Wigner functions and their 3D projections

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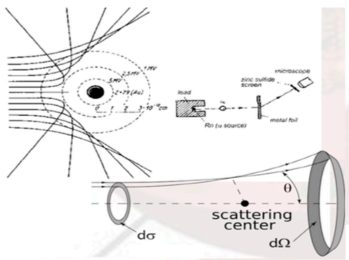
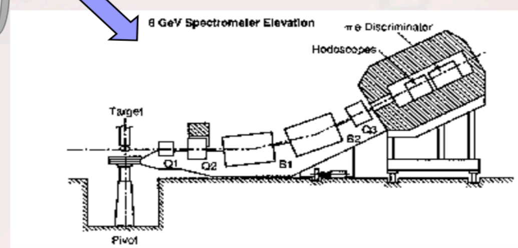
Exploring the nucleon/nucleus: Of fundamental importance in science



Rutherford Scattering, 1911

- Discovery of nucleus



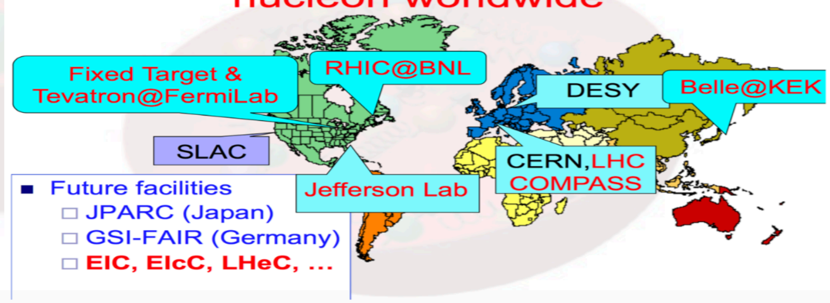
DIS at SLAC, 1960s

- Discovery of quarks

Quantum ChromoDynamics:

$$L = \bar{\psi}(i\gamma \cdot \partial - m_q)\psi - \frac{1}{4}F^{\mu\nu a}F_{\mu\nu a} - g_s\bar{\psi}\gamma \cdot A\psi$$

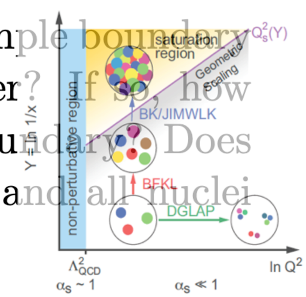
Exploring the partonic structure of nucleon worldwide



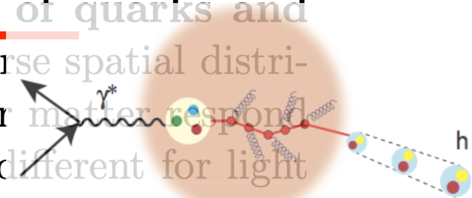
Big questions for EIC

- **How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?** How are these quark and gluon distributions correlated with overall nucleon properties, such as spin direction? What is the role of the orbital motion of sea quarks and gluons in building the nucleon spin?

- **Where does the saturation of gluon densities set in?** Is there a simple boundary that separates this region from that of more dilute quark-gluon matter? If so, how do the distributions of quarks and gluons change as one crosses the boundary? Does this saturation produce matter of universal properties in the nucleon and all nuclei viewed at nearly the speed of light?



- **How does the nuclear environment affect the distribution of quarks and gluons and their interactions in nuclei?** How does the transverse spatial distribution of gluons compare to that in the nucleon? How does nuclear matter respond to a fast moving color charge passing through it? Is this response different for light and heavy quarks?



Proton spin: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$

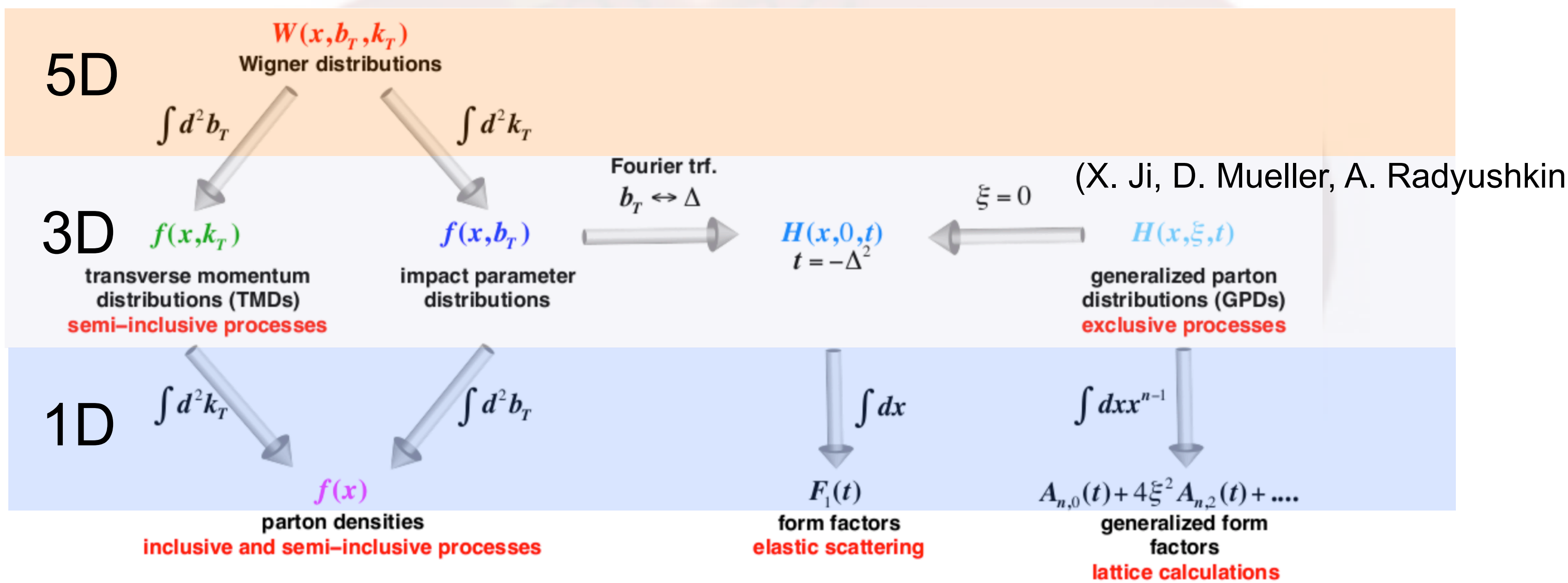
- We know fairly well how much quark helicity contribute to the proton spin: $\Delta\Sigma=0.3\pm0.05$
- With large errors we know gluon helicity contribution plays an important role
- No direct information on quark and gluon orbital angular momentum contributions

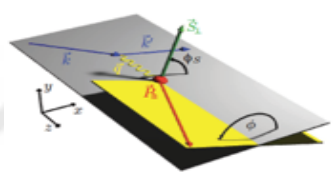
New ways to look at partons

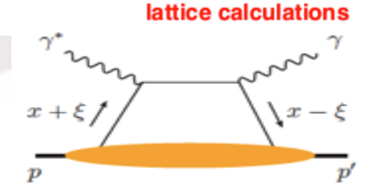
- We not only need to know that partons have long. momentum, but must have transverse degrees of freedom as well
- Partons in transverse coordinate space
  - Generalized parton distributions (GPDs)
- Partons in transverse momentum space
  - Transverse-momentum distributions (TMDs)
- Both? Wigner distributions!

Unified view of the Nucleon

❑ Wigner distributions (Belitsky, Ji, Yuan)

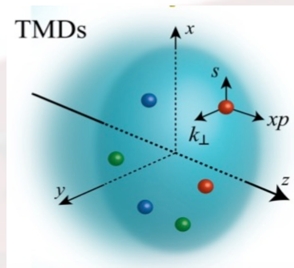


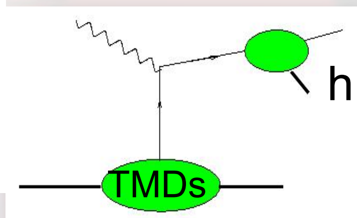




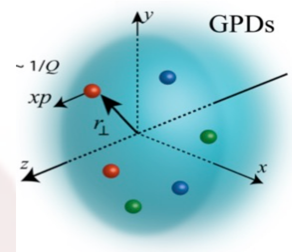
Zoo of TMDs & GPDs

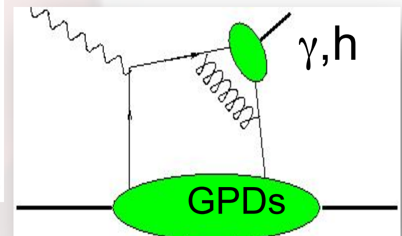
	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1, h_{1T}^\perp$





	U	L	T
U	H		$\mathcal{E}_T$
L		$\widetilde{H}$	
T	E		$H_T, \widetilde{H}_T$





- NOT directly accessible
- Their extractions require measurements of x-sections and asymmetries in **a large kinematic domain of** x_B, t, Q^2 (GPD) and x_B, P_T, Q^2 , z (TMD)

talks in this session

What can we learn

- 3D Imaging of partons inside the nucleon (non-trivial correlations)
  - Try to answer more detailed questions as Rutherford was doing 100 years ago
- QCD dynamics involved in these processes
  - Transverse momentum distributions: universality, factorization, evolutions,…
  - Small-x: BFKL vs Sudakov?

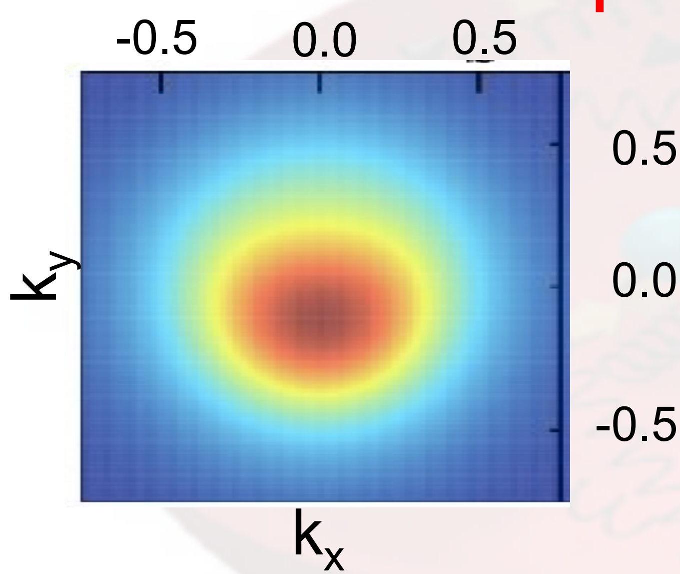
What we have learned

- Unpolarized transverse momentum (coordinate space) distributions from, mainly, DIS, Drell-Yan, W/Z boson productions, (HERA exp.)
- Indications of polarized quark distributions from low energy DIS experiments (HERMES, COMPASS, JLab)

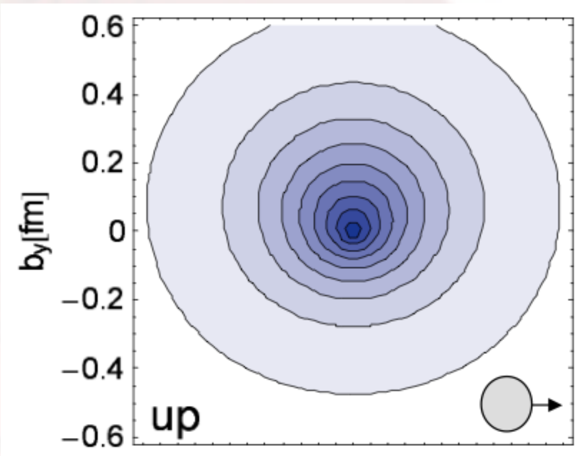
What we are missing

- Precise, detailed, mapping of polarized quark/gluon distribution
  - Universality/evolution more evident
- Spin correlation in momentum and coordinate space/ tomography
  - Crucial for orbital motion

Deformation when nucleon is transversely polarized



Quark Sivers function fit to the SIDIS Data, Anselmino, et al. 2009



Lattice Calculation of the transvese density Of Up quark, QCDSF/UKQCD Coll., 2006

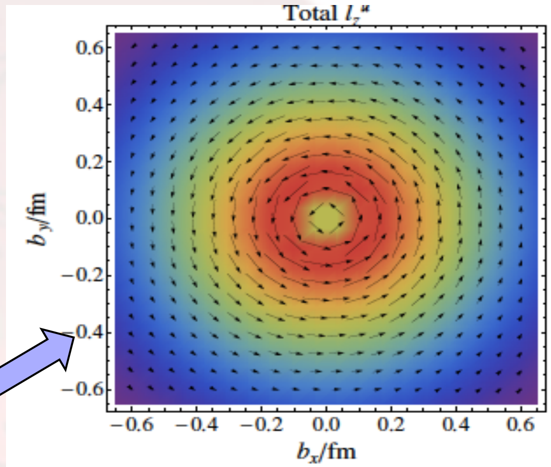
Parton's orbital motion through the Wigner Distributions

Phase space distribution:

Projection onto p (x) to get the momentum (probability) density

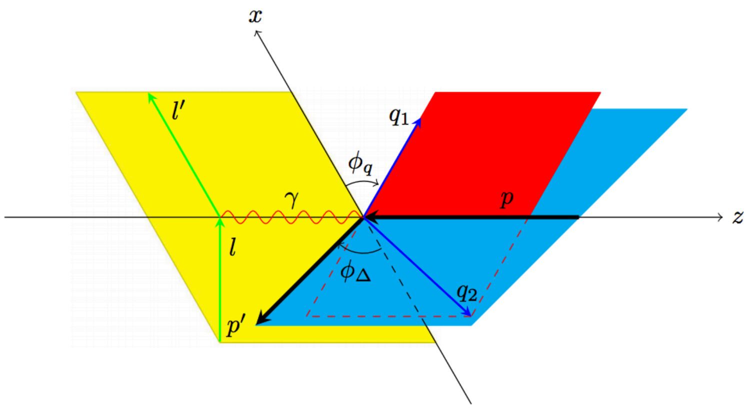
Quark orbital angular momentum

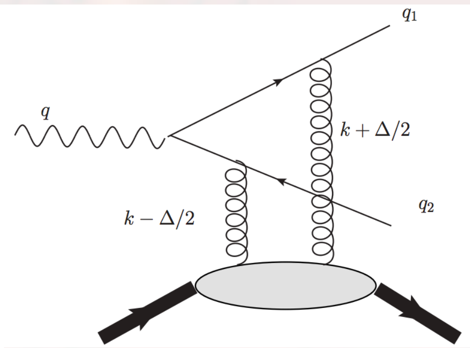
$$L(x) = \int (\vec{b}_\perp \times \vec{k}_\perp) W(x, \vec{b}_\perp, \vec{k}_\perp) d^2\vec{b}_\perp d^2\vec{k}_\perp$$



Well defined in QCD:
Ji, Xiong, Yuan, PRL, 2012; PRD, 2013
Lorce, Pasquini, Xiong, Yuan, PRD, 2012
Lorce-Pasquini 2011
Hatta 2011

Hunting the Gluon Orbital Angular Momentum





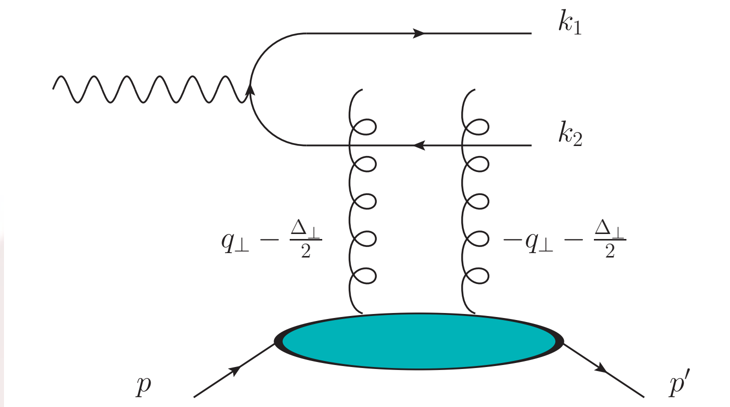
$$A_{\sin(\phi_q-\phi_\Delta)} \propto \frac{(\bar{z}-z)|\vec{q}_\perp||\vec{\Delta}_\perp|}{\vec{q}_\perp^2+\mu^2}\mathcal{L}_g(\xi,t)$$

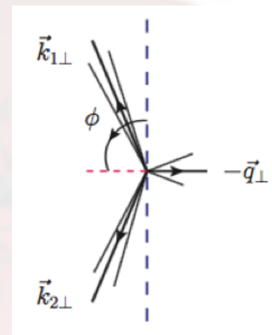
Ji,Yuan,Zhao, arXiv:1612.02438
Hatta,Nakagawa,Yuan,Zhao, arXiv:1612.02445

- The longitudinal spin asymmetry depends on the gluon OAM distribution
- More quantitative studies needed to show the impact from EIC measurements

Nucleon/Nucleus Tomography at Small-x

Hatta-Xiao-Yuan,1601.01585





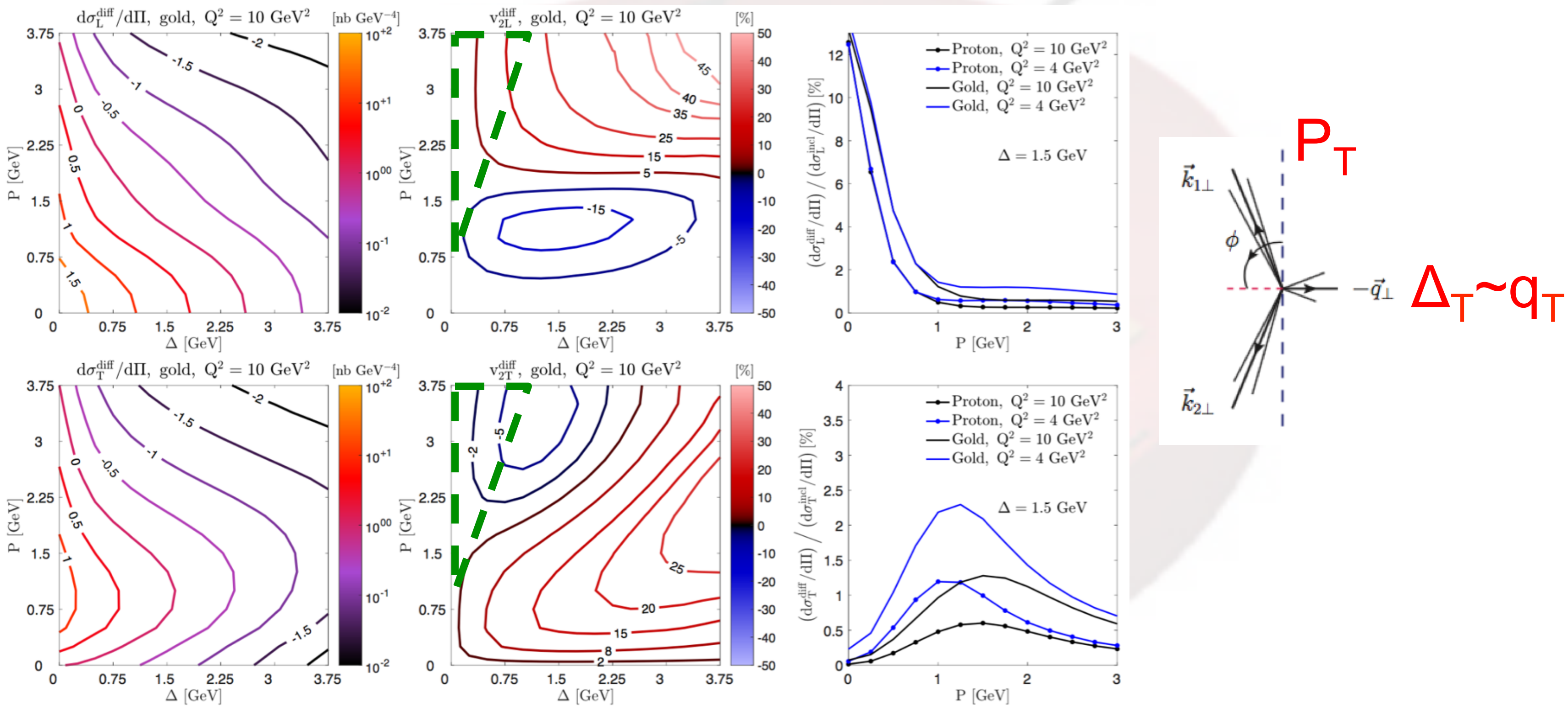
cos(2ϕ) anisotropy

- In the Breit frame, by measuring the recoil of final state proton, one can access Δ_T. By measuring jets momenta, one can approximately access q_T.
- The diffractive dijet cross section is proportional to the square of the Wigner distribution → nucleon/nucleus tomography

$$x\mathcal{W}_g^T(x, |\vec{q}_\perp|, |\vec{b}_\perp|) + 2\cos(2\phi)x\mathcal{W}_g^\epsilon(x, |\vec{q}_\perp|, |\vec{b}_\perp|)$$

→ **Anisotropy ~ few %**

This has generated a lot of interests…

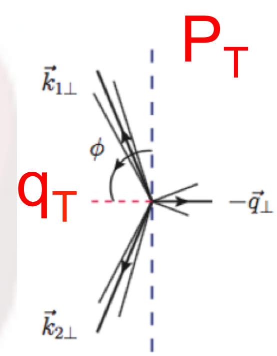


CGC calculations: Mäntysaari-Mueller-Salazar-Schenke, 1912.05586, 1902.05087; Mäntysaari-Roy-Salazar-Schenke 2011.02464

Soft gluon radiations can generate an azimuthal asymmetry

Catani-Grazzini-Sargsyan 2017

- Azimuthal angular asymmetries arise from soft gluon radiations
  - ϕ is defined as angle between total and different transverse momenta of the two final state particles
- Infrared safe but divergent
  - <cos(ϕ)>, <cos(2ϕ)>, … divergent, $\sim 1/q_T^2$
  - Examples discussed include Vj, top quark pair production

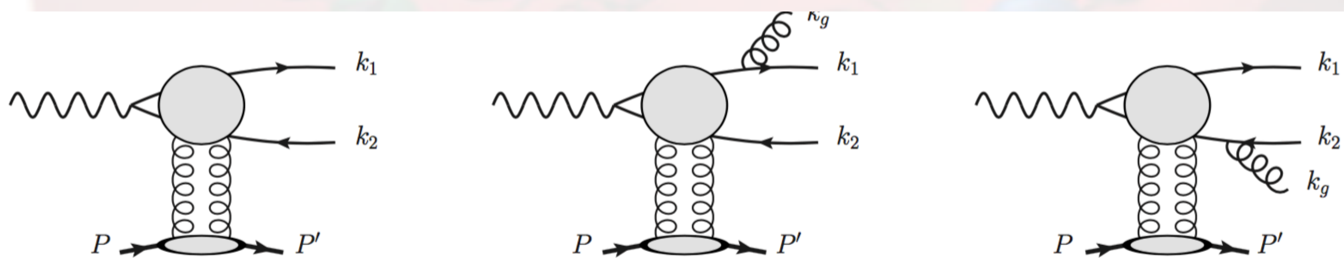


Diffractive dijet production

- Gluon radiation tends to be aligned with the jet direction

$$S_J(q_\perp) = \delta(q_\perp) + \frac{\alpha_s}{2\pi^2} \int dy_g \left(\frac{k_1 \cdot k_2}{k_1 \cdot k_g k_2 \cdot k_g} \right)_{\vec{q}_\perp = -\vec{k}_{g\perp}}$$

$$S_{J0}(|q_\perp|) + 2\cos(2\phi) S_{J2}(|q_\perp|) + \cdots$$



Hatta-Xiao-Yuan-Zhou, 2010.10774, 2106.05307
anisotropy was neglected in an earlier paper:
Hatta-Mueller-Ueda-Yuan, 1907.09491

Leading power contributions, explicit result at α_s

$$S_J(q_\perp) = S_{J0}(|q_\perp|) + 2\cos(2\phi) S_{J2}(|q_\perp|)$$

$$S_{J0}(q_\perp) = \delta(q_\perp) + \frac{\alpha_0}{\pi}\frac{1}{q_\perp^2}\ , \quad S_{J2}(q_\perp) = \frac{\alpha_2}{\pi}\frac{1}{q_\perp^2}\ ,$$

where

$$\alpha_0 = \frac{\alpha_s C_F}{2\pi} 2\ln\frac{a_0}{R^2}\ , \quad \alpha_2 = \frac{\alpha_s C_F}{2\pi} 2\ln\frac{a_2}{R^2}\ .$$

a_0,a_2 are order 1 constants, so,

in the small-R limit, <cos(2ϕ)> goes to 1

Additional gluon radiation contributions,

- In the momentum space, it will be a convolution
  - $q_T=k_{g1}+k_{g2}+\ldots$
  - Dominant contributions will be ϕ-independent
- It is convenient to perform resummation in Fourier-b space

$$\widetilde{S}_J(b_\perp) = \int d^2q_\perp e^{iq_\perp\cdot b_\perp} S_J(q_\perp)$$
$$= \widetilde{S}_{J0}(|b_\perp|) - 2\cos(2\phi_b)\widetilde{S}_{J2}(|b_\perp|) + \cdots$$

$$\widetilde{S}_{J0}(b_\perp) = 1 + \alpha_0 \ln(\mu_b^2/P_\perp^2)\ , \quad \widetilde{S}_{J2}(b_\perp) = \alpha_2$$

All order resummation, in Fourier-b space

$$\widetilde{S}_{J0}(b_\perp) = e^{-\Gamma_0(b_\perp)} \;, \quad \widetilde{S}_{J2}(b_\perp) = \alpha_2 e^{-\Gamma_0(b_\perp)} \quad \Gamma_0(b_\perp) = \int_{\mu_b^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2}\alpha_0$$

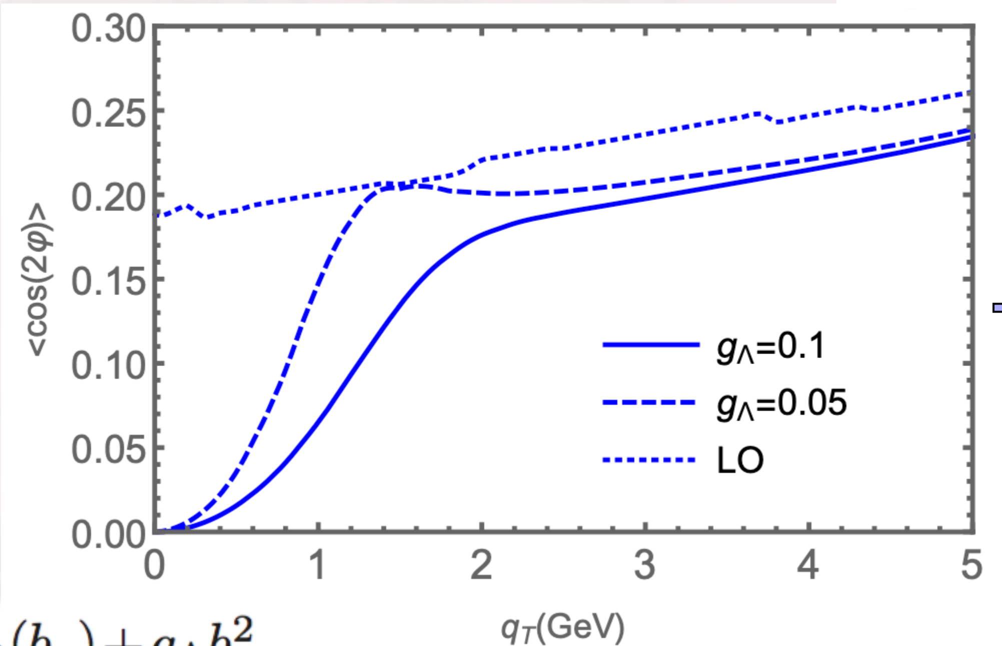
EIC Kinematics:

P_T~15GeV

R=0.4

$y_1=y_2$

Non-pert. input:

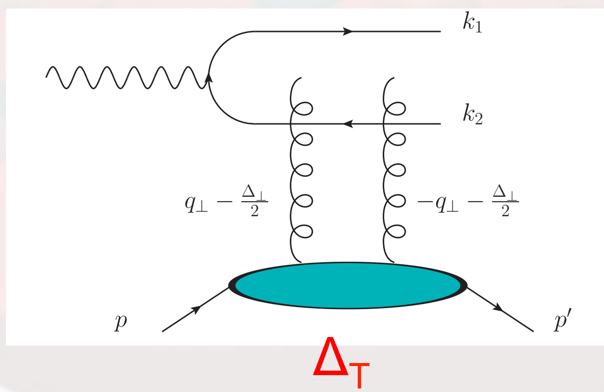


$\alpha_2/\alpha_0 \approx 0.14$

$$\Gamma_0(b_\perp) \Longrightarrow \Gamma_0(b_*) + g_\Lambda b_\perp^2$$

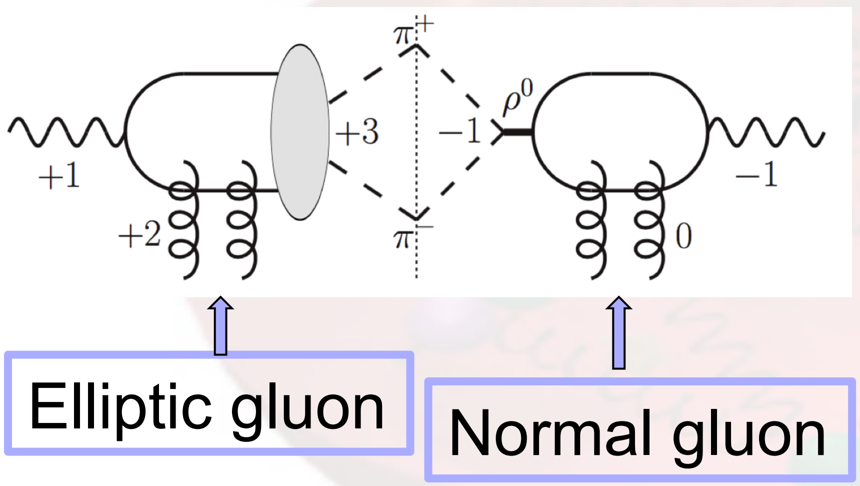
Comments

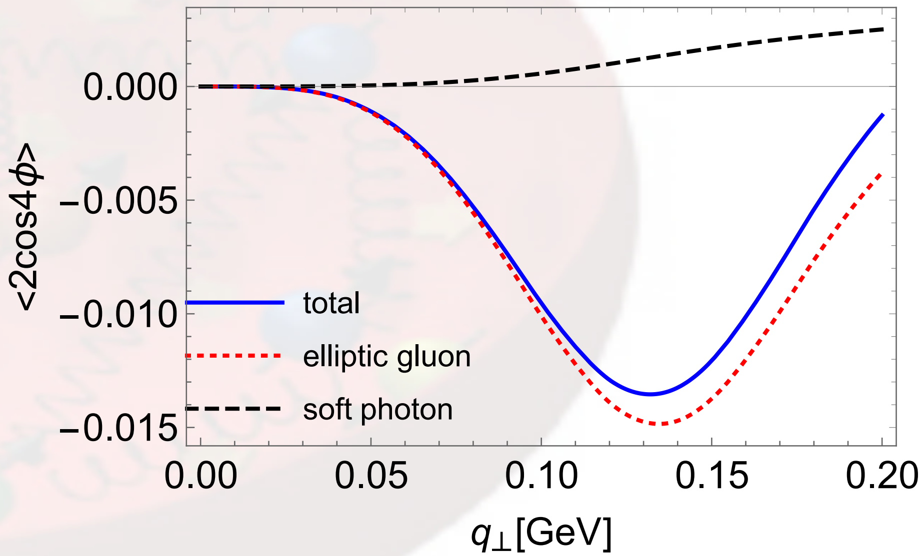
- To avoid the soft gluon radiation contribution, we need to reconstruct nucleon/nucleus recoil momentum to study the tomography



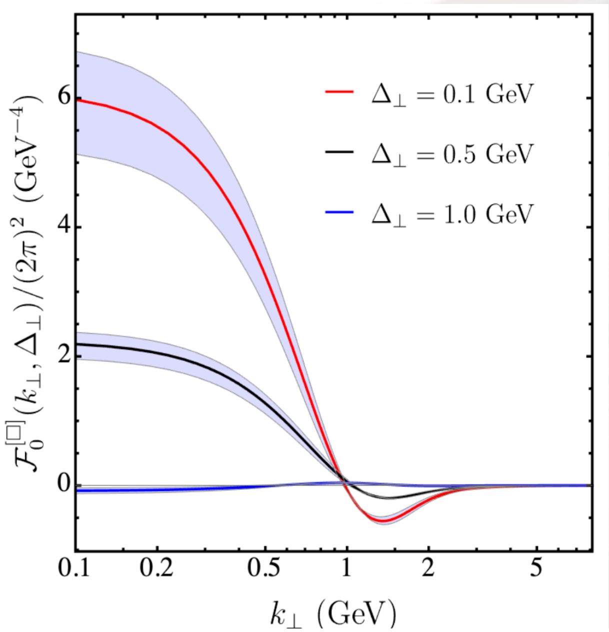
More exclusive process: γp→p'+$\pi^+\pi^-$

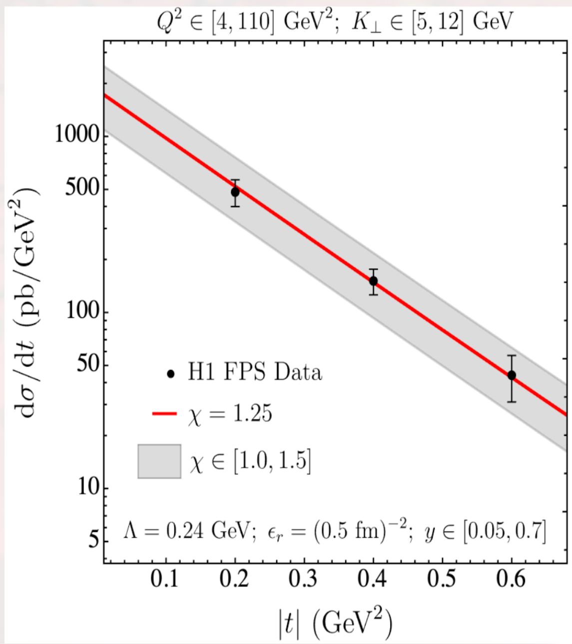
Hagiwara-Zhang-J.Zhou-Y.Zhou, 2106.13466

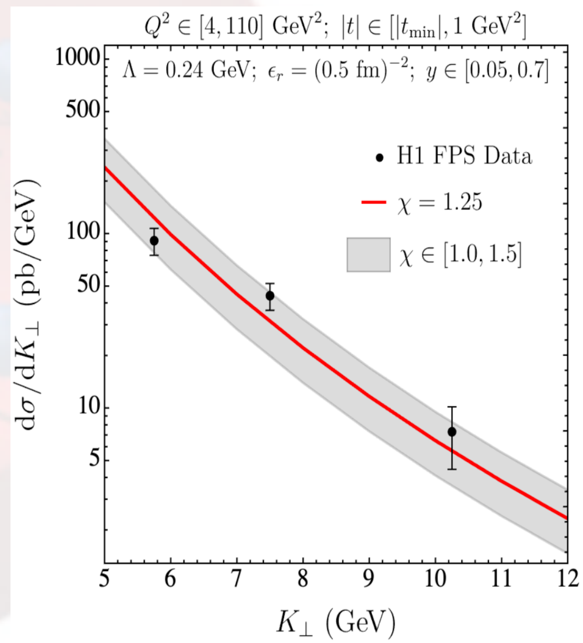




Small-x saturation model gives a reasonable description of HERA Data







CGC Model for the gluon Wigner distribution

Boer-Setyadi 2106.15148

Conclusion

- Great progress has been made in the last few years: probe nucleon tomography through parton Wigner distributions
- Further developments are needed to explore the full potential of the future electron-ion collider
  - Including challenges in the experimental set-up, such as the precision measurement of the recoil nucleon, particle ID, …