Recent progress in the lattice QCD calculation of parton physics

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Argonne NATIONAL LABORATORY

Outline

- Parton distribution functions (PDFs)
- Generalized parton distributions (GPDs)
- Transverse-momentum-dependent (TMD) distributions

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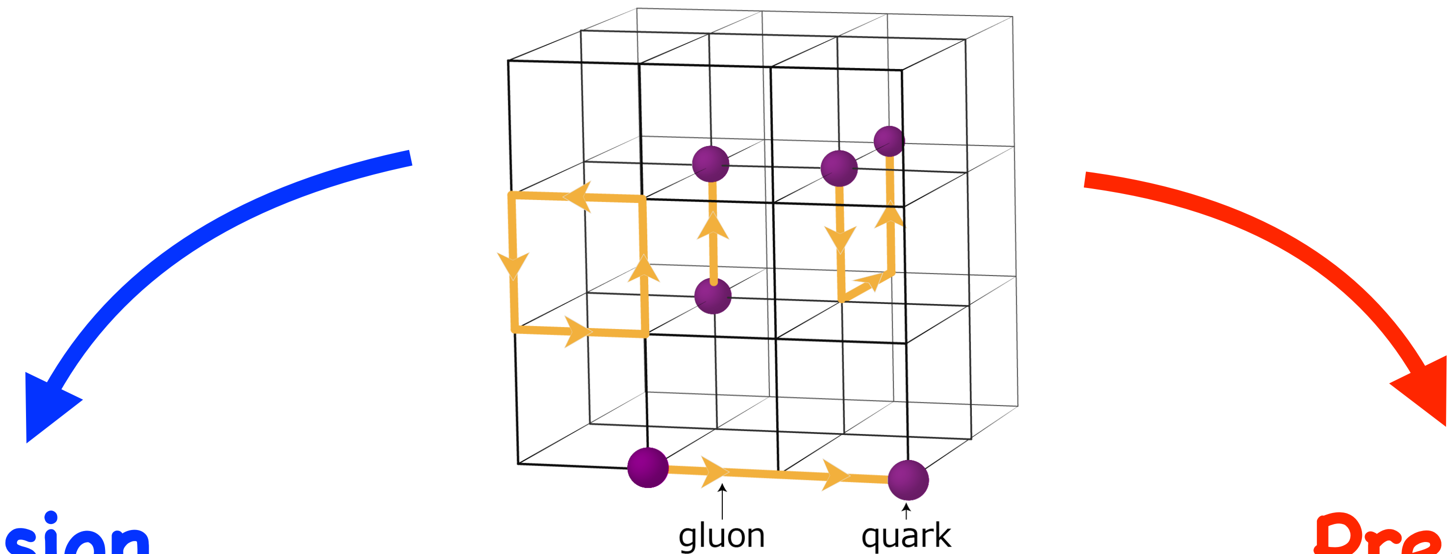
Parton distribution functions (PDFs)



Precision

Prediction

Controlling the systematic errors:

- Lattice systematics:
  - excited-state contamination,
  - $a \rightarrow 0$ (lattice renormalization),
  - physical m_π,
  - $L \rightarrow \infty$;
- Higher-order perturbative matching and resummation;
- Power corrections, range of reliable x.

Guiding information for the less-knowns:

- Sea quark;
- Spin-dependent PDFs, e.g., transversity;
- Gluon PDFs;
- Distribution amplitudes;
- Higher-twist PDFs;
- GPDs;
- TMDs.

Large-momentum effective theory (LaMET)

- The quasi-PDF:

X. Ji, PRL 110 (2013); SCPMA57 (2014).
X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, arXiv: 2004.03543

$$\tilde{f}(y, P^z, \mu) = \int_{-\infty}^{\infty} \frac{d\lambda}{4\pi} e^{iy\lambda}\, \tilde{h}(\lambda = zP^z, P^z, \mu) \equiv \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{iy\lambda} \langle P \,|\, \bar{\psi}(z)\Gamma W[z,0]\psi(0) \,|\, P \rangle$$

$$f(x, \mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) \tilde{f}(y, P^z, \mu) \; + \; \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

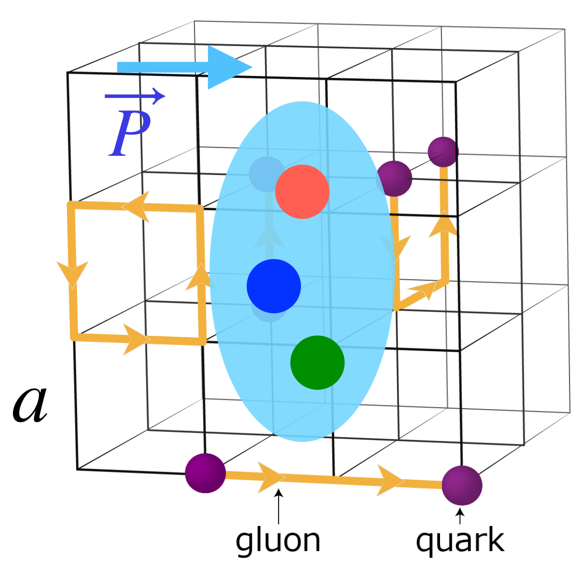
- X. Xiong, X. Ji, J.-H. Zhang and YZ, PRD 90 (2014);
- Y.-Q. Ma and J. Qiu, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and YZ, PRD98 (2018).

- Direct calculation of the x-dependence through a large-momentum expansion;
- Precision under control for $x \in [x_{min}, x_{max}]$.

Large-momentum effective theory (LaMET)

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) \tilde{f}(y, P^z, \mu) \;+\; \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

$\tilde{f}(x, P^z, a)$

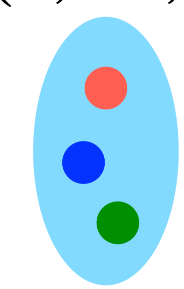


$P^z \gg \Lambda_{\rm QCD}$

Renormalization

$a \to 0$

$\tilde{f}(x, P^z, \mu)$



Perturbative matching and power corrections

$f(x, \mu)$

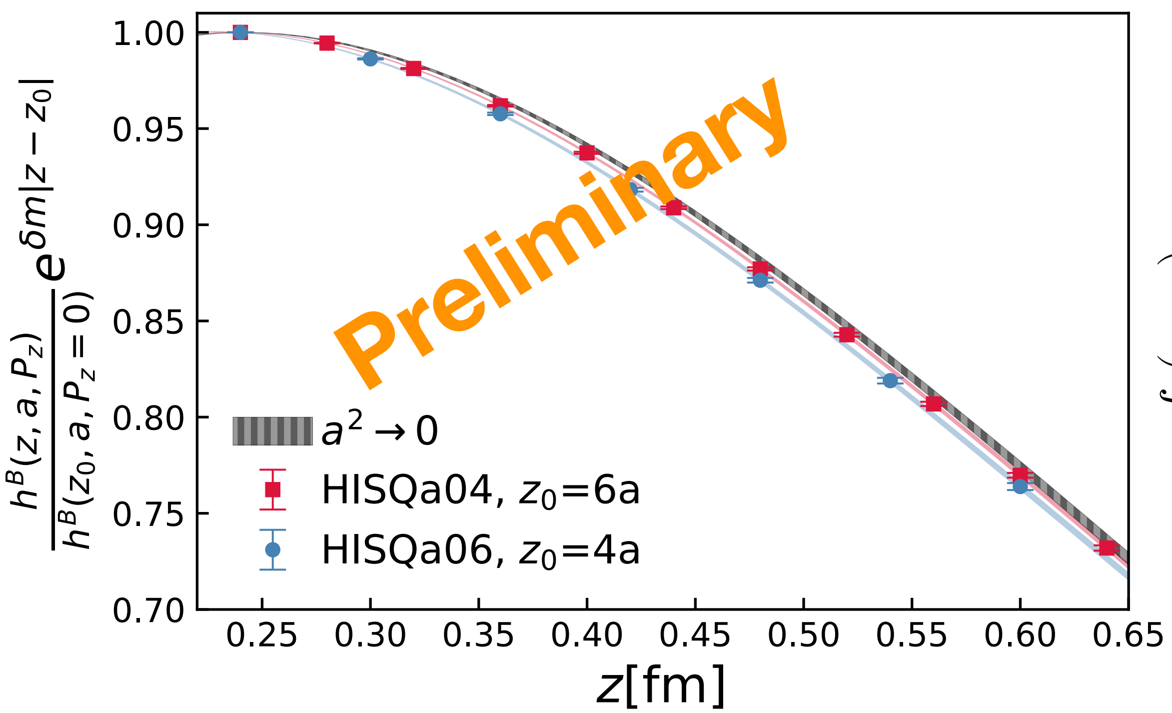
- excited-state contamination ✔
- physical m_π ✔
- L→∞ ✔ negligible

- Renormalization schemes:
  - RI/MOM, higher-twist effects at large z
  - Ratio, higher-twist effects at large z
  - RI/xMOM, cutoff effects at small z
  - Hybrid scheme, ✔
- Continuum extrapolation with multiple lattice spacings, ✔

- NNLO matching for non-singlet quark PDFs, ✔
- NLO matching for the other PDFs, ✔
- Kinematic target mass correction, ✔
- Dynamical higher-twist suppressed by P^z.

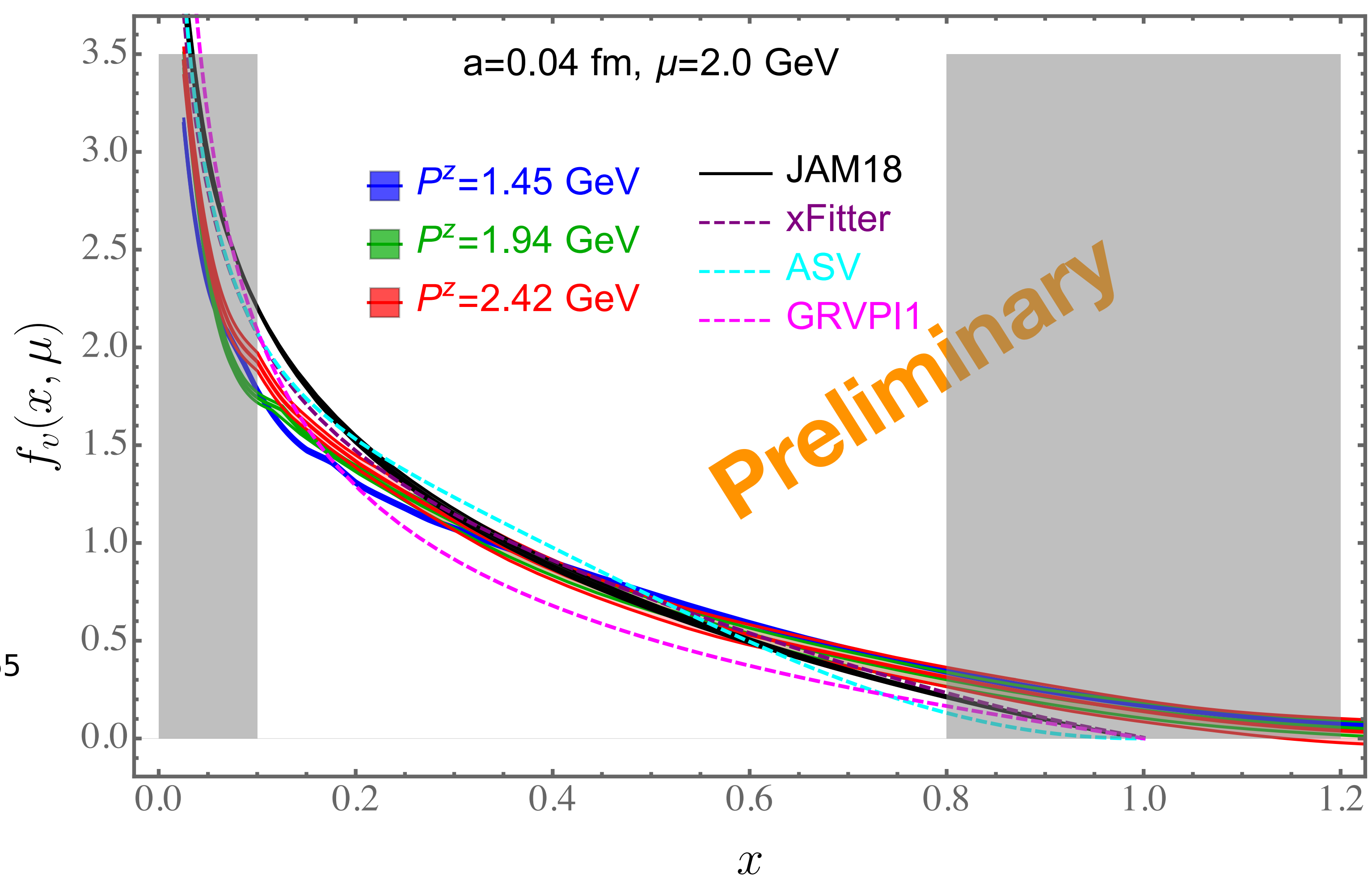
Pion valence PDF from hybrid scheme w. NNLO matching

Continuum extrapolation



a=0.06 fm and 0.04 fm

$m_\pi = 300$ MeV



Talk by YZ at 38th International Symposium on Lattice Field Theory

Gray band (preliminary):

Error at P^z=2.42 GeV > 15% or variation in P^z > 1% between P^z=1.94 and 2.42 GeV.

Coordinate-space factorization approaches

- Short distance expansion of
    - Current-current correlator
        - V. Braun and D. Mueller, Eur.Phys.J.C 55 (2008) 349
        - Y.-Q. Ma and J. Qiu, Phys.Rev.Lett. 120 (2018) 2, 022003
    - The equal-time quark bilinear (pseudo-PDF)

$$\tilde{h}(\lambda, z^2\mu^2) = \int_0^1 d\alpha\ \mathscr{C}(\alpha, z^2\mu^2)\ h(\alpha\lambda, \mu) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2)$$
$$= \int_0^1 d\alpha\ \mathscr{C}(\alpha, z^2\mu^2) \int dx\ e^{ix\alpha\lambda} f(y, \mu^2) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2)$$

- A. Radyushkin, Phys.Rev.D 96 (2017) 3, 034025
- K. Orginos, et al., Phys.Rev.D 96 (2017) 9, 094503

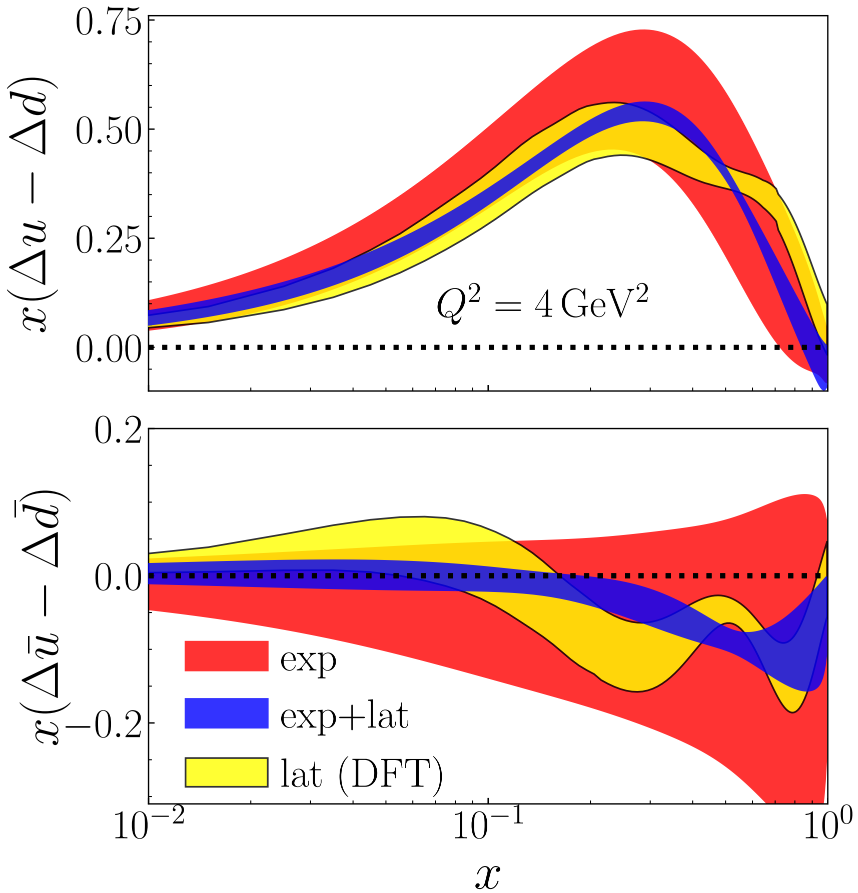
- Requirement for short z limits the largest λ, difficult to control the Fourier transform;

- Extraction of the PDF relies on parameterization/modeling of the PDFs.

Coordinate-space factorization approaches

$$\tilde{h}(\lambda, z^2\mu^2) = \int_0^1 d\alpha\, \mathscr{C}(\alpha, z^2\mu^2) \int dx\, e^{ix\alpha\lambda} f(y, \mu^2) \;+\; \mathcal{O}(z^2\Lambda^2_{\text{QCD}})$$

Proton isovector sea quark PDF



J. Bringewatt, N. Sato et al., Phys.Rev.D 103 (2021) 1, 016003

"Confronting lattice parton distributions with global QCD analysis",

J. Bringewatt, N. Sato et al., Phys.Rev.D 103 (2021) 1, 016003

"Neural-network analysis of Parton Distribution Functions from Ioffe-time pseudodistributions",

L. Del Debbio, T. Giani et al., JHEP 02 (2021) 138

Challenge in controlling the power corrections and model dependence:

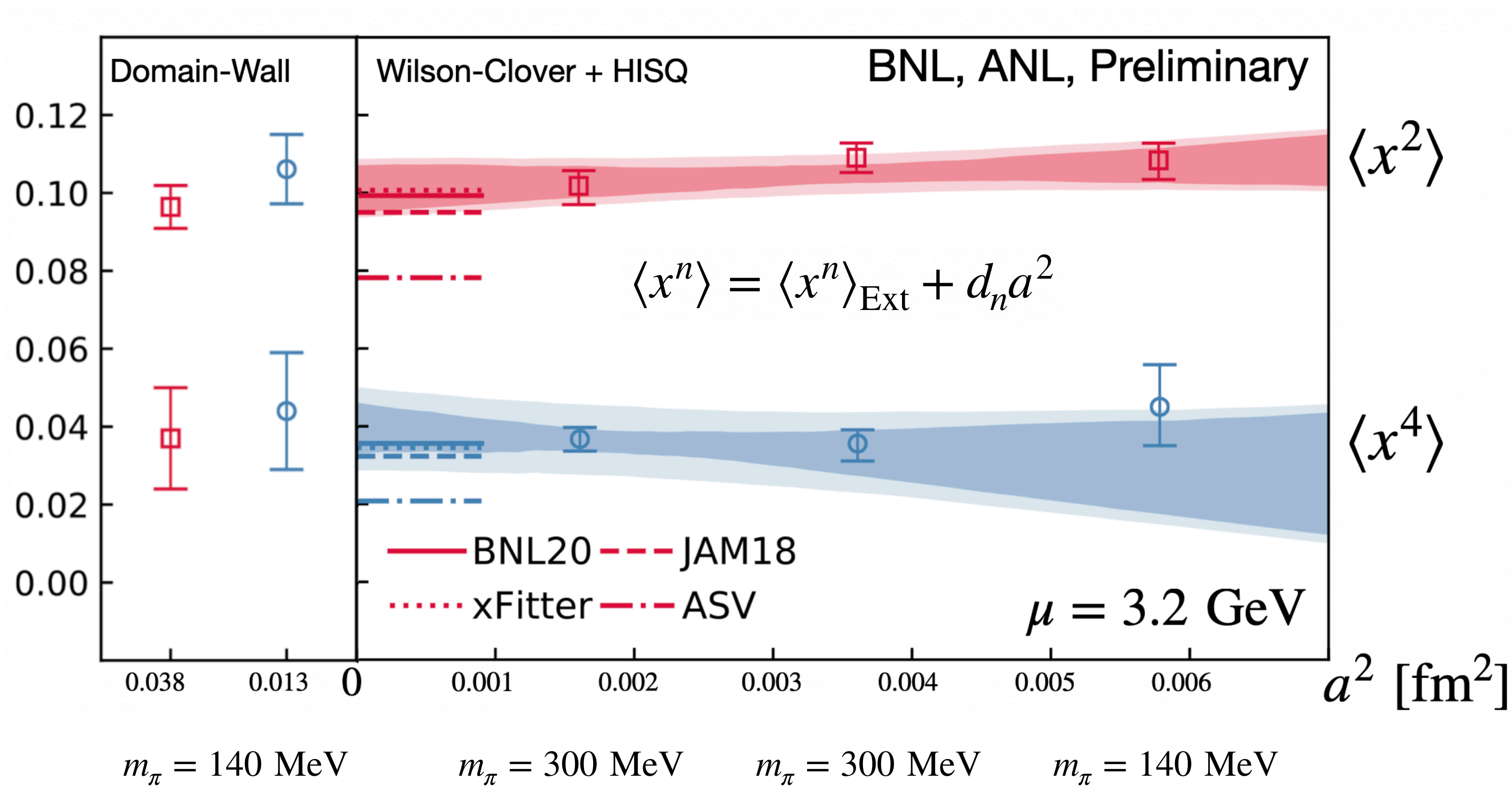
- $1/z^2$ corresponds to Q^2 in DIS and DY processes;
- To suppress lattice cutoff effects, z >> a ($z \geq 3a$?). In the literature, z_{max}=0.5-0.8 fm has been used, which corresponds to 0.08 GeV^2 < $1/z^2$ < 2.70 GeV^2 for a=0.04 fm, lower than most experimental cuts in Q^2.

Coordinate-space factorization approaches

- Model-independent determination of (higher) Mellin moments from operator product expansion (OPE):

$$\tilde{h}(\lambda, z^2\mu^2) = \sum_{n=0}^{\infty} \frac{(-i\lambda)^n}{n!} C_n\left(z^2\mu^2\right) a_n(\mu) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2)$$

Moments of the pion valence quark PDF with NNLO Wilson coefficients



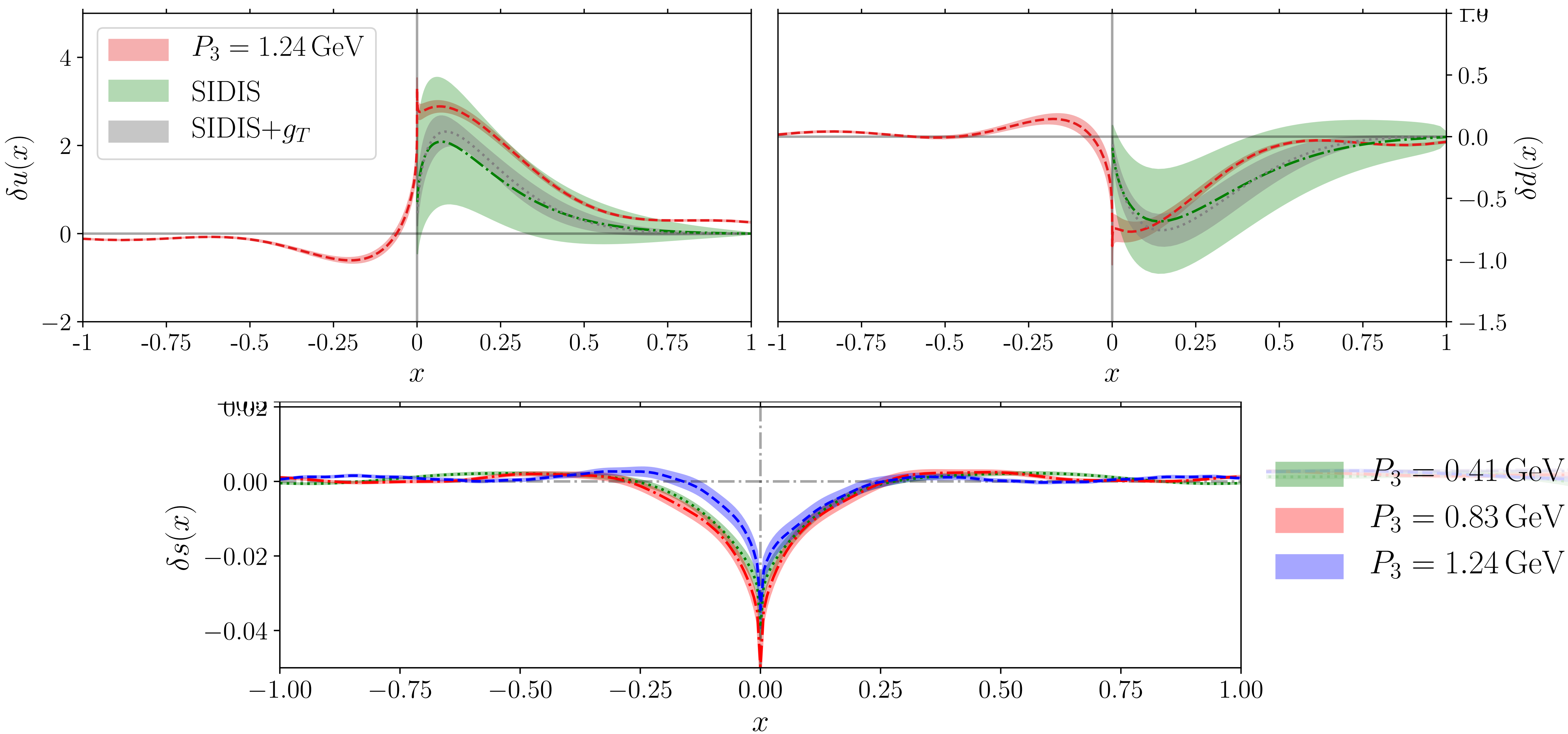
Xiang Gao's talk at GHP Workshop 2021.

Lattice calculation of less-knowns

- Proton transversity PDFs with flavor decomposition:

$a = 0.094$ fm, $L = 3.0$ fm, $m_\pi = 260$ MeV

RI/MOM renormalization and NLO matching

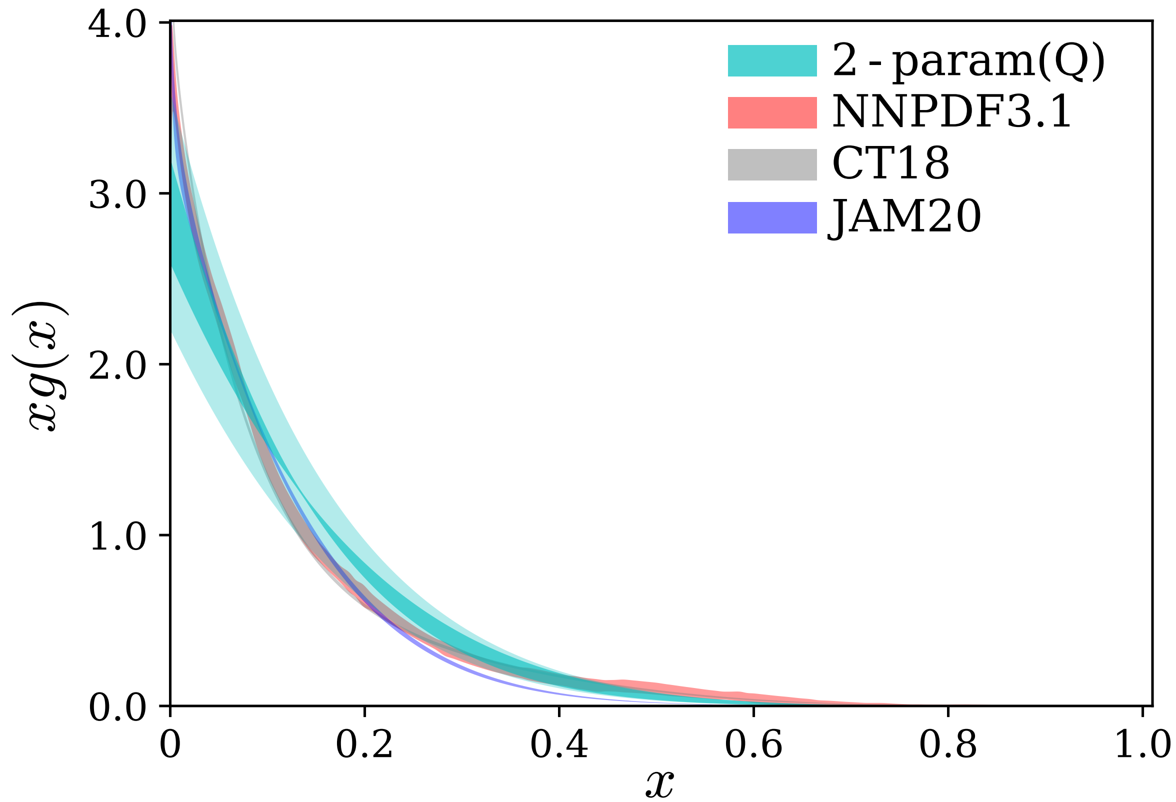


C. Alexandrou, et al. (ETMC), arXiv: 2106.16065

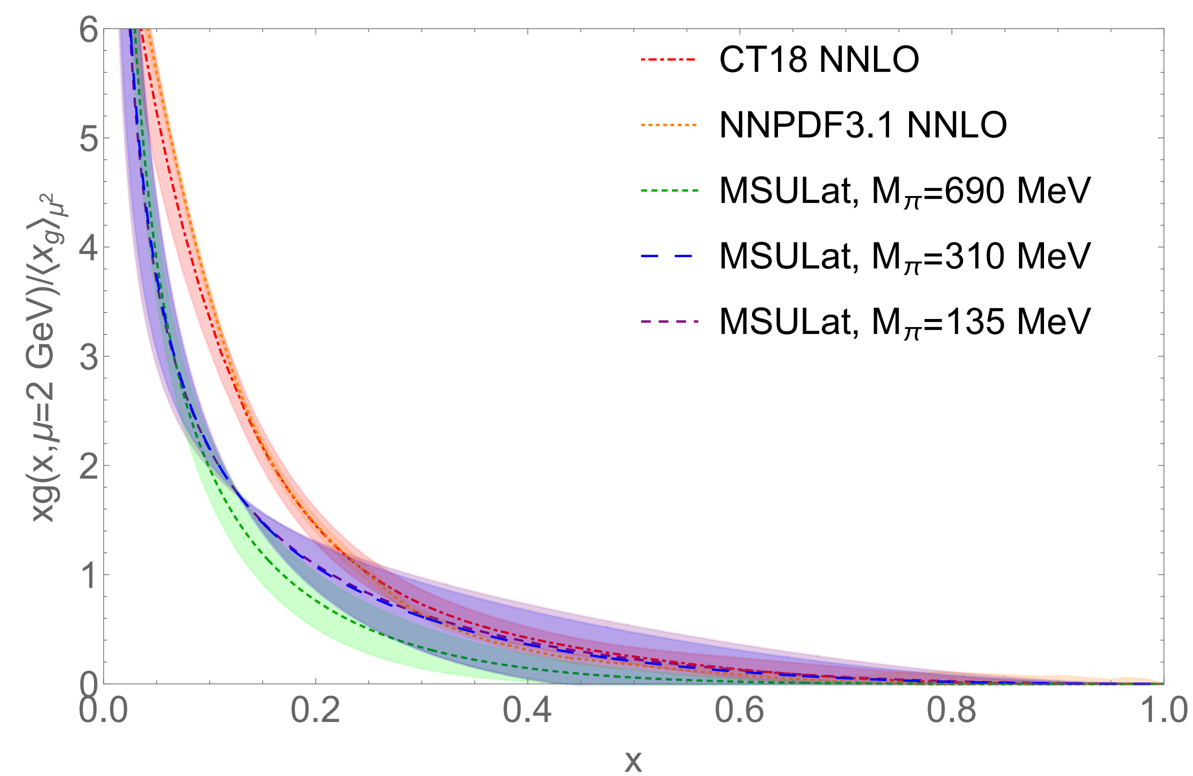
Lattice calculation of less-knowns

- Gluon PDFs:

pseudo-PDF approach w. NLO matching w.o. mixing to quark singlet channel



T. Khan, et al. (HadStruc), arXiv: 2107.08960



Z. Fan, R. Zhang and H.-W. Lin, Int.J.Mod.Phys.A 36 (2021) 13, 2150080

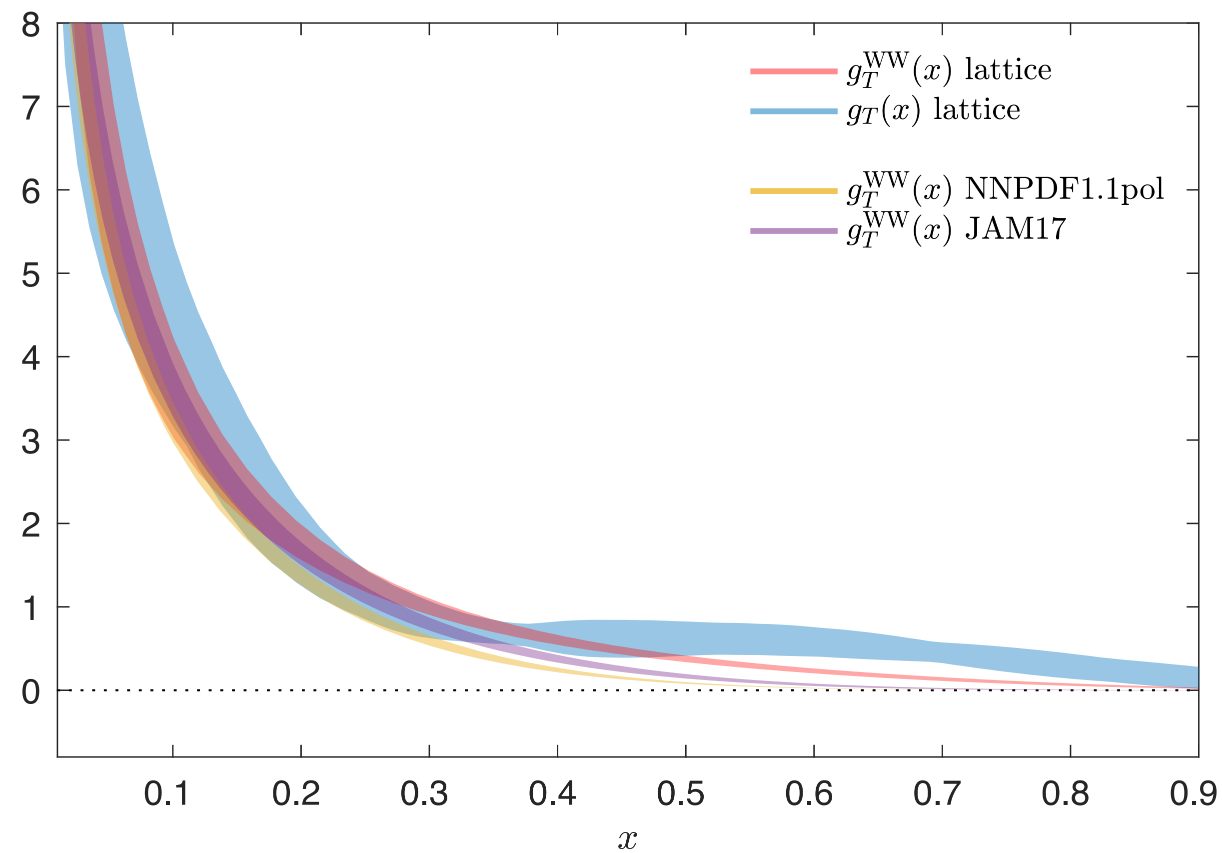
- a=0.094 fm
- z_{max}=0.56 fm
- 2-parameter model of the gluon PDF

- a=0.09 fm
- z_{max}=0.45 fm
- 2-parameter model of the gluon PDF

Lattice calculation of less-knowns

- Higher-twist PDFs:

First lattice QCD calculation of the twist-3 PDF $g_T(x)$ from quasi-PDF with RI/MOM renormalization



NLO matching, ignoring the mixing to three-particle correlations.

S. Bhattacharya, K. Cichy et al., Phys.Rev.D 102 (2020) 11, 111501

First results of $h_L(x)$ also became available recently:

S. Bhattacharya, K. Cichy et al., arXiv: 2107.02574

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LaMET approach

- GPDs from the quasi-GPDs:

Y.-S. Liu, YZ et al., Phys.Rev.D 100 (2019) 3, 034006

$$\tilde{F}(x,\tilde{\xi},t,\mu) = \frac{1}{2\bar{P}^0}\int \frac{d\lambda}{2\pi} e^{iy\lambda}\langle P'S'|O_{\gamma^0}(z)|PS\rangle$$

$$F(x,\xi,t,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{y\bar{P}^z}\right)\tilde{F}(y,\xi,t,\bar{P}^z,\mu)$$

$$+\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(x\bar{P}^z)^2},\frac{t}{(x\bar{P}^z)^2},\frac{\Lambda^2_{\rm QCD}}{((1-x)\bar{P}^z)^2},\frac{t}{((1-x)\bar{P}^z)^2}\right)$$

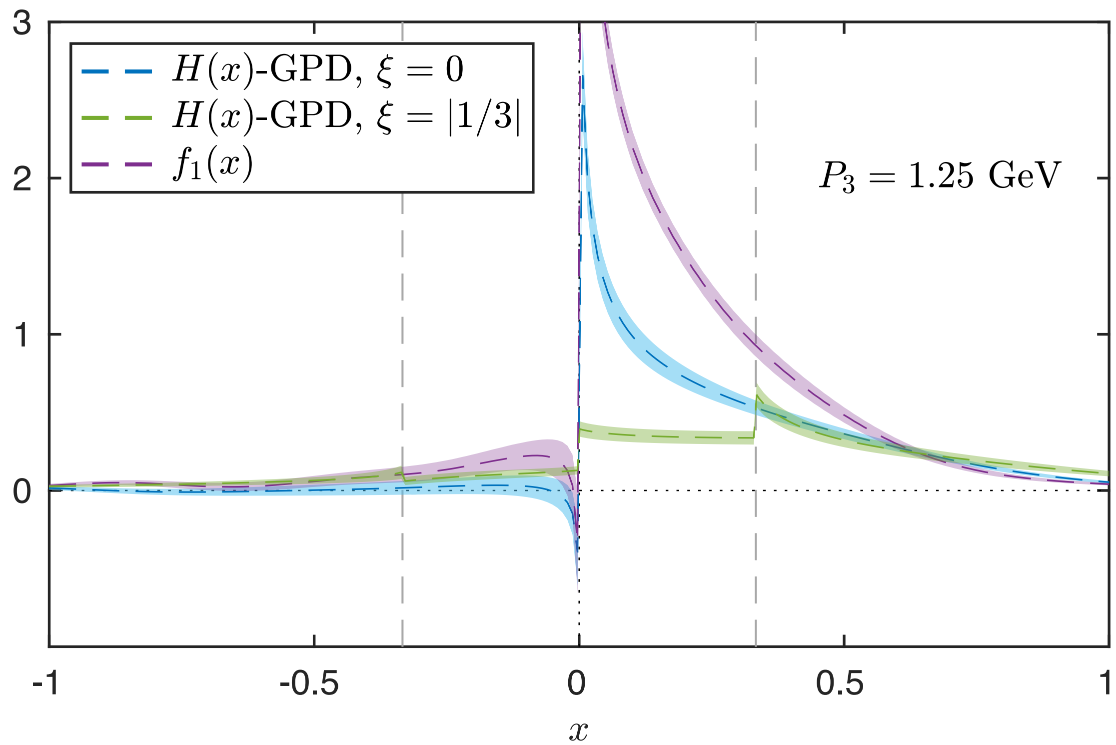
- More kinematic variables, less knowledge about model preference from phenomenology;

- Large-momentum expansion can still be used to directly calculate the (x, ξ, t)-dependence, with controlled systematic errors.

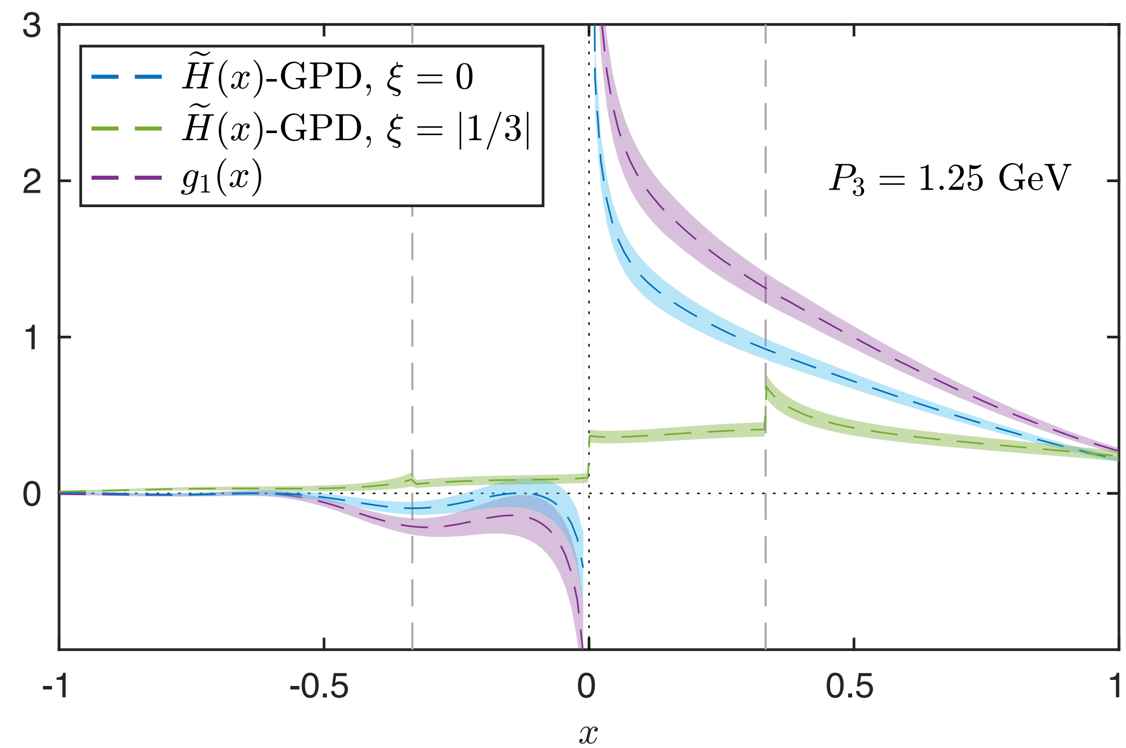
LaMET approach

- Lattice calculation of the proton isovector GPDs with RI/MOM renormalization and NLO matching

Unpolarized



Helicity



C. Alexandrou et al. (ETMC), Phys.Rev.Lett. 125 (2020) 26, 262001

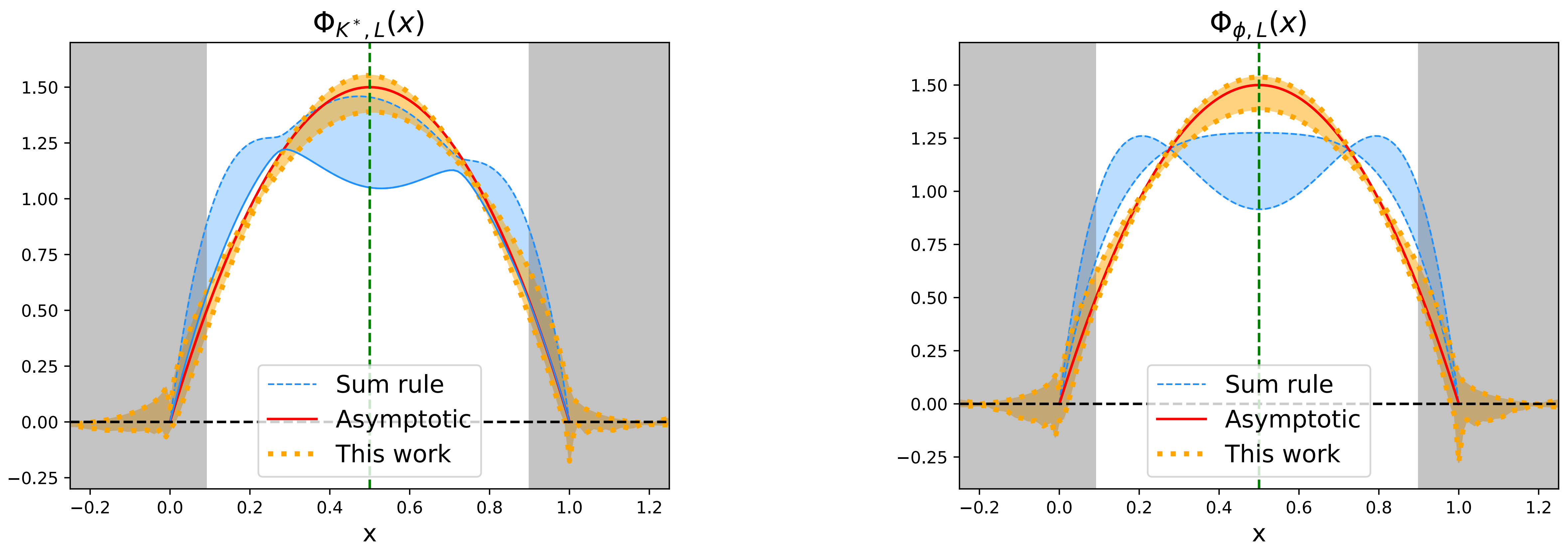
Practical question:

how to combine the lattice determination of GPDs with the extraction from Compton form factors?

$a = 0.094$ fm, $L = 3.0$ fm, $m_\pi = 260$ MeV

Light-cone distribution amplitudes (LCDAs)

- First lattice QCD results of LCDAs of K* and ϕ mesons:



J. Hua et al. (LPC), Phys.Rev.Lett. 127 (2021) 6, 062002

- Physical pion mass
- Lattice spacings a = 0.06, 0.09, 0.12 fm
- Largest momentum P^z=2.15 GeV.
- Hybrid scheme renormalization w. NLO matching

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- Parton distribution functions (PDFs)
- Generalized parton distributions (GPDs)
- **Transverse-momentum-dependent (TMD) distributions**

TMDPDF and quasi-TMDPDF

Factorization formula in LaMET:

- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and Y.Z., PRD99 (2019);
- M. Ebert, I. Stewart, Y.Z., PRD99 (2019), JHEP09 (2019) 037.
- Ji, Liu and Liu, Nucl.Phys.B 955 (2020), Phys.Lett.B 811 (2020).

$$\frac{1}{2}\zeta\frac{d\ln f_i^{\rm TMD}}{d\zeta} = \gamma_\zeta^i(\mu, b_T)$$

Rapidity anomalous dimension or Collins-Soper kernel

Quasi-TMDPDF: equal-time staple-shaped quark bilinear

$$\frac{\tilde{f}^{\rm TMD}(x, \vec{b}_T, \mu, P^z)}{\sqrt{S_r^q(b_T, \mu)}} = C^{\rm TMD}\left(\mu, xP^z\right) \exp\left[\frac{1}{2}\gamma_\zeta^q(\mu, b_T)\ln\frac{(2xP^z)^2}{\zeta}\right] \times f^{\rm TMD}(x, \vec{b}_T, \mu, \zeta) + \mathcal{O}\left(\frac{1}{(xb_TP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}\right)$$

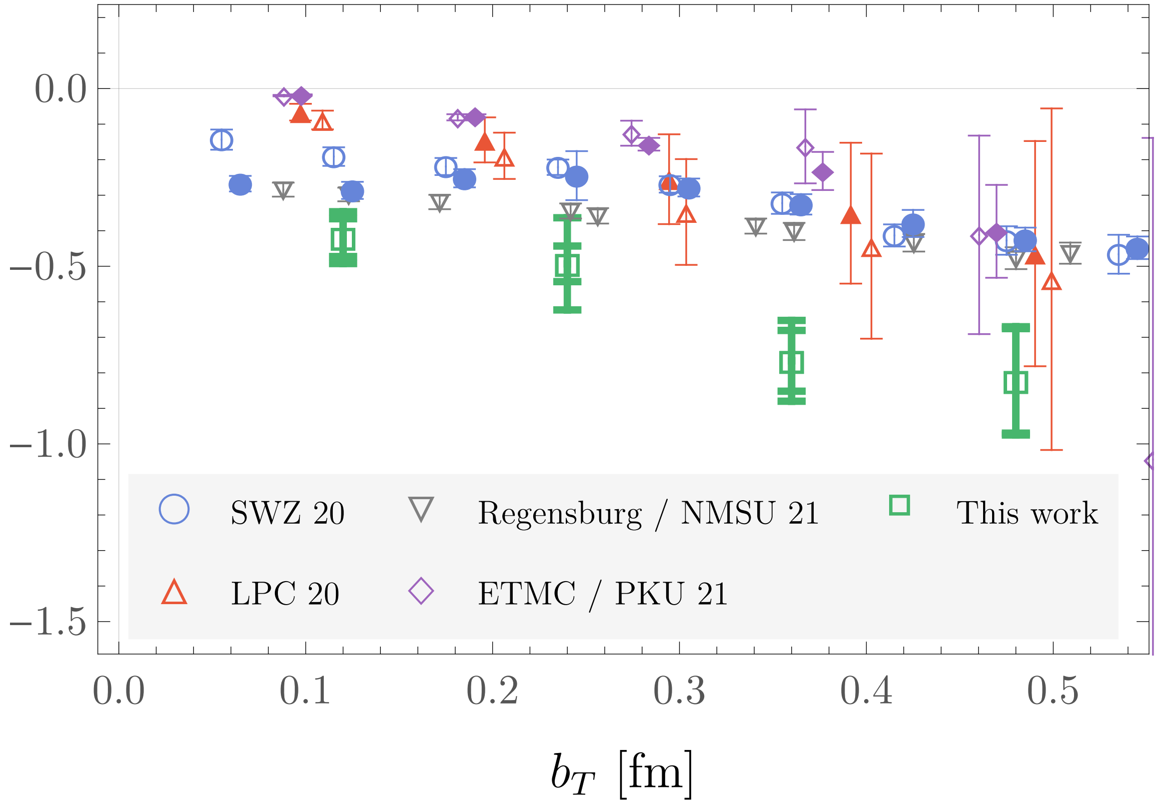
"Reduced soft function"

Collins-Soper kernel from quasi-TMDPDFs

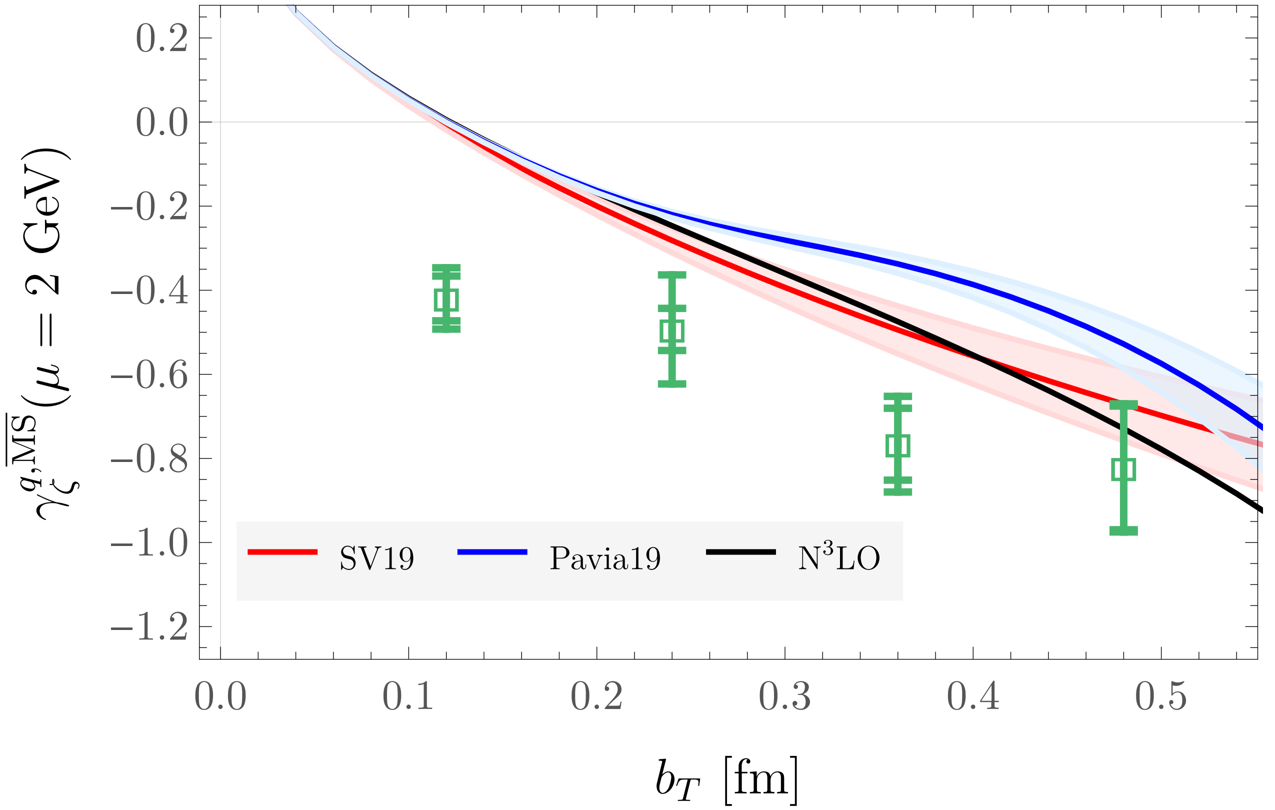
$$\gamma_\zeta^q(\mu, b_T) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{C^{\rm TMD}(\mu, xP_2^z)\tilde{f}^{\rm TMD}(x, \vec{b}_T, \mu, P_1^z)}{C^{\rm TMD}(\mu, xP_1^z)\tilde{f}^{\rm TMD}(x, \vec{b}_T, \mu, P_2^z)}$$

M. Ebert, I. Stewart, Y.Z., PRD99 (2019).

Results by different groups w/ different systematics.



Comparison with phenomenology



SV19: I. Scimemi and A. Vladimirov, JHEP 06 (2020) 137
Pavia19: A. Bacchetta et al., JHEP 07 (2020) 117

P. Shanahan, M. Wagman and Y.Z., arXiv: 2107.11930.

Reduced soft function from a light-meson form factor

- LaMET factorization of the light-meson form factor:

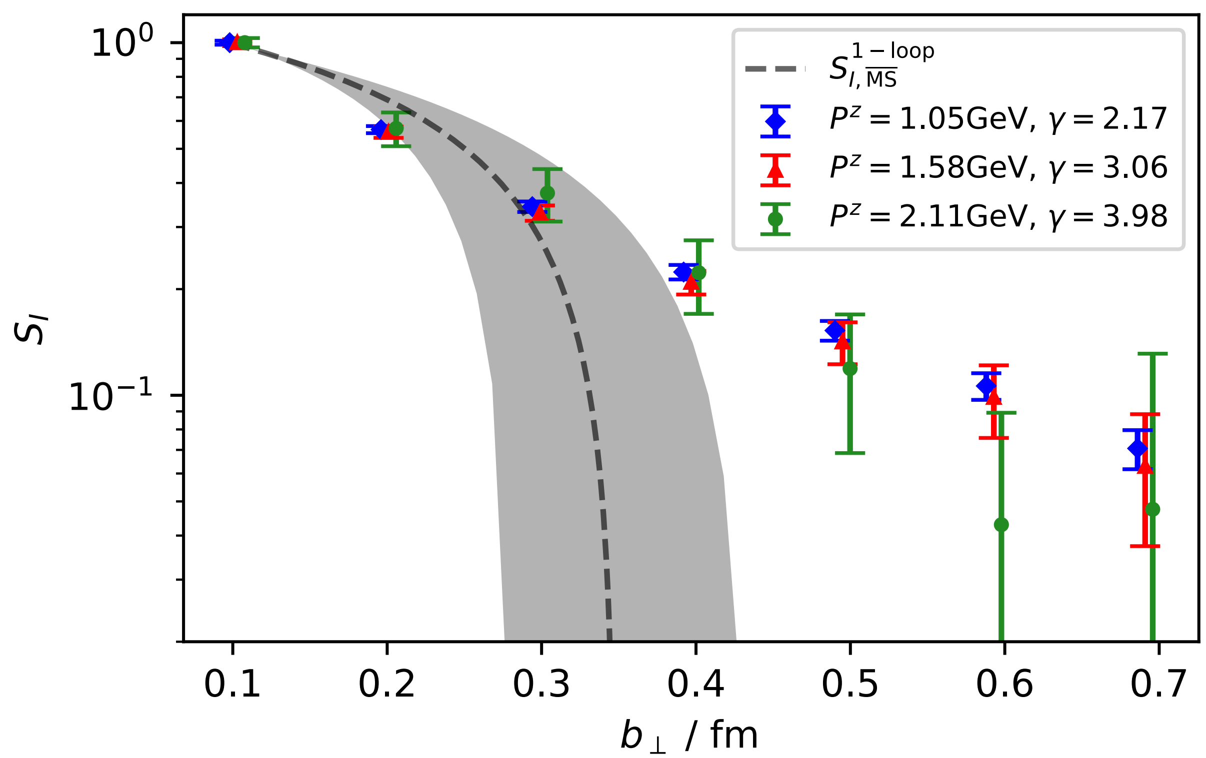
Ji, Liu and Liu, Nucl.Phys.B 955 (2020), Phys.Lett.B 811 (2020).

$$F(b_T, P^z) \equiv \langle \pi(-P) \,|\, j_1(b_T) j_2(0) \,|\, \pi(P) \rangle \qquad j = \bar{\psi}\Gamma\psi$$

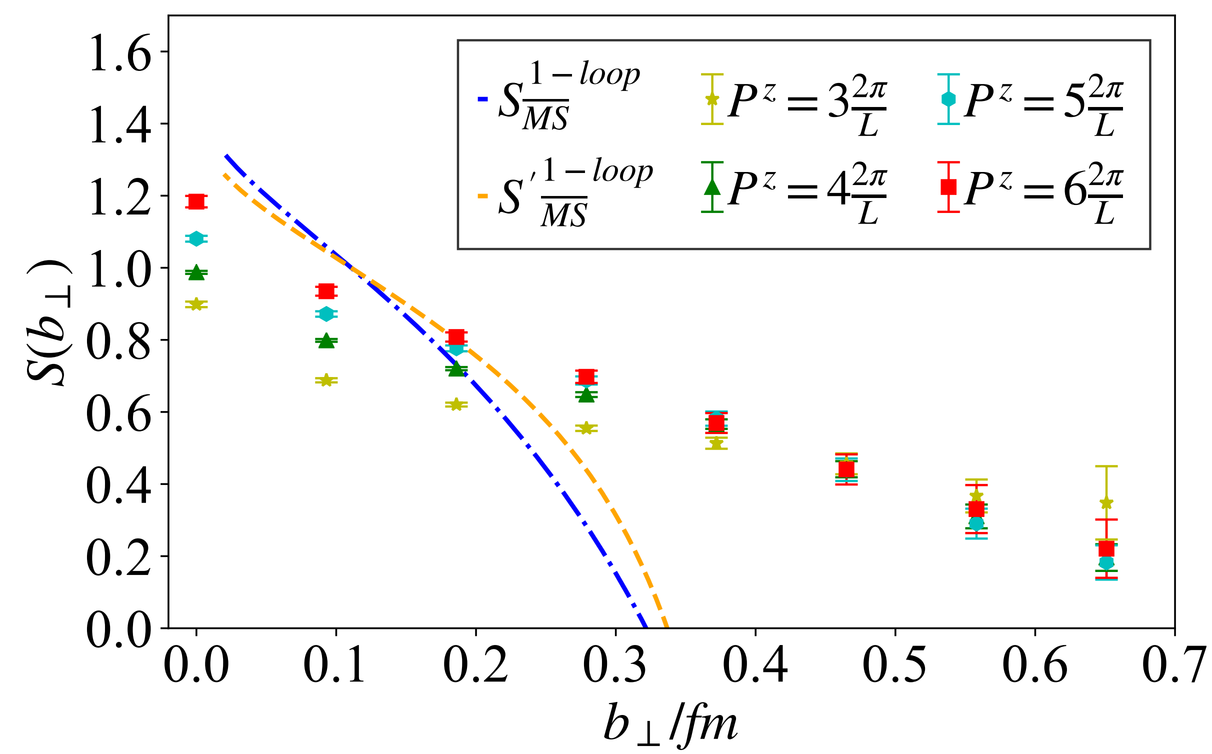
$$= S_q^r(b_T, \mu)\; H(x, \mu) \otimes \Phi^\dagger(x, b_T, -P^z) \otimes \Phi(x, b_T, P^z)$$

Φ: Quasi-TMD distribution amplitude

Lattice results with LO perturbative matching



Q.-A. Zhang, et al. (LPC), Phys.Rev.Lett. 125 (2020).



Y. Li et al., arXiv: 2106.13027.

Conclusion

- LaMET enables direct determination of the x-dependence of parton distributions, and the precision can be systematically improved;

- Much progress has been made towards PDFs, GPDs and TMDs using LaMET and other methods;

- To complement the EIC, controlling the precision is key for the future direction.

Summary of recent progress

- Finite volume effects
  - H.-W. Lin and R. Zhang, Phys.Rev.D 100 (2019) 7, 074502
  - W.-Y. Liu and J.-W. Chen, arXiv: 2011.13536
- Renormalization and continuum limit
  - RI/MOM scheme
    - C. Alexandrou et al. (ETMC), Phys.Rev.D 103 (2021) 094512
    - H.-W. Lin, J.-W. Chen and R. Zhang, arXiv: 2011.14971
  - Ratio scheme
    - C. Alexandrou et al. (ETMC), Phys.Rev.D 103 (2021) 094512
    - J. Karpie et al. (HadStruc), arXiv: 2105.13313
  - RI/xMOM scheme
    - J. Green, K. Jansen and F. Steffens, Phys.Rev.D 101 (2020) 7, 074509
    - C. Alexandrou et al. (ETMC), Phys.Rev.D 103 (2021) 094512
  - Hybrid scheme
    - X. Ji, YZ et al., Nucl.Phys.B 964 (2021) 115311
    - Huo et al. (LPC), Nucl.Phys.B 969 (2021) 115443
- Perturbative matching
  - Complete matching for quark and gluon PDFs at NLO
    - W. Wang et al., Phys.Rev.D 100 (2019) 7, 074509
  - NNLO matching for the unpolarized and helicity non-singlet (and pion valence) quark PDFs
    - L.-B. Chen, R.-L. Zhu and W. Wang, PRL126 (2021);
    - Z.-Y. Li, Y.-Q. Ma and J.-W. Qiu, PRL126 (2021).

Summary of recent progress

PDFs:

- Unpolarized, helicity and transversity PDFs with sea quarks and flavor separation
  - C. Alexandrou, et al. (ETMC), arXiv: 2106.16065
- Charm quark PDF
  - R. Zhang, H.-W. Lin and B. Yoon, arXiv: 2005.01124
- Gluon PDFs
  - Z. Fan, R. Zhang and H.-W. Lin, Int.J.Mod.Phys.A 36 (2021) 13, 2150080;
  - Z. Fan and H.-W. Lin, 2104.06372T
  - Khan, et al. (HadStruc), arXiv: 2107.08960
- Valence pion (and its excitation) PDF
  - X. Gao, YZ, et al., Phys.Rev.D 102 (2020) 9, 094513
  - X. Gao, YZ, et al., Phys.Rev.D 103 (2021) 9, 094510
  - X. Gao, YZ, et al., Phys.Rev.D 103 (2021) 9, 094504
- Higher-twist PDFs
  - S. Bhattacharya, K. Cichy et al., Phys.Rev.D 102 (2020) 11, 111501
  - S. Bhattacharya, K. Cichy et al., arXiv: 2107.02574

Summary of recent progress

GPDs:

- Proton isovector unpolarized and helicity GPDs
  - C. Alexandrou et al. (ETMC), Phys.Rev.Lett. 125 (2020) 26, 262001
  - H.-W. Lin, arXiv: 2008.12474
- Pion valence GPD
  - J.-W. Chen, H.-W. Lin and J.-H. Zhang, Nucl.Phys.B 952 (2020) 114940
- Meson LCDAs
  - R. Zhang, C. Honkala, H.-W. Lin and J.-W. Chen, Phys.Rev.D 102 (2020) 9, 094519
  - J. Hua et al. (LPC), Phys.Rev.Lett. 127 (2021) 6, 062002

Summary of recent progress

TMDs:

- Collins-Soper kernel
  - P. Shanahan, M. Wagman and Y.Z., arXiv: 2107.11930.
  - Q.-A. Zhang, et al. (LPC), Phys.Rev.Lett. 125 (2020).
  - M. Schlemmer, A. Vladimirov and A. Schaefer, *JHEP* 08 (2021) 004
  - Y. Li et al., arXiv: 2106.13027.
  - P. Shanahan, M. Wagman and Y.Z., arXiv: 2107.11930.
- Soft function
  - Q.-A. Zhang, et al. (LPC), Phys.Rev.Lett. 125 (2020).
  - Y. Li et al., arXiv: 2106.13027.