The Role of Lattice QCD in nucleon imaging

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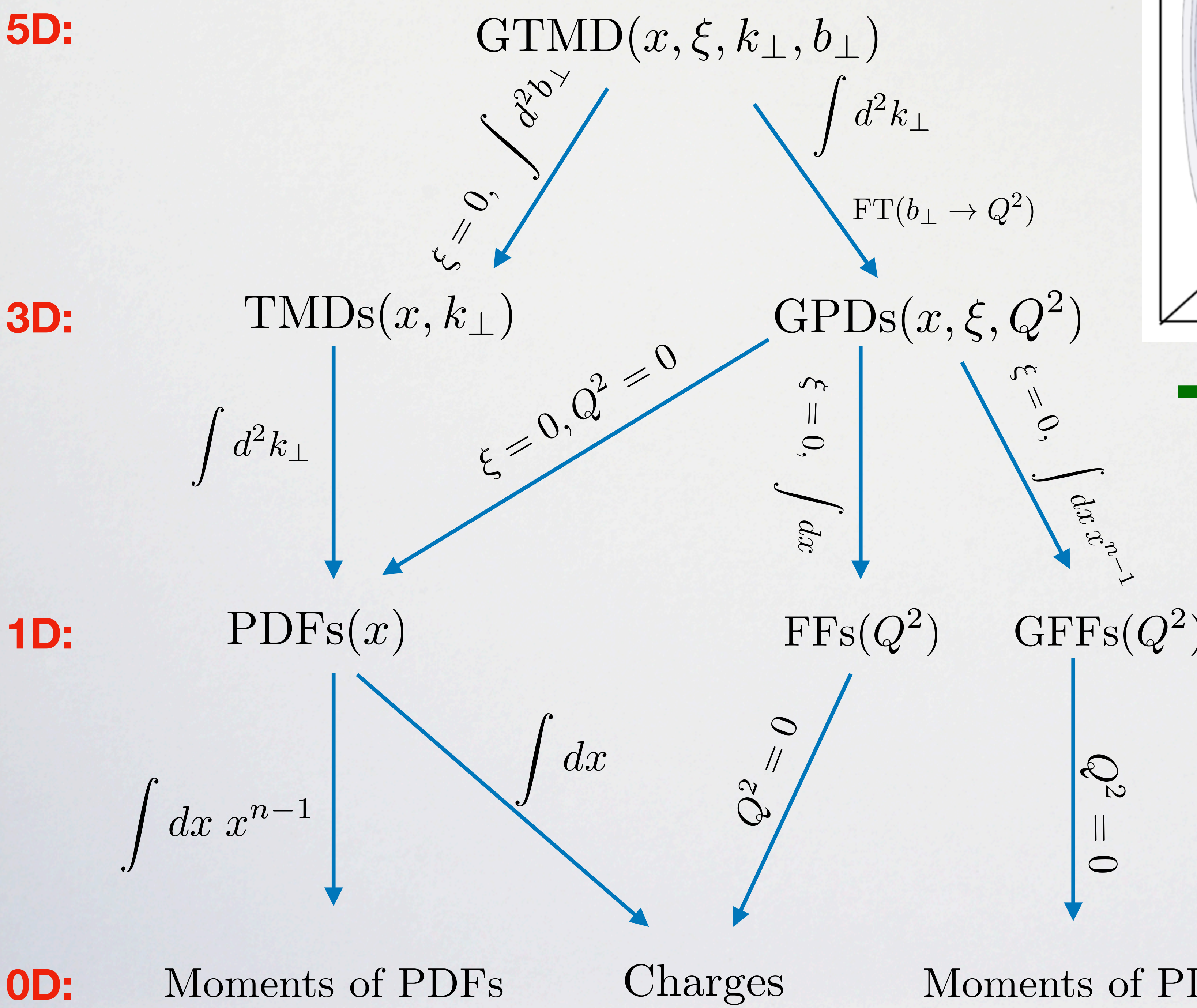
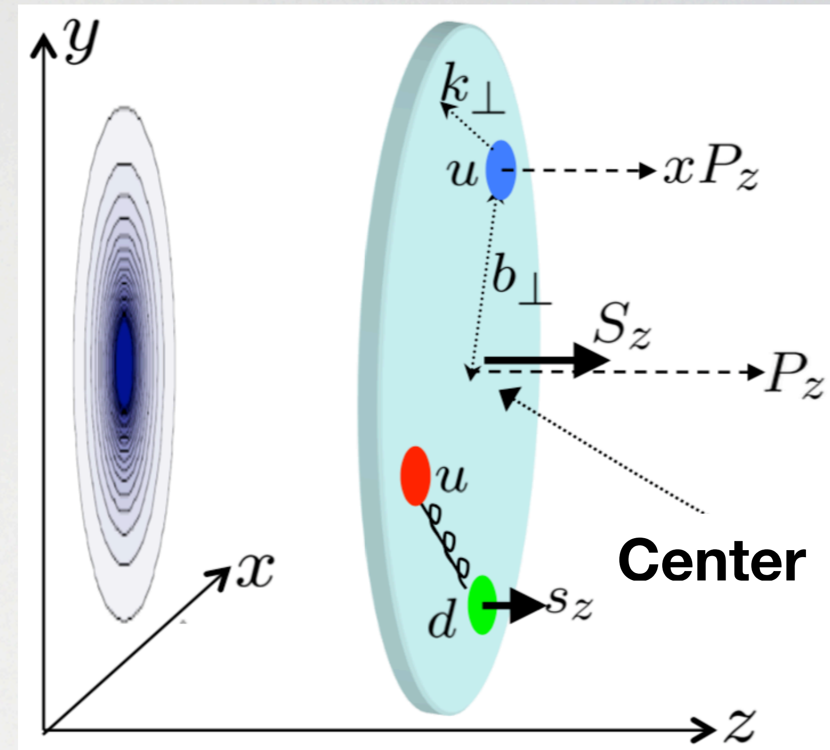
OUTLINE OF THE TALK

- *Motivation - The way from 0D to 5D structure*
- *Nucleon charges and first moments*
- *Nucleon Spin decomposition*
- *Form Factors and Parton Distribution Functions*
- *Generalized Parton Distribution Functions*
- *Summary Future Plans*

Motivation - The way from 0D to 5D

➡ Long way to go before we are able to do 5D imaging of the nucleon





➡ First steps in the calculation of 3D imaging, GPDs & TMDs

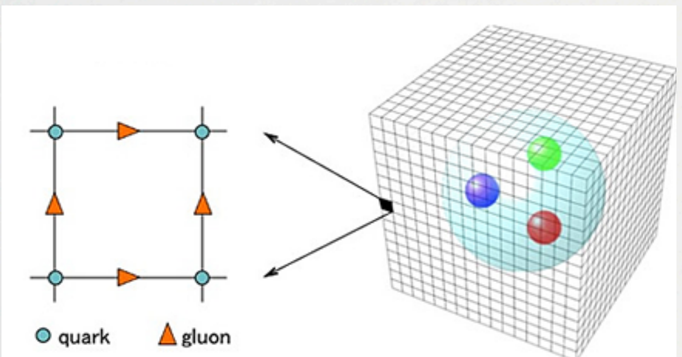
➡ Relatively only recently we are able to do PDFs directly on the lattice

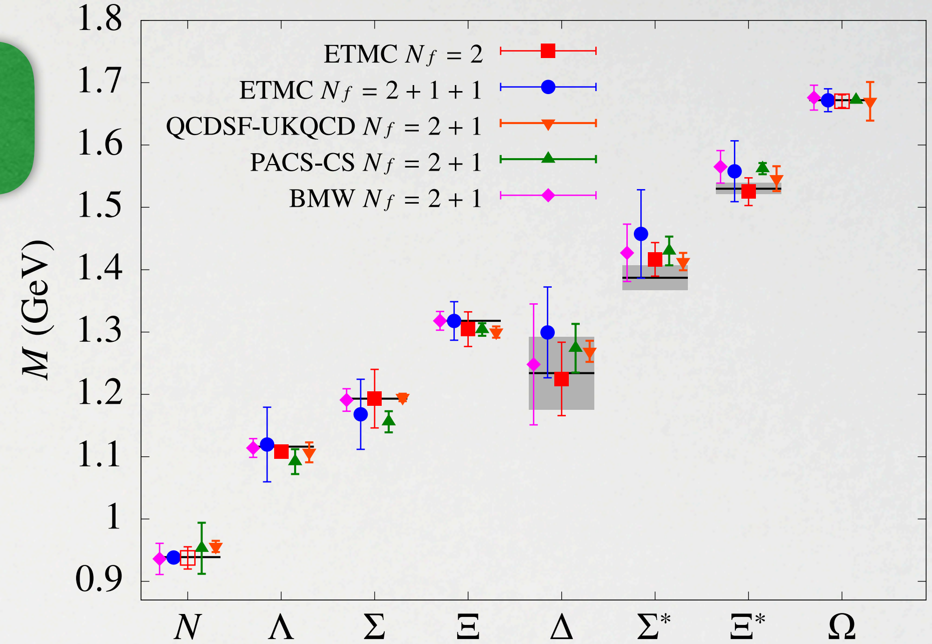
➡ In the past, lattice QCD focused $FFs(Q^2), GFFs(Q^2), \text{Charges}, \ \text{Moments}$

Computing observables on the Lattice

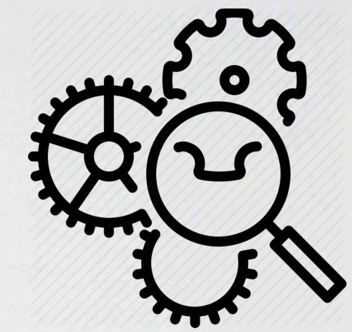
$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int_U \mathcal{O}(D^{-1}[U], U) \det(D[U]^{N_f}) e^{-S[U]}$$

LQCD



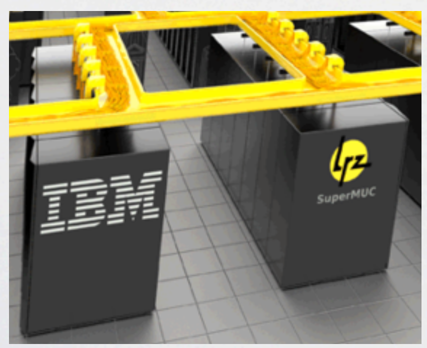


Phys. Rev. D96 (2017) no.3, 034511 1704.02647



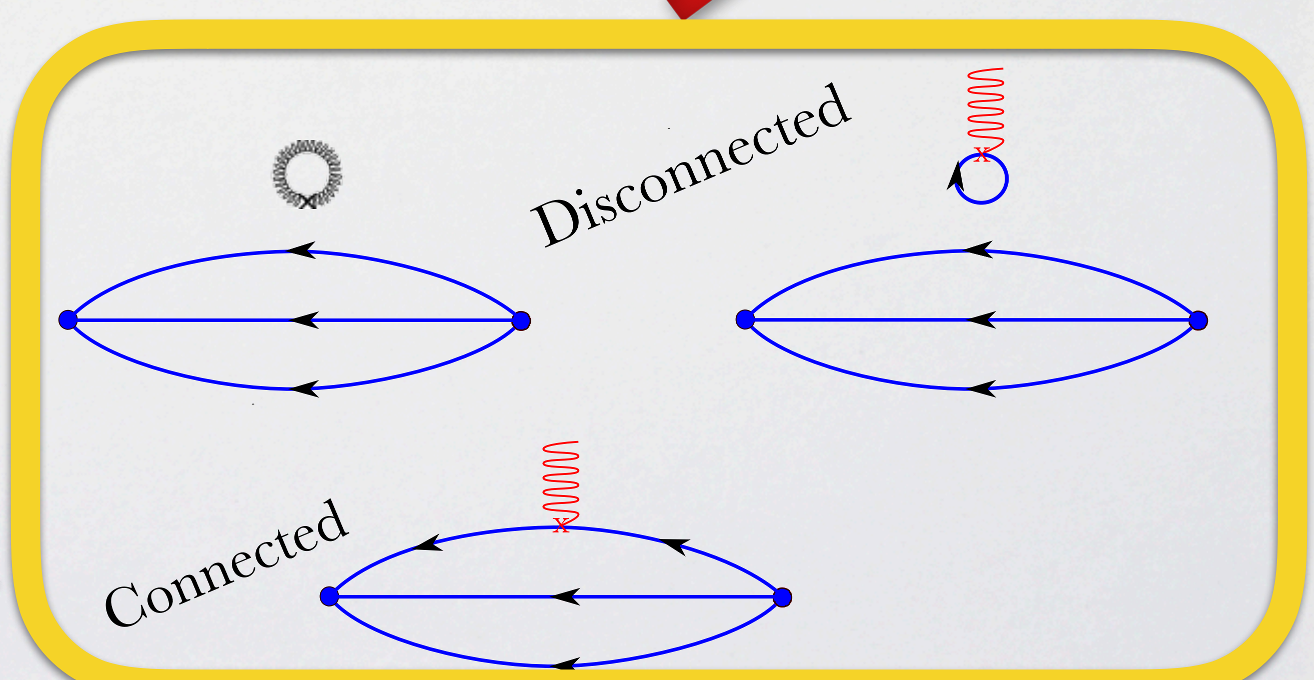
Data Analysis

Configurations Simulation



Quark Propagators

Contractions



0D quantities

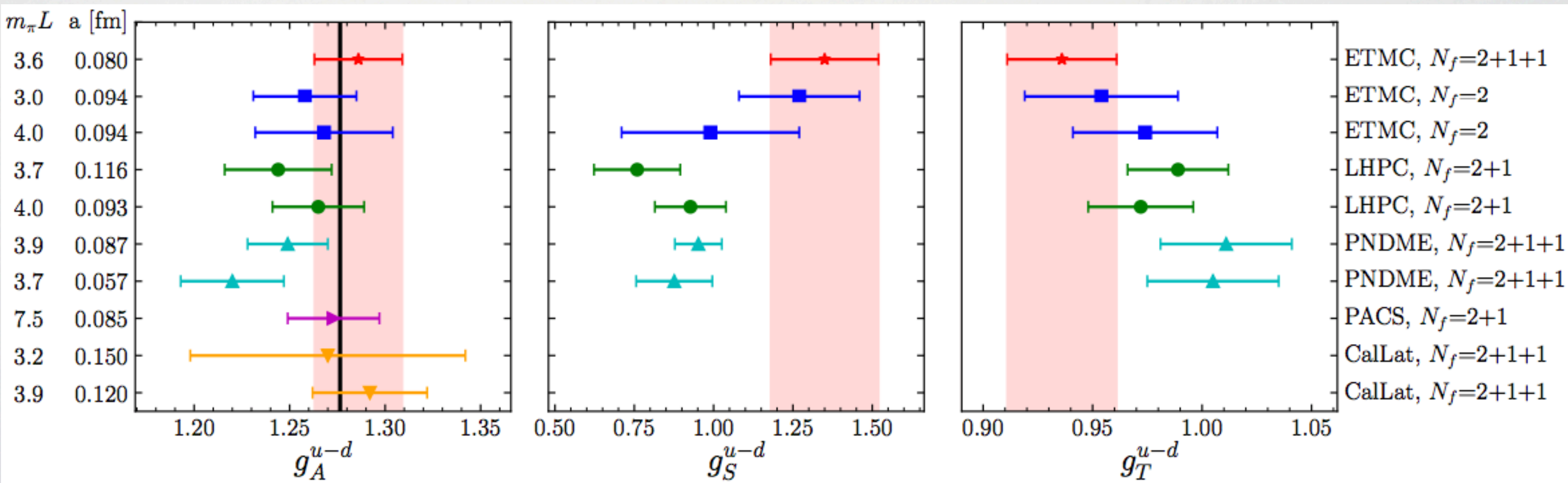
0D: Charges

$$\mathcal{O}(x) = \bar{\psi}(x)\Gamma\psi(x)$$

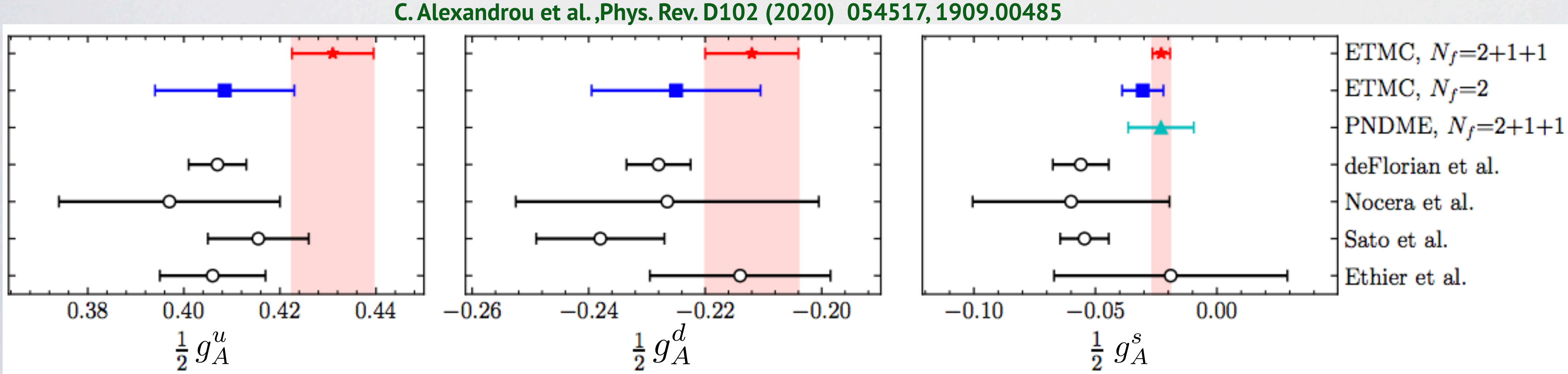
Matrix elements

$$\langle N|\mathcal{O}(x)|N\rangle \rightarrow g_{A,S,T}$$

- **Axial charge** ⟶ $\Gamma = \gamma_5\gamma_\mu$ ⟶ **First moment of the helicity PDF**
- **Scalar charge** ⟶ $\Gamma = 1$
- **Tensor charge** ⟶ $\Gamma = \sigma_{\mu\nu}$ ⟶ **First moment of the transversity PDF**



C. Alexandrou et al. ,Phys. Rev. D102 (2020) 054517, 1909.00485



0D: Second Moment of the unpolarized PDF

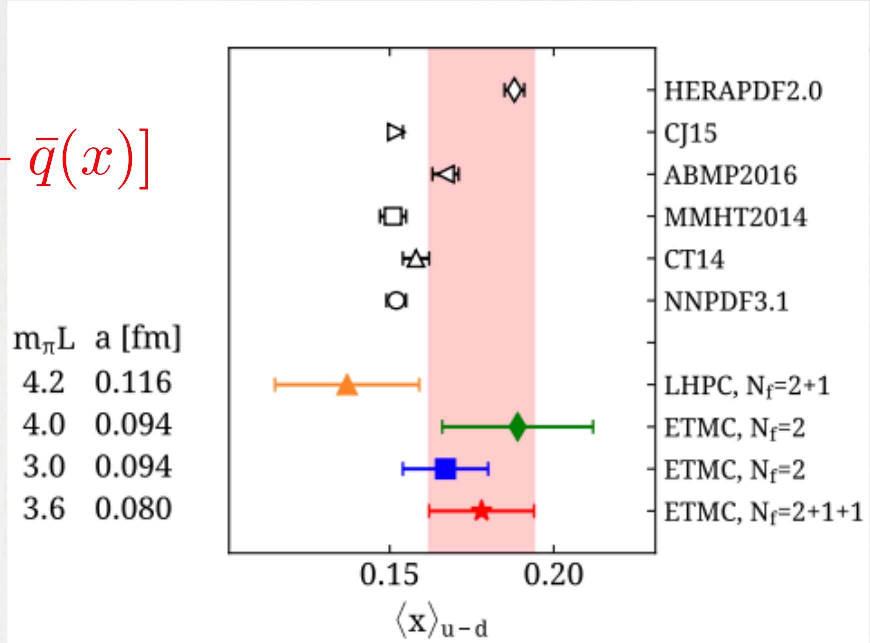
Direct access to moments of Parton Distribution Functions

$$\mathcal{O}^{\mu\nu}(x) = \bar{\psi}(x)\gamma^{\mu}D^{\nu}\psi(x)$$

$$\mathcal{O}^{\mu\nu}(x) = F^{\mu\rho}(x)F^{\nu}_{\rho}(x)$$

- Isovector combination is relatively easy to access
- Singlet is complicated because of the mixing and disconnected contributions

$$\langle x\rangle^{q^+} = \int dx\, x[q(x) + \bar{q}(x)]$$

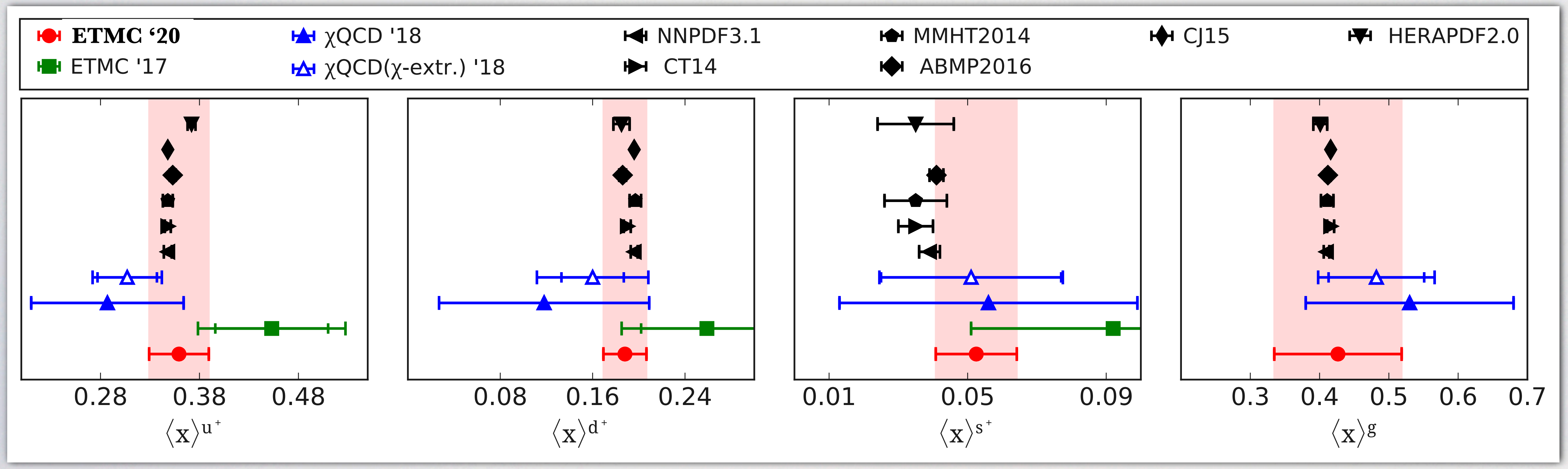


✳ Up and down quarks agree with phenomenology but lattice has very big errors
✳ Competitive results for the strange
✳ The gluon is still a challenge due to the large statistical errors

Quantities are given in $\overline{\mathrm{MS}}$ scheme at $\mu^2 = 4\ \mathrm{GeV}^2$

C. Alexandrou et al., Phys. Rev. D 101, 094513 (2020)

C. Alexandrou et al. ,Phys. Rev. D101 (2020) 034519, 1908.10706



1D quantities

Nucleon Vector matrix element describes the coupling of nucleon with a vector

$$\langle N(p',s')|j_\mu|N(p,s)\rangle \propto \bar{u}_N(p',s')\left(\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2)\right)u_N(p,s)$$

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{2m_N}F_2(q^2)$$

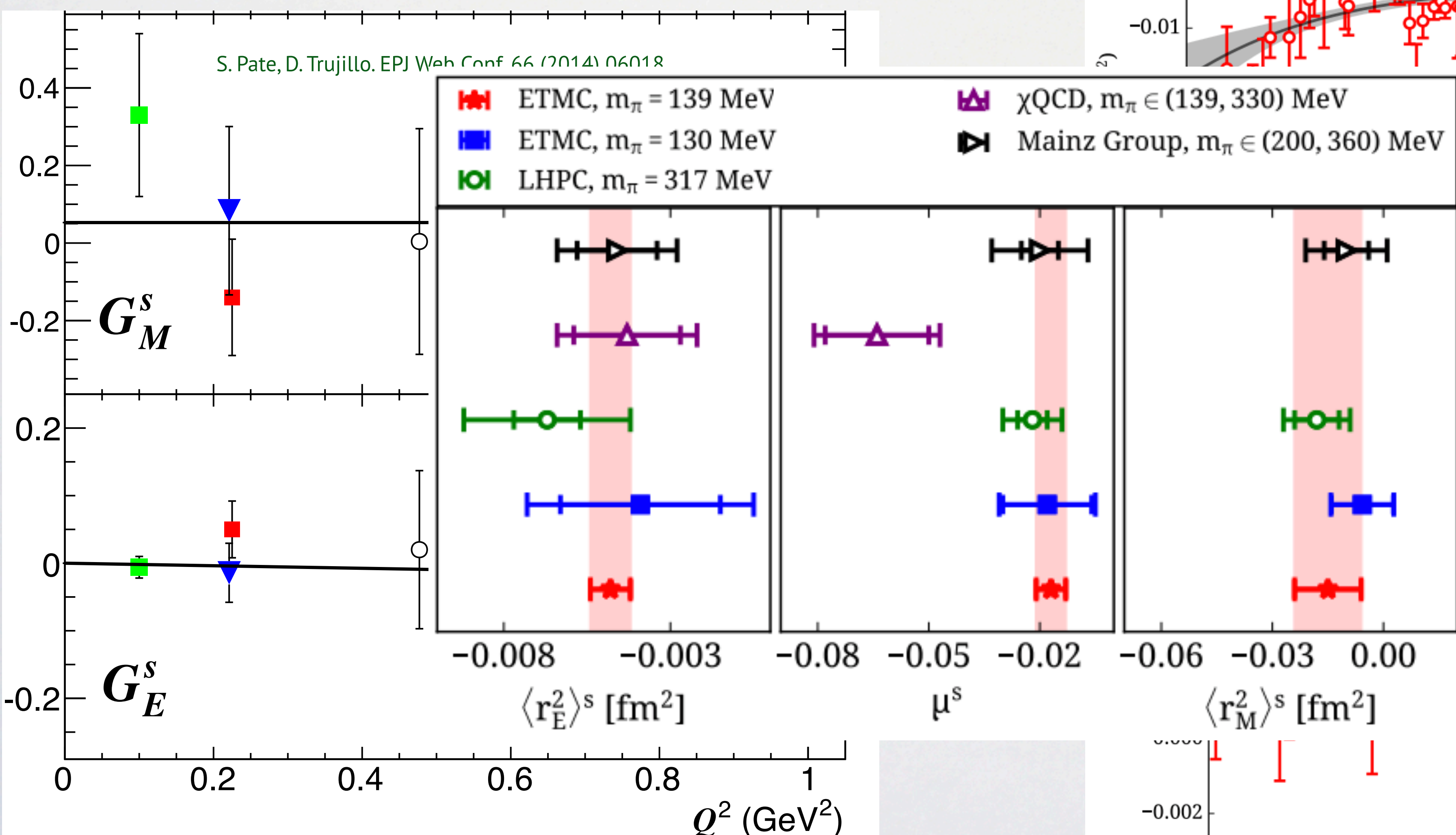
$$j^\mu(x) = \frac{2}{3}\bar{u}(x)\gamma_\mu u(x) - \frac{1}{3}\bar{d}(x)\gamma_\mu d(x) - \frac{1}{3}\bar{s}(x)\gamma_\mu s(x)$$

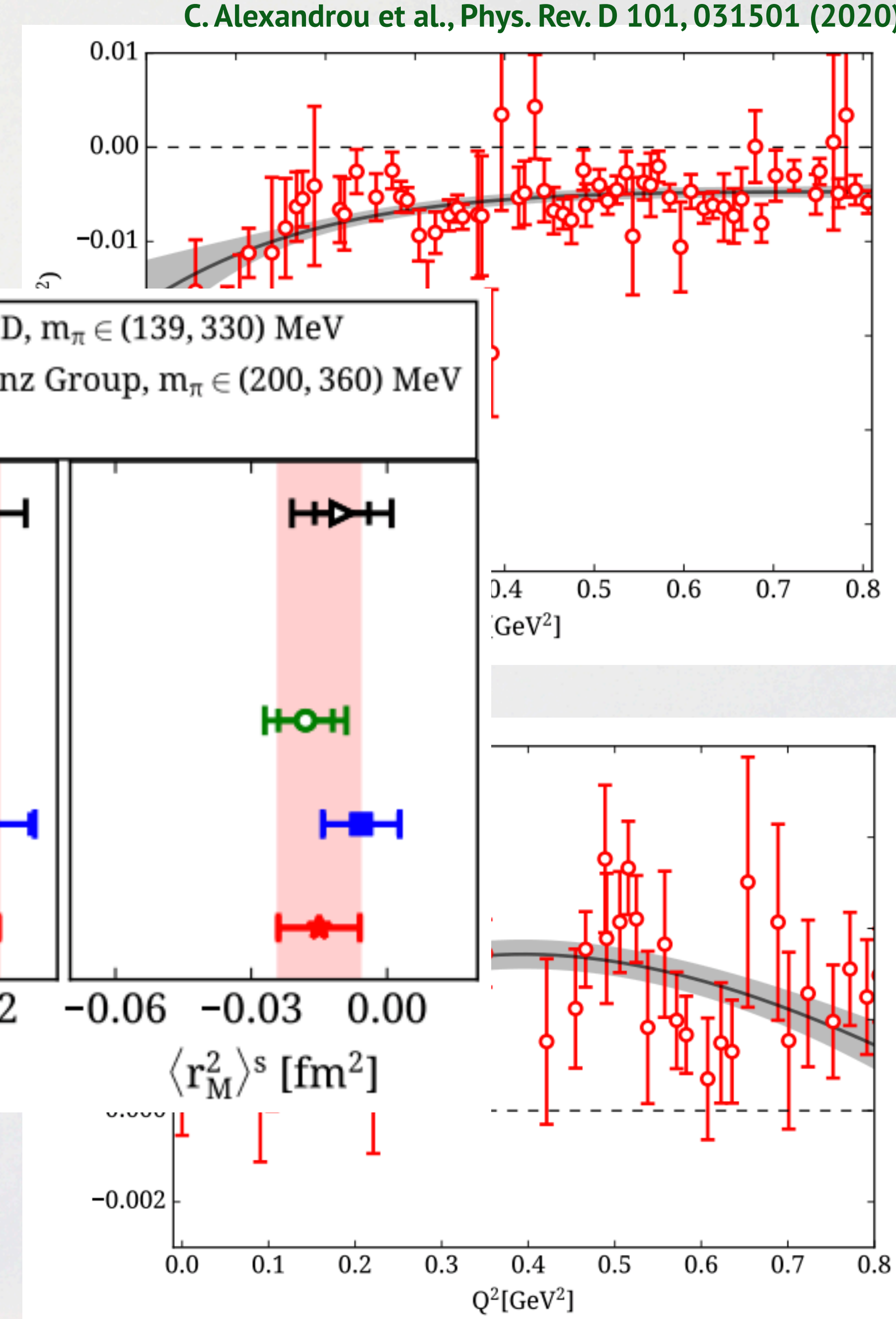
$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$

C. Alexandrou et al., Phys. Rev. D 101, 031501 (2020)

Parity-violating electron scattering data analysis

Experimental Data: E734, SAMPLE, HAPPEx,G0, PVA4



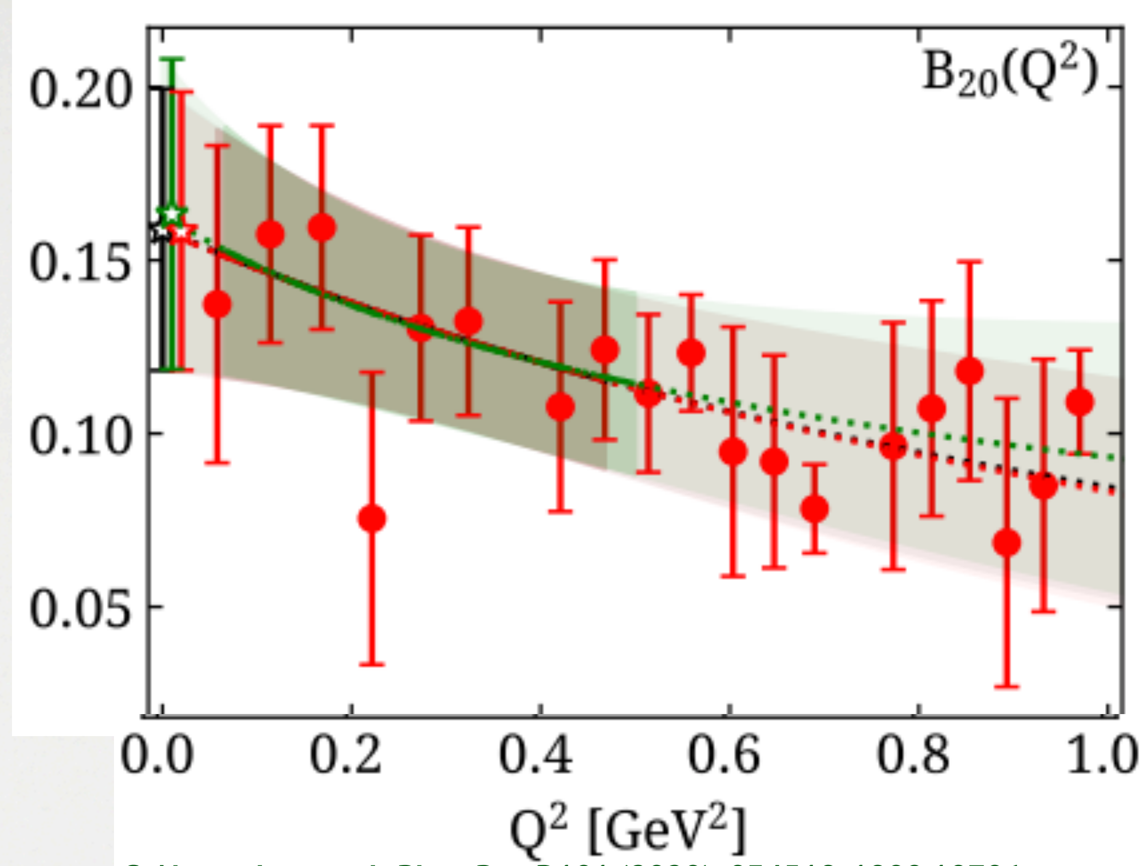


Results are consistent with zero

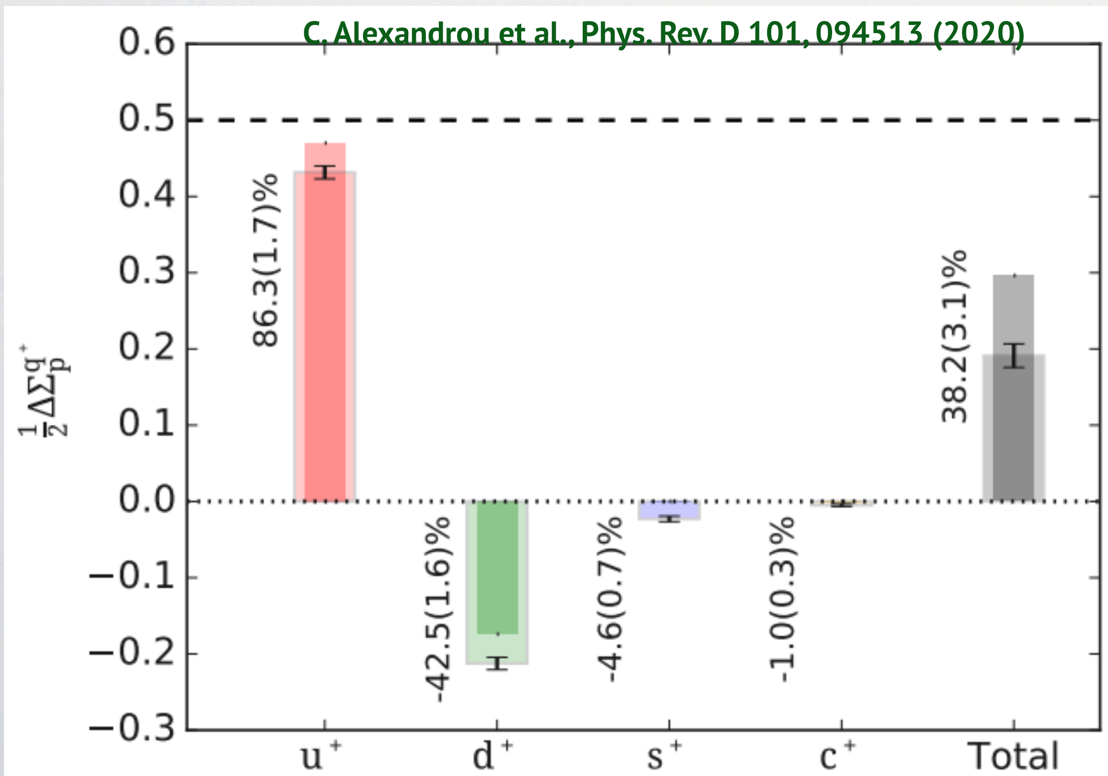
1D: Generalized FFs - Nucleon Spin

✱ Nucleon matrix element decomp.

$$\langle N(p',s')|\bar{T}^{\mu\nu;q,g}|N(p,s)\rangle = \bar{u}_N(p',s')\Big[A_{20}^{q,g}(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}^{q,g}(q^2)\frac{i\sigma^{\{\mu\rho}q_\rho P^{\nu\}}}{2m_N} + C_{20}^{q,g}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m_N}\Big]u_N(p,s)$$



C. Alexandrou et al.,Phys. Rev. D101 (2020) 034519, 1908.10706



- About 40% to the nucleon spin from intrinsic quark spin

X. Ji, Phys. Rev. Left. 78, 610 (1997)

✱ Summary Equations

$$J^{q,g} = \frac{1}{2}\left[A_{20}^{q,g}(0) + B_{20}^{q,g}(0)\right]$$

$$L^q = J^q - \frac{1}{2}\Delta\Sigma^q$$

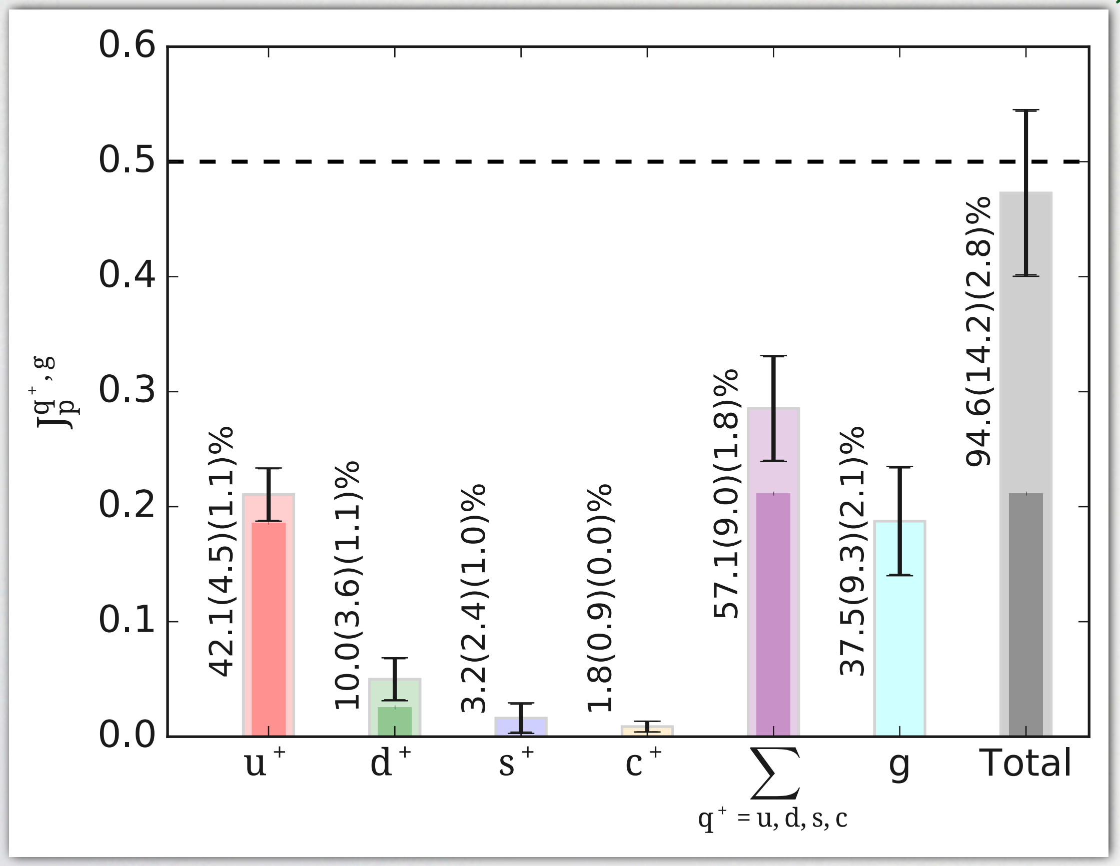
$$\frac{1}{2} = \sum_q \left(\frac{1}{2}\Delta\Sigma_{q+} + L_{q+}\right) + J_g$$?

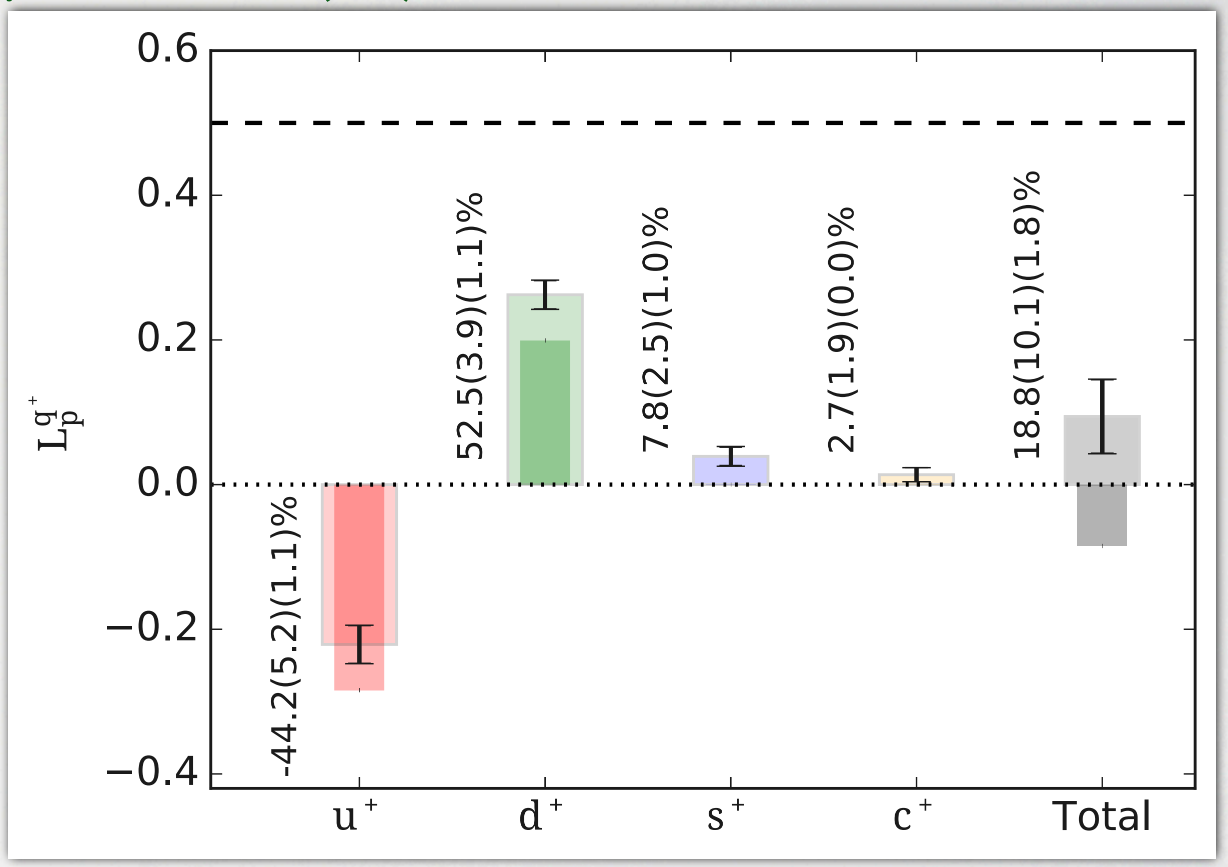
1D: Nucleon Spin

✱ **Angular momentum**

✱ **Orbital Angular momentum**

C. Alexandrou et al., Phys. Rev. D 101, 094513 (2020)





- Including up, down, strange and charm quarks we get about 60% of the proton spin
- About 40% comes from the gluon contribution but results still have large uncertainties

- The up quark is orbiting in the opposite direction compared to the proton spin like the down quark
- Up and down quarks are also cancelling each other
- Total has large uncertainties

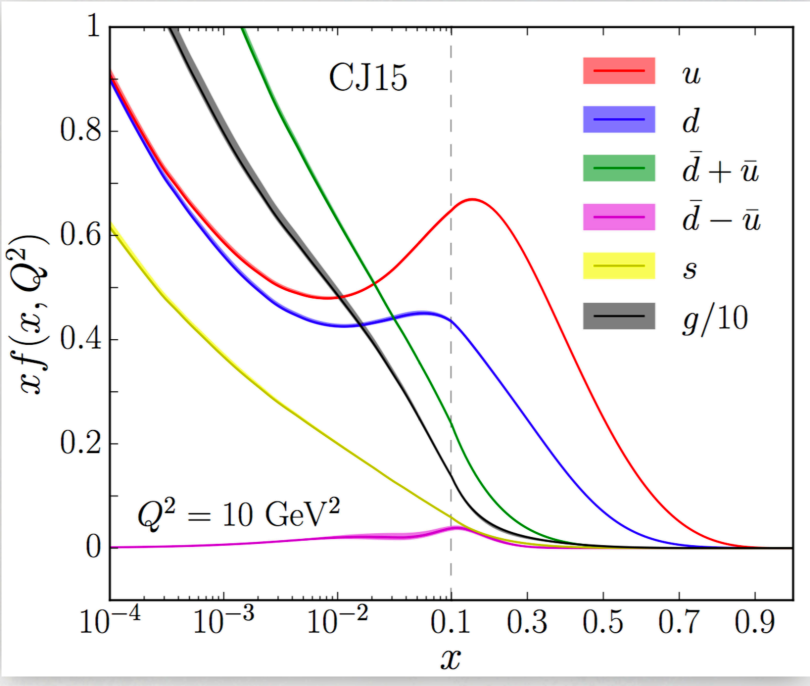
1D: Parton Distribution Functions (PDFs)

★ Universal tools to study hadron structure (1D)

★ Global fit analyses of DIS data: main source of information

★ Global fits not without ambiguities

A. Accardi et al., arXiv:1602.03154]



➡ Calculation from first principle is needed

★ PDFs parameterized in terms of off-forward matrix elements of non-local light-cone operators (Not accessible in Euclidean lattice)

★ Lattice QCD: long-standing history of moments of PDFs (via OPE), but reconstruction of PDFs not feasible (gauge noise, mixing)

★ Alternative approaches proposed, e.g.: quasi-PDFs, pseudo-PDFs, auxiliary heavy quark, hadronic tensor, OPE w/o OPE

★ All methods are under investigation in lattice QCD

1D: Accessing PDFs on a Euclidean

Matrix elements of spatial operators with fast moving hadrons

$$\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \langle N(P_3)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathcal{A}(z,0)\Psi(0)\,|\,N(P_3)\rangle_{\mu^2}$$

Contact with light-cone PDFs feasible:

- ★ **Matching procedure in large momentum ET (LaMET) to relate quasi-PDFs to light-cone PDF**

X. Ji, Phys. Rev. Left. 110 (2013) 262002

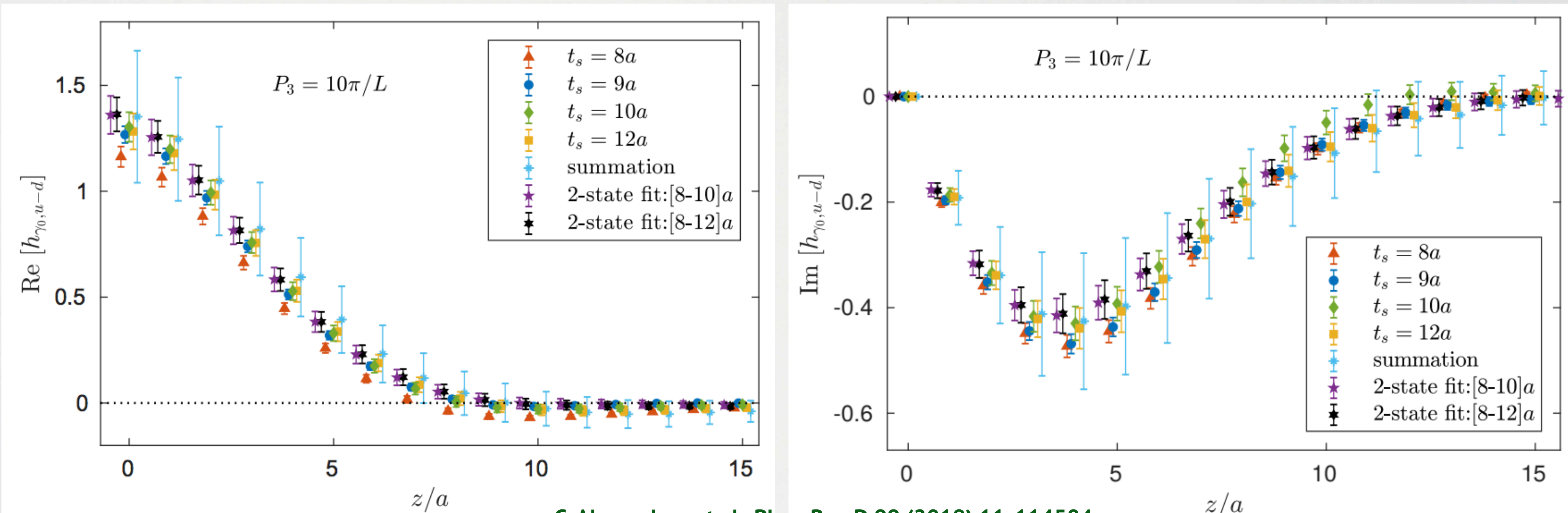
- ★ **Difference reduced as P_3 increases** $\mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, m_N^2/P_3^2\right)$

1D: Progress in isovector PDFs

★ **Prior 2017 lattice calculations missing two main ingredients, preventing comparison with phenomenological data on PDFs**

- **Proper renormalization prescription**
- **Appropriate matching expressions for lattice data**

- Unpolarized PDFs - Investigation of excited states cont.

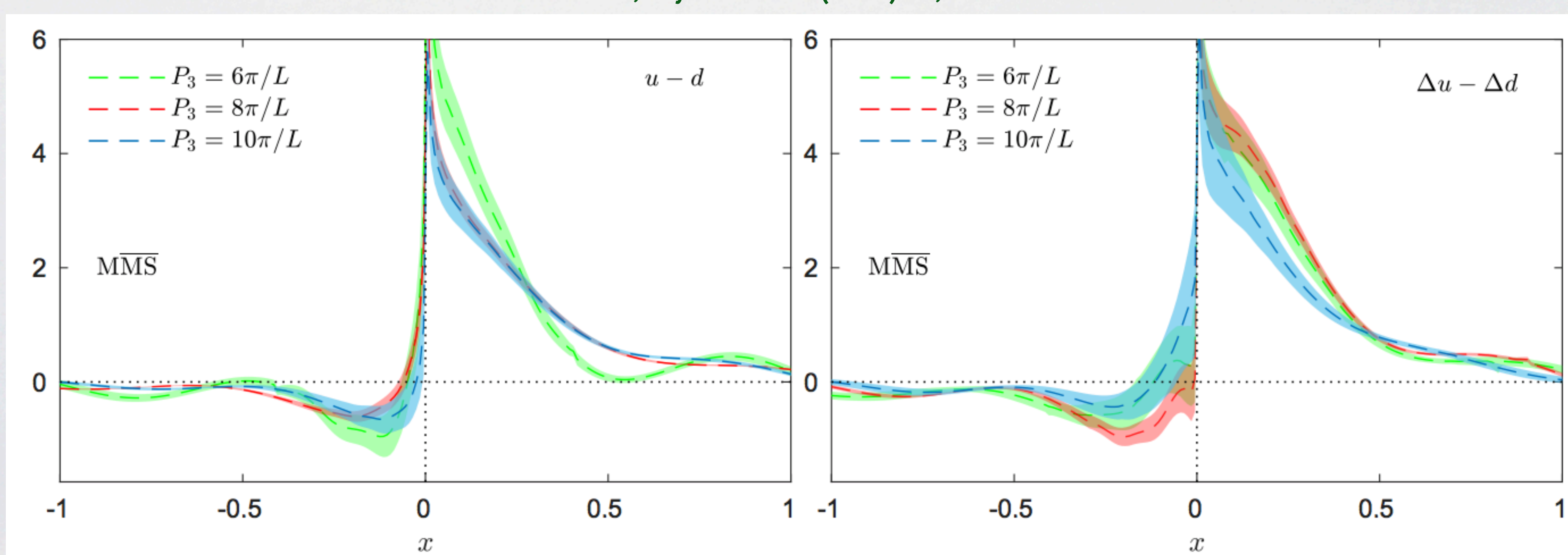


C. Alexandrou et al., Phys. Rev. D 99 (2019) 11, 114504

➡ Investigation of the momentum dependence of the PDFs

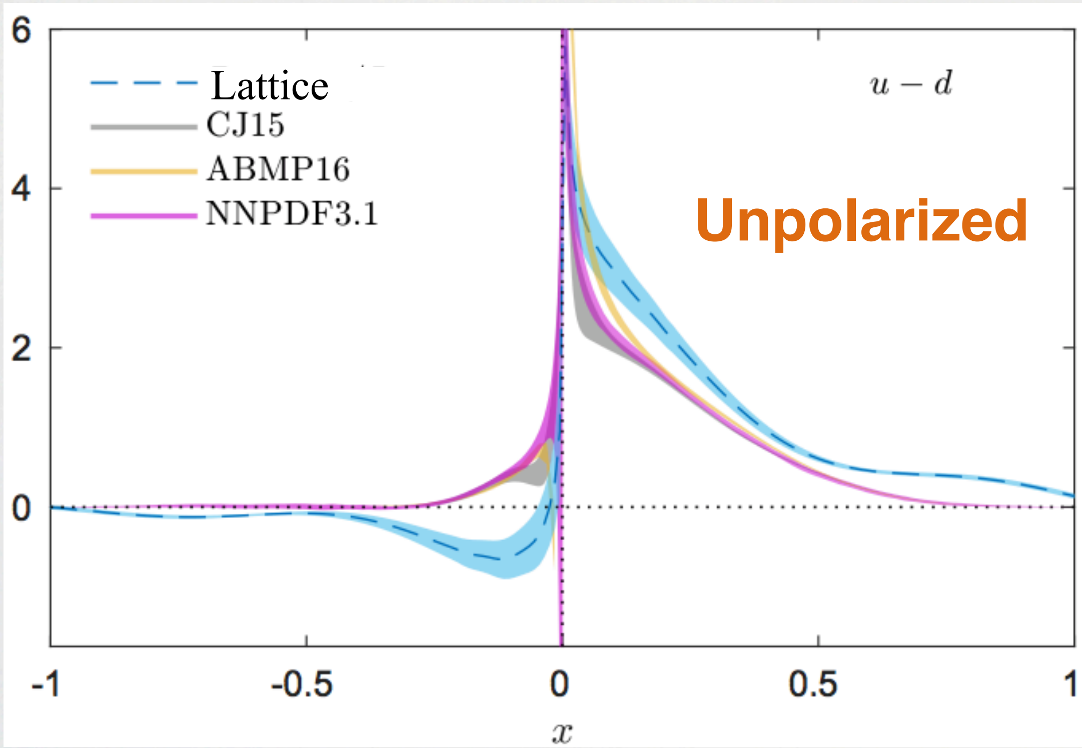
➡ Unphysical oscillations are reduced with momentum

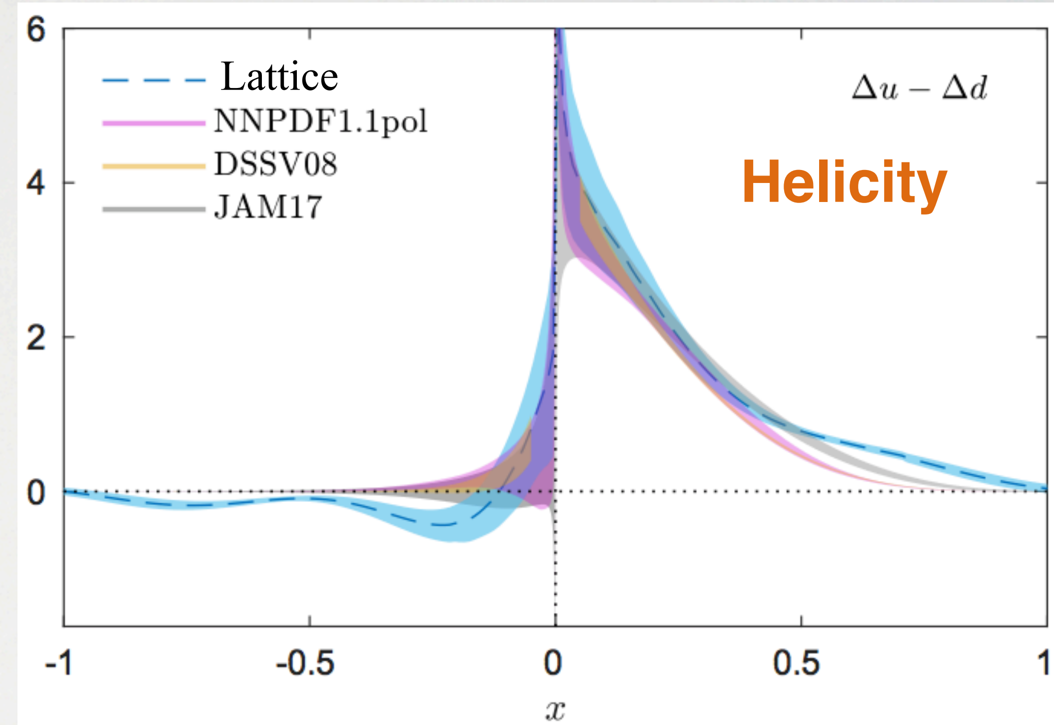
➡ Higher momenta are challenging



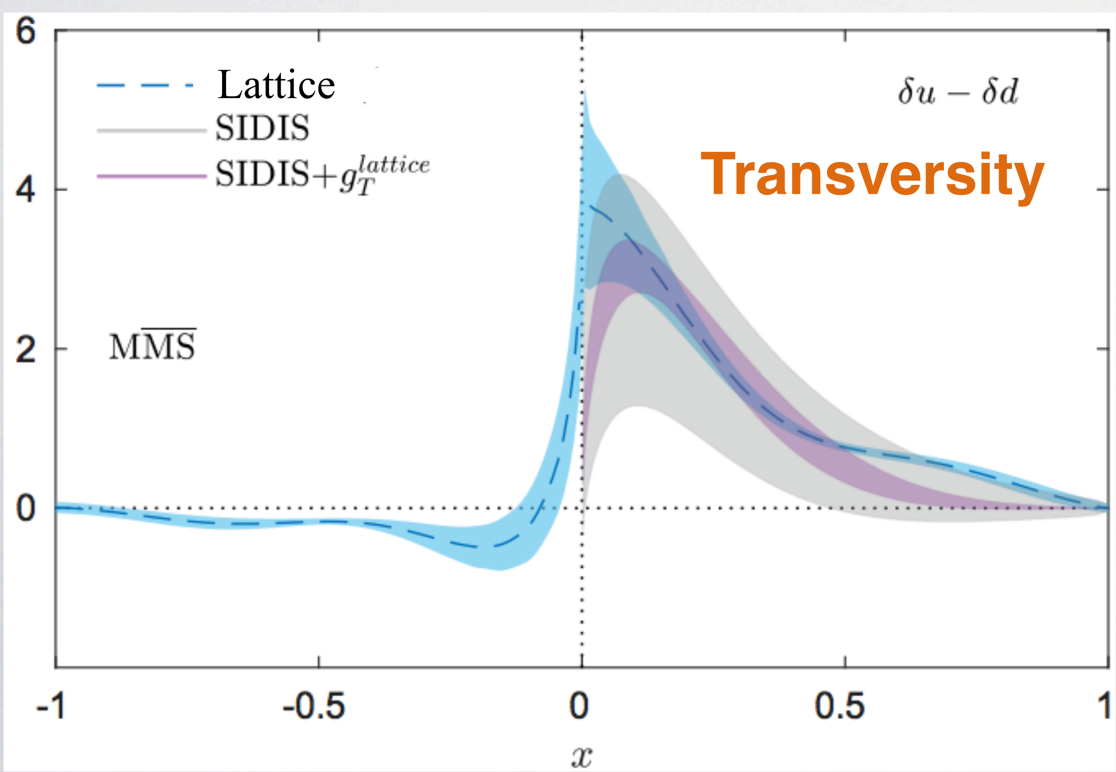
1D: Progress in isovector PDFs - Comparison

Comparison of lattice results with phenomenology





C. Alexandrou et al., Phys. Rev. D 99 (2019) 11, 114504

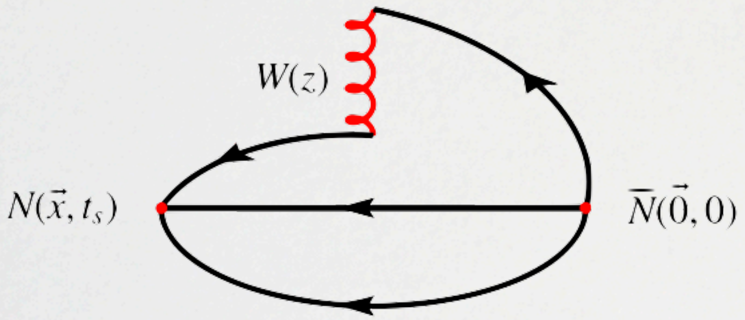


- ★ **Unpolarized case: Discrepancy with phenomenology - More thorough investigation is needed**
- ★ **Helicity case: Better behaved and mostly in agreement with phenomenology**
- ★ **Transversity case: Lattice results are much better constrained**

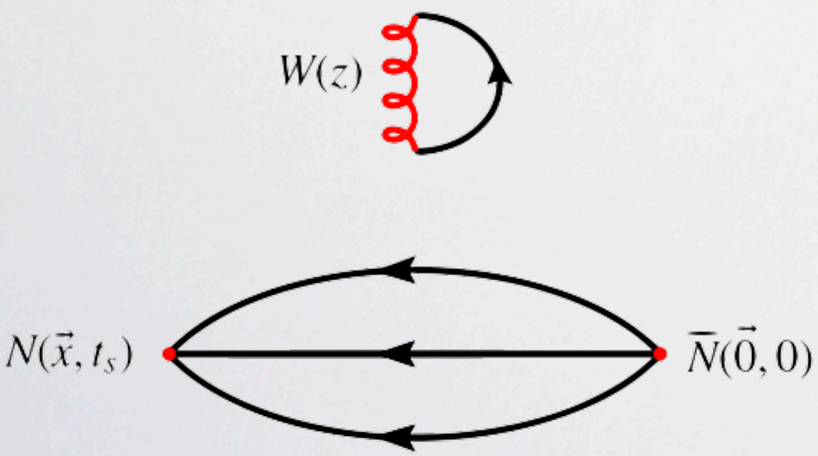
1D: Flavor decomposition - Helicity case

Up to now Lattice QCD has concentrated in the isovector combination of PDFS

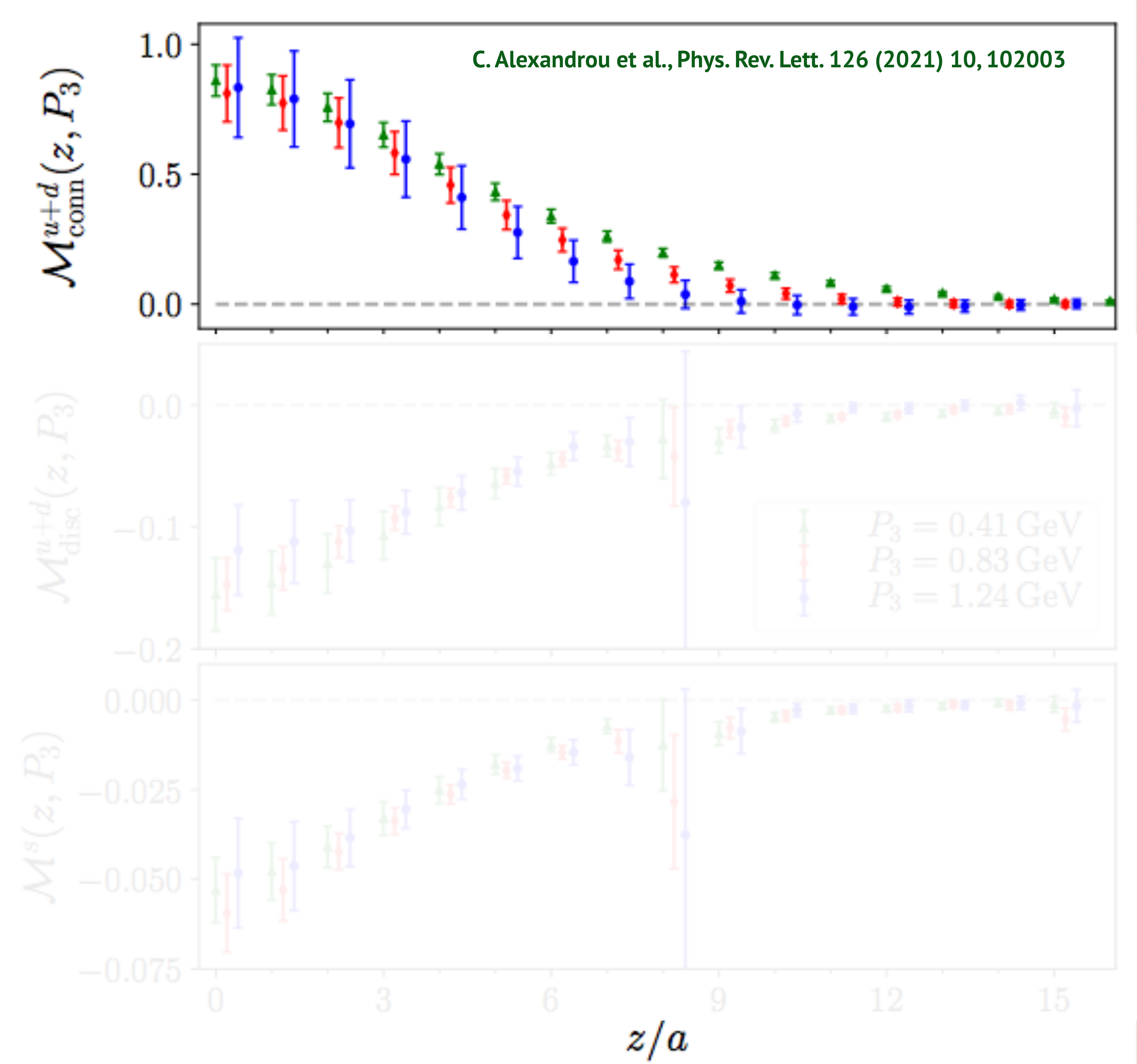
- Avoids expensive disconnected diagrams
- Avoids mixing with gluons



- Isoscalar connected diagram: Straight forward to implement, computationally cheap



- Isoscalar disconnected diagram: Quark loop with Wilson line needs sophisticated algorithms - Computationally very demanding
- Mixing with gluon is in progress…

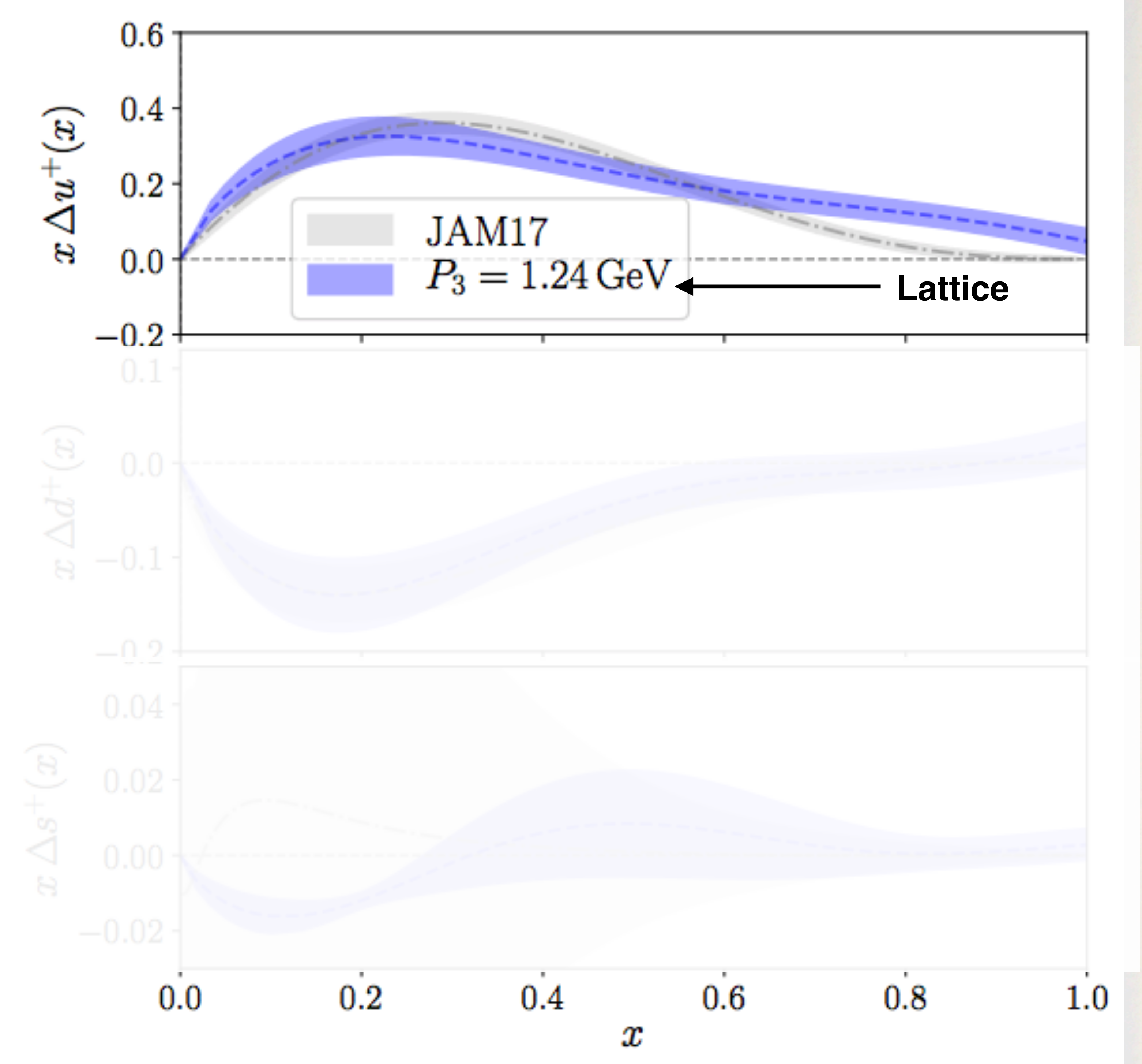


1D: Flavor decomposition - Helicity case

First results about the flavor decomposition of the helicity Parton Distribution Function

$$q^+(x) \equiv q(x) + \bar{q}(x)$$

- ★ **Calculation is done at a heavier than physical pion mass**
- ★ **For the up quark mostly compatible except for x>0.6**
- ★ **For the down quark the agreement is better but the errors are bigger**
- ★ **For the strange quark, JAM17 results are very noisy - Lattice has the potential to have competitive results**



3D quantities

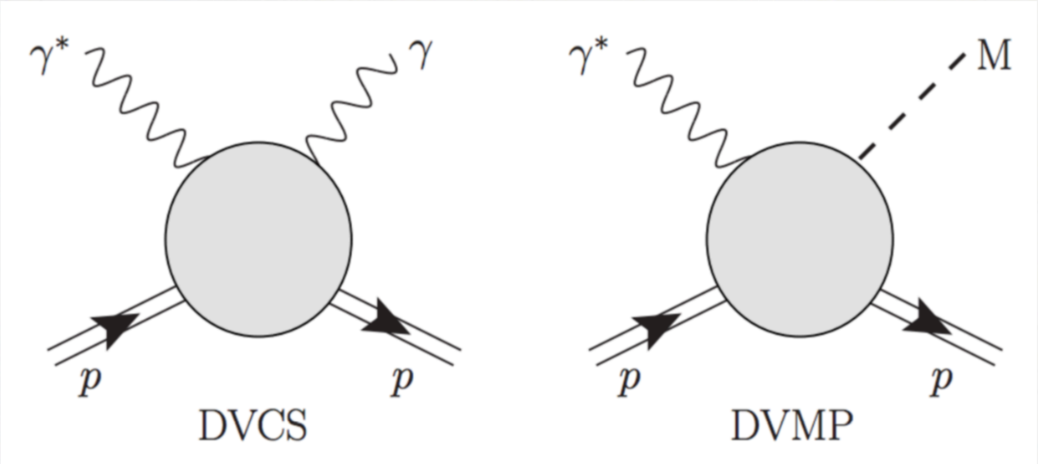
3D: Generalized Parton Distribution

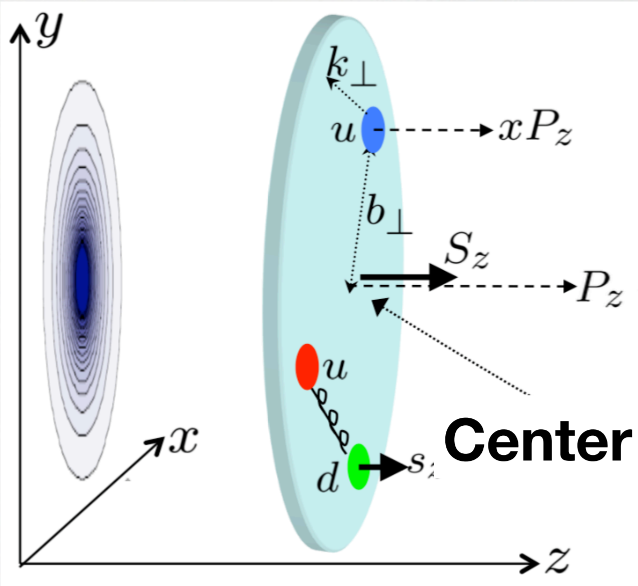
★ GPDs provide information on spatial distribution of partons inside the hadron, and its mechanical properties M. Burkardt, Phys. Rev. D 62 071503 (2000), hep-ph/0005108

★ GPDs can be accessed experimentally in Deeply Virtual Compton Scattering (DVCS) and Deeply Virtual Meson Production (DVMP)

X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249

(Halls A,B,C (JLab), PHENIX, STAR, HERMES, COMPASS, GSI, BELLE, J-PARC)





★ Experimentally, GPDs are not well-constrained:

- independent measurements to disentangle GPDs
- limited coverage of kinematic region
- data are available on certain GPDs
- GPDs phenomenology is more complicated than PDFs (multi-dimensional objects)

3D: Generalized Parton Distribution - Lattice

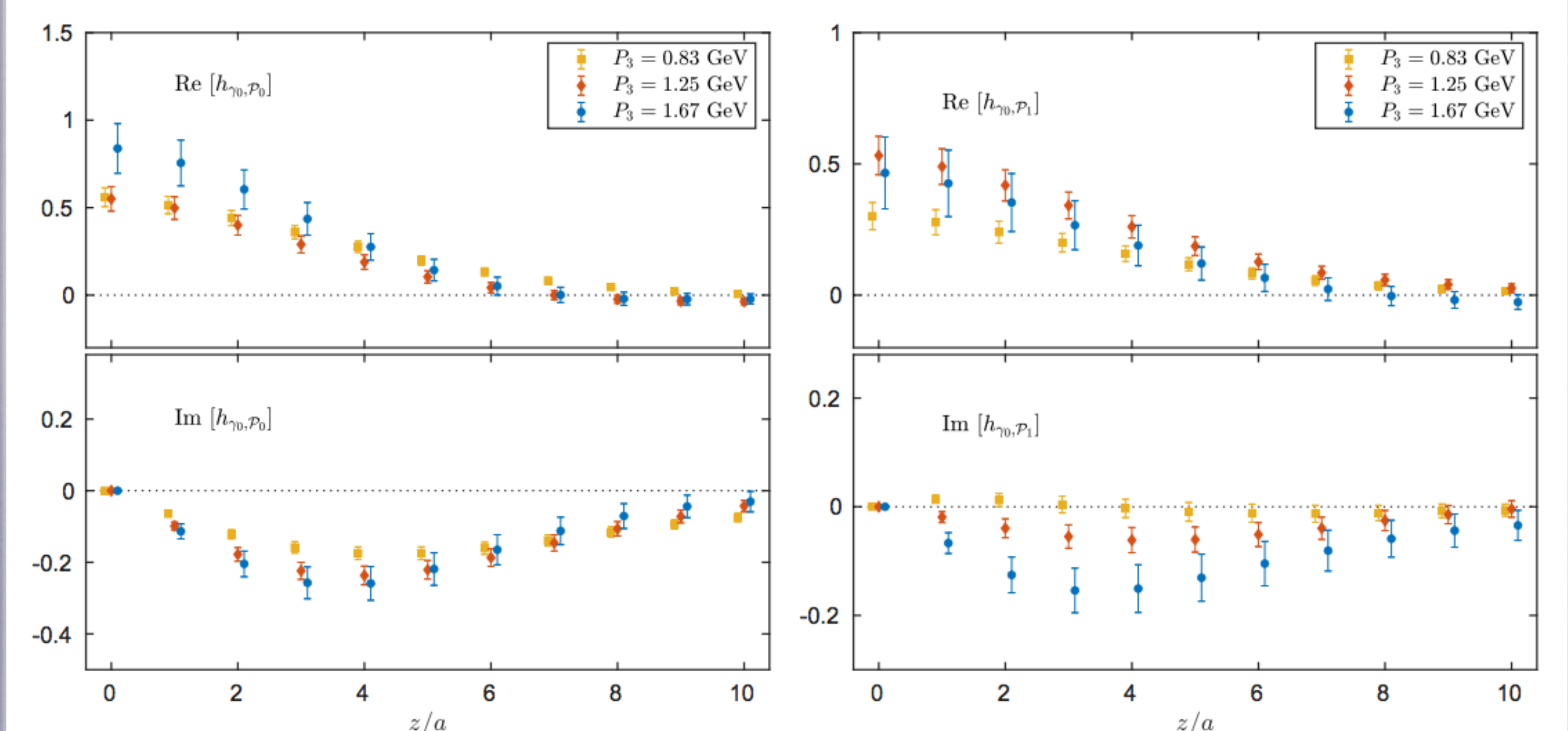
Decomposition of matrix elements to GPDs

Unpolarized

$$\Pi_{\gamma^0}(\mathcal{P}_0; P_f, P_i) = C\left[F_H(Q^2)\left(\frac{E_f E_i}{2m^2} + \frac{E_f+E_i}{4m} + \frac{P_{f_\rho}P_{i_\rho}}{4m^2} + \frac{1}{4}\right) + F_E(Q^2)\left(P_{f_\rho}P_{i_\rho}\left(\frac{E_f+E_i}{8m^3}+\frac{1}{4m^2}\right) - \frac{(E_f-E_i)^2}{8m^2} + \frac{E_f+E_i}{8m}+\frac{1}{4}\right)\right],$$

$$\Pi_{\gamma^0}(\mathcal{P}_j; P_f, P_i) = i\,\epsilon_{j0\rho\tau}C\left[F_H(Q^2)\frac{P_{f_\rho}P_{i_\tau}}{4m^2} + F_E(Q^2)\left(\frac{(E_f+E_i)P_{f_\rho}P_{i_\tau}}{8m^3} + \frac{P_{f_\rho}P_{i_\tau}-P_{f_\tau}P_{i_\rho}}{8m^2}\right)\right],$$

$$\Pi_{\gamma^3\gamma^5}(\mathcal{P}_j; P_f, P_i) = i\,C\left[F_{\tilde{H}}(Q^2)\left(\delta_{j3}\left(\frac{E_f+E_i}{4m} - \frac{P_{f_\rho}P_{i_\rho}}{4m^2} + \frac{1}{4}\right) + \frac{P_{i_3}P_{f_j}+P_{f_3}P_{i_j}}{4m^2}\right)\right.$$



C. Alexandrou et al., Phys.Rev.Lett. 125 (2020) 26, 262001

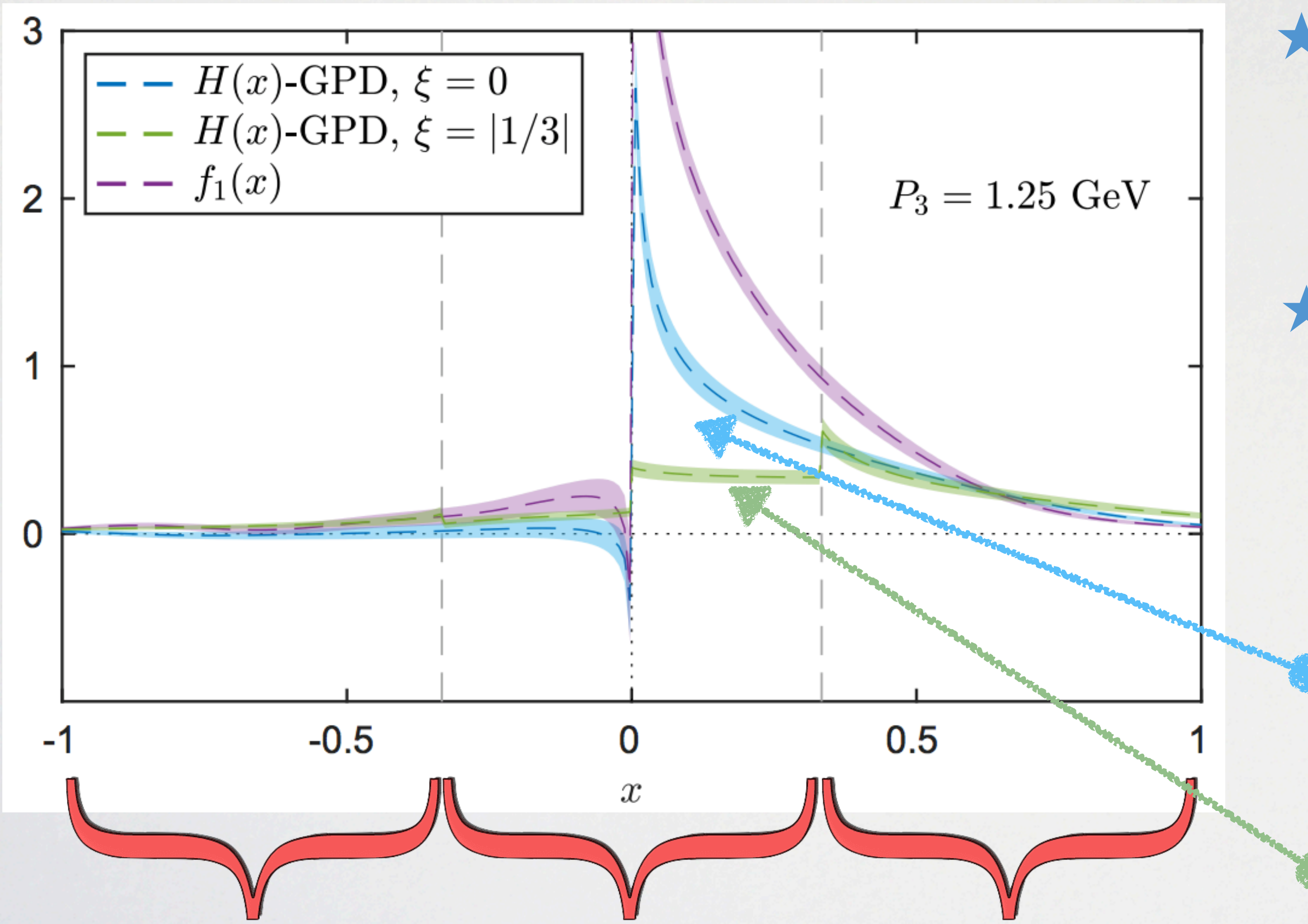
- Kinematic factors defined by calculation setup
- Decomposition similar to the way we decompose the form factors
- Two unpolarized GPDs $H(x,\xi,Q^2),\ E(x,\xi,Q^2)$
- Two helicity GPDs $\tilde{H}(x,\xi,Q^2),\ \tilde{E}(x,\xi,Q^2)$

$\xi = -\frac{Q_3}{2P_3}$ Skewness

- Results with zero Skewness
- Various combinations of matrix elements to isolate GPDs is needed

3D: Generalized Parton Distribution - Lattice

C. Alexandrou et al., Phys.Rev.Lett. 125 (2020) 26, 262001



DGLAP ERBL DGLAP

★ **ERBL region:** $(|x| < \xi)$

amplitude of removing a quark-antiquark pair with momentum -Q_3

★ **DGLAP region:** $(\xi < |x| < 1)$

amplitude of removing a quark-antiquark pair from the hadron and inserting it back with momentum increase Q_3

$\xi = 0, \quad Q^2 = 0.69 \text{ GeV}^2$

$|\xi| = 1/3, \quad Q^2 = 1.38 \text{ GeV}^2$

★ **ERBL region:** $(|x| < \xi)$

- ***f_1(x)*** dominant
- ***H(x,1/3)*** suppressed compared to ***H(x,0)***

★ **DGLAP region:** $(\xi < |x| < 1)$

- ***H(x,1/3)*** similar behavior as ***H(x,0)***
- ***H(x,0)*** is asymptotically equal to ***f_1(x)***

Summary

- LQCD can compute nucleon charges, second moments **(0D)** reliably at the physical point of the theory, (axial, scalar, tensor, average momentum fraction)

- Strange EM form factors **(1D)** have been computed in good precision compared to experimental determinations

- LQCD can compute isovector PDFs **(1D)** now even at the physical point
  - Sytematic errors are constantly improving

- First steps (recently) to do the flavor decomposition of PDFs **(1D)**
  - Mixing with the gluons need to be treated
  - Simulations at the physical point needed

- First results for isovector GPDs for unpolarized and helicity **(3D)**
  - Need to find a way to access smaller Q^2 values
  - Simulations at the physical point needed
  - Transversity GPD is on the way