Testing a hexaquark picture for $d^*(2380)$

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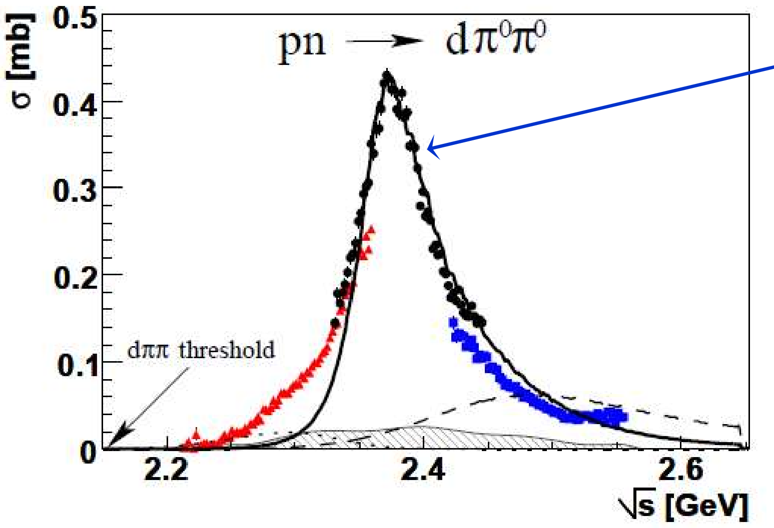
In this talk, we test the hexaquark ($qqqqqq$) wave functions for $d^*(2380)$.

Content

Introduction

- WASA-at-COSY collaboration reported $d^*(2380)$, with $(J, I) = (3,0), M{\sim}2380$ MeV, $\Gamma{\sim}70$ MeV.

PRL 106(2011)242302



Solid line from the s-channel contribution.

- - - - t-channel ΔΔ contribution

· · · · roper excitation process

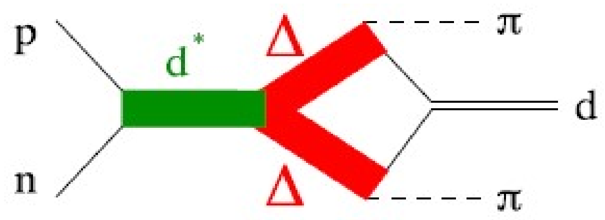
Hatched area: systematic uncertainties

- There are other reactions that also support $d^*(2380)$

$dd \to {}^4\text{He}\,\pi\,\pi$, $pd \to {}^3\text{He}\,\pi\,\pi$, $np \to np\pi^0\pi^0$, $np \to pp\pi^0\pi^+$, $np \to d\pi^0\pi^0, d\pi^+\pi^-$

[NPA825(2009) , PLB637(2006), PLB743(2015), PRC88(2013), PRL(2014)202301]

Production mechanism for $d^*(2380)$



NPA958(2017)

- $d^*(2380)$ is likely a six-quark state composed of u, d quarks only.
- Then a natural question is its structure.

ΔΔ system

$\Delta - \Delta$
$(qqq \in 1_c)\ (qqq \in 1_c)$

- The $d^*(2380)$ mass is only 80 MeV less than the ΔΔ mass.
- Just a composite state of two hadrons !

hexaquark

$qqqqqq \in 1_c$

- Additional hidden-color component.
- New multiquark !
  (q^6 is the 1st multiquark in this kind as q^4, q^5 are not possible.)

Theoretical works

$d^*(2380) \approx \Delta\Delta$

$$\underset{(qqq)}{\Delta} - \underset{(qqq)}{\Delta}$$

- Dyson and Xuong (PRL1964), purely from the SU(6) classification of two-baryon states, calculated the mass in the $(J, I) = (3,0)$ channel, $M = 2350$ MeV.
- The dynamics that supports this might be the attractive force between ΔΔ (PLB1980, Oka and Yazaki).

One problem is the small decay width, $\Gamma \approx 70$ MeV. (Bashkanov, PLB2013)

- The Δ in free space has $\Gamma \approx 115$ MeV.
  If $d^*(2380)$ is a ΔΔ state, why it has the small width like $\Gamma \approx 70$ MeV ?

$d^*(2380) \approx qqqqqq$

- Park et al. (PRD2015) calculated the hexaquark mass in a variational method.
  - ✓ Their calculated mass is too large (2630 or higher).
  - ✓ So they excluded the hexaquark possibility for $d^*(2380)$.
- However, this result seems to depend too much on kinetic energy and spatial separation of the constituent quarks.

- Shi et al. (EPJC2019) constructed the hexaquark w.f. in a diquark model, $d^*(2380) \sim qq - qq - qq$ using $qq \in \bar{3}_c, J = 1$ only.
  - ✓ Gal and Karliner (EPJC2019) refuted that the diquark model is problematic for $qqqqqq$ and the calculated mass there requires too much binding ~360 MeV.
  - ✓ Our view is that the diquark configuration is not better than others (later).

- Is the hexaquark picture not possible (??)

In this work

We further investigate a hexaquark possibility for $d^*(2380)$ after modifying the limitations mentioned above.

Hexaquark wave functions

In our construction, we assume that

- $d^*(2380) \approx qqqqqq$ with $q = (u, d)$.
- All the quarks are in an S-wave.
- No kinetic energy and no spatial dependence for the constituent quarks.
  ※ Similar in spirit as in Dyson and Xuong (PRL1964), Shi. et.al (EPJC2019) or SU(3) quark model.

We impose that

- totally, $qqqqqq \in \mathbf{1}_c,\ J = 3, I = 0$ as we study $d^*(2380)$.
- The $qqqqqq$ system should be antisymmetric totally (Fermion system).
  - ✓ The space part is symmetric (S-wave).
  - ✓ The spin part is also symmetric ($J = 3$).

    ⇒ The rest, the color-isospin combined, must be antisymmetric.

All we need is to construct the color-isospin part of the wave function.

To construct the color-isospin part

- Divide the six-quark into $(qq)(qq)(qq)$. (Not a diquark model !)
  ※ Several ways to divide the six-quark into three diquarks and they all must be equivalent.
- Each qq needs to be antisymmetric in the combined space of color-isospin.

qq in color		qq in isospin
$\mathbf{6}_c$: $S_{ab} = \frac{1}{\sqrt{2}}[q_a q_b + q_b q_a]$ (sym)	⇨	$I_{di} = 0$: $[ud] \equiv \frac{1}{\sqrt{2}}(ud - du)$ (antisym)
$\bar{\mathbf{3}}_c$: $T^a = \frac{1}{\sqrt{2}}\epsilon^{abc}[q_b q_c - q_c q_b]$ (antisym)	⇨	$I_{di} = 1$: $uu, dd, \{ud\} \equiv \frac{1}{\sqrt{2}}(ud + du)$ (sym)

- $(qq)(qq)(qq) = \begin{cases} (6\oplus\bar{3})\otimes(6\oplus\bar{3})\otimes(6\oplus\bar{3}) \Longrightarrow 1_c \text{: 5 configs in color} \\ (I_{di} = 0 \oplus I_{di} = 1)\otimes(I_{di} = 0 \oplus I_{di} = 1)\otimes(I_{di} = 0 \oplus I_{di} = 1) \Longrightarrow I = 0 \text{ : 5 configs in isospin} \end{cases}$

Possible configurations in color and isospin

Color, $|C_i\rangle$

$(\mathbf{6}_c \otimes \mathbf{6}_c \otimes \mathbf{6}_c) \Longrightarrow 1_c$

$|C_1\rangle = \frac{1}{12}\epsilon^{abc}\epsilon^{a'b'c'}\, S_{aa'}\; S_{bb'}\; S_{cc'}$

$(\bar{\mathbf{3}}_c \otimes \bar{\mathbf{3}}_c \otimes \bar{\mathbf{3}}_c) \Longrightarrow 1_c$

$|C_2\rangle = \frac{1}{8\sqrt{6}}\epsilon^{abc}\; T_a\; T_b\; T_c$

$(\mathbf{6}_c \otimes \bar{\mathbf{3}}_c \otimes \bar{\mathbf{3}}_c) \Longrightarrow 1_c$

$|C_3\rangle = \frac{1}{8\sqrt{3}}\; S_{ab}\, T^a\, T^b$

$(\bar{\mathbf{3}}_c \otimes \mathbf{6}_c \otimes \bar{\mathbf{3}}_c) \Longrightarrow 1_c$

$|C_4\rangle = \frac{1}{8\sqrt{3}} T^a\, S_{ab}\;\, T^b$

$(\bar{\mathbf{3}}_c \otimes \bar{\mathbf{3}}_c \otimes \mathbf{6}_c) \Longrightarrow 1_c$

$|C_5\rangle = \frac{1}{8\sqrt{3}} T^a\, T^b\, S_{ab}$

$\langle C_i | C_j\rangle = \delta_{ij}$

Isospin, $|I_i\rangle$

$(I_{di} = 0) \otimes (I_{di} = 0) \otimes (I_{di} = 0) \Longrightarrow I = 0$

$|I_1\rangle = [ud][ud][ud]$

$(I_{di} = 1) \otimes (I_{di} = 1) \otimes (I_{di} = 1) \Longrightarrow I = 0$

$|I_2\rangle = \frac{1}{\sqrt{6}}[(uu\{ud\} - \{ud\}uu)dd - (uudd - dduu)\{ud\} + (\{ud\}dd - dd\{ud\})uu]$

$(I_{di} = 0) \otimes (I_{di} = 1) \otimes (I_{di} = 1) \Longrightarrow I = 0$

$|I_3\rangle = \frac{1}{\sqrt{3}}([ud]uudd - [ud]\{ud\}\{ud\} + [ud]dduu)$

$(I_{di} = 1) \otimes (I_{di} = 0) \otimes (I_{di} = 1) \Longrightarrow I = 0$

$|I_4\rangle = \frac{1}{\sqrt{3}}(uu[ud]dd - \{ud\}[ud]\{ud\} + dd[ud]uu)$

$(I_{di} = 1) \otimes (I_{di} = 1) \otimes (I_{di} = 0) \Longrightarrow I = 0$

$|I_5\rangle = \frac{1}{\sqrt{3}}(uudd[ud] - \{ud\}\{ud\}[ud] + dduu[ud])$

$\langle I_i | I_j\rangle = \delta_{ij}$

Color-isospin configurations

$|\psi_i\rangle = |C_i\rangle \otimes |I_i\rangle$ $(i = 1, \cdots, 5)$

$\langle \psi_i | \psi_j \rangle = \delta_{ij}$

$$|\psi_1\rangle = \frac{1}{12}\epsilon_{abc}\epsilon_{a'b'c'}(S_{[ud]})^{aa'}(S_{[ud]})^{bb'}(S_{[ud]})^{cc'}$$

$$|\psi_2\rangle = \frac{1}{48}\epsilon_{abc}\Big[(T_{uu})^a(T_{\{ud\}})^b(T_{dd})^c - (T_{\{ud\}})^a(T_{uu})^b(T_{dd})^c - (T_{uu})^a(T_{dd})^b(T_{\{ud\}})^c + (T_{dd})^a(T_{uu})^b(T_{\{ud\}})^c + (T_{\{ud\}})^a(T_{dd})^b(T_{uu})^c - (T_{dd})^a(T_{\{ud\}})^b(T_{uu})^c\Big]$$

diquark picture

$$|\psi_3\rangle = \frac{1}{24}\Big[(S_{[ud]})^{ab}(T_{uu})_a(T_{dd})_b - (S_{[ud]})^{ab}(T_{\{ud\}})_a(T_{\{ud\}})_b + (S_{[ud]})^{ab}(T_{dd})_a(T_{uu})_b\Big]$$

$$|\psi_4\rangle = \frac{1}{24}\Big[(T_{uu})_a(S_{[ud]})^{ab}(T_{dd})_b - (T_{\{ud\}})_a(S_{[ud]})^{ab}(T_{\{ud\}})_b + (T_{dd})_a(S_{[ud]})^{ab}(T_{uu})_b\Big]$$

$$|\psi_5\rangle = \frac{1}{24}\Big[(T_{uu})_a(T_{dd})_b(S_{[ud]})^{ab} - (T_{\{ud\}})_a(T_{\{ud\}})_b(S_{[ud]})^{ab} + (T_{dd})_a(T_{uu})_b(S_{[ud]})^{ab}\Big]$$

others

- The diquark tensors, $(S_{[ud]})_{ab}$, $(T_{uu})^a$, $(T_{dd})^a$, $(T_{\{ud\}})^a$ are the building blocks.

  ※ For example, $(S_{[ud]})_{ab}$ denotes the diquark symmetric in color and antisymmtric in u, d flavor.

$[(q_1 q_2)\ (q_3 q_4)\ (q_5 q_6)]$

- By construction, $|\psi_i\rangle$ is antisymmetric only under $(1 \leftrightarrow 2,\ 3 \leftrightarrow 4, 5 \leftrightarrow 6)$.
- $|\psi_i\rangle$ is not fully antisymmetric.

Example

antisym under $u \leftrightarrow d$, a $\leftrightarrow b$

$$|\psi_5\rangle \propto (T_{uu})_a (T_{dd})_b (S_{[ud]})^{ab} - (T_{\{ud\}})_a (T_{\{ud\}})_b (S_{[ud]})^{ab} + (T_{dd})_a (T_{uu})_b (S_{[ud]})^{ab}$$

No sym under exchange of the two quarks.
But the quarks are combination of symmetric and antisymmetric parts.

- But $|\psi_i\rangle$ must form all the components of the fully antisymmetric wave function.

- The fully antisymmetric wave function can be obtained by linearly combining the five $|\psi_i\rangle$ as

$$|\Psi\rangle = \sum_{i=1}^{5} a_i |\psi_i\rangle \qquad \text{with } \langle\Psi|\Psi\rangle = 1.$$

To find a_i

- $|\Psi\rangle = \sum_{i=1}^{5} a_i|\psi_i\rangle$: $(q_1q_2)(q_3q_4)(q_5q_6) \Longrightarrow$ (12),(34),(56) are antisym.

$$(q_1\ q_2)\ (q_3\ q_4)\ (q_5\ q_6)$$

(13), (24) : combination of sym and antisym parts

- Rewrite $|\Psi\rangle$ in (13),(24),(56) pairs.
- Adjust a_i such a way that the symmetric pairs go away.☞
- Repeat this for the other pairs like, (14)(23)(56), (15)(34)(26), etc.

This leads to $a_1 = a_2 = -a_3 = -a_4 = -a_5 = \frac{1}{\sqrt{5}}$

Then the fully antisymmetric color-isospin part is

$$|\Psi\rangle = \sum_{i=1}^{5} a_i |\psi_i\rangle = \frac{1}{\sqrt{5}}[|\psi_1\rangle + |\psi_2\rangle - |\psi_3\rangle - |\psi_4\rangle - |\psi_5\rangle]$$

- This shows that all the five configurations are equally important.
- Obviously, the diquark model, which relies on either $|\psi_1\rangle$ or $|\psi_2\rangle$, does not provide a fully antisymmetric wave function.

- Q: $|\Psi\rangle$, in this construction, has nothing to do with the potential of the system.
  Is $|\Psi\rangle$ an energy eigenstate of the system ?
  $V_{eff}|\Psi\rangle = \lambda|\Psi\rangle$ (??) (We will check this point later !)

Effective potential and the mass formula

- To test the fully antisymmetric wave function, $|\Psi\rangle$, we take

Effective potential composed of three terms,

$$V_{eff} = \underbrace{v_0 \sum_{i<j} \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i m_j}}_{\equiv V_{CS}} + \underbrace{v_1 \sum_{i<j} \frac{\lambda_i \cdot \lambda_j}{m_i m_j}}_{\equiv V_{CE}} + v_2$$

λ_i: Gell-Mann matrix for color
J_i: spin
m_i: constituent quark mass

※ V_{eff} is a contact type that acts on two quarks at the same spatial point.

- And calculate the hexaquark mass using the mass formula

$$M_H = \sum_i m_i + \langle V_{eff} \rangle$$

The baryon masses can be generated well.

$$V_{eff} = v_0 \sum_{i<j} \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i m_j} + v_1 \sum_{i<j} \frac{\lambda_i \cdot \lambda_j}{m_i m_j} + v_2$$

$$M_H = \sum_i m_i + \langle V_{eff} \rangle$$

Baryon	$M_{\rm expt}$	$M_{\rm theory}$			Each term in Case III			
		Case I	Case II	Case III	$\sum m_q$	V_{CS}	V_{CE}	v_2
N	940	844	940 (input)	940 (input)	990	−146.0	−26.5	
Δ	1232	1136	1232 (input)	1232 (input)	990	146.0	−26.5	
Λ	1116	1014	1088	1116 (input)	1160	−146.0	−20.5	
Σ	1193	1080	1154	1182	1160	−79.8	−20.5	122.5
Σ^*	1385	1273	1279	1375	1160	112.9	−20.5	
Ξ	1320	1223	1347	1330	1330	−107.3	−15.5	
Ξ^*	1531	1415	1472	1522	1330	85.4	−15.5	
Ω	1672	1564	1605	1675	1500	63.6	−11.5	

Case I: v_0 only; Case II: v_0, v_1; Case III: v_0, v_1, v_2

For Case III

$v_0 = (-199.6\ \text{MeV})^3 \quad v_1 = (71.2\ \text{MeV})^3 \quad v_2 = 122.5\ \text{MeV}$

$$m_N = 3m_u + 2\frac{v_0}{m_u^2} - 8\frac{v_1}{m_u^2} + v_2\ ,$$

$$m_\Delta = 3m_u - 2\frac{v_0}{m_u^2} - 8\frac{v_1}{m_u^2} + v_2\ ,$$

$$m_\Lambda = 2m_u + m_s + 2\frac{v_0}{m_u^2} - \frac{8}{3}v_1\left[\frac{1}{m_u^2} + \frac{2}{m_u m_s}\right] + v_2\ ,$$

$$m_\Sigma = 2m_u + m_s - \frac{8}{3}v_0\left[\frac{1}{4m_u^2} - \frac{1}{m_u m_s}\right] - \frac{8}{3}v_1\left[\frac{1}{m_u^2} + \frac{2}{m_u m_s}\right] + v_2\ ,$$

$$m_{\Sigma^*} = 2m_u + m_s - \frac{8}{3}v_0\left[\frac{1}{4m_u^2} + \frac{1}{2m_u m_s}\right] - \frac{8}{3}v_1\left[\frac{1}{m_u^2} + \frac{2}{m_u m_s}\right] + v_2\ ,$$

$$m_\Xi = m_u + 2m_s - \frac{8}{3}v_0\left[-\frac{1}{m_u m_s} + \frac{1}{4m_s^2}\right] - \frac{8}{3}v_1\left[\frac{2}{m_u m_s} + \frac{1}{m_s^2}\right] + v_2\ ,$$

$$m_{\Xi^*} = m_u + 2m_s - \frac{8}{3}v_0\left[\frac{1}{2m_u m_s} + \frac{1}{4m_s^2}\right] - \frac{8}{3}v_1\left[\frac{2}{m_u m_s} + \frac{1}{m_s^2}\right] + v_2$$

$$m_\Omega = 3m_s - 2\frac{v_0}{m_s^2} - 8\frac{v_1}{m_s^2} + v_2\ .$$

- We applied V_{eff} and the mass formula to the tetraquark system also.

Isospin	Heavy nonet	Light nonet	ΔM_{exp}	$\Delta\langle V_{eff}\rangle$, (SSC)
$I = 1$	$a_0(1450)$	$a_0(980)$	494	472
$I = 1/2$	$K_0^*(1430)$	$K_0^*(800)$	601	566
$I = 0\ (\sim \mathbf{8}_f)$	$f_0(1370)$	$f_0(500)$	875	612
$I = 0\ (\sim \mathbf{1}_f)$	$f_0(1500)$	$f_0(980)$	515	542

(H.Kim et al. EPJC 2017, PRD 2018 etc).

Further simplification of V_{eff} when it applies to this hexaquark.

$$V_{eff} = v_0 \sum_{i<j} \lambda_i \cdot \lambda_j \frac{J_i \cdot J_j}{m_i m_j} + v_1 \sum_{i<j} \frac{\lambda_i \cdot \lambda_j}{m_i m_j} + v_2$$

- This hexaquark is composed of u, d quarks only $\Longrightarrow$ $m_i = m_j \equiv m$.
- Any quark pair must have spin-1 $\Longrightarrow$ $\langle J_i \cdot J_j \rangle = \frac{1}{4}$.

- The effective potential is simplified as

$$V_{eff} = \left[\frac{v_0}{4m^2} + \frac{v_1}{m^2}\right] \sum_{i<j} \lambda_i \cdot \lambda_j + v_2$$

- For $\langle V_{eff} \rangle$, we need to evaluate just $\langle \Psi | \sum_{i<j} \lambda_i \cdot \lambda_j | \Psi \rangle$

The hexaquark mass

To test $|\Psi\rangle$, we evaluate $\langle\Psi|V_{eff}|\Psi\rangle$
and determine the mass through $M_H = \sum_i m_i + \langle\Psi|V_{eff}|\Psi\rangle$.

$\langle C_i | V_{eff} | C_i \rangle$ is the same regardless of $|C_i\rangle$

$|\Psi\rangle = \sum_{i=1}^{5} a_i |\psi_i\rangle$

$a_i = \pm\frac{1}{\sqrt{5}}$

$$\langle\Psi|V_{eff}|\Psi\rangle \doteq \sum_{i,j} a_i a_j \langle\psi_i|V_{eff}|\psi_j\rangle \doteq \frac{1}{5}\sum_i \langle C_i|V_{eff}|C_i\rangle \doteq \langle C_i|V_{eff}|C_i\rangle \text{ !}$$

⇩

$\delta_{ij}\langle C_i|V_{eff}|C_i\rangle$ because $|\psi_i\rangle = |C_i\rangle \otimes |I_i\rangle$

$\langle C_k | \sum_{i<j} \lambda_i \cdot \lambda_j | C_k \rangle = -16$

$$= -16\left[\frac{v_0}{4m^2} + \frac{v_1}{m^2}\right] + v_2$$

$$= 361.5 \text{ MeV}$$

$$V_{eff} = \left[\frac{v_0}{4m^2} + \frac{v_1}{m^2}\right]\sum_{i<j} \lambda_i \cdot \lambda_j + v_2$$

$|\Psi\rangle = \sum_i a_i |\psi_i\rangle$ with $a_i = \pm\frac{1}{\sqrt{5}}$

Inputs are fixed from baryon spectrum

$v_0 = (-199.6 \text{ MeV})^3$ $\quad v_1 = (71.2 \text{ MeV})^3$ $\quad v_2 = 122.5$ MeV

with the quark mass, $m = 330$ MeV

m (MeV)	$\langle V_{eff} \rangle$ (MeV)	M_H(MeV)
300	568	2368
330	362	2342
350	198	2300

Sensitivity on m
(Note, v_0, v_1, v_2 depend on m too)

The hexaquark mass is

$$M_H = \sum_i m_i + \langle V_{eff} \rangle = 2341.5 \text{ MeV}$$

- This value is somewhat close to the $d^*(2380)$ mass !
  (Also similar to that of Dyson and Xuoung.)
- So the hexaquark scenario may not be ruled out for $d^*(2380)$.

Comments on the hexaquark W.F.

1. We found, $\langle\psi_i|V_{eff}|\psi_i\rangle = \langle C_i|V_{eff}|C_i\rangle \equiv A_i$ are the same for all $|\psi_i\rangle$

   $A_1 = A_2 = A_3 = A_4 = A_5 \equiv A = \langle\Psi|V_{eff}|\Psi\rangle$

   - This is a natural consequence coming from the freedom in dividing the six-quark into three diquarks ! (see the digression)

2. This implies that $|\Psi\rangle$ forms an eigenstate of V_{eff}. (So our result is trustworthy.)

   - Since there is no mixing, $\langle\psi_i|V_{eff}|\psi_j\rangle = 0$ $(i \neq j)$, $V_{eff}|\psi_i\rangle = A|\psi_i\rangle$ for all $|\psi_i\rangle$.

   $$V_{eff}|\Psi\rangle = V_{eff}\left(\sum_i a_i|\psi_i\rangle\right) = A|\Psi\rangle$$

3. The diquark model, which is commonly adopted for multiquark studies, does not have privilege over other configurations in the hexaquark with $J = 3$!

   - $|\psi_1\rangle$, $|\psi_2\rangle$ are w.f. based on diquark model.
     [$|\psi_1\rangle$ is composed of the $\mathbf{6}_c$ diquark, $|\psi_2\rangle$ is of the $\bar{\mathbf{3}}_c$ diquark.]
   - They form the same energy state as others, $A_1 = \cdots = A_5$ in addition to that $|\psi_1\rangle$, $|\psi_2\rangle$ are not fully antisymmetric.

digression

- Consider $|C_1\rangle$ in the (12)(34)(56) division

$$|C_1\rangle = \frac{1}{12}\epsilon^{abc}\epsilon^{a'b'c'}(S_{12})_{aa'}(S_{34})_{bb'}(S_{56})_{cc'}$$

- The two diquarks, (12),(34) are in $\mathbf{6}$ in this division but (13) or (24) pair is either in $\bar{\mathbf{3}}, \mathbf{6}$.
- We may rewrite the $|C_1\rangle$ above in a new division like (13)(24)(56) by moving q_2, q_3.

$|C_1\rangle$ type in (13)(24)(56) division, $|C_1'\rangle$

$|C_5'\rangle$

$$|C_1\rangle = \frac{1}{24}\epsilon^{abc}\epsilon^{a'b'c'}(S_{13})_{aa'}(S_{24})_{bb'}(S_{56})_{cc'} - \frac{1}{16}(T_{13})^a(T_{24})^b(S_{56})_{ab}$$

$$\Longrightarrow |C_1\rangle = \frac{1}{2}|C_1'\rangle - \frac{\sqrt{3}}{2}|C_5'\rangle$$

$\langle V_{eff}\rangle$ w.r.t this state, $\langle C_1|V_{eff}|C_1\rangle = \frac{1}{4}\langle C_1'|V_{eff}|C_1'\rangle + \frac{3}{4}\langle C_5'|V_{eff}|C_5'\rangle$

$$\Longrightarrow A_1 = \frac{1}{4}A_1' + \frac{3}{4}A_5'$$

- Due to the freedom mentioned above, $A_1 = A_1',\ A_5 = A_5'$.

$$\Longrightarrow A_1 = A_5$$

- Other equalities can be proven similarly.

Summary

- We have constructed the hexaquark w.f. for $d^*(2380)$ by dividing the six quarks into three diquarks of either type, $(\bar{3}_c, I = 1), (6_c, I = 0)$.
- There are five configurations and the fully antisymmetric wave function was constructed by linearly combining the five.
- We test this wave function as well as the five configurations by calculating the mass using the effective potential, $V_{eff} = V_{CS} + V_{CE} + const.$
- The mass is found to be the same regardless of the configurations being used including the fully antisymmetric one.
  ⇒ This is due to the freedom in choosing three diquarks in the construction of its wave function
- The diquark model, which is commonly used for the multiquark studies, may not be adequate for the hexaquark system with $J = 3$.
- The hexaquark mass is found to be around 2342 MeV, and this is indeed close to the experimental mass of $d^*(2380)$.
- Therefore, the hexaquark picture is still promising for $d^*(2380)$ as far as the mass is concerned.

Thank you for your attention !