The quest for gravitational properties of subatomic particles

2nd PSQ@EIC Meeting: Precision Studies on QCD at EIC

July 21, 2021

Oleg Teryaev
JINR, Dubna

Main Topics

- Equivalence Principle: way to merge strongest and weakest interactions
- GPDs and (gravitational) formfactors: EP for spin and its Extension
- D-term, pressure and inflation
- Spin-1 and average shear

Strong interactions and gravity

- Gravitational interaction is strongly suppressed ~ $(\Lambda/M_{Pl})^2$
- Equivalence Principle

- I: Acceleration <-> Gravity
- HIC: a ~ Λ, a/g ~ $\frac{c^2}{v_\oplus^2} \cdot \frac{R_\oplus}{R_A} \sim 10^{30}$)
- M_{Pl} -> Λ ("GeV Gravity")

- II: Coupling to Energy-Momentum Tensor

Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle \quad = \quad \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M\Big]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- No M_{Pl}! May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons Ji's SRs
- Describe interaction with both classical and TeV gravity

Ji's and 1st moment "Charge" SRs: GPDs imply models for both EM and Gravitational Formfactors (Selyugin,OT '09)

- Smaller mass square radius (attraction vs repulsion ☺): follows from Regge behaviour of GPDs ~ $x^{\alpha(t)}$ (cf AdS QCD)

$$\rho(b) = \sum_q e_q \int dx q(x,b) \qquad = \int d^2 q F_1(Q^2 = q^2) e^{i\vec{q}\,\vec{b}}$$

$$= \int_0^\infty \frac{q dq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1+\tau}$$

$$\rho_0^{\mathrm{Gr}}(b) = \frac{1}{2\pi} \int_\infty^0 dq q J_0(qb) A(q^2)$$

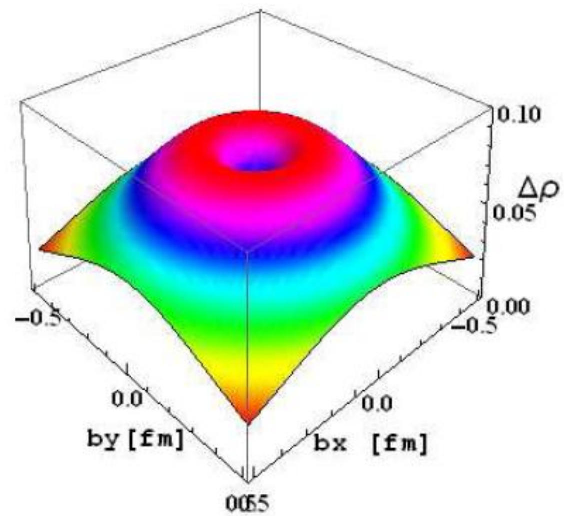


FIG. 17: Difference in the forms of charge density F_1^P and "matter" density (A)

Electromagnetism vs Gravity (OT'99)

- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q)$$

$$M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P | J_q^\mu | P \rangle = 2 e_q P^\mu$$

$$\sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2 P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2 e_q M \phi(q)$$

$$M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

- Mass as charge – equivalence principle

EP and hadron structure

- “Microscopic” EP (coupling of gravity to EMT)
- +
- Conservation law
  (Momentum SR to get local from LC:
  $\int dx\, x\, (\Sigma\, q(x) + G(x))=1$)
- =
- “Macroscopic” EP (universal falling) :
- Tested VERY precisely

Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G}\langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

  $\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$ spin dragging twice smaller than EM
- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM $\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot\vec{g}$
- Orbital and Spin momenta dragging – the same - Equivalence principle

Equivalence principle

- Newtonian – “Falling elevator” – well known and checked (also for elementary particles)
- Post-Newtonian – gravity action on (quantum!) SPIN – known since 1962 (Kobzarev and Okun’; ZhETF paper contains acknowledgment to Landau: probably his last contribution to theoretical physics before car accident); rederived from conservarion laws - Kobzarev and V.I. Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way

Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

PHYSICAL REVIEW LETTERS

VOLUME 68 | 13 JANUARY 1992 | NUMBER 2

Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson

Physics Department, FM-15, University of Washington, Seattle, Washington 98195

(Received 25 September 1991)

- If (CP-odd!) GEDM=0 -> constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background

$$\mathcal{H} = -g\mu_N \boldsymbol{B}\cdot\boldsymbol{S} - \zeta\hbar\boldsymbol{\omega}\cdot\boldsymbol{S}, \quad \zeta = 1+\chi$$

$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \quad (95\%\mathrm{C.L.})$$

EP and quantum measurement

- If spin is just a geometric vector, EP for Earth's rotation is "trivial": spin rotates with Earth's angular velocity like Foucault pendulum
- Non-trivial if quantum measurement (quite practical here) is performed in the rotating frame

Indirect probe of spin-gravity coupling

- Matrix elements of energy-momentum tensors may be extracted from accurate high-energy experiments ("3D nucleon picture")
- Allow to probe the couplings to quarks and gluons separately

Equivalence principle for moving particles EPII vs EPI

- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER

$$\mathcal{M}_g = (\epsilon^2 + p^2) h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations: $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, OT, '05)
- Arbitrary fields – Obukhov, Silenko, OT '09,'11,'13,16,17

Gravity vs accelerated frame for spin and helicity

- Spin precession – well known factor 3 (Probe B; spin at satellite – probe of PNEP!) – smallness of relativistic correction (~**P**^{2}) is compensated by 1/ **P**^{2} in the momentum direction precession frequency
- Helicity flip – the same!
- No helicity flip in gravitomagnetic field – another formulation of PNEP (OT'99) and
- Flip by "gravitoelectric" field: relic neutrino? Black hole?

$$\frac{d\sigma_{+-}}{d\sigma_{++}} = \frac{tg^2(\frac{\phi}{2})}{(2\gamma - \gamma^{-1})^2}$$

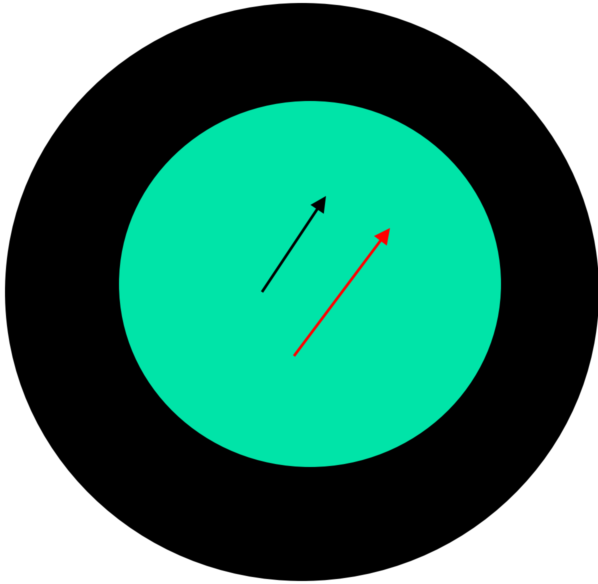
Gyromagnetic and Gravigyromagnetic ratios

- Free particles – coincide
- $\langle P+q|T^{mn}|P-q\rangle = P^{\{m}\langle P+q|J^{n\}}|P-q\rangle/e$ up to the terms linear in q
- Gravitomagnetic g=2 for any spin
- Special role of g=2 for ANY spin (asymptotic freedom for vector bosons)

- Should Einstein know about PNEP, the outcome of his and de Haas experiment would not be so surprising
- Recall also g=2 for Black Holes. Indication of "quantum" nature?!

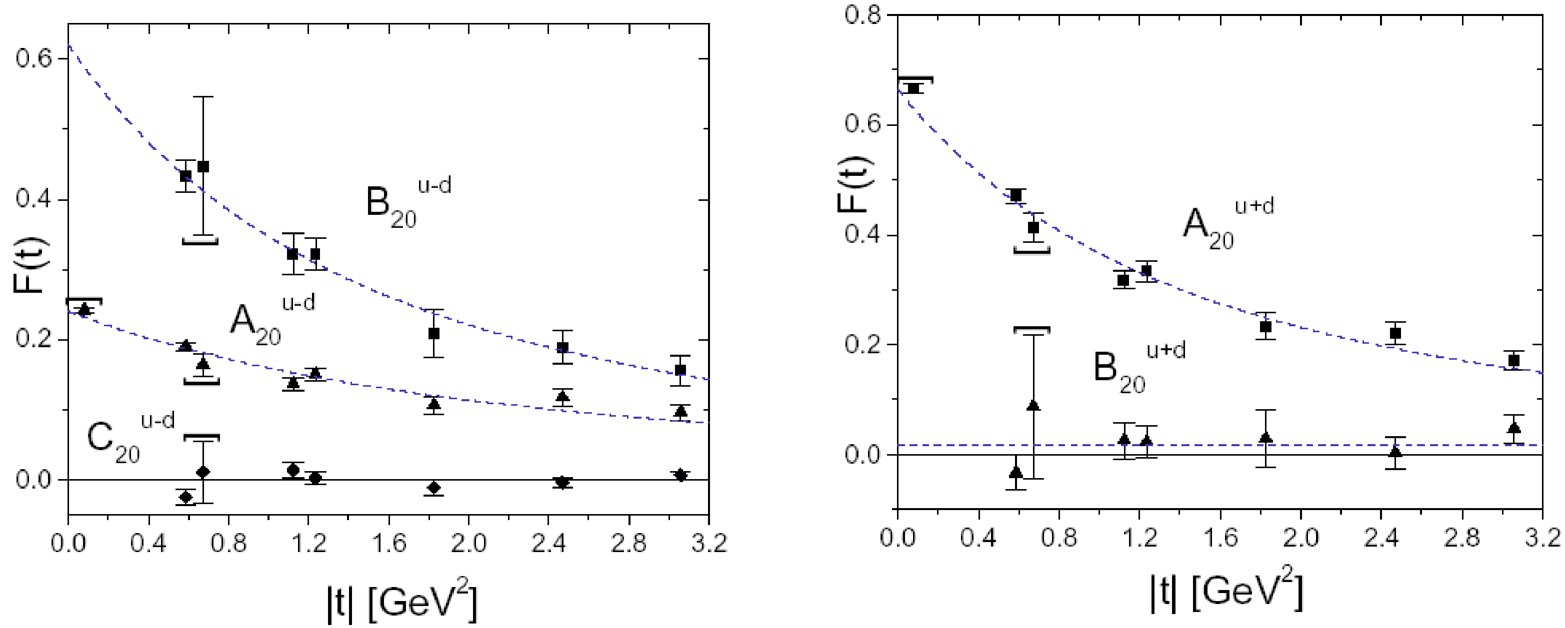
Cosmological implications of PNEP

- Necessary condition for Mach's Principle (in the spirit of Weinberg's textbook) -
- Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For flat "Universe" - precession frequency equal to that of shell rotation
- Simple observation-Must be the same for classical and quantum rotators – PNEP!
- More elaborate models - Tests for cosmology ?!



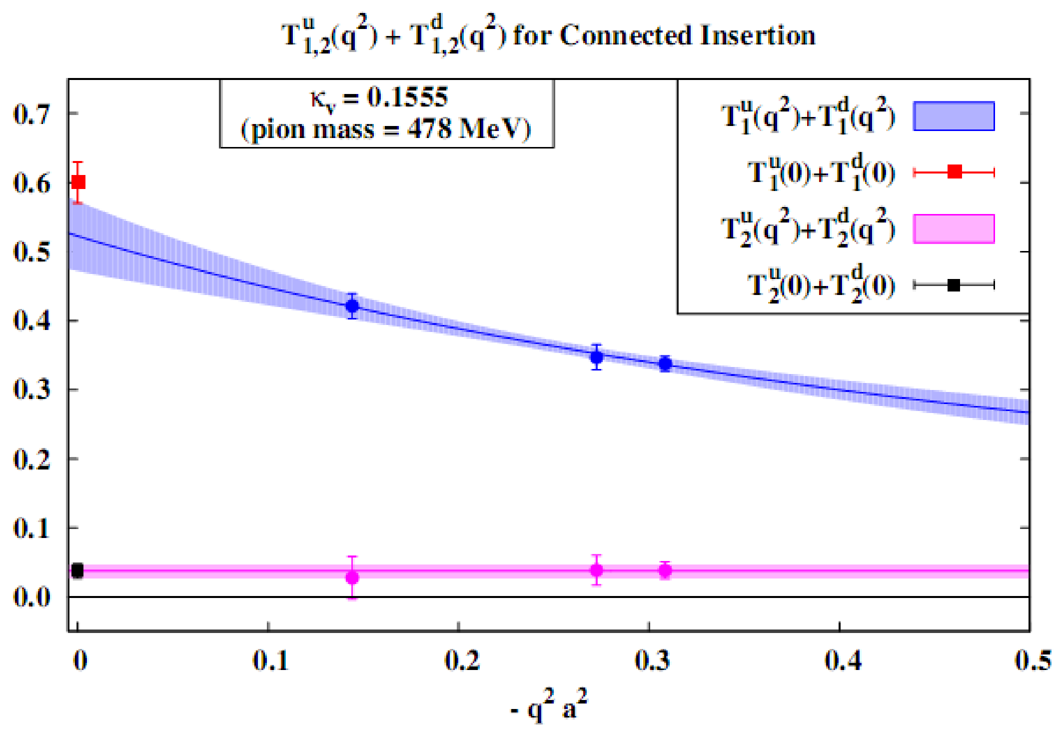
Generalization of Equivalence principle

- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)



More recent lattice study (M. Deka,…K.-F. Liu et al. Phys.Rev. D91 (2015) no.1, 014505)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence Principle=Exact EquiPartition

- In NLO pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravityproof confinement?! Nucleons do not break even by black holes?! Match BH complementarity?! “GeV gravity”?
- Support by recent observation of smallness of EP-forbidden “Cosmological Constant” (cf talk K.-F. Liu)

Exact Equipartition and Pivot

- Important notion introduced by C. Lorce to relate transverse spin SR's of Ji&Yuan and Leader et al.
- Naïve interpretation of ExEP: common (approximately, averagely) pivot for quarks and gluons:
- $\langle J_{T(q,G)} \rangle = \langle x_0 \rangle \langle P_{L(q,G)} \rangle$
- Can this be satisfied for some of pivot choices?

One more gravitational formfactor

- Quadrupole

$$\langle P+q/2|T^{\mu\nu}|P-q/2\rangle = C(q^2)(g^{\mu\nu}q^2 - q^\mu q^\nu) + \ldots$$

- Cf vacuum matrix element – cosmological constant (vacuum pressure)

$$\langle 0|T^{\mu\nu}|0\rangle = \Lambda g^{\mu\nu}$$

$$\Lambda = C(q^2)q^2$$

- Inflation ~ annihilation ($q^2>0$) OT'15
- How to measure experimentally – D-term in Deeply Virtual Compton Scattering

D-term interpretation: Inflation and annihilation

- Quadrupole gravitational FF

$$\langle P+q/2|T^{\mu\nu}|P-q/2\rangle = C(q^2)(g^{\mu\nu}q^2 - q^\mu q^\nu) + \ldots$$

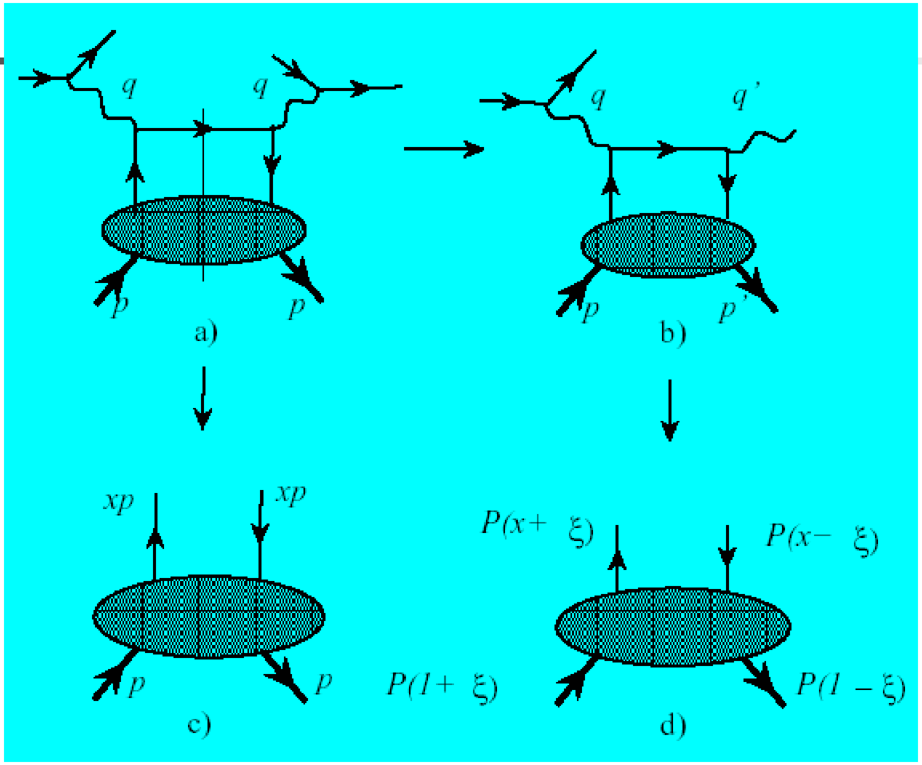
- Moment of D-term – positive
- Vacuum – Cosmological Constant $\langle 0|T^{\mu\nu}|0\rangle = \Lambda g^{\mu\nu}$

- 2D effective CC – negative in scattering, positive in annihilation

$$\Lambda = C(q^2)q^2$$

- Similarity of inflation and Schwinger pair production – Starobisnky, Zel'dovich
- Was OUR Big Bang resulting from one graviton annihilation at extra dimensions??! Version of "ekpyrotic" ("pyrotechnic") universe
- Traceless+Trace =
- M_I: (3/4+1/4) X.Ji'96
- M_{Gr}: (3/2-1/2) OT'99 ("Antigravity": seen in trace part of Einstein Eqs)

Way to D-term: cf QCD Factorization for DIS and DVCS (AND VM production)



- Manifestly spectral

$$\mathcal{H}(x_B) = \int_{-1}^{1} dx \frac{H(x)}{x - x_B + i\epsilon}$$

- Extra dependence on ξ

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x - \xi + i\epsilon},$$

Unphysical regions

- DIS : Analytical function – polynomial in $1/x_B$ if $1 \le |x_B|$

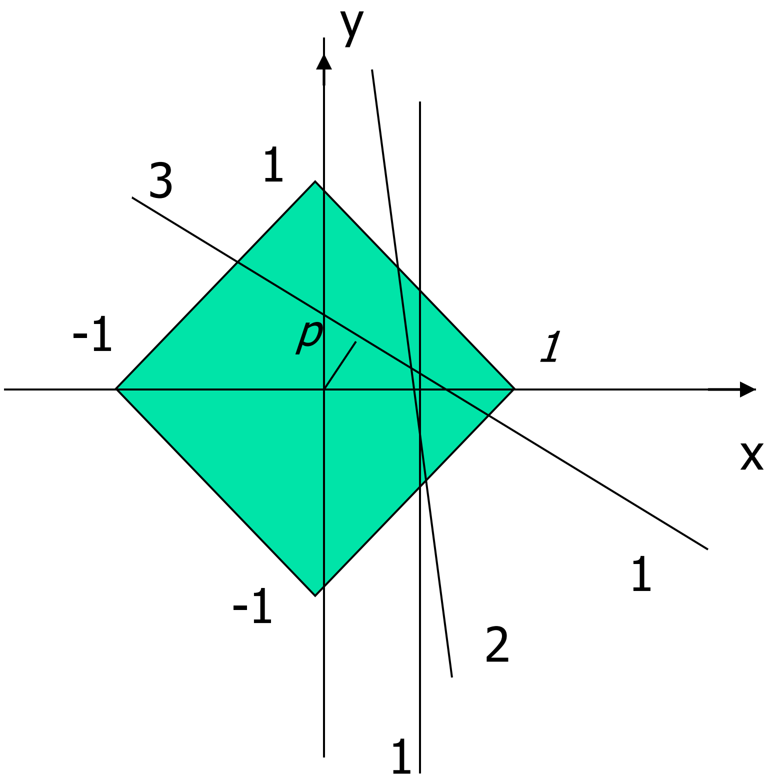
$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- DVCS – additional problem of analytical continuation of $H(x,\xi)$
- Solved by using of Double Distributions Radon transform

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

Double distributions and their integration

- Slope of the integration line-skewness
- Kinematics of DIS: $\xi = 0$
  ("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$
  - line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography

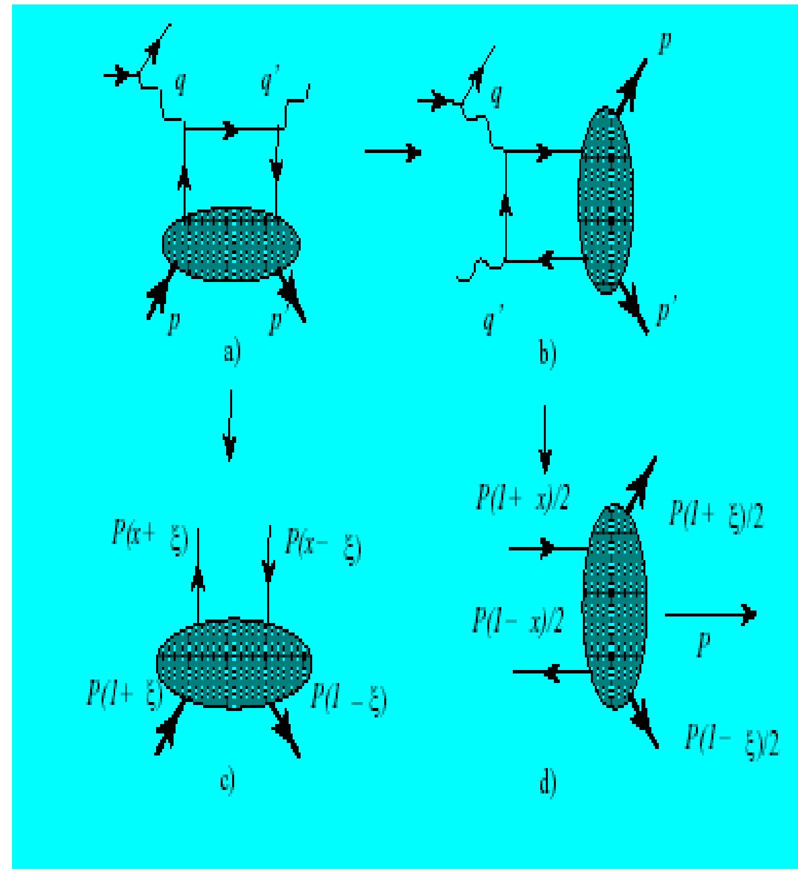


$$f(x,y) = -\frac{1}{2\pi^2}\int_0^\infty \frac{dp}{p^2}\int_0^{2\pi} d\phi |cos\phi| (H(p/cos\phi + x + ytg\phi, tg\phi) - H(x + ytg\phi, tg\phi)) =$$

$$= -\frac{1}{2\pi^2}\int_{-\infty}^\infty \frac{dz}{z^2}\int_{-\infty}^\infty d\xi (H(z + x + y\xi, \xi) - H(x + y\xi, \xi))$$

Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes
- Duality between s and t channels (Polyakov,Shuvaev, Guzey, Vanderhaeghen)



GDA -> back to unphysical regions for DIS and DVCS

- Recall DIS

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- Non-positive powers of x_B

- DVCS

$$H(\xi) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

- Polynomiality (general property of Radon transforms): moments - integrals in *x* weighted with x^n - are polynomials in $1/\xi$ of power *n+1*
- As a result, analyticity is preserved: only non-positive powers of ξ appear

Holographic property (OT'05)

- Factorization Formula ->

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x - \xi + i\epsilon}$$

- Analyticity -> Imaginary part -> Dispersion relation:

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,x)}{x - \xi + i\epsilon}$$

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x,x) - H(x,\xi)}{x - \xi + i\epsilon}$$

- "Holographic" equation (DVCS AND VM)

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} \int_{-1}^{1} H(x,\xi) dx (x-\xi)^{n-1} = const$$

Holographic property - II

- Directly follows from double distributions

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

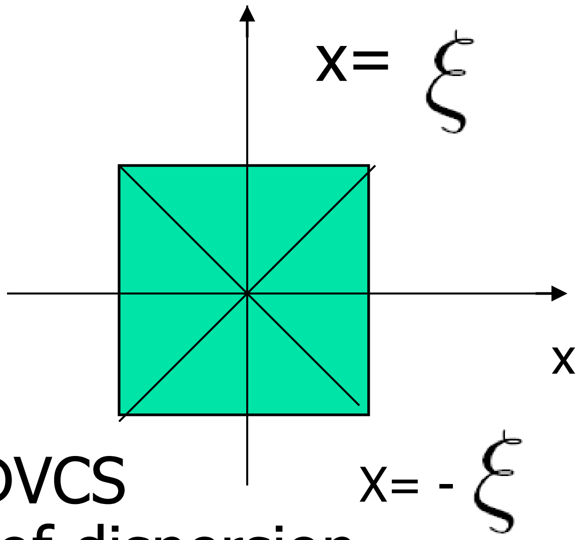
- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term G(x,y)

$$\Delta\mathcal{H}(\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy \frac{G(x,y)}{1-y}$$

=-($\int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z-1} = const$)

Holographic property - III

- 2-dimensional space -> 1-dimensional section!
- Momentum space: any relation to holography in coordinate space ?!



- Strategy (now adopted) of GPD's
studies: start at diagonals
(through SSA due to imaginary part of DVCS amplitude) and restore by making use of dispersion relations + subtraction constants

Analyticity of Compton amplitudes in energy plane (Anikin,OT'07)

- Finite subtraction implied

$$\mathrm{Re}\mathcal{A}(\nu,Q^2)=\frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu',Q^2)}{(\nu'^2-\nu^2)}+\Delta \qquad \Delta=2\int_{-1}^{1}d\beta\,\frac{D(\beta)}{\beta-1}$$

$$\Delta^p_{\mathrm{CQM}}(2)=\Delta^n_{\mathrm{CQM}}(2)\approx 4.4, \qquad \Delta^p_{\mathrm{latt}}\approx\Delta^n_{\mathrm{latt}}\approx 1.1$$

- Numerically close to Thomson term for real proton (but NOT neutron) Compton Scattering!

- Duality (sum of squares vs square of sum; proton: 4/9+4/9+1/9=1)?!

From D-term to pressure

- *Inverse -> 1st moment (model)*
- *Kinematical factor: weighted pressure C~<p r^4> (<p r^2> =0)* M.Polyakov'03

$$T^Q_{\mu\nu}(\vec{r},\vec{s}) = \frac{1}{2E}\int \frac{d^3\Delta}{(2\pi)^3}\, e^{i\vec{r}\cdot\vec{\Delta}}\, \langle p', S'|\hat{T}^Q_{\mu\nu}(0)|p, S\rangle$$

$$T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\delta_{ij}$$

- *Justification: (Fourier inversed) consistency principle for Born gravitational scatterring? 2D<->3D?*

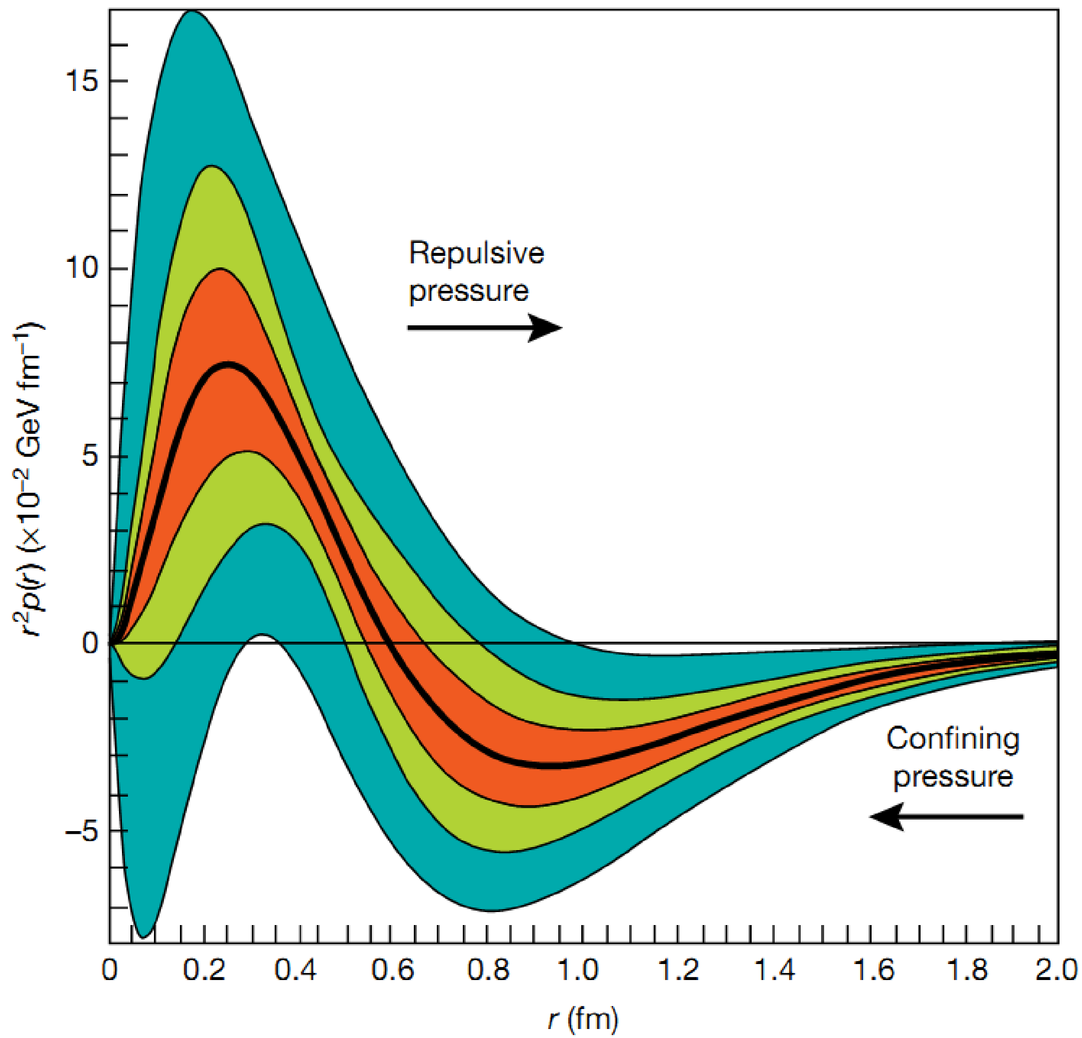
LETTER

NATURE | www.nature.com/nature

https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]



Gravitational formfactors and pressure in hadron pairs production

- Back to GDA region
- -> moments of H(x,x) - define the coefficients of powers of cosine!– $1/\xi$
- Higher powers of cosine in t-channel – threshold in s -channel
- Larger for pion than for nucleon pairs because of less fast decrease at x ->1
- Stability defines the sign of GDA

$$\mathscr{H}(\xi) = -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

$$= -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,x) \frac{x^n}{\xi^{n+1}} + \Delta\mathscr{H}.$$

Quantum roots of classical stability

- GPDs

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x,x) - H(x,\xi)}{x - \xi + i\epsilon}$$

- =-($\int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z-1} = const$)
- Sufficient condition: positive (because of forward limit!) H is a decreasing function of ʒ at any x

- GDA's $H(z,\xi) = sign(\xi)\Phi(\frac{z}{\xi}, \frac{1}{\xi})$

$$\mathscr{H}(\xi) = -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

$$= -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,x) \frac{x^n}{\xi^{n+1}} + \Delta\mathscr{H}.$$

- Positivity of GDA balance between unitarity and stability
- Soft PION theorem – positivity of DA!?

Gravitational FFs from Belle data on GDAs

S. Kumano, Qin-Tao Song and O. Teryaev, PRD **97** (2018) 014020.

- Gravitational FFs are related to twist-2 GDAs

$$A_{\lambda_1\lambda_2} = T_{\mu\nu}\varepsilon^{\mu}(\lambda_1)\varepsilon^{\nu}(\lambda_2)/e^2$$

$$A_{++} = \sum_q \frac{e_q^2}{2}\int_0^1 dz \frac{2z-1}{z(1-z)}\Phi^q(z,\xi,W^2)$$

$$\int dz(2z-1)\Phi_q^+(z,\xi,W^2) = \frac{2}{(P^+)^2}\langle \pi^+(p_1)\pi^-(p_2)|T_q^{++}(0)|0\rangle$$

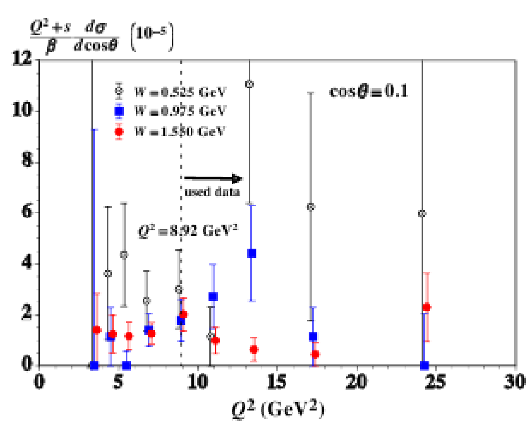
$$\langle \pi^0(p_1)\pi^0(p_2)|T^{\mu\nu}(0)|0\rangle = \frac{1}{2}\left[(sg^{\mu\nu} - P^{\mu}P^{\nu})\Theta_1 + \Delta^{\mu}\Delta^{\nu}\Theta_2\right]$$

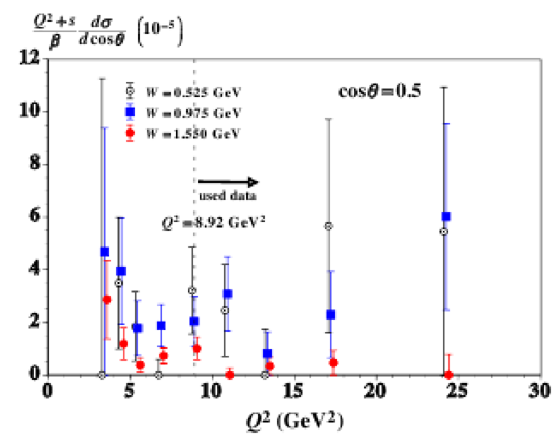
M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003

$$P = p_1 + p_2\ ,\ \Delta = p_1 - p_2$$

- Belle data and scaling : W=0.525,0.975, 1.55 GeV

$$\frac{(Q^2+s)d\sigma}{\beta d|\cos\theta|} \propto \left|\Phi^{\pi^0\pi^0}(z,\cos\theta,W,Q)\right|^2$$





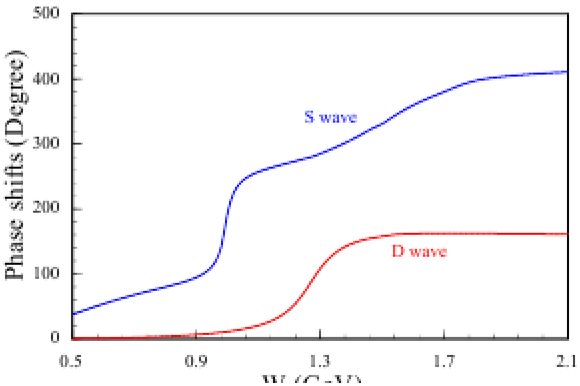
Phase shifts and resonances

- Leading harmonics

$$\sum_q \Phi_q^+\left(z,\xi,W^2\right)=18n_f z(1-z)(2z-1)[B_{10}(W)+B_{12}(W)P_2(2\xi-1)]$$
$$=18n_f z(1-z)(2z-1)[\tilde{B}_{10}(W)+\tilde{B}_{12}(W)P_2(\cos\theta)]$$

$$\tilde{B}_{10}(W)=\bar{B}_{10}(W)e^{i\delta_0},\tilde{B}_{12}(W)=\bar{B}_{12}(W)e^{i\delta_2}$$

- S/D shifts



- $f_0(500)$, $f_2(1270)$ contributions

$$\bar{B}_{12}(W)=\beta^2\frac{10g_{f_2\pi\pi}f_{f_2}M_{f_2}^2}{9\sqrt{2}\sqrt{(M_{f_2}^2-W^2)^2-\Gamma_{f_2}^2M_{f_2}^2}}$$

$$\bar{B}_{10}(W)=\frac{5g_{f_0\pi\pi}f_{f_0}}{3\sqrt{2}\sqrt{(M_{f_0}^2-W^2)^2-\Gamma_{f_0}^2M_{f_0}^2}}$$

Fits and results

- Collection

$$\Phi_q^+(z,\xi,W^2) = N_h z^\alpha (1-z)^\alpha (2z-1)[\tilde{B}_{10}(W) + \tilde{B}_{12}(W) P_2(\cos\theta)]$$

$$\tilde{B}_{10}(W) = [\frac{-3+\beta^2}{2}\frac{5R_\pi}{9} F_h(W^2) + \frac{5 g_{f_0\pi\pi} f_{f_0}}{3\sqrt{2}\sqrt{(M_{f_0}^2 - W^2)^2 - \Gamma_{f_0}^2 M_{f_0}^2}}] e^{i\delta_0}$$

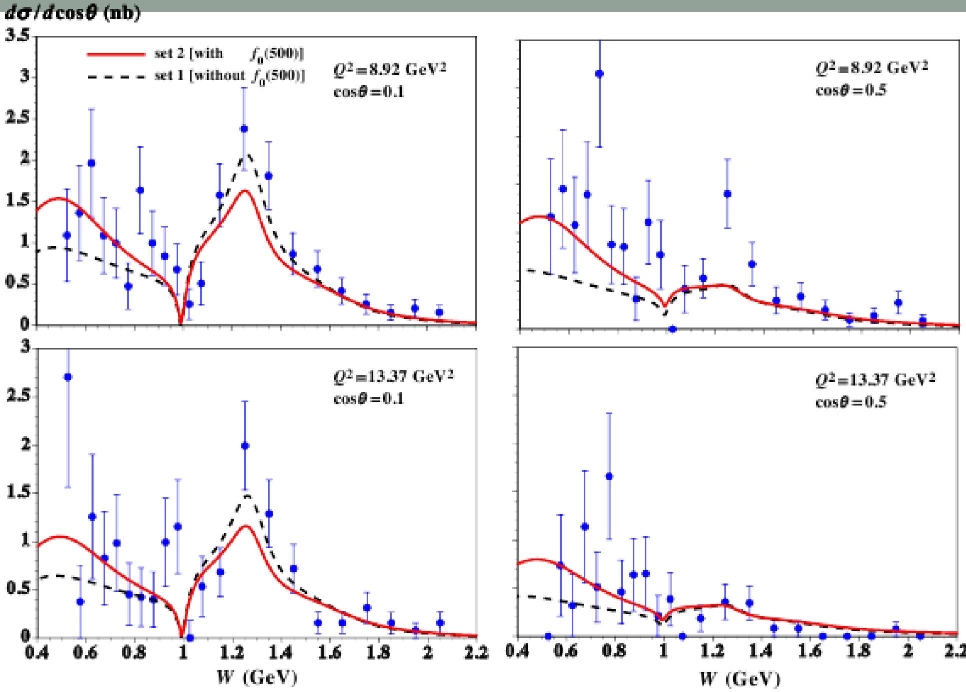
$$\tilde{B}_{12}(W) = [\beta^2 \frac{5R_\pi}{9} F_h(W^2) + \beta^2 \frac{10 g_{f_2\pi\pi} f_{f_2} M_{f_2}^2}{9\sqrt{2}\sqrt{(M_{f_2}^2 - W^2)^2 - \Gamma_{f_2}^2 M_{f_2}^2}}] e^{i\delta_2}$$

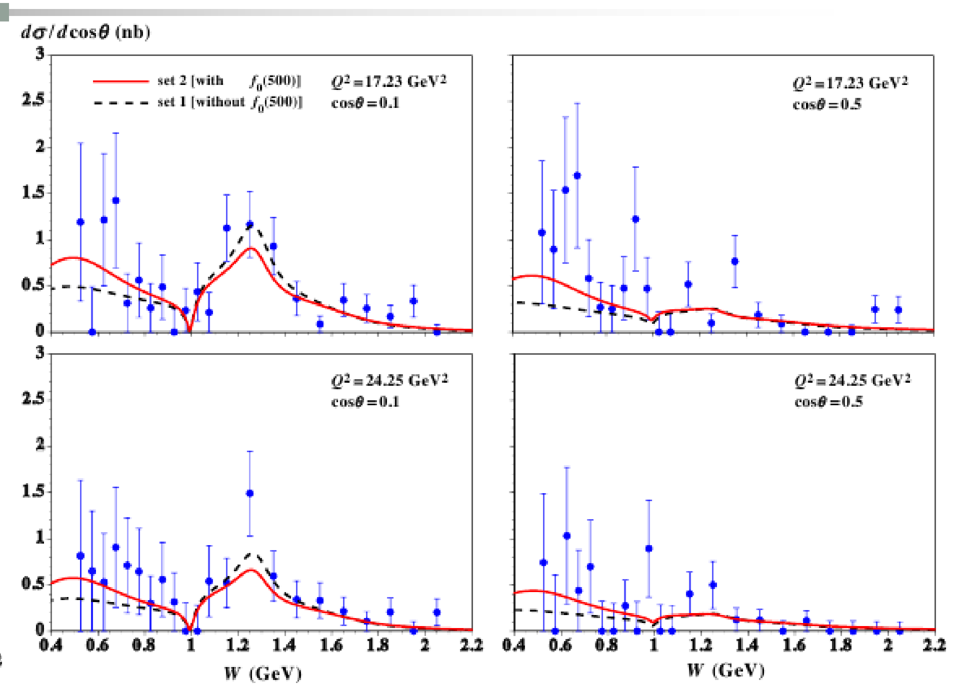
$$F_h(W^2) = \frac{1}{\left[1 + \frac{W^2 - 4m_\pi^2}{\Lambda^2}\right]^{n-1}}$$

- Best fit with (2) and without (1) f_0

	Set 1	Set 2
α	0.801±0.042	1.157± 0.132
Λ	1.602±0.109	1.928±0.213
a	3.878± 0.165	3.800± 0.170
b	0.382± 0.040	0.407± 0.041
f_{f0}	------	0.0184± 0.034
	$\frac{\chi^2}{NOF} = 1.22$	$\frac{\chi^2}{NOF} = 1.09$

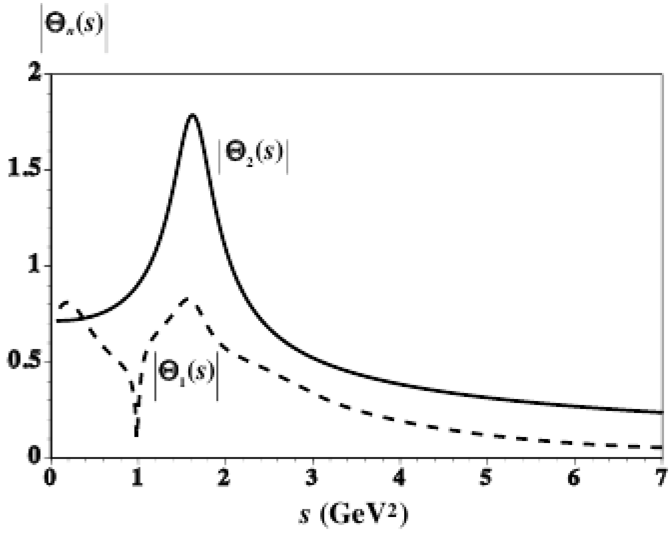
Description of data





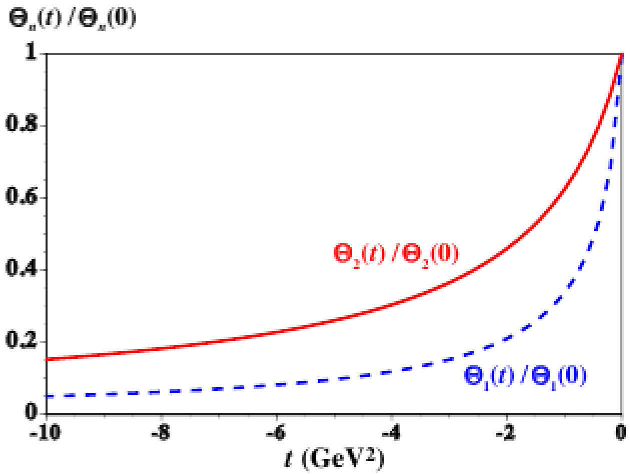
Formfactors

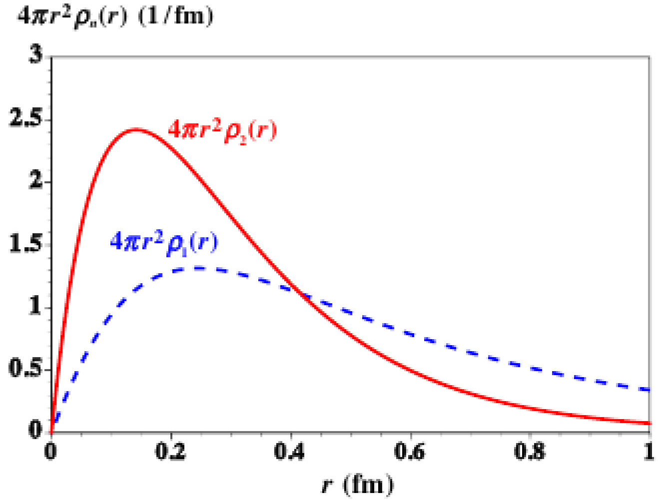
- Resonance structure in pressure – related Θ_1



Time-like -> space-like

- Dispersion relation and Fourier transform





$$< r^2 >= 6\int_{4m^2}^{\infty}\frac{\mathrm{Im}(F(s))}{s^2}$$

- Mass radius

$$\sqrt{\langle r^2 \rangle} = 0.69 \text{ fm for } \Theta_2$$

Spin 1 EMT and inclusive processes

- Forward matrix element -> density matrix
- Contains P-even term: tensor polarization $S^{\alpha\beta}$
- Symmetric and traceless: correspond to (average) shear forces
- For spin ½: P-odd vector polarization requires another vector (q) to form vector product

SUM RULEs

- Efremov,OT'81 : zero sum rules:
- Current conservation: 1st moment: also in parton model by Close and Kumano (90)
- EMT conservation: 2nd moment (forward analog of Ji's SR: AGM =<A_T>=0)
- Average shear force (compensated between quarks and gluons)
- Gravity and (Ex)EP (zero average shear separately for quarks and gluons) – OT'09

Manifestation of post-Newtonian (Ex)EP for spin 1 hadrons

- Tensor polarization - coupling of EMT to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$A_T = \frac{\sigma_+ + \sigma_- - 2\sigma_0}{3\bar{\sigma}}$$

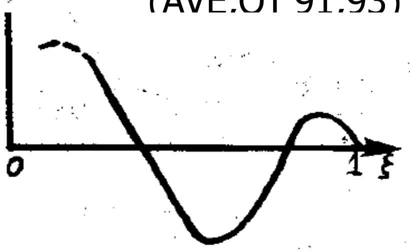
$$\int_0^1 C_i^T(x)dx = 0$$

$$\langle P,S|\bar{\psi}(0)\gamma^\nu D^{\nu_1}...D^{\nu_n}\psi(0)|P,S\rangle_{\mu^2} = i^{-n}M^2 S^{\nu\nu_1}P^{\nu_2}...P\nu_n \int_0^1 C_q^T(x)x^n dx$$

(AVE,OT'91,93)

$$\sum_q \langle P,S|T_i^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu(1-\delta(\mu^2)) + 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

$$\langle P,S|T_g^{\mu\nu}|P,S\rangle_{\mu^2} = 2P^\mu P^\nu\delta(\mu^2) - 2M^2 S^{\mu\nu}\delta_1(\mu^2)$$

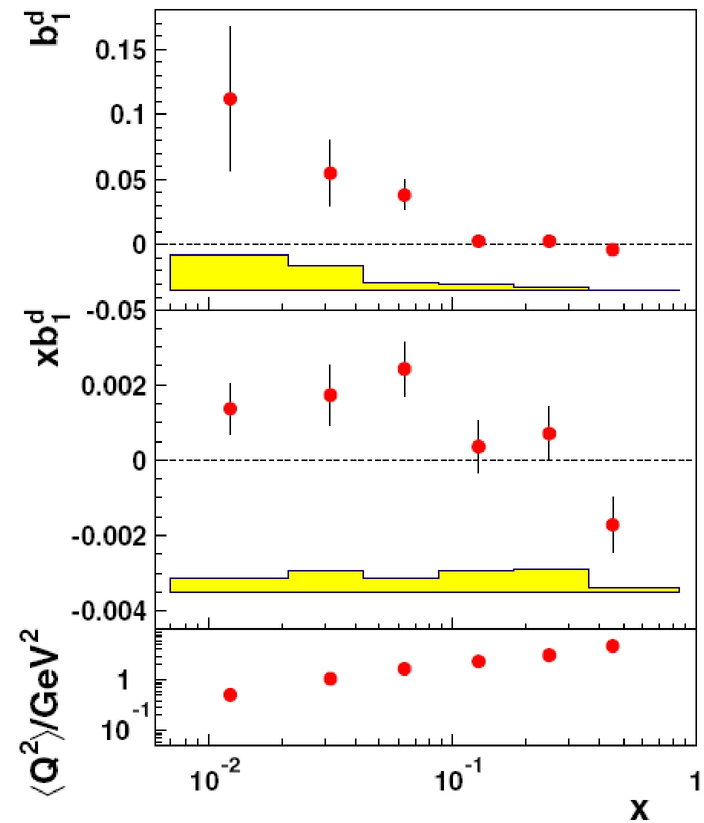


$$\sum_q \int_0^1 C_i^T(x)xdx = \delta_1(\mu^2)$$ =0 for ExEP

HERMES – data on tensor spin structure function

PRL **95**, 242001 (2005)

- Isoscalar target – proportional to the sum of u and d quarks – combination required by (Ex)EP
- Second moments – compatible to zero better than the first one (collective tensor polarized glue << sea)



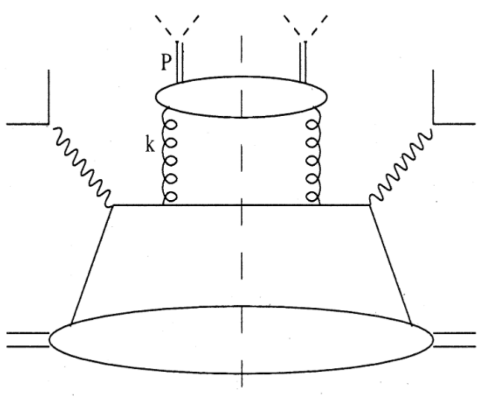
Where else to test?

- EIC
- DY@J-PARC
- ET'81-any hard process ("multi-messenger")
- Possibility: hadronic tensor SSA@NICA

Fragmentation functions

- Tensor polarized fragmentation functions: (Szymanowski, Schaefer, OT'99)

A. Schäfer et al. / Physics Letters B 464 (1999) 94–100



- Suggestion'21: zero SRs (analogous to momentum SR) may probe the (Ex)EP for hadrons inside partons (EIC: gluons)

More on vector mesons and ExEP

- J=1/2 -> J=1. QCD SR/model/lattice calculation of Rho's AMM gives g close to 2 (g=2 exactly in AdS QCD).
- Why?
- Maybe because of similarity of moments and ExEP
- $g-2=\langle E_u(x)\rangle$; $B=\langle xE_u(x)\rangle$
- Directly for charged Rho (combinations like p+n for nucleons unnecessary!). Not reduced to non-extended EP: Gluons momentum fraction sizable

CONCLUSIONS/OUTLOOK

- EIC is also a "Gravity lab"
- Separate couplings of quarks and gluons to gravity
- ExEP, pressure, shear, cosmological constant
- Comparison to QCD matter ("GeV gravity")
- Unruh radiation (Becattini; Prokhorov, Zakharov, OT)
- EoS (Goldstein,Liuti; Fukushima et al.}

BACKUP

Is D-term independent?

- Fast enough decrease at large energy -> $$\mathrm{Re}\,\mathcal{A}(\nu) = \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2-\nu^2} + \mathrm{C}_0$$

$$\mathrm{C}_0 = \Delta - \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2}$$
$$= \Delta + \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,x)}{x}.$$

- FORWARD limit of Holographic equation

$$\Delta = \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,0) - H^{(+)}(x,x)}{x}$$
$$= 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0) - H(x,x)}{x},$$

$$\mathrm{C}_0(t) = 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0,t)}{x}$$

"D – term" 30 years before…

- Cf Brodsky, Close, Gunion'72 (seagull ~ pressure) – but NOT DVMP
- D-term – a sort of renormalization constant
- May be calculated in effective theory if we know fundamental one
- OR
- Recover through special regularization procedure (D. Mueller)?

ExEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible

ExEP and Sivers function

- Sivers function – process dependent (effective) one
- T-odd effect in T-conserving theory- phase
- FSI – Brodsky-Hwang-Schmidt model
- Unsuppressed by M/Q twist 3
- Process dependence- colour factors
- After Extraction of phase – relation to universal (T-even) matrix elements

ExEP and Sivers function -II

- Qualitatively similar to OAM and Anomalous Magnetic Moment (talk of S. Brodsky)
- Quantification : weighted TM moment of Sivers PROPORTIONAL to GPD E (OT'07, **hep-ph/0612205**): $x f_T(x) : xE(x)$

- Burkardt SR for Sivers functions is then related to Ji's SR for E and, in turn, to Equivalence Principle

$$\sum_{q,G} \int dxx f_T(x) = \sum_{q,G} \int dxxE(x) = 0$$

ExEP and Sivers function for deuteron

- EEP - smallness of deuteron Sivers function
- Cancellation of Sivers functions – separately for quarks (before inclusion gluons)
- Equipartition + small gluon spin – large longitudinal orbital momenta (BUT small transverse ones –Brodsky, Gardner)

Another relation of Gravitational FF and NP QCD (first reported at 1992: **hep-ph/9303228**)

- BELINFANTE (relocalization) invariance :

decreasing in coordinate –

$$M^{\mu,\nu\rho} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J^5_{S\sigma} + x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu}$$

smoothness in momentum space

$$M^{\mu,\nu\rho} = x^\nu T^{\mu\rho}_B - x^\rho T^{\mu\nu}_B$$

- Leads to absence of massless pole in singlet channel – U_A(1)

$$\epsilon_{\mu\nu\rho\alpha} M^{\mu,\nu\rho} = 0.$$

- Delicate effect of NP QCD

$$(g_{\rho\nu} g_{\alpha\mu} - g_{\rho\mu} g_{\alpha\nu}) \partial^\rho (J^\alpha_{5S} x^\nu) = 0$$

- Equipartition – deeply related to relocalization invariance by QCD evolution

$$q^2 \frac{\partial}{\partial q^\alpha} \langle P | J^\alpha_{5S} | P + q \rangle = (q^\beta \frac{\partial}{\partial q^\beta} - 1) q_\gamma \langle P | J^\gamma_{5S} | P + q \rangle$$

$$\langle P, S | J^5_\mu(0) | P + q, S \rangle = 2M S_\mu G_1 + q_\mu (Sq) G_2,$$

$$q^2 G_2|_0 = 0$$

Holography vs NLO

- Depends on factorization scheme
- Special role of scheme preserving the coefficient function
- Nucleon as (scheme dependent) black hole – 3D information encoded in 2D

C vs Cbar (=Λ)

- Cancellations of Cbars – negative pressure
- Cf Chaplygin gas: ($p=-A/\varrho$) – analog of cosmological constant
- Cancellation in vacuum; Pauli (divergent), Zel'dovich (finite)
- Flavour structure of pressure: DVMP!