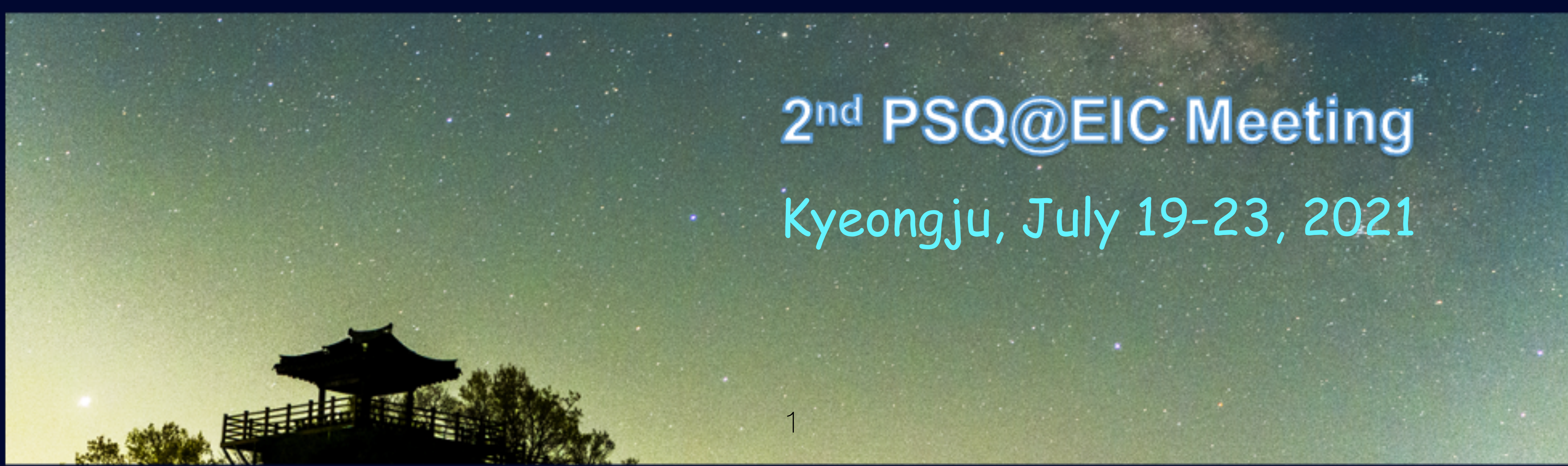
A New look at the Nucleon structure from a modern perspective

Hyun-Chul Kim
Department of Physics,
Inha University



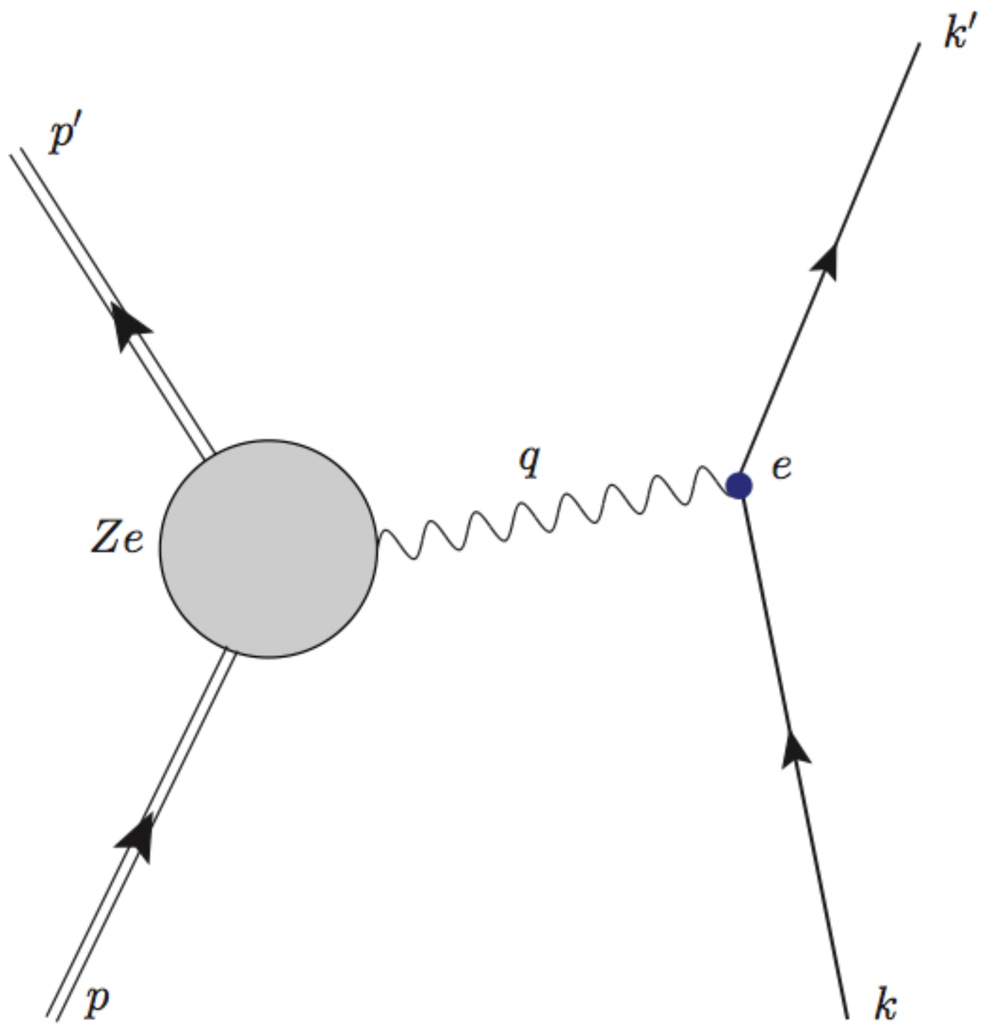
Modern view vs. Traditional view

Form factors: Traditional Understanding

Form factors tell you how the corresponding particle looks like in various aspects.

Historical example: Rutherford scattering

Rutherford, PRS (1911)



$$\frac{d\sigma}{d\Omega} = \frac{V^2 E'^2}{(2\pi)^2(\hbar c)^4} \left|\langle\psi_f|\mathcal{H}_{\rm int}|\psi_i\rangle\right|^2$$

$$\langle\psi_f|\mathcal{H}_{\rm int}|\psi_i\rangle = \frac{Z4\pi\alpha\hbar^3 c}{|\mathbf{q}|^2\cdot V}\int \rho(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}/\hbar}\,d^3x$$

Form factor of a particle

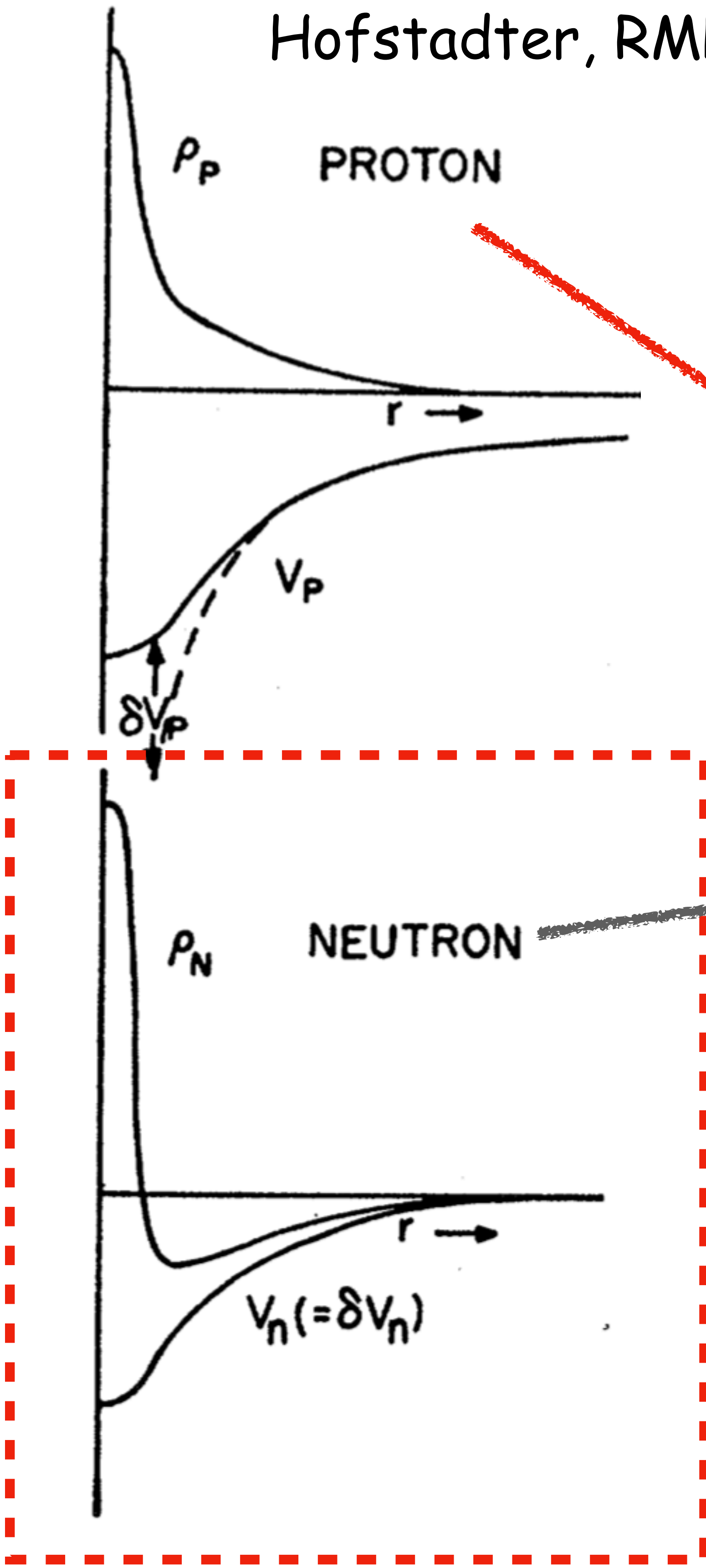
$$F(\mathbf{q}) = \int \rho(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}/\hbar}\,d^3x$$

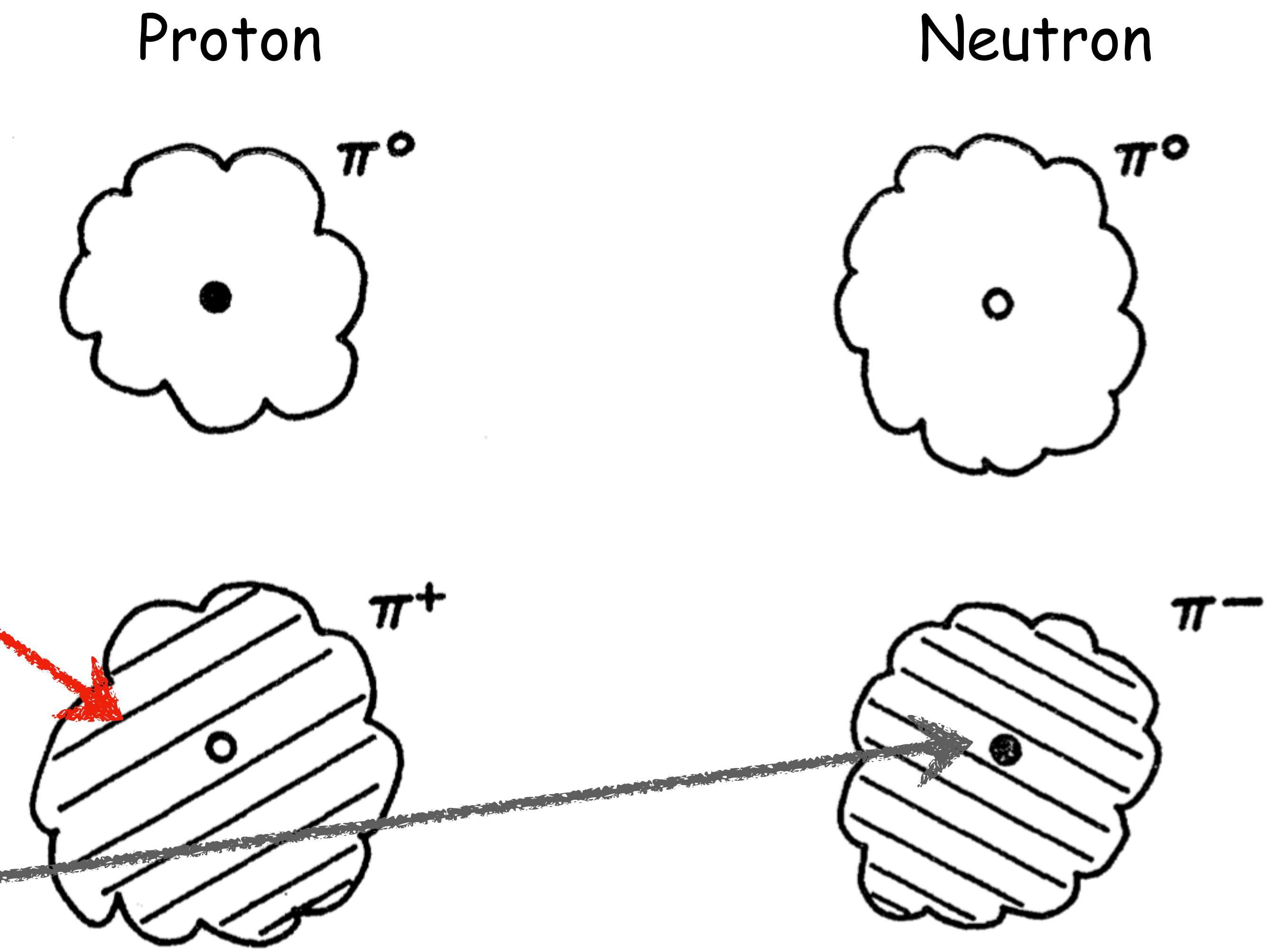
➡ This is the text expression for the definition of a form factor.

Caveat: Location r is uncertain by the Compton wavelength: $\delta r \sim \dfrac{\hbar}{Mc}$

Form factors: Traditional Understanding

Hofstadter, RMP 28 (1956)





We will soon critically discuss this neutron charge density. (Yennie et al. already knew the problem in 1957, related to the Breit frame (BF).

Yennie et al. RMP 29, 144 (1957).

Critical view on Nucleon form factors

Traditional interpretation of the nucleon form factors

$$F_1(Q^2) = \int d^3x e^{i\mathbf{Q}\cdot\mathbf{x}} \rho(\mathbf{r}) \rightarrow \quad \rho(\mathbf{r}) = \sum \psi^\dagger(\mathbf{r})\psi(\mathbf{r})$$

Particle number fixed.

- This is valid for atoms and nuclei: $\frac{\delta r}{r} = \frac{m_e \alpha}{M} \sim 10^{-5}$

Crucial criticism on the traditional definition of the nucleon form factors.

- It is not valid anymore for the nucleon: $r \sim 0.8\,\mathrm{fm}$ $\quad \delta r \sim \frac{\hbar}{M_N c} \approx 0.2\,\mathrm{fm}$

  $\delta r / r \sim 0.25$

The nucleon is a **relativistic** particle!

Particle creation and annihilation inside a nucleon

⇨
- Validity of the nucleon 3D distributions was put into question.
- View on the nucleon form factors has been modernized.

M. Burkardt, PRD 62 (2000) [**66** (2002)]

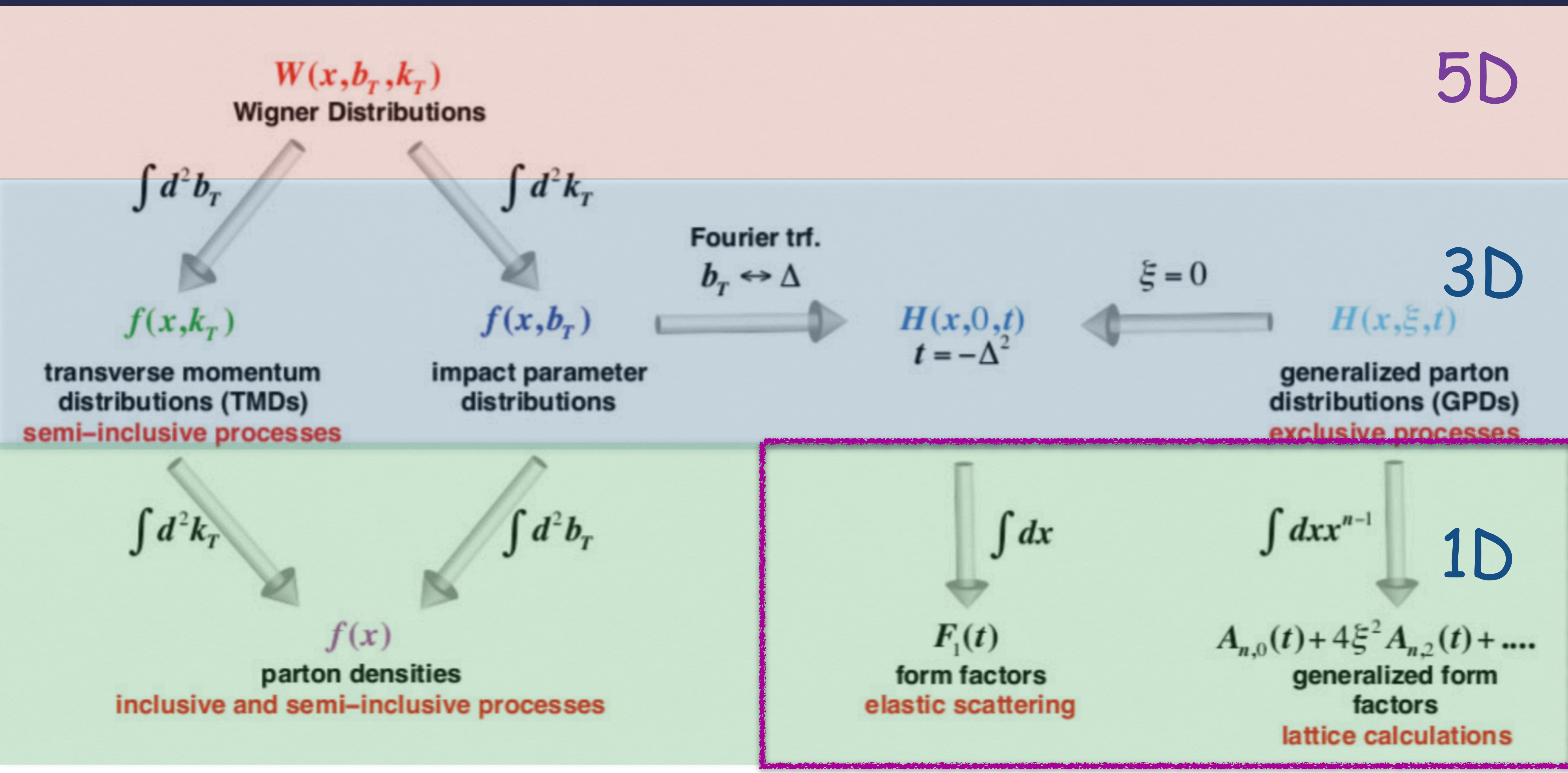
Belitsky & Radyushkin, Phys.Rept. **418** (2005)

G.A. Miller, PRL **99** (2007)

C. Lorce, PRL **125** (2020)

R. L. Jaffe, PRD **103** (2021)

Stitching together a 5D Image of the Nucleon

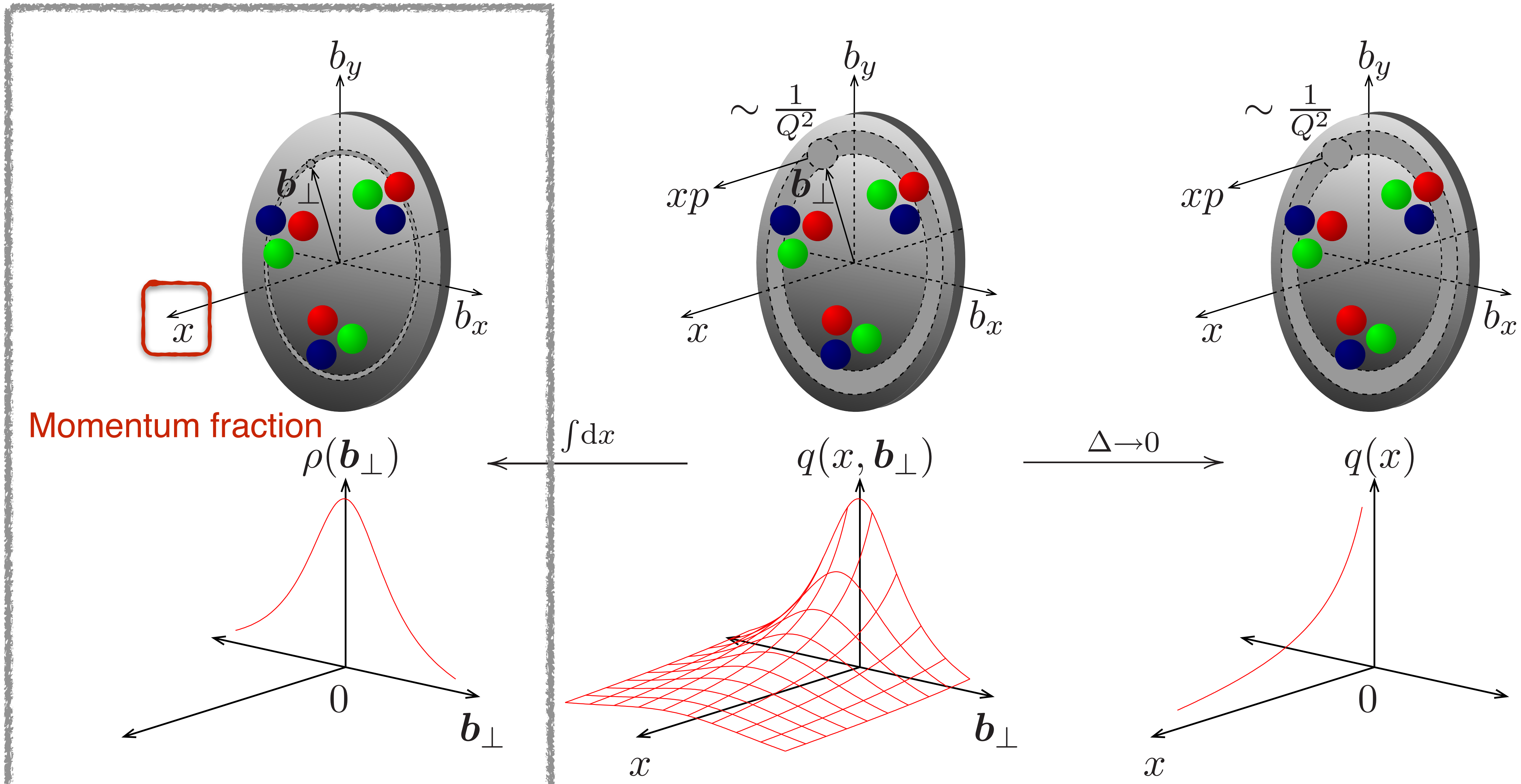


Yuan's talk on Tuesday

I will concentrate on 2D transverse distributions.

Figure taken from Eur. Phys. J. A (2016) 52: 268

Modern Understanding on Nucleon form factors



Transverse densities of Form factors

GPDs
Nucleon Tomography

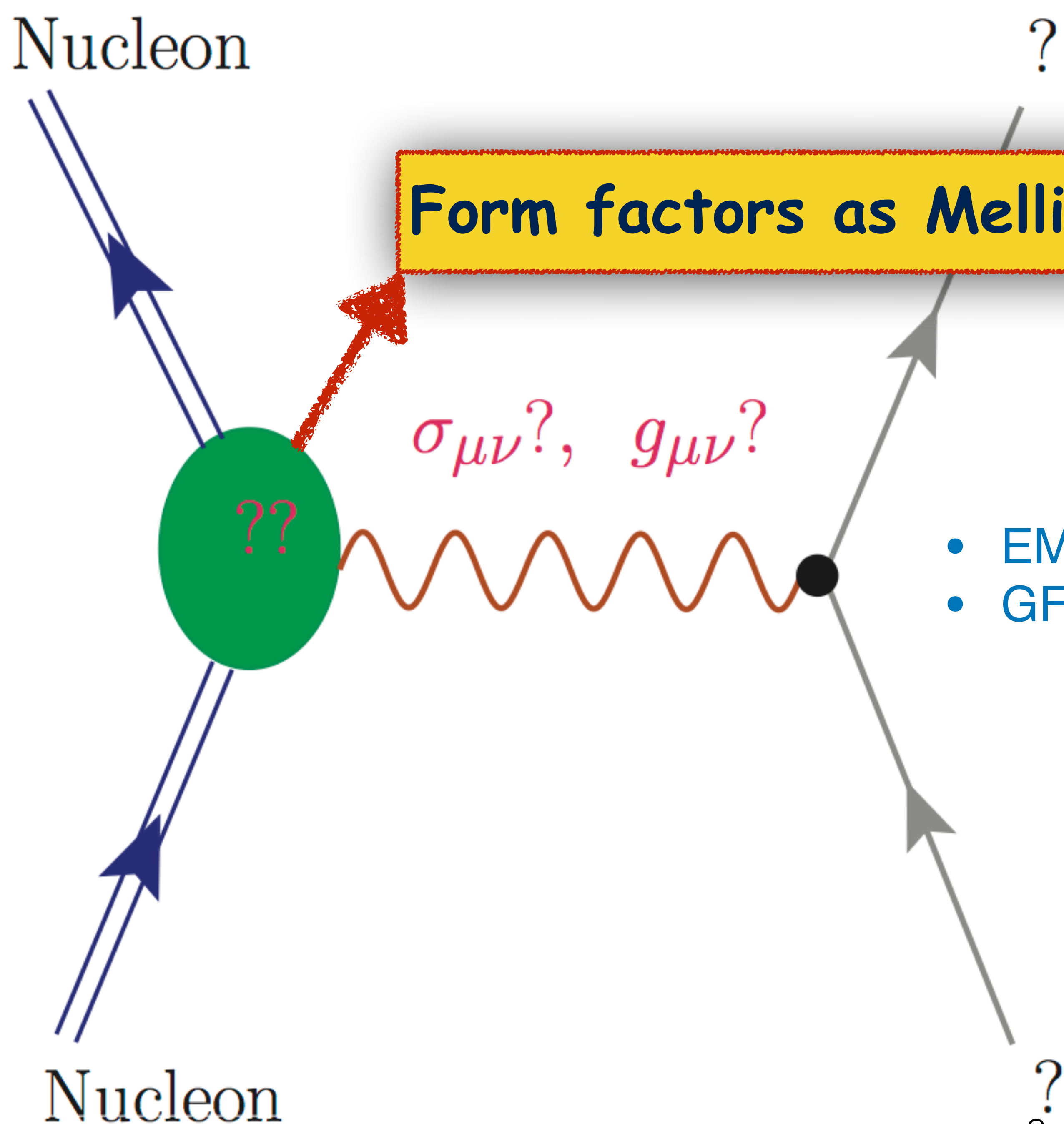
Structure functions
Parton distributions

D. Brömmel, Dissertation (Regensburg U.)

Modern Understanding on Nucleon form factors

Probes are unknown for **Tensor form factors**
and the **Gravitational form factors!**

Form factors as Mellin moments of the GPDs



- EMFFs: the first moments of the vector GPDs
- GFFs: the second moments of the isoscalar GPDs

D. Müller et al. Fortschr. Phys. 42 (1994).

X. D. Ji, PRL 78, PRD 55 (1997).

A. V. Radyushkin, PLB 380 (1996)

Modern Understanding on Nucleon form factors

- GPDs

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P'|\overline{\psi}(-\lambda n/2)\gamma^\mu \psi(\lambda n/2)|P\rangle = H(x,\xi,\Delta^2)\overline{U}(P')\gamma^\mu U(P) + E(x,\xi\Delta^2)\overline{U}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}U(P) + \cdots,$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P'|\overline{\psi}(-\lambda n/2)\gamma^\mu \gamma_5 \psi(\lambda n/2)|P\rangle = \widetilde{H}(x,\xi,\Delta^2)\overline{U}(P')\gamma^\mu \gamma_5 U(P) + \widetilde{E}(x,\xi,\Delta^2)\overline{U}(P')\frac{\gamma_5 \Delta^\mu}{2M}U(P) + \cdots,$$

- Mellin moments of the GPDs

- The first moments of the GPDs H & E yield the well-known EM form factors

$$\int_{-1}^{1} dx H(x,\xi,\Delta^2) = F_1(\Delta^2), \quad \int_{-1}^{1} dx E(x,\xi,\Delta^2) = F_2(\Delta^2)$$

- The second moments of the GPDs H & E give the gravitational (EMT) FFs (Ji's sum rules).

$$\int_{-1}^{1} dx\ x \sum_q H^q(x,\xi) = M_2^Q + \frac{4}{5} d_1 \xi^2 ,$$

$$\int_{-1}^{1} dx\ x \sum_q E^q(x,\xi) = \left(2J^Q - M_2^Q\right) - \frac{4}{5} d_1 \xi^2$$

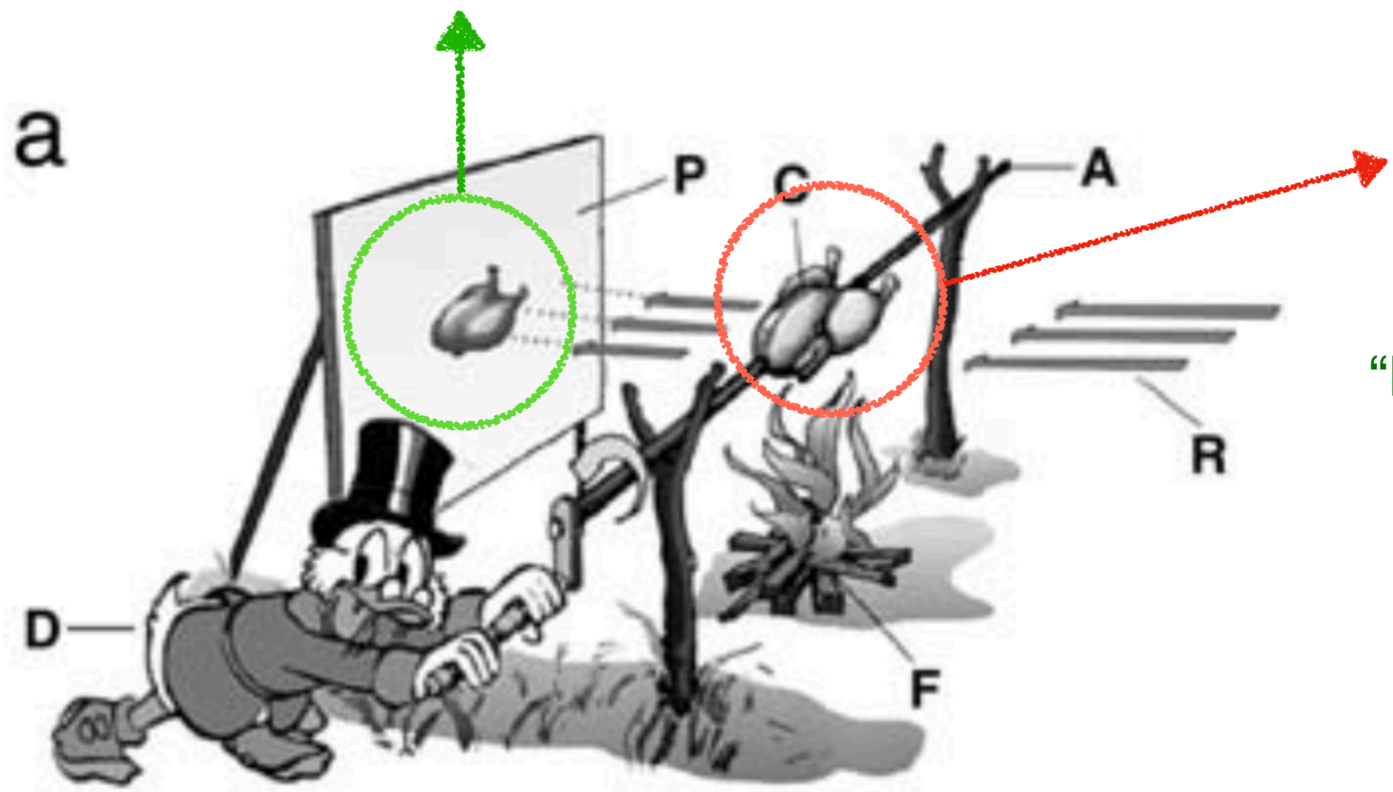
D. Müller et al. Fortschr. Phys. 42 (1994).

X. D. Ji, PRL 78, PRD 55 (1997).

A. V. Radyushkin, PLB 380 (1996)

Abel & Radon transforms & Nucleon tomography

2D transverse distributions in the IMF (QM probabilistic)



3D distributions in the BF
(Quasi-probabilistic)

"Electron tomography, edited by J. Frank"

- Abel transformation maps 3D distributions of a particle with spin 0 or 1/2 at rest onto 2D transverse plane in the IMF.
(Radon transform is required for that with higher spin.)

3D distributions ⟷ 2D distributions

This is the subject of the present talk.

M. Burkardt, PRD 62 (2000) [66 (2002)]

G. A. Miller, PRL. 99 (2007).

Carlson & Vanderhaeghen, PRL 100 (2008)

C. Lorce, PRL 125 (2020).

Panteleeva & Polyakov, ArXiv: 2102.10902

Mechanical properties
of
Baryons

Gravitational form factors

- EMT current in QCD & GFFs

Kobzarev et al. 1962; Pagels, 1966

$$T_q^{\mu\nu} = \frac{1}{4}\overline{\psi}_q\left(-i\overleftarrow{\mathcal{D}}^\mu\gamma^\nu - i\overleftarrow{\mathcal{D}}^\nu\gamma^\mu + i\overrightarrow{\mathcal{D}}^\mu\gamma^\nu + i\overrightarrow{\mathcal{D}}^\nu\gamma^\mu\right)\psi_q - g^{\mu\nu}\overline{\psi}_q\left(-\frac{i}{2}\overleftarrow{\not{\mathcal{D}}} + \frac{i}{2}\overrightarrow{\not{\mathcal{D}}} - m_q\right)\psi_q,$$

$$T_g^{\mu\nu} = F^{a,\mu\eta}\,F^{a,}{}_\eta{}^\nu + \frac{1}{4}g^{\mu\nu}F^{a,\kappa\eta}\,F^{a,}{}_{\kappa\eta}.$$

D(Druck)-term

Weiss & Polyakov, 1999

$$\langle p'|T^{\mu\nu}(0)|p\rangle = \bar{u}(p')\left[A^a(t)\frac{P^\mu P^\nu}{M_N} + J^a(t)\frac{iP^{\{\mu\sigma_\nu\}\rho}\Delta_\rho}{2M_N} + D^a(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M_N} + M_N\bar{c}^a(t)g^{\mu\nu}\right]u(p)$$

δg^{00} → Mass: $\sum_a A^a(0) = 1$

δg^{0i} → Spin: $\sum_a J^a(0) = \frac{1}{2}$

δg^{ij} → Deformation of space = mechanical properties of the nucleon

Non-conservation of EMT pieces

Pressure & Shear-force distributions (pressure anisotropy)

Pressure & Shear-force distributions

$$T^a_{ij}(\boldsymbol{r},\sigma',\sigma) = p^a(r)\delta^{ij}\delta_{\sigma'\sigma} + s^a(r)\left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)\delta_{\sigma'\sigma}$$

- 3D Shear-force density in the BF

$$s^a(r) = -\frac{1}{4M_B} r \frac{d}{dr}\frac{1}{r}\frac{d}{dr}\tilde{D}^a(r)$$

- 3D Pressure density in the BF   M.V. Polyakov, PLB555 (2003)

$$p^a(r) = \frac{1}{6M_B}\frac{1}{r^2}\frac{1}{dr}r^2\frac{d}{dr}\tilde{D}^a(r) - M_B\int\frac{d^3\Delta}{(2\pi)^3}e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}}\bar{c}^a(t)$$

$$\tilde{D}^a(r) = \int\frac{d^3\Delta}{(2\pi)^3}e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}}D^a(t)$$

- This term is related to forces between quark and gluon subsystems (Polyakov & Son, 2018).
- It contributes to gluon and quark parts of energy density (mass decomposition). (Lorce, 2018)
- It vanishes for Goldstone bosons (P. Schweitzer & M.V. Polyakov, 2019).

Abel transforms

- Abel transform from 3D in the BF to 2D in the IMF (Also invertible)

$$\mathcal{E}(x_\perp) = 2\int_{x_\perp}^{\infty} \left(\varepsilon(r) + \frac{3}{2}p(r) + \frac{1}{4m}\partial^2\left[\tilde{A}(r) - 2\tilde{J}(r)\right]\right)\frac{rdr}{\sqrt{r^2 - x_\perp^2}}$$

$$\rho_J^{(2D)}(x_\perp) = 3\int_{x_\perp}^{\infty} \frac{\rho_J(r)}{r}\frac{x_\perp^2\, dr}{\sqrt{r^2 - x_\perp^2}}$$

$$\mathcal{S}(x_\perp) = \int_{x_\perp}^{\infty} \frac{s(r)}{r}\frac{x_\perp^2\, dr}{\sqrt{r^2 - x_\perp^2}}$$

$$\frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp) = \frac{1}{2}\int_{x_\perp}^{\infty}\left(\frac{2}{3}s(r) + p(r)\right)\frac{rdr}{\sqrt{r^2 - x_\perp^2}}$$

Abel, J. Reine und Angew. Math. 1 (1826)

- Abel transform is used for tomography of spherically symmetric systems (spin 0 & 1/2 hadrons).
- For non-spherical objects (spin > 1/2), the Radon transform comes into play.

Panteleeva & Polyakov, ArXiv: 2102.10902

J. Y. Kim & HChK, ArXiv 2105.10279

Equivalence of the 3D BF & 2D LF distributions

- Von Laue Conditions

$$\int_0^\infty dr\ r^2 p(r) = 0 \quad \Longleftrightarrow \quad \int d^2x_\perp \mathcal{P}(x_\perp) = 0$$

- Local stability Conditions

$$\frac{2}{3}s(r) + p(r) > 0 \quad \Longleftrightarrow \quad \frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp) > 0$$

Geometric factor

$$\Omega_d = \frac{\sqrt{\pi}}{2}\frac{\Gamma\left(\frac{d+1}{2}\right)}{\Gamma\left(\frac{d+2}{2}\right)}$$

- Mechanical radius

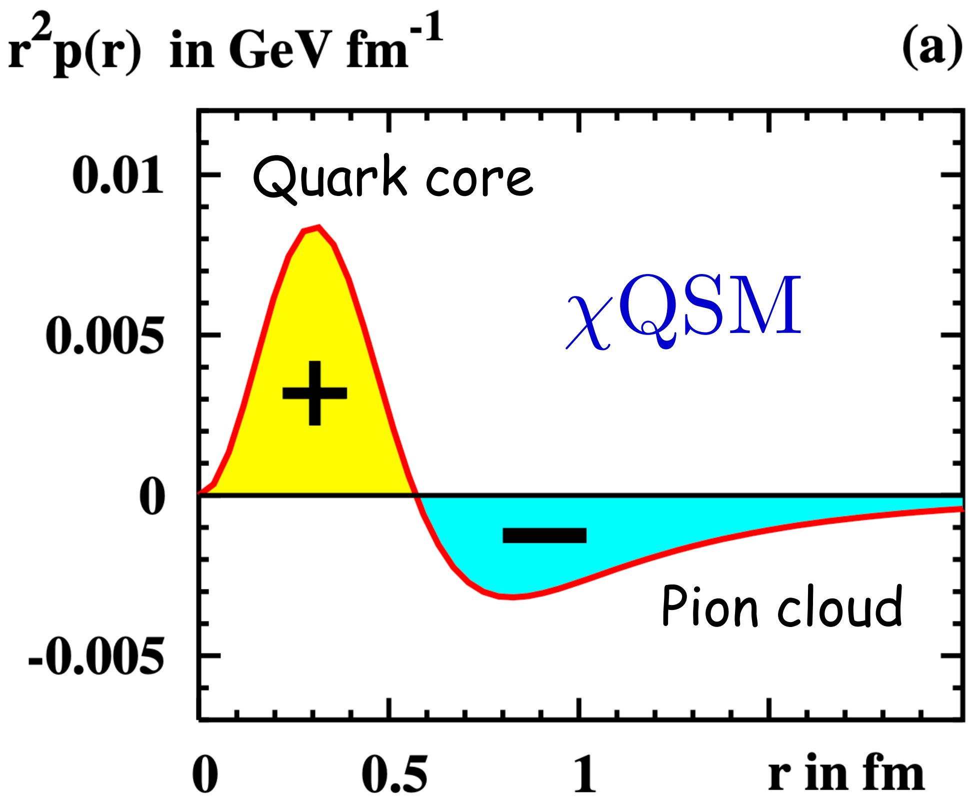
$$\langle x_\perp^2 \rangle_{\text{mech}} = \frac{\int d^2x_\perp x_\perp^2 \left(\frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp)\right)}{\int d^2x_\perp \left(\frac{1}{2}\mathcal{S}(x_\perp) + \mathcal{P}(x_\perp)\right)} = \frac{4D(0)}{\int_{-\infty}^0 dt D(t)} = \frac{2}{3}\langle r^2 \rangle_{\text{mech}}$$

- D(Druck)-terms

$$D(0) = -\frac{4M_N}{15}\int d^3r r^2 s(r) = m\int d^3r r^2 p(r) \quad \Longleftrightarrow \quad D(0) = -m\int d^2x_\perp x_\perp^2 \mathcal{S}(x_\perp) = 4m\int d^2x_\perp x_\perp^2 \mathcal{P}(x_\perp)$$

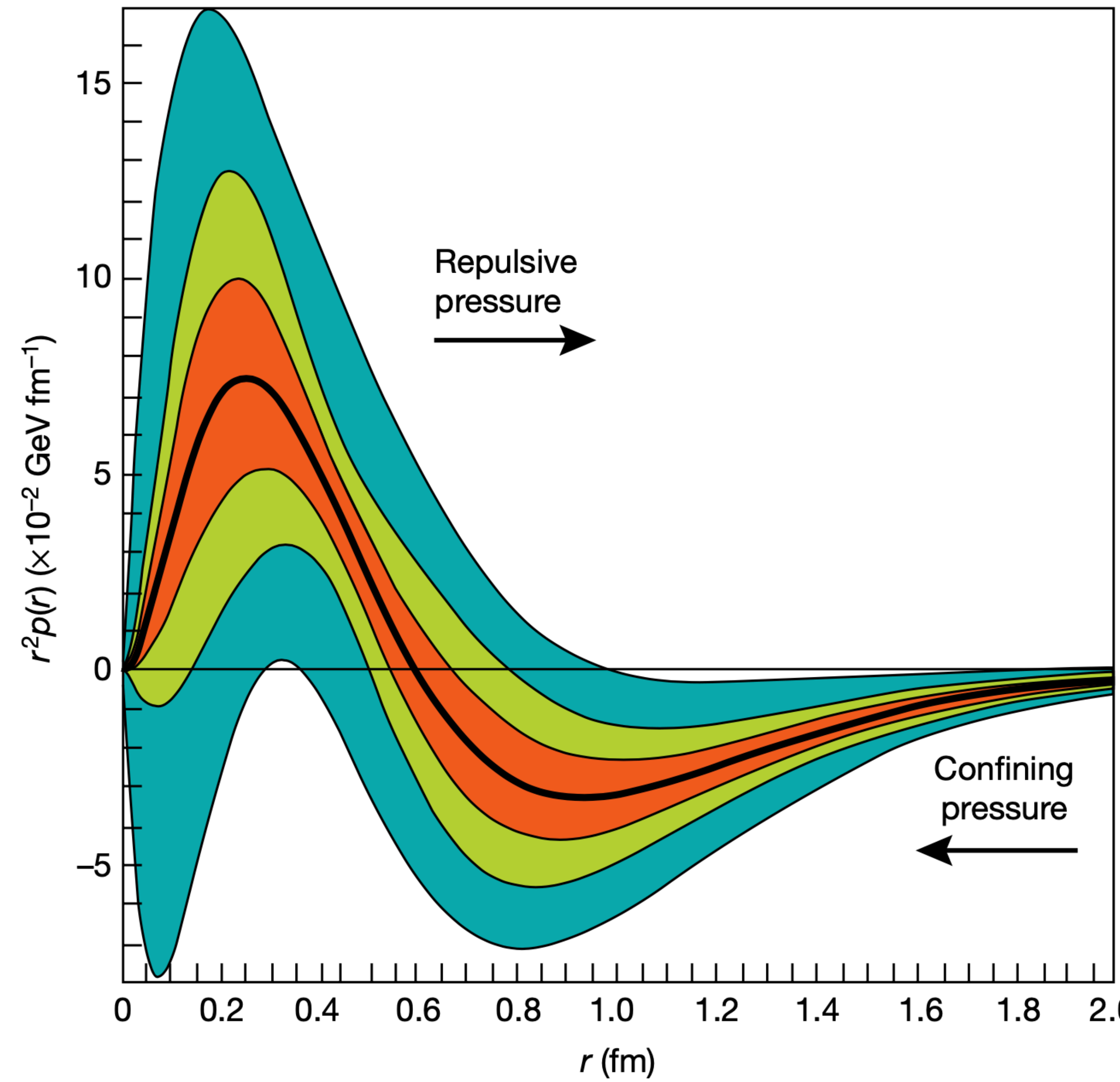
Panteleeva & Polyakov, ArXiv: 2102.10902

The 3D BF pressure density



Goeke et al., PRD 75 (2007)

Kim, HChK, H. Son, M. Polyakov
PRD 103 (2021):
Extended to a singly heavy baryon



V. Burkert et al., Nature 557, 396 (2018)

Global stability condition (von Laue)

$$\int_0^\infty dr\, r^2 p(r) = 0$$

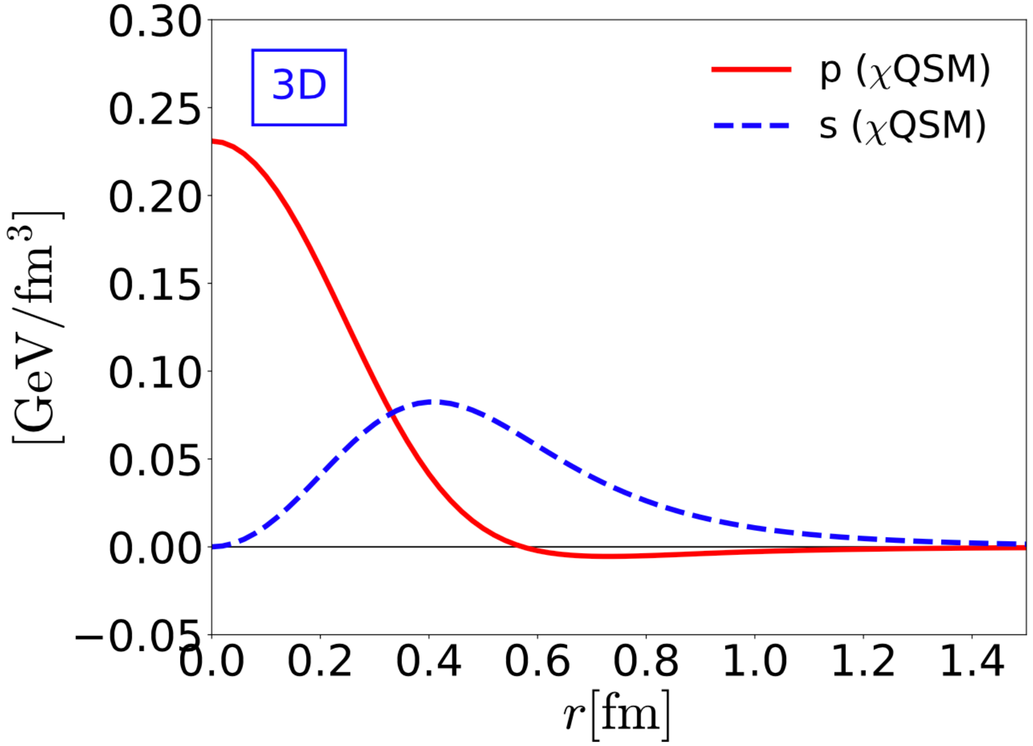
The 3D & 2D pressure & shear-force densities

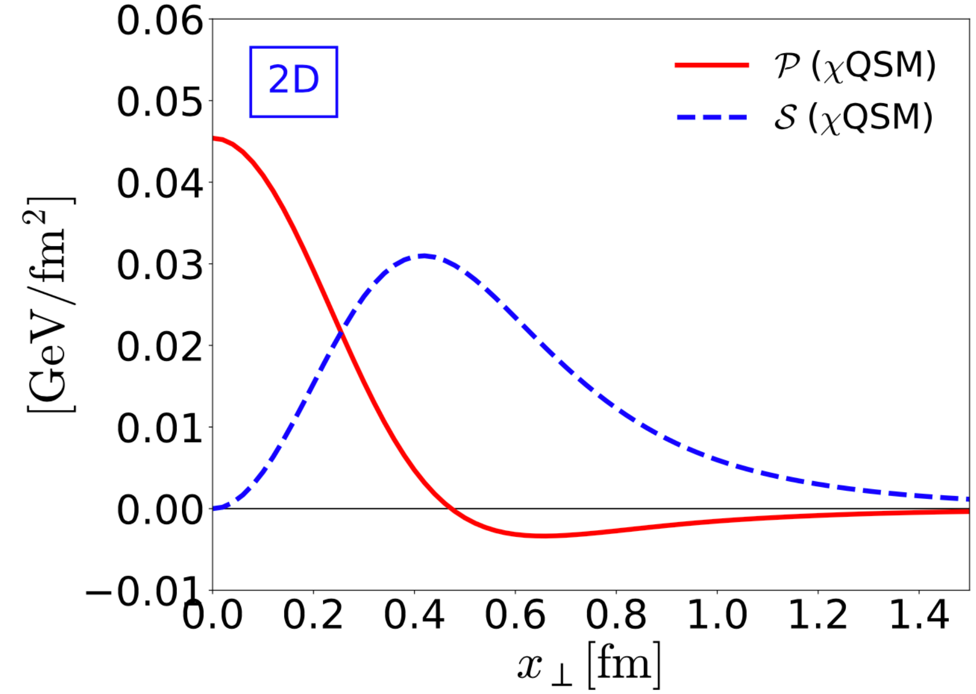
3D in BF

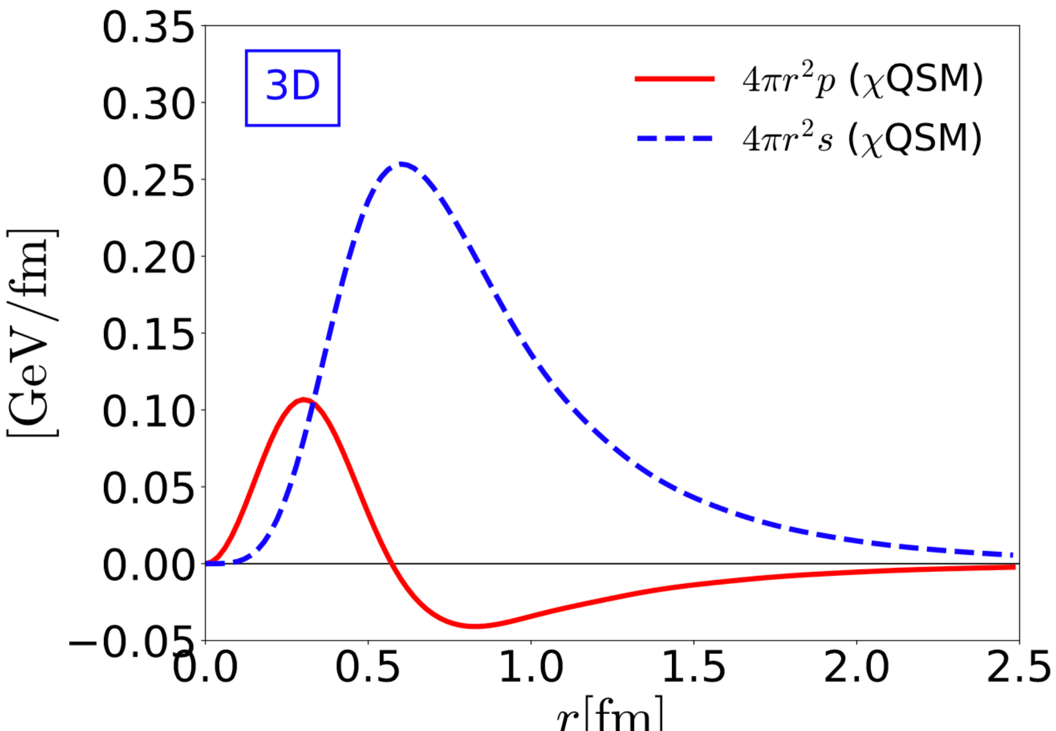
χQSM

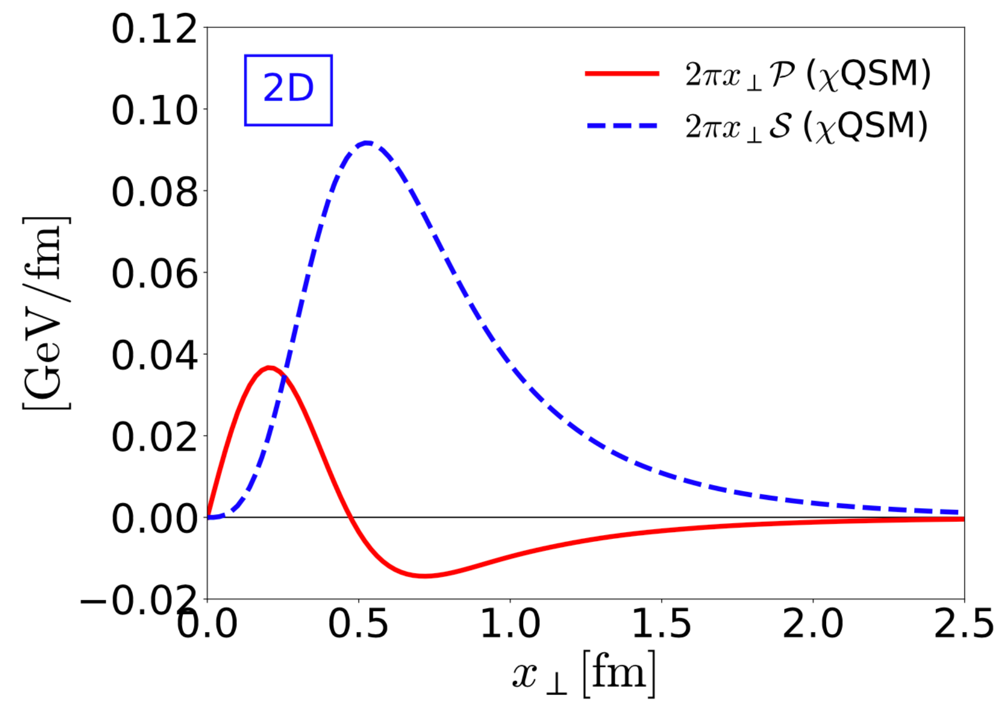
2D in IMF

Abel transforms









J. Y. Kim & HChK, ArXiv 2105.10279

Radii of the proton

$$\langle x_\perp^2 \rangle_{\rm mass} < \langle x_\perp^2 \rangle_{\rm mech} < \langle x_\perp^2 \rangle_{\rm charge} < \langle x_\perp^2 \rangle_J \quad (\text{2D } \chi\text{QSM})$$

$$\langle r^2 \rangle_{\rm mech} < \langle r^2 \rangle_{\rm mass} < \langle r^2 \rangle_{\rm charge} < \langle r^2 \rangle_J \quad (\text{3D } \chi\text{QSM})$$

$$\langle x_\perp^2 \rangle_{\rm mass} = \frac{1}{m} \int d^2 x_\perp x_\perp^2 \mathcal{E}(x_\perp) = \frac{2}{3} \langle r^2 \rangle_{\rm mass} + \frac{D(0)}{m^2} \qquad (D(0) < 0)$$

Note that 2D mass radius is smaller than the 3D one.

$\langle x_\perp^2 \rangle_{\rm mass}$ (fm^2)	$\langle x_\perp^2 \rangle_J$ (fm^2)	$\langle x_\perp^2 \rangle_{\rm mech}$ (fm^2)	$\langle x_\perp^2 \rangle_{\rm charge}$ (fm^2)
0.39	1.19	0.42	0.58

$\langle r^2 \rangle_{\rm mass}$ (fm^2)	$\langle r^2 \rangle_J$ (fm^2)	$\langle r^2 \rangle_{\rm mech}$ (fm^2)	$\langle r^2 \rangle_{\rm charge}$ (fm^2)
0.66	1.49	0.63	0.86

J. Y. Kim & HChK, ArXiv 2105.10279

Stability conditions

- Conservation of the statice EMT current → Global & local stability conditions

$$\partial^i T_{ij} = \frac{r_j}{r}\left[\frac{2}{3}\frac{\partial s(r)}{\partial r} + \frac{2s(r)}{r} + \frac{\partial p(r)}{\partial r}\right] = 0$$

- Von Laue condition: Global stability condition

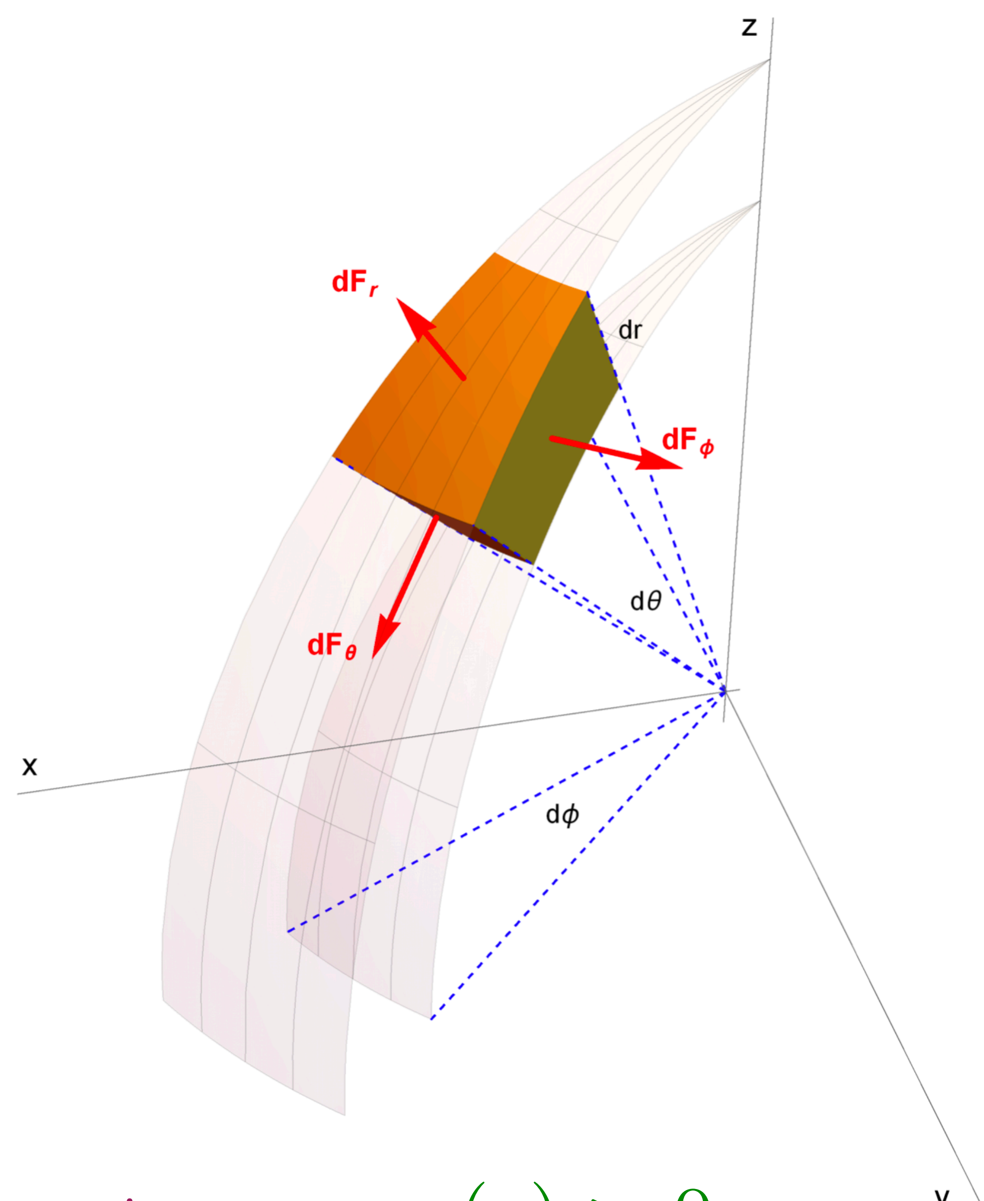
$$\int_0^\infty dr\ r^2 p(r) = 0$$

$$dF^i_{(r,\theta,\phi)} = T^{ij} dS_{(r,\theta,\phi)} \boldsymbol{e}^j_{(r,\theta,\phi)}$$

$$p_r(r) := \frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r),$$

$$p_\theta(r) := \frac{dF_\theta}{dS_\theta} = -\frac{1}{3}s(r) + p(r),$$

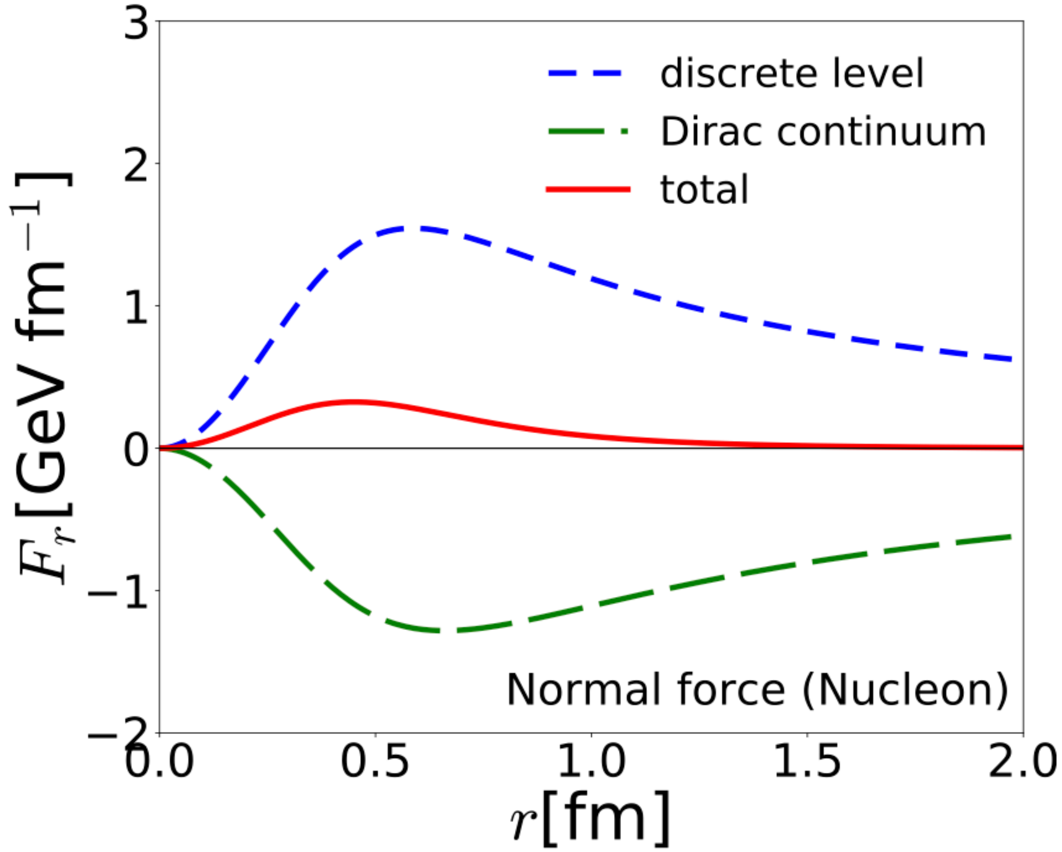
$$p_\phi(r) := \frac{dF_\phi}{dS_\phi} = -\frac{1}{3}s(r) + p(r)$$



Local stability conditions $p_r(r) > 0$

A. Perevalova, M. V. Polyakov and P. Schweitzer, PRD 94 (2016)

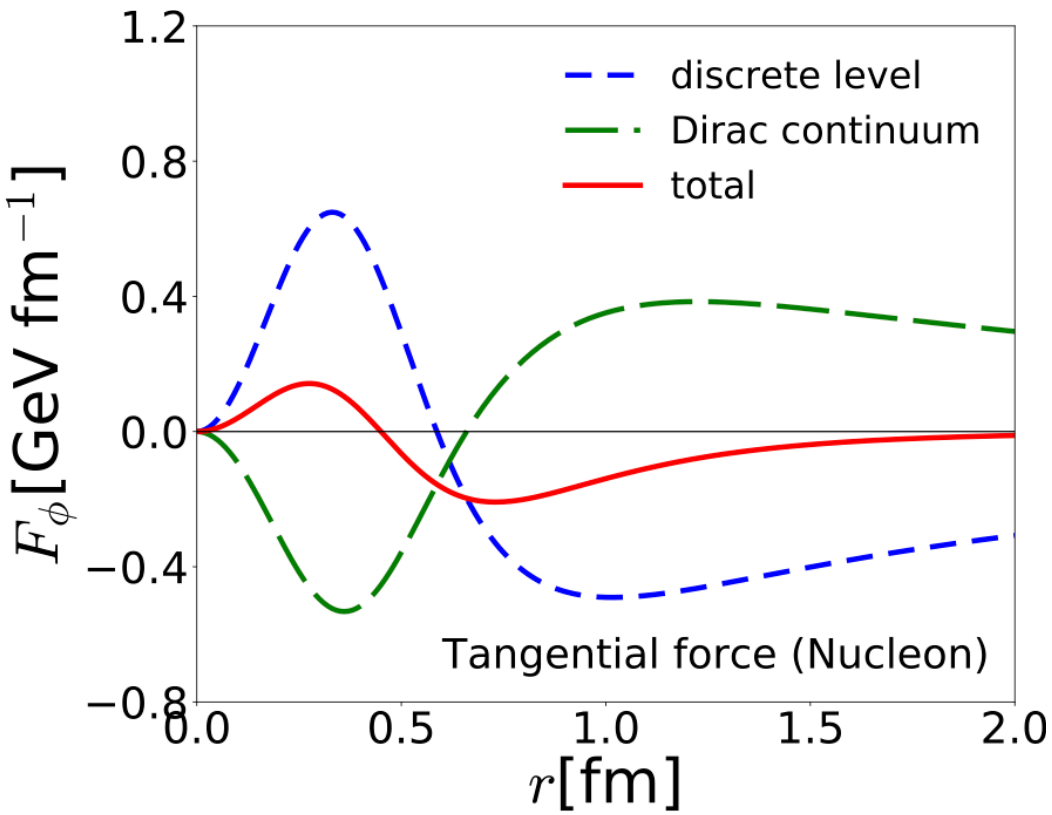
3D force fields & local stability



- Normal force is always positive:

$$F_r(r) > 0$$

The discrete level overcomes the Dirac continuum.



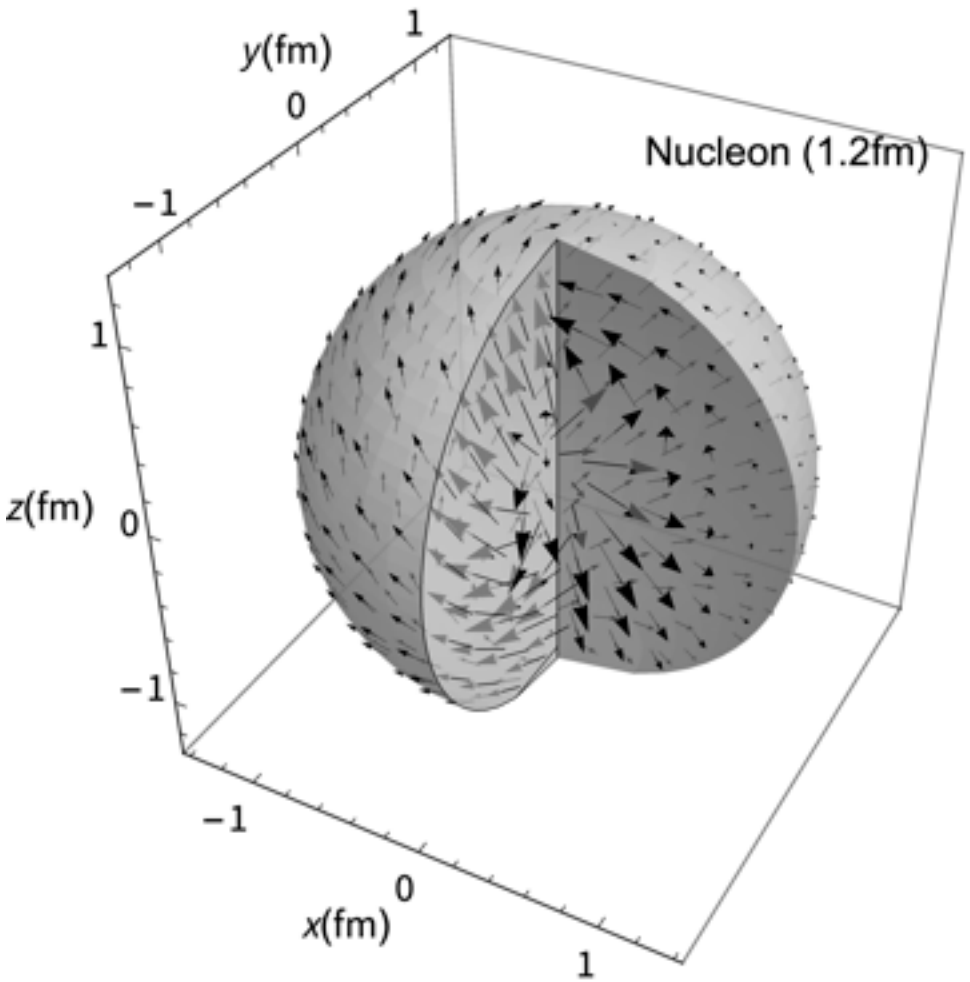
- Tangential force should at least have one nodal point.

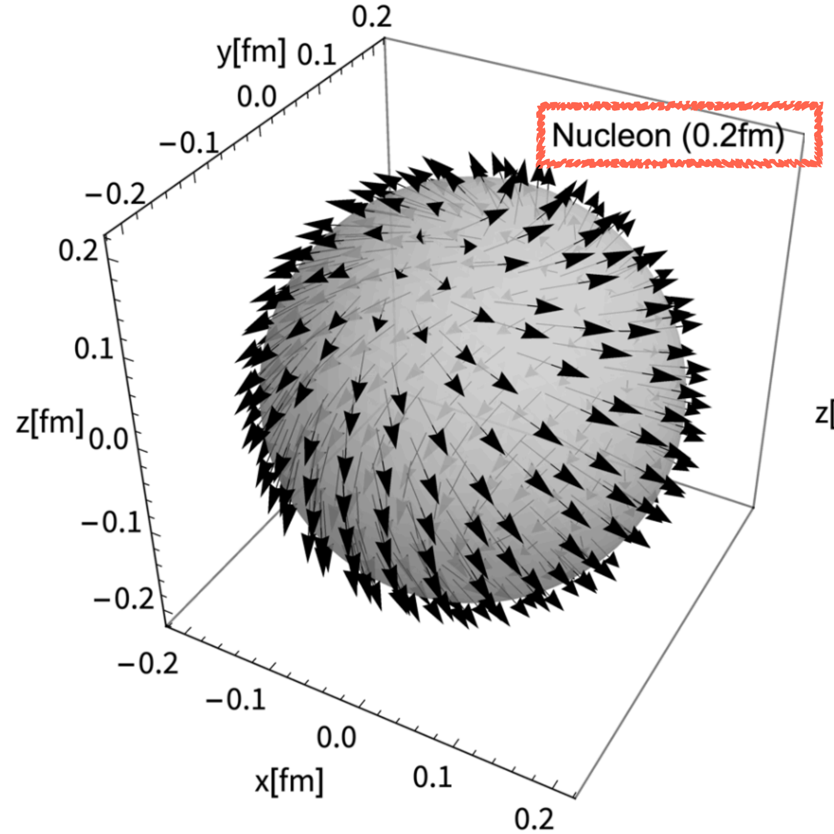
→ Inner part of the tangential force is opposite to its outer part.

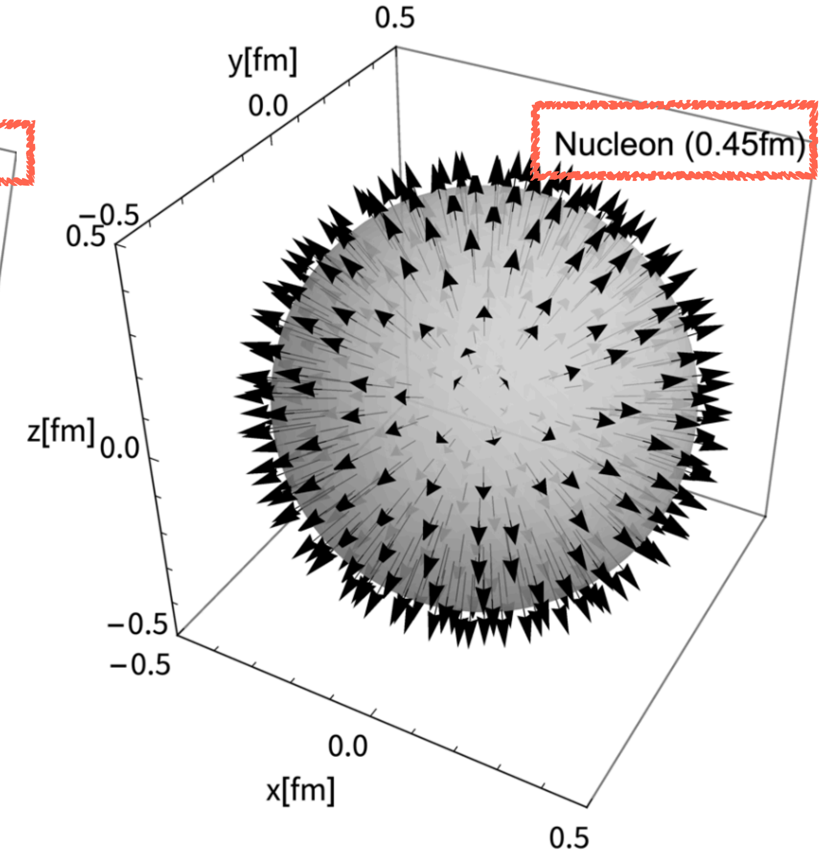
Kim, HChK, H. Son, M. Polyakov PRD 103 (2021)

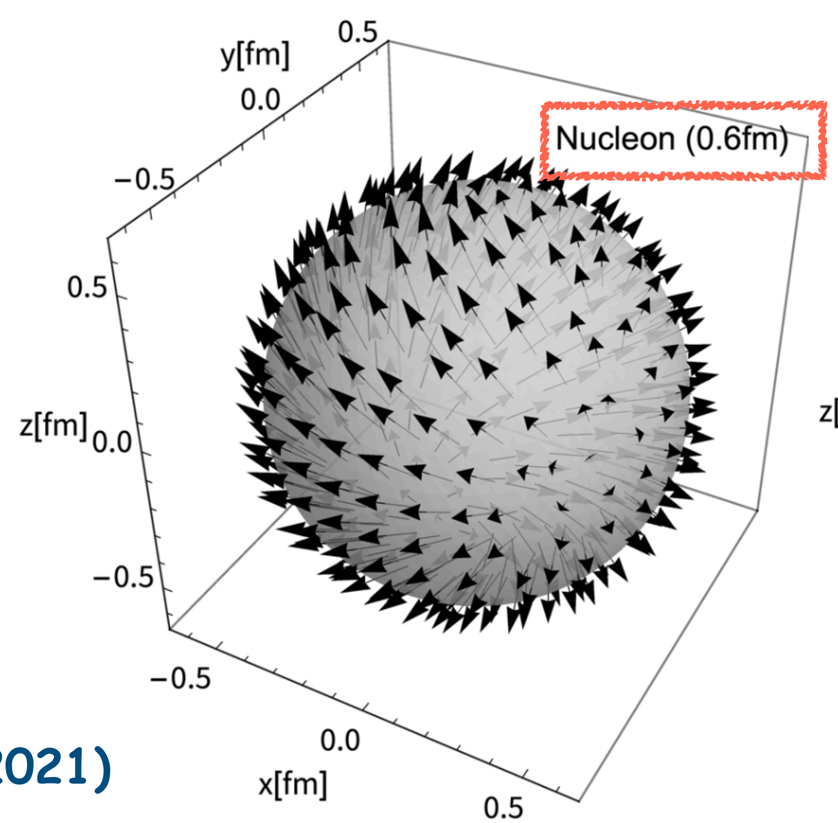
3D force fields & local stability

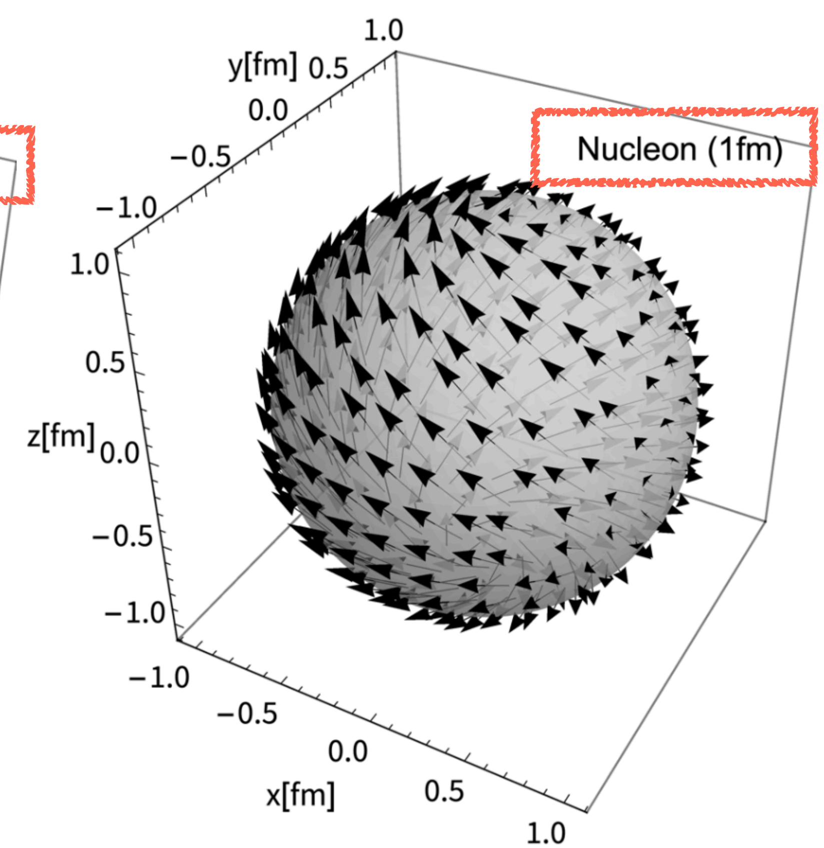
- Looking into the nucleon: How the force fields act locally to acquire the stability of the nucleon.







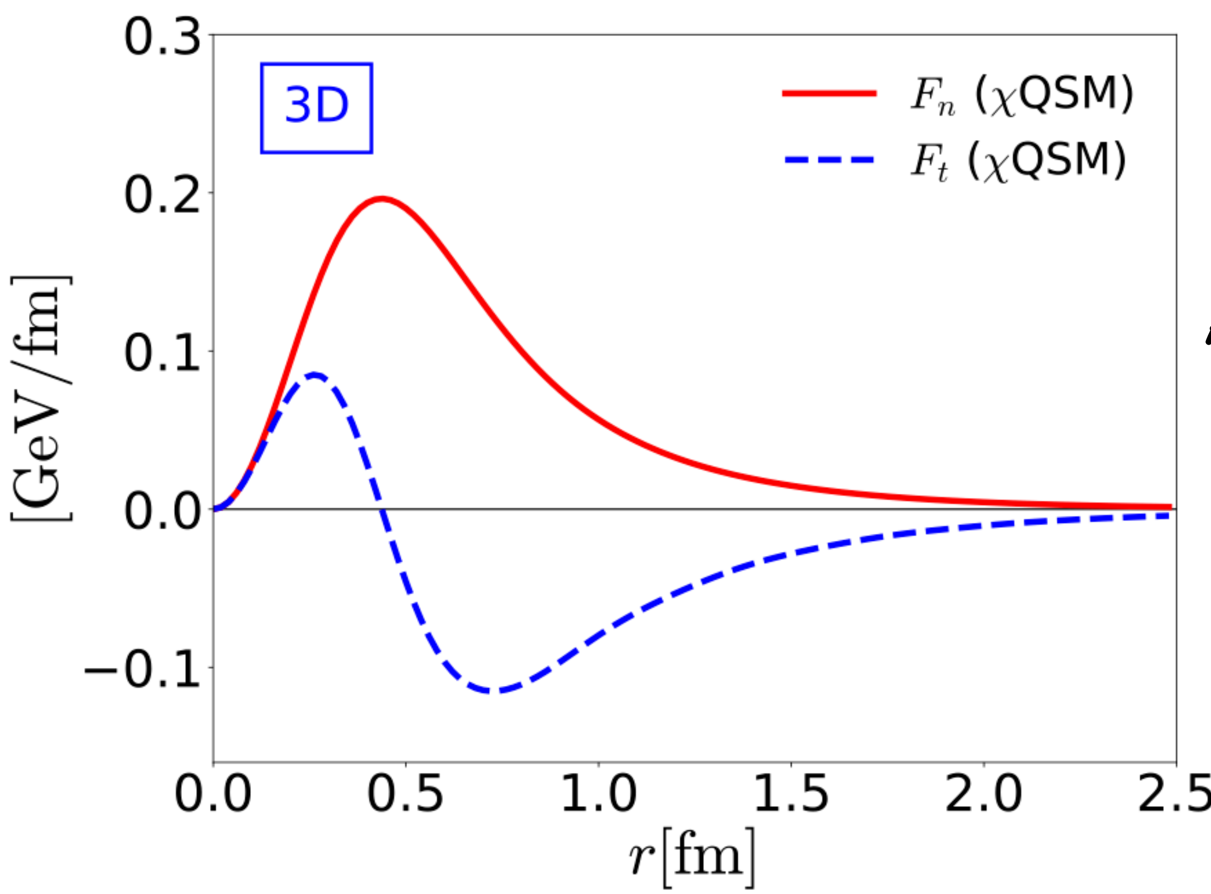




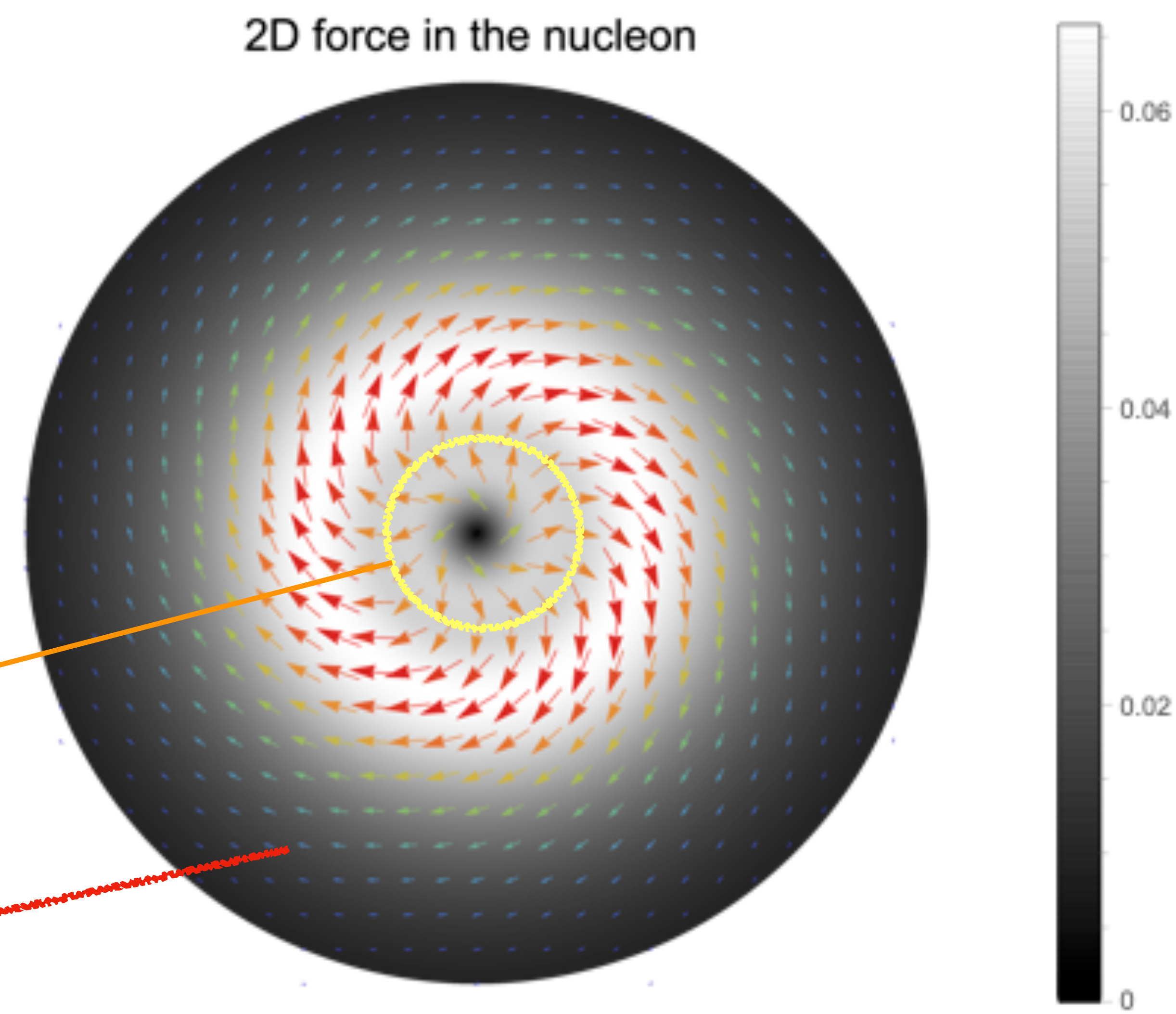
Kim, HChK, H. Son, M. Polyakov PRD 103 (2021)

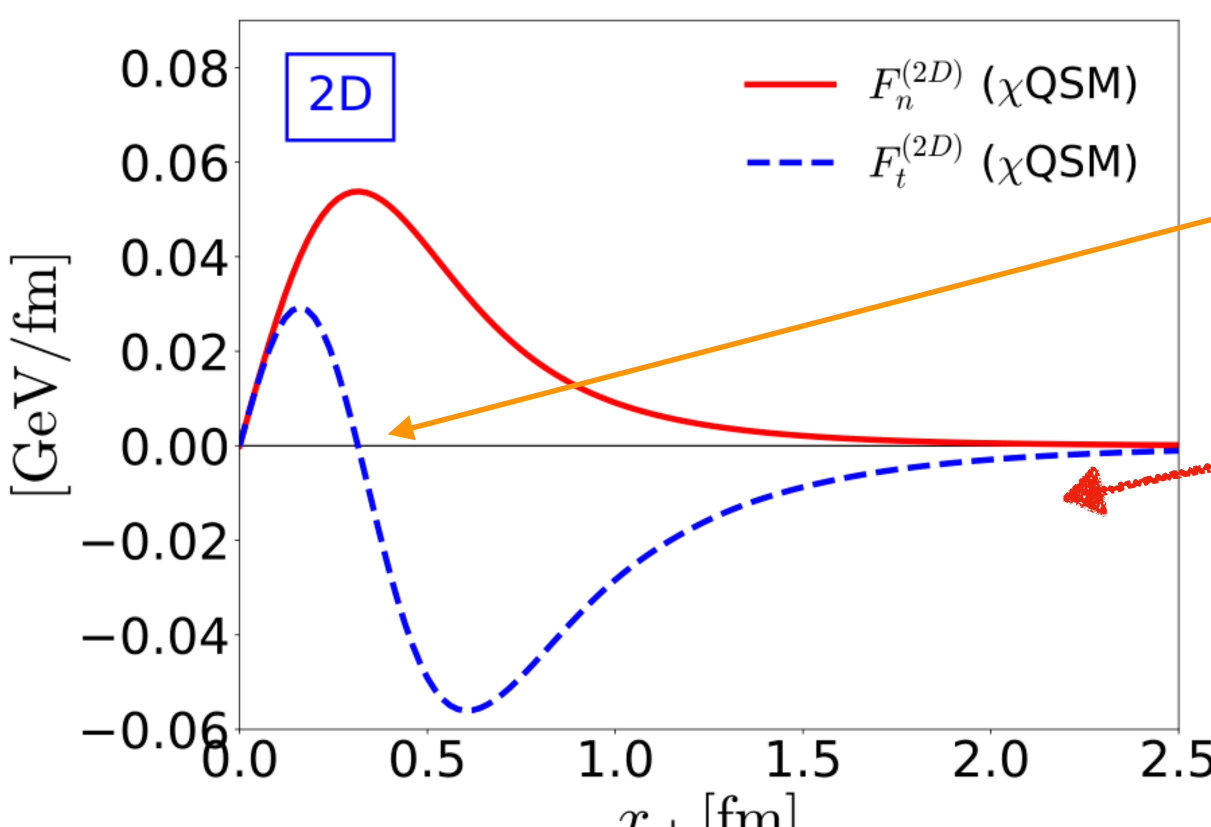
2D force fields & local stability

J. Y. Kim & HChK, ArXiv 2105.10279



Abel transformation



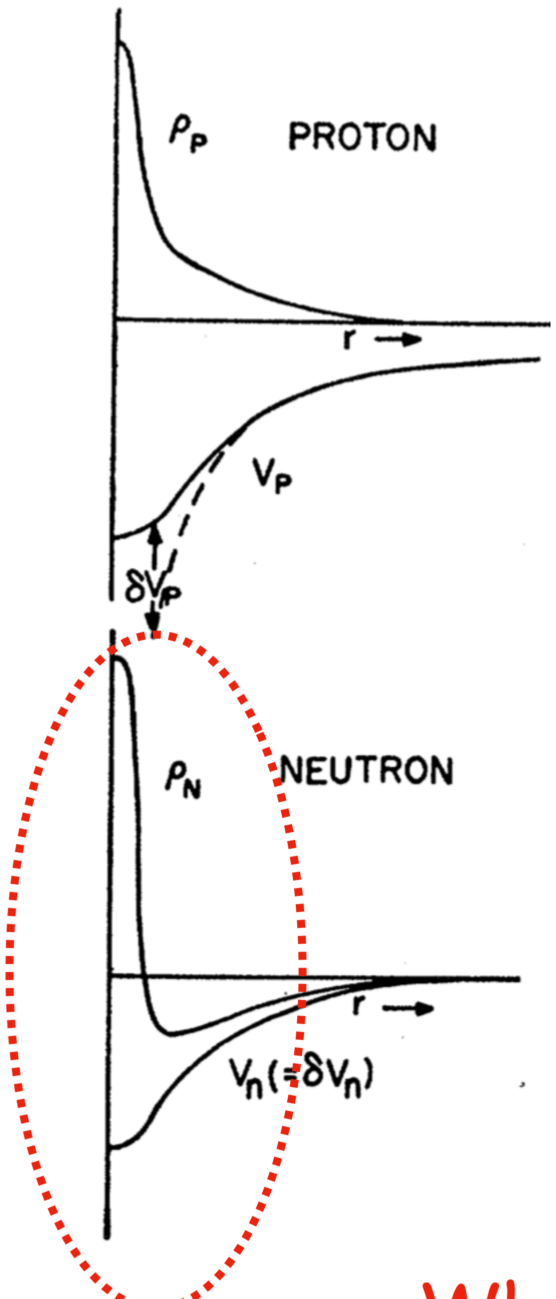


Outer part is governed by
the tangential force. (Stability is acquired).

Transverse charge distribution of the polarized Neutron

Charge distributions of the nucleon

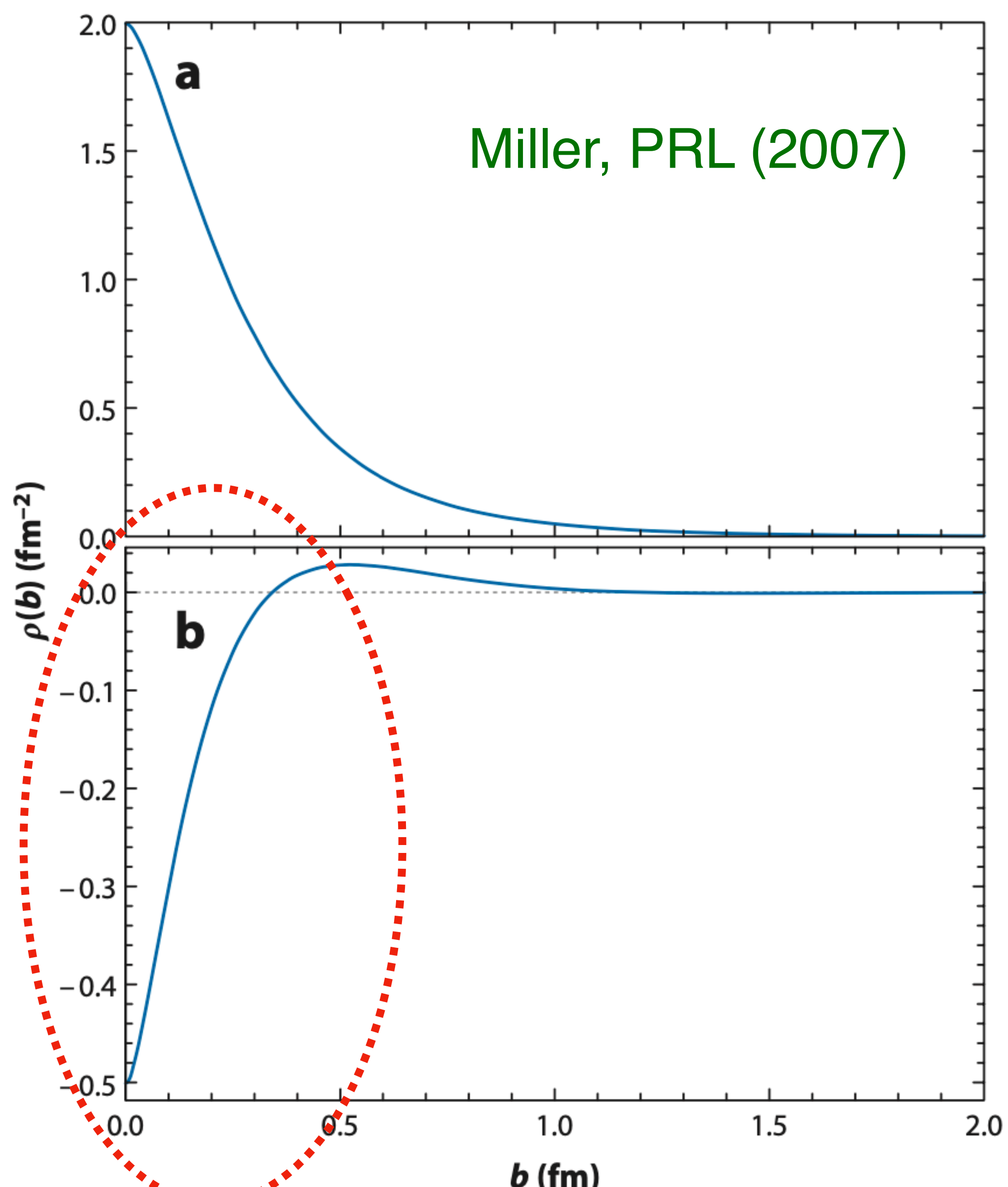
3D charge densities of the nucleon in the BF



Yennie et al., RMP (1957)

Abel transforms that will be discussed soon.

2D Transverse charge densities of the nucleon in the IMF



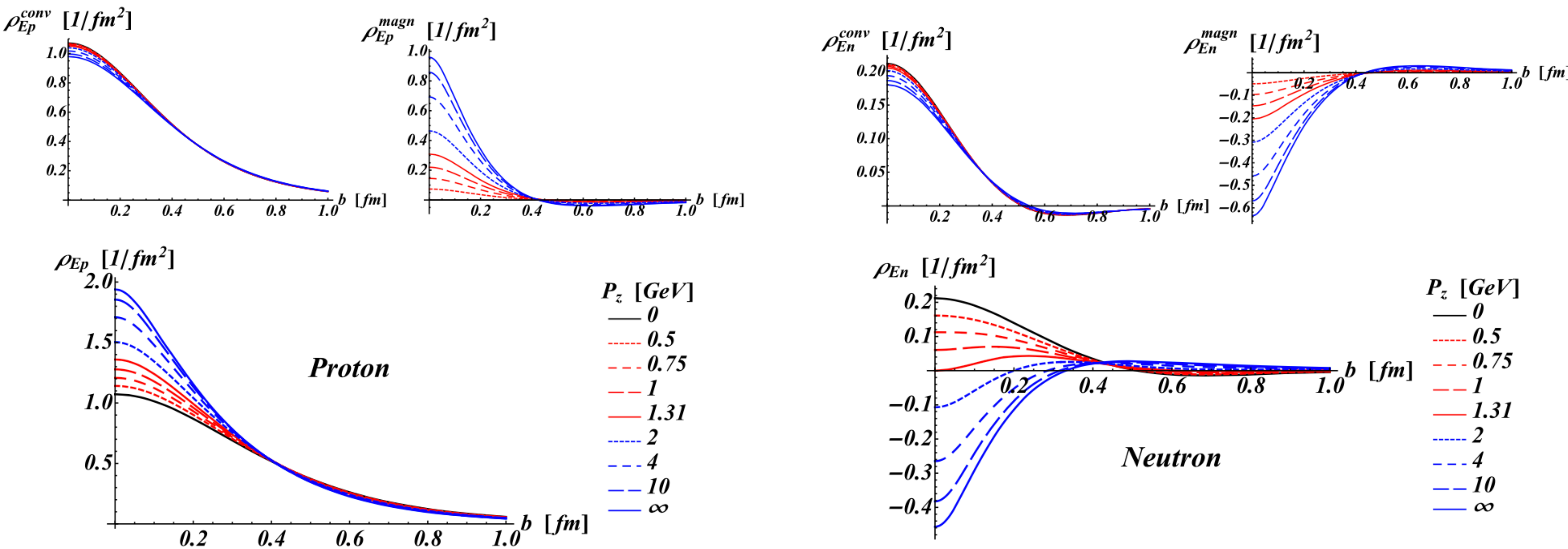
Miller, PRL (2007)

What's wrong with this?

(Actually, nothing wrong.)

2D density exhibits correctly QM probabilistic meaning.

Charge distributions of the nucleon



$$\langle p', s' | \hat{j}^\mu(0) | p, s \rangle = \sum_{s'_B, s_B} D^{*(j)}_{s'_B s'}(p'_B, \Lambda) D^{(j)}_{s_B s}(p_B, \Lambda) \times \Lambda^\mu{}_\nu \langle p'_B, s'_B | \hat{j}^\nu(0) | p_B, s_B \rangle,$$

$$\tilde{\rho}_E = \tilde{\rho}_E^{\rm conv} + \tilde{\rho}_E^{\rm magn}$$

$$\rho_E^X(b; P_z) = \int_0^\infty \frac{dQ}{2\pi} Q J_0(Qb) \tilde{\rho}_E^X(Q; P_z)$$

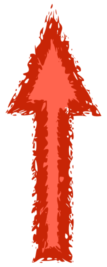
$$\tilde{\rho}_E^{\rm conv}(Q; P_z) = \frac{P^0 + M(1+\tau)}{(P^0 + M)(1+\tau)} G_E(Q^2),$$

$$\tilde{\rho}_E^{\rm magn}(Q; P_z) = \frac{\tau P_z^2}{P^0(P^0 + M)(1+\tau)} G_M(Q^2)$$

C. Lorce, PRL 125 (2020)

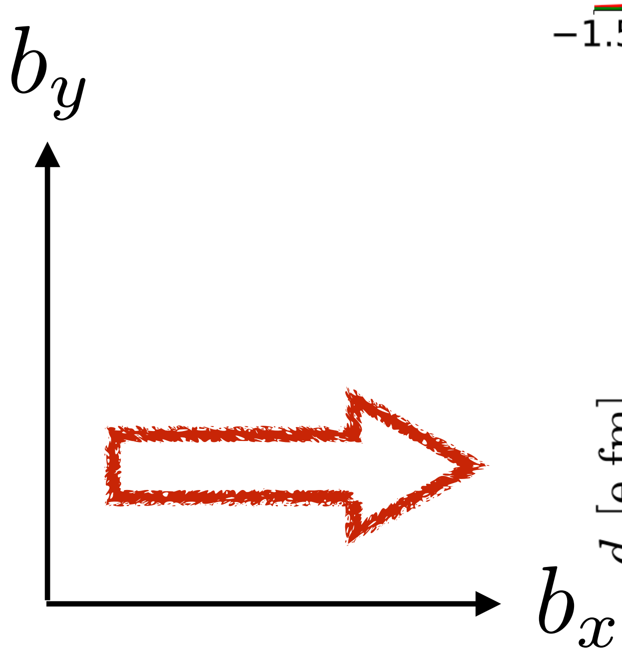
Charge distributions of the tr. polarized nucleon

Carlson & Vanderhaghen, PRL

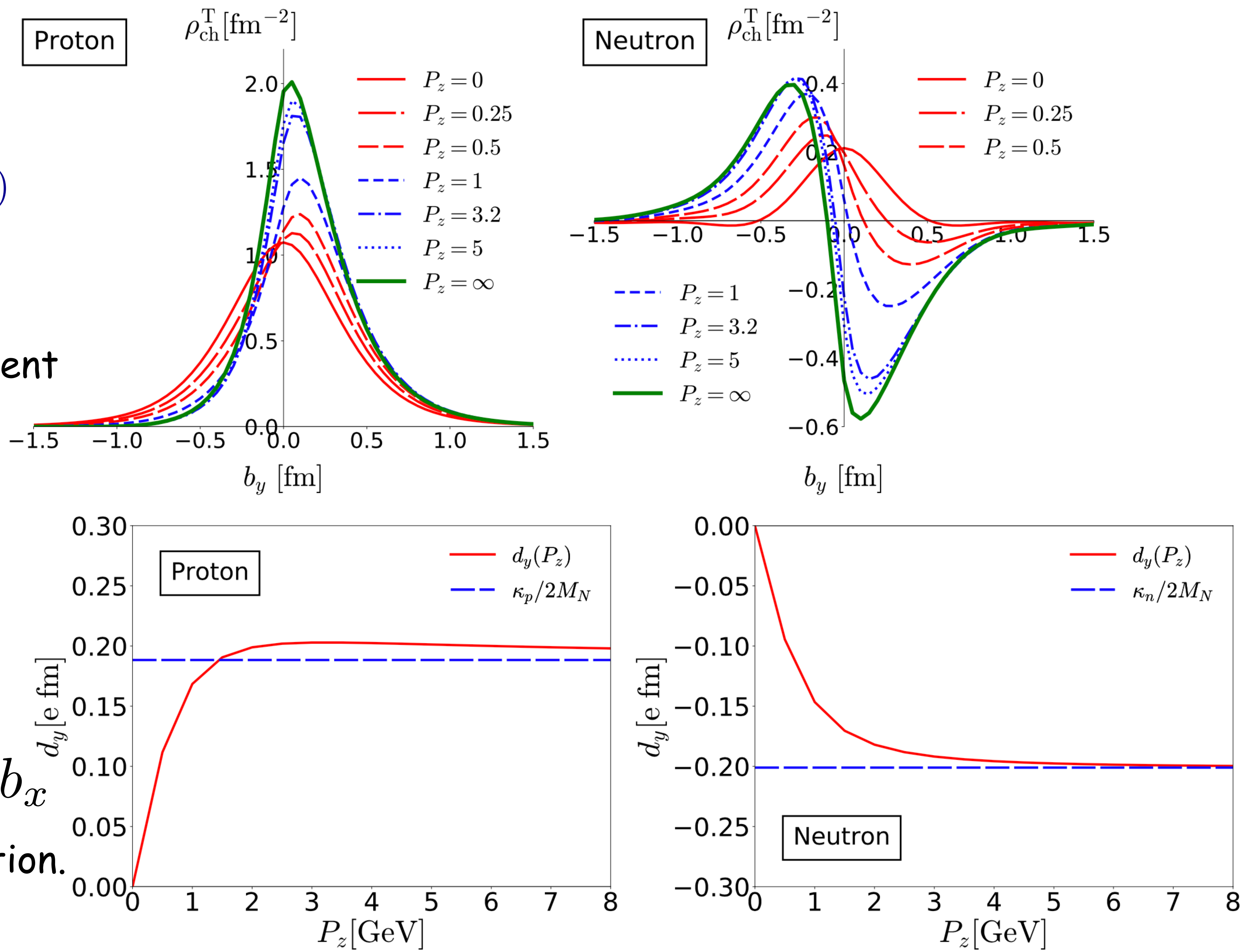


$$\boldsymbol{E}' = \gamma(\boldsymbol{v} \times \boldsymbol{B})$$

Induced electric dipole moment



Polarized in x direction.



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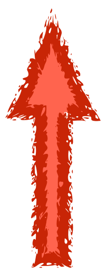
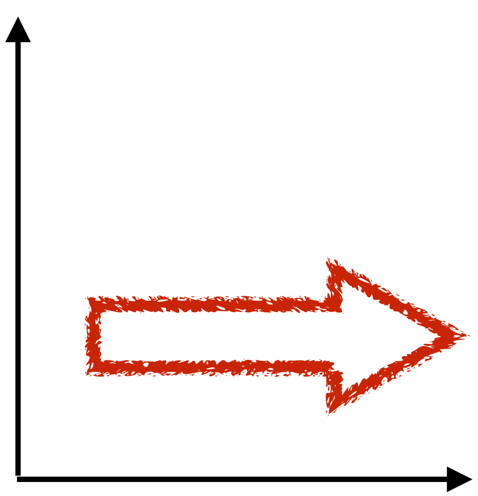
Charge distributions of the tr. polarized proton



$$\boldsymbol{E}' = \gamma(\boldsymbol{v} \times \boldsymbol{B})$$

Induced electric dipole moment

b_y



b_x

Polarized in x direction.

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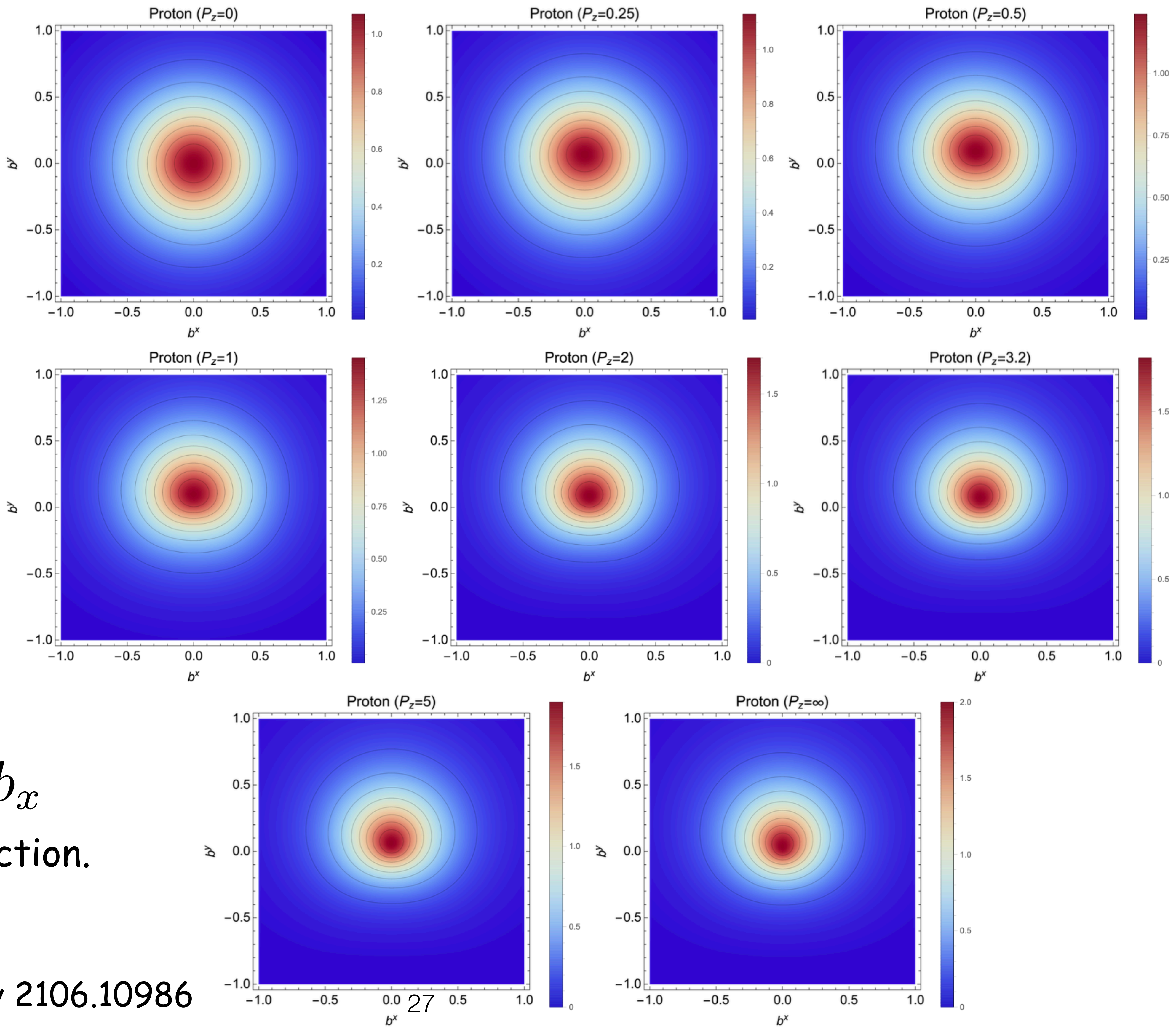
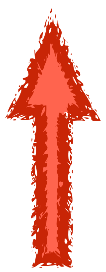
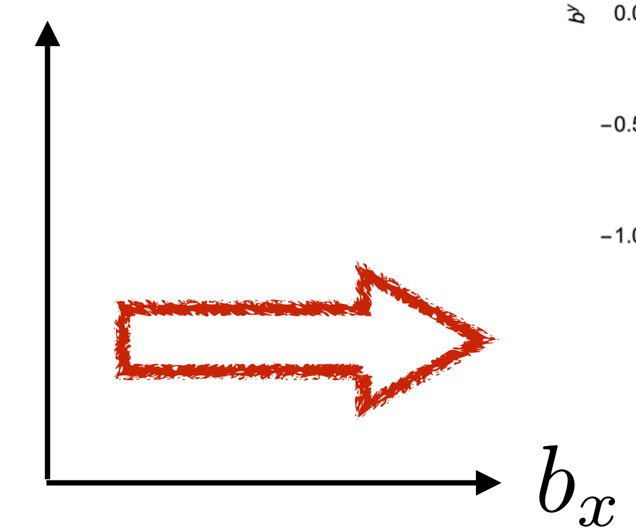


Charge distributions of the tr. polarized neutron

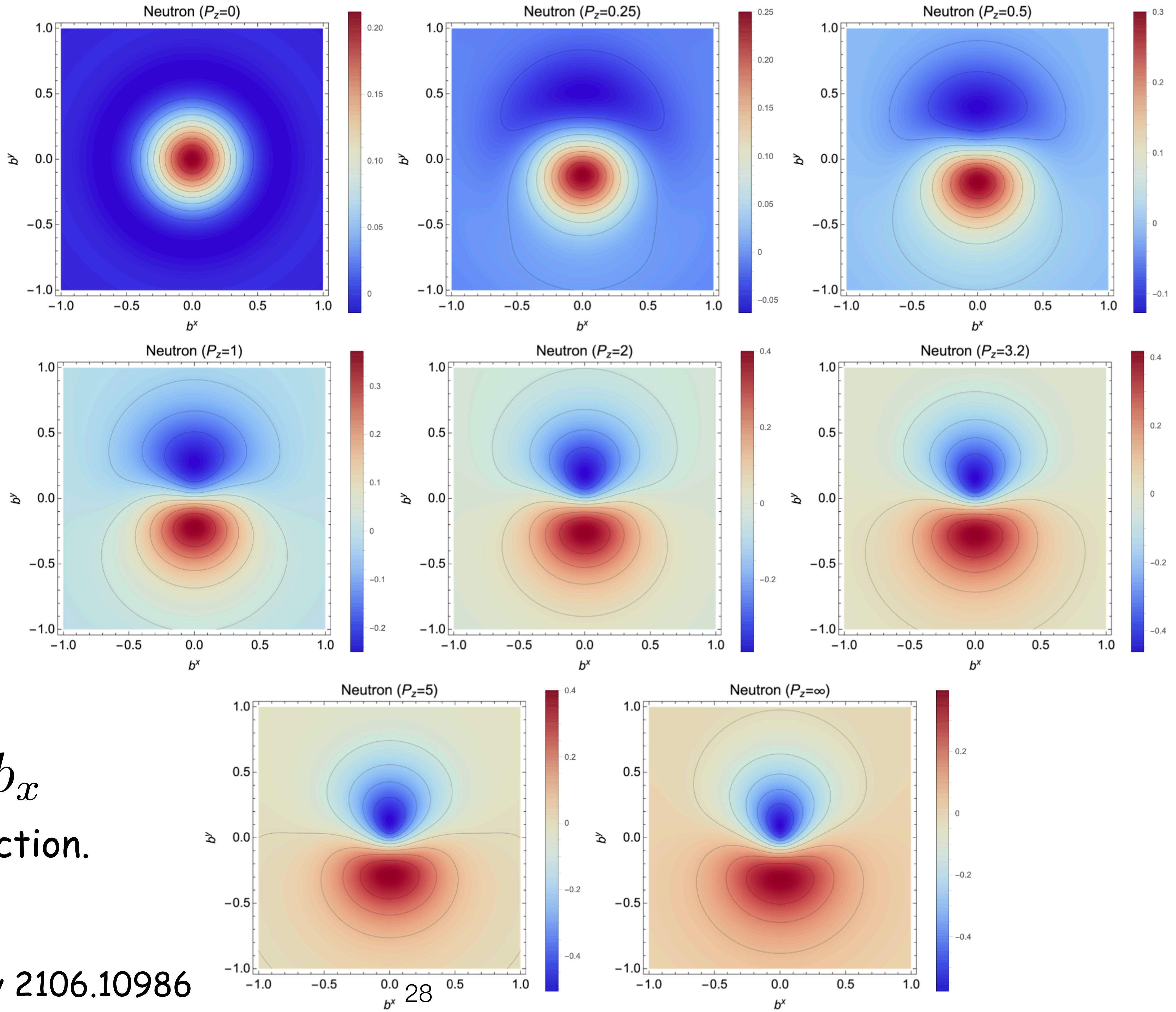


$$\boldsymbol{E}' = \gamma(\boldsymbol{v} \times \boldsymbol{B})$$

Induced electric dipole moment



Polarized in x direction.



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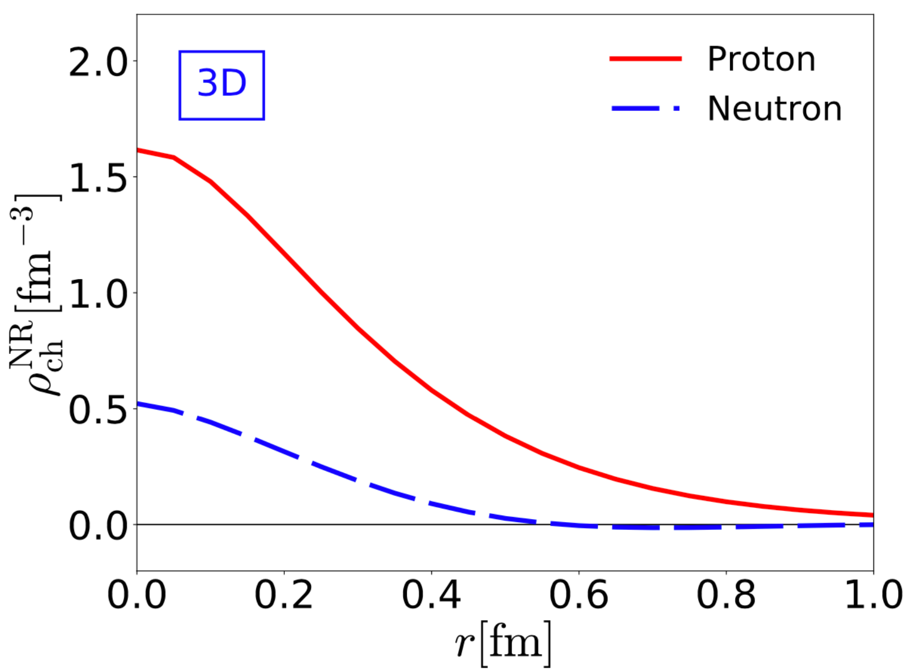
Abel transforms of charge & magnetization distributions

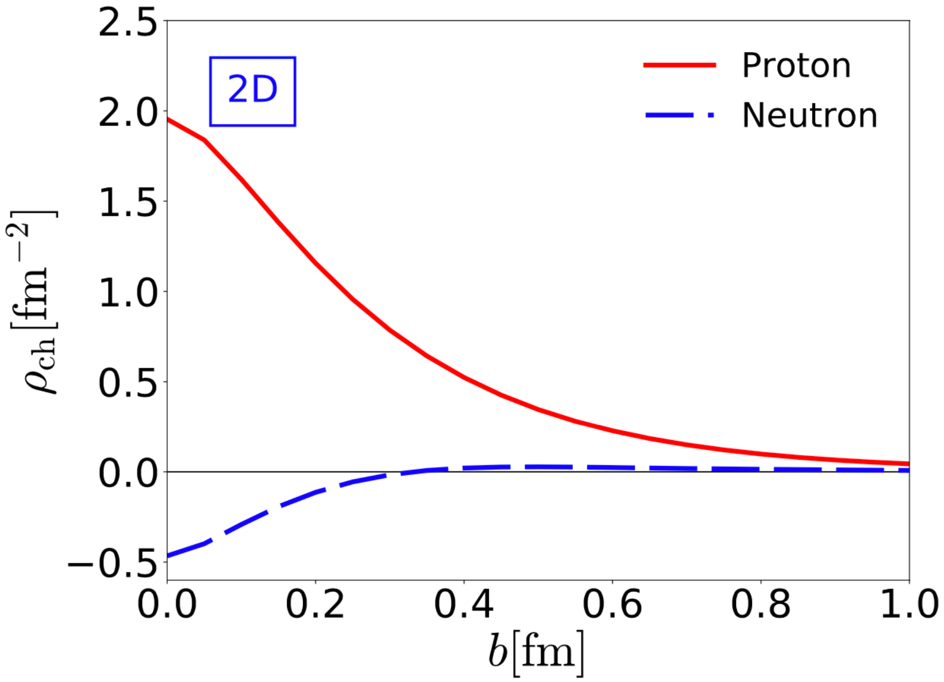
$$\rho_{\rm ch}(b) + \frac{1}{4M_N^2}\partial_\perp^2 \rho_{\rm M}(b) = \int_b^\infty \frac{2r\,dr}{\sqrt{r^2-b^2}}\rho_{\rm ch}^{\rm NR}(r), \quad \rho_{\rm ch}(b) + \rho_M(b) = \int_b^\infty \frac{2r\,dr}{\sqrt{r^2-b^2}}\rho_{\rm M}^{\rm NR}(r)$$

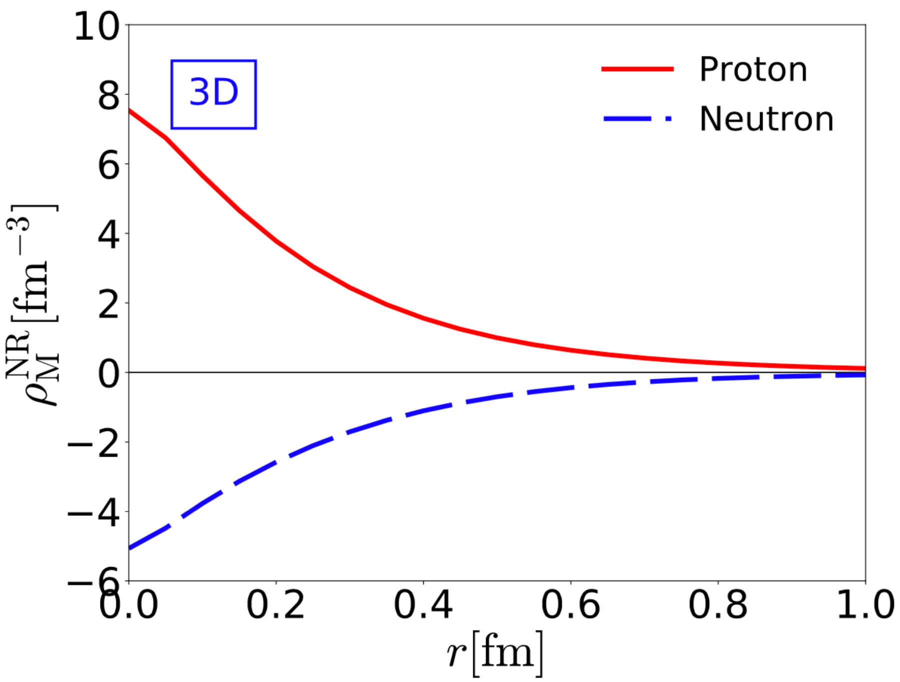


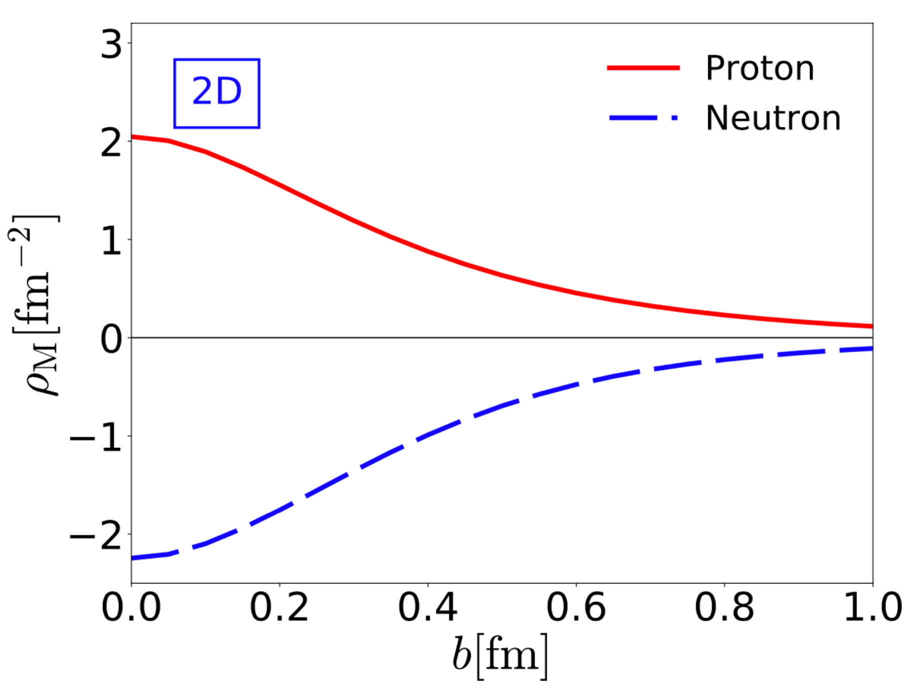
From 3D at rest

to 2D in IMF

Abel transforms







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Summary & Conclusions

2D transverse structure of the Nucleon

- The nucleon is *per se* a relativistic particle.
- The 3D BF distributions have a quasi-probabilistic meaning in a Wigner sense.
- Abel transform makes 3D BF densities equivalent to 2D IMF ones.

  In 2D, we restore quantum mechanically probabilistic meaning of the densities.
- **The 3D global & local stability conditions are all conveyed to the 2D ones!**
- **3D distributions in BF still provide physical intuitions, even though they have only a quasi-probabilistic meaning.**
- Higher-spin baryons are under investigation by using the Radon transform.

Though this be madness, yet there is method in it.

Hamlet Act 2, Scene 2

by Shakespeare

Thank you very much for the attention!