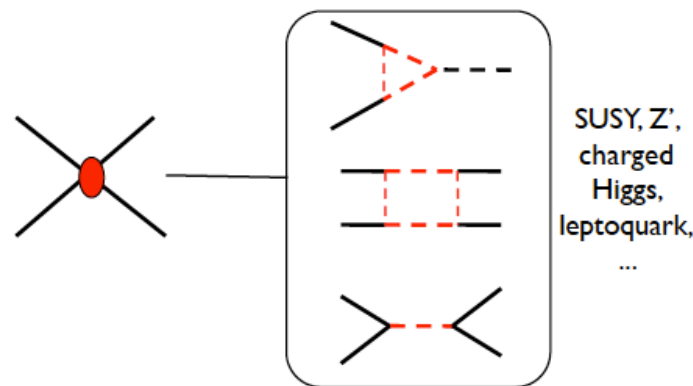


- The only source of information on tensor charge of the nucleon

$$\delta q \equiv g_T^q = \int_0^1 dx \left[ h_1^q(x, Q^2) - h_1^{\bar{q}}(x, Q^2) \right]$$

- Tensor couplings, not present in the SM Lagrangian, could be the footprints of new physics at higher scales

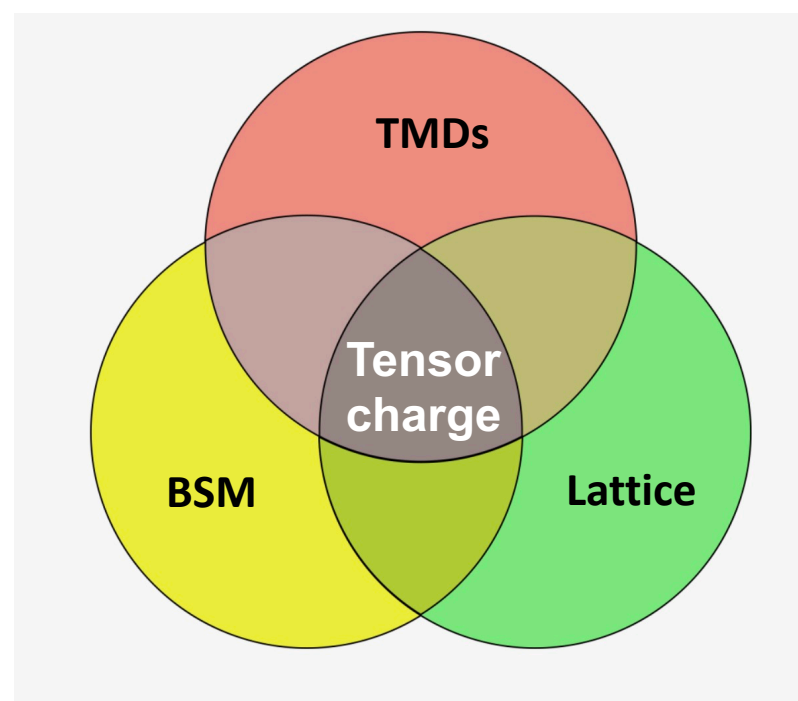
$$\epsilon_T g_T \approx M_W^2 / M_{\text{BSM}}^2$$



Bhattacharya et al, PRD 85 (12)  
Pattie et al., P.R. C88 (13)

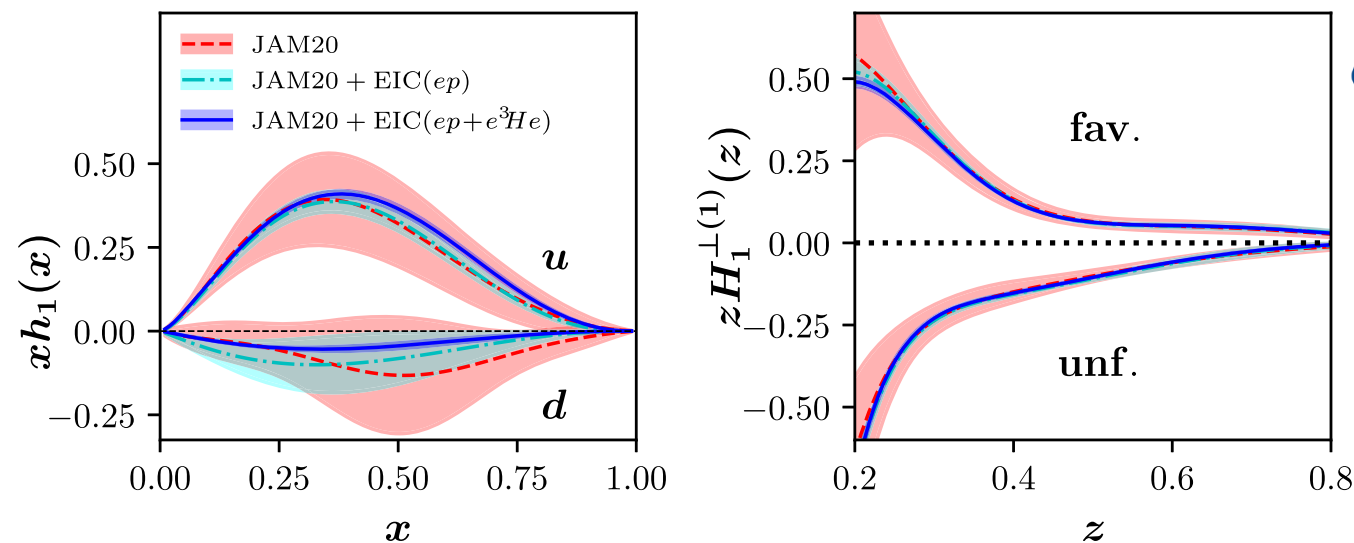
- Tensor charge is extensively studied on the lattice

Gupta et al, (18), Alexandrou et al., (19)

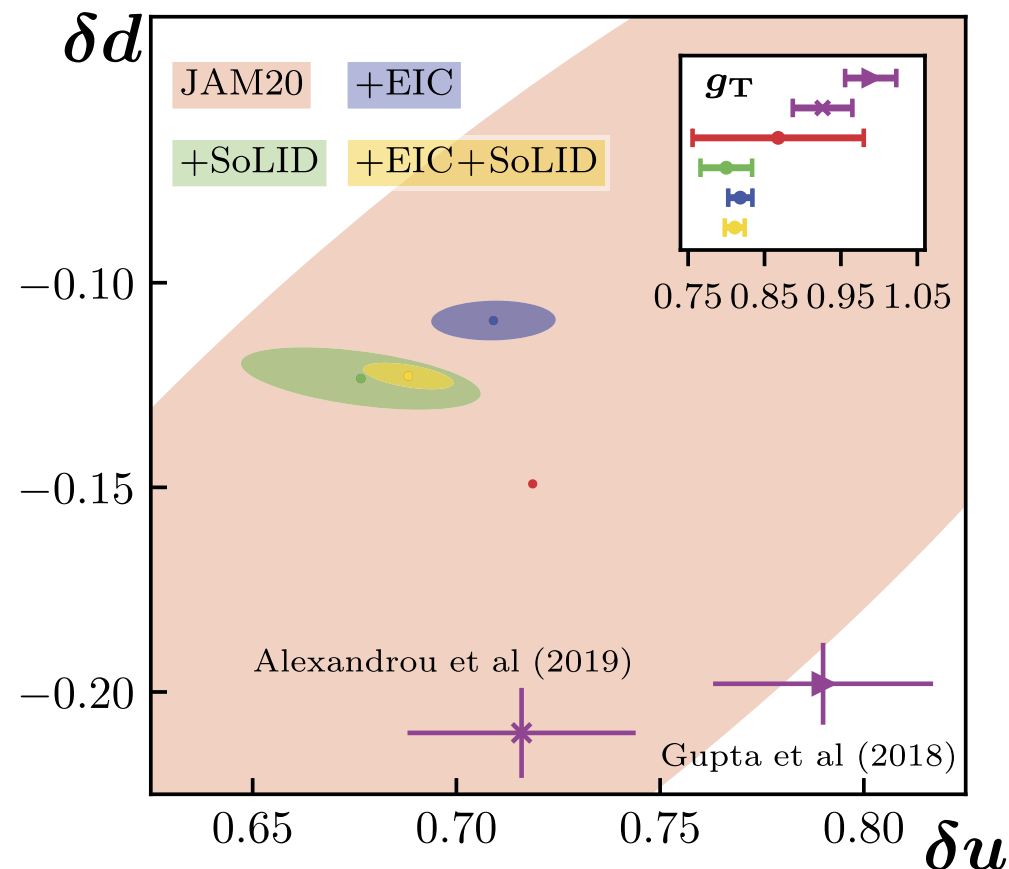


# TENSOR CHARGE AT THE EIC AND JLAB

L. Gamberg, Z. Kang, D. Pitonyak, A. Prokudin, N. Sato Phys.Lett.B 816 (2021)



JAM20: Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato, Phys.Rev.D 102 (2020)



EIC data will allow to have  $g_T$  extraction at the precision at the level of lattice QCD calculations

JLab 12 data will allow to have complementary information on tensor charge to test the consistency of the extraction and expand the kinematical region

# WIGNER DISTRIBUTIONS AND THE SPIN

- Wigner distributions have information on both position and motion

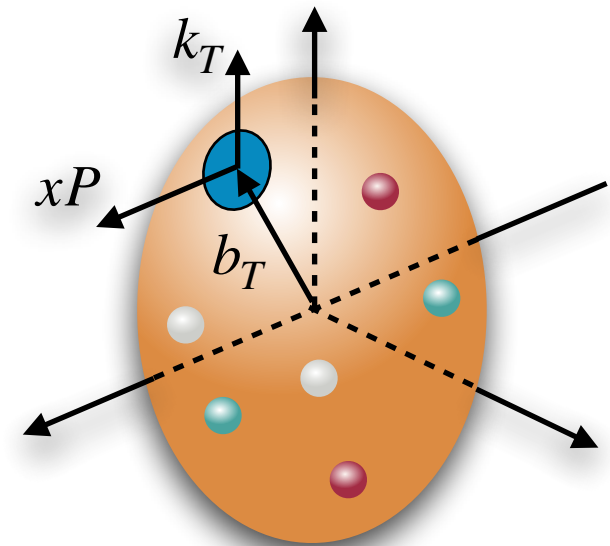
$$W(x, \vec{k}, \vec{b}) = \int \frac{dz^- d\vec{z}}{2(2\pi)^3} \int \frac{d\vec{\Delta}}{(2\pi)^2} e^{ixP^+ z^- - i\vec{k} \cdot \vec{z}} \langle P' S | \bar{\psi}(\vec{b} - \vec{z}/2) \gamma^+ \psi(\vec{b} + \vec{z}/2) | PS \rangle$$

- The most intuitive definition of OAM involves Wigner functions

$$L_z = \int dx d^2 \vec{b} d^2 \vec{k} (\vec{b} \times \vec{k})_z W(x, \vec{b}, \vec{k})$$

C. Lorcé, B. Pasquini, Phys. Rev. D 84, (2011)

C. Lorcé, B. Pasquini, X. Xiong and F. Yuan, Phys. Rev. D 85, (2012)



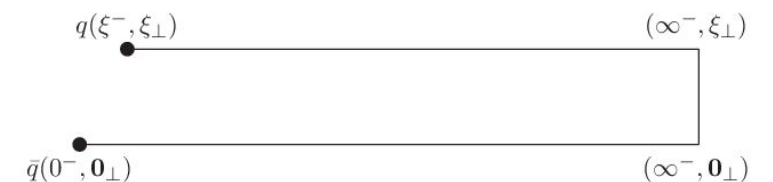
- The gauge link makes this definition gauge invariant and the two choices:

Straight link - kinetic OAM, Ji:  $L_q$

X. Ji, X. Xiong and F. Yuan, Phys. Rev. D 88, no. 1, (2013)

Y. Hatta, PLB 708 (2012) 186

Staple-like link - canonical OAM, Jaffe-Manohar:  $\mathcal{L}_q$



- The difference between the two  $\mathcal{L}_q - L_q$  is related to the torque force experienced by the struck quark and generated by the final state interactions M. Burkardt, Phys. Rev. D 88 (2013)

- How to fully access Wigner distributions in experiments is still to be explored

# CONCLUSIONS

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- There has been a lot of progress in understanding of the spin in the last decades
- Spin studies are synergistic with many other areas, in particular with lattice QCD
- The EIC is going to have a huge impact on our understanding of the sea quarks and gluons, the spin, and the multi-dimensional nucleon structure
- In this talk I concentrated on the spin of the nucleon. Equally exiting and interesting discussion is about the spin 1 systems such as deuteron
- In preparation of the talk I benefited a lot from discussions with my colleagues: Martha Constantinou, Cedric Lorce, Yoshitaka Hatta, Daniel Pitonyak, and Yuri Kovchegov