Lattice calculations of hadron structure in the pseudo-PDF framework, and their impact in resolving the internal structure of mesons and baryons

David Richards (Jefferson Lab)

for HadStruc Collaboration

2nd PSQ@EIC Meeting

HadStruc Collaboration

Robert Edwards, Jianwei Qiu, David Richards, Eloy Romero, Frank Winter, Nikhil Karthik

Jefferson Lab

Carl Carlson, Colin Egerer, Christos Kallidonis,
Tanjib Khan, Christopher Monahan, Kostas Orginos, Raza Sufian

College of William and Mary

Wayne Morris, Anatoly Radyushkin

Old Dominion University

Joe Karpie

Columbia University

Savvas Zafeiropoulos

Aix Marseille Univ, Marseille, France

Yan-Qing Ma

Peking University, Beijing, China

Balint Joo

ORNL

Hadron Structure: No-go Theorem?

- First Challenge:
  - Euclidean lattice precludes calculation of light-cone/time-separated correlation functions

PDFs, GPDs, TMDs

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P \mid \bar{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)}\psi(0) \mid P\rangle$$

So…. …Use *Operator-Product-Expansion to formulate in terms of Mellin Moments* with respect to Bjorken x.

$$\longrightarrow \quad \langle P \mid \bar{\psi}\gamma_{\mu_1}(\gamma_5)D_{\mu_2}\dots D_{\mu_n}\psi \mid P\rangle \to P_{\mu_1}\dots P_{\mu_n}a^{(n)}$$

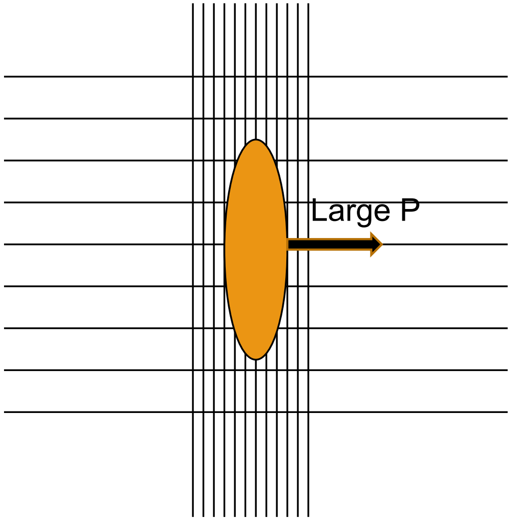
- Second Challenge:
  - Discretised lattice: power-divergent mixing for higher moments

Moment Methods

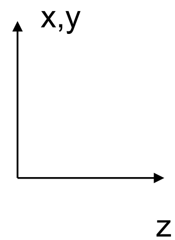
Recent work by ETMC

  - Extended operators: Z.Davoudi and M. Savage, PRD 86,054505 (2012)
  - Valence heavy quark: W.Detmold and W.Lin, PRD73, 014501 (2006)

PDFs from Euclidean Lattice



Large-Momentum Effective Theory (LaMET)



"Equal time" correlator

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

$$q(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz'\, A^z(z')}\psi(0) \mid P > + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

$$\Big\downarrow$$

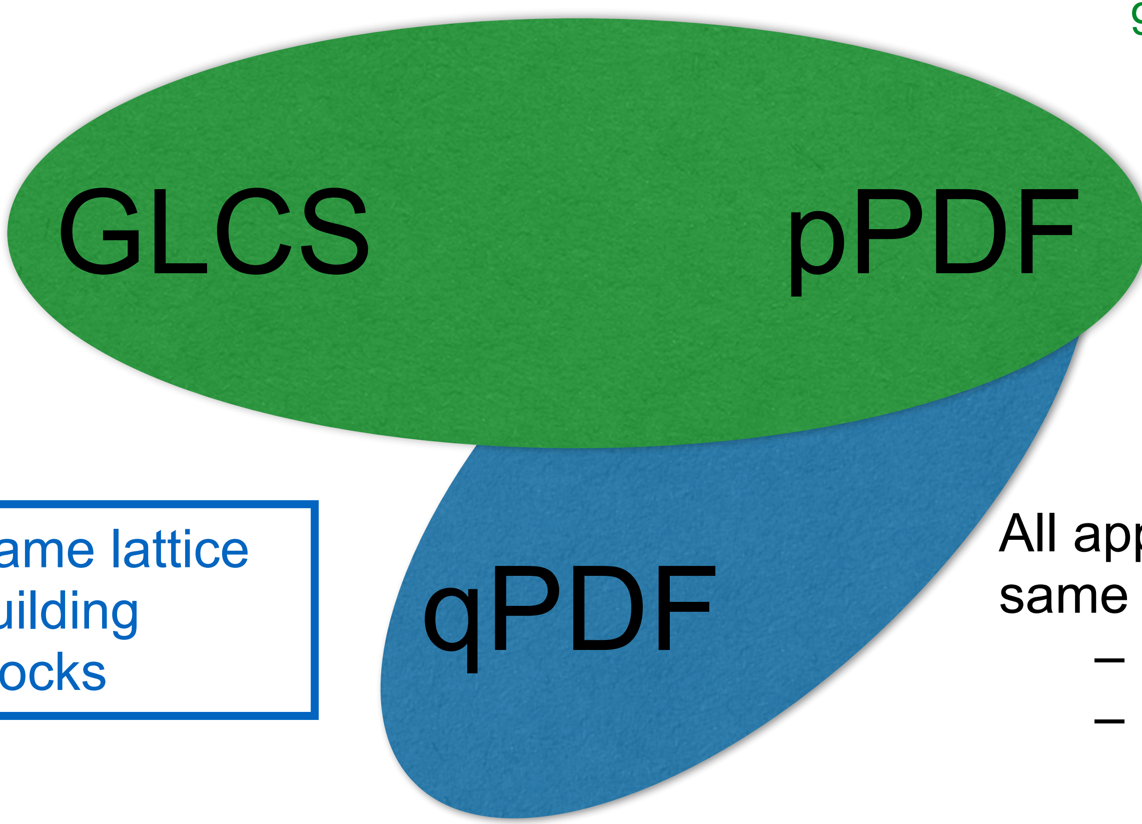
$$q(x,\mu^2,P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

PDFs, GPDs and TMDs

Ma and Qiu, Phys. Rev. Lett. 120 022003

A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

Light cone reduces to a point



Characterized by *short-distance factorization*

Same lattice building blocks

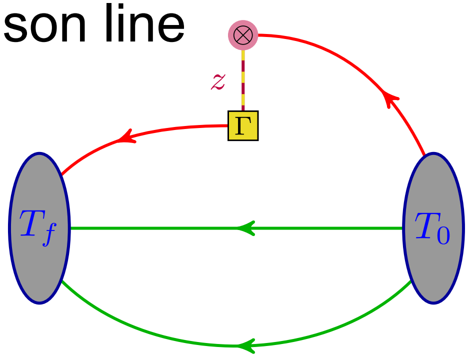
All approaches should give same after:
- Finite volume
- Discretization Uncertainies
- *Infinite momentum*

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

Nucleon iso-vector PDFs

Pseudo-PDFs

Wilson line



A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time.* $\nu = p \cdot z$

B.Ioffe, PL39B, 123 (1969); V.Braun *et al*, PRD51, 6036 (1995)

$$M^\alpha(p,z) = \langle p \mid \bar{\psi}\gamma^\alpha U(z;0)\psi(0) \mid p\rangle$$

$p = (p^+, m^2/2p^+, 0_T)$ $\qquad$ $z = (0, z_-, 0_T)$

$$\Downarrow$$

$$M^\alpha(z,p) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

Ioffe-time pseudo-Distribution (pseudo-ITD) generalization to *space-like z*

Lattice "building blocks" that of quasi-PDF approach.

Pseudo-PDFs

To deal with UV divergences, introduce reduced distribution

$$\mathfrak{M} = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0,z^2)} \equiv \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu,0)}\right) / \left(\frac{\mathcal{M}(0,z^2)}{\mathcal{M}(0,0)}\right)$$

$$\mathfrak{M}(\nu, z^2) = \int_0^1 du\, K(u, z^2\mu^2, \alpha_s) Q(u\nu, \mu^2)$$

Computed on lattice

Perturbatively calculable

Ioffe-time Distribution

$$Q(\nu,\mu) = \mathfrak{M}(\nu, z^2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[\ln\left(z^2\mu^2 \frac{e^{2\gamma_E+1}}{4}\right) B(u) + L(u)\right] \mathfrak{M}(u\nu, z^2).$$

K. Orginos et al., PRD96 (2017), 094503

Match data at different *z*

$$\Downarrow$$

$$Q(\nu) = \int_{-1}^{1} dx\, q(x) e^{i\nu x}$$

$$q(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} Q(\nu)$$

Need data for all ν, *or additional physics input*

Inverse problem

Ioffe-Time Distribution to PDF

J.Karpie, K.Orginos, A.Radyushkin, S.Zafeiropoulos, Phys.Rev.D 96 (2017)

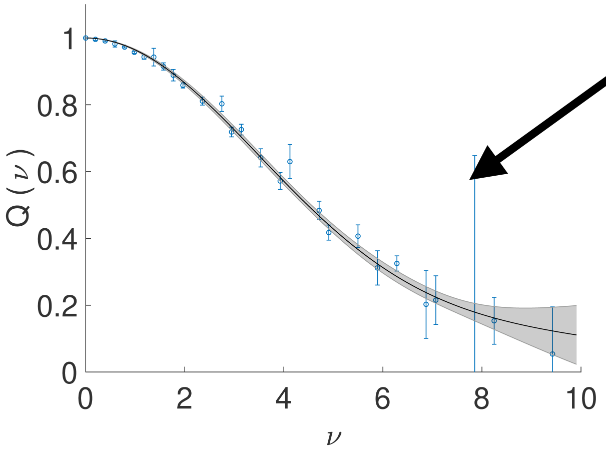
B.Joo *et al.*, HEP 12 (2019) 081, J.Karpie et al., *Phys.Rev.Lett.* 125 (2020) 23, 232003

To extract PDF requires additional information - *use a phenomenologically motivated parametrization*

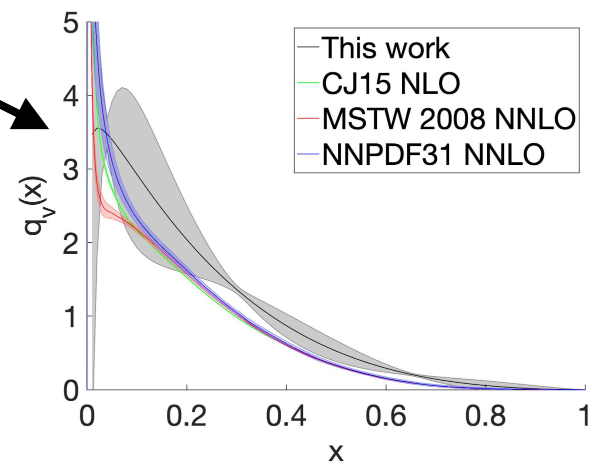
$$f(x) = x^a(1-x)^b P(x)$$

MSTW, CJ

$$P(x) = \frac{1 + c\sqrt{x} + dx}{B(a+a, b+1) + cB(a+1.5, b+1) + dB(a+2, b+1)}$$

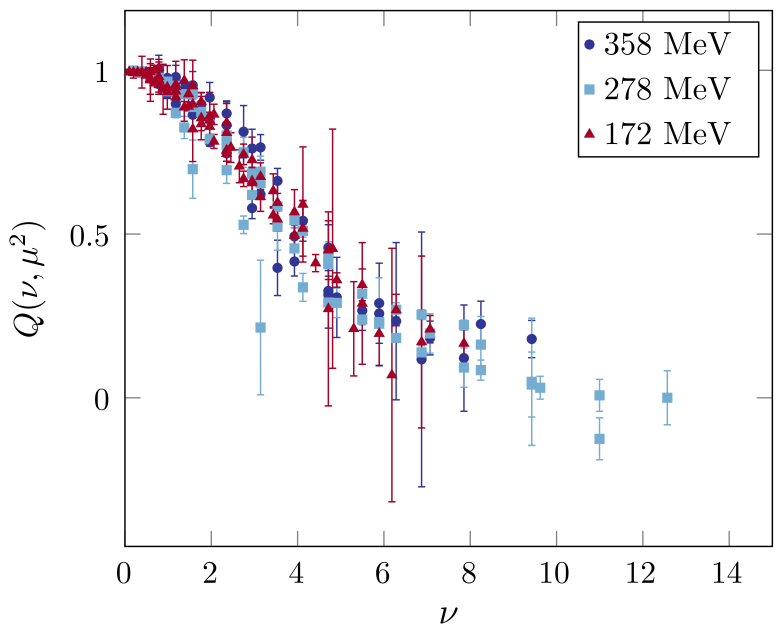


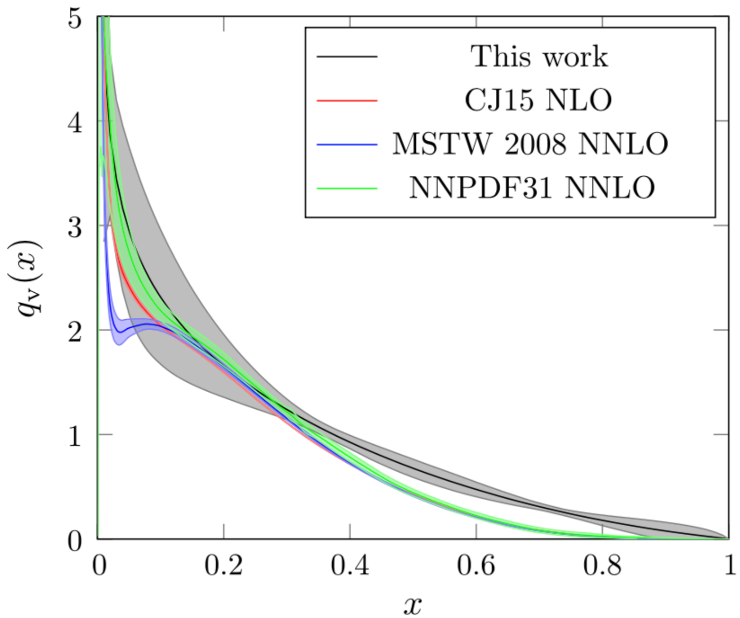
a127m415L



PDFs at Physical Quark Masses

ID	a(fm)	M_π(MeV)	β	c_{SW}	am_l	am_s	$L^3 \times T$	N_{cfg}
$a094m360$	0.094(1)	358(3)	6.3	1.20536588	-0.2350	-0.2050	$32^3 \times 64$	417
$a094m280$	0.094(1)	278(3)	6.3	1.20536588	-0.2390	-0.2050	$32^3 \times 64$	500
$a091m170$	0.091(1)	172(6)	6.3	1.20536588	-0.2416	-0.2050	$64^3 \times 128$	175





B.Joo *et al.*,Phys.Rev.Lett. 125 (2020) 23, 232003

Physical pion

$$q_v(x,\mu^2,m_\pi) = q_v(x,\mu^2,m_0) + a\Delta m_\pi + b\Delta m_\pi^2$$

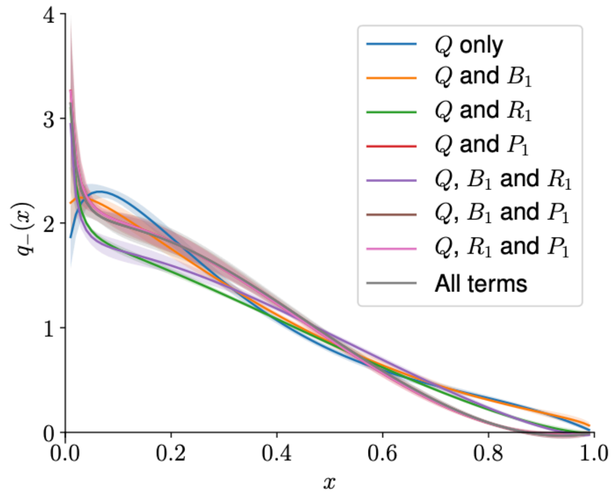
Pseudo-PDF in Precision Era

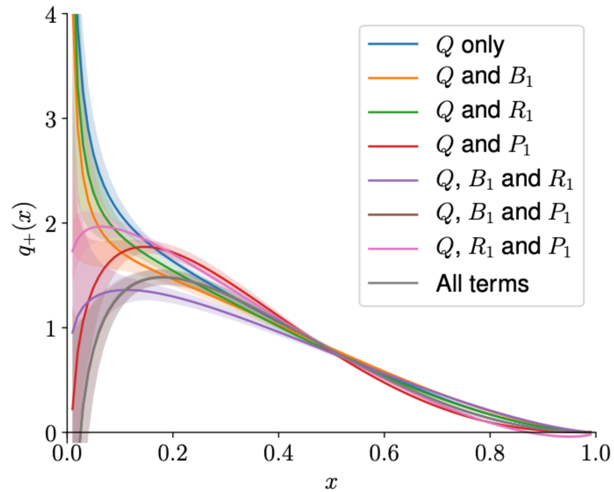
J.Karpie,K.Orginos,A.Radyushkin,S.Zafeiropoulos
arXiv:2105.13313

ID	a(fm)	M_π(MeV)	β	c_{SW}	κ	$L^3 \times T$	N_{cfg}
$\widetilde{A}5$	0.0749(8)	446(1)	5.2	2.01715	0.13585	$32^3 \times 64$	1904
E5	0.0652(6)	440(5)	5.3	1.90952	0.13625	$32^3 \times 64$	999
N5	0.0483(4)	443(4)	5.5	1.75150	0.13660	$48^3 \times 96$	477

$$\mathfrak{M}(p,z,a) = \mathfrak{M}_{\text{cont}}(\nu,z^2) + \sum_{n=1} \left(\frac{a}{|z|}\right)^n P_n(\nu) + (a\Lambda_{\text{QCD}})^n R_n(\nu)\,.$$ Lattice spacing

$$\mathfrak{M}_{\text{cont}}(\nu,z^2) = \mathfrak{M}_{\text{lt}}(\nu,z^2) + \sum_{n=1} (z^2\Lambda_{\text{QCD}}^2)^n B_n(\nu)\,.$$ Higher twist



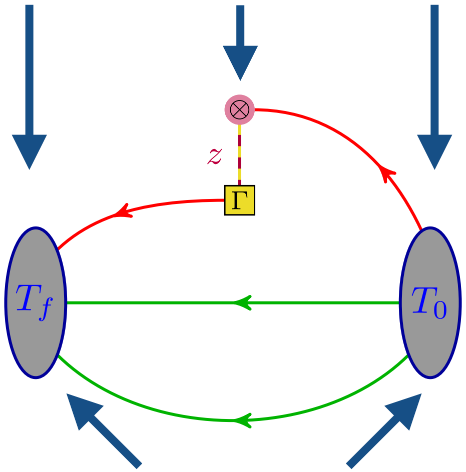


Pseudo-PDF in Precision Era

To control systematic uncertainties, need precise computations over a wide range of momentum.

- Use a low-mode projector to capture states of interest "distillation" M.Peardon *et al* (Hadspec), Phys.Rev.D 80 (2009) 054506

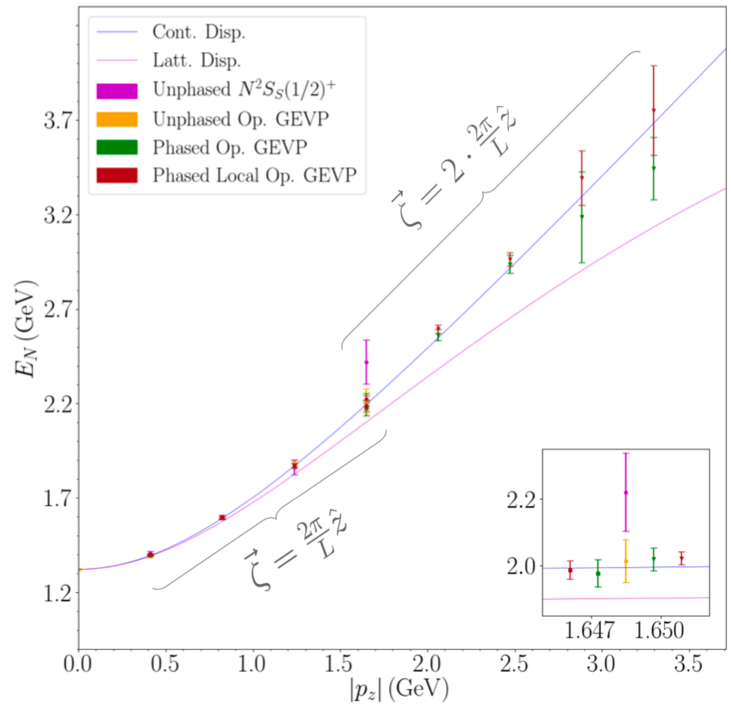
Momentum projection



Variational basis

+ momentum smearing

G.Bali *et al*, Phys.Rev.D 93 (2016) 9, 094515



C.Egerer *et al* (Hadstruc), Phys. Rev. D 103, 034502 (2021)

Expand the x-dependence in terms of (shifted) Jacobi Polynomials

$$\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \mathfrak{Re}\int_0^1 dx\, \mathcal{K}_{\mathrm{v}}\left(x\nu, z^2\mu^2\right) x^\alpha (1-x)^\beta\, \Omega_n^{(\alpha,\beta)}(x)$$

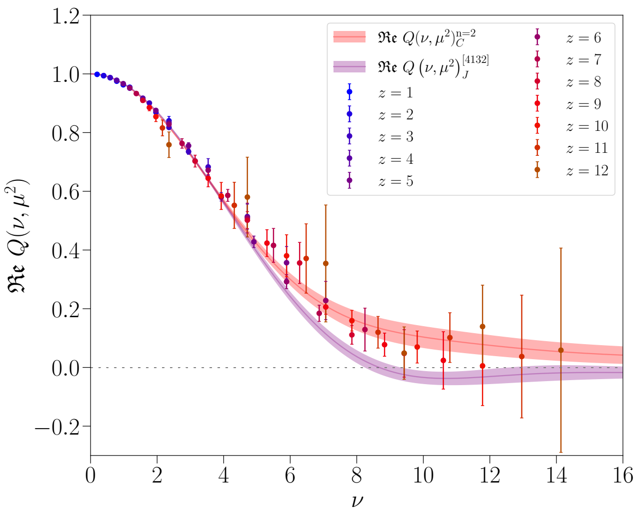
Matching kernel

J.Karpie,K.Orginos,A.Radyushkin,S.Zafeiropoulos, arXiv:2105.13313

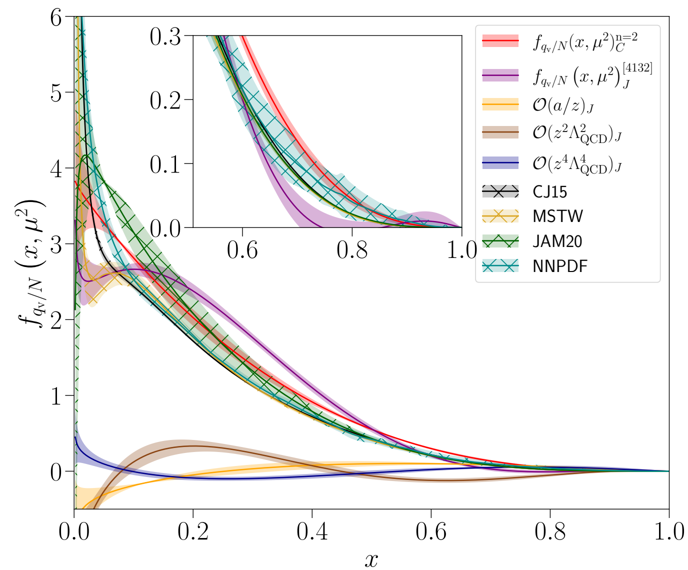
$$\mathfrak{Re}\,\mathfrak{M}_{\mathrm{fit}}\left(\nu, z^2\right) = \sum_{n=0}^{\infty} \sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{\mathrm{v},n}^{lt\,(\alpha,\beta)} + \left(\frac{a}{z}\right)\sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{az\,(\alpha,\beta)} + z^2\Lambda_{\mathrm{QCD}}^2 \sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{\mathrm{v},n}^{t4\,(\alpha,\beta)}$$

Discretization

Higher twist



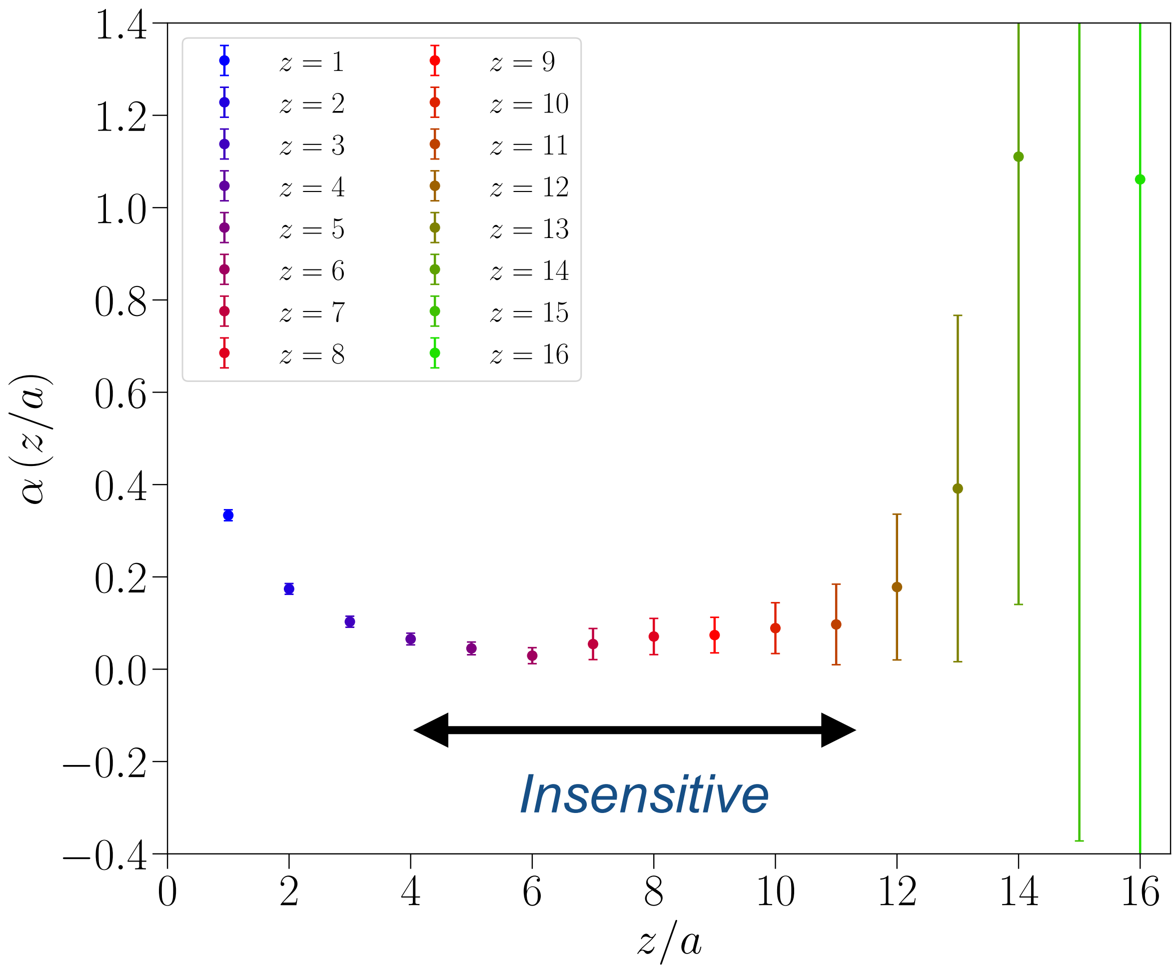
$m_\pi \simeq 358$ MeV



C.Egerer *et al.* (hadstruc), arXiv:2107.05199

DGLAP Evolution

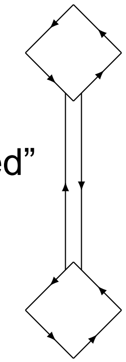
- Look at two-parameter, matched fit



Nucleon Iso-singlet PDFs

Gluon Contribution to unpolarized PDF

“disconnected”

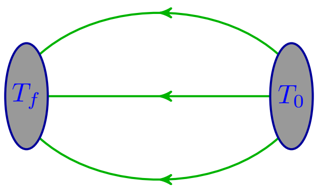


T.Khan *et al.* (Hadstruc), arXiv:2107.08960

$$M_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p|\, G_{\mu\alpha}(z)\, W[z,0]\, G_{\lambda\beta}(0)\, |p\rangle$$

$$\Downarrow$$

$$O_g(z) = G_{ji}(z)\, U(z,0)\, G_{ij}(0)\, U(0,z) - \; G_{ti}(z)\, U(z,0)\, G_{it}(0)\, U(0,z).$$



Two-point functions as in isovector case

Reduced matrix element: $\mathfrak{M}(\nu,z^2) = \left(\frac{\mathcal{M}(\nu,z^2)}{\mathcal{M}(\nu,0)|_{z=0}}\right) \Big/ \left(\frac{\mathcal{M}(0,z^2)|_{p=0}}{\mathcal{M}(0,0)|_{p=0,z=0}}\right)$

Flavor-singlet quantities are subject to severe signal-to-noise problems compared with isovector measures:

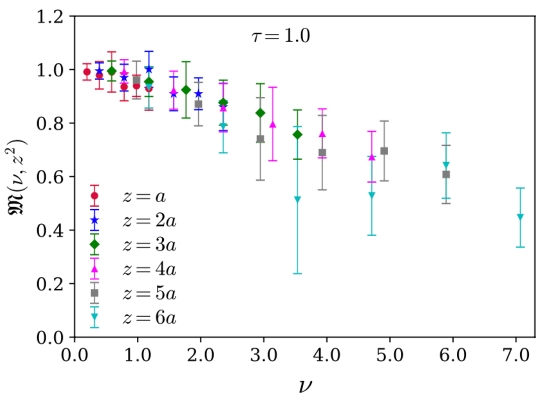
- Use distillation and many more measurements per configuration - *sampling of lattice*
- Use of summed Generalized Eigenvalue Problem (sGEVP) - *better control over excited state contributions*
- Use of *Gradient Flow - smoothing of short-distance fluctuations*

Well-defined way to smooth fields, M. Luscher, JHEP 08, 071 (2010).

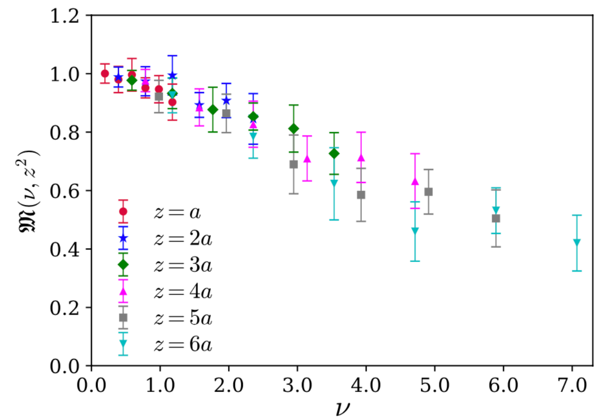
Flow time

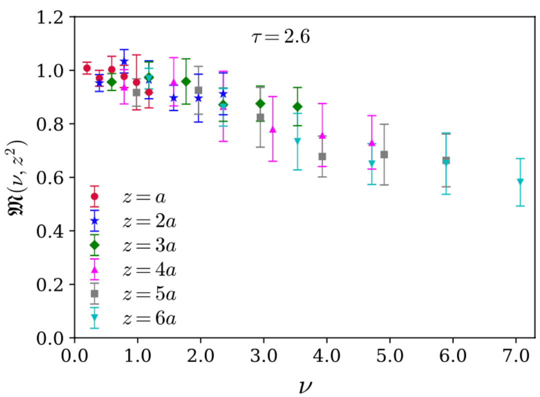
$$\dot{B}_\mu = D_\nu G_{\nu\mu} \quad , \quad D_\mu = \partial_\mu + [B_\mu, \cdot\,]$$
$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]\,,$$

$$\dot{V}_\mu(\tau, x) = -g_0^2\{\partial_{x,\mu} S(V_\mu(\tau, x))\} V_\mu(\tau, x)$$



$$\tau \longrightarrow 0$$





ITD to PDF

Matching: I.Balitsky,W.Morris,A.Radyushkin,Phys.Lett.B 808 (2020) 135621

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{I}_g(\nu,\mu^2)}{\mathcal{I}_g(0,\mu^2)} - \frac{\alpha_s N_c}{2\pi}\int_0^1 du\, \frac{\mathcal{I}_g(u\nu,\mu^2)}{\mathcal{I}_g(0,\mu^2)} \left\{ \ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right) B_{gg}(u) + 4\left[\frac{u+\ln(\bar{u})}{\bar{u}}\right]_+ + \frac{2}{3}\left[1-u^3\right]_+ \right\}$$

N.B neglecting quark-gluon mixing

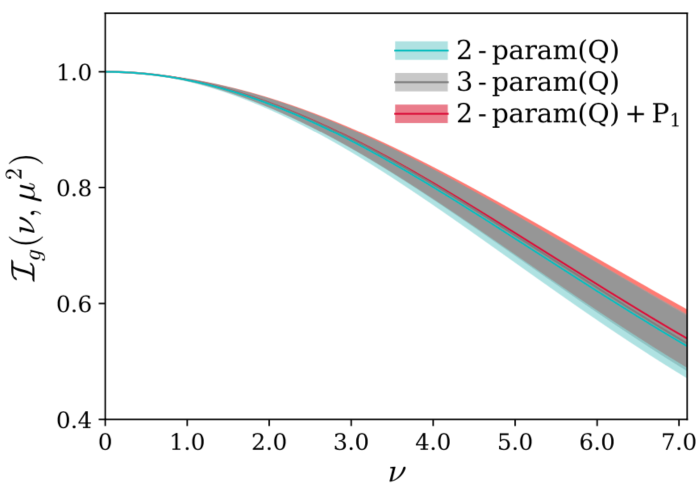
Implementation for obtaining the PDFs follows that of the isovector distribution

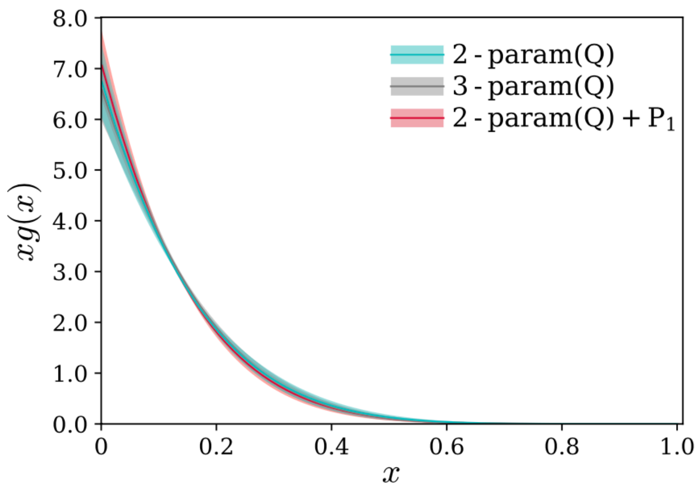
– *Expand in Jacobi Polynomials*

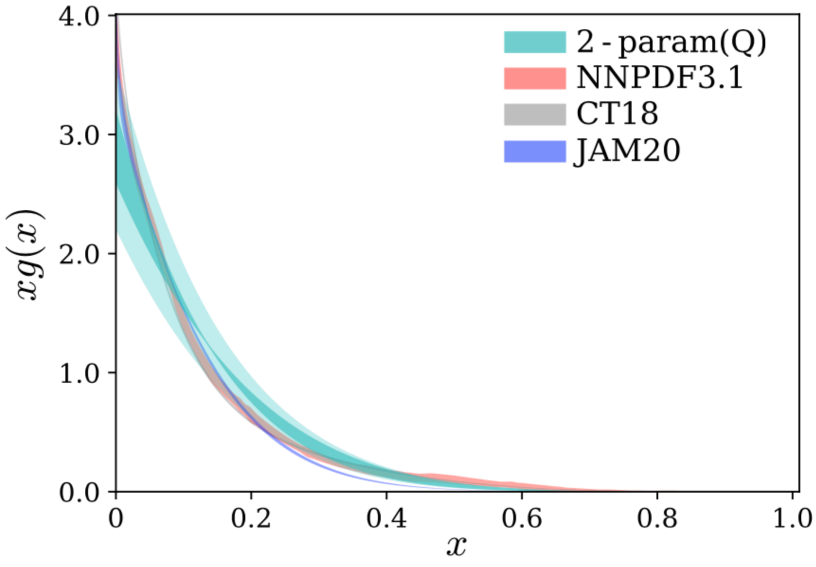
$x^{\alpha}(1-x)^{\beta}$

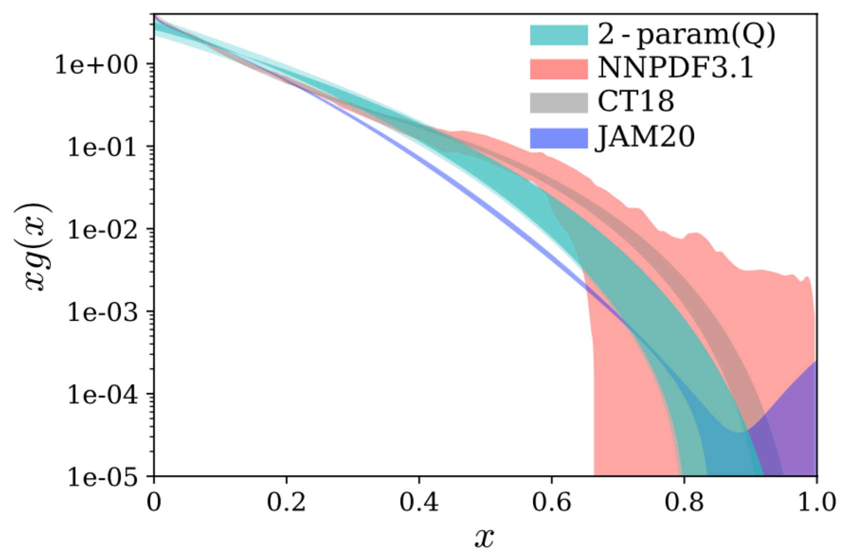
$+J_1^{\alpha,\beta}$

$+a/|z|$









Require normalization of $xg(x)$ $\langle x \rangle_g^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV}) = 0.427(92)$

C.Alexandrou et al., Phys. Rev. Lett. 119, 142002 (2017)

PION STRUCTURE

Lattice QCD studies a *free, isolated pion*

Journal of Physics G: Nuclear and Particle Physics

PAPER

Revealing the structure of light pseudoscalar mesons at the electron–ion collider

J Arrington[1], C Ayerbe Gayoso[2], P C Barry[3,4], V Berdnikov[5], D Binosi[6], L Chang[7], M Diefenthaler[3], M Ding[6], R Ent[3], T Frederico[8] + Show full author list

Published 25 May 2021 • © 2021 IOP Publishing Ltd

Journal of Physics G: Nuclear and Particle Physics, Volume 48, Number 7

Citation J Arrington *et al* 2021 *J. Phys. G: Nucl. Part. Phys.* **48** 075106

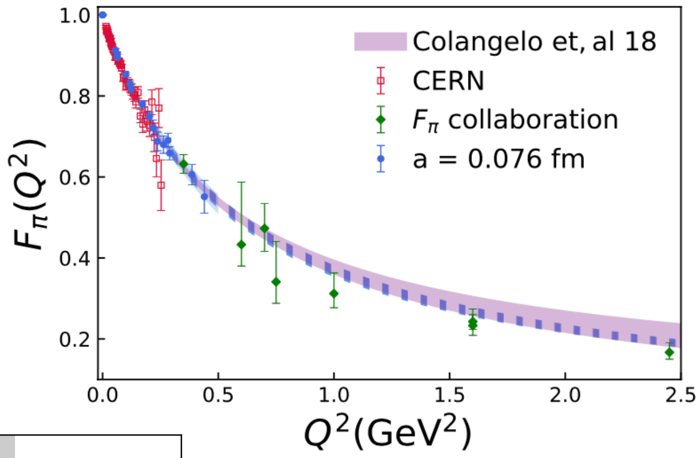
Pion and Kaon Structure

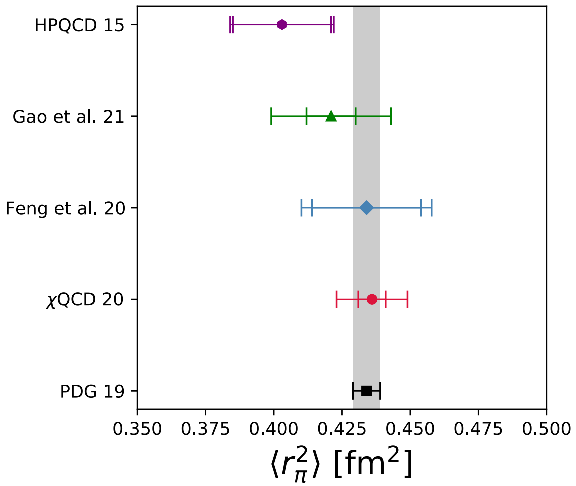
- Lattice QCD can explore a lone, free pion or kaon
- Paradigm computation of hadron structure

X.Gao, N.Karthik *et al.*, arXiv:2102.06047

Charge Radius

Partonic degrees of freedom





Challenges mirror those of experimental measurement.

$$F_\pi(Q^2) = \sum_{k=0}^{k_{\max}} a_k z^k$$

z-expansion

"Good Lattice Cross Sections"

$$\sigma_n(\nu, \xi^2, P^2) = \langle P \mid T\{\mathcal{O}_n(\xi)\} \mid P\rangle$$

Ma and Qiu, Phys. Rev. Lett. 120 022003

Expressed in coordinate space

Short distance scale

where

$$\sigma_n(\nu, \xi^2, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(x\nu, \xi^2, x^2P^2, \mu^2) + \mathcal{O}(\xi^2 \Lambda_{\mathrm{QCD}}^2)$$

Calculated in LQCD

Parton Distribution function

Calculated in perturbation theory ("process dependent")

$$\mathcal{O}(\xi) = \bar{\psi}(0)\Gamma W(0, 0+\xi)\psi(\xi)$$

Encompasses qPDF/pPDF

Gauge-Invariant Currents

$$\begin{aligned} \mathcal{O}_S(\xi) &= \xi^4 Z_S^2 [\bar{\psi}_q \psi_q](\xi)[\bar{\psi}_q \psi](0) \\ \mathcal{O}_{V'}(\xi) &= \xi^2 Z_{V'}^2 [\bar{\psi}_q \xi \cdot \gamma \psi_{q'}](\xi)[\bar{\psi}_{q'} \xi \cdot \gamma \psi](0) \end{aligned}$$

Flavor-changing

+ analogous gluon operators

Good Lattice Cross Section

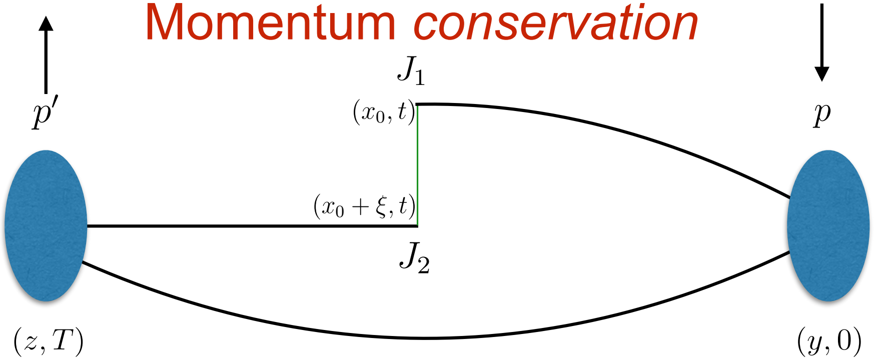
Sufian *et al.*, Phys. Rev. D 99, 074507 (2019); arXiv:2001.04960

Sequential-Source Approach

Momentum *conservation*

Momentum projection

Momentum projection



Process, i.e. current, dependent

$$\frac{1}{2}[\sigma^{\mu\nu}_{V,A}(\xi,p) + \sigma^{\mu\nu}_{A,V}(\xi,p)]$$

$$\boxed{\equiv \epsilon^{\mu\nu\alpha\beta}\xi_\alpha p_\beta T_1(\nu,\xi^2) + (p^\mu\xi^\nu - \xi^\mu p^\nu)T_2(\nu,\xi^2)}$$

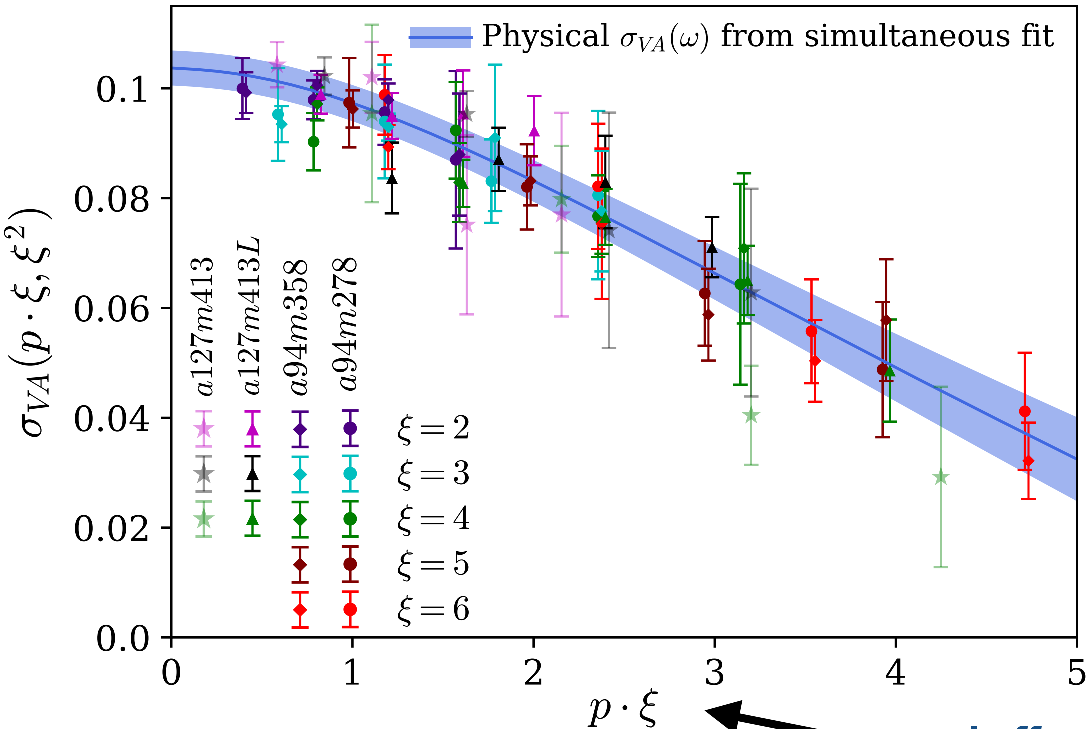
Perturbative kernel:

N.B. We're inconsistent $\omega \leftrightarrow \nu$!

$$\widetilde{\sigma}^{q(1)}_{VA}(\widetilde{\omega},q^2) = \int_0^1 \frac{dx}{x}\,\widetilde{K}^{(1)}(x\widetilde{\omega},q^2,\mu^2) f^{(0)}_{q_v/q}(x,\mu^2) + \int_0^1 \frac{dx}{x}\,\widetilde{K}^{(0)}(x\widetilde{\omega},q^2,\mu^2) f^{(1)}_{q_v/q}(x,\mu^2).$$

Y-Q Ma

Lattice Cross Sections



Ioffe time

"Ioffe Time Distribution"

"Z-expansion fit"

$$\sigma_{VA}(\omega,\xi^2) = \sum_{k=0}^{k_{\max}=4} \lambda_k \tau^k + b_1 m_\pi + b_2 a + b_3 \xi^2 + b_4 a^2 p^2 + b_5 e^{-m_\pi(L-\xi)}$$

$$\tau = \frac{\sqrt{\omega_{\text{cut}}+\omega} - \sqrt{\omega_{\text{cut}}}}{\sqrt{\omega_{\text{cut}}+\omega} + \sqrt{\omega_{\text{cut}}}}$$

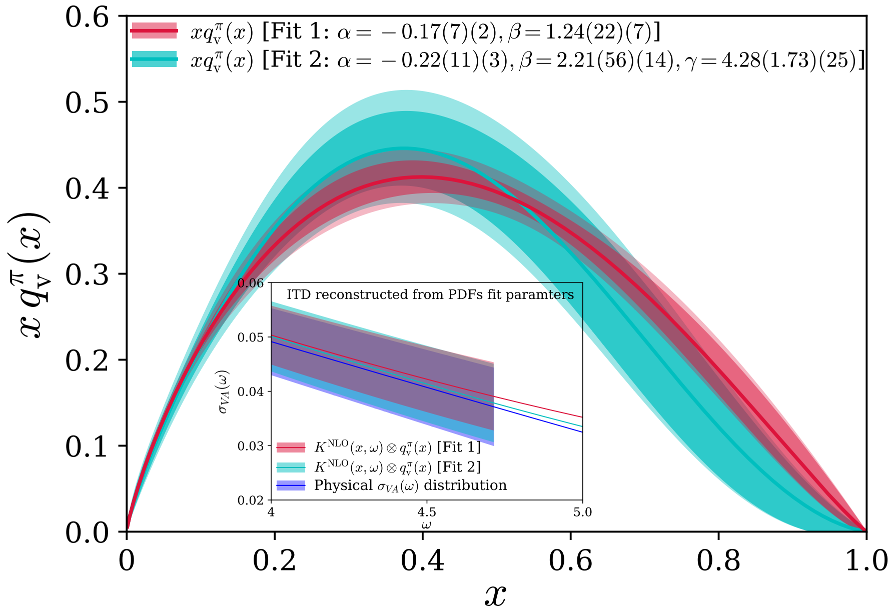
Pion PDF

"Inverse Problem" - ill-posed inverse Fourier transform.

$$\sigma_n(\nu,\xi^2,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) K_n^a(x\nu,\xi^2,x^2P^2,\mu^2) + \mathcal{O}(\xi^2\Lambda^2_{\rm QCD})$$

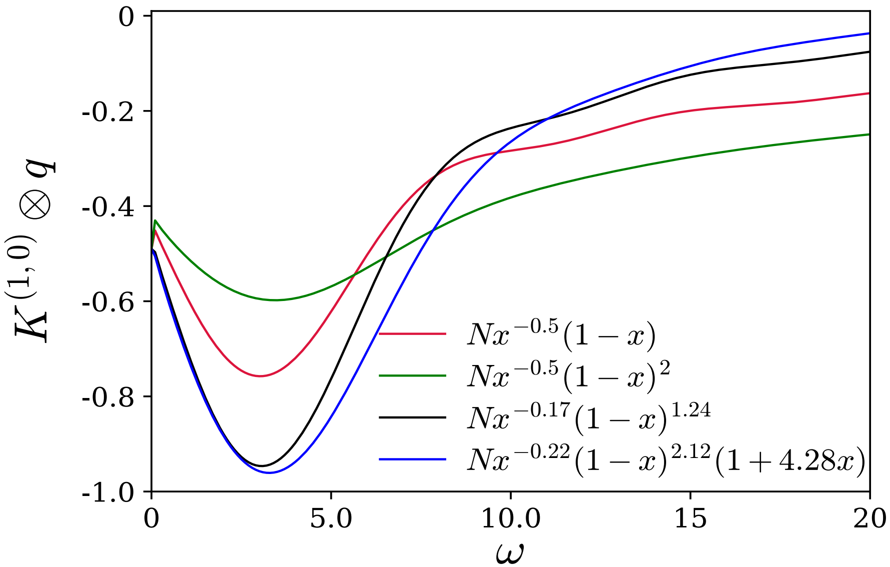
Calculate on Lattice — Extract PDF? — Calculate in PQCD

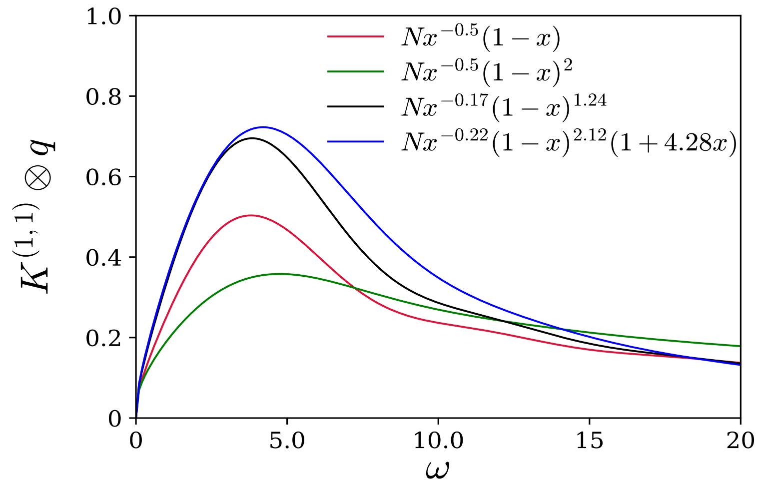
Similar challenge to global fitting community!



$$q_v^\pi(x) = \frac{x^\alpha(1-x)^\beta(1+\gamma x)}{B(\alpha+1,\beta+1)+\gamma B(\alpha+2,\beta+1)}$$

NLO term well-controlled

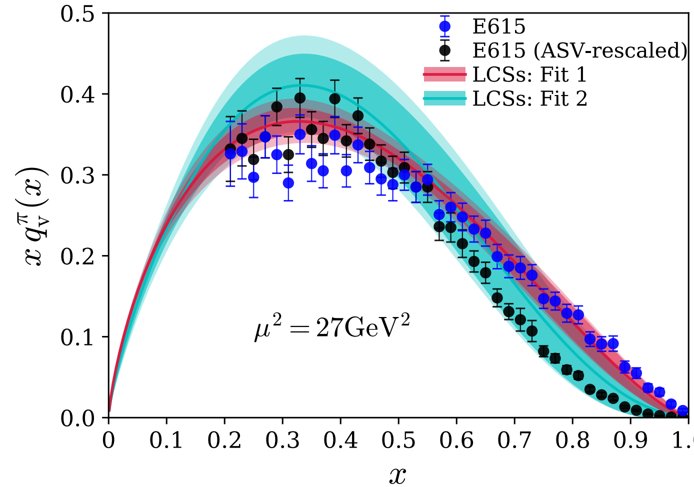




Pion Valence Quark Distribution at Large x from Lattice QCD

Raza Sabbir Sufian,[1] Colin Egerer,[2] Joseph Karpie,[3] Robert G. Edwards,[1] Bálint Joó,[1] Yan-Qing Ma,[4,5,6] Kostas Orginos,[1,2] Jian-Wei Qiu,[1] and David G. Richards[1]

Sufian *et al.*, Phys. Rev. D102, 05408 (2020)



Determine large-x behavior $\rightarrow$ *need for finer resolution and reach in Ioffe time.*

FUTURE DIRECTIONS AND THE EIC

Lattice ↔ Global Analysis

Analysis *within NNPDF Framework*

Neural-network analysis of Parton Distribution Functions from Ioffe-time pseudodistributions

Luigi Del Debbio,[a] Tommaso Giani,[a,1] Joseph Karpie,[b] Kostas Orginos,[c,d] Anatoly Radyushkin[c,e] and Savvas Zafeiropoulos[f]

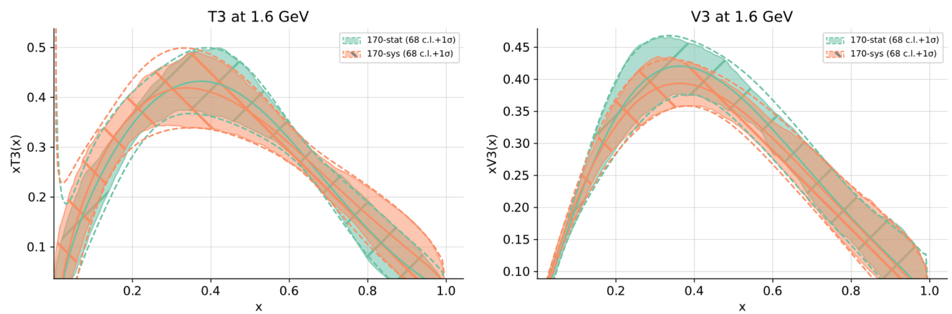
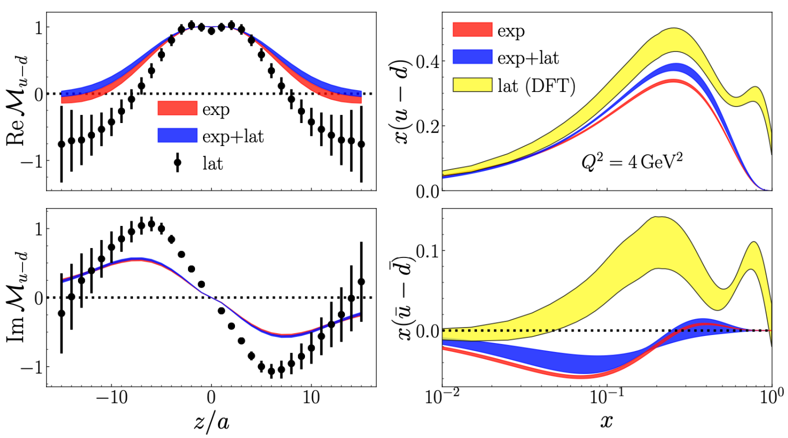


Figure A.2. PDFs from the fits *170-stat* and *170-sys*.



1. arXiv:2010.00548 [pdf, other] hep-ph hep-lat nucl-th doi 10.1103/PhysRevD.103.016003

Confronting lattice parton distributions with global QCD analysis

Authors: J. Bringewatt, N. Sato, W. Melnitchouk, Jian-Wei Qiu, F. Steffens, M. Constantinou

Abstract: We present the first Monte Carlo based global QCD analysis of spin-averaged and spin-dependent parton distribution functions (PDFs) that includes nucleon isovector matrix elements in coordinate space from lattice QCD. We investigate the degree of universality of the extracted PDFs when the lattice and experimental data are treated under the same conditions within the Bayesian likelihood analysis.... ▽ More

Submitted 1 October, 2020; **originally announced** October 2020.

Comments: 23 pages, 4 figures

Report number: JLAB-THY-20-3257

Journal ref: Phys. Rev. D 103, 016003 (2021)

Hadron Tomography

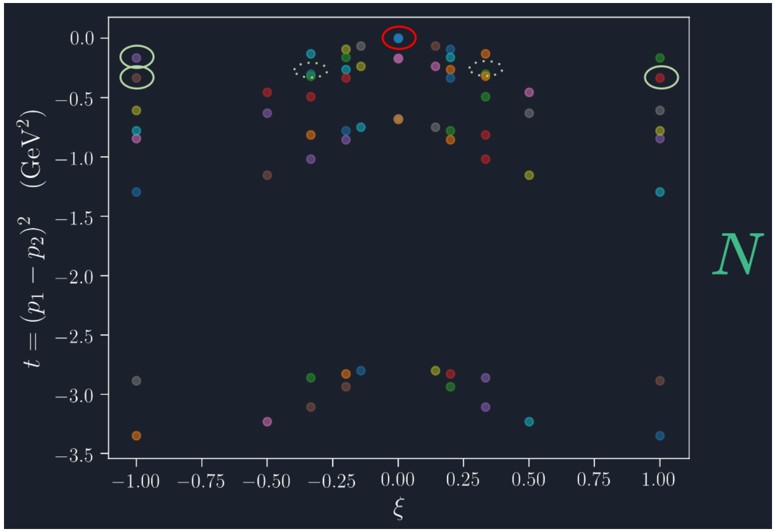
- Three-dimensional imaging of hadrons

Generalized Parton Distributions

Transverse Momentum Dependent dist

Thanks, Colin Egerer

A.Radyushkin, PRD100, 116011 (2019)



N

pseudo-GITD

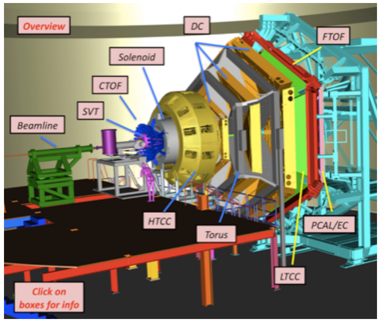
pseudo-GPD

$$\mathscr{M}(\nu, \xi, t; z^2) = e^{i\xi\nu} \int_{-1}^{1} dx\, e^{ix\nu} \mathscr{H}(x, \xi, t, z^2)$$

$$\nu = (\nu_1 + \nu_2)/2 \qquad \xi = \frac{\nu_1 - \nu_2}{\nu_1 + \nu_2}$$

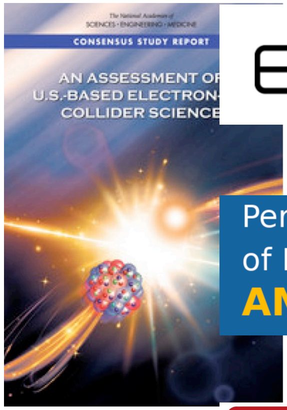
Lattice is complementary to experiment and essential!

Hadron Tomography



JLab@12GeV

Lattice QCD

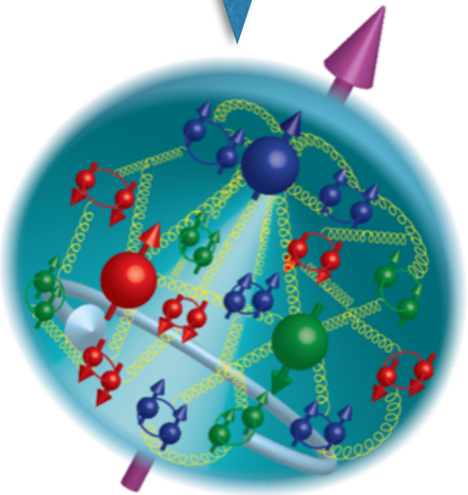


ECP EXASCALE COMPUTING PROJECT

Perceiving the Emergence of Hadron Mass through **AMBER@CERN**

Electron **I**on **C**ollider in **C**hina, EicC

CENTER *for* NUCLEAR FEMTOGRAPHY



3D Image of nucleon and nuclei at the femtoscale

Summary

- Important progress at understanding the internal structure of nucleons and pions within the pseudo-PDF framework.
  - Helicity and transversity distributions
  - Contribution of disconnected diagrams to complete the isoscalar picture
- Increasing interplay of the lattice and global-fitting communities
- No where is this interplay more important than in nuclear femtography