Quarkonia and heavy-light mesons in a Bethe-Salpeter approach

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Light-Cone Distribution Amplitudes

QCD factorization in B decays involves matrix elements which are convolution integrals:

$$\langle \pi^+\pi^- | (\bar{u}b)_{\rm V-A} (\bar{d}u)_{\rm V-A} | \bar{B}_d \rangle \to \int_0^1 d\xi du dv \, \Phi_B(\xi) \, \Phi_\pi(u) \, \Phi_\pi(v) \, T(\xi, u, v; m_b)$$

The integrals are over a (hard) scattering kernel $T\,(\xi,u,v,m)$ and light-cone distribution amplitudes (LCDA) expanded in Gegenbauer polynomials:

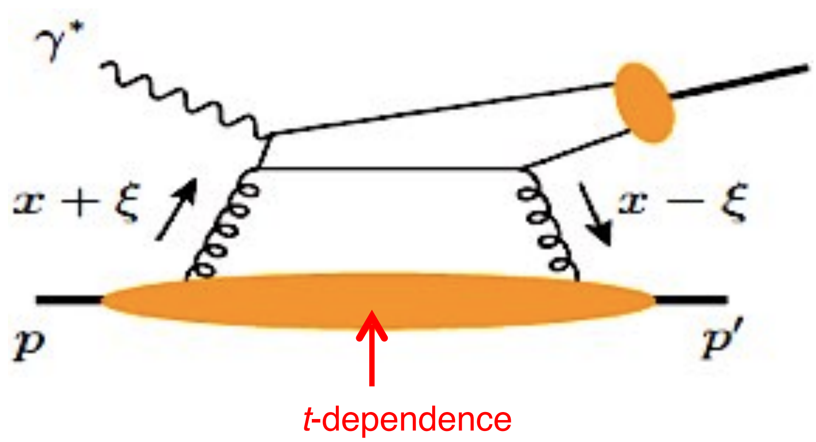
$$\phi_M(x) \neq \phi^{\rm asy}(x) = 6x(1-x)$$

- LCDA were poorly known for light mesons, in recent years improved determinations of the first two Gegenbauer moments of the pion and kaon, RQCD Collaboration, Bali et al. (2019).
- Next to nothing was known about heavy-light mesons, mostly models and asymptotic LCDA used.

Spatial imaging of glue in a nucleon/nucleus

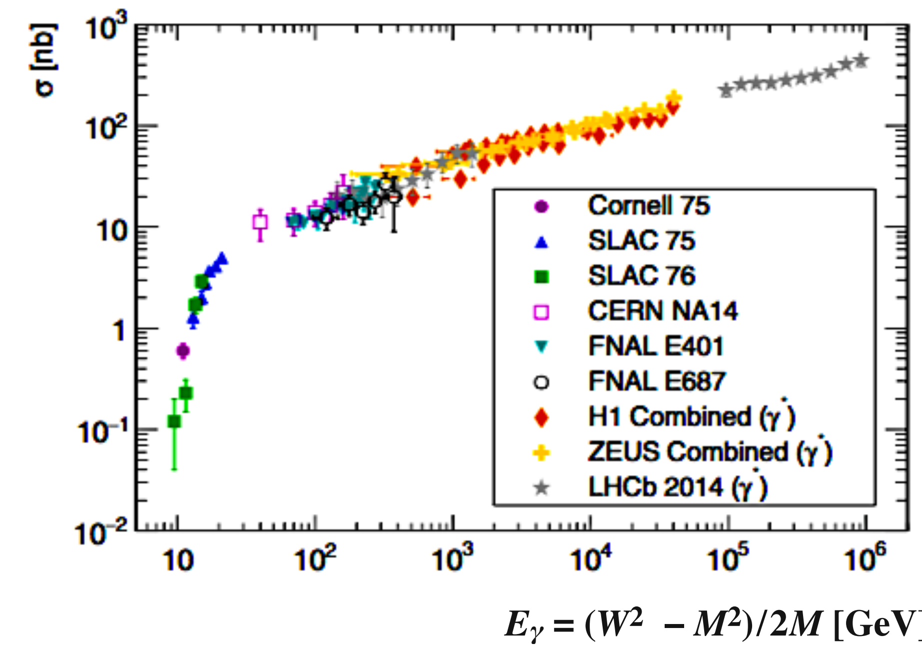
Exclusive electroproduction: hard-scattering mechanism $E_\gamma > 10$ GeV

J. C. Collins, L. Frankfurt, M. Strikman, Phys.Rev.D 56 (1997)
S.J. Brodsky, E. Chudakov, P. Hoyer, J.M. Laget, Phys. Lett. B (2001).
D.Y. Ivanov, A. Schäfer, L. Szymanowski and G. Krasnikov, Eur. Phys. J. C 34 (2004)



$J/\Psi, \Phi, \ldots$

$$Q^2 = -\left(p_e^i - p_e^f\right)^2$$
$$W = \sqrt{2\nu \cdot M_p + M_p^2 - Q^2}$$
$$t = \left(p_p^i - p_p^f\right)^2 = (p_\gamma - p_{J/\psi})^2$$
$$\xi = (P' - P) \cdot n/2$$



- Exclusive *charmonium production*: narrow quarkonium exchanges gluons with the nucleon's light quarks.
- Cross section is proportional to the square of the gluon density in the hadron.
- At large virtualities $Q^2 \gg m_Q{}^2$, electroproduction of heavy mesons is expressed in terms of *meson distribution amplitudes* in the factorization theorem (Collins, Frankfurt & Strikman 1997).

See also talks by Andreas Schäfer and Bowen Xiao

Charged current production of light and charmed mesons

- Initial work by Marat Siddikov and Iván Schmidt for pion and rho production, Phys. Rev. D 99 (2019).
- Claim that their numerical estimates show that production could be studied in neutrino- or electron-induced processes at EIC.
- Recently extended do D_s and D_s^* mesons in collinear QCD where generalized gluon distributions factorize from perturbatively calculable coefficient functions.
- *Required input*: pseudoscalar and vector meson *distribution amplitudes*.

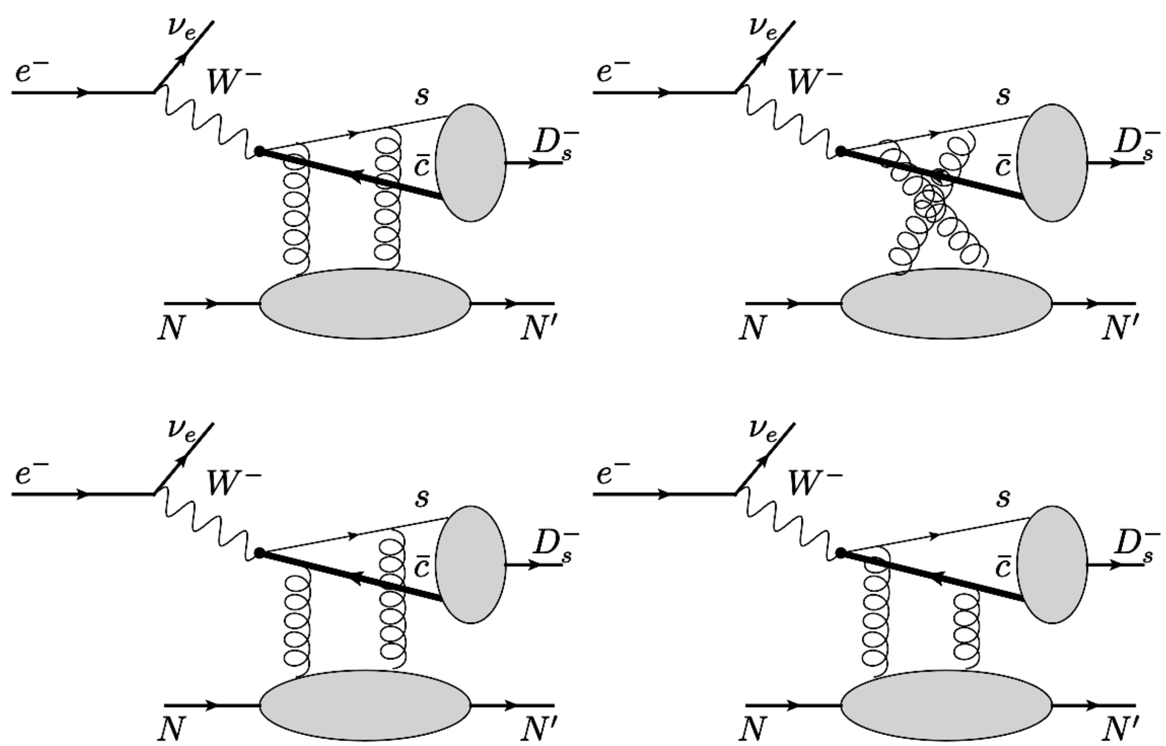
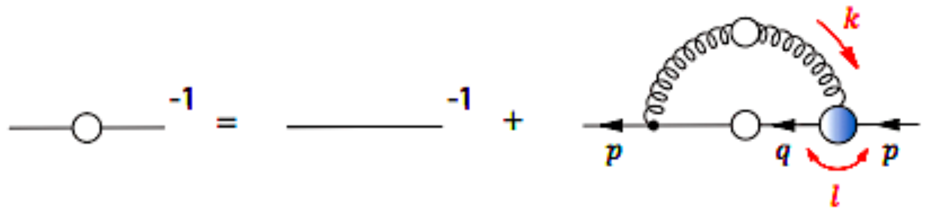
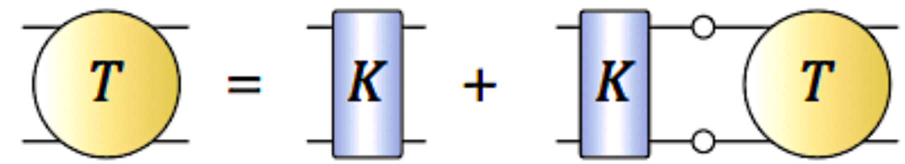


FIG. 1: Feynman diagrams for the factorized amplitude for the $e^- + N \to \nu_e + D_s^- + N'$ process involving the gluon GPDs; the thick line represents the heavy anti-quark $\bar{c}$.

B. Pire, L. Szymanowski, and J. Wagner, arXiv:2104.04944 (2021)

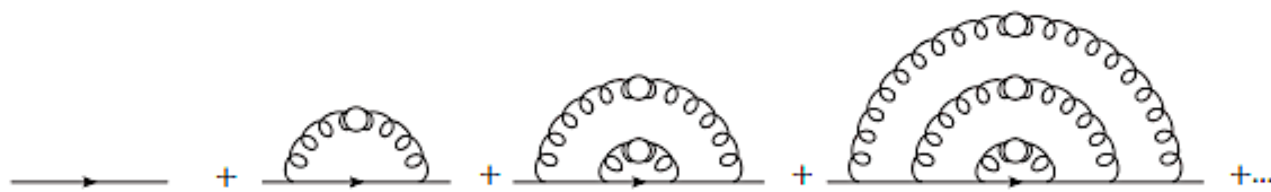
Bethe-Salpeter Equation for QCD Bound States



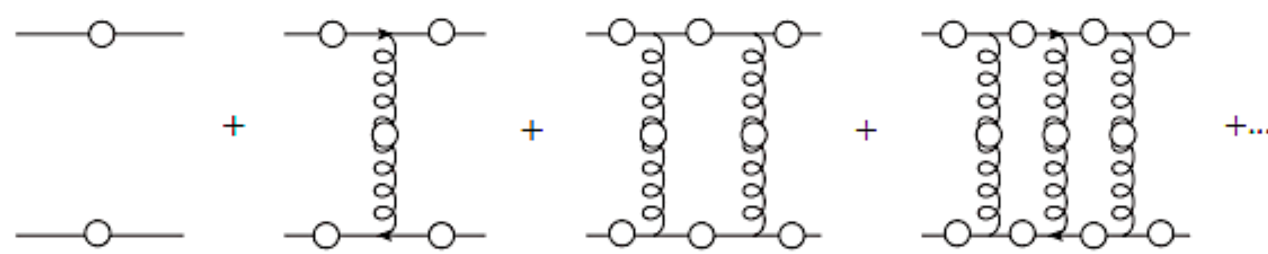


Rainbow-ladder truncation (leading symmetry-preserving approximation)

$S(p) =$



$G^4(k,q,P) =$



- Quark propagators are obtained by solving the gap equation (DSE) for space-like momenta.
- In solving the BSE in Euclidean space, the propagators are functions of $(k+P)^2$, $P=(0,0,0,iM)$.
- Extension to complex plane via Cauchy's integral theorem.

Bethe-Salpeter Equation for QCD Bound States

$$\Gamma_M^{fg}(k,P) = \int^{\Lambda} \frac{d^4q}{(2\pi)^4} K_{fg}(k,q;P) S_f(q_\eta) \Gamma_M^{fg}(q,P) S_g(q_{\bar{\eta}})$$

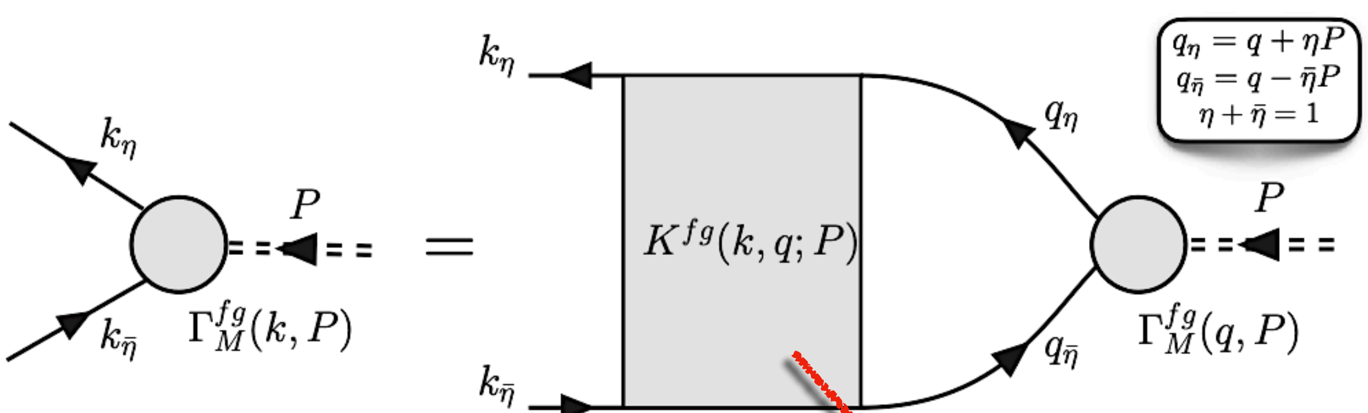
$K_{fg}(q,k;P)$ = Quark-antiquark scattering kernel

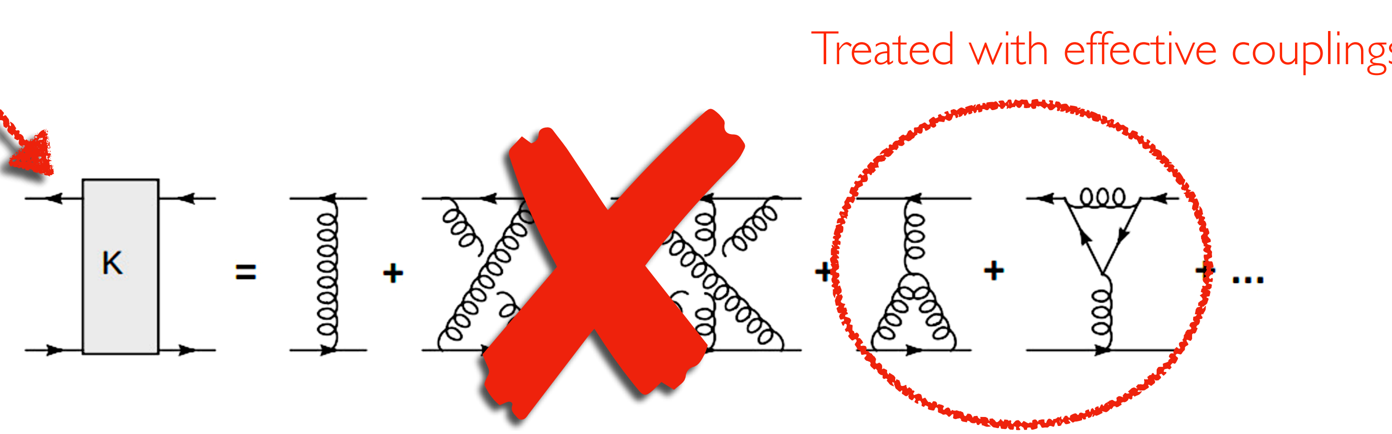
$S_f(q_\eta)$ = Dressed quark propagator

$\Gamma_M^{fg}(k,P)$ = Meson's Bethe-Salpeter Amplitude (BSA)

Poincaré invariant BSA solution for pseudoscalar mesons:

$$\Gamma_M^{fg}(k,P) = \gamma_5 \Big[iE_M^{fg}(k,P) + \gamma\cdot P\, F_M^{fg}(k,P) + \gamma\cdot k\; k\cdot P\, G_M^{fg}(k,P) + \sigma_{\mu\nu} k_\mu P_\nu\, H_M^{fg}(k,P) \Big]$$





Pseudoscalar Meson Spectrum

Mesons/Observables	m_M	$m_M^{\text{exp.}}$	ϵ_r^m [%]	f_M	$f_M^{\text{exp./lQCD}}$	ϵ_r^f [%]
$\pi(u\bar{d})$	0.136	0.140	2.90	$0.094^{+0.001}_{-0.001}$	0.092(1)	2.17
$K(s\bar{u})$	0.494	0.494	0.0	$0.110^{+0.001}_{-0.001}$	0.110(2)	0.0
$D_u(c\bar{u})$	$1.867^{+0.008}_{-0.004}$	1.870	0.11	$0.144^{+0.001}_{-0.001}$	0.150(0.5)	4.00
$D_s(c\bar{s})$	$2.015^{+0.021}_{-0.018}$	1.968	2.39	$0.179^{+0.004}_{-0.003}$	0.177(0.4)	1.13
$\eta_c(c\bar{c})$	$3.012^{+0.003}_{-0.039}$	2.984	0.94	$0.270^{+0.002}_{-0.005}$	0.279(17)	3.23
$\eta_b(b\bar{b})$	$9.392^{+0.005}_{-0.004}$	9.398	0.06	$0.491^{+0.009}_{-0.009}$	0.472(4)	4.03

Mesons/Observables	m_M	$m_M^{\text{exp.}}$	ϵ_r^m [%]	f_M	f_M^{lQCD}	ϵ_r^f [%]
$B_u(b\bar{u})$	$5.277^{+0.008}_{-0.005}$	5.279	0.04	$0.132^{+0.004}_{-0.002}$	0.134(1)	4.35
$B_s(b\bar{s})$	$5.383^{+0.037}_{-0.039}$	5.367	0.30	$0.128^{+0.002}_{-0.003}$	0.162(1)	20.50
$B_c(b\bar{c})$	$6.282^{+0.020}_{-0.024}$	6.274	0.13	$0.280^{+0.005}_{-0.002}$	0.302(2)	7.28
$\eta_b(b\bar{b})$	$9.383^{+0.005}_{-0.004}$	9.398	0.16	$0.520^{+0.009}_{-0.009}$	0.472(4)	10.17

F. Serna, R. Correa da Silveira, J.J. Cobos Martínez, B.E., E. Rojas, Eur. Phys. J. C 80 (2020)

Vector Meson Spectrum

Mesons/Properties	m_M	$m_M^{\rm exp}$	ϵ_r^m %	f_M	$f_M^{exp/\rm lQCD}$	ϵ_r^f %
$\rho(u\bar{d})$	0.730	0.775	5.810	0.145	0.153	5.229
$\phi(s\bar{s})$	1.070	1.019	5.197	0.187	0.168	11.309
$K^*(s\bar{u})$	0.942	0.896	5.134	0.177	0.159	11.321
$J/\Psi(c\bar{c})$	3.124	3.097	0.872	0.277	0.294	5.782
$\Upsilon(b\bar{b})$	9.411	9.460	0.518	0.594	0.505	17.624

Mesons/Properties	m_M	$m_M^{\rm exp}$	ϵ_r^m %	f_M	$f_M^{exp/\rm lQCD}$	ϵ_r^f %
$D_u^*(c\bar{u})$	2.021	2.009	0.597	0.165	0.158	4.430
$D_s^*(c\bar{s})$	2.169	2.112	2.699	0.205	0.190	7.895

F. Serna, R. Correa da Silveira, B.E., in preparation

Light-Cone Distribution Amplitudes

- We do not obtain the LCDA directly from the light-front wave functions:

$$f_M\,\phi_M(x,\mu) = \int^{\mu^2} \frac{d^2 k_\perp}{16\pi^2}\,\psi_M(x,k_\perp)$$

- Instead, we compute the Mellin moments:

$$\langle x^m \rangle = \int_0^1 dx\, x^m \phi_M(x,\mu) \qquad \langle x^0 \rangle = \int_0^1 dx\, \phi_M(x,\mu) = 1$$

$$f_M \phi_M(x,\mu) = Z_2\, \mathrm{tr}_{\mathrm{CD}} \int \frac{d^4 k}{(2\pi)^4}\, \delta(n\cdot k - x\, n\cdot P)\,\gamma_5\,\gamma\cdot n\, S(k)\Gamma_\pi(k;P)S(k-P)$$

$$f_M (n\cdot P)^{m+1} \int_0^1 dx\, x^m \phi_M(x,\mu) = Z_2\, \mathrm{tr}_{\mathrm{CD}} \int \frac{d^4 k}{(2\pi)^4}\,(n\cdot k)^m\,\gamma_5\,\gamma\cdot n\,\chi_\pi(k;P)$$

Bethe-Salpeter wave function

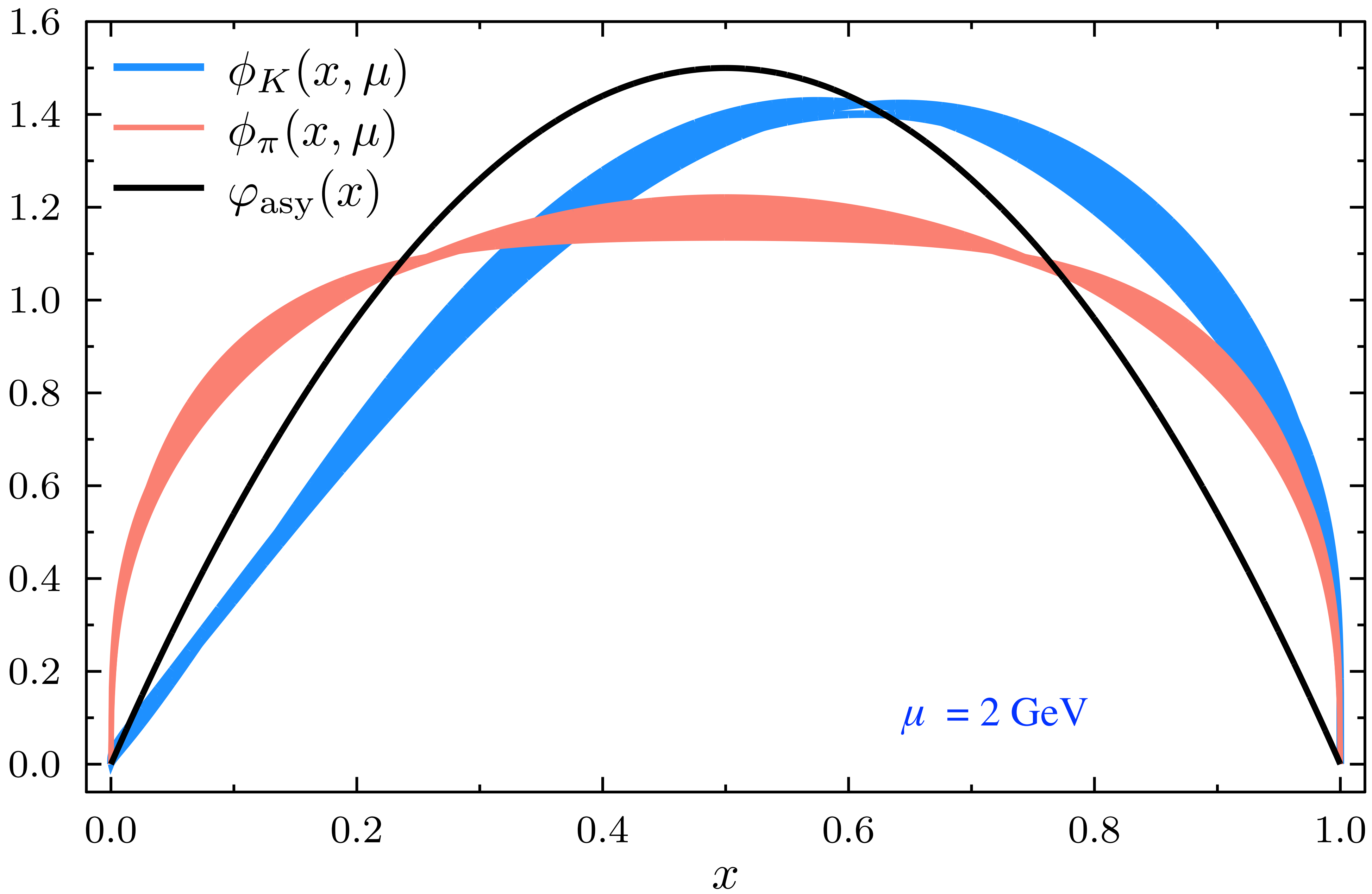
L. Chang, I.C. Cloët, J.J. Cobos-Martínez, C.D. Roberts, S.M. Schmidt, P. C. Tandy, Phys. Rev. Lett. 110 (2013)
D. Binosi, L. Chang, M. Ding, F. Gao, J. Papavassiliou and C. D. Roberts, Phys. Lett. B 790 (2019)

- We reconstruct the distributions amplitudes by means of a Gegenbauer-like expansion or other functional forms:

$$\phi_\pi^{\mathrm{rec.}}(x,\mu) = \mathcal{N}(\alpha)\,[x\bar{x}]^{\alpha-1/2}\,[1 + a_2 C_2^\alpha(2x-1)]$$

$$\phi_H^{\mathrm{rec.}}(x,\mu) = \mathcal{N}(\alpha,\beta)\,4x\bar{x}\,e^{4\,\alpha x\bar{x} + \beta(x-\bar{x})}$$

Light-Cone Distribution Amplitudes



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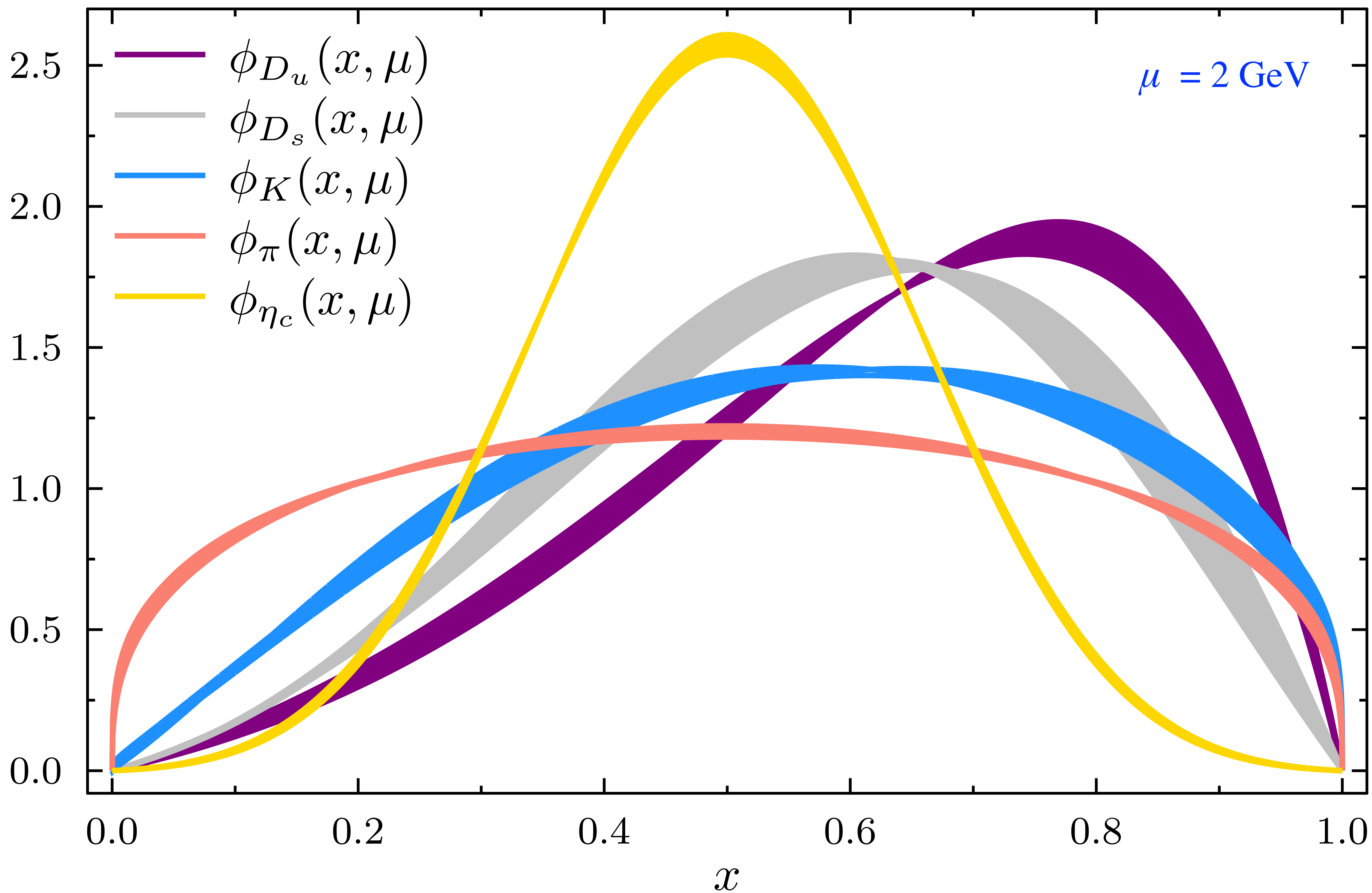

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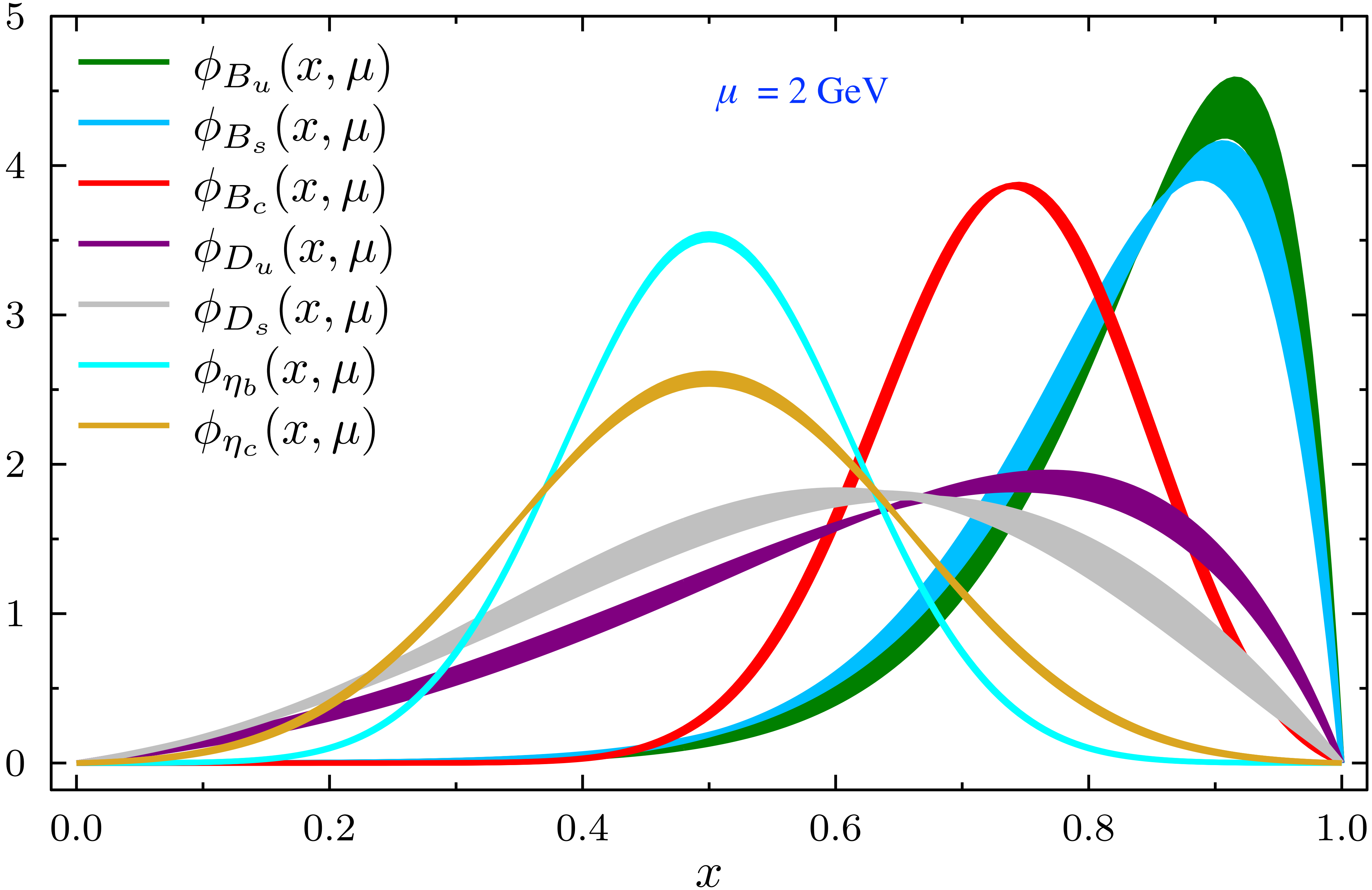

F. Serna, R. Correa da Silveira, J.J. Cobos Martínez, B.E., E. Rojas, Eur. Phys. J. C 80 (2020)

Only valid for full QCD !

Application to HQET requires the use of heavy-quark expansion of charm and bottom propagator in the Bethe-Salpeter equation.

$$\langle 0|\bar{u}(zn)\,\mathcal{W}\,[z,0]\,\gamma\cdot n\,\gamma_5\,h_v(0)|\bar{B}(v)\rangle$$

$$S_Q(p) = \frac{\gamma\cdot p + m_Q}{p^2 - m_Q^2} \xrightarrow{m_Q\to\infty} \frac{1+\gamma\cdot v}{2\,v\cdot k} + \mathcal{O}\left(\frac{k}{m_Q}\right),$$

$$p_\mu = m_Q v_\mu + k_\mu$$
$$k \sim \Lambda_{\rm QCD}$$

Conclusions & Progress

- Much progress was made from QCD based modeling toward nonperturbative numerical solutions of quark propagators and quark-antiquark bound states for flavored mesons satisfying chiral symmetry and Poincaré covariance.
- Good reproduction of charmonium and bottonium as well as D and B meson mass spectrum and their weak decay constants.
- Improvements in Bethe-Salpeter kernels beyond ladder truncation underway …
  $\Longrightarrow$ needed for scalar and axialvector channels and their higher radially excited states, as well as better control of quark correlation functions on complex plane.
- LCDA of vector quarkonia and D and D_s mesons underway.
- Calculation of scalar D and D_s mesons & excitations (PANDA-FAIR, Jlab, EIC ?)