Measures of pion and Kaon structure: mass and pressure distributions via GPDs

Khépani Raya Montaño

Lei Chang

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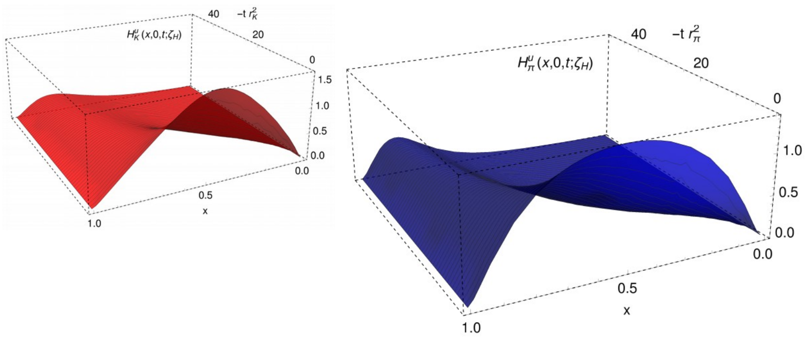
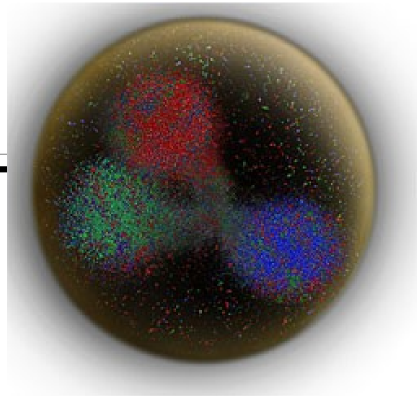
José Rodríguez Quintero...



2nd PSQ@EIC Meeting

July 19-23, 2021. APCTP-CFNS - Korea (online)

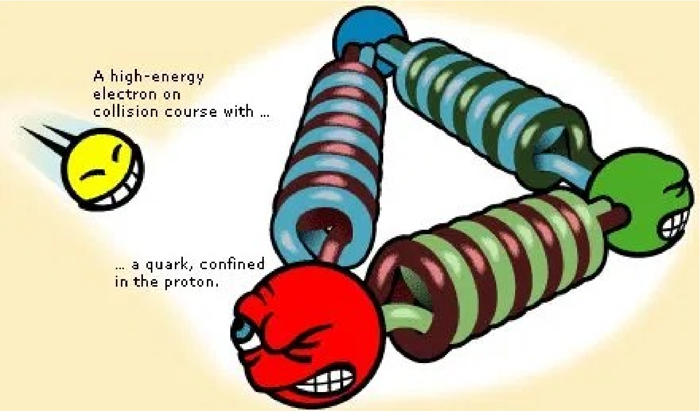
QCD: Basic Facts

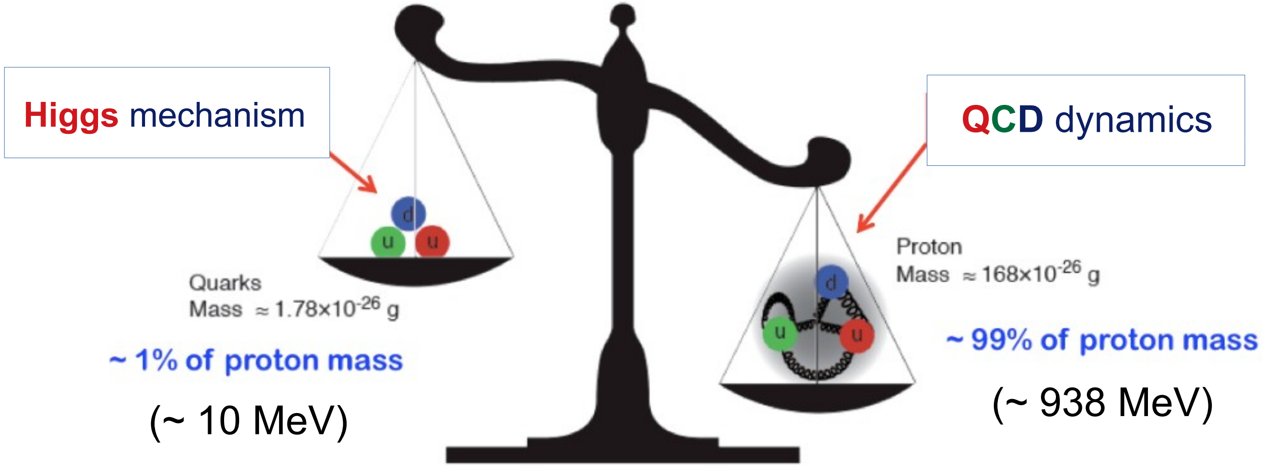


- ➢ **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

- ◆ Quarks and gluons not *isolated* in nature.
- ➔ Formation of colorless bound states: "**Hadrons**"

- ◆ Emergence of hadron masses (EHM) from QCD **dynamics**





QCD: Basic Facts

➢ **QCD** is characterized by two **emergent** phenomena: confinement and dynamical generation of mass (DGM).

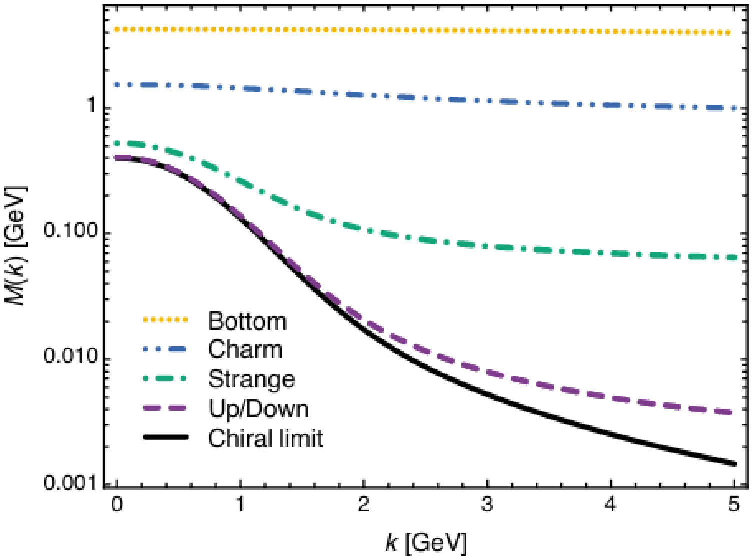
$$\mathcal{L}_{\rm QCD} = \sum_{j=u,d,s,\ldots} \bar{q}_j[\gamma_\mu D_\mu + m_j]q_j + \tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu},$$
$$D_\mu = \partial_\mu + ig\tfrac{1}{2}\lambda^a A^a_\mu\,,$$
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu + \partial_\nu A^a_\mu - \underline{g f^{abc} A^b_\mu A^c_\nu},$$

Can we trace them down to fundamental d.o.f?

- Emergence of hadron masses (EHM) from QCD **dynamics**

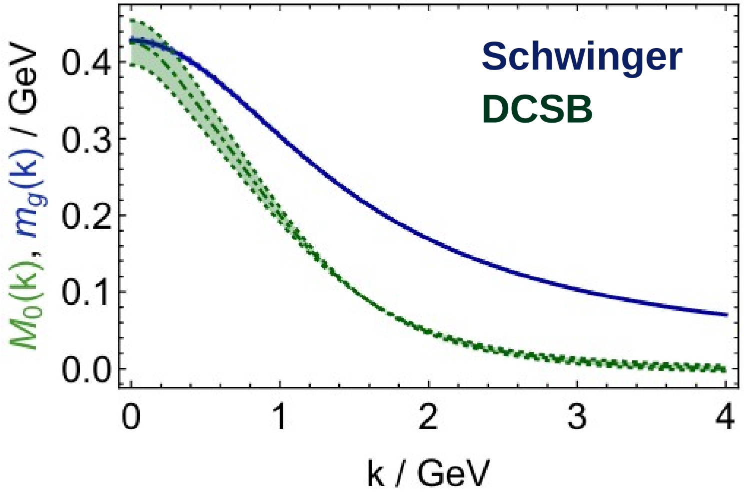
Dynamical masses

(Dynamical Chiral Symmetry Breaking)



"Higgs" masses

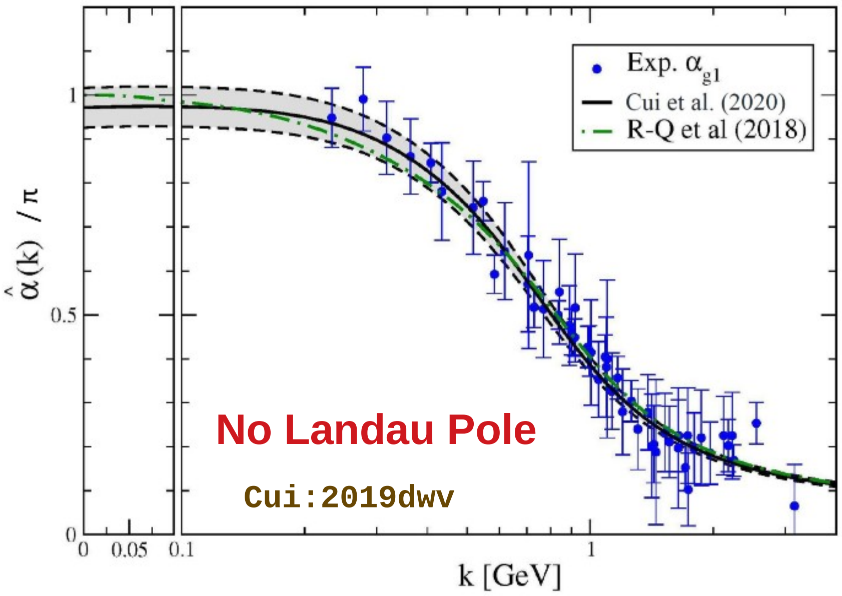
$$S_f^{-1}(p) = Z_f^{-1}(p^2)(i\gamma\cdot p + \mathbf{M_f(p^2)})$$

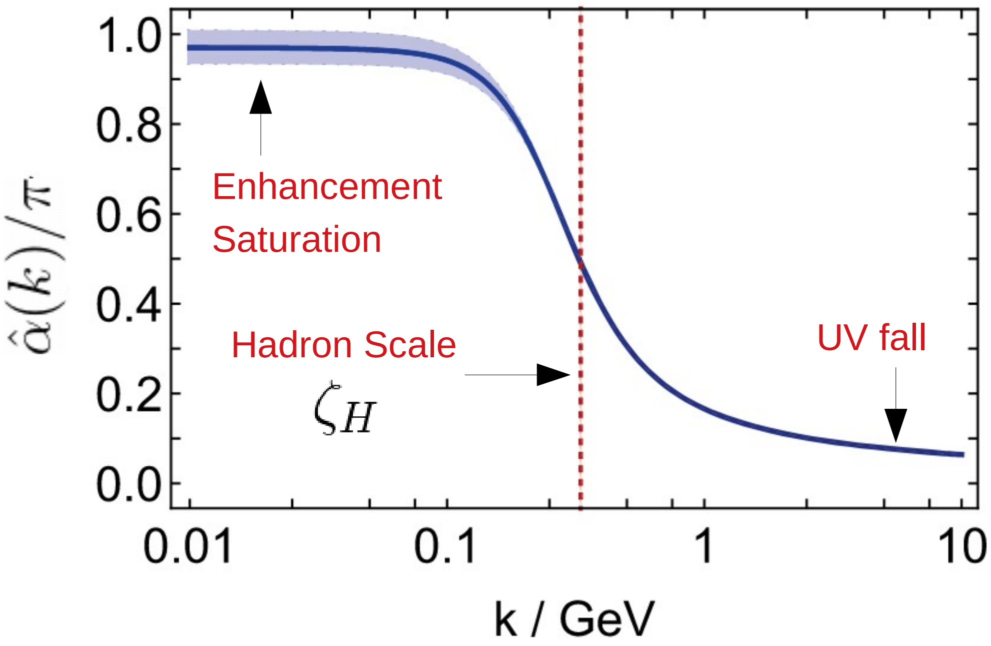


Gluon and quark *running masses*

QCD: Basic Facts

➢ **Confinement** and the **EHM** are tightly connected with **QCD's running coupling**.





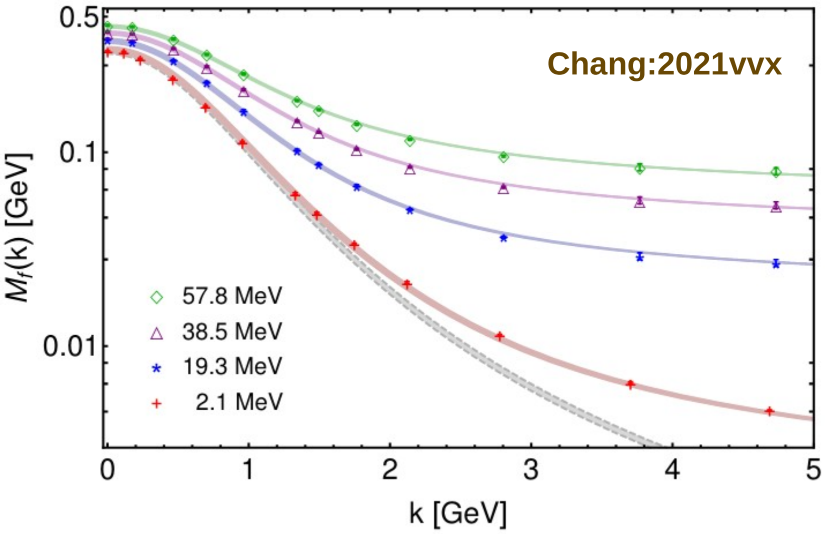
Modern picture of **QCD** coupling. 'Effective Charge'

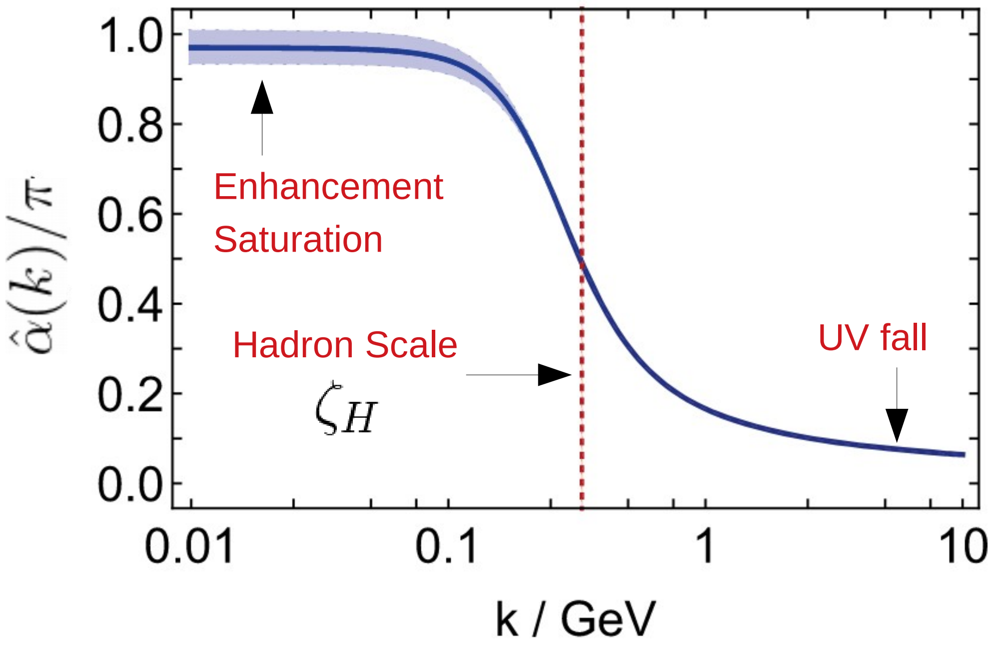
Combined continuum + lattice QCD analysis

ζ_H: Fully **dressed valence** quarks express all hadron's properties

QCD: Basic Facts

➢ **Confinement** and the **EHM** are tightly connected with **QCD's running coupling**.





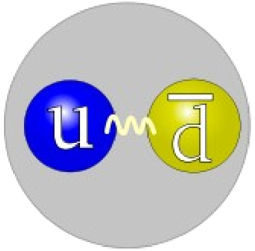
The **Effective Charge** connects **Lattice QCD** and **continuum** mass functions.

Same charge we shall use for **DGLAP evolution.**

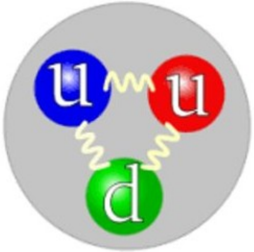
… which defines ζ_H

Why pions and Kaons?

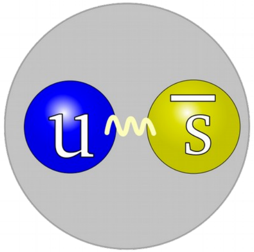
- **Pions** and **Kaons** emerge as QCD’s (pseudo)-**Goldstone** bosons.



$m_\pi \approx 0.140\ \mathrm{GeV}$



$m_p \approx 0.940\ \mathrm{GeV}$



$m_K \approx 0.490\ \mathrm{GeV}$

‘Higgs’ masses

$m_{u/d} \approx 0.004\ \mathrm{GeV}$

$m_s \approx 0.095\ \mathrm{GeV}$

- Their study is crucial to understand the **EHM** and the ***hadron structure*.**

- Dominated by **QCD** dynamics

  Simultaneously explains the mass of the proton and the *masslessness* of the pion

- Interplay between **Higgs** and **strong** mass generating mechanisms.

The light-front wave function approach

"One ring to rule them all"

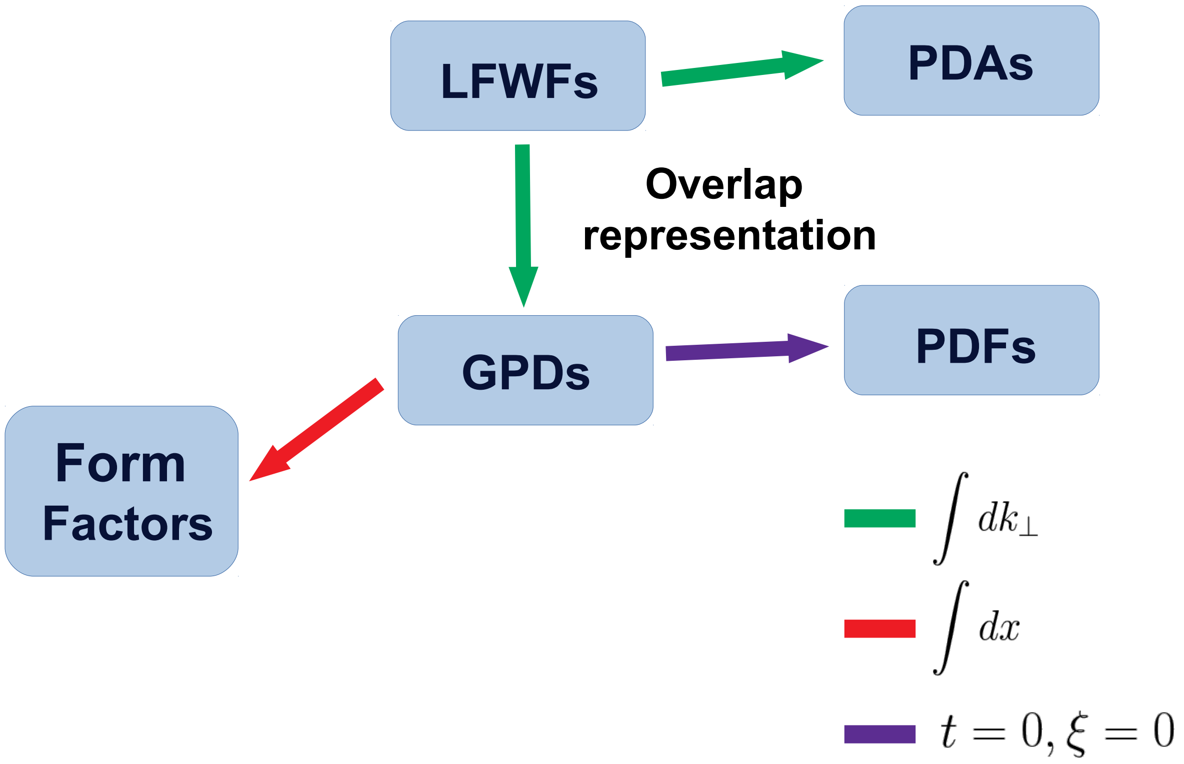
$$\psi^q_{\mathrm{M}}\left(x, k_\perp^2\right) = \mathrm{tr}\int_{dk_\parallel} \delta^x_n(k_{\mathrm{M}})\gamma_5\gamma\cdot n\,\chi_{\mathrm{M}}(k_-, P)$$

Bethe-Salpeter wave function

- Yields a **variety** of **distributions**.

Light-front wave function approach

- **Goal:** get a **broad picture** of the pion and Kaon structure.



The idea:
Compute ***everything*** from the **LFWF.**

LFWF approach

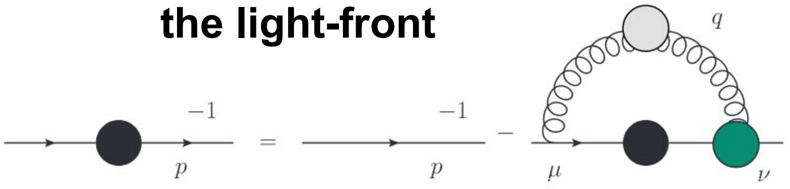
$$\psi_{\mathrm{M}}^{q}\left(x, k_{\perp}^{2}\right)=\operatorname{tr} \int_{d k_{\|}} \delta_{n}^{x}(k_{\mathrm{M}}) \gamma_{5} \gamma \cdot n\, \chi_{\mathrm{M}}(k_{-}, P)$$

- **Goal:** get a **broad picture** of the pion and Kaon structure.

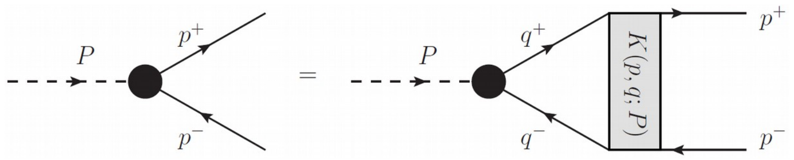
BSWFs → **LFWFs**

Project onto the light-front

Connection with Continuum Schwinger Methods (**CSM**)



Quark DSE



Meson BSE

Truncation required!

The idea:
Compute ***everything*** from the **LFWF.**

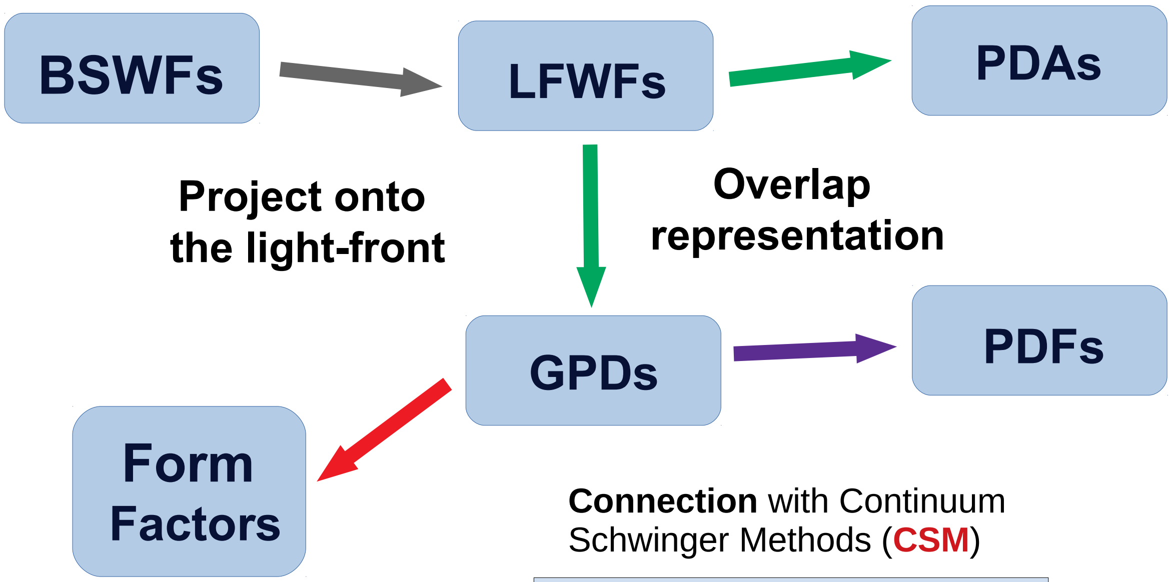
The inputs:
Solutions from quark **DSE** and meson **BSE**.

- Numerically challenging, but doable
- Already on the market: PDAs, PDFs, Form factors...

K. Raya *et al.*,
arXiv: 1911.12941 [nucl-th]

Light-front wave function approach

- **Goal:** get a **broad picture** of the pion and Kaon structure.



Connection with Continuum Schwinger Methods (**CSM**)

PDA and **PDF** as **benchmarks**

The idea:
Compute ***everything*** from the **LFWF.**

The inputs:
Solutions from quark **DSE** and meson **BSE**.

The alternative inputs:
Model **BSWF** from realistic DSE ***predictions***.

LFWF: Nakanishi model

- A Nakanishi-like representation for the **BSWF**:

$$n_K \chi_K(k_-^K, P_K) = \underbrace{\mathcal{M}(k,P)}_{1} \int_{-1}^{1} dw\, \underbrace{\rho_K(w)}_{2} \underbrace{\mathcal{D}(k,P)}_{3}$$

(Kaon as example)

1: Matrix structure (leading BSA):

$$\mathcal{M}(k;P_K) = -\gamma_5[\gamma \cdot P_K M_u + \gamma \cdot k(M_u - M_s) + \sigma_{\mu\nu} k_\mu P_{K\nu}]\,,$$

Equivalent to considering the **leading** Bethe-Salpeter amplitude:

$$\Gamma_{\mathrm{M}}(q;P) = i\gamma_5 E_{\mathrm{M}}(q;P)$$

(from a total of **4**)

(others can be **incorporated** systematically)

S-S Xu et al., PRD 97 (2018) no.9, 094014.

LFWF: Nakanishi model

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$$\mathcal{M}(k;P_K) = -\gamma_5[\gamma \cdot P_K M_u + \gamma \cdot k(M_u - M_s) + \sigma_{\mu\nu} k_\mu P_{K\nu}] \, ,$$

2: Spectral weight: Tightly connected with the meson properties.

3: Denominators: $\mathcal{D}(k;P_K) = \Delta(k^2, M_u^2)\Delta((k-P_K)^2, M_s^2)\hat{\Delta}(k_{\omega-1}^2, \Lambda_K^2) \, ,$

where: $\Delta(s,t) = [s+t]^{-1}, \; \hat{\Delta}(s,t) = t\Delta(s,t) \, .$

S-S Xu et al., PRD 97 (2018) no.9, 094014.

LFWF: Nakanishi model

- Recall the expression for the **LFWF**:

$$\psi^q_{\rm M}\left(x,k_\perp^2\right) = {\rm tr}\int_{dk_{||}} \delta^x_n(k_{\rm M})\gamma_5\gamma\cdot n\,\chi_{\rm M}(k_-,P)$$

$$<x>^q_{\rm M} := \int_0^1 dx\; x^m \psi^q_{\rm M}(x,k_\perp^2)$$

- Algebraic manipulations yield:

+ Uniqueness of Mellin moments

$$\Rightarrow \psi^q_{\rm M}(x,k_\perp) \sim \int dw\; \rho_{\rm M}(w)\cdots$$

✓ Compactness of this result is a merit of the AM.

- Thus, $\rho_{\rm M}(w)$ determines the profiles of, *e.g.* **PDA** and **PDF**: (it also works the **other way around**)

$$f_{\rm M}\phi^q_{\rm M}(x;\zeta_H) = \int \frac{d^2k_\perp}{16\pi^3}\psi^q_{\rm M}(x,k_\perp;\zeta_H)$$

$$q_{\rm M}(x;\zeta_H) = \int \frac{d^2k_\perp}{16\pi^3}\left|\psi^q_{\rm M}(x,k_\perp;\zeta_H)\right|^2$$

Chiral limit / Factorized model

- In the **chiral limit**, the **Nakanishi model** reduces to:

$$\psi_{\rm M}^q(x,k_\perp^2;\zeta_H) \sim \tilde{f}(k_\perp)\phi_{\rm M}^q(x;\zeta_H) \sim f(k_\perp)[q_{\rm M}(x;\zeta_H)]^{1/2}$$

"Factorized model"

$$[\phi_{\rm M}^q(x;\zeta_H)]^2 \sim q_{\rm M}(x;\zeta_H)$$

- ✓ Sensible assumption as long as:

$m_{\rm M}^2 \approx 0$ (meson mass) $M_{\bar{h}}^2 - M_q^2 \approx 0$ (*h*-antiquark, *q*-quark masses) ζ_H

→ Produces **identical** results as Nakanishi model for pion

- Therefore:

$$\psi_{\rm M}^q(x,k_\perp^2;\zeta_H) = [q^{\rm M}(x;\zeta_H)]^{1/2}\left[4\sqrt{3}\pi\frac{M_q^3}{(k_\perp^2+M_q^2)^2}\right]$$

Single parameter!

$M_q \sim r_{\rm M}^{-1}$

(charge radius)

No need to determine the spectral weight !

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Single parameter!

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(charge radius)

Systematically improvable !

(account for other BSAs, x-k correlations, for example)

Chiral limit / Factorized model

$$\psi_{\rm M}^{q}(x,k_\perp^2;\zeta_H) = [q^{\rm M}(x;\zeta_H)]^{1/2}\left[4\sqrt{3}\pi\frac{M_q^3}{(k_\perp^2+M_q^2)^2}\right]$$

"Chiral M1"

$$M_q \sim r_{\rm M}^{-1}$$

$$M_u = 0.31\ {\rm GeV} \Leftrightarrow r_\pi = 0.66\ {\rm fm}$$

- We can also consider a **"Gaussian model"**:

$$\psi_{\rm M}^{q}(x,k_\perp^2;\zeta_H) = [q^{\rm M}(x;\zeta_H)]^{1/2}\left(\frac{32\pi^2 r_{\rm M^2}}{\chi_{\rm M}^2(\zeta_H)}\right)^{1/2}\exp\left[-\frac{r_{\rm M}^2 k_\perp^2}{2\chi_{\rm M}^2(\zeta_H)}\right]$$

$$\chi_{\rm P}^2(\zeta_{\mathcal{H}}) = \langle x^2\rangle_{\bar h}^{\zeta_{\mathcal{H}}} + \tfrac{1}{2}(1-d_{\rm P})\langle x^2\rangle_u^{\zeta_{\mathcal{H}}}$$

Asymmetry factor $\sim M_{\bar h}^2 - M_u^2$

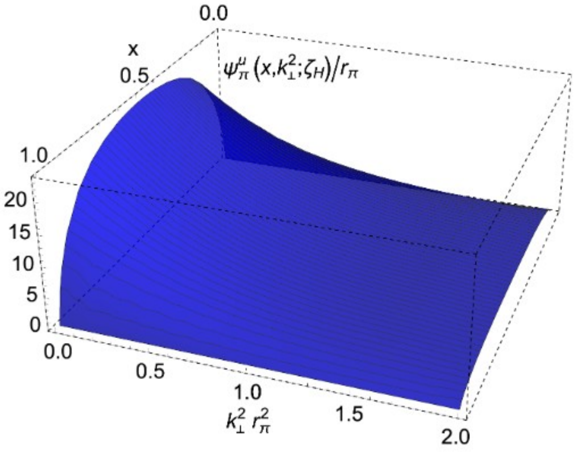
(h-antiquark, q-quark masses)

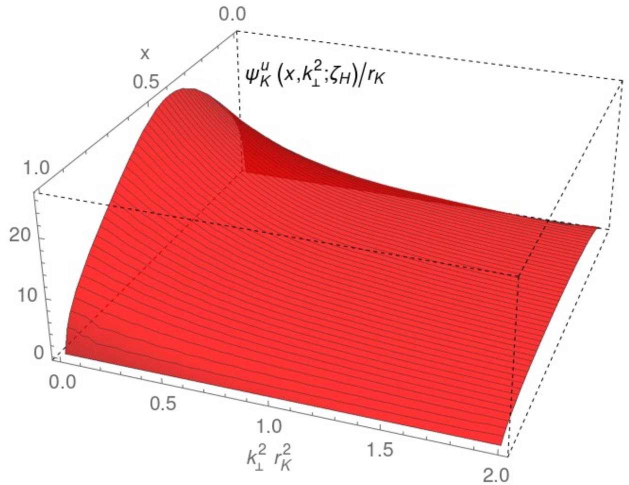
- **One parameter** to determine **both** models:
  - Either M_q or $r_{\rm M}$ (charge radius)

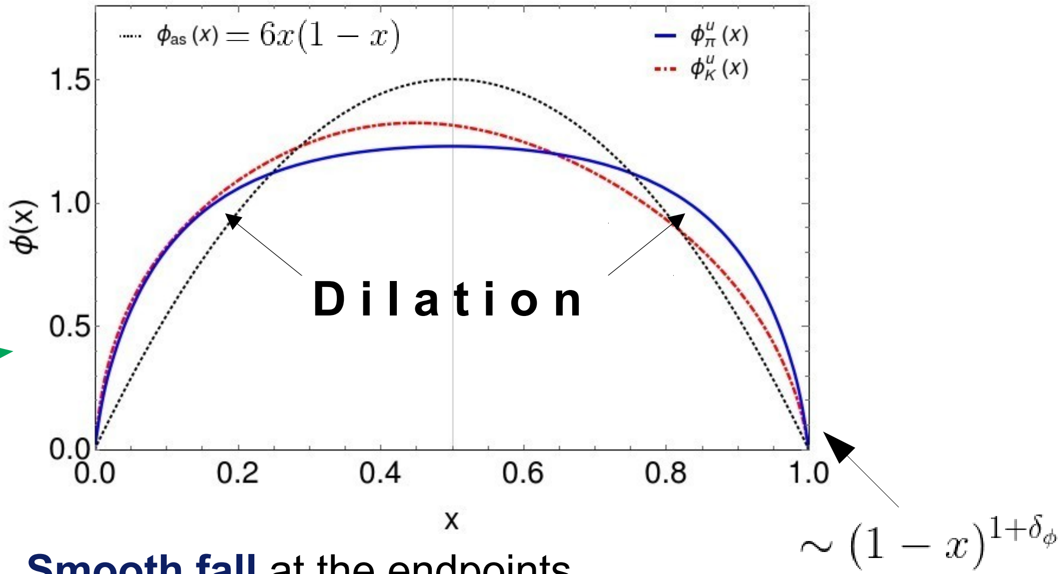
- Unless specified otherwise, **Nakanishi model** results will be shown.
- By construction, **PDA** and **PDF** are the **same** in any presented model.
- In general, **Chiral M1** ≈ **Nakanishi** (for pion)

LFWFs and PDAs

$$f_M \phi_M^q(x;\zeta_H) = \int \frac{d^2 k_\perp}{16\pi^3} \psi_M^q(x, k_\perp; \zeta_H)$$







- **Smooth fall** at the endpoints
- **Broad** and concave functions of ***x***
  - Consequence of **DCSB**
- **Higgs** induced asymmetry for **Kaon**:
  - Moduled by the difference $M_s - M_u$

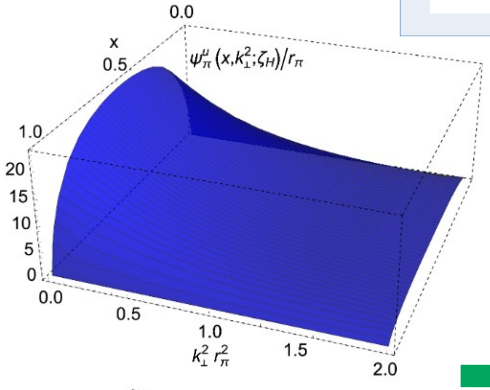
LFWFs and GPDs

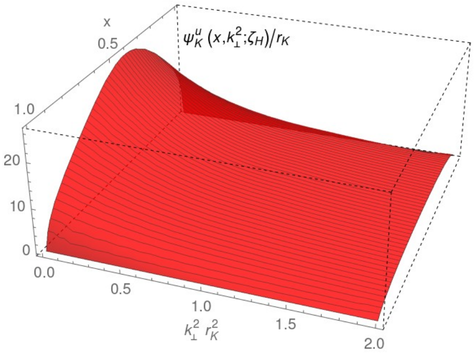
LFWFs → **GPDs**

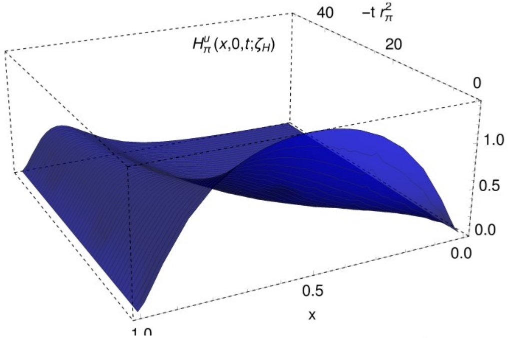
- In the **overlap representation,** the valence-quark **GPD** reads as:

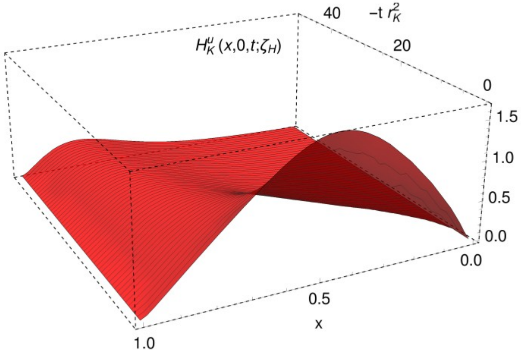
$$H^q_{\rm M}(x,\xi,t) = \int \frac{d^2k_\perp}{16\pi^3}\,\psi^{q*}_{\rm M}\left(x^-,(\mathbf{k}_\perp^-)^2\right)\psi^{q}_{\rm M}\left(x^+,(\mathbf{k}_\perp^+)^2\right)$$

ζ_H









- ✓ ***Valid*** in the **DGLAP** region
- ✓ ***Positivity*** fulfilled
- ✓ Can be ***extended*** to the **ERBL** region $|x| \leq \xi$

Chouika:2017rzs

- ✓ ***Analytic*** in our factorized models.

GPD: factorized model

$$H_M^q(x,\xi,t) = \int \frac{d^2k_\perp}{16\pi^3} \psi_M^{q*}\left(x^-, (\mathbf{k}_\perp^-)^2\right) \psi_M^q\left(x^+, (\mathbf{k}_\perp^+)^2\right)$$

Overlap representation

Factorized **LFWF**

$$\psi_M^q(x, k_\perp^2; \zeta_H) = \left[q^M(x;\zeta_H)\right]^{1/2} \widetilde{\psi}_M(k_\perp^2;\zeta_H)$$

PDF controls (mostly) the *x*-dependence

$$H_M^q(x,\xi,t;\zeta_H) = \theta(x_-)\left[q^M(x_-;\zeta_H)q^M(x_+;\zeta_H)\right]^{1/2}\Phi_M(z;\zeta_H)$$

$$\Phi_M(z;\zeta_H) = \int \frac{d^2k_\perp}{16\pi^3}\widetilde{\psi}_M(k_\perp^2;\zeta_H)\widetilde{\psi}_M\left((k_\perp - s_\perp)^2;\zeta_H\right)$$

t-dependence, evaluated **analytically**

$$x_\pm = \frac{x \pm \xi}{1 \pm \xi}$$

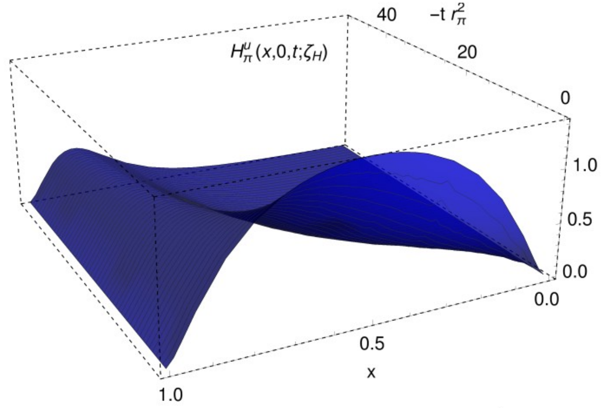
$$z = s_\perp^2 = \frac{-t(1-x)^2}{1-\xi^2}$$

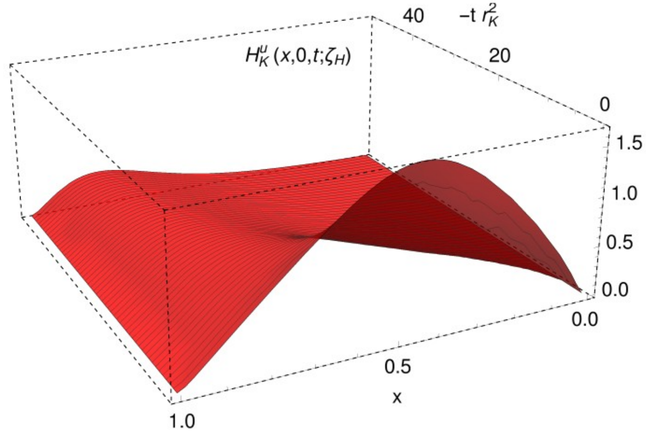
LFWFs and PDFs

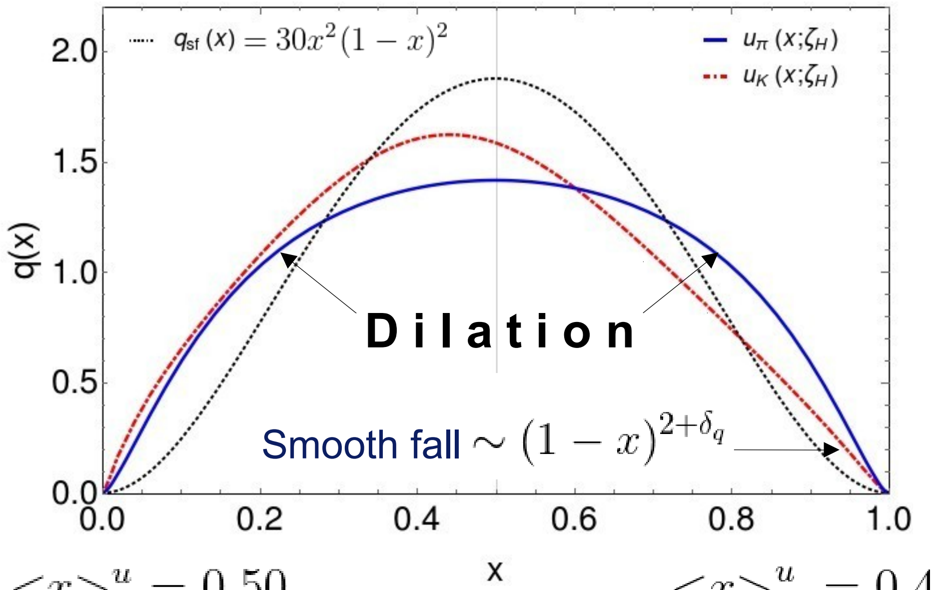
GPD → PDF

- The **PDF** is obtained from the **forward limit** of the **GPD**.

$$q(x) = H(x,0,0)$$







$<x>_\pi^u = 0.50$ $<x>_K^u = 0.47$

- ζ_H: meson properties determined by the fully-dressed valence-quarks.
- **Broad** + **Higgs**-induced *asymmetry*

DGLAP evolution: The idea

Idea. Use QCD's **effective charge** to define an *all orders* evolution.

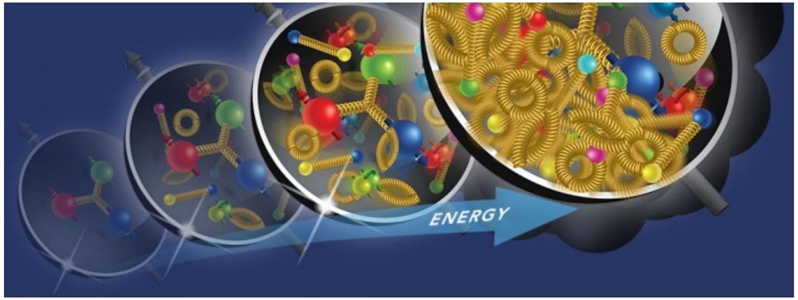
Starting from fully-dressed **quasiparticles**, at ζ_H

(at which **valence quarks** carry ***all*** meson's **properties**)

Sea and **Gluon** content unveils, as prescribed by **QCD**

$$\left\{\zeta^2 \frac{d}{d\zeta^2}\int_0^1 dy\delta(y-x) - \boxed{\frac{\alpha(\zeta^2)}{4\pi}}\int_x^1 \frac{dy}{y}\begin{pmatrix} P_{qq}^{\rm NS}\left(\frac{x}{y}\right) & 0 \\ 0 & \mathbf{P}^{\rm S}\left(\frac{\mathbf{x}}{\mathbf{y}}\right)\end{pmatrix}\right\}\begin{pmatrix} H_\pi^{{\rm NS},+}(y,t;\zeta) \\ \mathbf{H}_\pi^{\rm S}(y,t;\zeta)\end{pmatrix} = 0$$

Exact equation → **"All orders scheme"**



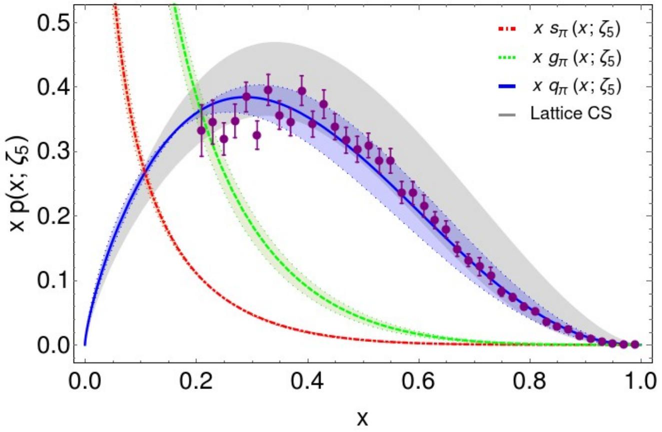
Details are found here:

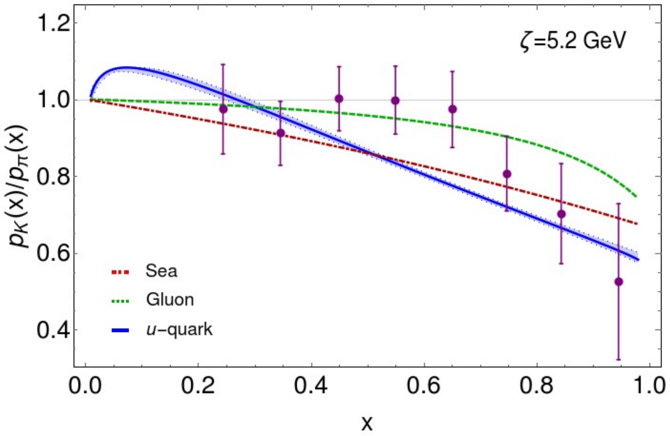
"Kaon and pion parton distributions"

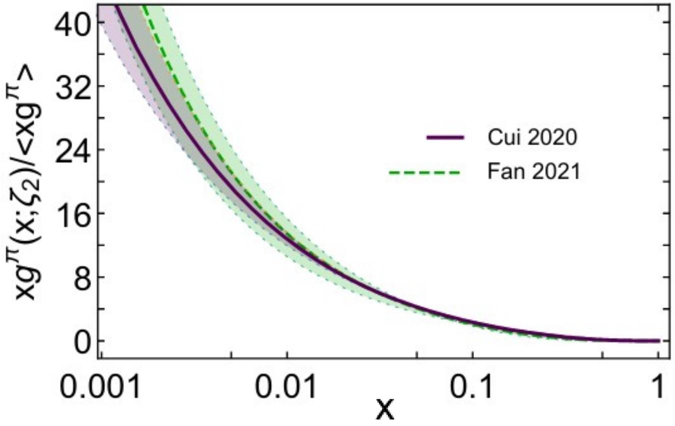
Eur.Phys.J.C 80 (2020) 11, 1064. Cui *et al*.

Evolved PDFs

GPD → PDF







- **Not tuned**, initial scale for evolution $\zeta_H = 0.331\ \mathrm{GeV}$
- Valence at **2 GeV**

	$\langle x\rangle_u^\pi$	$\langle x^2\rangle_u^\pi$	$\langle x^3\rangle_u^\pi$
lQCD [53]	0.21(1)	0.16(3)	
lQCD [54]	0.254(03)	0.094(12)	0.057(04)
Ref. [102]	0.24	0.098	0.049
Refs. [39, 40]	0.24(2)	0.098(10)	0.049(07)
Herein	0.24(2)	0.094(13)	0.047(08)

- In **agreement** with:
  - ✓ **ASV** analysis — Aicher:2010cb
  - ✓ **Lattice** CS — Sufian:2020vzb, Sufian:2019bol
  - ✓ **DSEs** — Cui:2020tdf
- Valence at **5.2 GeV** →

- **Gluon** in pion: — Chang:2021utv
  - ✓ **Lattice** MSU — Fan:2021bcr

$$<\mathbf{x}>_\pi^{\mathrm{val}} = 0.41(4)$$

$$<\mathbf{x}>_K^{\mathrm{val}} = 0.43(4)$$

Pion EFF

GPD → FFs

- **Electromagnetic** form factor is obtained from the **t-dependence** of the **0-th moment**:

$$F_M^q(-t=\Delta^2) = \int_{-1}^{1} dx\, H_M^q(x,\xi,t)$$

Can safely take **ξ = 0**

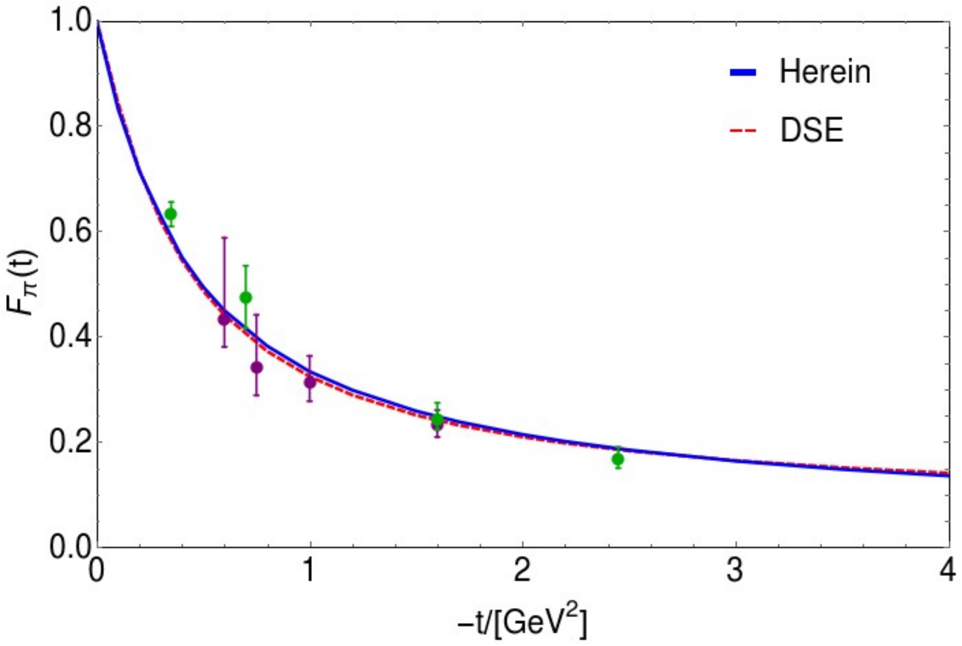
"Polinomiality"

$$F_M(\Delta^2) = e_u F_M^u(\Delta^2) + e_{\bar{f}} F_M^{\bar{f}}(\Delta^2)$$

Weighed by electric charges

→ **Isospin symmetry**

$$F_{\pi^+}(-t) = F_{\pi^+}^u(-t)$$



Data: G.M. Huber *et al*. PRC 78 (2008) 045202

DSE: L. Chang *et al*. PRL 111 (2013) 14, 141802

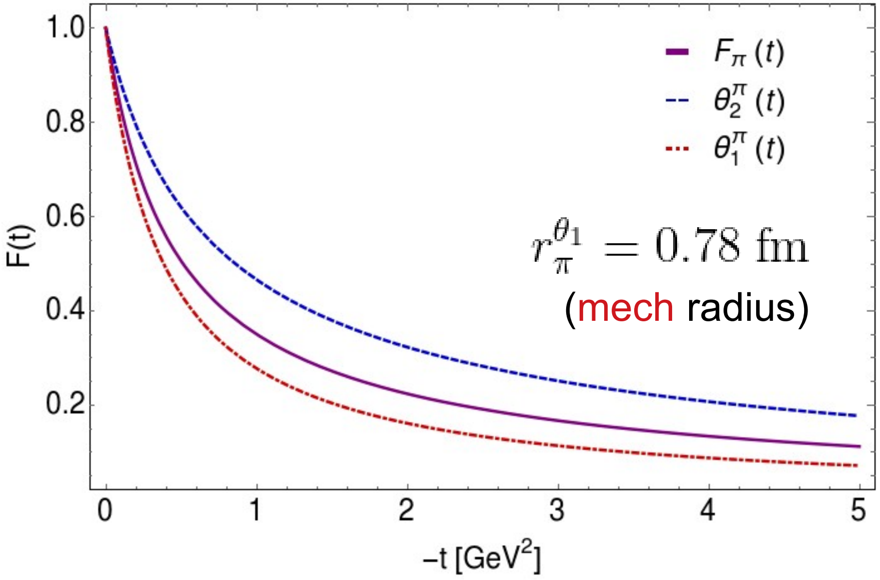
Pion Gravitational FFs

GPD → FFs

- **Gravitational** form factors are obtained from the **t-dependence** of the **1-st moment**:

$$J_M(t,\xi) = \int_{-1}^{1} dx\, x H_M(x,\xi,t) = \Theta_2^M(t) - \xi^2\Theta_1^M(t)$$

- ✓ Directly obtained if **ξ = 0**
- ✓ Only **DGLAP** GPD is required
- ✗ **ERBL** GPD needed



- Sophisticated techniques exist. Chouika:2017dhe
- But a sound expression can be constructed:

$$\theta_1^{P_q}(\Delta^2) = c_1^{P_q}\theta_2^{P_q}(\Delta^2) + \int_{-1}^{1} dx\, x \left[H_P^q(x,1,0)P_{M_q}(\Delta^2) - H_P^q(x,1,-\Delta^2)\right]$$

"Soft pion theorem"

Zhang:2021mtn

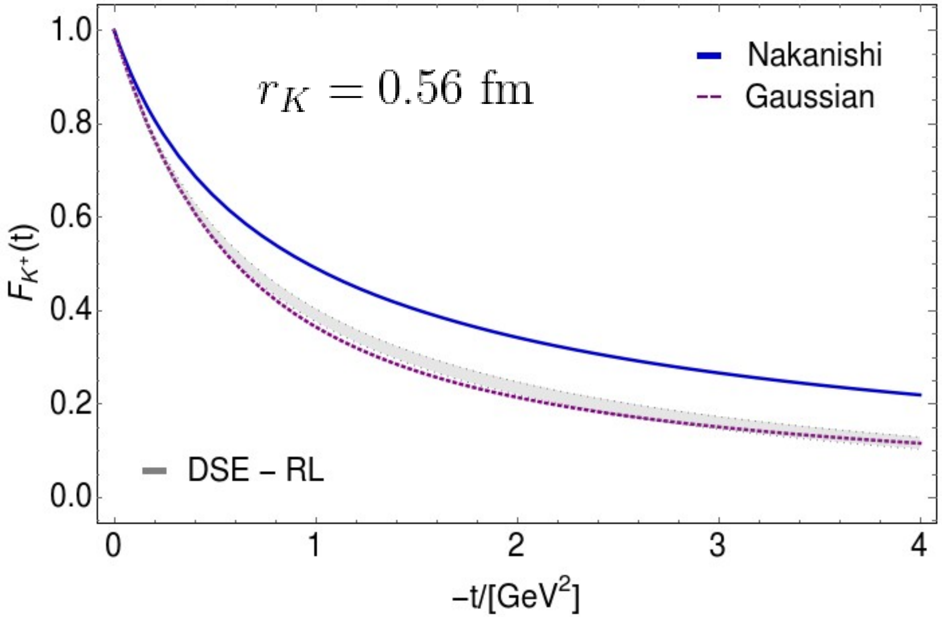
$$r_\pi^E = 0.68 \text{ fm}\,, \quad r_\pi^{\theta_2} = 0.56 \text{ fm}$$

(charge radius) (mass radius)

Kaon EFF

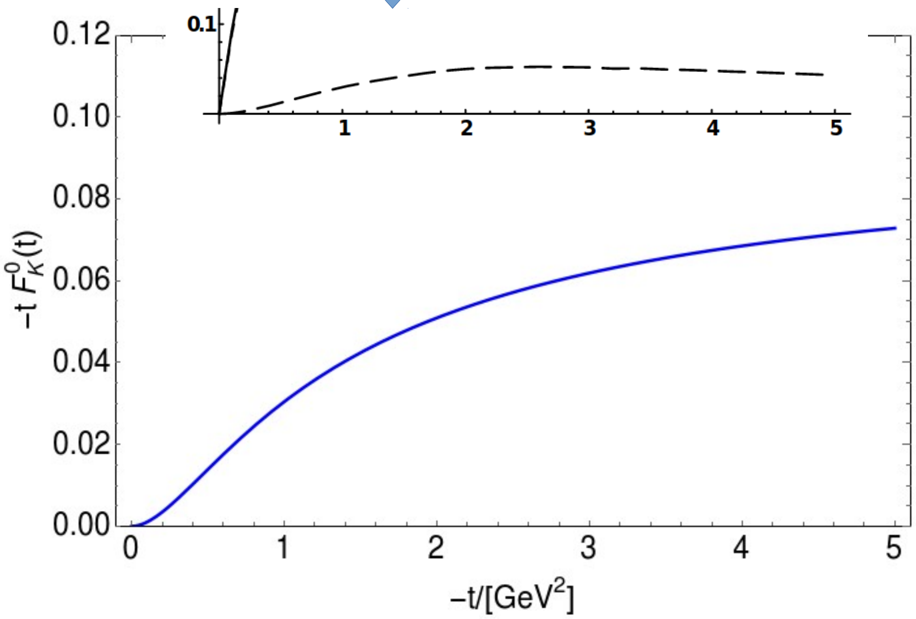
GPD → FFs

- **Electromagnetic** form factor: **charged** and **neutral** kaon



DSE - K⁺: Gao:2017mmp, Eichmann:2019bqf

DSE - K⁰: Burden:1995ve



Kaon is more **compressed**

$$r_K^j \approx 0.85\, r_\pi^j$$

j = *mech, charge, mass*

Charge and mass distributions

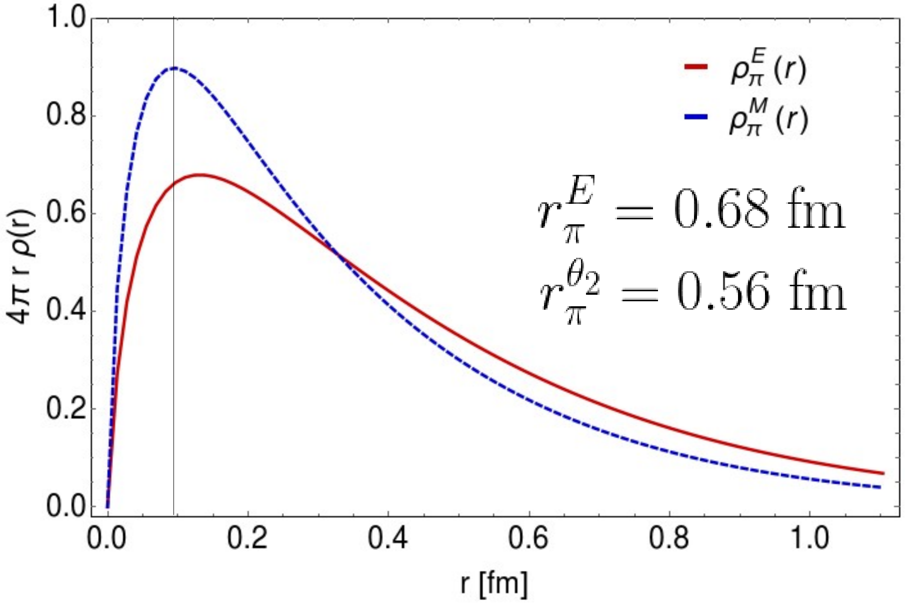
$$\rho_{\rm P}(b) = \frac{1}{2\pi}\int_0^\infty d\Delta\,\Delta J_0(\Delta\, b) F_{\rm P}(\Delta^2)$$

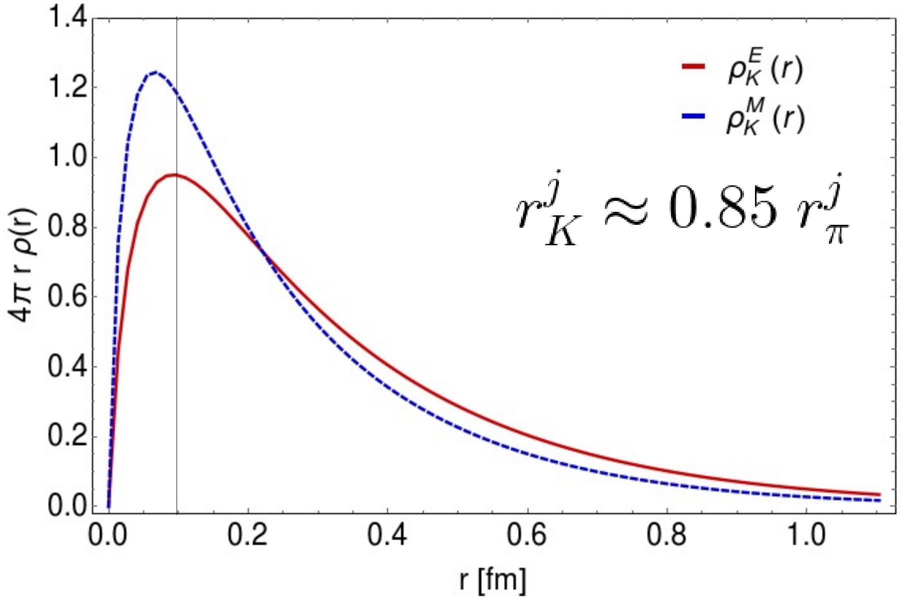
$$F_{\rm P}^E(\Delta^2) \to \rho_{\rm P}^E(b)$$

$$\theta_2^{\rm P}(\Delta^2) \to \rho_{\rm P}^M(b)$$

- **Intuitively**, we expect the meson to be localized at a **finite space**.
- **Charge** effect span over a **larger domain** than **mass** effects.

More **massive** hadron → More **compressed**





On the Radii

GPD → **FFs**

$$H_{\mathrm{P}}^{u}(x,\xi,t;\zeta_H) = \theta(x_-)\left[u^{\mathrm{P}}(x_-;\zeta_H)u^{\mathrm{P}}(x_+;\zeta_H)\right]^{1/2}\Phi_{\mathrm{P}}(z;\zeta_H)$$

- In the **factorized** models:

$$\frac{\partial^n}{\partial^n z}\Phi_{\mathrm{P}}^{u}(z;\zeta_{\mathcal{H}})\Big|_{z=0} = \frac{1}{\langle x^{2n}\rangle_{\bar{h}}^{\zeta_{\mathcal{H}}}}\frac{d^n F_{\mathrm{P}}^{u}(\Delta^2)}{d(\Delta^2)^n}\Big|_{\Delta^2=0}$$

PDF moments — Derivatives of **EFF**

$$\frac{\partial}{\partial z}\Phi_{\mathrm{P}}^{u}(z;\zeta_{\mathcal{H}})\Big|_{z=0} = -\frac{r_{\mathrm{P}}^2}{4\chi_{\mathrm{P}}^2(\zeta_{\mathcal{H}})},$$

$$\frac{\partial}{\partial z}\Phi_{\mathrm{P}}^{\bar{h}}(z;\zeta_{\mathcal{H}})\Big|_{z=0} = (1-d_{\mathrm{P}})\frac{\partial}{\partial z}\Phi_{\mathrm{P}}^{u}(z;\zeta_{\mathcal{H}})\Big|_{z=0}$$

Asymmetry term = **0** for **pion**

GPD can be built from:

- Distribution **amplitude** / Distribution **function**
- Derivatives of the electromagnetic **form factor**

Reminder:

$$\left[\phi_{\mathrm{M}}^{q}(x;\zeta_H)\right]^2 \sim q_{\mathrm{M}}(x;\zeta_H)$$

On the Radii

GPD → FFs

$$H_P^u(x,\xi,t;\zeta_H) = \theta(x_-)\left[u^P(x_-;\zeta_H)u^P(x_+;\zeta_H)\right]^{1/2}\Phi_P(z;\zeta_H)$$

- In the **factorized** models:

$$\frac{\partial^n}{\partial^n z}\Phi_P^u(z;\zeta_{\mathcal{H}})\Big|_{z=0} = \frac{1}{\langle x^{2n}\rangle_{\bar{h}}^{\zeta_{\mathcal{H}}}}\frac{d^n F_P^u(\Delta^2)}{d(\Delta^2)^n}\Big|_{\Delta^2=0}$$

PDF moments — Derivatives of **EFF**

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$$\frac{\partial}{\partial z}\Phi_P^{\bar{h}}(z;\zeta_{\mathcal{H}})\Big|_{z=0} = (1-d_P)\frac{\partial}{\partial z}\Phi_P^u(z;\zeta_{\mathcal{H}})\Big|_{z=0}$$

Asymmetry term = **0** for **pion**

- In the **Chiral M1** model:

$$\frac{r_P{}^2}{6\langle x^2\rangle_{\zeta_H}} = \frac{3}{5M_q{}^2}$$

Clear **connection**:

- Constituent mass ***M***
- Charge **radius**
- PDF **moment**

(at hadron scale)

Sensible values

$$M_u = 0.31\ \text{GeV} \Leftrightarrow r_\pi = 0.66\ \text{fm}$$

On the Radii

GPD → FFs

$$H_{\mathrm{P}}^{u}(x,\xi,t;\zeta_H)=\theta(x_-)\left[u^{\mathrm{P}}(x_-;\zeta_H)u^{\mathrm{P}}(x_+;\zeta_H)\right]^{1/2}\Phi_{\mathrm{P}}(z;\zeta_H)$$

- In the **factorized** models:

$$\frac{\partial^n}{\partial^n z}\Phi_{\mathrm{P}}^u(z;\zeta_{\mathcal{H}})\Big|_{z=0}=\frac{1}{\langle x^{2n}\rangle_{\bar{h}}^{\zeta_{\mathcal{H}}}}\frac{d^n F_{\mathrm{P}}^u(\Delta^2)}{d(\Delta^2)^n}\Big|_{\Delta^2=0}$$

PDF moments — Derivatives of **EFF**

$$\frac{\partial}{\partial z}\Phi_{\mathrm{P}}^u(z;\zeta_{\mathcal{H}})\Big|_{z=0}=-\frac{r_{\mathrm{P}}^2}{4\chi_{\mathrm{P}}^2(\zeta_{\mathcal{H}})},$$

$$\frac{\partial}{\partial z}\Phi_{\mathrm{P}}^{\bar{h}}(z;\zeta_{\mathcal{H}})\Big|_{z=0}=(1-d_{\mathrm{P}})\frac{\partial}{\partial z}\Phi_{\mathrm{P}}^u(z;\zeta_{\mathcal{H}})\Big|_{z=0}$$

Asymmetry term = **0** for **pion**

- Therefore, the **mass *radius***:

$$r_{\mathrm{P}_u}^{\theta_2 2}=\frac{3r_{\mathrm{P}}^2}{2\chi_{\mathrm{P}}^2}\langle x^2(1-x)\rangle_{\mathrm{P}_{\bar{h}}},$$

$$r_{\mathrm{P}_{\bar{h}}}^{\theta_2 2}=\frac{3r_{\mathrm{P}}^2}{2\chi_{\mathrm{P}}^2}(1-d_{\mathrm{P}})\langle x^2(1-x)\rangle_{\mathrm{P}_u}$$

$$\left(\frac{r_\pi^{\theta_2}}{r_\pi^E}\right)^2=\frac{<x^2(1-x)>_{\zeta_H}^q}{<x^2>_{\zeta_H}^q}\approx\left(\frac{4}{5}\right)^2$$

Determined from **PDF** moments!

Impact parameter space GPDs

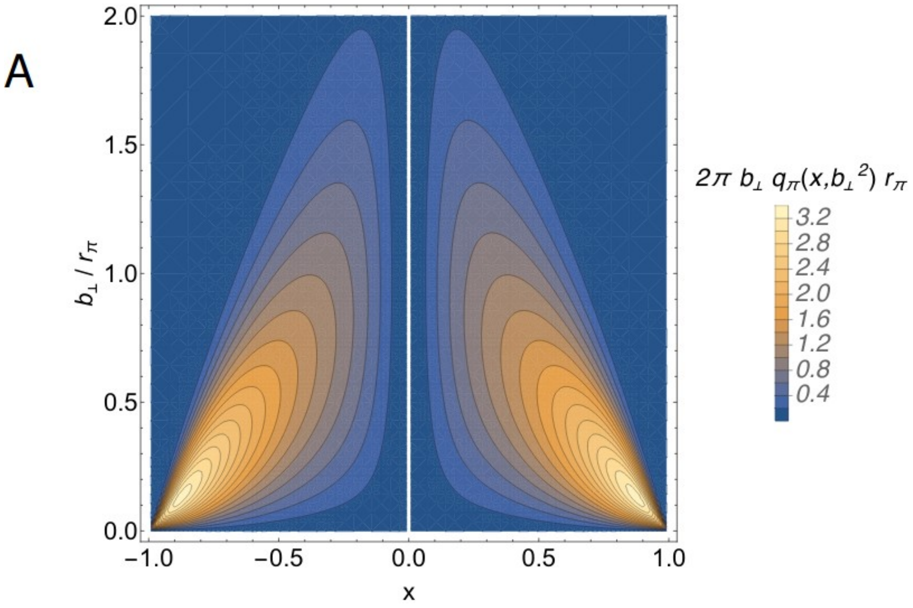
$$u^{\mathrm{P}}(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta)\, H_{\mathrm{P}}^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$

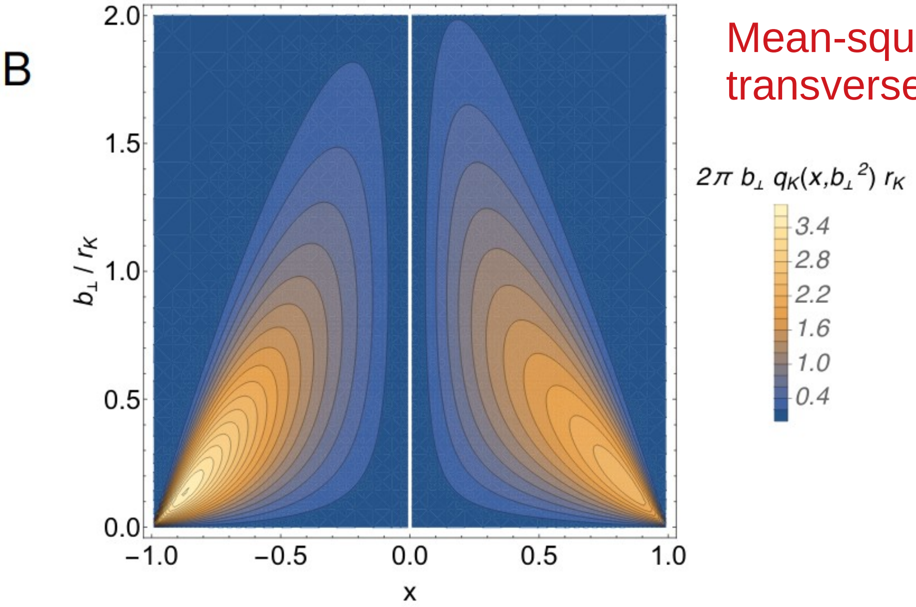
Algebraic derivation!

$$\langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_u^\pi = \boxed{\frac{2}{3}} r_\pi^2 = \langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_{\bar{d}}^\pi ,$$

$$\langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_u^K = 0.71 r_K^2 , \langle b_\perp^2(\zeta_{\mathcal{H}})\rangle_{\bar{s}}^K = 0.58 r_K^2 .$$

Mean-square transverse extent

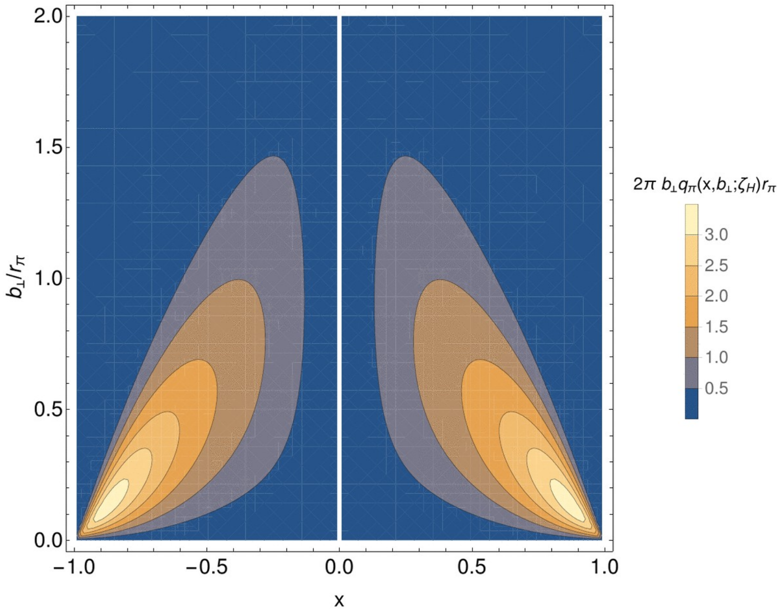




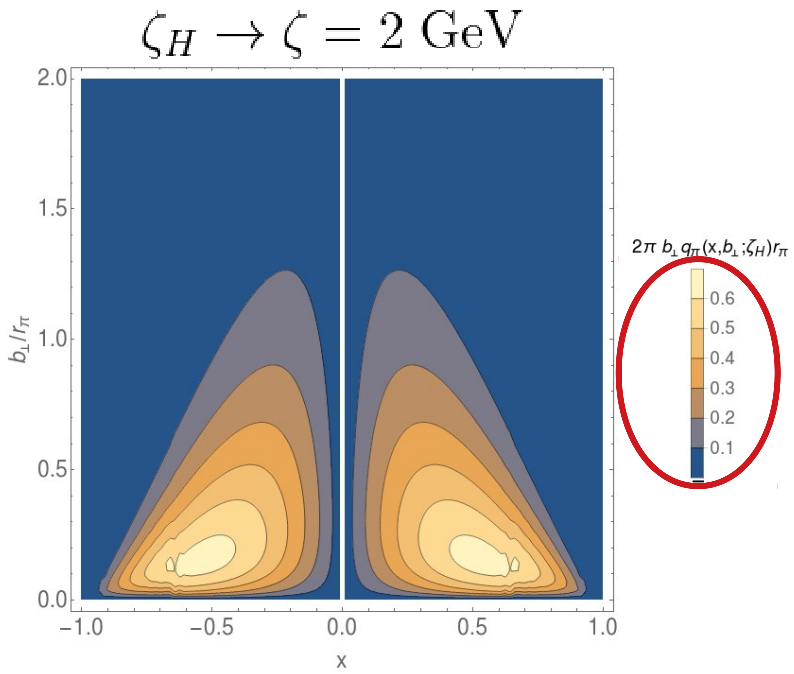
- Likelihood of finding a valence-**quark** with **momentum fraction x**, at **position b**.

Evolved IPS-GPD: Pion Case

$$u^{\mathsf{P}}(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta)\, H_{\mathsf{P}}^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$



- **Likelihood** of finding a parton with LF momentum **x** at transverse position **b**



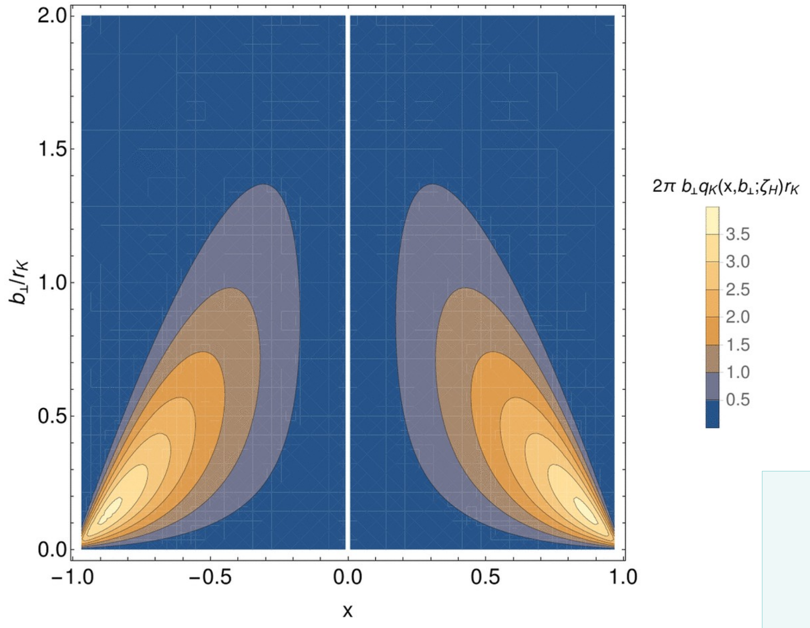
- Peaks **broaden** and **maximum drifts:**

$$\text{max}: \; 3.29 \to 0.55$$

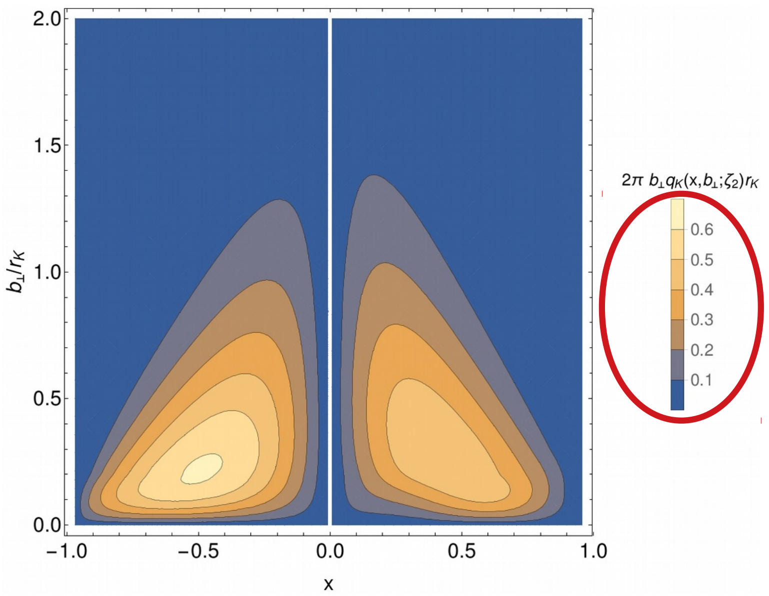
$$(|x|, b) = (0.88, 0.13) \to (0.47, 0.23)$$

Evolved IPS-GPD: Kaon Case

$$u^{\mathsf{P}}(x, b_\perp^2; \zeta_{\mathcal{H}}) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(b_\perp \Delta)\, H_{\mathsf{P}}^u(x, 0, -\Delta^2; \zeta_{\mathcal{H}})$$



$\zeta_H \to \zeta = 2\ \mathrm{GeV}$



$$\max_{(s,u)} : (3.61, 2.38) \to (0.61, 0.49)$$
$$(x, b)_u = (0.84, 0.17) \to (0.41, 0.28)$$
$$(x, b)_s = (-0.87, 0.13) \to (-0.48, 0.22)$$

- **Likelihood** of finding a parton with LF momentum **x** at transverse position **b**

Pressure distributions

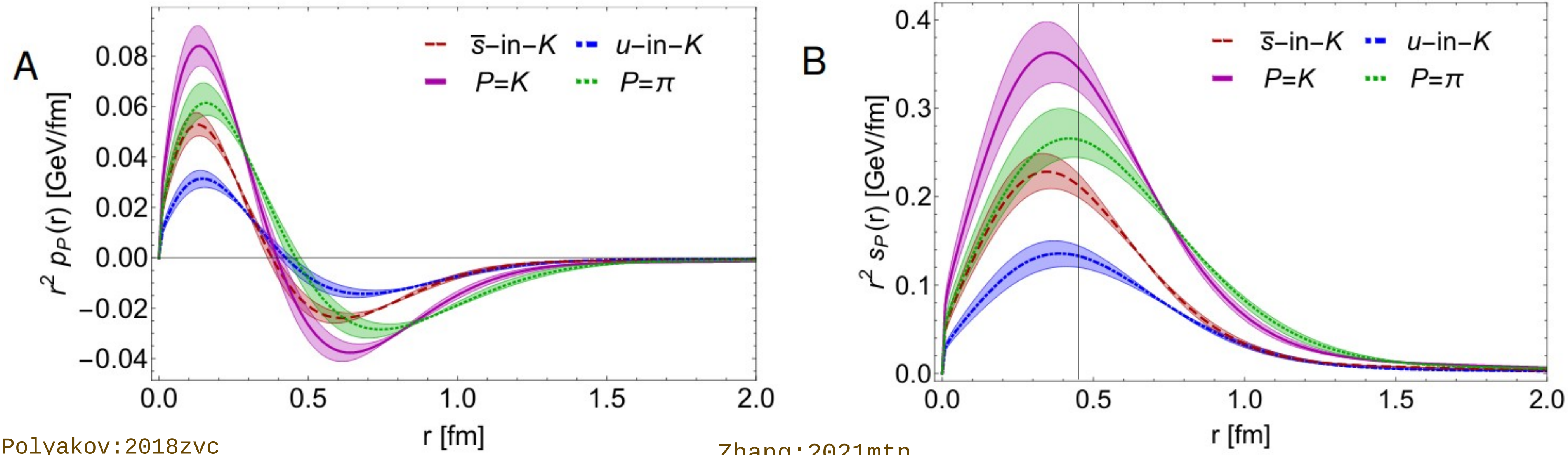
$$p_K^u(r) = \frac{1}{6\pi^2 r}\int_0^\infty d\Delta \, \frac{\Delta}{2E(\Delta)} \, \sin(\Delta r)[\Delta^2 \theta_1^{K_u}(\Delta^2)]\,,$$

$$s_K^u(r) = \frac{3}{8\pi^2}\int_0^\infty d\Delta \, \frac{\Delta^2}{2E(\Delta)} \, j_2(\Delta r)\,[\Delta^2 \theta_1^{K_u}(\Delta^2)]\,,$$

"Pressure" Quark attraction/repulsion

CONFINEMENT

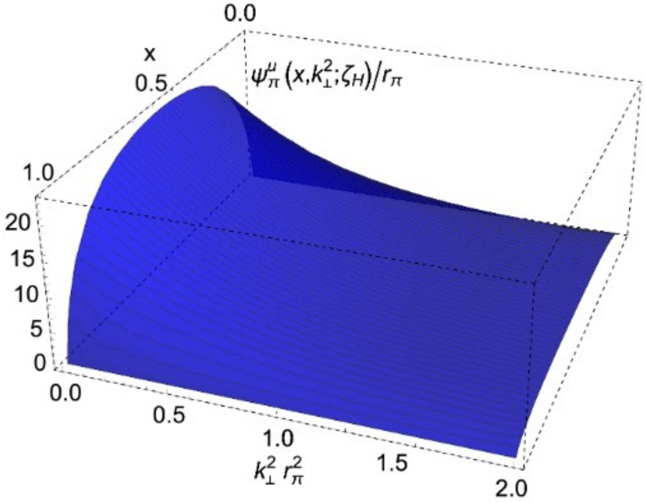
"Shear" *Deformation* QCD forces

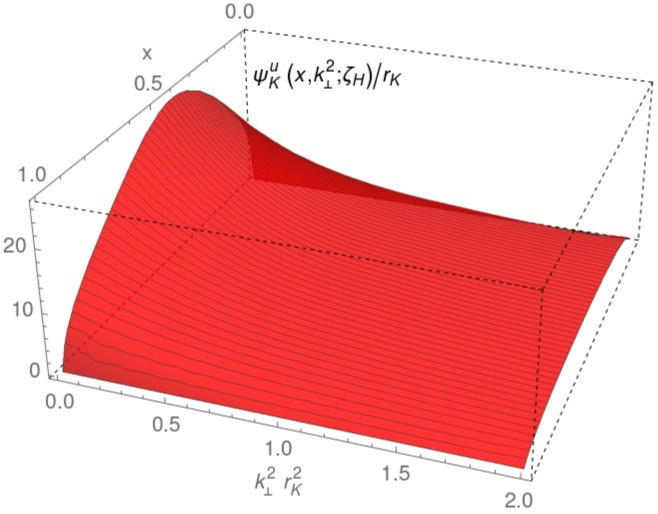


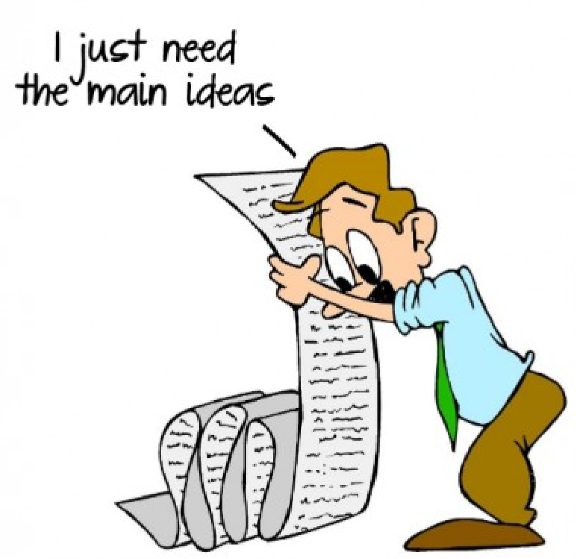
Polyakov:2018zvc

Zhang:2021mtn

Summary and Highlights







Summary

- Focusing on the **pion** and **Kaon**, we discussed a variety of **parton distributions**:

Light-front wave function
(encodes many properties)

↔ **Parton distribution amplitudes**
(momentum distribution)

← Inputs from **Continuum Schwinger Methods**
C. D. Roberts's talk

↓

Generalized parton distributions
(leads to many other objects)

→ **Impact parameter space GPD**
(spatial distributions)

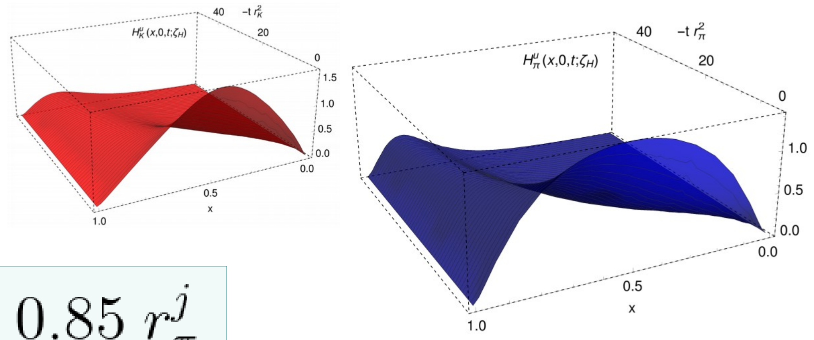
- ✓ Parton distribution functions
  (momentum distribution, valence-sea-gluon)
- ✓ Electromagnetic form factors
  (charge and momentum distributions)
- ✓ Gravitational form factors
  (mass distribution, mechanical properties)

The role of **evolution:**
Generate **sea** and **glue** from pure **valence**

→ **Pressure distributions**
(spatial distribution, mechanical properties)

Highlights

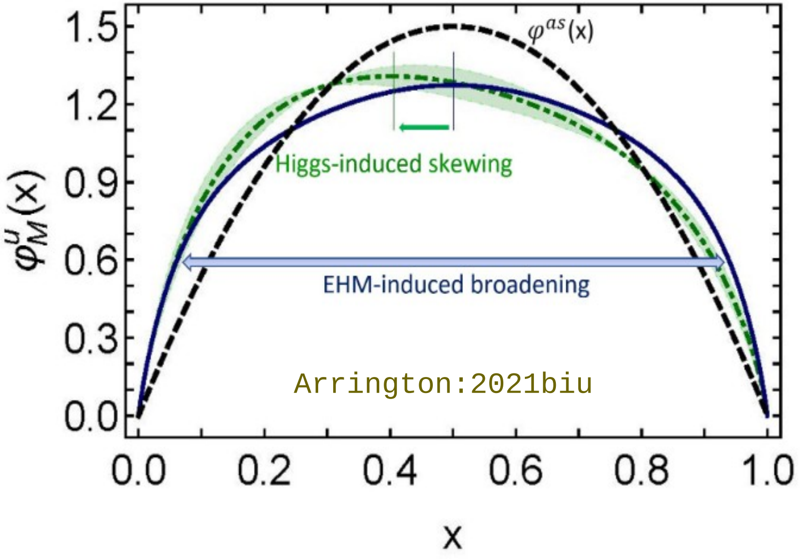
- **QCD's EHM** produce **broad π-K** distributions.
- **Interplay** between **QCD** and **Higgs** mass generation:
  - Slightly *skewed* Kaon distributions.



- The **ordering** of **radii**: $r_\pi^{\theta_1} > r_\pi^{E} > r_\pi^{\theta_2}$ $\quad r_K^j \approx 0.85\, r_\pi^j$
- **Gluon** and **sea** revealed through **evolution**.
  - **Definition** of ζ_H

Valence dressed quasiparticles

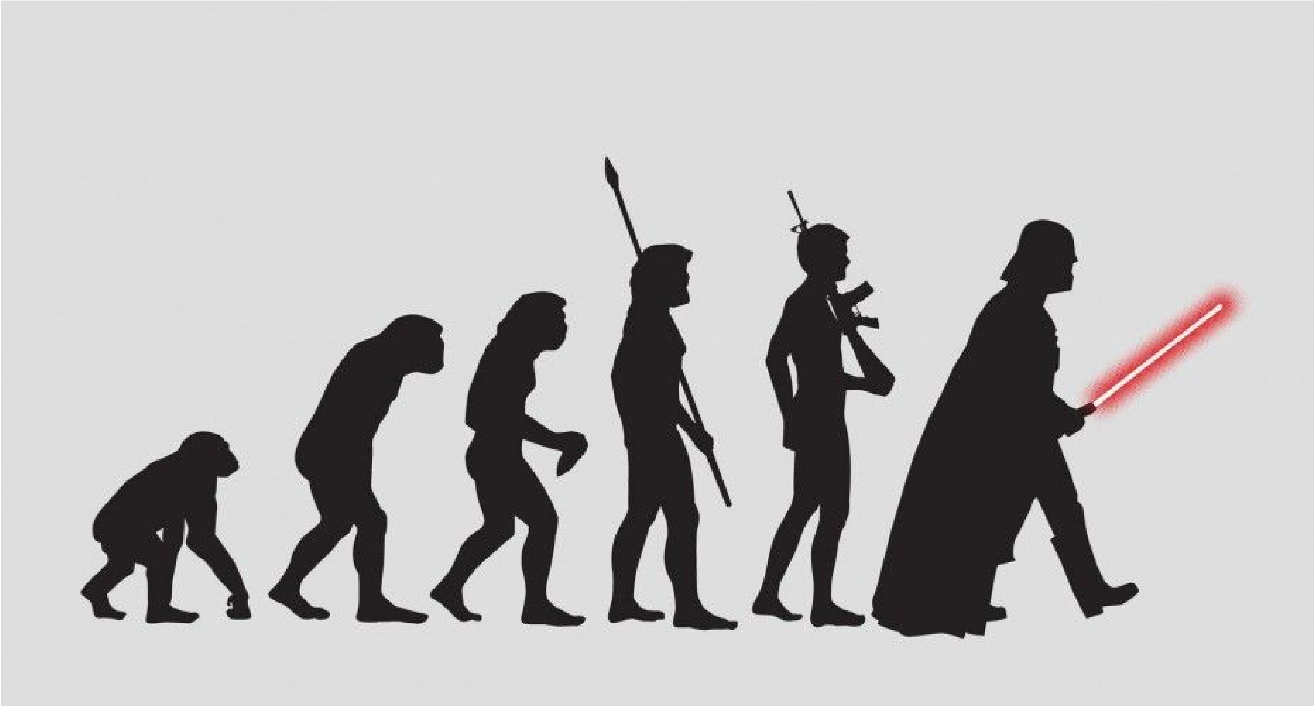
'**All orders**' scheme
- **QCD** effective charge.



> - Mass, gluon/sea, pressure, charge **distributions** addressed through **LFWFs** and **GPDs**
>
>   ... in anticipation to experiments in modern facilities


NEED BACKUP!
memegenerator.net

REGARDING EVOLUTION...



DGLAP + Effective Coupling

Idea. Define an **effective** coupling such that:

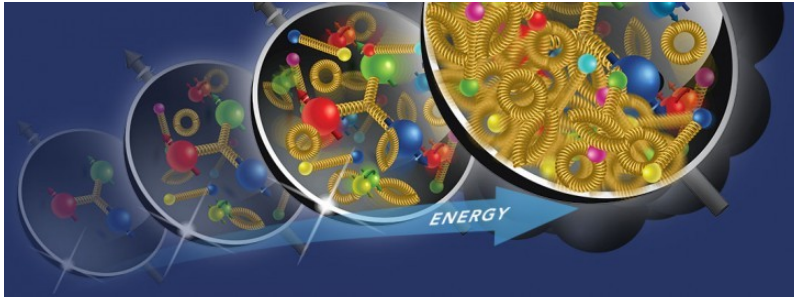
Starting from fully-dressed **quasiparticles**, at ζ_H

(at which **valence quarks** carry ***all*** meson's **properties**)

Sea and **Gluon** content unveils, as prescribed by **QCD**

$$\left\{\zeta^2\frac{d}{d\zeta^2}\int_0^1 dy\delta(y-x) - \boxed{\frac{\alpha(\zeta^2)}{4\pi}}\int_x^1 \frac{dy}{y}\begin{pmatrix} P_{qq}^{\rm NS}\left(\frac{x}{y}\right) & 0 \\ 0 & \mathbf{P}^{\rm S}\left(\frac{\mathbf{x}}{\mathbf{y}}\right)\end{pmatrix}\right\}\begin{pmatrix} H_\pi^{{\rm NS},+}(y,t;\zeta) \\ \mathbf{H}_\pi^{\rm S}(y,t;\zeta)\end{pmatrix} = 0$$

Exact equation → **"All orders hypothesis"**



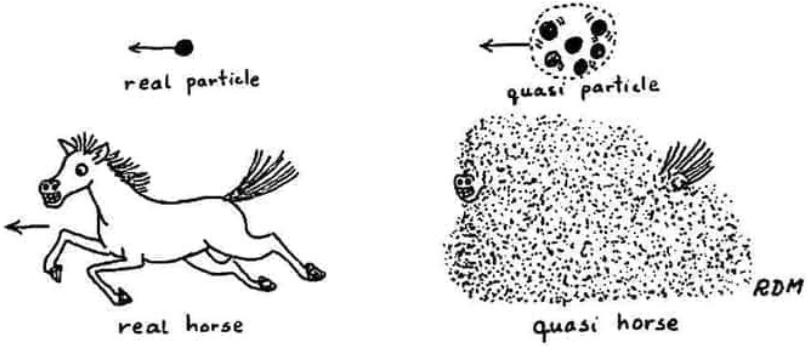


Fig. 0.4 *Quasi Particle Concept*

DGLAP + Effective Coupling

Idea. Define an **effective** coupling such that:

Starting from fully-dressed **quasiparticles**, at ζ_H (at which **valence quarks** carry ***all*** meson's **properties**) → **Sea** and **Gluon** content unveils, as prescribed by **QCD**

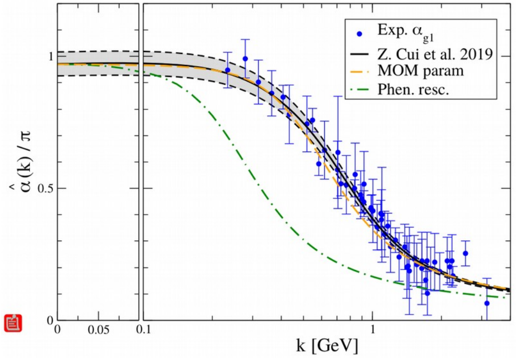
$$\left\{ \zeta^2 \frac{d}{d\zeta^2} \int_0^1 dy\,\delta(y-x) - \frac{\alpha(\zeta^2)}{4\pi} \int_x^1 \frac{dy}{y} \begin{pmatrix} P_{qq}^{\rm NS}\left(\frac{x}{y}\right) & 0 \\ 0 & \mathbf{P}^{\rm S}\left(\frac{\mathbf{x}}{\mathbf{y}}\right) \end{pmatrix} \right\} \begin{pmatrix} H_\pi^{{\rm NS},+}(y,t;\zeta) \\ \mathbf{H}_\pi^{\rm S}(y,t;\zeta) \end{pmatrix} = 0$$

"All orders hypothesis"

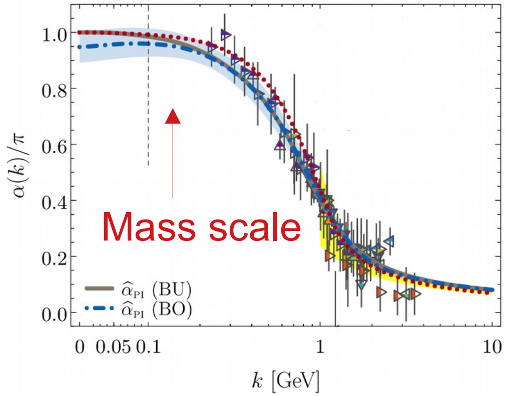
- Features of **QCD effective** charge lead to the **answer**.
- And ζ_H can be **properly defined**.

Not tuned!

Cui:2019dwv



Rodriguez-Quintero:2018wma



DGLAP + Effective Coupling

Cui:2020tdf

Idea. Define an effective coupling such that:

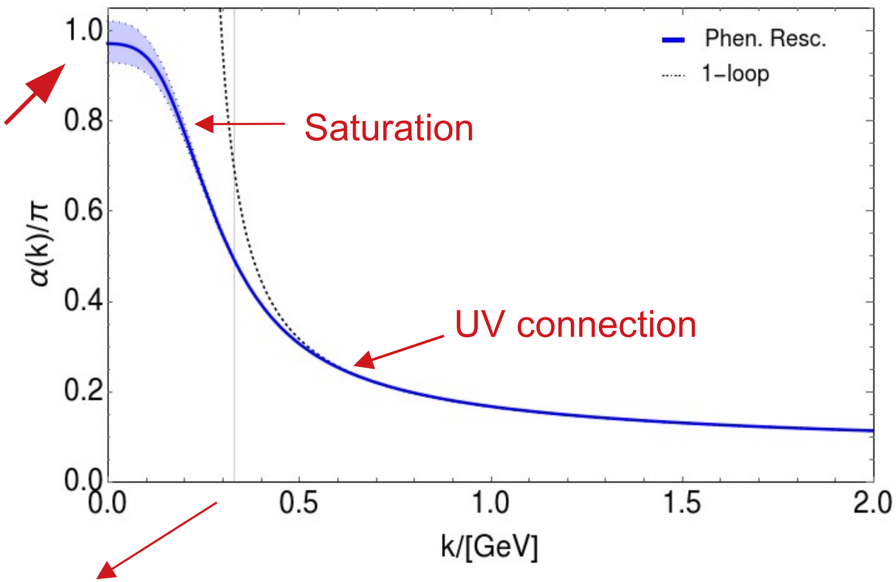
$$\frac{d}{dt}q(x;t) = -\frac{\alpha(t)}{4\pi}\int_x^1 \frac{dy}{y}q(y;t)P\left(\frac{x}{y}\right)$$

"All orders hypothesis"

- The ***coupling***:

$$\alpha(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{M}^2(k^2)}{\Lambda^2_{\rm QCD}}\right]} \; ; \; \frac{\alpha(0)}{\pi} = 0.97(4)$$

IR fixed point



→ Where $\mathcal{M}(k^2 = \Lambda^2_{\rm QCD}) := m_G = 0.331(2)\ \text{GeV}$ defines a screening mass.

- We identify: $\zeta_H := m_G(1 \pm 0.1)$ ⟸ 10% uncertainty

(fully dressed quasiparticles are the correct degrees of freedom)

All orders hypothesis

Implication 1: valence-quark PDF

$$\langle x^n(\zeta_f)\rangle_q = \exp\left(-\frac{\gamma_{qq}^{(n)}}{4\pi} S(\zeta_0,\zeta_f)\right)\langle x^n(\zeta_0)\rangle_q = \langle x^n(\zeta_H)\rangle_q \left(\frac{\langle x(\zeta_f)\rangle_q}{\langle x(\zeta_H)\rangle_q}\right)^{\gamma_{qq}^{(n)}/\gamma_{qq}^{(1)}}$$

$q = u, \bar{d}$

$$S(\zeta_0,\zeta_f) = \int_{2\ln(\zeta_0/\Lambda_{QCD})}^{2\ln(\zeta_f/\Lambda_{QCD})} dt\, \alpha(t)$$

This ratio encodes the information of the charge

Implication 2: glue and sea-quark DFs $(n_f = 4)$

Momentum sum rule:

$$\langle 2x(\zeta_f)\rangle_q + \langle x(\zeta_f)\rangle_{sea} + \langle x(\zeta_f)\rangle_g = 1$$

$$\langle 2x(\zeta_f)\rangle_q = \exp\left(-\frac{8}{9\pi} S(\zeta_H,\zeta_f)\right), \qquad q = u, \bar{d};$$

$$\langle x(\zeta_f)\rangle_{sea} = \langle x(\zeta_f)\rangle_{\sum_q q+\bar{q}} - (\langle x(\zeta_f)\rangle_u + \langle x(\zeta_f)\rangle_{\bar{d}}),$$

$$= \frac{3}{7} + \frac{4}{7}\langle 2x(\zeta_f)\rangle_u^{7/4} - \langle 2x(\zeta_f)\rangle_u$$

$$\langle x(\zeta_f)\rangle_g = \frac{4}{7}\left(1 - \langle 2x(\zeta_f)\rangle_u^{7/4}\right);$$

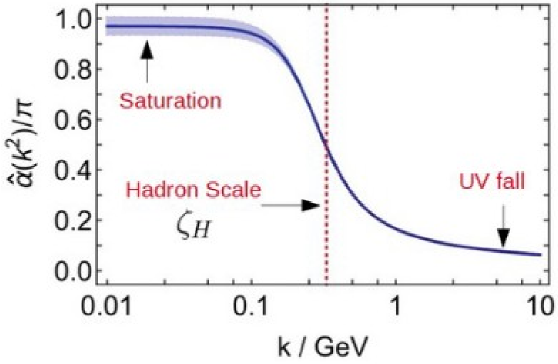
$\zeta_f/\zeta_H \to \infty$

A textbook result:
G. Altarelli, Phys. Rep. 81, 1 (1982)

J. Rodriguez-Quintero's slides (EHM - Amber 2021)

- Closed **algebraic** relations between momentum fractions
- **Recovery** of sum rule and asymptotic limits
- Clear connection with the **hadron scale**.

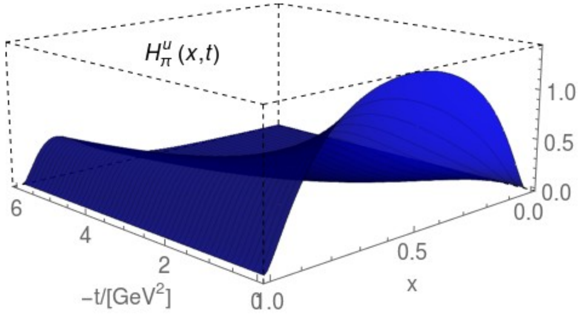
→ Therefore, the scale is **unambiguously** defined (**not tuned**)



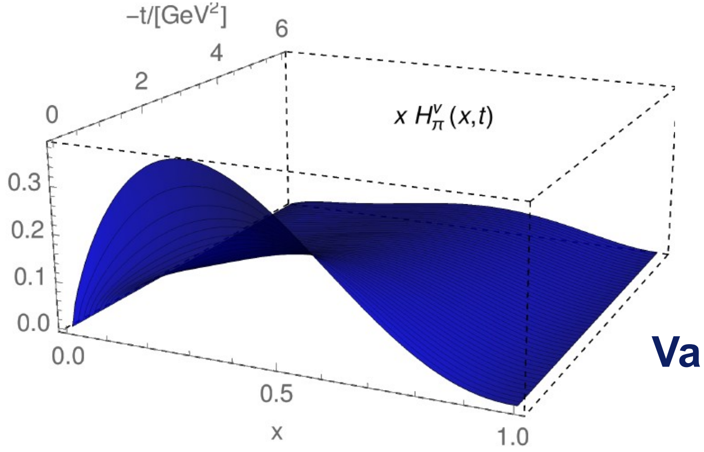
Evolved distributions: GPDs

$\zeta = 5.2\ \text{GeV}$

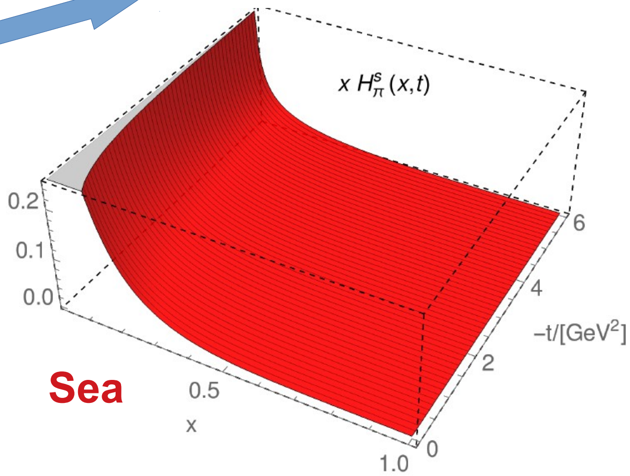
- Starting with **valence** distributions, at *hadron scale*, generate **gluon** and **sea** distributions via **all orders** evolution equations.



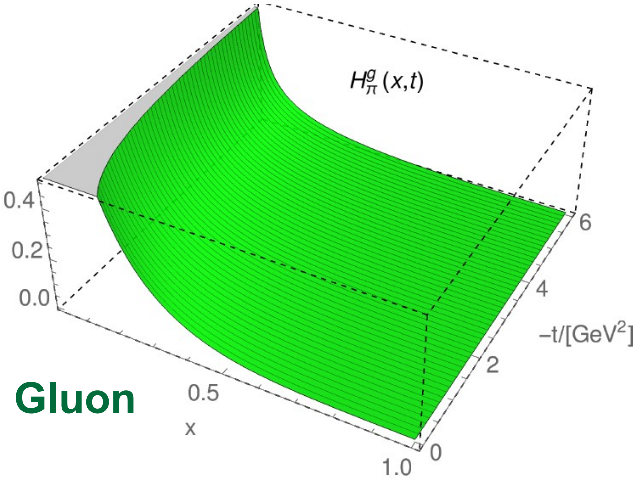
$\zeta_H = 0.331\ \text{GeV}$



Valence



Sea



Gluon

On the large-x behavior

On the large-x behavior

Assuming a theory in which the quarks interact via an exchange of a **vector boson,** asymptotically damped as: $\sim \left[\frac{1}{k^2}\right]^{\beta}$

- The **EFF**: $F(Q^2 \to \infty) \sim \left[\frac{1}{Q^2}\right]^{\beta}$
- The **PDF**: $q(x \to 1; \zeta_H) \sim (1-x)^{2\beta}$

$\forall \zeta > \zeta_H$

$x \simeq 1 \Rightarrow q^{\pi}(x;\zeta) \propto (1-x)^{2+\gamma}, \gamma > 0$

And we can actually know how the exponent evolves:

Kaon and pion parton distributions

Eur.Phys.J.C 80 (2020) 11, 1064 Cui:2020tdf

The way the **PDF** reaches (or not) such behavior is under **scrutiny**.

Courtoy:2020fex

Original **LO** analysis *seems* to **contradict QCD**. Conway:1989fs

Lattice QCD, **LFHQCD** and **DSEs** support the **ASV** analysis.

Chang:2020kjj Aicher:2010cb