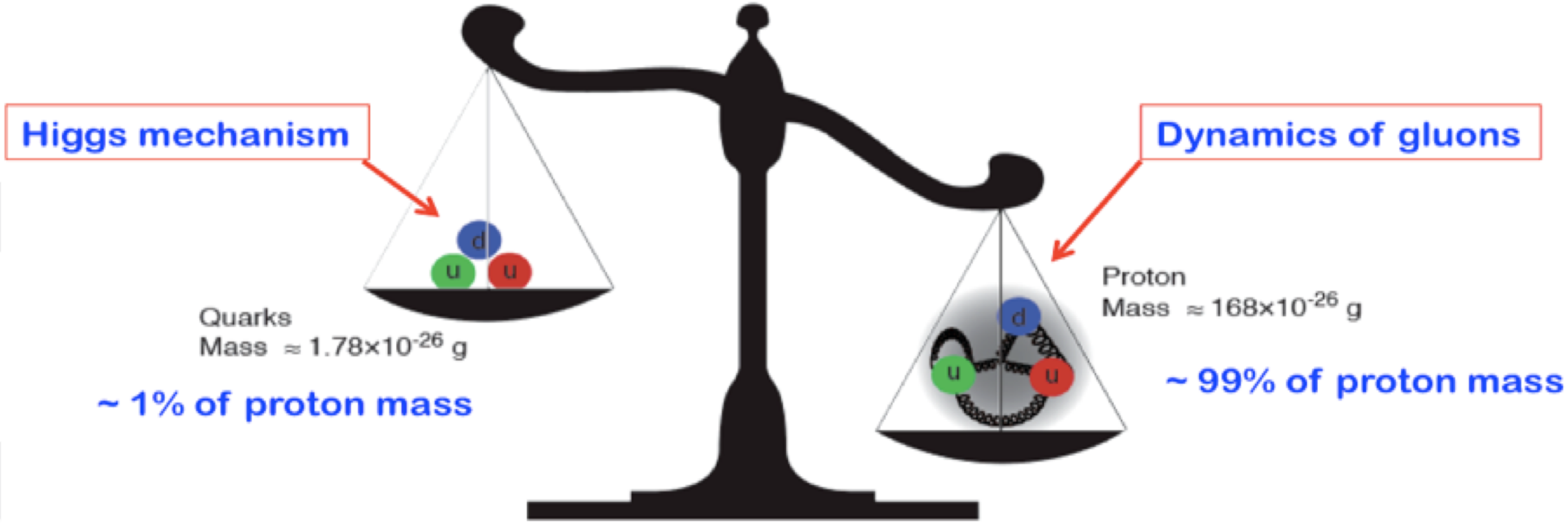
# High-Level Physics Presentation (Mass)

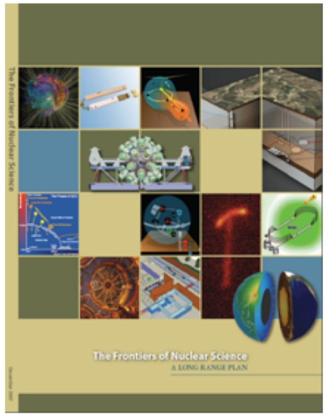
**Jianwei Qiu**
**Jefferson Lab, Theory Center**



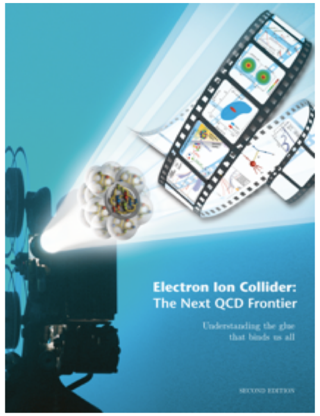
# Mass of Nucleon?

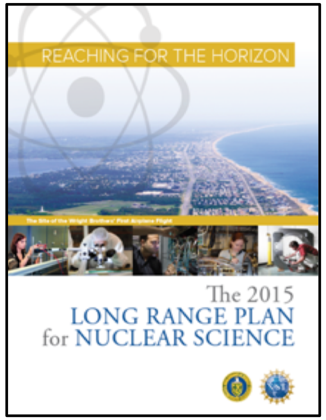
- One of the profound questions that U.S. EIC is built to address:

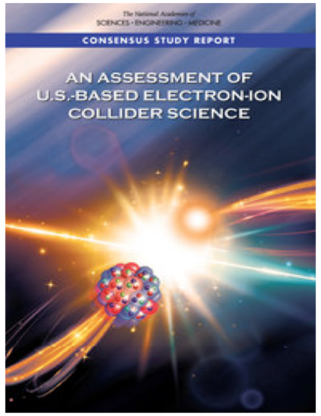
…







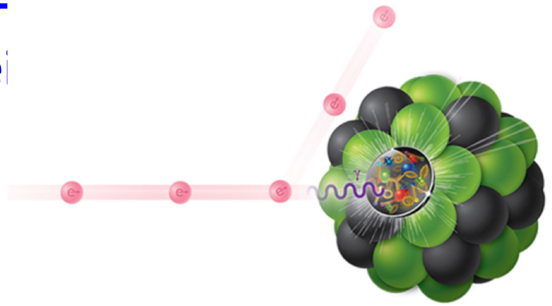

…

"… answer science questions that are compelling, fundamental, and timely, and help maintain U.S. scientific leadership in nuclear physics."

**Finding 1:**

An EIC can uniquely address three profound questions about nucleon – nucleons and protons – and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?



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# Mass of Nucleon?

## ❑ Nucleon Mass:

$m = E/c^2$ from the A. Einstein's famous equation E = mc²

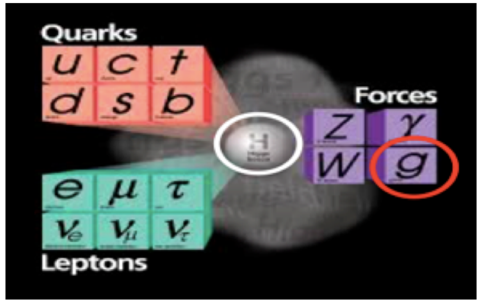
**Mass is the Energy of the nucleon when it is at Rest!**

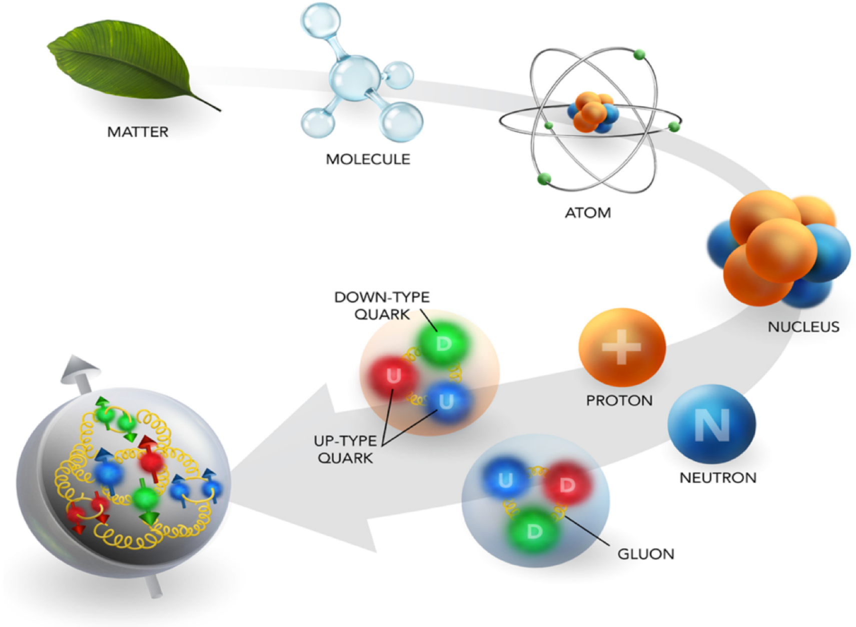
$$M_n = \left. \frac{\langle P|H_{\rm QCD}(\psi, A)|P\rangle}{\langle P|P\rangle} \right|_{\rm at\ rest}$$

## ❑ Nucleon is not elementary:

**Nucleon is a strongly interacting, relativistic bound state of quarks and gluons of QCD**

**Our understanding of the nucleon has been evolving, and will continue to evolve,**




***Understanding it fully is still beyond the best mind that we have!***

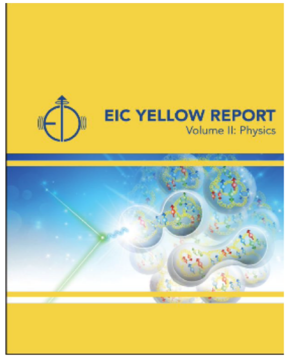
$H_{\rm QCD}(\psi, A)$ **is known, but not** $|P\rangle = ?$

# How does Nucleon Mass arise?

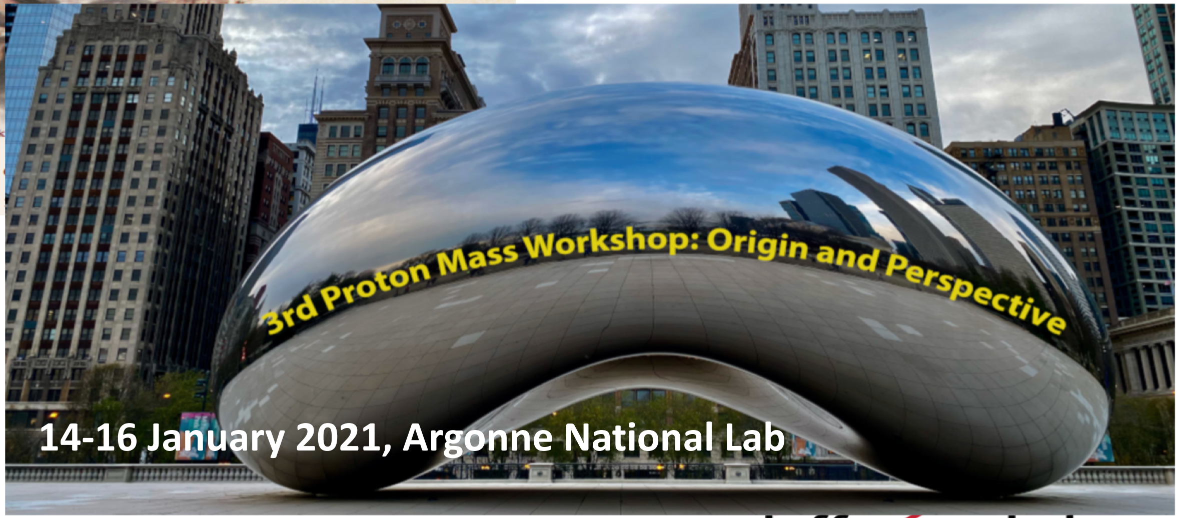
- ❑ **A true international interest and devoted effort:**



*One of the key questions that EIC is built to address!*




(> 200 participants!)




*A focused INT workshop has been planned*

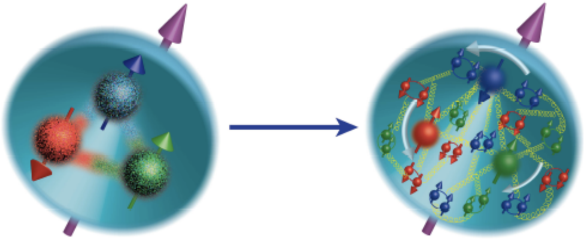
https://indico.phy.anl.gov/event/2/

# Mass of Nucleon in QCD

## ❑ Mass without mass:

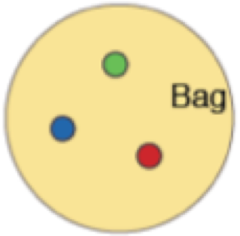
- QCD Lagrangian does not have mass dimension parameters, other than the masses of current quarks, $m_q \ll M_p$
- Asymptotic freedom ⟷ confinement:

➡ A dynamical scale, $\Lambda_{\rm QCD}$ , consistent with $\frac{1}{R} \sim 200\ {\rm MeV}$



## ❑ A consistent check:

- Bag model:

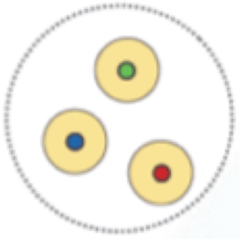



✧ Kinetic energy of three quarks: $K_q \sim 3/R$

✧ Bag energy (bag constant B): $T_b = \frac{4}{3}\pi R^3\, B$

✧ Minimize total energy $K_q + T_b$: $M_p \sim \frac{4}{R} \sim \frac{4}{0.88 fm} \sim 912{\rm MeV}$

- Constituent quark model:



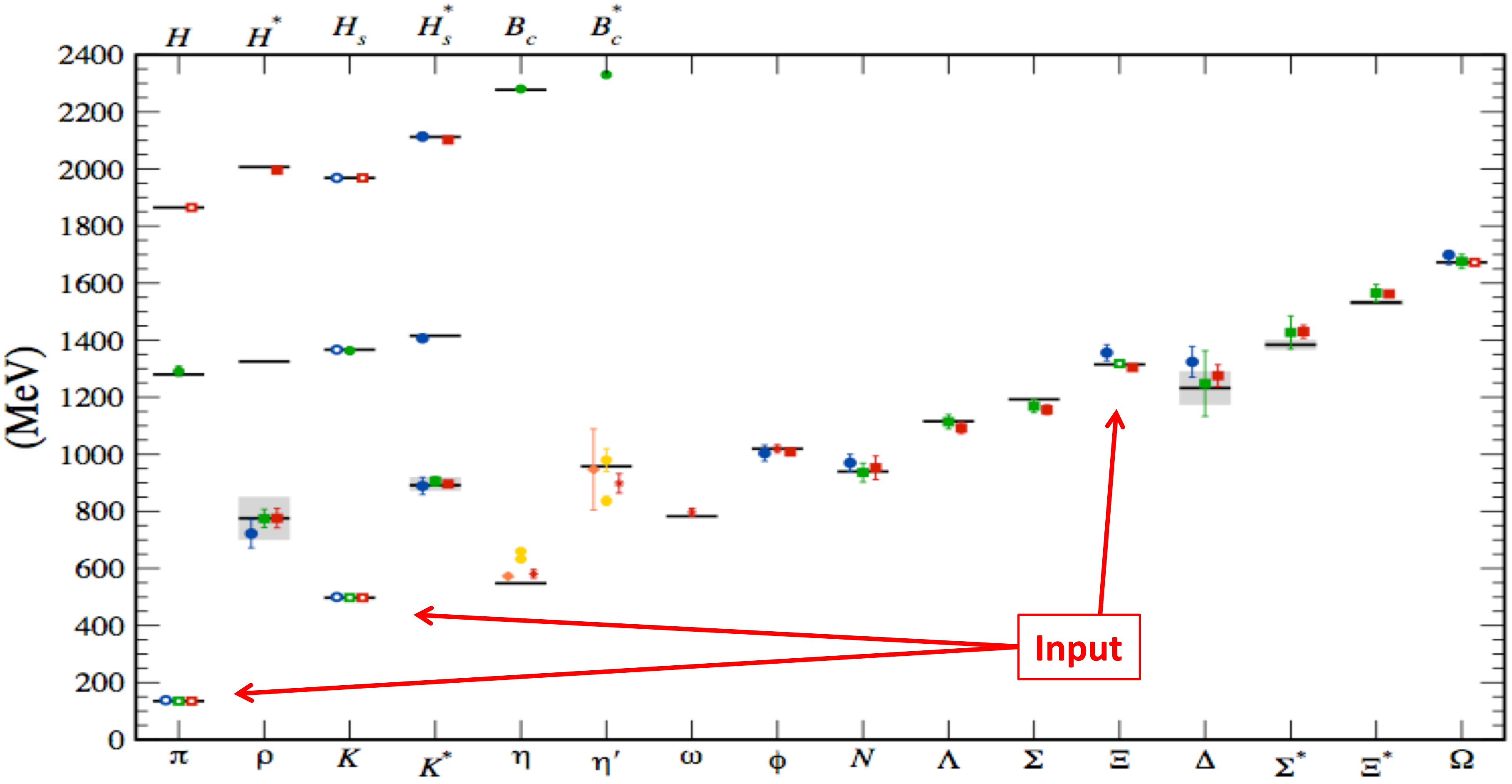
✧ Spontaneous chiral symmetry breaking:

***Massless quarks gain ~300 MeV mass when traveling in vacuum***

➡ $M_p \sim 3\, m_q^{\rm eff} \sim 900\ {\rm MeV}$

# Mass of Nucleon in QCD

❑ **From Lattice QCD:**




*How does QCD generate this? The role of quarks vs. that of gluons?*

*If we do not understand proton mass, we do not understand QCD!*



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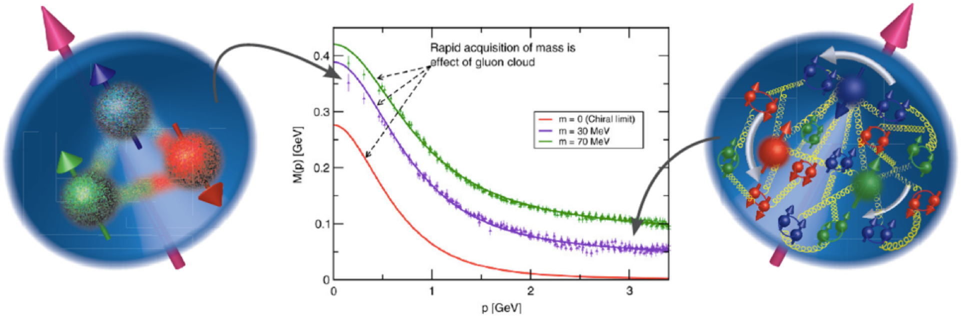
# Beyond Lattice QCD

- **Three-pronged theory approach to explore the origin nucleon mass:**
  - **Mass decomposition – roles of the constituents – but, not unique!**

    **Matching individual terms to physical observables with controllable approximations – Factorization!**
  - **lattice QCD – calculations of individual terms**
  - **Model calculation – approximated analytical approach**




$$M_p = 2m_u^{\rm eff} + m_d^{\rm eff}$$

$$M_n = \sum_{f=q,g} \left. \frac{\langle P|T_f^{00}(0)|P\rangle}{2P^0} \right|_{\rm cm}$$

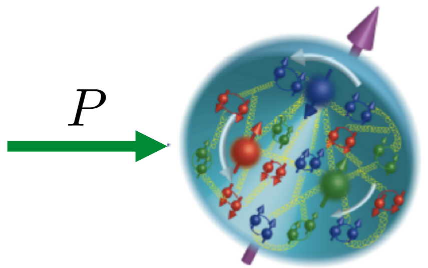
$$= M_q + M_g + M_m + M_a$$

**Not unique, none of these terms are physical observables!**

- **Experimental measurements of individual terms with reliable theory matching:**
  - **Paton momentum fractions**
  - **Nucleon sigma-terms**
  - **Trace anomaly – matching to heavy quarkonium production near the threshold – JLab12 + EIC**

# Mass of Nucleon in QCD

## ❑ Role of quarks and gluons:




**QCD Energy-Momentum Tensor (EMT):**

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}$$

**With** $T_q^{\mu\nu} = \overline{\psi}\,\gamma^\mu \frac{1}{2}\, i\overleftrightarrow{D}^\nu\, \psi$ **and** $T_g^{\mu\nu} = -F^{\mu\sigma}F^{\nu}{}_{\sigma} + \frac{1}{4} g^{\mu\nu} F^2$

**Expectation values:**

$$\langle T_f^{\mu\nu}\rangle = \frac{\langle P|\int d^3r\, T_f^{\mu\nu}(r)|P\rangle}{\langle P|P\rangle} = \frac{\langle P|T_f^{\mu\nu}(0)|P\rangle}{2P^0}$$

with $\langle P|P'\rangle = 2P^0(2\pi)^3\delta^3(P-P')$, $f = q, g$ and $\int d^3r = (2\pi)^3\delta^3(0)$

**Form factors:**

$$\langle P|T_f^{\mu\nu}(0)|P\rangle = 2P^\mu P^\nu A_f(0) + 2M_n^2\, g^{\mu\nu}\,\overline{C}_f(0)$$

$$A_q(0) + A_g(0) = 1$$
$$\overline{C}_q(0) + \overline{C}_g(0) = 0$$

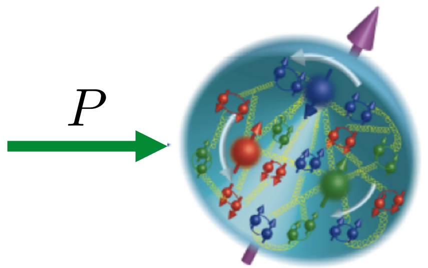
$$P^2 = M_n^2$$

➡ $\langle P|T^{\alpha}{}_{\alpha}(0)|P\rangle = 2M_n^2$

# Mass of Nucleon in QCD

## ❑ Decomposition of the trace of EMT:

**Trace of the QCD energy-momentum tensor:**




$$T^{\alpha}_{\ \alpha} = \underbrace{\frac{\beta(g)}{2g} F^{\mu\nu,a} F^{a}_{\mu\nu}}_{\text{QCD trace anomaly}} + \sum_{q=u,d,s} \underbrace{m_q(1+\gamma_m)\overline{\psi}_q \psi_q}_{\text{Chiral symmetry breaking}}$$

$$\beta(g) = -(11 - 2n_f/3)\, g^3/(4\pi)^2 + \dots$$

$$\langle P|T^{\alpha}_{\ \alpha}(0)|P\rangle = 2P^2 = 2M_n^2$$

$$\Rightarrow \quad \langle T^{\alpha}_{\ \alpha}\rangle = \frac{\langle P|T^{\alpha}_{\ \alpha}(0)|P\rangle}{2P^0} = \frac{M_n^2}{P^0}$$

$$\Rightarrow \quad M_n = \langle T^{\alpha}_{\ \alpha}\rangle|_{\text{at reest}}$$

Without separate the quark from gluon contribution to EMT

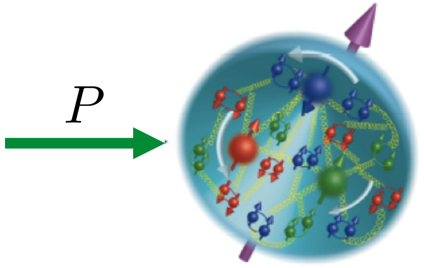
**In the nucleon's rest frame,** $\underbrace{\langle \int d^3r\, T^{\mu}_{\ \mu}\rangle}_{=M} = \underbrace{\langle \int d^3r\, T^{00}\rangle}_{=M} - \sum_i \underbrace{\langle \int d^3r\, T^{ii}\rangle}_{=0}$

**Nucleon mass:** ***Gluon quantum effect + Chiral symmetry breaking!***

**The sigma-term can be calculated in LQCD, Need the trace anomaly to test the sum rule!**

# Mass of Nucleon in QCD

## ❑ Decomposition of "energy"-operator of EMT:




**Mass is the Energy of the nucleon when it is at Rest!**

$$M_n = P^\mu\, v_{\mathrm{cm},\mu} = (P_q^\mu + P_g^\mu)\, v_{\mathrm{cm},\mu}$$

**Velocity of the Center of Mass**

**Momentum operator:**

$$\hat{P}^\mu = \hat{P}_q^\mu + \hat{P}_g^\mu \qquad \textbf{With} \qquad \hat{P}_f^\mu = \int d^3r\, T_f^{0,\mu}(r) \qquad \textbf{where} \qquad f = q, g$$

**Nucleon momentum:**

$$P^\mu = \langle \hat{P}_q^\mu \rangle + \langle \hat{P}_g^\mu \rangle \qquad \textbf{With} \qquad \langle \hat{P}_f^\mu \rangle = \frac{\langle P|\hat{P}_f^\mu|P\rangle}{\langle P|P\rangle} = \frac{\langle P|T_f^{0\mu}(0)|P\rangle}{2P^0}$$

**Nucleon mass:**

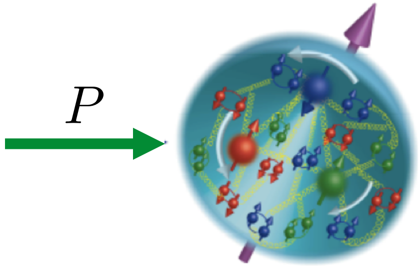
$$M_n = \sum_{f=q,g} \left. \frac{\langle P|T_f^{00}(0)|P\rangle}{2P^0} \right|_{\mathrm{cm}}$$

**Note:** $\langle P|T_f^{00}(0)|P\rangle$ **is NOT a physical observable!**

# Mass of Nucleon in QCD

## ❑ Decomposition of "energy"-operator of EMT:

$$M_n = \sum_{f=q,g} \frac{\langle P|T_f^{00}(0)|P\rangle}{2P^0}\bigg|_{\rm cm}$$




- Decompose the RHS into a sum of several gauge invariant terms
- Decomposition is not unique, since only the sum is a physical observable
- Usefulness – Each term can be related to physical observables with controllable approximations
- Individual contribution to the nucleon mass – physical interpretation of each term (?)

## ❑ Ji's decomposition:

**Let** $T^{\mu\nu} = \overline{T^{\mu\nu}} + \widehat{T^{\mu\nu}}$ **With** $\overline{T^{\mu\nu}} = T^{\mu\nu} - \frac{1}{4}g^{\mu\nu}T^{\alpha}_{\ \alpha}$ **and** $\widehat{T^{\mu\nu}} = \frac{1}{4}g^{\mu\nu}T^{\alpha}_{\ \alpha}$

$\overline{T^{\mu\nu}},\ \widehat{T^{\mu\nu}}$ **Renormalized separately, in different Lorentz representations**

$$\Rightarrow\quad M_n = \sum_{f=q,g} \frac{\langle P|T_f^{00}(0)|P\rangle}{2P^0}\bigg|_{\rm cm} = M_q + M_g + M_m + M_a$$

- Relativistic motion: $M_q + M_g$
- χ Symmetry Breaking: $M_m$
- Quantum fluctuation: $M_a$
- Trace Anomaly: $M_a$, $\langle \widehat{T}_g^{00}\rangle$
- Quark Energy: $M_q$, $\langle \overline{T}_q^{00}\rangle$
- Gluon Energy: $M_g$, $\langle \overline{T}_g^{00}\rangle$
- Quark Mass: $M_m$, $\langle \widehat{T}_q^{00}\rangle$

# Mass of Nucleon in QCD

## ❑ Physics interpretation:

**Quark Energy** $\langle T_q^{00} \rangle$ : $$M_q = \frac{3}{4}\left(M\sum_q \langle x\rangle_q - \sum_q \sigma_q\right)$$

**Gluon Energy** $\langle T_g^{00} \rangle$ : $$M_g = \frac{3}{4}M\langle x\rangle_g$$

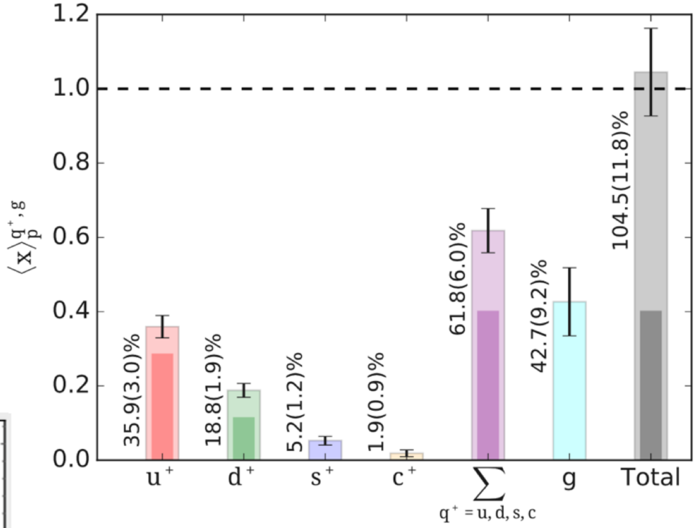
**Quark Mass** $\langle \widehat{T}_q^{00} \rangle$ : $$M_m = \sum_q \sigma_q$$

**Trace Anomaly** $\langle \widehat{T}_g^{00} \rangle$ : $$M_a = \frac{\gamma_m}{4}\sum_q \sigma_q - \frac{\beta(g)}{4g}(E^2+B^2)$$

Note: $\langle x\rangle_f$ and $\sigma_q$ are calculable in lattice QCD

**Parton momentum fraction:**

$$\langle x\rangle_f = \int_0^1 dx\, x f(x,\mu^2)$$

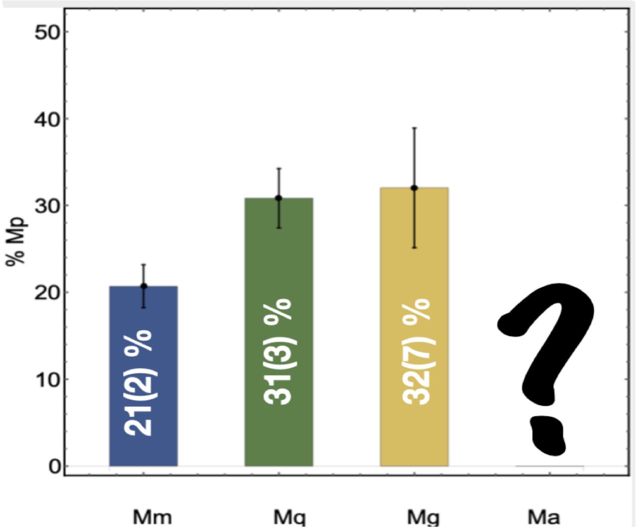



## ❑ LQCD calculation:

**Quark sigma-term:**

$$\sigma_q = \frac{\langle P|\overline{\psi}_q(0) m_q \psi_q(0)|P\rangle}{2P^0}$$

| | $u+d$ | $s$ | $c$ |
|---|---|---|---|
| $\sigma$ [MeV] | 41.6(3.8) | 45.6(6.2) | 107(22) |




**Access the trace anomaly Indirectly?**

$$M_a = \frac{M}{4} - \sum_q \frac{\sigma_q}{4}$$

**Or by experiment?**

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# Mass of Nucleon in QCD

## ❑ Other decompositions:

EPJC78 (2018),JHEP09 (2020) PRD102 (2020)

$$M = \underbrace{\left[\langle \int d^3r\, \overline{\psi}\gamma^0 iD^0\psi\rangle - \langle \int d^3r\, \overline{\psi}m\psi\rangle\right]}_{\text{Quark kinetic and potential energy}} + \underbrace{\langle \int d^3r\, \overline{\psi}m\psi\rangle}_{\text{Quark rest mass energy}} + \underbrace{\langle \int d^3r\, \tfrac{1}{2}(\vec{E}^2 + \vec{B}^2)\rangle}_{\text{Gluon total energy}}$$

**Without separate EMT into traceless piece – mixing of terms via renormalizations**

## ❑ Compare to Ji's decomposition:

C.L., EPJC78 (2018)

$$T_a^{00} = \underbrace{\bar{T}_a^{00}}_{= \frac{3}{4}T_a^{00} + \frac{1}{4}\sum_i T_a^{ii}} + \underbrace{\hat{T}_a^{00}}_{= \frac{1}{4}T_a^{00} - \frac{1}{4}\sum_i T_a^{ii}} \qquad a = q, g$$

$$M_q = \frac{3}{4}\left(a - \frac{b}{1+\gamma_m}\right)M \neq \langle \int d^3r\, \psi^\dagger i\vec{D}\cdot\vec{\alpha}\psi\rangle \qquad M_m = \frac{4+\gamma_m}{4(1+\gamma_m)}\, b\, M = \langle \int d^3r\, \left(1 + \tfrac{1}{4}\gamma_m\right)\overline{\psi}m\psi\rangle$$

$$M_g = \frac{3}{4}(1-a)\,M \neq \langle \int d^3r\, \tfrac{1}{2}(\vec{E}^2 + \vec{B}^2)\rangle \qquad M_a = \frac{1}{4}(1-b)\,M = \langle \int d^3r\, \tfrac{1}{4}\frac{\beta(g)}{2g}G^2\rangle$$

**Matter of interpretations! Key is how can we measure each term with controllable approximations!**

# The Role of the Trace Anomaly

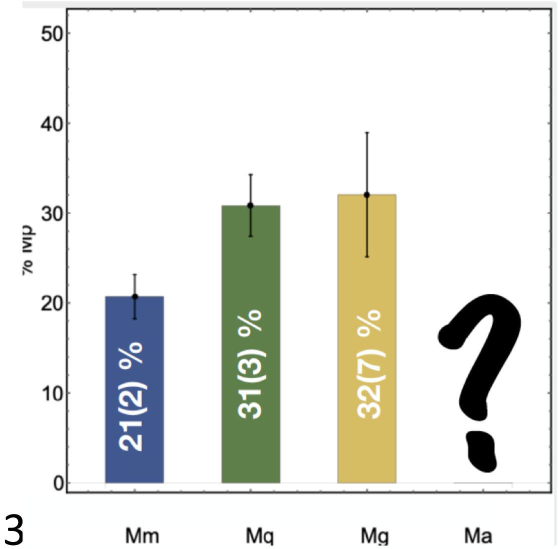
## ❑ The Trace Anomaly from LQCD:

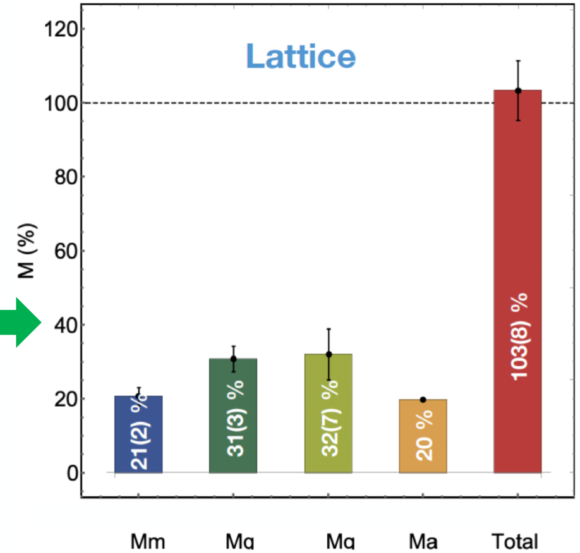
"Well-defined operator, but complicated renormalization pattern, and suppressed signal-to-noise ratio"

M. Constantinou
@ the 3rd Proton Mass workshop

"Use sum rules to extract trace anomaly indirectly" - *a consistent check*

**Sum rule:** $$M_a = \frac{M_p}{4} - \sum_q \frac{\sigma_q}{4} \sim 19.83(0.07)\,\%$$




## ❑ The Trace Anomaly from JLab12 + EIC:

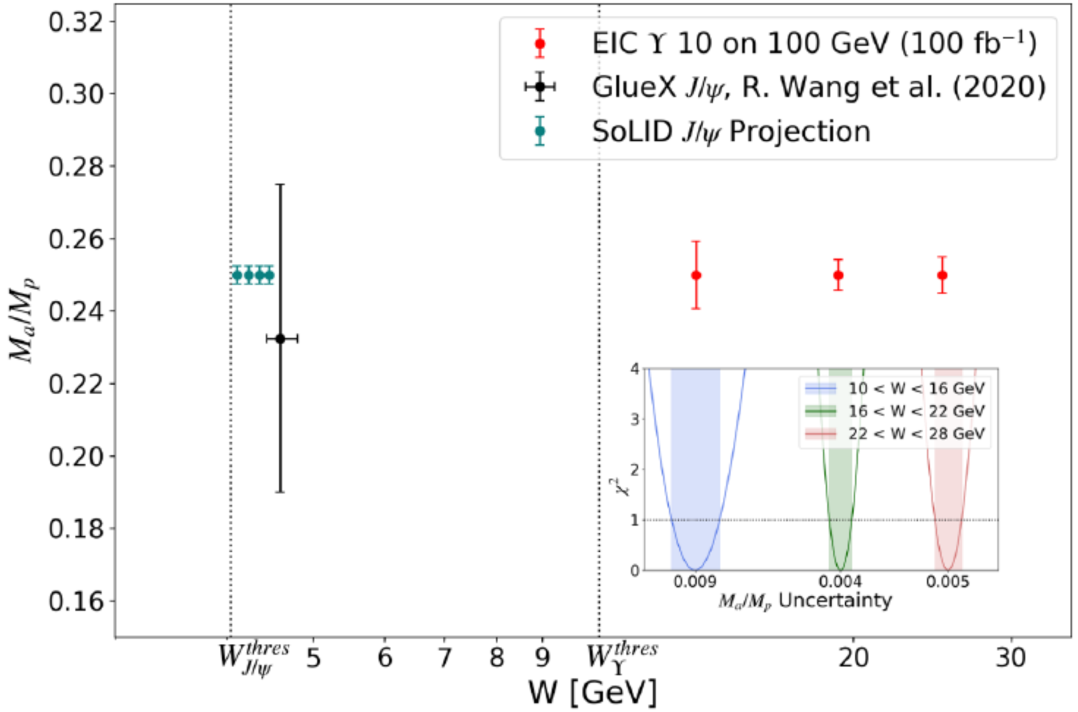



Figure 7.26: Projection of the trace anomaly contribution to the proton mass (Ma/Mp) with ϒ photoproduction on the proton at the EIC

EIC Yellow Report



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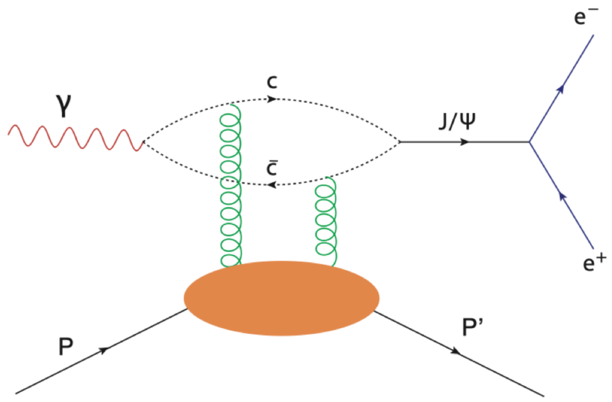
# Extract the Trace Anomaly from Experiments

## ❑ QCD Trace Anomaly:

$$T^{\alpha}_{\alpha} = \underbrace{\frac{\beta(g)}{2g} F^{\mu\nu,a} F^{a}_{\mu\nu}}_{\text{QCD trace anomaly}} + \sum_{q=u,d,s} m_q (1+\gamma_m) \overline{\psi}_q \psi_q$$

$F^{\mu\nu,a} F^{a}_{\mu\nu}$ Is a scalar, and high twist!

## ❑ Diffractive heavy quarkonium production:

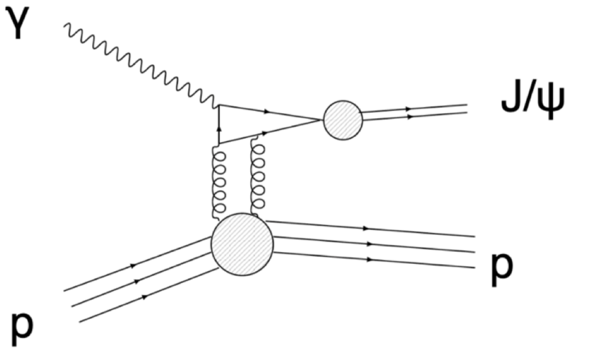



Two gluons may not be factorized into

$$F^{\mu\nu,a} F^{a}_{\mu\nu}$$

**Using a slow-moving dipole approximation to measure the scale field response** $\langle P | F^2 | P \rangle$

**D. Kharzeev 1996**




Near threshold, dominance of

$$g^2 \mathbf{E}^{a2} = \frac{8\pi^2}{b} \theta^{\mu}_{\mu} + g^2 \theta^{(G)}_{00}$$

Assuming the validity of vector meson dominance, can relate photoproduction to quarkonium scattering amplitude and probe the mass of the proton

DK, Satz, Syamtomov, Zinovjev '99

Other approaches to threshold photoproduction:

Hatta, Yang '18; Hatta, Rajan, Yang '19; Mamo, Zahed '19

# Extract the Trace Anomaly from Experiments

- **Is the VMD valid near threshold?**

**Near threshold:** $$t_{min} = -\frac{M_\psi^2 M}{M_\psi + M} \simeq -2.23\ \text{GeV}^2 \simeq -(1.5\ \text{GeV})^2$$

**Might be too large for the VMD approximation!**

- **Taking advantage of the measured t-distribution to get the nucleon's mass radius:**

$$\langle \mathbf{p}_1 | T_{\mu\nu} | \mathbf{p}_2 \rangle = \left(\frac{M^2}{p_{01}\ p_{02}}\right)^{1/2} \frac{1}{4M}\, \bar{u}(p_1, s_1)\Big[G_1(q^2)(p_\mu\gamma_\nu + p_\nu\gamma_\mu) + G_2(q^2)\frac{p_\mu p_\nu}{M} + G_3(q^2)\frac{(q^2 g_{\mu\nu} - q_\mu q_\nu)}{M}\Big] u(p_2, s_2),$$

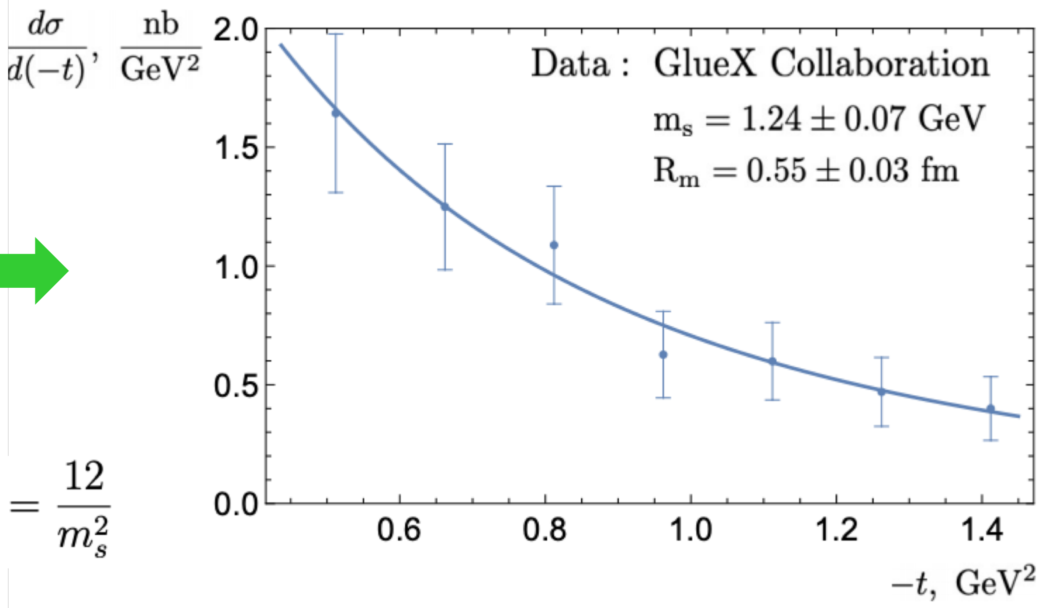
**Define:**

$$G(q^2) = G_1(q^2) + G_2(q^2)\left(1 - \frac{q^2}{4M^2}\right) + G_3(q^2)\frac{3q^2}{4M^2}$$

**Mass radius:**

$$\langle R_{\rm m}^2 \rangle = \frac{6}{M}\frac{dG}{dt}\Big|_{t=0}$$

**Dipole model for the Form Factor:** $$G(t) = \frac{M}{\left(1 - \frac{t}{m_s^2}\right)^2} \quad \Rightarrow \quad \langle R_{\rm m}^2 \rangle = \frac{12}{m_s^2}$$

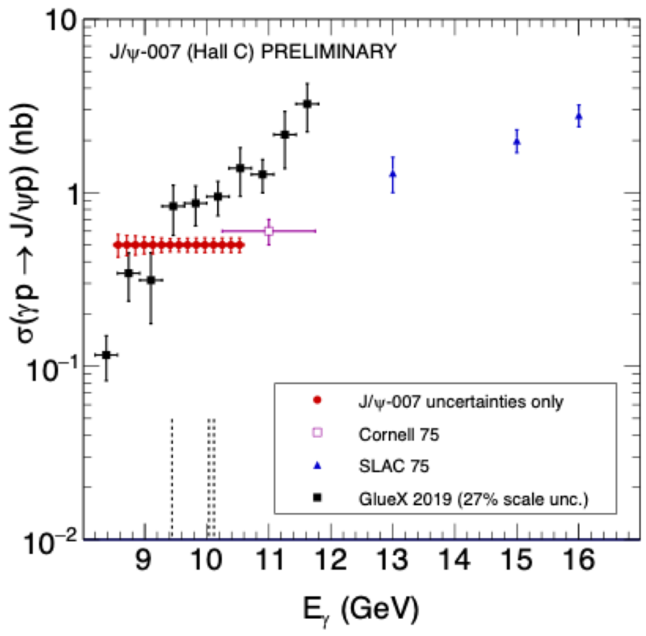


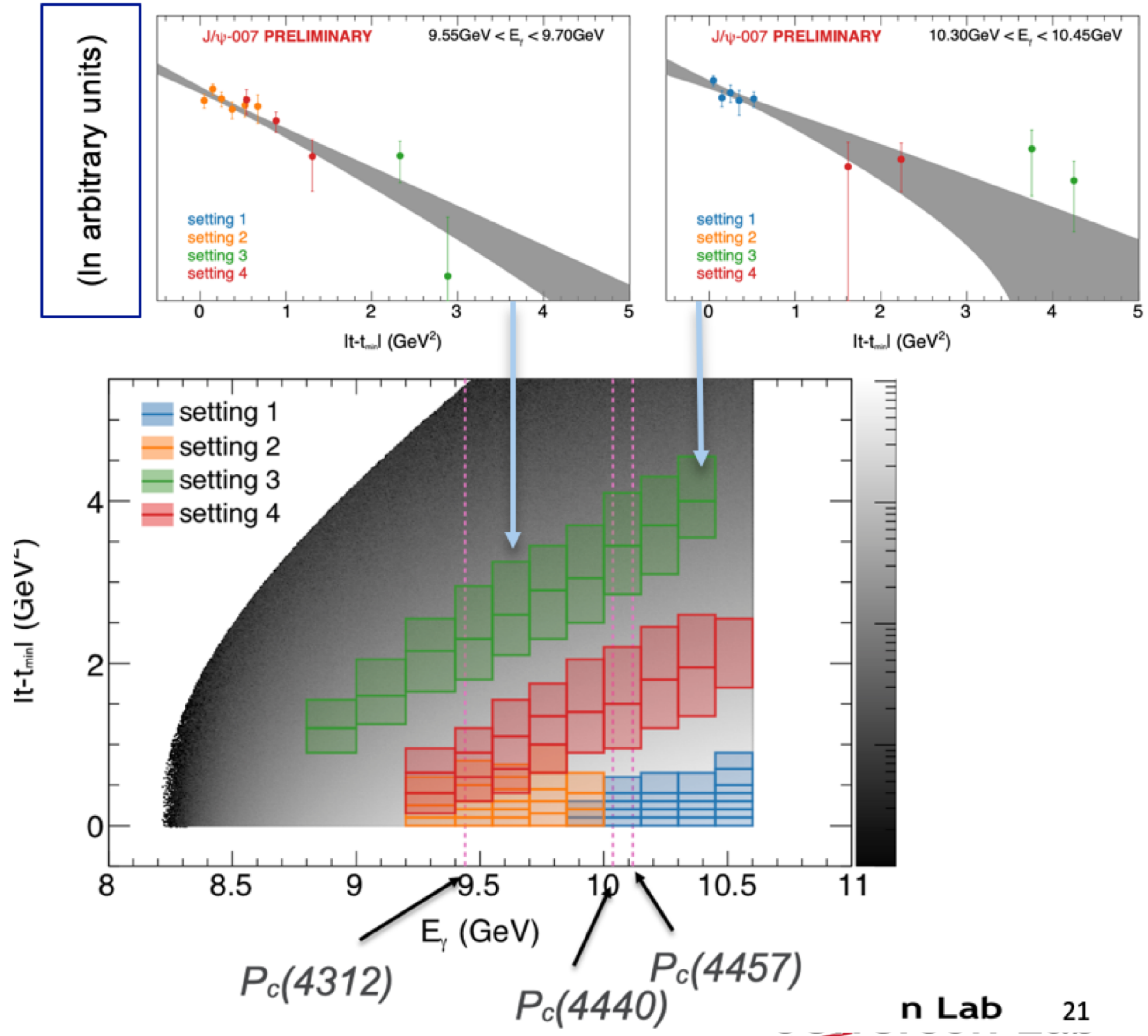
# Extract the Trace Anomaly from Experiments

❑ **Hall C also measured J/psi photo-production near the threshold:**

Z.-E. Meziani

- **First ever determination of the *t*-dependence of the cross section as a function of photon energy near threshold.**
- **Highly sensitive to *s*-channel resonances**
- **Only showing electron data, muon data is an independent experiment with same statistics but different systematics**



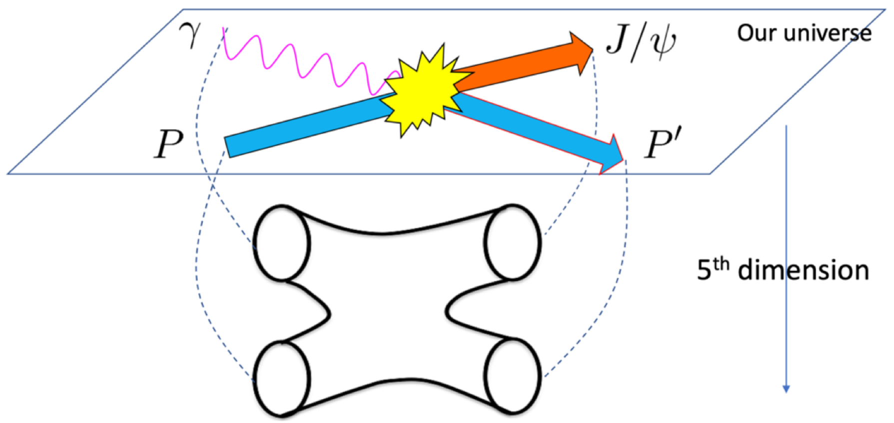
# Extract the Trace Anomaly from Experiments

- **Holographic approach:**

Boussarie & Hatta et al, Phys.Rev.D 101 (2020) 11, 114004)

$$\frac{d\sigma}{dt} = \frac{\alpha_{em}}{4(W^2 - M_p^2)^2} \overline{\Sigma}_{pol} \overline{\Sigma}_{spin} \left| \langle P | \vec{\epsilon} \cdot \vec{J}(0) | P' p \rangle \right|^2$$

$$\sigma_{tot} = \int_{t_{min}}^{t_{max}} \frac{d\sigma}{dt} \, dt$$




**How to calculate the scattering amplitude?**

$$\langle P | \vec{\epsilon} \cdot \vec{J}(q) | P' p \rangle = (2\pi)^4 \delta^4 (P + q - P' - p) \langle P | \vec{\epsilon} \cdot \vec{J}(0) | P' p \rangle$$

**Y. Hatta, D.L. Yang (1801.02163) – gauge/string duality:**

$$\langle P | \epsilon \cdot J(0) | P' k \rangle = -\frac{2\kappa^2}{f_\psi R^3} \int_0^{z_m} dz \frac{\delta S_{D7}(q,k,z)}{\delta g_{\mu\nu}} \frac{z^2 R^2}{4} \langle P | T^{gTT}_{\mu\nu} | P' \rangle + \frac{2\kappa^2}{f_\psi R^3} \frac{3}{8} \int_0^{z_m} dz \frac{\delta S_{D7}(q,k,z)}{\delta \phi} \frac{z^4}{4} \langle P | \frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a | P' \rangle$$

Gluon from Traceless $T^{\mu\nu}$

Trace Anomaly

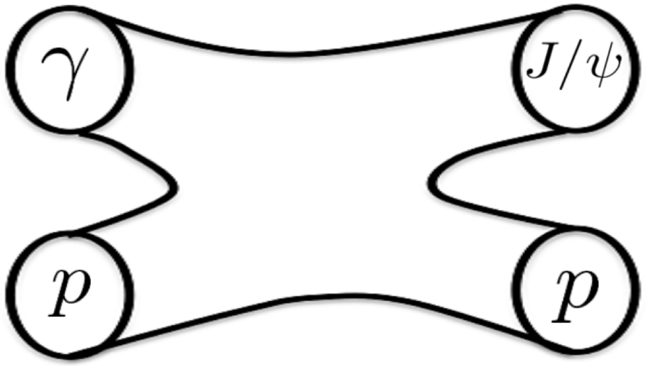
Dilaton field

$2\kappa^2 = \frac{8\pi^2}{N_c^2} R^3$ – 5D gravitational constant

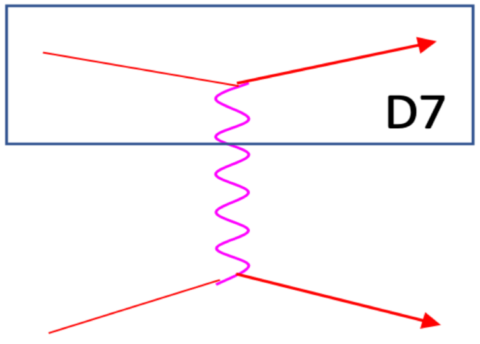
# Extract the Trace Anomaly from Experiments

## ❑ Holographic approach - dilton:

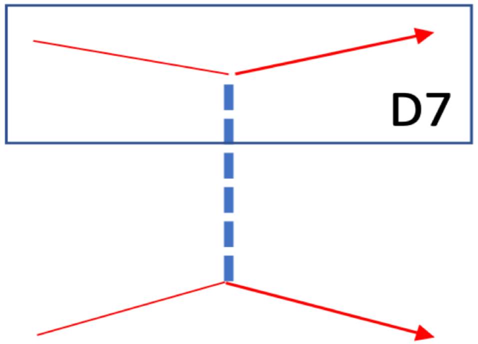
The operator $F^{\mu\nu}F_{\mu\nu}$ is dual to a massless string called dilaton




$\approx$




+




Suppressed compared to graviton exchange at high energy, but not at very low energy!

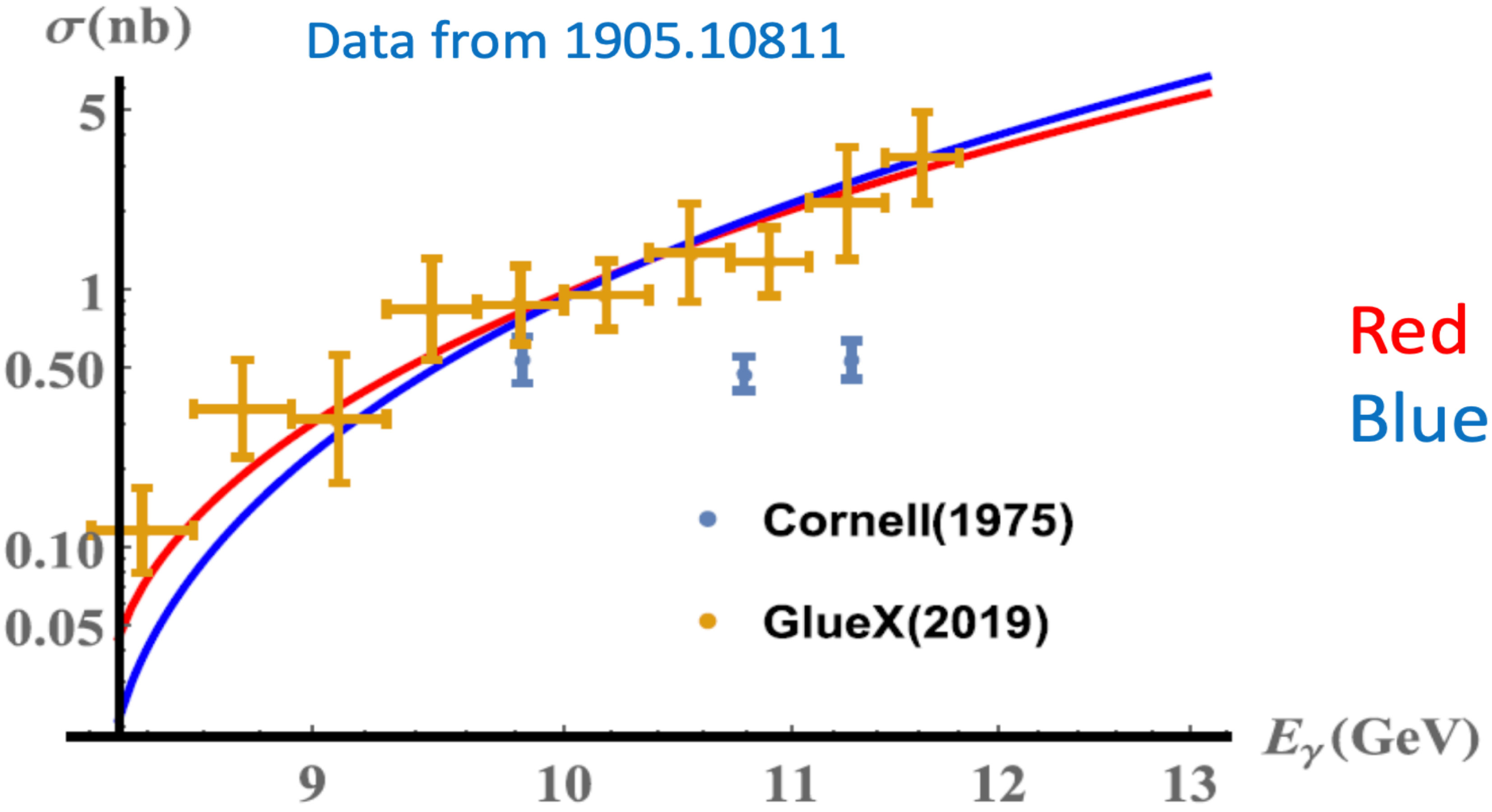
$$\langle P|\epsilon \cdot J(0)|P'k\rangle \approx -\frac{2\kappa^2}{f_\psi R^3}\int_0^{z_m} dz \frac{\delta S_{D7}(q,k,z)}{\delta g_{\mu\nu}} \frac{z^2R^2}{4}\langle P|T^{gTT}_{\mu\nu}|P'\rangle + \frac{2\kappa^2}{f_\psi R^3}\frac{3}{8}\int_0^{z_m} dz \frac{\delta S_{D7}(q,k,z)}{\delta\phi}\frac{z^4}{4}\langle P|\frac{1}{4}F^{\mu\nu}_a F^a_{\mu\nu}|P'\rangle$$

Gluon condensate (nonforward version)

# Extract the Trace Anomaly from Experiments

❑ **Photo-production – fitting GlueX data:**

$$M_m = \frac{1}{4}\frac{\langle P|m(1+\gamma_m)\bar{\psi}\psi|P\rangle}{2M} \equiv \frac{b}{4}M \qquad M_a = \frac{1}{4}\frac{\langle P|\frac{\beta}{2g}F^2|P\rangle}{2M} \equiv \frac{1-b}{4}M$$



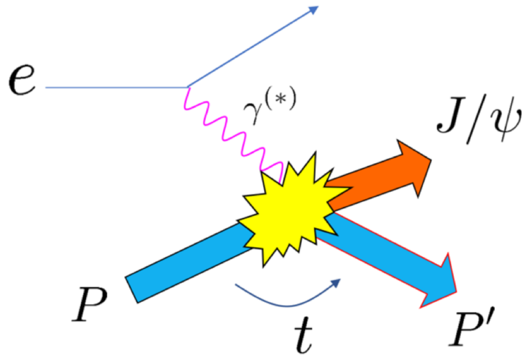

Red $b = 0$

Blue $b = 1$

# Extract the Trace Anomaly from Experiments

## ❑ Lepton production at high $Q^2$ – Need EIC:

Boussarie & Hatta et al,
Phys.Rev.D 101 (2020) 11, 114004)



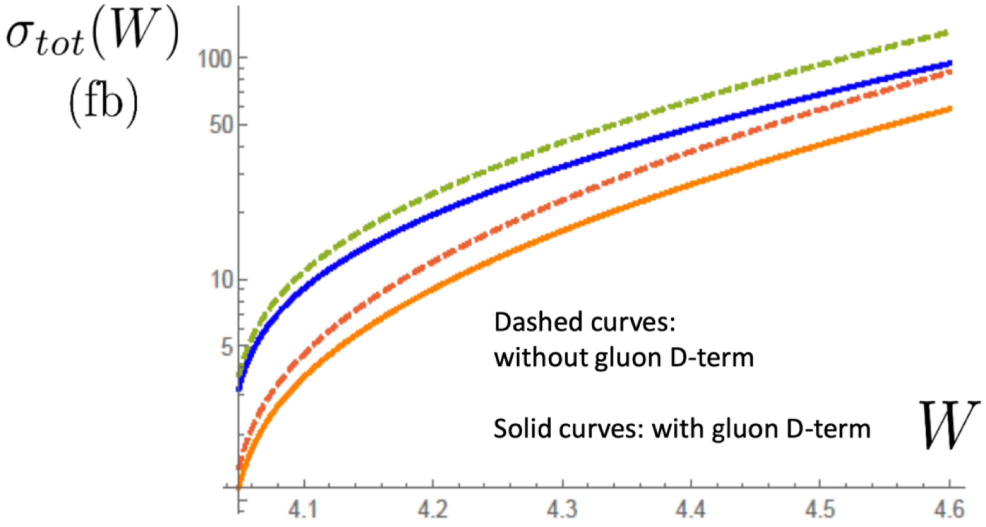

**High Q2 allows to use OPE**

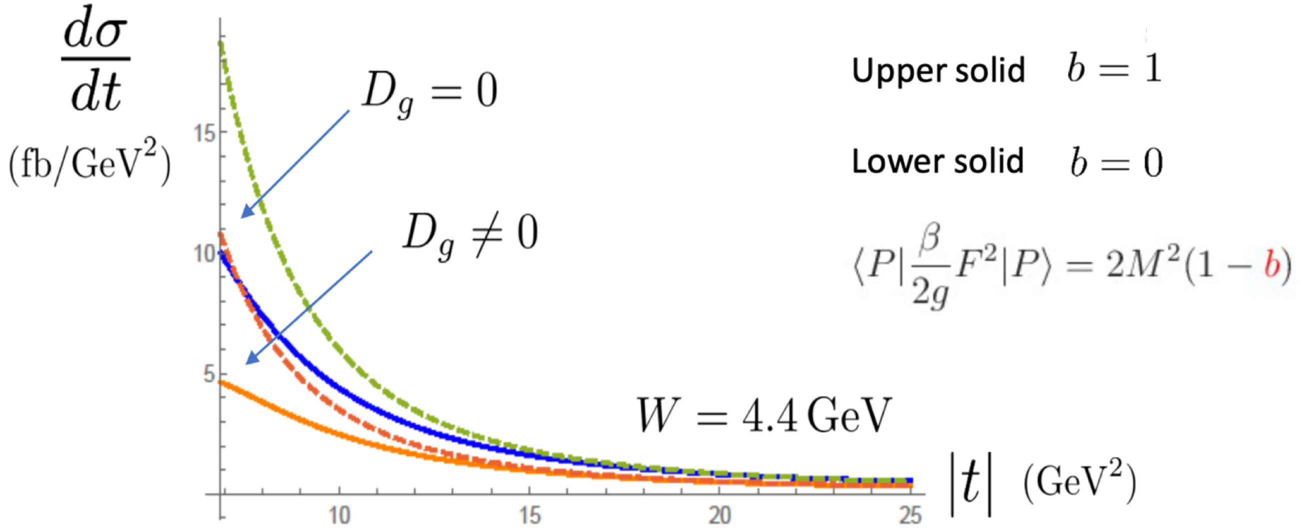
**Large $S_{l\text{-}p}$ can still be near threshold:**

$$W^2 = y(S_{ep} - m_N^2) + m_N^2 - Q^2$$

$$Q^2 \gg M_V^2 \gg m_N^2 \qquad Q^2 \gg |t|$$

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{u}(P')\left[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{C}_{q,g}Mg^{\mu\nu}\right]u(P)$$




$J/\psi$ $\quad Q^2 = 64\,\mathrm{GeV}^2 \quad \sqrt{S_{ep}} = 20\,\mathrm{GeV}$ (plots revised from 2004.12715)

# Summary and Outlook

- ❑ Nucleon mass is the charge of gravitational force, impacts every sectors of our physical world!

- ❑ Nucleon mass closely connected to quantum anomalies

  Non-perturbative QCD generates a new scale:

- ❑ Need a three-pronged approach to explore the origin nucleon mass
  - Mass decomposition – roles of the constituents – but, not unique!
  - lattice QCD – calculations of individual terms
  - Experimental measurements of individual terms – EIC is capable of measuring the trace anomaly, but more theory work is needed

- ❑ Questions and challenges:
  - Can we find a way to calculate the trace anomaly in LQCD?
  - How well can we control the approximations?
  - How well can we extract the trace anomaly from experiments?
  - … …

**Thanks!**