Origin of Confinement – How the EIC can provide insight

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In collaboration with:

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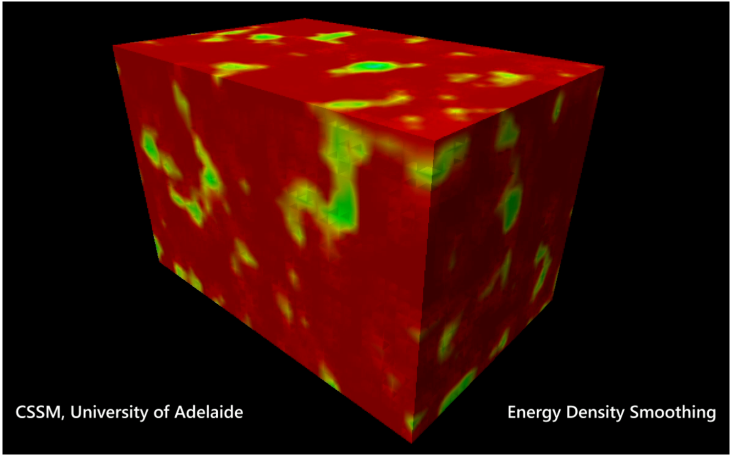
Amalie Trewartha, & Adam Virgili

Introduction



Centre-Vortices capture the essence QCD Vacuum Structure

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  - Gluon propagator – J. C. Biddle, W. Kamleh and DBL, Phys. Rev. D **98** (2018) 094504.
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- What is the origin of confinement in QCD?

Vortex Structure in the Colour Fields of the QCD Vacuum

What Are Centre Vortices?

- Centre vortices in 3D are tube-like topological defects present in the QCD vacuum.
- We locate thin vortex lines on the lattice.
- The vortex line can be thought of as the 'axis of rotation' of the vortex.

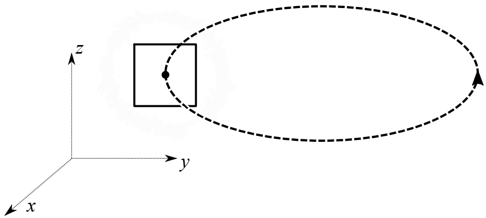


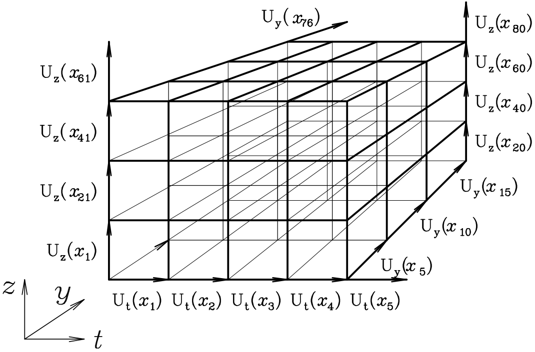
Figure: A centre vortex (dashed line) intersecting a lattice plaquette (solid square).

What Are Centre Vortices?

- On the lattice, the gluon-field is encoded in terms of the link variable

$$U_\mu^{ab}(x) \simeq \exp\Big(i\,a\,g\,A_\mu^{ab}(x)\Big) \;,$$

  a 3×3 complex special-unitary matrix.



1. Maximal Centre Gauge

- Gauge transformations are applied to the link variables.
- The gauge links are brought as close as possible to an element of the group *Centre*, $Z(3)$.
- The centre elements of $SU(3)$ are the cube roots of 1, namely

$$Z(3) = \exp\left(\frac{2\pi i}{3}\, m\right) I\,, \;\; m \in \{-1, 0, +1\}$$

- This is done by maximising the functional

$$R = \sum_x \sum_\mu |\, \mathrm{tr}[U_\mu(x)]\,|^2$$

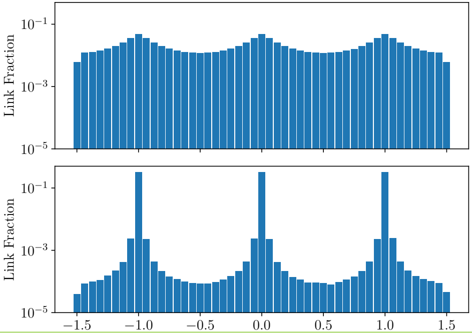
- This is called **Maximal Centre Gauge**

1. Maximal Centre Gauge

- Distribution of link phases following gauge fixing to Maximal Centre Gauge.

$$\mathrm{tr}\, U_\mu^{\mathrm{MCG}}(x) = \underbrace{r_\mu(x)}_{\text{real}} \exp\underbrace{\left(\frac{2\pi i}{3}\phi_\mu(x)\right)}_{\text{phase}}, \quad -\frac{3}{2} < \phi_\mu(x) \le \frac{3}{2}.$$

- Before gauge fixing.
- After gauge fixing.



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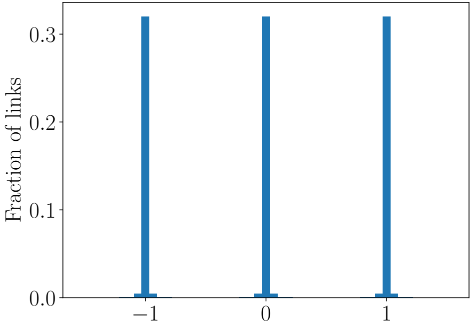


2. Centre Projection

- We then project onto $Z(3)$

$$\phi_\mu(x) \to m_\mu(x) \in \{-1,\, 0,\, +1\}$$

such that

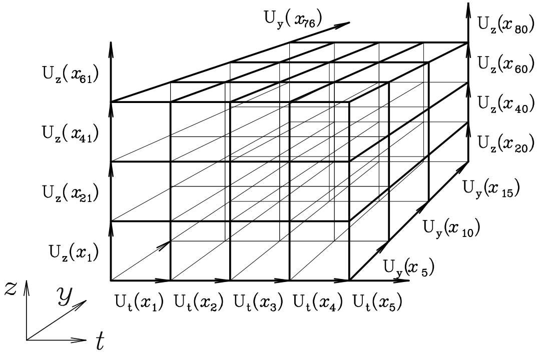
$$U_\mu^{\mathrm{MCG}}(x) \to Z_\mu(x) = \exp\left(\frac{2\pi i}{3}\, m_\mu(x)\right) I\,, \;\; m_\mu(x) \in \{-1,\, 0,\, +1\}\,,$$

i.e. a cube-root of 1 times the identity matrix.

- Eight degrees of freedom are replaced by one of the three cube-roots of 1.

3. Identifying Vortices

- Examine the product of $Z_\mu(x)$ around each elementary square (plaquette).
- Each plaquette takes a value from $Z(3)$.



3. Identifying Vortices

- Examine the product of $Z_\mu(x)$ around each elementary square (plaquette).
- Each plaquette takes a value from $Z(3)$.
- Non-trivial plaquettes with values

$$\exp\left(\frac{2\pi i}{3}\, m\right) I\,, \quad m \in \{-1, +1\}\,,$$

  identify our thin vortices.
- Thin vortices locate the centre of the physical thick vortices

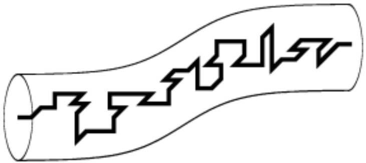


Figure: An example of a vortex path embedded within a thick vortex. M. Engelhardt, H. Reinhardt, Nuclear Physics B **585** (2000) 597

Configurations

- This projection allows us to define 3 sets of configurations:
  - Untouched - $U_\mu(x)$
  - Vortex Only - $Z_\mu(x)$
  - Vortex Removed - $R_\mu(x) = Z_\mu^\dagger(x)\, U_\mu(x)$

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- 4 ensembles
  - $20^3 \times 40$ pure gauge (PG), spacing $a = 0.125$ fm
  - $32^3 \times 64$ pure gauge (PG), spacing $a = 0.100$ fm
  - $32^3 \times 64$ dynamical $2+1$ flavour, spacing $a = 0.1022$ fm, $m_\pi = 701$ MeV
  - $32^3 \times 64$ dynamical $2+1$ flavour, spacing $a = 0.0933$ fm, $m_\pi = 156$ MeV
  - S. Aoki, *et al.* (PACS-CS), Phys. Rev. D **79**, 034503.

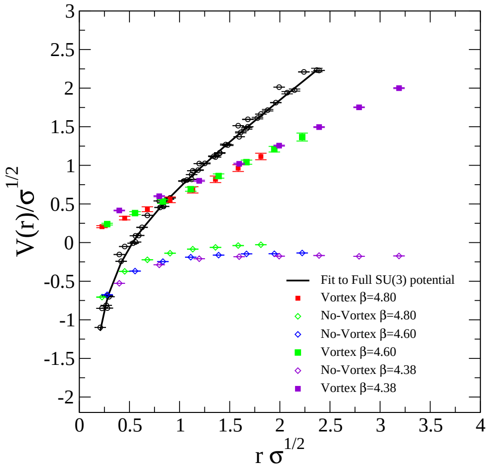
Phenomenology of Centre Vortices

Static Quark Potential

- Measures the potential energy between two massive, static quarks at separation r.
- Serves as an indicator of confining behaviour in the form of a linear long-range potential.
- Typically described via the Cornell potential

$$V(r) = V_0 - \frac{\alpha}{r} + \sigma\, r$$

Centre Vortices and Confinement – Pure Gauge Sector



MCG procedure cannot simultaneously identify all SU(3) vortex matter.

O'Cais et al, Phys. Rev. D **82**, *114512 (2010).*

Figure from

Bowman et al, Phys. Rev. D **84**, *034501 (2011).*

Centre Vortices and the Landau-Gauge Gluon Propagator

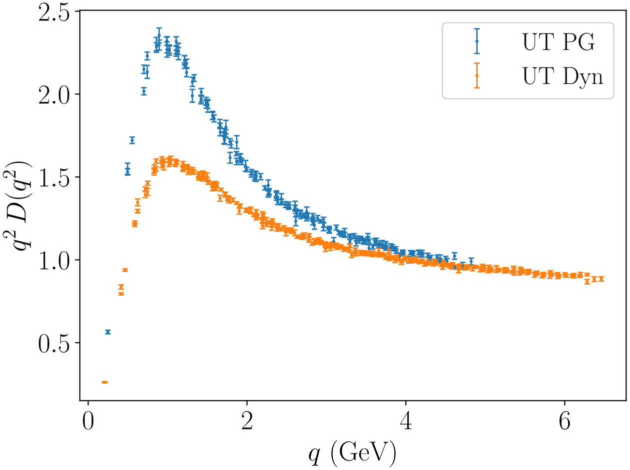
- The nonperturbative scalar gluon propagator in momentum space is

$$D(q^2) \equiv \frac{Z(q^2)}{q^2} \to \frac{1}{q^2} \text{ at tree level.}$$

- Consider the renormalisation function

$$Z(q^2) = q^2 \, D(q^2) \, .$$

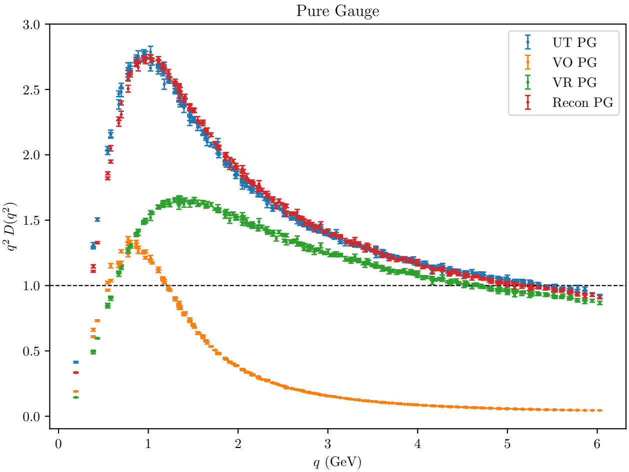
- Renormalise by setting $Z(q^2) = 1$ at $q = 4.4$ GeV.
- $32^3 \times 64$ lattices. $m_\pi = 156$ MeV.



J. C. Biddle, W. Kamleh and DBL, in preparation.

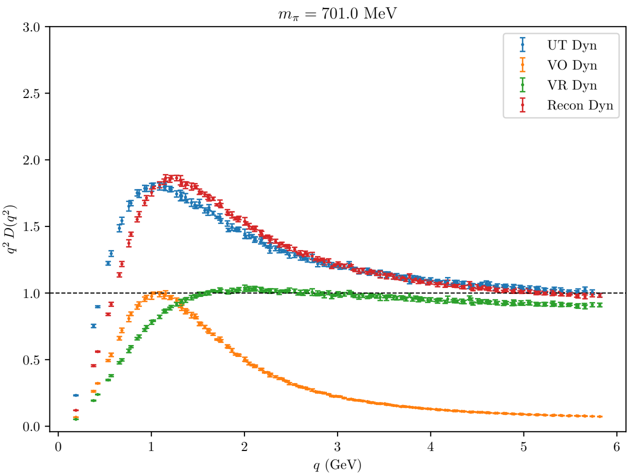
Gluon Propagator – Pure Gauge Sector

- Vortex Removal (VR) suppresses infrared enhancement whilst preserving UV perturbative behaviour.
- Vortex-Only (VO) configurations capture the long-distance physics.
- Reconstruction of the propagator as a linear combination of the vortex-modified parts recovers full propagator.
- Residual infrared enhancement in the vortex-removed result is undesirable.



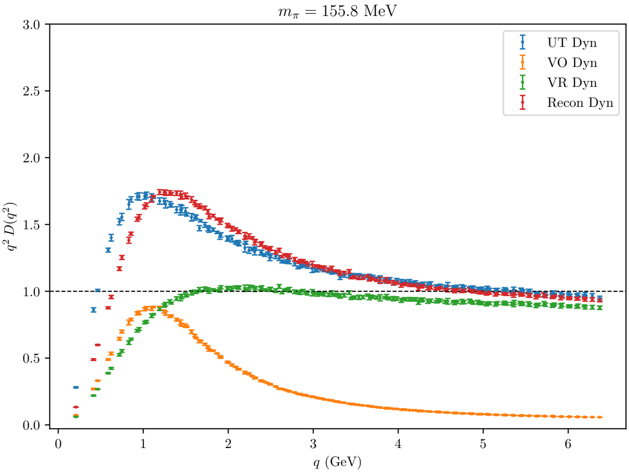
Gluon Propagator – Dynamical Fermions $m_\pi = 701$ MeV

- Dynamical fermions (UT) suppress the overall infrared strength.
- Vortex Removal (VR) almost eliminates infrared enhancement.
- Vortex-Only (VO) configurations capture the long-distance physics.
- Reconstruction is less perfect.



Gluon Propagator – Dynamical Fermions $m_\pi = 156$ MeV

- Lighter dynamical *u* and *d* quarks further suppress the infrared enhancement.
- Centre Vortex degrees of freedom are able to capture the effects of dynamical fermions in QCD.



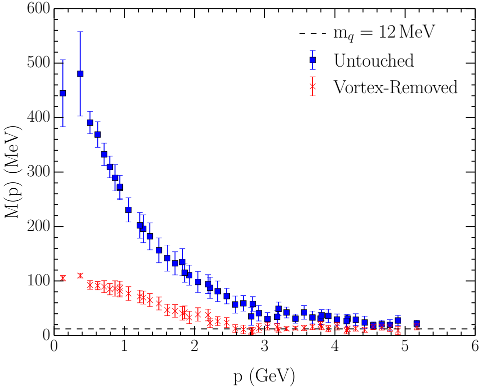
Centre Vortices and the Landau-Gauge Quark Propagator

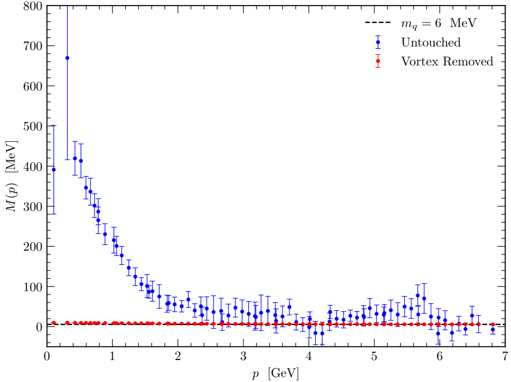
- Probe dynamical mass generation using the quark propagator

$$S(p) = \frac{Z(p)}{i\not{p} + M(p)} \,,$$

- Enhancement of the mass function, $M(p)$, at low momenta indicates dynamical mass generation.
- Renormalisation function, $Z(p)$, is typically infrared suppressed.
- Consider the Overlap-Dirac fermion action
  - Provides a lattice implementation of chiral symmetry,
  - No additive mass renormalisation,
  - Sensitive to the topological structure of the gauge fields.

Pure Gauge vs Dynamical: Quark Mass function





Restoration of Chiral Symmetry

- If vortices are responsible for $D\chi SB$, then their removal should restore chiral symmetry

$$\mathsf{SU}\,2_{\mathrm{L}} \times \mathsf{SU}\,2_{\mathrm{R}} \times \mathrm{U}(1)_{\mathrm{A}}$$

- Expect hadrons related by chiral transformations to become degenerate

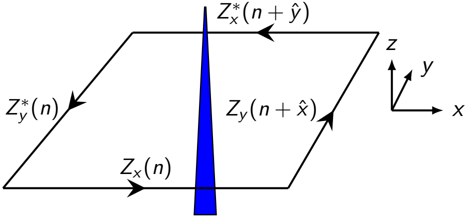
$$\begin{array}{ccc} \pi & \xleftrightarrow{\mathrm{U}(1)_{\mathrm{A}}} & a_0 \\ \rho & \xleftrightarrow{\mathsf{SU}\,2_{\mathrm{L}} \times \mathsf{SU}\,2_{\mathrm{R}}} & a_1 \\ N & \xleftrightarrow{\mathsf{SU}\,2_{\mathrm{L}} \times \mathsf{SU}\,2_{\mathrm{R}}} & \Delta \end{array}$$

- At light quark masses, all symmetries are observed to be restored.
- A. Trewartha, W. Kamleh and DBL, J. Phys. G **44** (2017) 125002 [arXiv:1708.06789 [hep-lat]].

What do centre vortices look like?

Rendering Projected Vortices

- Vortex directions are indicated using a right-handed coordinate system.
- For example,
  - An $m = +1$ vortex in the x-y plane is plotted in the $+\hat{z}$ direction as a blue jet.



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- For example,
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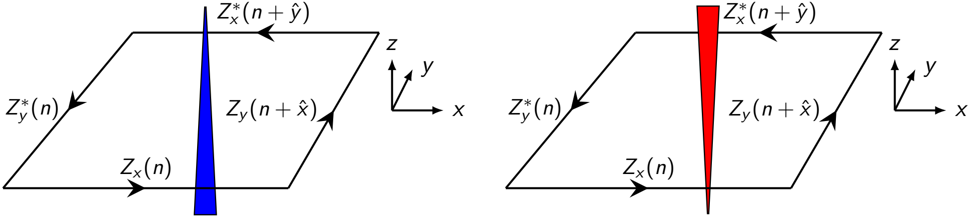
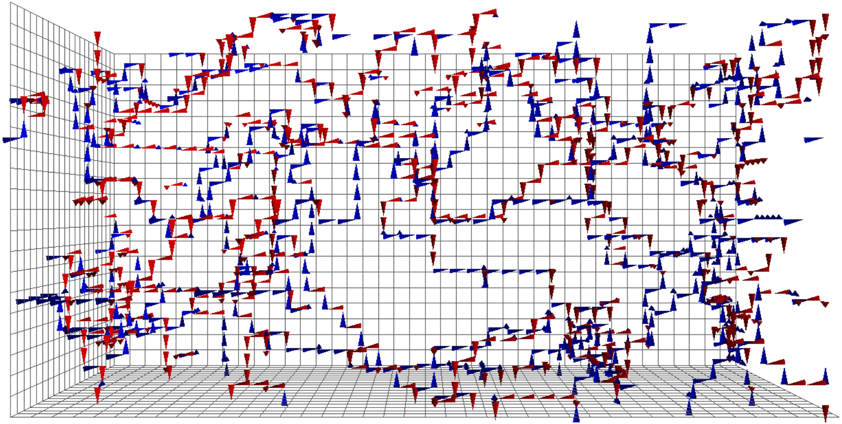
  - An $m = -1$ vortex in the x-y plane is plotted in the $-\hat{z}$ direction as a red jet.

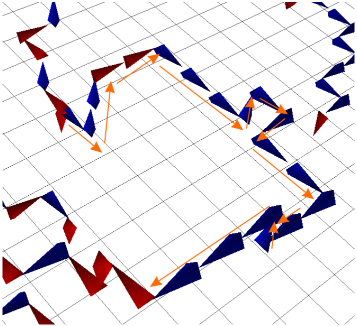


$t = 1$ J. Biddle *et. al.* Phys. Rev. D **102**, 034504

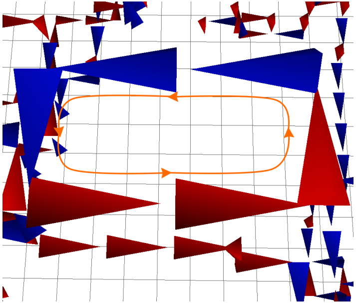


Key features

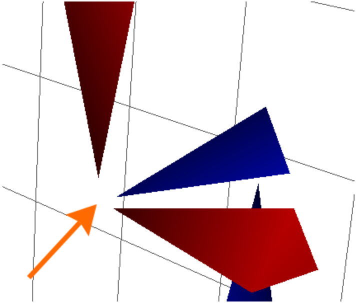
Vortex lines



Closed loops



Vortex Monopole



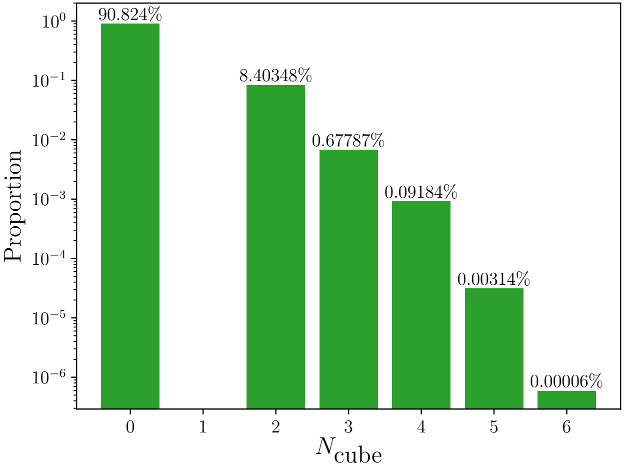
Visualising Centre Vortices

- Consider the number of vortices entering a 3D cube on the dual lattice.

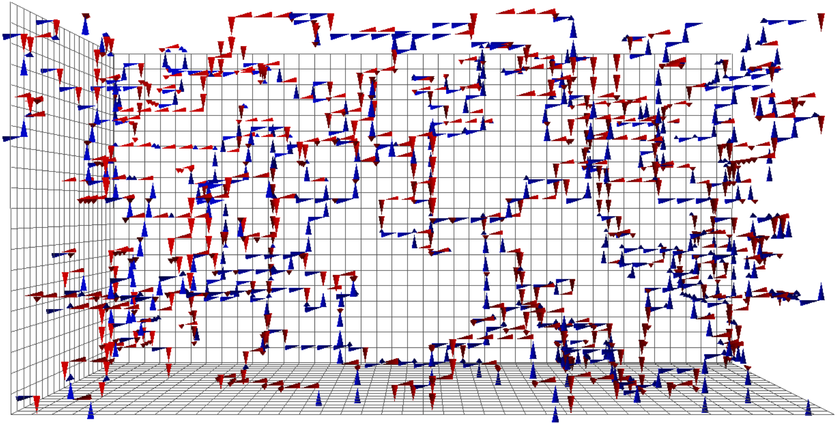
$N_{\text{cube}}(\tilde{x})$	Interpretation
0	No vortices present.
1	Terminating vortex, forbidden by Bianchi*.
2	Vortex line flowing through the cube.
3	Simple three-way vortex monopole.
4	Vortex intersection.
5	Complex five-way monopole path.
6	Vortex intersections or double monopoles.

*Bianchi identity implies a continuous flow of centre vortex flux through a spatial cube.

Visualising Centre Vortices

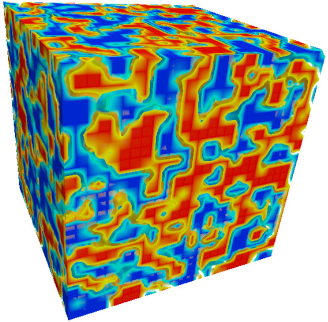


Time slice $t = 2$



Impact of Dynamical Fermions on Centre Vortex Structure

Visualisations of the Topological Charge Density



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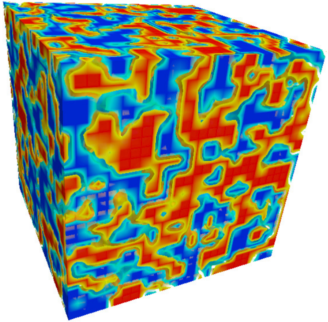

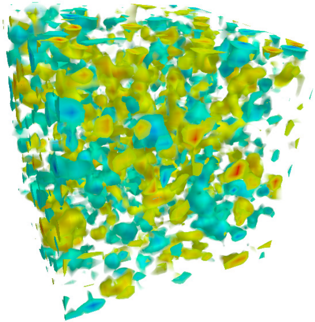

- See "Buried treasure in the sand of the QCD vacuum," P. J. Moran and DBL, [arXiv:0805.4246 [hep-lat]].

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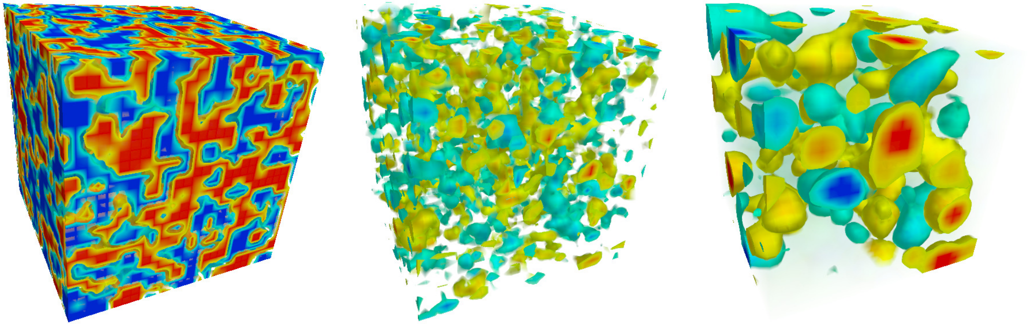

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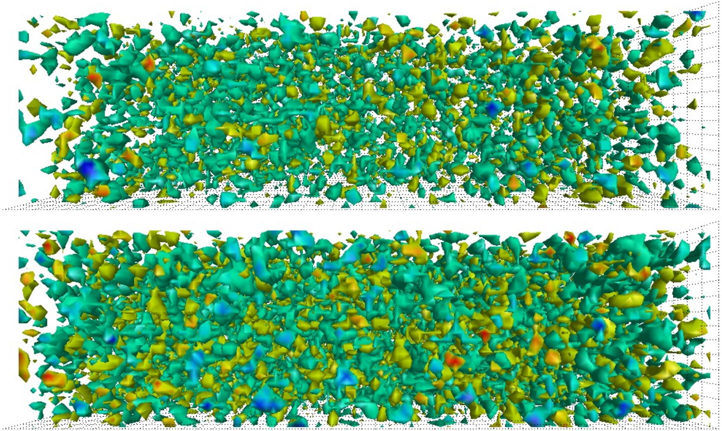
Pure Gauge versus MILC 2 + 1-Flavour QCD: $m_{u,d} = 27.1$ MeV



- "Impact of Dynamical Fermions on QCD Vacuum Structure," P. J. Moran and DBL, Phys. Rev. D **78** (2008) 054506 [arXiv:0801.2016 [hep-lat]].

Impact of Dynamical Fermions on Centre Vortex Structure

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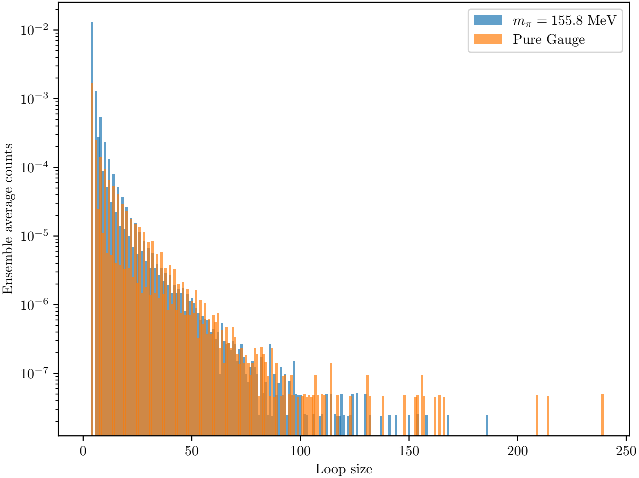
  - $3,277 \pm 156$ vortices in the Pure Gauge theory.
- There is enhancement in of secondary loop structure,
  - Both in number and in complexity via monopoles.

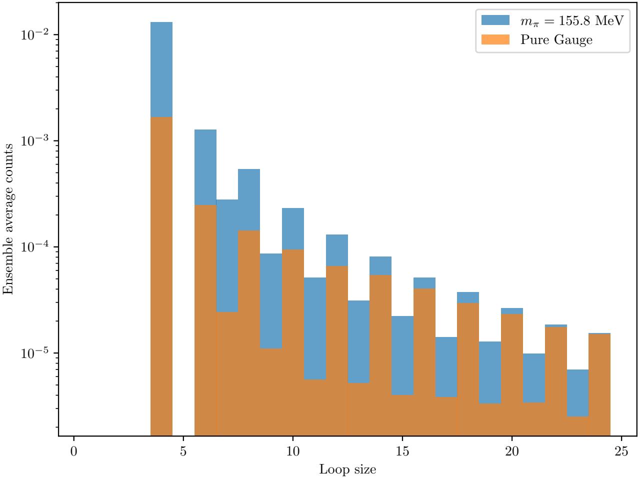
Impact of Dynamical Fermions: Secondary Loop Lengths

- Relative distribution of link paths within an ensemble.
- Moderate size loops are exponentially distributed.
- There is an order of magnitude increase in the number of small loops in QCD.
- Secondary loops can be large in path length.
- Comb-like structure indicates lower probability of monopoles.



Impact of Dynamical Fermions: Secondary Loop Lengths

- Path lengths 1, 2, 3 and 5 don't close.
  - Bianchi identity forbidden.
- Odd link-path lengths require a monopole-antimonopole pair.
- Greater relative abundance of monopole-antimonopole pairs in QCD.
  - Tines of the comb are shorter in QCD.



Static Quark Potential in QCD

Static Quark Potential on the Lattice

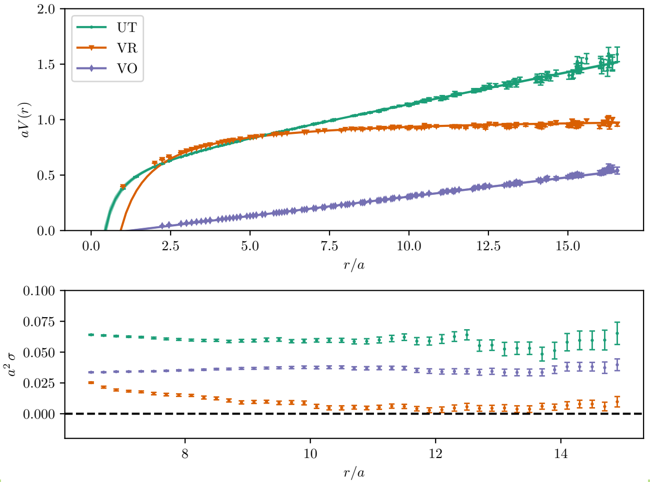
- On the lattice, the static quark potential is calculated by considering Wilson loops $W(r,t)$ with spatial extent r and temporal extent t.
- The static quark potential V_0 is obtained through the relationship:

$$\langle W(r,t) \rangle = \sum_{i=0} a_i \, e^{-V_i(r)\, t} \, .$$

- A variational analysis is used to isolate the ground-state potential.

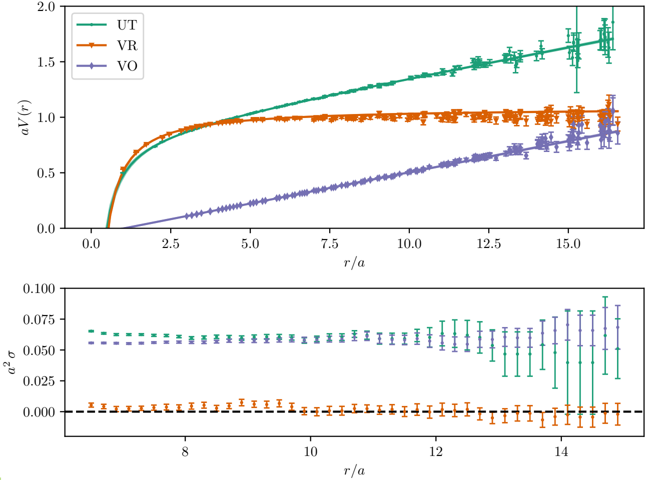
Static Quark Potential – Pure Gauge Sector

- Vortex-only (VO) reproduces only 62% of the original (UT) static quark potential.
- Vortex removal (VR) only suppresses confinement.
- Lower plot reports the local slope from fits to $V(r)$ over a window of $r \pm \frac{3}{2}\, a$.



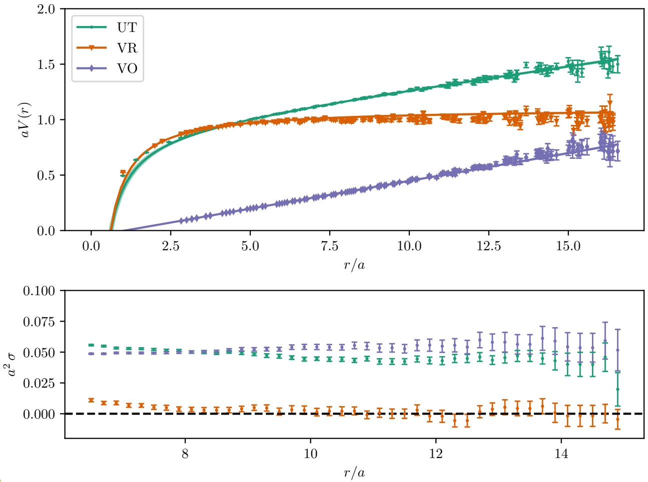
Introducing Dynamical Fermions ($m_\pi = 701\text{MeV}$)

- In the presence of dynamical fermions, vortices capture the full string tension.
- Vortex removal leaves no residual confining potential.
- Centre vortices are the origin of confinement in QCD.



Lighter dynamical fermions ($m_\pi = 156\text{MeV}$)

- Lighter quark masses screen the confining potential.
- Vortices continue to capture the full string tension.
- Vortex removal leaves no residual confining potential.
- Centre vortices are the origin of confinement in QCD.



Conclusions

Visualisations reveal:

- High density of vortices and complexity in their long-distance percolating structure.
- The proliferation of monopoles in $SU(3)$ gauge theory with further enhancement in QCD.
- Enhancement of small vortex paths upon introducing dynamical fermion degrees of freedom.

Conclusions

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  - Eliminates dynamical mass generation in the mass function of the quark propagator.
  - Eliminates the confinement potential of the static quark potential.

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- In QCD, the consideration of vortices alone:
  - Captures the infrared enhancement of the gluon propagator.
  - Fully reproduces the confining potential of the static quark potential.

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  - Captures the infrared enhancement of the gluon propagator.
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Conclusions

- Dynamical fermions radically alter the centre-vortex structure of the vacuum fields.
- In QCD, vortex removal:
  - Suppresses the infrared enhancement of the gluon propagator.
  - Eliminates dynamical mass generation in the mass function of the quark propagator.
  - Eliminates the confinement potential of the static quark potential.
- In QCD, the consideration of vortices alone:
  - Captures the infrared enhancement of the gluon propagator.
  - Fully reproduces the confining potential of the static quark potential.
- Conclude that centre vortices provide a common origin for
  - Confinement, and the
  - Dynamical generation of mass via chiral symmetry breaking.
- Centre Vortices capture the essence of nonperturbative QCD.

Interactive 3D Visualisation Techniques

- Rendered in AVS Express Visualisation Edition.
  `http://www.avs.com/solutions/express/`
- Exported in VRML.
- Converted to U3D format via pdf3d ReportGen.
  `https://www.pdf3d.com/products/pdf3d-reportgen/`
- Imported into LaTeX via media9 package.
- Viewed in Adobe acroread (Linux, use 9.4.1 when 3D support was maintained).
  `ftp://ftp.adobe.com/pub/adobe/reader/unix/9.x/9.4.1/`

Lighter dynamical fermions ($m_\pi = 156$MeV)

- Lattice QCD results characterised well by simple models.
- Original (UT):

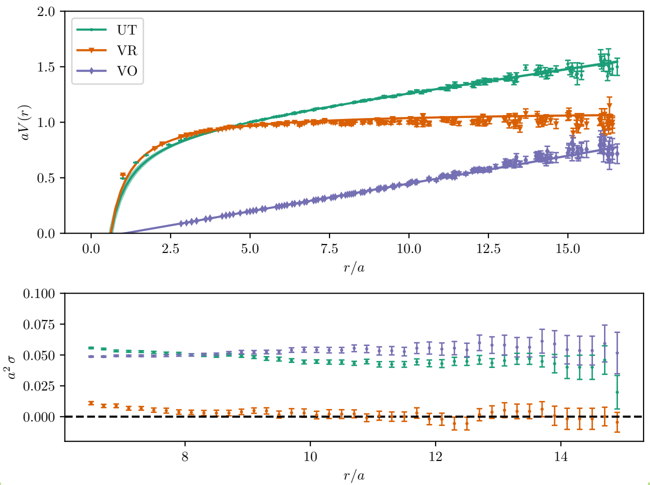
$$V(r) = V_0 - \frac{\alpha}{r} + \sigma\, r$$

- Vortex-removed (VR):

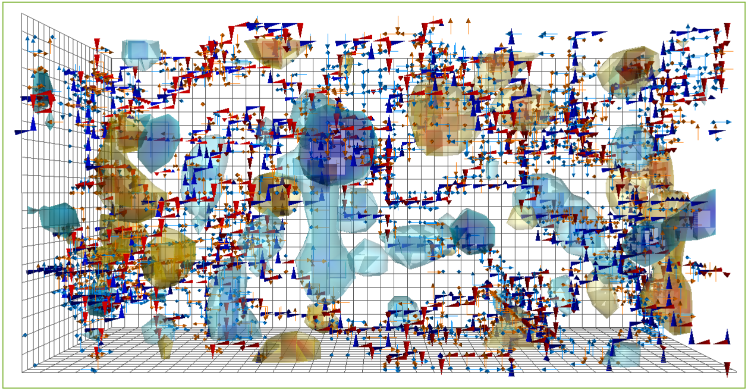
$$V(r) = V_0 - \frac{\alpha}{r}$$

- Vortex-only (VO):

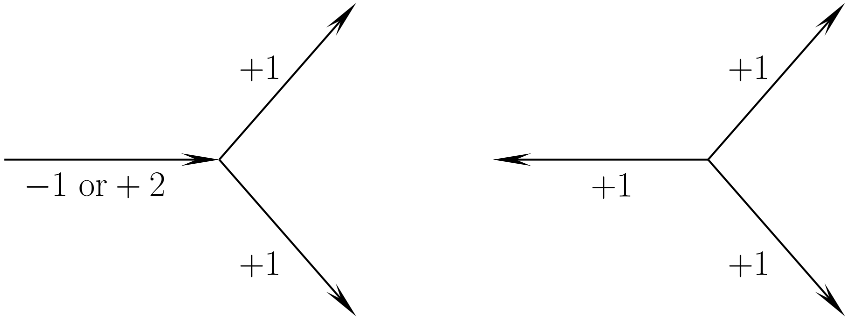
$$V(r) = V_0 + \sigma\, r$$



Animation of Centre Vortex Structure Google: YouTube CSSM Visualisations



Visualising Centre Vortices



- A vortex branching point with centre charge $+2$ flowing into a vertex (left) is equivalent to a vortex monopole with charge $+1$ flowing out of the vertex (right). Arrows indicate the direction of flow for the labelled charge.
- Our convention illustrates the directed flow of charge $m = +1$.

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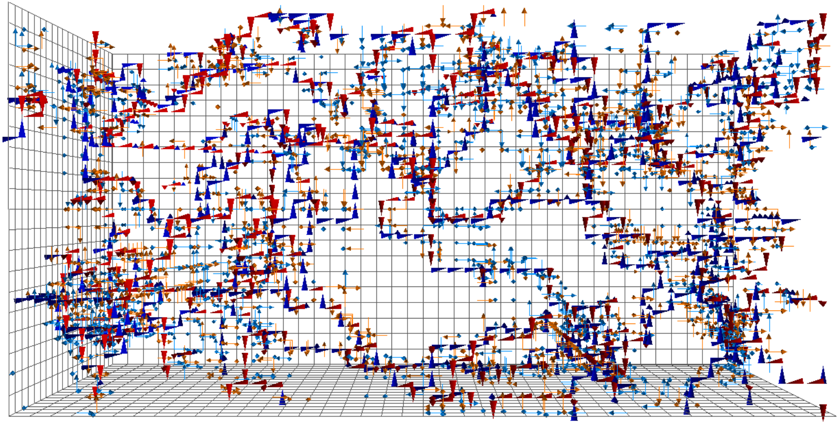
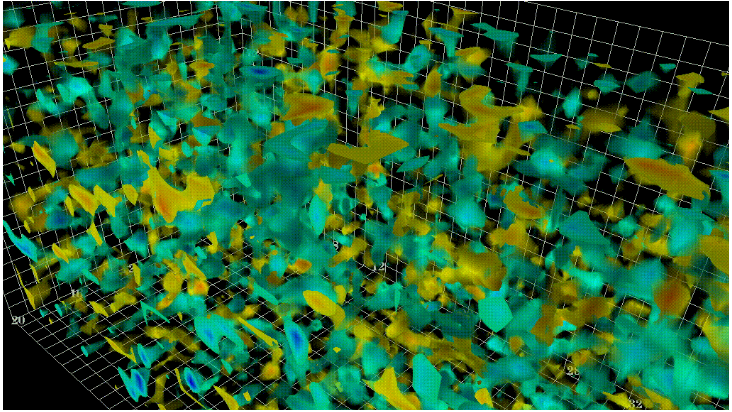
- See "Visualization of center vortex structure," to link vortices to topological charge.

  J. C. Biddle, W. Kamleh and DBL, Phys. Rev. D **102** (2020) 034504 [arXiv:1912.09531 [hep-lat]].

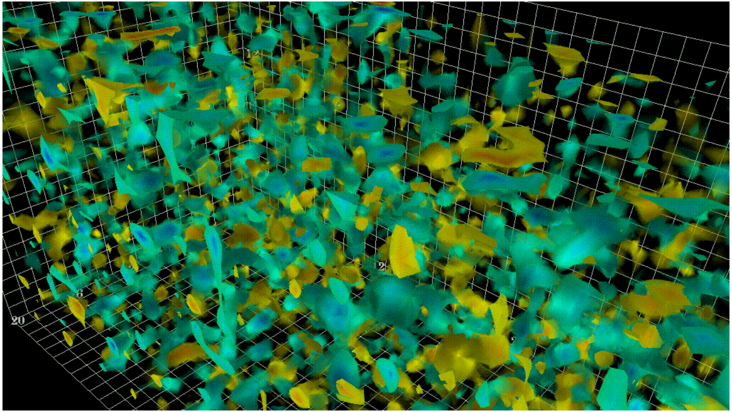
Time slice $t = 1$



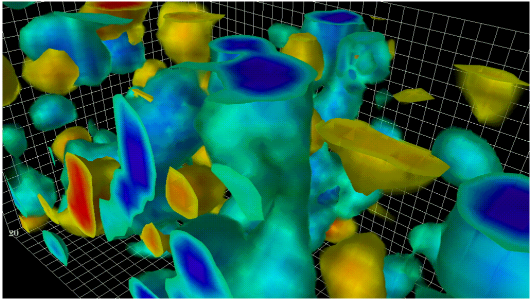
Untouched Configurations with Cooling

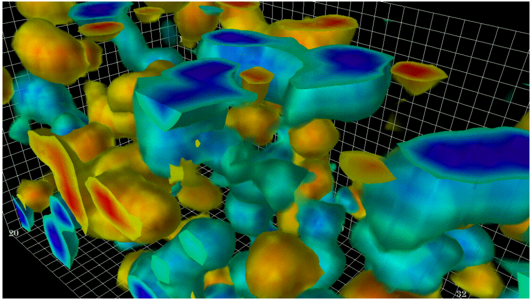


Vortex Removed Configurations with Cooling



Untouched (left) and Vortex-Only comparison (10 sweeps)





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  - J. E. Mandula and M. Ogilvie, Phys. Lett. **B185**, 127 (1987).
  - C. A. Aubin and M. C. Ogilvie, Phys. Lett. **B570**, 59 (2003), hep-lat/0306012.
  - P. O. Bowman, *et al.* Phys. Rev. D **76**, 094505 (2007) [arXiv:hep-lat/0703022 [hep-lat]].