Extraction of Transverse momentum dependent distributions

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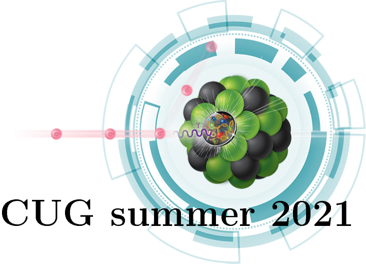
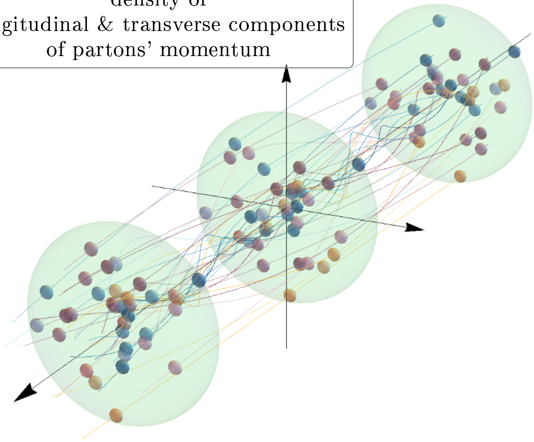


EICUG summer 2021 meeting

Transverse momentum dependent = TMD

TMD distributions measure density of longitudinal & transverse components of partons' momentum



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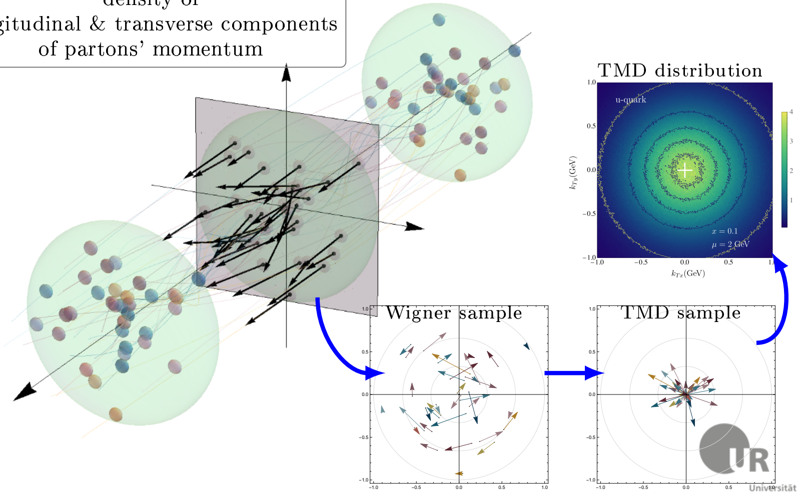

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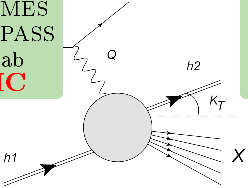
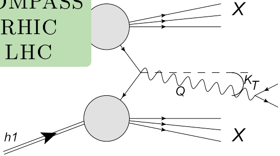
TMD factorization describes p_T-spectrum of the "double-inclusive processes" at small-p_T, where the transverse momentum is dominantly generated by the intrinsic motion of partons.

HERMES
COMPASS
JLab
EIC



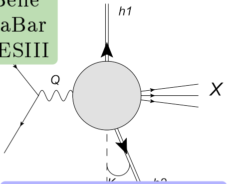
SIDIS

FermiLab
COMPASS
RHIC
LHC



Drell-Yan

Belle
BaBar
BESIII



$e^+e^- \to h_1 + h_2 + X$

q is momentum of initiating EW-boson
$q^2 = \pm Q^2$
q_T^μ transverse component

$$\begin{cases} Q^2 \gg \Lambda^2_{QCD} \\ Q^2 \gg q_T^2 \end{cases}$$

TMD factorization formula (in ζ-prescription)

Rapidity anomalous dimension (**CS-kernel**)

TMDs
$F \sim \langle P|\bar{q}[\text{staple link}]q|0\rangle$

$$\boxed{\frac{d\sigma}{d\text{PS}} = \sum_{ff'} H_{ff'}\left(\frac{Q}{\mu}\right) \int d^2 b\, e^{i(\mathbf{b}\cdot\mathbf{q}_T) - 2\mathcal{D}(b,\mu)\ln\left(\frac{Q^2}{\zeta_Q}\right)} F_{f\leftarrow h}(x_1, b) F_{f'\leftarrow h}(x_2, b)}$$

- Formulated in position space (evolution!)
- Each data-point is a convolution of **three independent nonperturbative** functions
- Each function is responsible for a separate kinematic variable

DY

$d\text{PS} = dQ\, dq_T\, dy$

$\mathcal{D}(b,\mu)$ $\quad$ $F(x,b)$

SIDIS

$d\text{PS} = dQ\, dq_T\, dx\, dz$

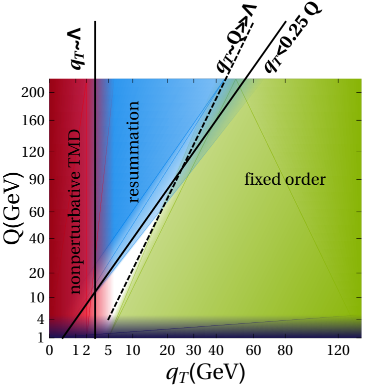
$\mathcal{D}(b,\mu)$ $\quad$ $F(x,b)$ $\quad$ $D(z,b)$

Kinematic region of TMD factorization

Factorization regions

$q_T \lesssim 0.25Q$ TMD factorization $= \begin{cases} q_T \lesssim \Lambda & \text{nonpertrubative regime} \\ q_T \gg \Lambda & \text{"resummation" regime} \end{cases}$

$q_T \sim Q \gg \Lambda$ collinear factorization



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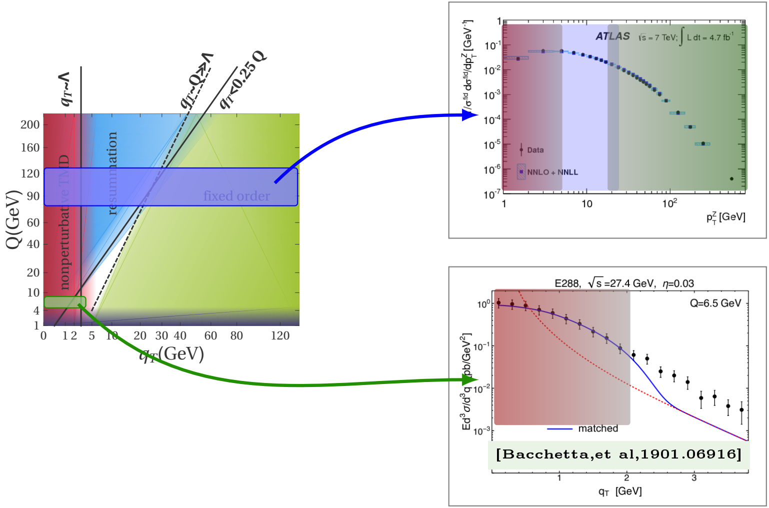
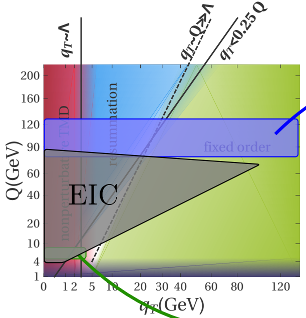
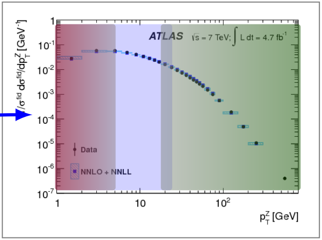


Currently the commonly used cut is $q_T < 0.25Q$

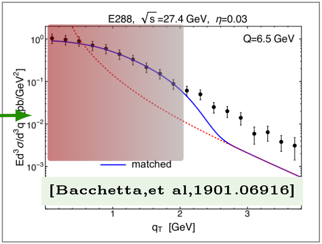
To push it further one needs power corrections

[Balitsky,17-21; Beneke, et al, 17-18; Ebert, et al, 20; ...]





EIC will perfect place
to investigate
the transition region



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To push it further one needs power corrections
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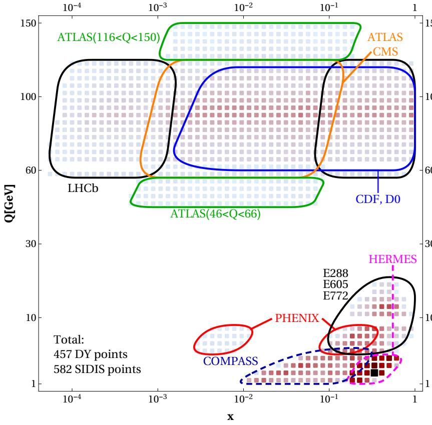
Extraction of a TMD requires many experiments

Data points for unpolarized TMDPDF



- $2 < Q < 150$GeV
- Drell-Yan (LHC,Tevatron, RHIC, fix targets)
- SIDIS (Hermes, Compass, JLab)
- SIA (Belle, Babar)

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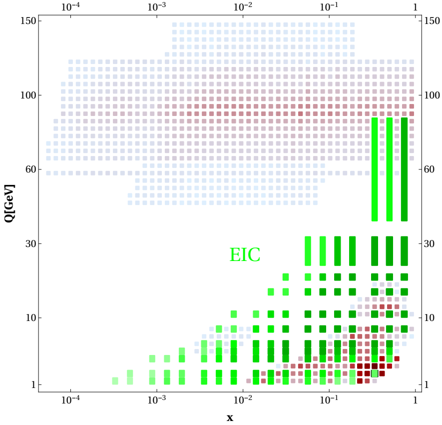


- $2 < Q < 150$GeV
- Drell-Yan (LHC,Tevatron, RHIC, fix targets)
- SIDIS (Hermes, Compass, JLab)
- SIA (Belle, Babar)
- **EIC** will cover the gap

Many different TMDs already at the leading power
Multiple physics topics (some are not transparent)

CS-kernel

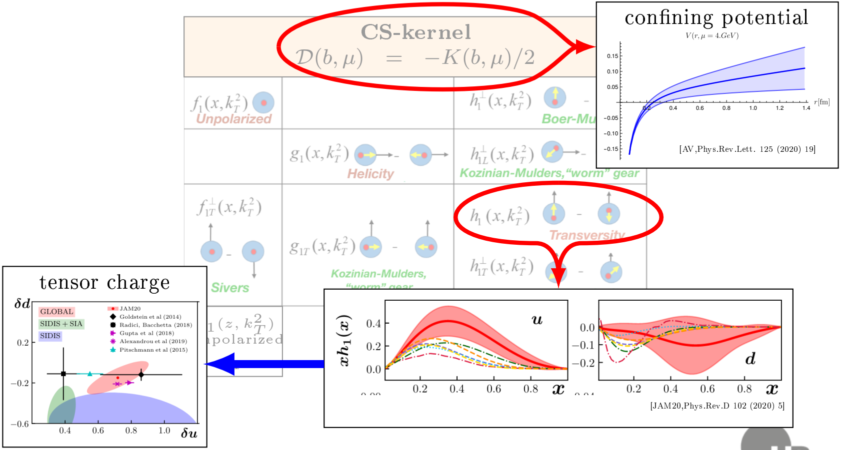
$$\mathcal{D}(b,\mu) \quad = \quad -K(b,\mu)/2$$

$f_1(x,k_T^2)$ Unpolarized		$h_1^\perp(x,k_T^2)$ - Boer-Mulders
	$g_1(x,k_T^2)$ - Helicity	$h_{1L}^\perp(x,k_T^2)$ - Kozinian-Mulders,"worm" gear
$f_{1T}^\perp(x,k_T^2)$ - Sivers	$g_{1T}(x,k_T^2)$ - Kozinian-Mulders, "worm" gear	$h_1(x,k_T^2)$ - Transversity $h_{1T}^\perp(x,k_T^2)$ - Pretzelosity
$D_1(z,k_T^2)$ unpolarized FF		$H_1^\perp(z,k_T^2)$ Collins FF

Nowadays,
we know something
about every TMD.

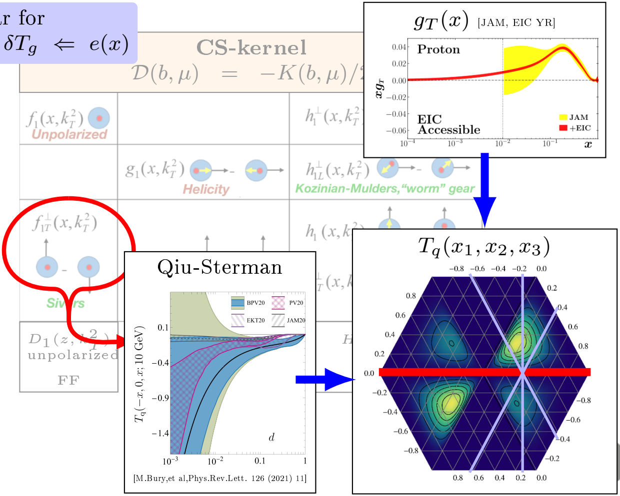
What can we learn from TMDs?

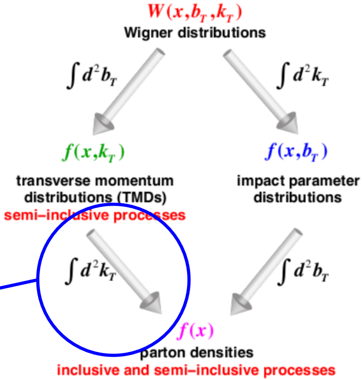
Many different TMDs already at the leading power
Multiple physics topics (some are not transparent)



Many different TMDs already at the leading power
Multiple physics topics (some are not transparent)

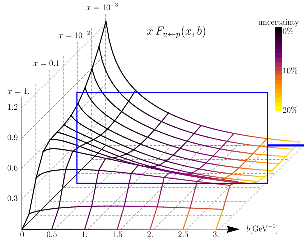
Similar for
$h_1^{\perp}(x,p_T) \Rightarrow \delta T_g \Leftarrow e(x)$

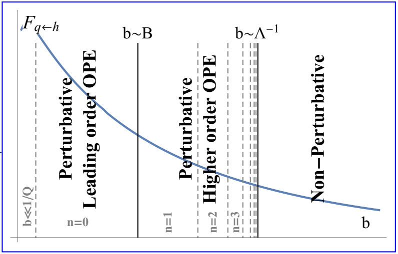




$$\lim_{b \to 0} f(x, b) = f(x)$$

TMD distributions are nonperturbative 3D functions
However, they match 1D PDFs at $b \to 0$ boundary





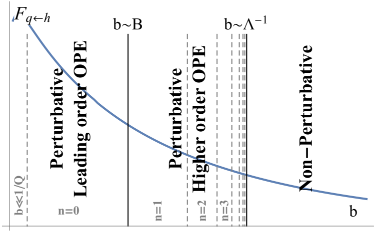
$$F(x,b) = \underbrace{\left[q(x) + \alpha_s\left(p(x)\ln(b^2\mu^2) + ...\right) + \alpha_s^2 ...\right]} + b^2 ... + ...$$

Lead.power OPE
up N^{3}LO

Higher power OPE
e.g. [V.Moos,AV,2008.01744]

$$\boxed{F(x,b) = C(x,b) \otimes q(x) f_{NP}(x,b)}$$

The theory side of the TMD→PDF matching is known very accurately



Great progress in last 2-3 years

- TMD evolution (CS kernel) **N^{3}LO** [AV,17; Li,Zhu,17]
- unpolarized TMDs at **N^{3}LO** [Ebert,et al,20; Luo,et al,19]
- twist-3/4 computations

table from [Moos,AV,20]

Name	Function	Twist of leading matching	Twist-2 distributions in matching	Twist-3 distributions in matching	Order of leading power coef.function	Ref.
unpolarized	$f_1(x,b)$	tw-2	$f_1(x)$	–	N^{3}LO (α_s^3)	[21, 22]
Sivers	$f_{1T}^{\perp}(x,b)$	tw-3	–	$T(-x,0,x)$	NLO (α_s^1)	[23]
helicity	$g_{1L}(x,b)$	tw-2	$g_1(x)$	$\mathcal{T}_g(x)$	NLO (α_s^1)	[16, 17]
worm-gear T	$g_{1T}(x,b)$	tw-2/3	$g_1(x)$	$\mathcal{T}_g(x)$	LO (α_s^0)	[13, 14]
transversity	$h_1(x,b)$	tw-2	$h_1(x)$	$\mathcal{T}_h(x)$	NNLO (α_s^2)	[19]
Boer-Mulders	$h_1^{\perp}(x,b)$	tw-3	–	$\delta T_\epsilon(-x,0,x)$	LO (α_s^0)	[14]
worm-gear L	$h_{1L}^{\perp}(x,b)$	tw-2/3	$h_1(x)$	$\mathcal{T}_h(x)$	LO (α_s^0)	[13, 14]
pretzelosity	$h_{1T}^{\perp}$	tw-3/4	–	$\mathcal{T}_h(x)$	LO (α_s^0)	eq.(4.8)

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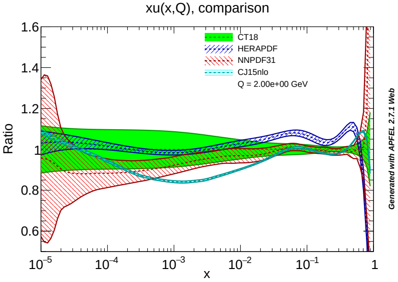
Matching to PDF leads to high predictive power, but is also a pitfall →



Result of a TMD fit is 100% dependent on PDF in use!

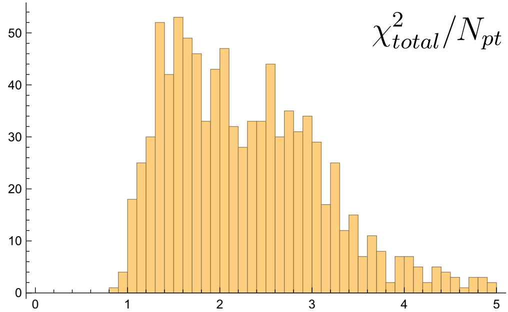
- Different PDF set are different
  - Especially in a "TMD-important" region $x \sim 0.1 - 0.7$
  - Different flavor decomposition
- As the result: SIDIS+DY fit [SV19]

PDF & FF sets	χ^2/N_{pt}
HERA20 & DSS	0.76
HERA20 & JAM19	0.93
NNPDF31 & DSS	1.00
NNPDF31 & JAM19	1.65
HERA20 & DSS (N^3LO)	0.88
NNPDF31 & DSS (N^3LO)	1.31

Obviously, one must include PDF uncertainty into the fit

Including PDF uncertainty "straightforwardly"

- PDF uncertainty is **larger** than the experimental precision
  - e.g. LHC 5-7% vs. 1%
- TMD factorization formula (in comparison to DIS) is sensitive to different x-domain
- **Strongly depends on the PDF set**



$\leftarrow$ SV19 fit
NNPDF+DSS
SIDIS+DY
$N_{pt} = 1039$
fit made for central replica
$\leftarrow$ distribution of χ^2 for 1000 replicas of NNPDF

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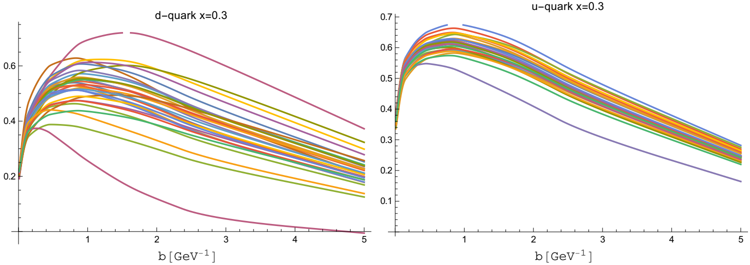
PDF essentially changes behavior of TMD = **PDF-bias**



SV19 fit, 40 random replicas of NNPDF3.1 (same f_{NP})

f_{NP} must be fit for each PDF replica
f_{NP} must be flavor dependent
In some sense, f_{NP} should **capable to compensate issues of PDFs**
So, together they form a TMD distribution with uncertainty band

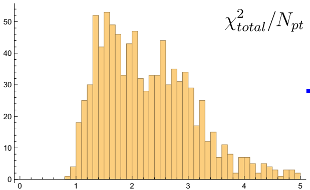
Flavor dependence of unpolarized TMD

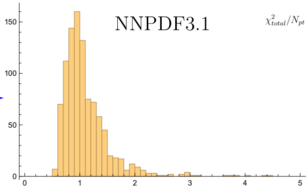
- Complecated profile
- 5 parameters
- no-flavor dependance

- Simple profile
- Flavor dependent
- 2 parameters per flavor (+1 global)

$$f_{NP}(x,b) = \exp\left(-\frac{\lambda_1(1-x)+\lambda_2 x + x(1-x)\lambda_5}{\sqrt{1+\lambda_3 x^{\lambda_4}\boldsymbol{b}^2}}\boldsymbol{b}^2\right)$$

$$f_{NP}(x,b) = \exp\left(-\frac{\lambda_1(1-x)+\lambda_2 x}{\sqrt{1+\lambda_0 x^2 \mathbf{b}^2}}\mathbf{b}^2\right)$$





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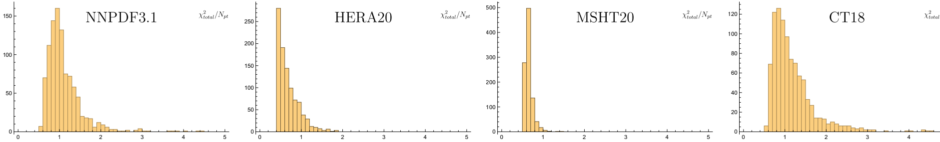
Flavor dependence of unpolarized TMD

DY only (457 points)
SV19 model

input PDF .	χ^2/N_{pt}
HERA20	0.97
NNPDF31	1.14
CT18	1.26
MSHT20	1.39

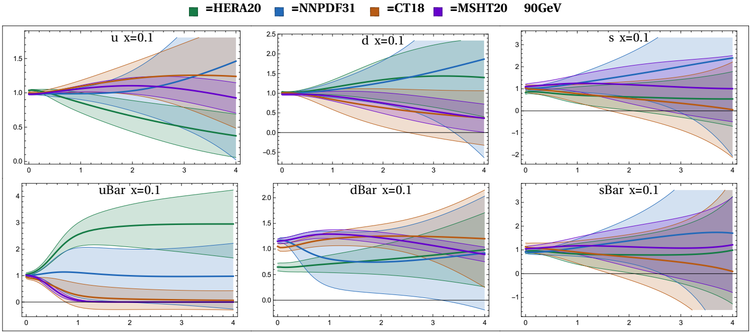
DY only (457 points)
flavor-dependent model

input PDF .	χ^2/N_{pt}
HERA20	0.90
NNPDF31	0.97
CT18	0.98
MSHT20	0.89



Similar fit-quality for different PDFs!

Towards realistic uncertainty band for unpolarized TMD

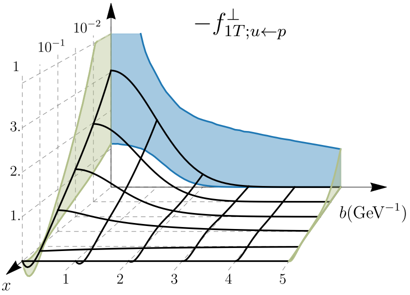


The final uncertanty band is $\sim \times 5$ original SV19

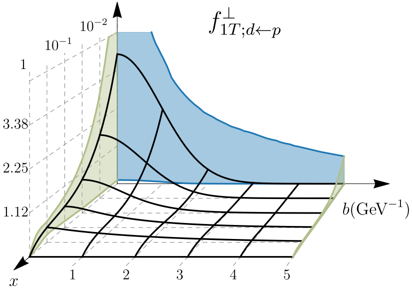
Future direction: simultaneous fit of TMDs and PDFs

Despite PDF-bias problem small-b matching essentially reduce parametric freedom.
It is not so for non-unpolarized TMDs, where collinear counter parts are unknown
In this case, one can use only the data as input (or models)
Example: Sivers function

u-quark



d-quark

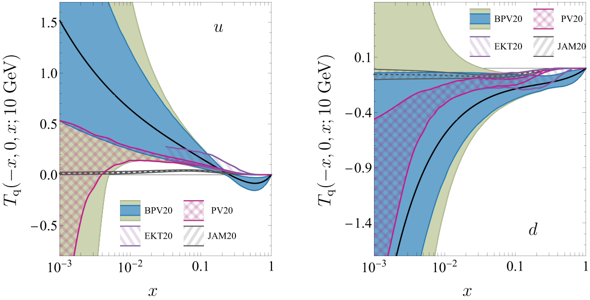


- Notably huge uncertainties
- Can be used to determine collinear part $\Rightarrow$ Qiu-Sterman function

$$b = 0.11/\mathrm{GeV} \Rightarrow \mu = 10\mathrm{GeV}$$

$$T_q(-x,0,x;\mu_b) = -\frac{1}{\pi}\left(1 + C_F a_s(\mu_b)\frac{\pi^2}{6}\right) f^{\perp}_{1T;q\leftarrow h}(x,b)$$
$$-\frac{a_s(\mu_b)}{\pi}\int_x^1 \frac{dy}{y}\left[\frac{\bar y}{N_c} f^{\perp}_{1T;q\leftarrow h}\left(\frac{x}{y},b\right) + \frac{3y^2\bar y}{2x} G^{(+)}\left(-\frac{x}{y},0,\frac{x}{y};\mu_b\right)\right] + \mathcal{O}(a_s^2) + \mathcal{O}(b^2)$$

$$\boxed{G^{(+)} = \pm(|T_d| + |T_u|)}$$



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Extracting Qiu-Sterman function

$$b = 0.11/\text{GeV} \Rightarrow \mu = 10\text{GeV}$$

$$T_q(-x,0,x;\mu_b) = -\frac{1}{\pi}\left(1 + C_F a_s(\mu_b)\frac{\pi^2}{6}\right) f_{1T;q\leftarrow h}^{\perp}(x,b)$$

$$-\frac{a_s(\mu_b)}{\pi}\int_x^1 \frac{dy}{y}\left[\frac{\bar{y}}{N_c} f_{1T;q\leftarrow h}^{\perp}\left(\frac{x}{y},b\right) + \frac{3y^2\bar{y}}{2x} G^{(+)}\left(-\frac{x}{y},0,\frac{x}{y};\mu_b\right)\right] + \mathcal{O}(a_s^2) + \mathcal{O}(b^2)$$

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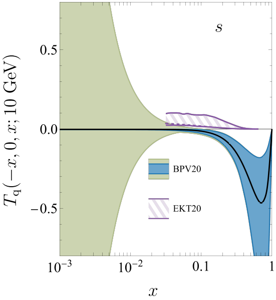

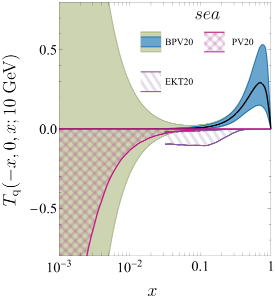

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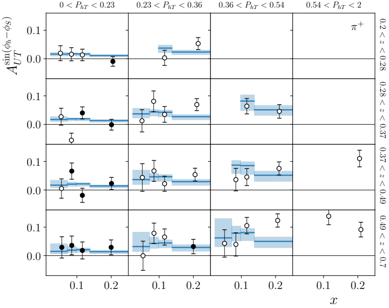
Discussion point: anti-quark Sivers function ∼ quark Sivers function

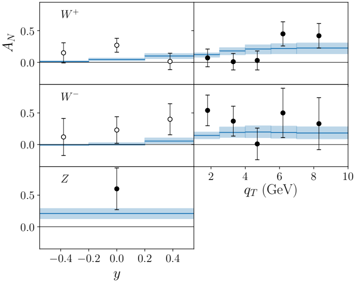
- describes SIDIS/DY puzzle
- Follows from models (e.g. [V.Braun, et al, 09])
- Not forbidden by intuition (Sivers is qgq-interference term)

HERMES π^+



SSA in SIDIS ∼ 1-5%

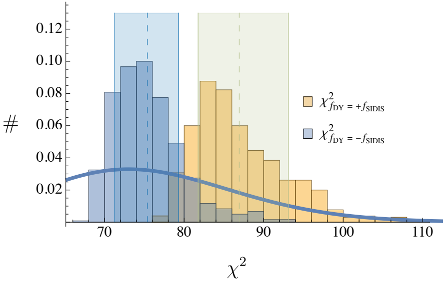
STAR $W^{\pm}/Z$

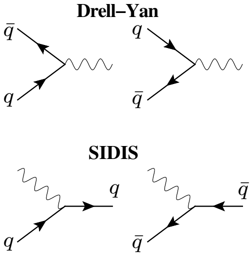


SSA in DY ∼ 10-30%

Discussion point: anti-quark Sivers function $\sim$ quark Sivers function
$\Rightarrow$ one cannot confirm Sivers sign-change

$$f_{1T}^{\perp}(SIDIS) = -f_{1T}^{\perp}(DY)$$





$$f_{1T}^{\perp}(sea) \to -f_{1T}^{\perp}(sea)$$

More accurate data is needed

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Conclusions

The TMD factorization is vastly expanding topic

- On-going theory development
- Increasing detailing of phenomenology
- More and more connections to other QCD topics

Collinear PDF ⇔ TMDs interplay

- TMD extractions are sensitive to collinear physics
- Unpolarized TMDs could reduce the precision of PDFs
- Polarized TMDs potentially more precise information for higher-twist distributions

A model-independent extraction of TMD distributions needs accurate data (EIC, JLab12)