Brookhaven National Laboratory

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Proton mass and D-term in near-threshold quarkonium production

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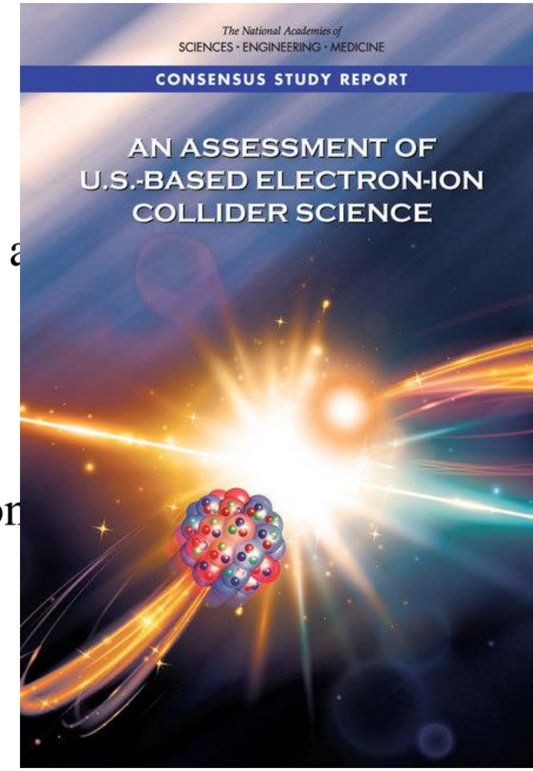
Outline

- Proton mass decomposition(s)
- Gravitational form factors
- Near-threshold quarkonium production
- Recent theory developments ← new!

Refs.
YH, Yang, 1808.02163
YH, Rajan, Tanaka, 1810.05116
YH, Rajan, Yang, 1906.00894
Boussarie, YH, 2004.12715
YH, Zhao, 2006.02798
YH, Strikman, 2102.12631

Origin of nucleon mass

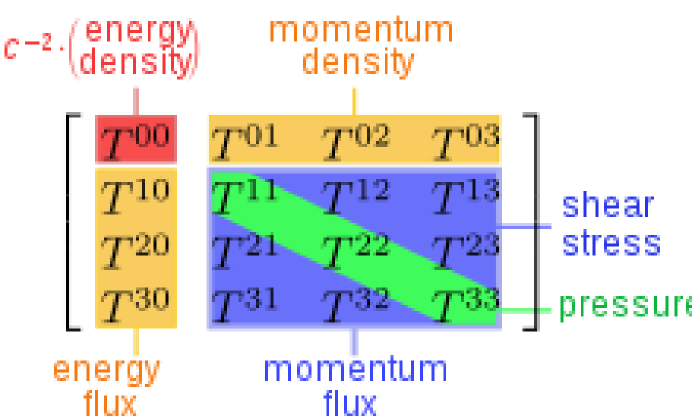


Finding 1: An EIC can uniquely address three profound questions a
protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluor

Study of mass $\subset$ Study of the QCD energy momentum tensor

$$M = \int d^3x T^{00}$$



Proton gravitational form factors

$$\langle P'|T^{\mu\nu}|P\rangle = \bar{u}(P')\left[A(t)\gamma^{(\mu}\bar{P}^{\nu)} + B(t)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M}\right]u(P)$$

Proton mass and spin decompositions

Proton mass decomposition Ji (1995)

quark mass (quark condensate)

$$M = M_q + M_g + M_a + M_m$$

kinetic energy

trace anomaly (gluon condensate)

Proton spin decomposition Jaffe, Manohar (1990)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

helicity orbital angular momentum

Can we determine individual components in experiment?
Usual strategy: First extract associated PDFs, then integrate over x.

$$\Delta G = \int_0^1 dx \Delta G(x) \qquad L^{q,g}_{can} = \int_0^1 dx L^{q,g}_{can}(x)$$

PDFs for proton mass?

$$M = M_q + M_g + M_a + M_m$$

twist-2 PDF

$xq(x), xG(x)$

??

twist-3 $e(x)$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

twist-2 helicity PDF

$\Delta q(x), \Delta G(x)$

twist-3 OAM PDF $L^{q,g}_{can}(x)$

Gluon condensate PDF

Ji (2020)
YH, Zhao (2020)

Anomaly term related to the gluon condensate

$$M_a \propto \langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$$

Introduce a twist-4 PDF

$$F(x) = \frac{P^+}{2M^2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P|F_{\mu\nu}(0)W[0,z]F^{\mu\nu}(z^-)|P\rangle$$

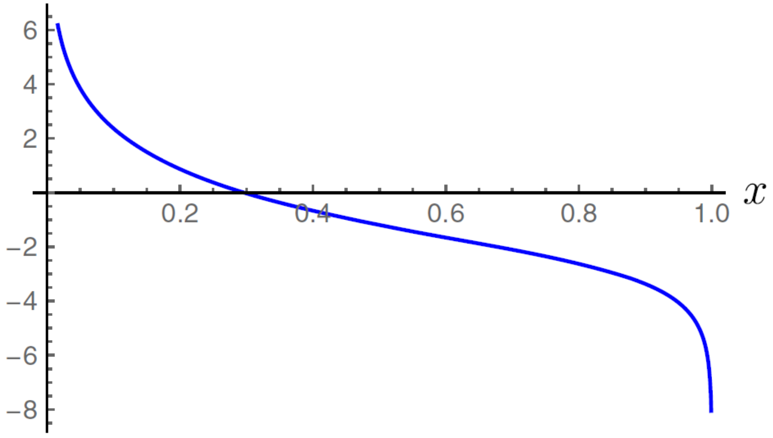
Most likely there's a delta function at $x = 0$, impossible to measure?

Origin of mass connected to the zero-mode problem in light-front quantization.

Calculable on a lattice? Ji (2020)

Can we directly access the x-integral instead?

$$\int_0^1 dx F(x) \sim \langle P|F^2|P\rangle$$



Proton mass decomposition: A different look

QCD trace anomaly

$$T^{\mu}_{\mu} = (T_q)^{\mu}_{\mu} + (T_g)^{\mu}_{\mu} = \frac{\beta}{2g} F^2 + m(1+\gamma_m)\bar{\psi}\psi$$

$$\langle P | T^{\mu}_{\mu} | P \rangle = 2M^2$$

$$M^2 = M_q^2 + M_g^2 \qquad M_{q,g}^2 \equiv \frac{1}{2} \langle P | (T_{q,g})^{\mu}_{\mu} | P \rangle$$

Compute $(T_q)^{\mu}_{\mu}$ and $(T_g)^{\mu}_{\mu}$ separately.

Result in $\overline{\rm MS}$ at two and three-loops

$$\eta_{\mu\nu}\left(T_g^{\mu\nu}\right)_R = \frac{\alpha_s}{4\pi}\left(\frac{14}{3}C_F\left(m\bar\psi\psi\right)_R - \frac{11}{6}C_A\left(F^2\right)_R\right) + \left(\frac{\alpha_s}{4\pi}\right)^2$$
$$\times\left[\left(C_F\left(\frac{812C_A}{27} - \frac{22n_f}{27}\right) + \frac{85C_F^2}{27}\right)\left(m\bar\psi\psi\right)_R + \left(\frac{28C_An_f}{27} - \frac{17C_A^2}{3} + \frac{5C_Fn_f}{54}\right)\left(F^2\right)_R\right]$$
$$+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{368\zeta(3)}{9} - \frac{25229}{729}\right)C_F^2 - \frac{2}{243}(4968\zeta(3)+1423)C_AC_F\right)\right.\right.$$
$$+\left(\frac{32\zeta(3)}{3} - \frac{91753}{1458}\right)C_AC_F^2 + \left(\frac{294929}{1458} - \frac{32\zeta(3)}{9}\right)C_A^2C_F - \frac{554}{243}C_Fn_f^2$$
$$\left.+\left(\frac{95041}{729} - \frac{64\zeta(3)}{9}\right)C_F^3\right\}\left(m\bar\psi\psi\right)_R$$
$$\left.+\left\{n_f\left(\left(\frac{1123}{162} - \frac{52\zeta(3)}{9}\right)C_AC_F + \left(4\zeta(3) + \frac{293}{36}\right)C_A^2 + \frac{16}{729}(81\zeta(3)-98)C_F^2\right) + n_f^2\left(\frac{655C_A}{2916} - \frac{361C_F}{729}\right) - \frac{2857C_A^3}{108}\right\}\left(F^2\right)_R\right]$$

YH, Rajan, Tanaka, (2018) (2-loop)

Tanaka (2018) (3-loop)

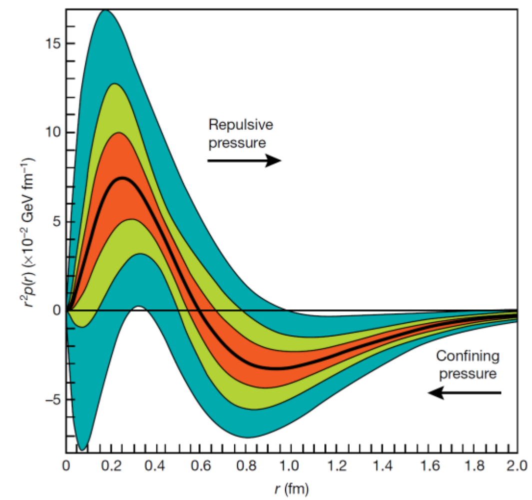
$$\eta_{\mu\nu}\left(T_q^{\mu\nu}\right)_R = \left(m\bar\psi\psi\right)_R + \frac{\alpha_s}{4\pi}\left(\frac{4}{3}C_F\left(m\bar\psi\psi\right)_R + \frac{1}{3}n_f\left(F^2\right)_R\right) + \left(\frac{\alpha_s}{4\pi}\right)^2$$
$$\times\left[\left(C_F\left(\frac{61C_A}{27} - \frac{68n_f}{27}\right) - \frac{4C_F^2}{27}\right)\left(m\bar\psi\psi\right)_R + \left(\frac{17C_An_f}{27} + \frac{49C_Fn_f}{54}\right)\left(F^2\right)_R\right]$$
$$+\left(\frac{\alpha_s}{4\pi}\right)^3\left[\left\{n_f\left(\left(\frac{64\zeta(3)}{9} - \frac{8305}{729}\right)C_F^2 - \frac{2}{243}(864\zeta(3)+1079)C_AC_F\right)\right.\right.$$
$$-\frac{8}{729}(972\zeta(3)+143)C_AC_F^2 + \left(\frac{32\zeta(3)}{9} + \frac{6611}{729}\right)C_A^2C_F - \frac{76}{243}C_Fn_f^2$$
$$\left.+\frac{8}{729}(648\zeta(3)-125)C_F^3\right\}\left(m\bar\psi\psi\right)_R$$
$$+\left\{n_f\left(\left(\frac{52\zeta(3)}{9} - \frac{401}{324}\right)C_AC_F + \left(\frac{134}{27} - 4\zeta(3)\right)C_A^2 + \left(\frac{2407}{1458} - \frac{16\zeta(3)}{9}\right)C_F^2\right)\right.$$
$$\left.\left.+n_f^2\left(-\frac{697C_A}{729} - \frac{169C_F}{1458}\right)\right\}\left(F^2\right)_R\right],$$

Alternative scheme

Metz, Pasquini, Rodini (2020)

D-term: the last global unknown

Burkert, Elouadrhiri, Girod (2018)



$D(t=0)$ is a conserved charge of the nucleon, just like mass and spin!

$$D = D_u + D_d + D_s + D_g + \cdots$$

$D_{u,d}$ from DVCS, related to the subtraction constant in the dispersion relation for the Compton form factor

Teryaev (2005)

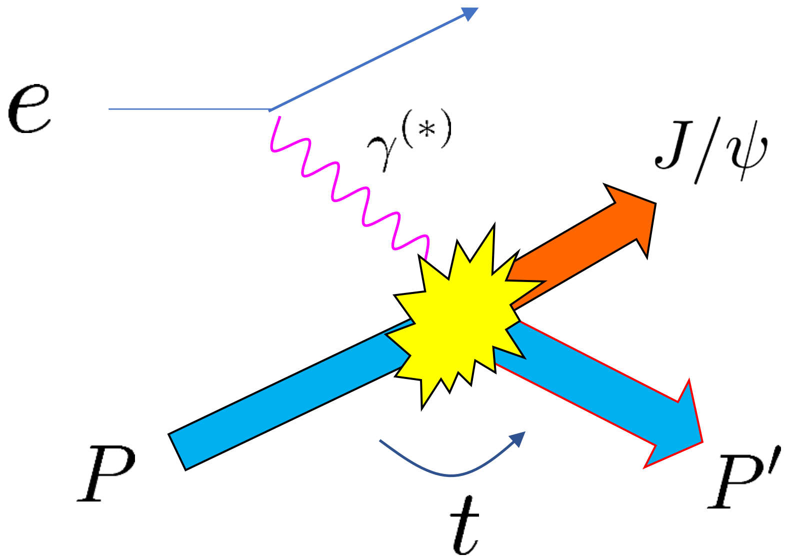
$$\mathrm{Re}\mathcal{H}_q(\xi,t) \;=\; \frac{1}{\pi}\int_{-1}^{1} dx \mathrm{P}\frac{\mathrm{Im}\mathcal{H}_q(x,t)}{\xi - x} + 2\int_{-1}^{1} dz \frac{D_q(z,t)}{1-z}$$

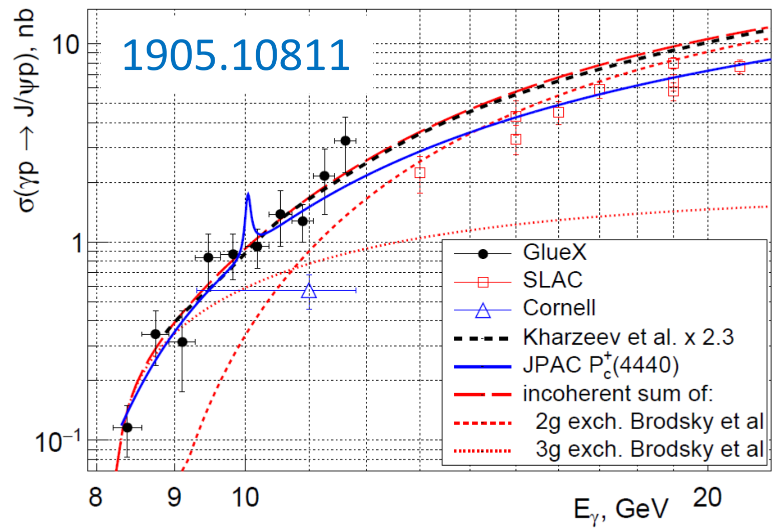
HOWEVER, this is not directly proportional to what we want $\int_{-1}^{1} dz z D_q(z,t) = D_q(t)$

The challenge for the community is to extract the spin-2 component (energy momentum tensor) from observables sensitive to all spins → large leverage in Q^2 required

What about gluon D-term?

Photo-production of J/ψ, Υ near threshold





Ongoing experiments at Jlab (GlueX)
Future measurement at EIC, EIcC? (also at RHIC?)

Main motivation: Insight into the origin of proton mass

Is the connection clear?
Can we make this precision science?

Near-threshold photo-production: theory approaches

Kharzeev, Satz, Syamtomov, Zinovjev (1998)

vector meson dominance + HQET, connection to gluon condensate

Brodsky, Chudakov, Hoyer, Laget (2001)

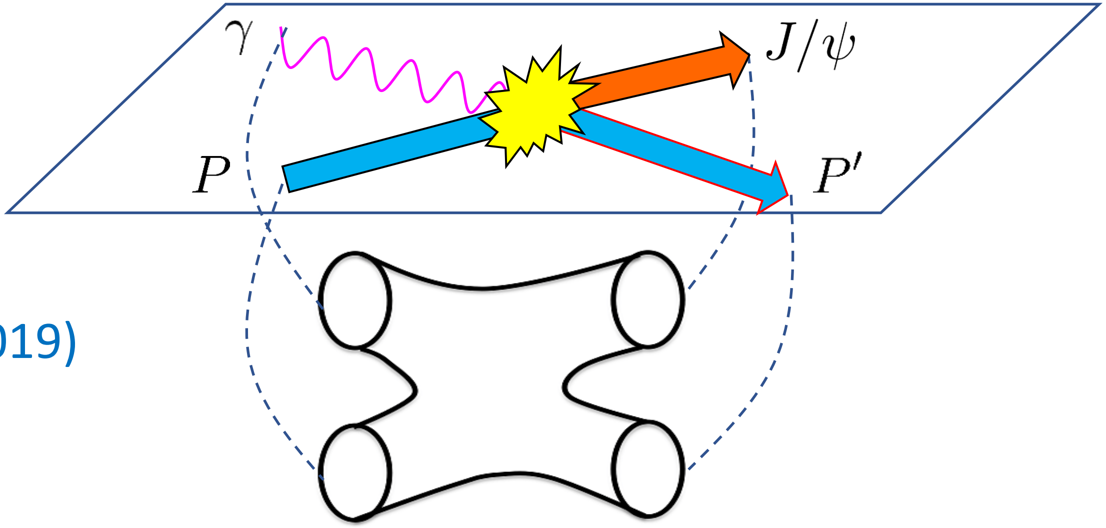
two-gluon, three-gluon exchanges

Frankfurt, Strikman (2002)

two-gluon form factors

YH, Yang (2018); Mamo, Zahed (2019)

AdS/CFT,
connection to gluon D-term

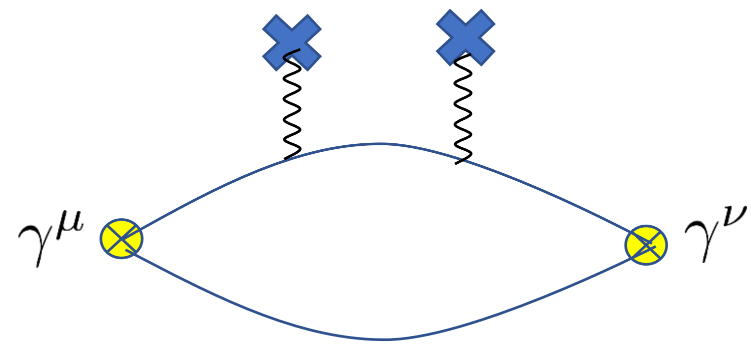


Caveat: None of these approaches are fully systematic

Leptoproduction at high-Q^2

Boussarie, YH (2020)

Use the local OPE between photon and J/psi interpolating operators



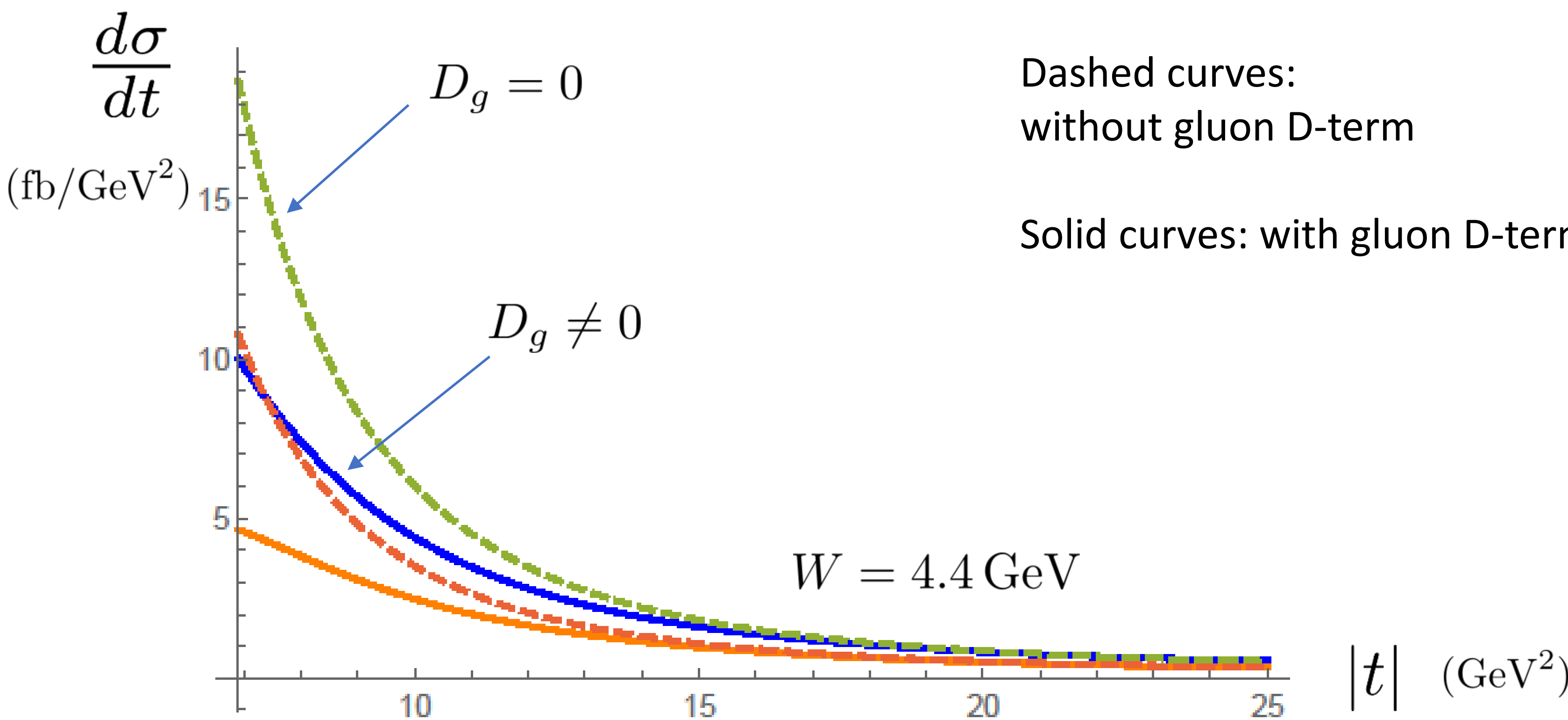
$$
\begin{aligned}
i \int d^4 r e^{ir\cdot q} \bar{c}\gamma^\mu c(0)\bar{c}\gamma^\nu c(-r) \\
\approx -\frac{\alpha_s(\mu_R)}{3\pi q^2}\Bigg[2\ln(-q^2/\mu_R^2)\Bigg\{\left(g^{\mu\alpha}-\frac{q^\mu q^\alpha}{q^2}\right)\left(g^{\nu\beta}-\frac{q^\nu q^\beta}{q^2}\right)+\frac{q^\alpha q^\beta}{q^2}\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right)\Bigg\}\hat{T}^g_{\alpha\beta}(0) \\
-2\frac{q^\alpha q^\beta}{q^2}\left(g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}\right)\hat{T}^g_{\alpha\beta}(0)+3\frac{q_\alpha q_\beta}{q^2}F^{\mu\alpha}F^{\nu\beta}(0)\Bigg],
\end{aligned}
$$

gluon EMT (traceless part)

Connection to gluon condensate, gluon D-term very clear

Twist-2, higher-spin operators are parametrically of the same order (the same problem as in the extraction of quark D-term in DVCS)

$J/\psi \quad Q^2 = 64\,\mathrm{GeV}^2 \quad \sqrt{S_{ep}} = 20\,\mathrm{GeV}$ (plots revised from 2004.12715)



Dashed curves:
without gluon D-term

Solid curves: with gluon D-term

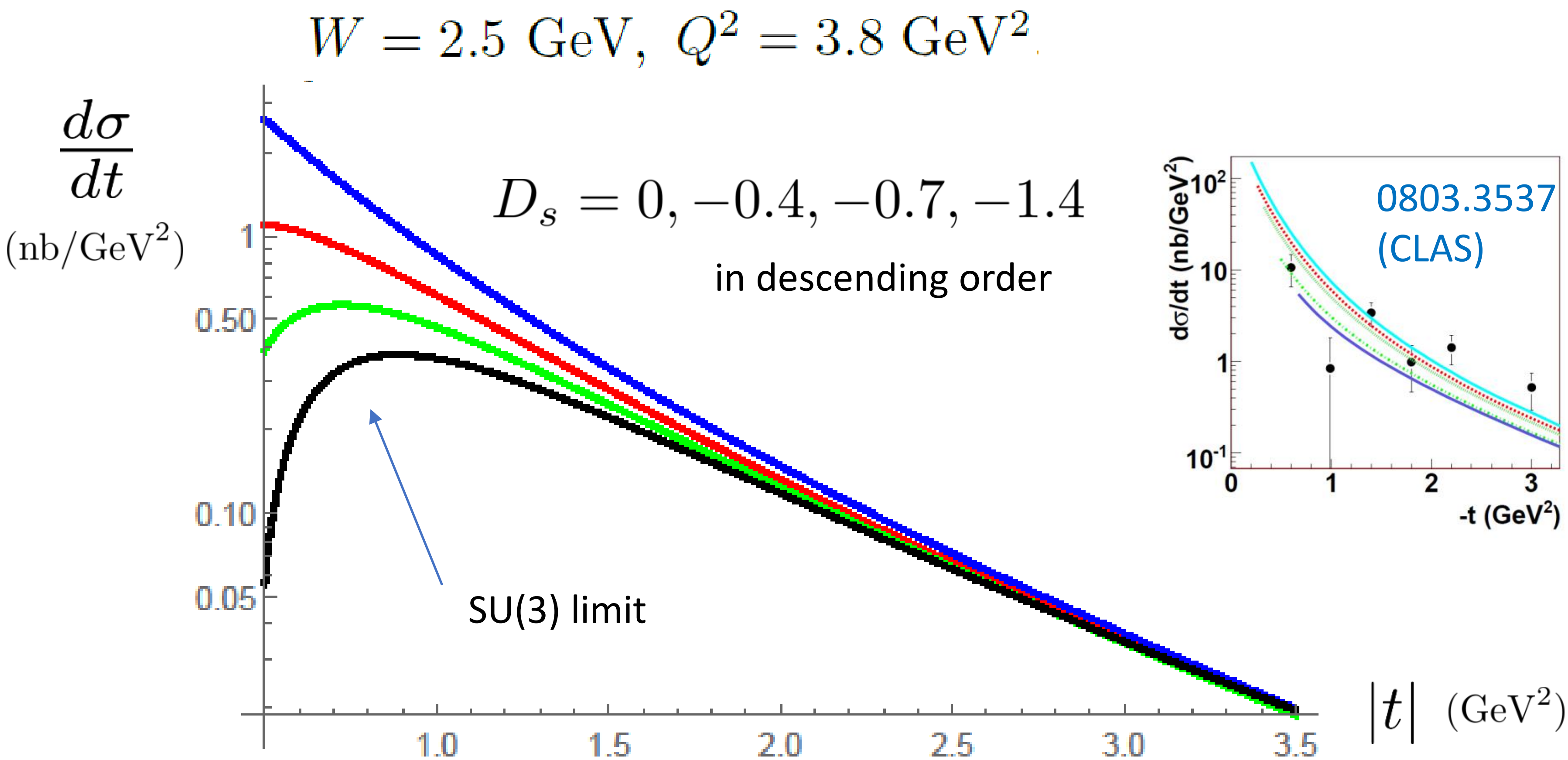
Upper solid $b = 1$

Lower solid $b = 0$

$$\langle P|\frac{\beta}{2g}F^2|P\rangle = 2M^2(1-b)$$

Strangeness D-term from ϕ -leptoproduction

YH, Strikman (2021)



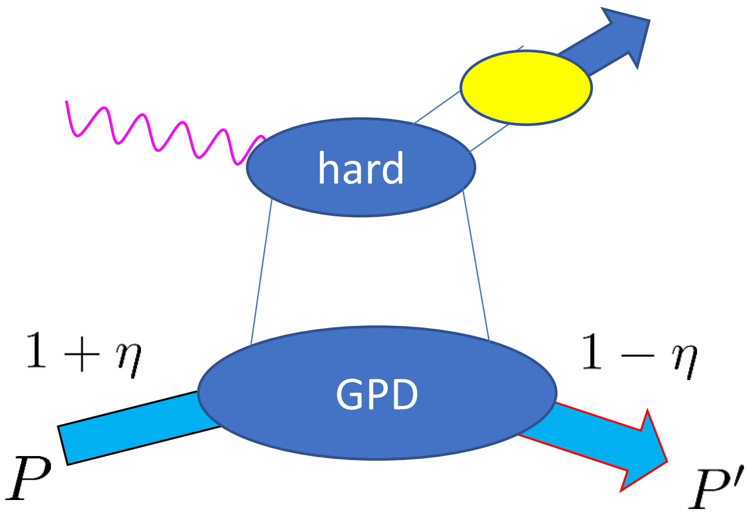
Possible flattening or bump in the $|t| < 1\,\text{GeV}^2$ region due to the strangeness D-term

QCD factorization for threshold production?

Light-cone dominance when $Q^2 \to \infty$ or $M_{QQ} \to \infty$ → GPD description?

OK at high energy, Collins, Frankfurt, Strikman (1996)
Ivanov, Schafer, Szymanowski, Krasnikov (2004)

but what about near the threshold?



Amplitude proportional to Compton form factor

$$\int_{-1}^{1} \frac{dx}{x} \left(\frac{1}{\eta - x - i\epsilon} - \frac{1}{\eta + x - i\epsilon} \right) H_g(x, \eta, t)$$

Gluon GPD

Skewness $\eta = \frac{P^+ - P'^+}{P'^+ + P^+}$

Near-threshold region corresponds to $x_B \approx \eta \approx 1$ Proton makes a full stop!

Back to PDF (GPD) framework, connection to EMT lost (or so it seems)

Energy momentum tensor strikes back

YH, Strikman (2021) (appendix)
Guo, Ji, Liu (2021)

If (and only if) $\eta \approx 1$, one can Taylor expand.

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots$$

spin=2 (energy momentum tensor)

$$\int_{-1}^{1} \frac{dx}{x} \left(\frac{1}{\eta - x - i\epsilon} - \frac{1}{\eta + x - i\epsilon} \right) H_g(x,\eta,t) \approx 2 \int dx (1 + x^2 + x^4 + \cdots) H_g(x,\eta,t)$$

spin=4 spin=6

Asymptotic form $H_g(x, \eta = 1) \approx (1-x^2)^2$

all spins $\displaystyle \int dx \frac{H_g(x,\eta=1,t)}{1-x^2} \sim \int_0^1 dx \frac{(1-x^2)^2}{1-x^2} = \frac{2}{3}$

spin-2 only $\displaystyle \int_0^1 dx (1-x^2)^2 = \frac{8}{15}$ ← 80% of the total (100% in AdS/CFT)

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Boussarie, YH (2020)

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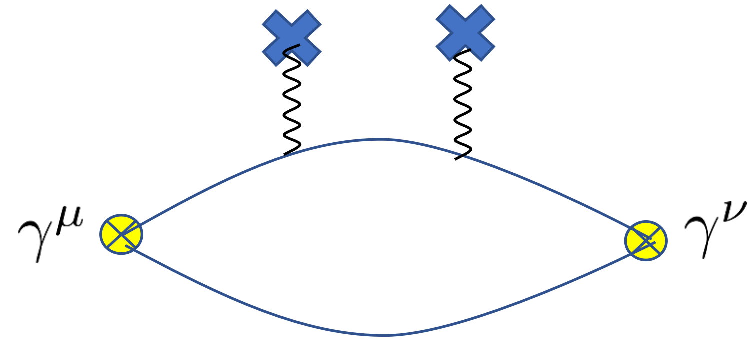

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- 2\frac{q^\alpha q^\beta}{q^2}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\hat{T}^g_{\alpha\beta}(0) + 3\frac{q_\alpha q_\beta}{q^2} F^{\mu\alpha}F^{\nu\beta}(0) \Bigg],
\end{aligned}
$$

gluon EMT (traceless part)

☺ Connection to gluon condensate, gluon D-term very clear

☺ Twist-2, higher-spin operators may be small and under control (this is not the case for the extraction of quark D-term in DVCS)

Conclusions

Threshold quarkonium production is a new, exciting frontier.
Can be studied at Jlab, RHIC, EIC…

Recently, a possible breakthrough paving the way for 1^{st}-principle calculations

Connection to GPDs at $\eta = 1$, largely unexplored.
Energy momentum tensor dominates over all the other twist-2 operators combined.

Unique opportunity to probe gluon gravitational form factors, D-term and trace anomaly (see however, Du et al, (2020); Sun, Tong, Yuan (2021))