GTMD Model Predictions for Diffractive Dijet Production at EIC [2106.15148]

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Outline

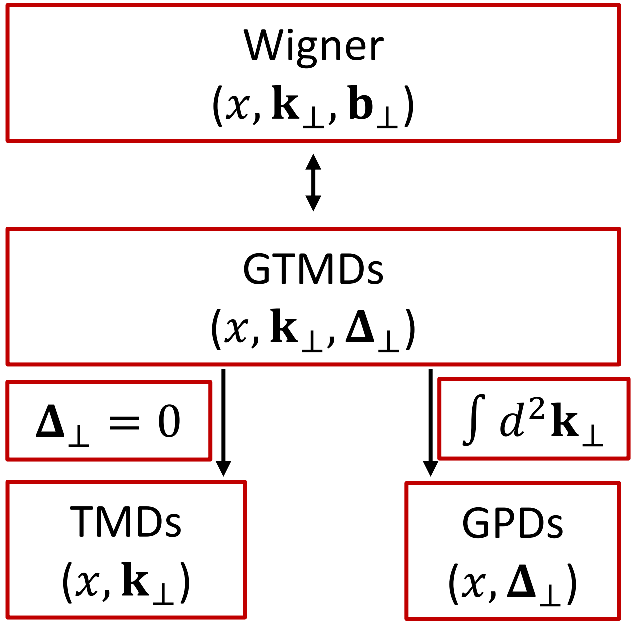
- ☐ Definitions of GTMDs
- ☐ Probing Gluon GTMDs
- ☐ Model for the unpolarized gluon GTMD at small x.
- ☐ Exclusive diffractive dijet production
- ☐ Model Fit to H1 Data
- ☐ Model Predictions for EIC
- ☐ Conclusions

Definitions of GTMDs

- Generalized Transverse Momentum Dependent parton distributions (GTMDs) are 5-dimensional parton distributions, a function of: [Ji 2003]; [Belitsky, Ji, Yuan, 2004]
    - parton’s longitudinal momentum fraction x (1-dimension),
    - parton's transverse momentum $\mathbf{k}_\perp$(2-dimensions), and
    - parton's transverse off-forwardness $\boldsymbol{\Delta}_\perp$(2-dimensions).

- Two definitions of GTMDs : [Soper, 1977]; [Diehl, 2002]; [Burkardt, 2016]
    1. The off-forward generalization of TMD.
    2. Transverse momentum dependent generalization of the impact parameter dependent GPD (the Fourier transform of the Wigner distribution).

- Accesssible in experiment, such as in diffractive dijet production DIS (gluon GTMDs). [Hatta, Xiao, Yuan, 2016]



Wigner $(x, \mathbf{k}_\perp, \mathbf{b}_\perp)$

↕

GTMDs $(x, \mathbf{k}_\perp, \boldsymbol{\Delta}_\perp)$

$\boldsymbol{\Delta}_\perp = 0$

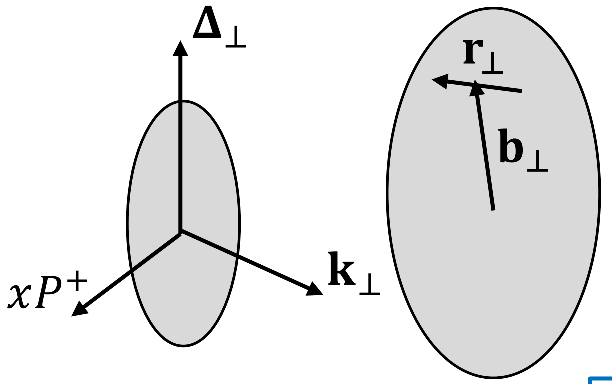
$\int d^2\mathbf{k}_\perp$

TMDs $(x, \mathbf{k}_\perp)$

GPDs $(x, \boldsymbol{\Delta}_\perp)$

Dipole size ← $\mathbf{r}_\perp \leftrightarrow \mathbf{k}_\perp$

Impact parameter ← $\mathbf{b}_\perp \leftrightarrow \boldsymbol{\Delta}_\perp$



Probing gluon GTMDs

- ☐ Impact parameter dependent GPD

Gauge link/Wilson lines

$$q(x,\boldsymbol{b}_\perp) = \int \frac{d\lambda}{2\pi P^+} e^{i\lambda x} \langle P^+, \boldsymbol{R}_\perp = 0|\, \overline{\psi}(-\frac{\lambda}{2}n + \boldsymbol{b}_\perp)\, \gamma^+ \,\mathcal{L}\, \psi(\frac{\lambda}{2}n + \boldsymbol{b}_\perp)\, |P^+, \boldsymbol{R}_\perp = 0\rangle$$

- ☐ Wigner distribution

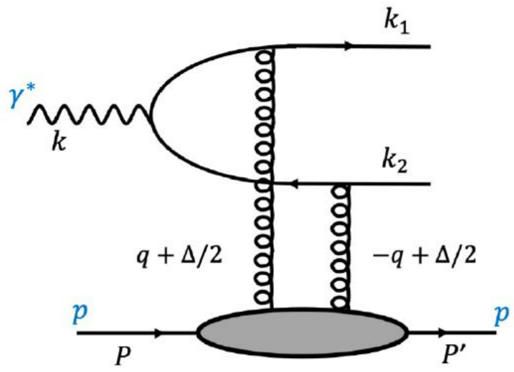
$$W(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp) \equiv \int \frac{d\lambda}{2\pi P^+} d^2\boldsymbol{r}_\perp\, e^{i\lambda x} e^{i\boldsymbol{k}_\perp\cdot \boldsymbol{r}_\perp} \langle P^+, \boldsymbol{R}_\perp = 0|\, \overline{\psi}(-\frac{\lambda}{2}n + \boldsymbol{b}_\perp - \frac{\boldsymbol{r}_\perp}{2})\, \gamma^+ \,\mathcal{L}\, \psi(\frac{\lambda}{2}n + \boldsymbol{b}_\perp + \frac{\boldsymbol{r}_\perp}{2})\, |P^+, \boldsymbol{R}_\perp = 0\rangle$$

- ☐ Define GTMDs

$$q_W(x,\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp) \equiv \int \frac{d^2\boldsymbol{b}_\perp}{(2\pi)^2} e^{i\boldsymbol{b}_\perp\cdot\boldsymbol{\Delta}_\perp} W(x,\boldsymbol{k}_\perp,\boldsymbol{b}_\perp)$$

$$G^{[+,-]}(x,\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp) = \frac{2}{P^+}\int \frac{dz^-}{2\pi}\frac{d^2\boldsymbol{z}_\perp}{(2\pi)^2} e^{ik\cdot z} \langle P'|\, \mathrm{Tr}\left[F^{+i}\left(-\frac{z}{2}\right) U^{[+]} F^{+i}\left(\frac{z}{2}\right) U^{[-]}\right]\, |P\rangle\Big|_{z^+=0}$$

- ☐ Small x limit: diffractive dijet production in $ep(A)$ collisions probes gluon GTMDs



$$G^{[+,-]}(\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp) = \frac{1}{2\pi g^2}\left[\boldsymbol{k}_\perp^2 - \frac{\boldsymbol{\Delta}_\perp^2}{4}\right] G^{[\Box]}(\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp)$$

$$G^{[\Box]}(\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp) = \frac{4N_c}{\langle P|P\rangle}\int \frac{d^2\boldsymbol{r}_\perp d^2\boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \langle P'|\, \underbrace{S^{[\Box]}(\boldsymbol{b}_\perp - \frac{\boldsymbol{r}_\perp}{2}, \boldsymbol{b}_\perp + \frac{\boldsymbol{r}_\perp}{2})}_{\text{Dipole scattering amplitude}}\, |P\rangle$$

[Dominguez-Marquet-Xiao-Yuan, 2011]
[Boer, van Daal, Mulders, Petreska, 2018]

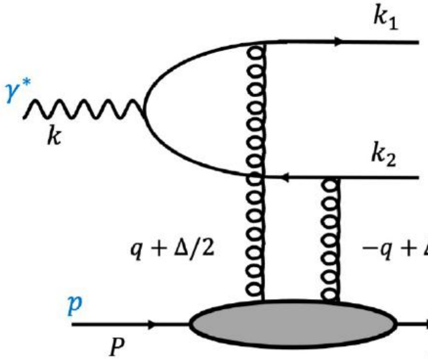
Model for the unpolarized gluon GTMD at small x

- ☐ Dipole scattering amplitude

$$\boxed{S^{[\Box]}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)} \rightarrow \boxed{1 - S^{[\Box]}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)} \Rightarrow \boxed{\mathcal{F}^{[\Box]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = \frac{4N_c}{\langle P|P\rangle} \int \frac{d^2\boldsymbol{r}_\perp d^2\boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} \langle P'|\, 1 - S^{[\Box]}(\boldsymbol{b}_\perp - \frac{\boldsymbol{r}_\perp}{2}, \boldsymbol{b}_\perp + \frac{\boldsymbol{r}_\perp}{2})\, |P\rangle}$$

$G^{[\Box]}$ $\qquad$ $\mathcal{F}^{[\Box]}$

- ☐ Impact parameter dependent McLerran-Venugopalan (*ep* collisions)



$$\boxed{\mathcal{F}^{[\Box]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = 4N_c \int \frac{d^2\boldsymbol{r}_\perp d^2\boldsymbol{b}_\perp}{(2\pi)^2} e^{-i\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp} e^{i\boldsymbol{\Delta}_\perp\cdot\boldsymbol{b}_\perp} e^{-\epsilon_r r_\perp^2} \left[1 - \exp\left(-\frac{1}{4} r_\perp^2 \chi Q_s^2(b_\perp) \ln\left[\frac{1}{r_\perp^2\Lambda^2} + e\right]\right)\right]}$$

$$\boxed{Q_s^2(b_\perp) = \frac{4\pi\alpha_s^2 C_F}{N_c} T_p(b_\perp)}$$ ⇒ Saturation scale

$$\boxed{T_p(b_\perp) = \exp\left(-b_\perp^2/(2R_p^2)\right)}$$ ⇒ Proton profile

[Salazar, Schenke, 2019]

free parameter

[Hagiwara, Hatta, Ueda, 2016]
[Pelicer, De Oliveira, Pasechnik, 2019]

free parameter

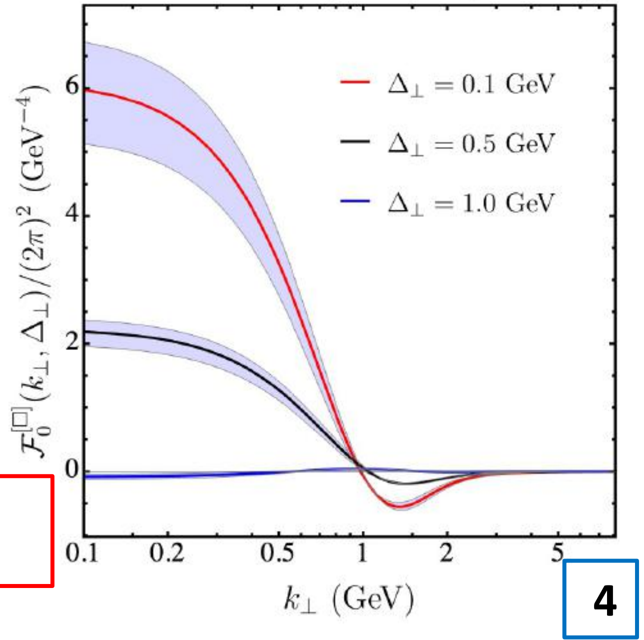
- ☐ Expansion of $\mathcal{F}^{[\Box]}$

$$\mathcal{F}^{[\Box]}(\boldsymbol{k}_\perp, \boldsymbol{\Delta}_\perp) = \underbrace{\mathcal{F}_0^{[\Box]}(k_\perp, \Delta_\perp)}_{\text{Angular independent}} + 2\underbrace{\mathcal{F}_2^{[\Box]}(k_\perp, \Delta_\perp)}_{\text{Elliptic, small w.r.t } \mathcal{F}_0} \cos 2\theta_{k\Delta} + \ldots,$$

[Hagiwara, Hatta, Ueda, 2016]
[Salazar, Schenke, 2019]
[Mäntysaari, Mueller, Schenke, 2019]

$$K = \frac{k_1 - k_2}{2}$$

$$\Delta = k_1 + k_2$$


$\Lambda = 0.24$ GeV; $\epsilon_r = (0.5\ \text{fm})^{-2}$; $\chi = 1.25^{+0.25}_{-0.25}$

Small contribution from
$\boldsymbol{\Delta}_\perp > 1$ GeV

Exclusive diffractive dijet production

- ☐ Relate: $\gamma^* p$ to ep (diffractive dijet in DIS)

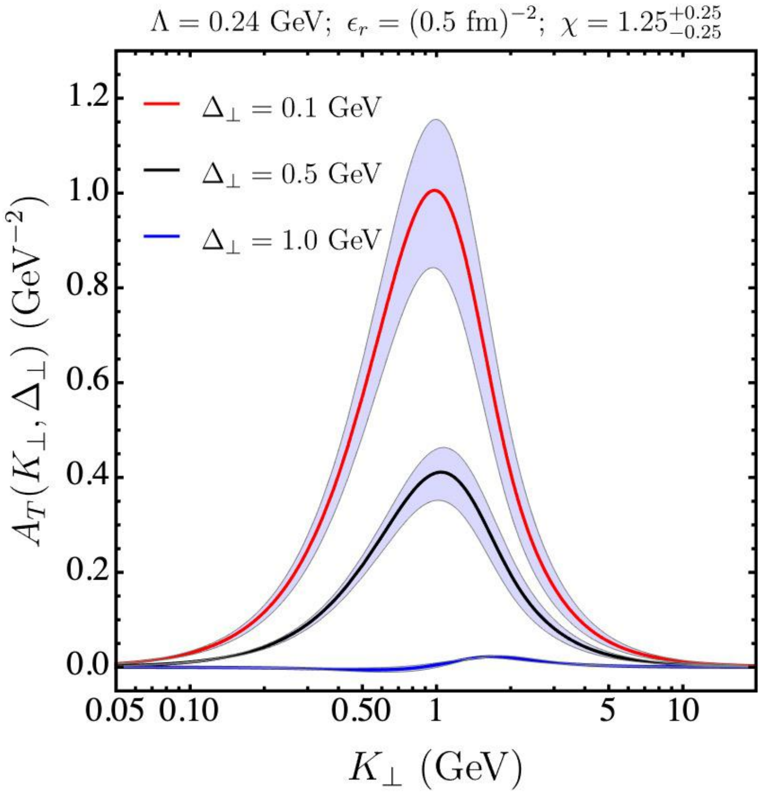
$$\boxed{\frac{d\sigma^{ep}}{dx\ dQ^2} = \frac{\alpha_{em}}{\pi x Q^2}\left[\left(1-y+\frac{y^2}{2}\right)\sigma_T^{\gamma^* p} + (1-y)\,\sigma_L^{\gamma^* p}\right]}$$

- ☐ **Photoproduction**: $Q^2 = 0$

$$\boxed{\frac{d\sigma^{\gamma p}}{dz_1 dz_2 dK_\perp d\Delta_\perp^2} = \frac{(2\pi)^4\alpha_{em}}{16N_c}\sum_f e_f^2\ \delta(z_1+z_2-1)\left[z_1^2+z_2^2\right]\frac{A_T^2(K_\perp,\Delta_\perp)}{K_\perp}.}$$

$$\boxed{A_T(K_\perp,\Delta_\perp) = \frac{1}{(2\pi)^2}\int_0^{K_\perp} dq_\perp\, q_\perp\, \mathcal{F}_0^{[\Box]}(q_\perp,\Delta_\perp)}$$

[Hagiwara,Hatta, Pasechnik, Tasevsky, Teryaev, 2017]
[Pelicer, De Oliveira, Pasechnik, 2019]
with different convention

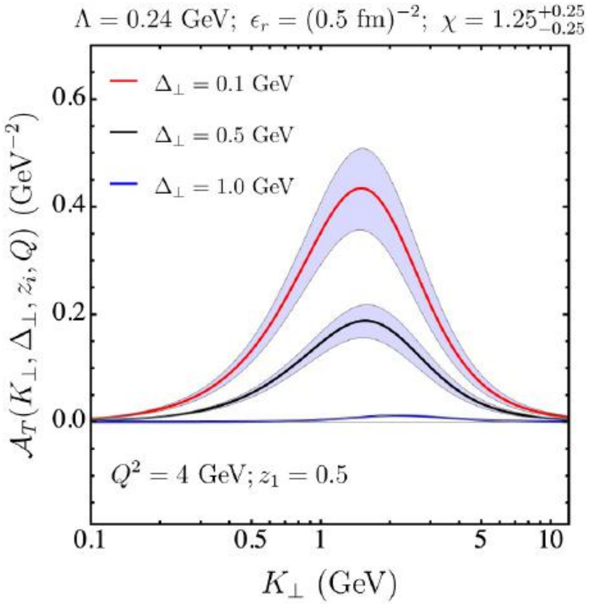


Exclusive diffractive dijet production

- **Electroproduction**: $Q^2 > 0$
  - ○ Transverse

$$\frac{d\sigma_T^{\gamma^* p}}{dz_1 dz_2 dK_\perp d\Delta_\perp^2} = \frac{(2\pi)^4 \alpha_{em}}{16 N_c} \sum_f e_f^2\, \delta(z_1 + z_2 - 1)\left[z_1^2 + z_2^2\right] \frac{\mathcal{A}_{\rm T}^2(K_\perp, \Delta_\perp, z_i, Q)}{K_\perp}$$

$$\mathcal{A}_{\rm T}(K_\perp, \Delta_\perp, z_i, Q) = \frac{1}{(2\pi)^2}\int_0^\infty dq_\perp \frac{q_\perp\, \mathcal{F}_0^{[\square]}(q_\perp, \Delta_\perp)}{2}\left[1 + \frac{K_\perp^2 - q_\perp^2 - z_1 z_2 Q^2}{\sqrt{(K_\perp^2 + q_\perp^2 + z_1 z_2 Q^2)^2 - (2K_\perp q_\perp)^2}}\right]$$

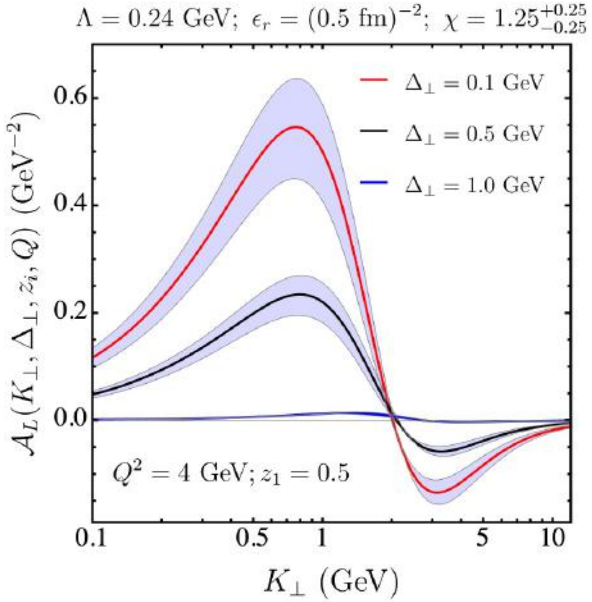


  - ○ Longitudinal

small

$$\frac{d\sigma_L^{\gamma^* p}}{dz_1 dz_2 dK_\perp d\Delta_\perp^2} = \frac{(2\pi)^4 \alpha_{em}}{4 N_c} \sum_f e_f^2\, \delta(z_1 + z_2 - 1)\, z_1^2 z_2^2 \frac{\mathcal{A}_{\rm L}^2(K_\perp, \Delta_\perp, z_i, Q)}{K_\perp}$$

$$\mathcal{A}_{\rm L}(K_\perp, \Delta_\perp, z_i, Q) = \frac{1}{(2\pi)^2}\int_0^\infty dq_\perp\, q_\perp\, \mathcal{F}_0^{[\square]}(q_\perp, \Delta_\perp)\left[\frac{K_\perp Q}{\sqrt{(K_\perp^2 + q_\perp^2 + z_1 z_2 Q^2)^2 - (2K_\perp q_\perp)^2}}\right]$$

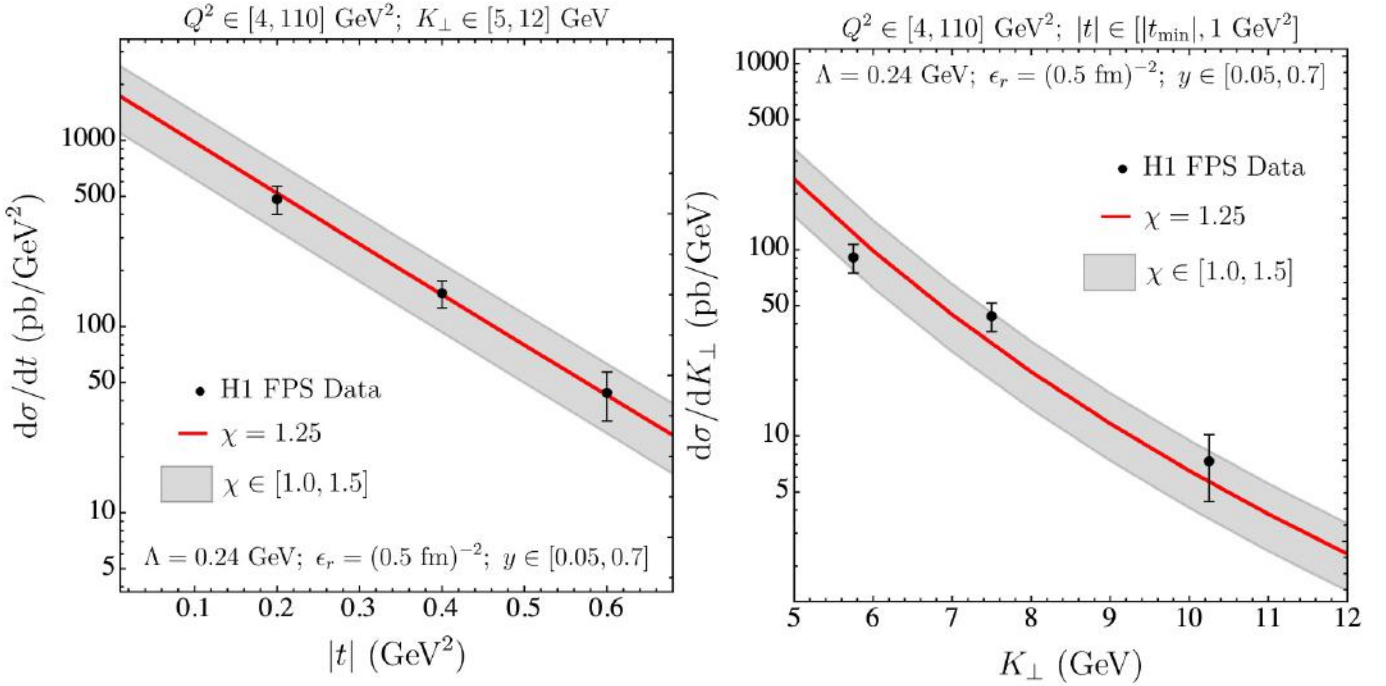


Model Fit to H1 Data

- ☐ Fit to H1 data for the production of two central jets [Aaron *et al*, 2012].
- ☐ Fairly good agreement with the data for t and $K_\perp$-dependence simultaneously.

- ☐ ***t*-dependence** :
  - ○ The slope is controlled by the proton profile e^{-bt} with $b\sim 6\ \mathrm{GeV}^{-2}$.
  - ○ Central value: $\chi = 1.25, \epsilon_r = (0.5\ \mathrm{fm})^{-2}$.

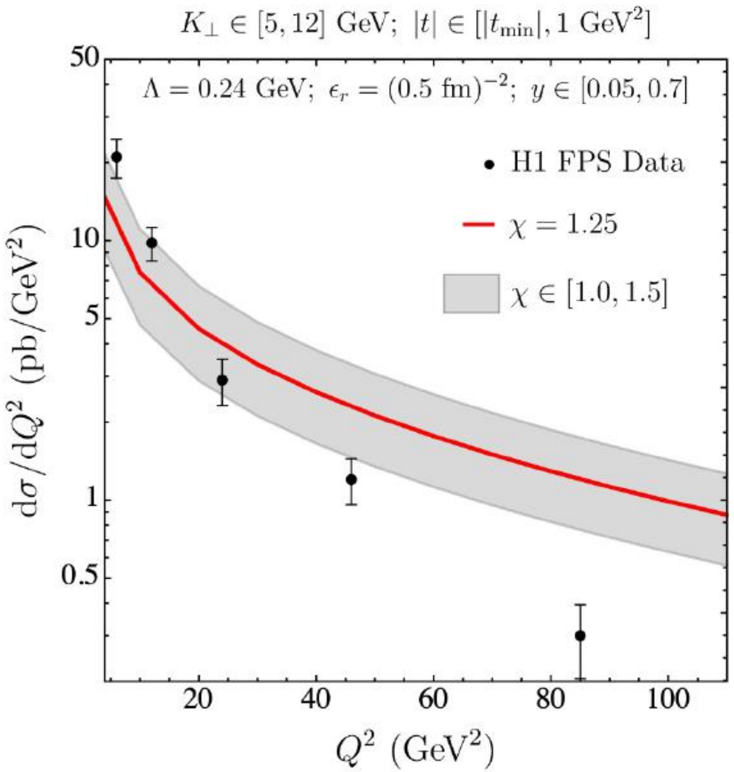
- ☐ **$K_\perp$-dependence :**
  - ○ Expect $q_\perp \ll\ K_\perp$ for large $K_\perp$.
  - ○ Approximate $K_\perp \approx k_{1_\perp}$.
  - ○ Small contribution from longitudinal part: $\sim$10% for $K_\perp$=12 GeV, becomes smaller for small $K_\perp$.

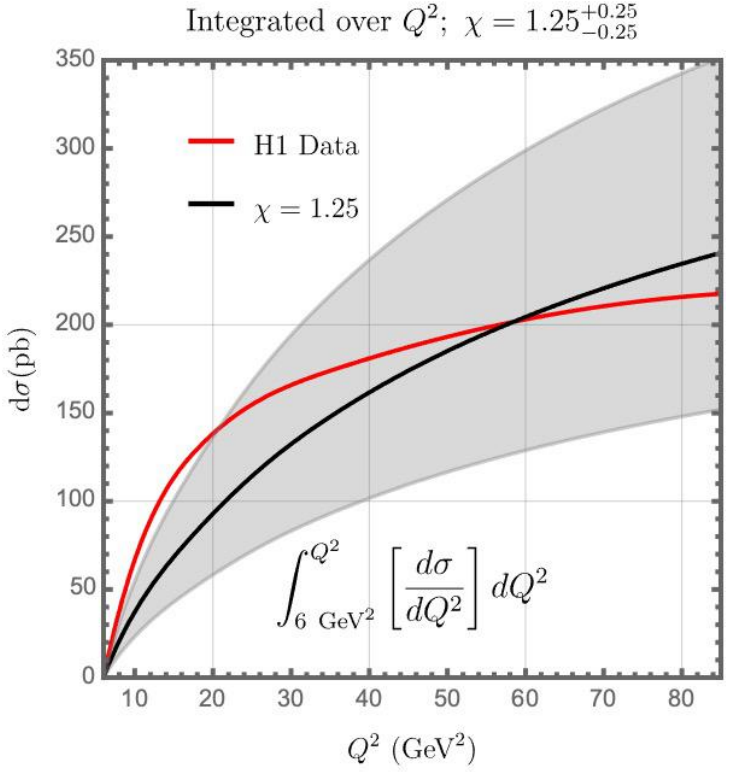


Lower-band : $\chi = 1.0$
Upper-band : $\chi = 1.5$
$t_{\min} \sim 0\ \mathrm{GeV}^2$

- H1 data: at least two jets.
- Assumption: the dijet contribution dominates and the corrections are α_s suppressed.

Model Fit to H1 Data





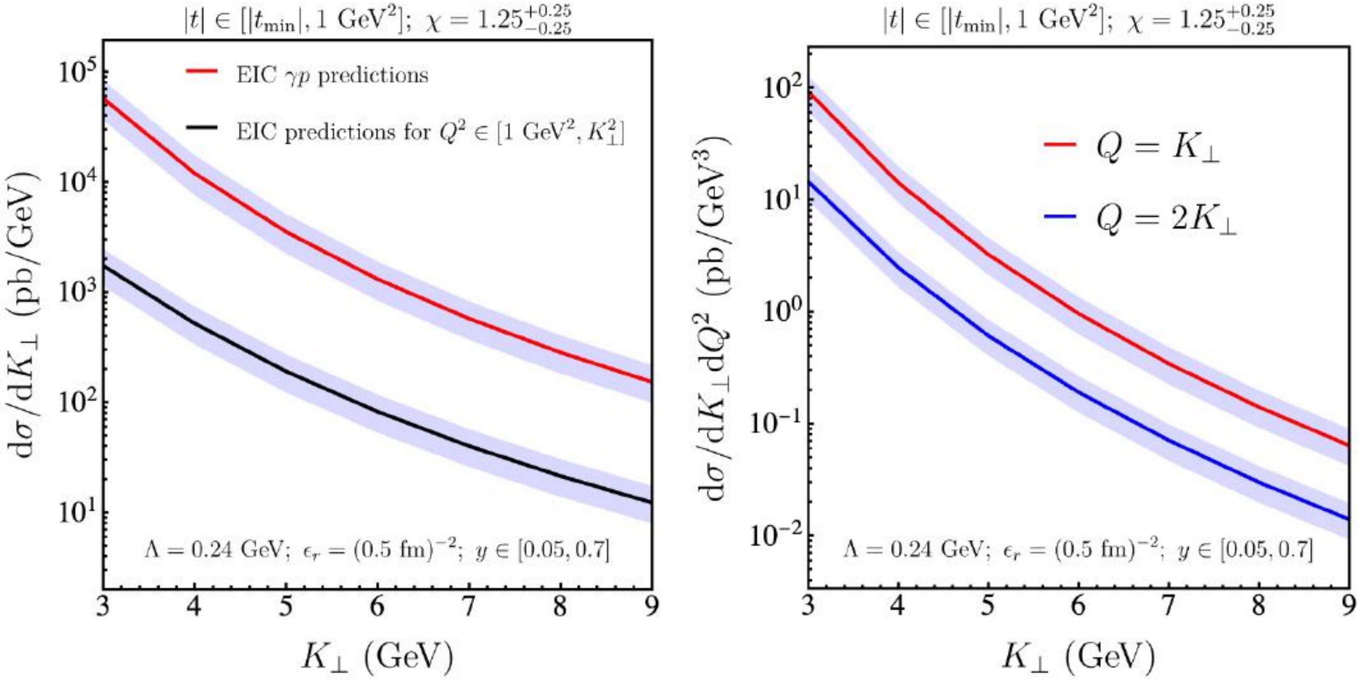
- ❑ Q^2**-dependence** :
  - ○ [Left figure] Expect $\chi > 1.25$ for small Q^2 (smaller x), and $\chi < 1.25$ for large Q^2 (larger x).
  - ○ [Right figure] The central values of the H1 data as function of Q^2 fall within the range of the Q^2 integral of the model.

- ❑ GBW, saturation scale corresponding x values: $Q_s^2(x) = A^{\frac{1}{3}}\left(\frac{3.10^{-4}}{x}\right)^{0.3}$ [Golec-Biernat, Wüsthoff, 1998]
- ❑ $Q_s^2(x) \to \chi Q_s^2(x)$, the free parameter associated with the saturation scale:

$$\chi = 2\left(\frac{3.10^{-4}}{x}\right)^{0.3} \to \chi = 1.25 \pm 0.25 \text{ corresponds to } x \sim (1-3)10^{-3}.$$

Model Predictions for EIC

- ❑ **[Left figure]**
  - ○ [Red line: photoproduction] γp predictions: $Q^2 = 0$ GeV.
  - ○ [Black line: leptoproduction] $Q^2 \in [1\ GeV^2, K_\perp^2]$ predictions
- ❑ **[Right figure]** $\frac{d\sigma}{dK_\perp\, dQ^2}$ for Q depend on $K_\perp$
  - ○ Smaller cross section for large Q^2 .
  - ○ Expect $\chi > 1.25$ for small Q^2, and $\chi < 1.25$ for large Q^2.



- ❑ Center of mass energy of EIC lower than HERA: select $3 \leq K_\perp \leq 9$ GeV.
- ❑ Fixed flavor case $N_f = 4$.
- ❑ Focus on small Δ ($t \in [t_{\min}, 1\ \mathrm{GeV}^2]$) as in HERA.

Conclusions

- The small-x (MV-like) gluon GTMD model can describe the $\Delta_{\perp}$ and $\mathrm{K}_{\perp}$ dependence of the diffractive dijet production H1 data fairly well and simultaneously.
- The Q^2 dependence requires consideration of an x dependent model.
- This GTMD description can be further tested at the EIC.

Thanks …