The Anomalous Magnetic Moment of the Muon

W. Morse - BNL

Physics

- I’m not trying to “teach physics” in this talk.
- You have many great professors who will do that.
- I’m just trying to convince you that physics is interesting.
- If you have a question, “raise your hand” or use the “chat”.
- Don’t wait until the end of the talk!
- Wlodek will tell me when there is a question.

Matter Particles circa 1930s

Particle	Mass (MeV)	Charge	Force	Size	Spin (h)
Proton p	938.3	+	S, E, W, G	10^{-15} m	1/2
Neutron n	939.6	0	S, W, G	10^{-15} m	1/2
Electron e	0.511	-	E, W, G	$< 10^{-20}$ m	1/2
Neutrino ν	$\approx 10^{-7}$	0	W, G	$< 10^{-20}$ m	1/2

Spin = h/2

- In 1920s, two young physicists could understand their data if the electron was spinning.
- Wrote a paper.
- Then the big boss came back from sabbatical.
- They tried to pull the paper, but it was already published.
- The reason the electron has spin 1/2 is so that we could be here to discover …….

Gauge Symmetries circa 1970s
$SM = SU(3) \times SU(2) \times U(1)$

Force	Carrier	Spin
Strong	Gluon	1
Electro-Magnetic	Photon	1
Weak	W	1
Gravity	Graviton	2
Mass	H	0

Weak Force

- Free neutron decay half-life = 10 minutes. $n \to pe\bar{\nu}$
- If earth to sun filled with Pb, most neutrinos would still get through.
- Strong $N^* \to p\,\pi$ 10^{-22} sec
- Electromagnetic $\pi \to \gamma\,\gamma$ 10^{-18} sec
- Hydrogen $p\,e \to n\,\nu$
- Without the weak interaction, there would be no energy from the sun, no elements but Big Bang H, He.
- The strength of the weak force determines the lifetime of the sun.
- Why don't neutrons in nucleus decay?

Physics

- Binding energy of the deuteron = 2.2 MeV.
- If neutron were 0.1% heavier, deuterion would be radioactive!
- If proton were heavier than the neutron, hydrogen would be radioactive!
- Physics is exactly what it has to be in order for us to be here to discover the laws of physics.

Quantum Mechanics

- Developed 1910 – 1950 by:
- Niels Bohr – “Anyone who thinks they understand QM, and is not deeply disturbed by it, doesn’t understand QM.”
- Albert Einstein – “God doesn’t play dice.”
- Erwin Schrödinger – “I wish I never discovered these damn wave functions.”
- I love QM! It is not trivial!

Quantum Mechanics

- Electron is described by Schrödinger wavefunction: Ψ
- QM: Rotate by an angle θ: $\Psi' = e^{iS\theta}\Psi = \left(\cos(S\theta) + i\sin(S\theta)\right)\Psi$
- $\theta = 360°, \Psi' = \cos(2\pi S)\,\Psi$
- Spin-statistics symmetry, Chemistry, ….
- Spin ½ are matter particles!
- Spin 0, 1, 2 are force particles!
- Very simple, but profound.

Magnetic Moment

- Particle is spinning,
- Particle is charged,
- Spinning charge creates a magnetic field:
- $\mu = \frac{gQS}{M}$
- Dirac Equation: *g* = 2 for a spin ½ point particle.
- Proton: *g* = 5.6, finally explained by quark model.
- Electron: *g* = 2.0
- Oppenheimer et al. calculated the first order correction to 2 to be infinity.

Spin ½ Particles
Three Generations!

Particle	Mass (MeV)	Particle	Mass (MeV)	Particle	Mass (MeV)
u	312	c	1750	t	171200
d	313	s	490	b	5620
e	0.5	$\mu \to e\nu\bar{\nu}$	105	τ	1777
ν_e	10^{-8}	ν_μ	10^{-7}	ν_τ	10^{-7}

More Symmetries

- C symmetry – changes particle to anti-particle.
- P symmetry – changes x to $-x$.
- T symmetry – changes t to $-t$.
- Good symmetries for strong, EM, gravity.
- Discovered in 1950 – 60s: P, T, C are broken symmetries in the weak interaction.
- Without weak interaction, we wouldn't be here.

Theory 1970s

- In SM we can only get this if there are three, or more, ways for a given reaction to go, and get QM interference with the three amplitudes, with at least one imaginary amplitude.
- Need at least three generations.
- In the big bang all the non-neutrino particles/anti-particles should have finally annihilated to photons.
- Due to symmetry breaking, p/photon ≈ 10^{-9}.

$$a = \frac{g-2}{2} = 0.0011$$

- 1948 I.I. Rabi, Conference at Shelter Island.
- Schwinger et al.,
- Quantum Electrodynamics (QED).
- "Renormalization" gets rid of the infinities.
- For any new theory, the first question: "Is it renormalizable".

QED Renormalization

- **Renormalization** is a collection of techniques in quantum field theory, the statistical mechanics of fields, and the theory of self-similar geometric structures, that are used to treat infinities arising in calculated quantities by altering values of these quantities to compensate for effects of their **self-interactions**.
- For example, an electron theory may begin by postulating an electron with an initial mass and charge. In quantum field theory a cloud of virtual particles, such as photons, positrons, and others surrounds and interacts with the initial electron. Accounting for the interactions of the surrounding particles (e.g. collisions at different energies) shows that the electron-system behaves as if it had a different mass and charge than initially postulated. Renormalization, in this example, mathematically replaces the initially postulated mass and charge of an electron with the experimentally observed mass and charge.

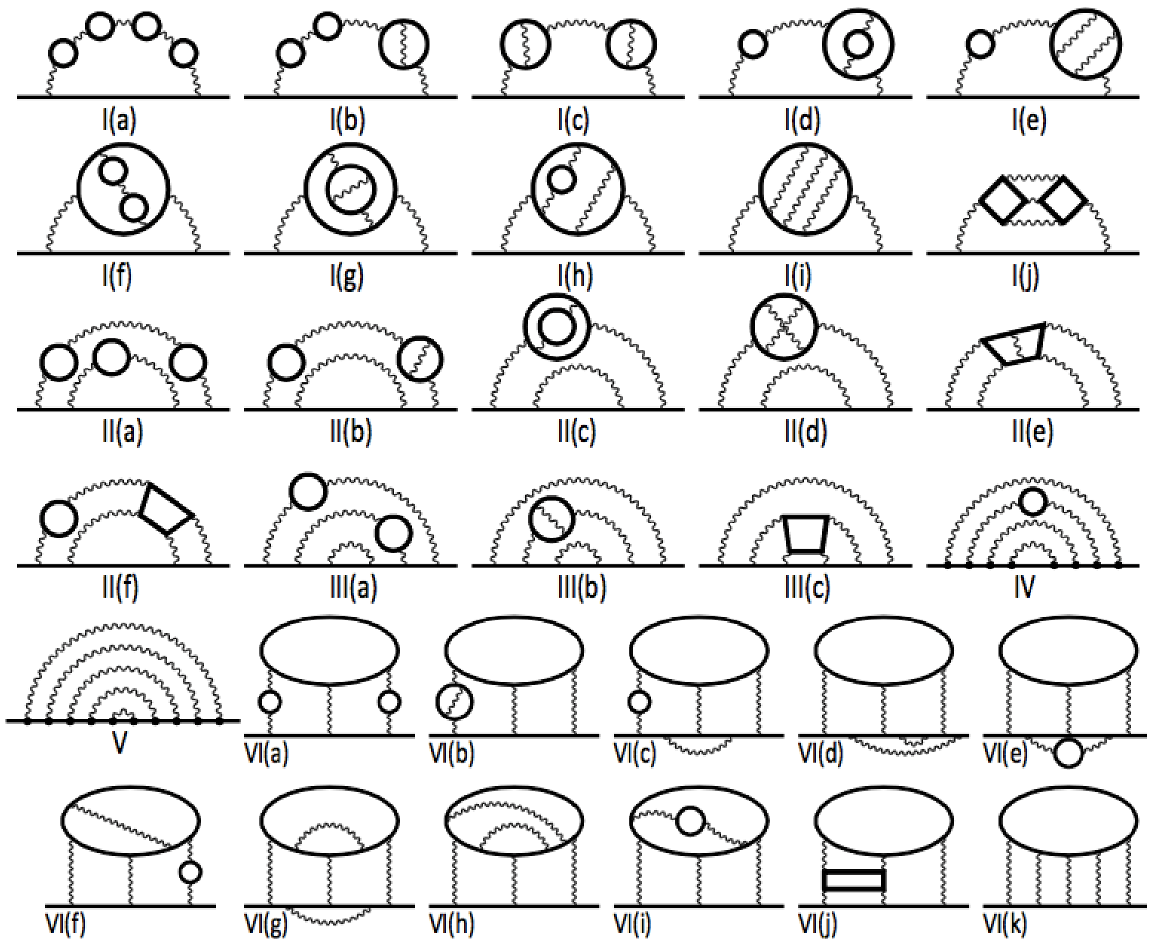
Quantum Mechanics

- Anomalous magnetic moment is due to QM.
- The energy of the vacuum classically is zero.
- QM: $dE \times dt = h$
- The problem with zero is that it has no uncertainty.

Quantum Electrodynamics Not Boring!

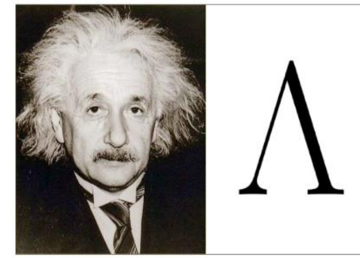
10^{th}
12672
diagrams

$dE \times dt = h$



What about Einstein's General Theory Relativity

Einsteins' Cosmological Constant



- GR has cosmological constant.
- Einstein at first had static universe.
- Chose value of Λ to stop the universe from collapsing.
- Then Hubble et al. found that the universe was expanding!
- No need for cosmological constant.
- Einstein: "Biggest blunder of my life".

“Dark Energy”

- Theorists calculated QM of the vacuum in 1960s: $\Lambda \approx 10^{120}$.
- Steve Weinberg: need $\Lambda < 2$ for us to exist.
- “Dark energy” discovered in 1990s: $\Lambda = 0.7$.
- aka cosmological constant.
- “Worst theoretical embarrassment in the history of physics” – Ed Witten.
- A new symmetry would reduce it, ….

Quantum Mechanics

- All particles exist in the vacuum.
- All particles contribute to the anomalous magnetic moment.
- Are there new particles beyond the SM?
- Are there new symmetries?
- E821 measure anomalous magnetic moment to 0.5 parts per million (ppm).
- Theorists: calculate to 0.5ppm. Compare.

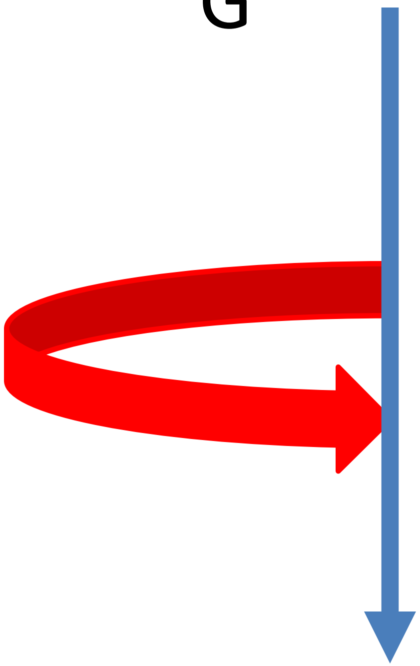
BNL E821 Experiment

- Accelerate protons through 24 billion volts.
- Electrical outlet = $110V$.
- Energy = 24 billion eV (GeV).
- Protons hit target.
- $E = mc^2$
- Convert energy into mass
- $P + P \rightarrow$ muons, anti-muons, electrons, anti-electrons, ….. Mini-big bang.
- Muon $m = 0.105\ GeV$.
- Beamline selects particles.

Spin Precession

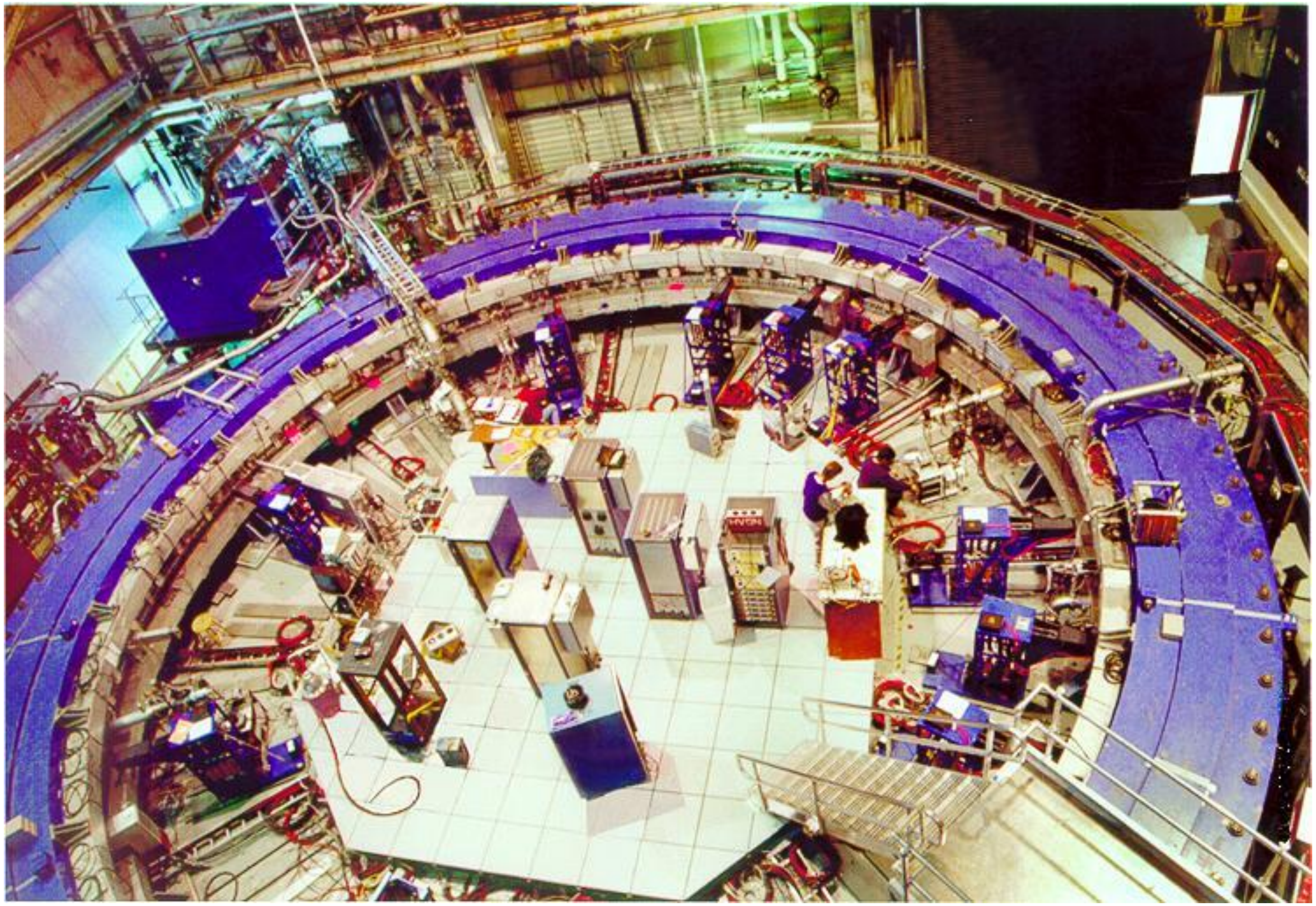
spinning top μ magnetic moment

G B



$$\omega_a = \frac{eaB}{m}$$

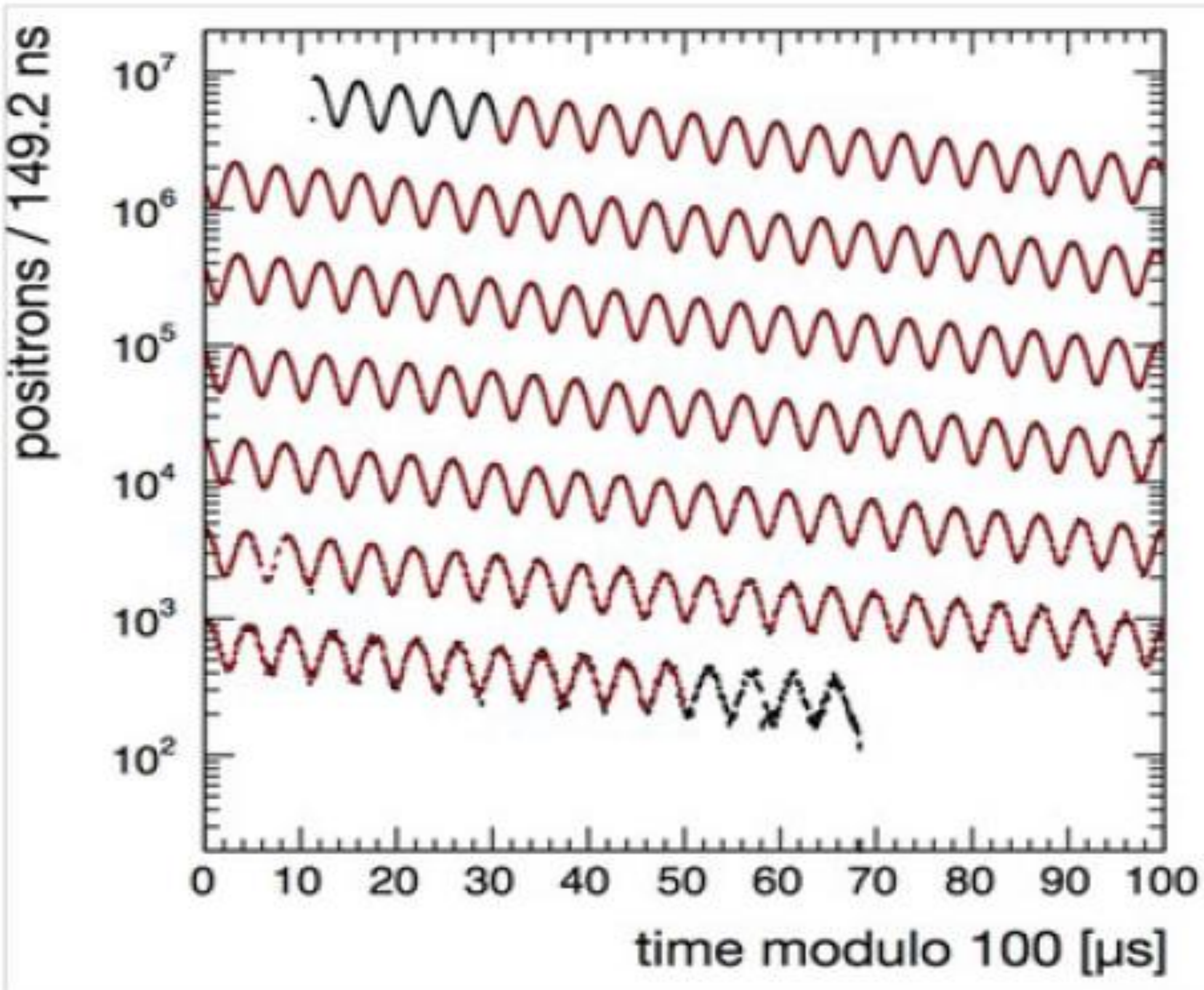
Storage Ring Magnet at BNL



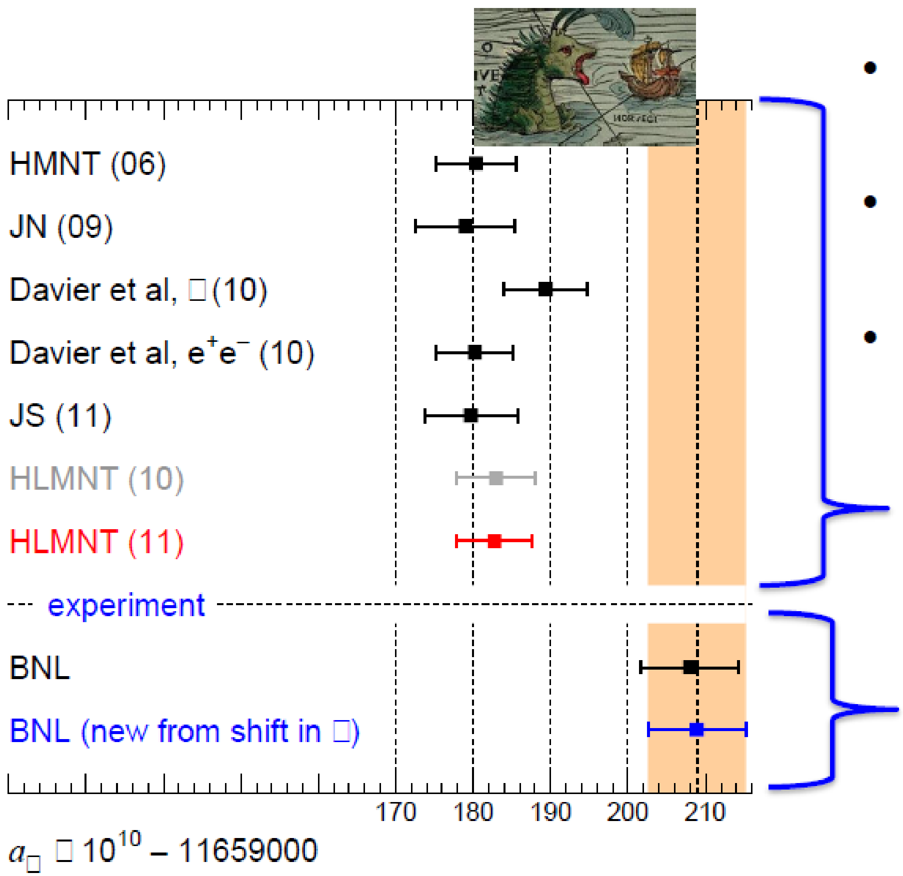
$\mu \rightarrow e\nu\bar{\nu}$

- Weak decay.
- The high energy positrons go preferentially in the direction of the muon spin.
- Just measure the high energy positrons.

High Energy Positrons vs. Decay Time



BNL E821



- The last a_μ experiment ended at BNL in 2001
- 3σ discrepancy with the theoretical expectation
- Goal of FNAL g-2 is to reduce the experimental error by a factor of 4

Theoretical evaluations

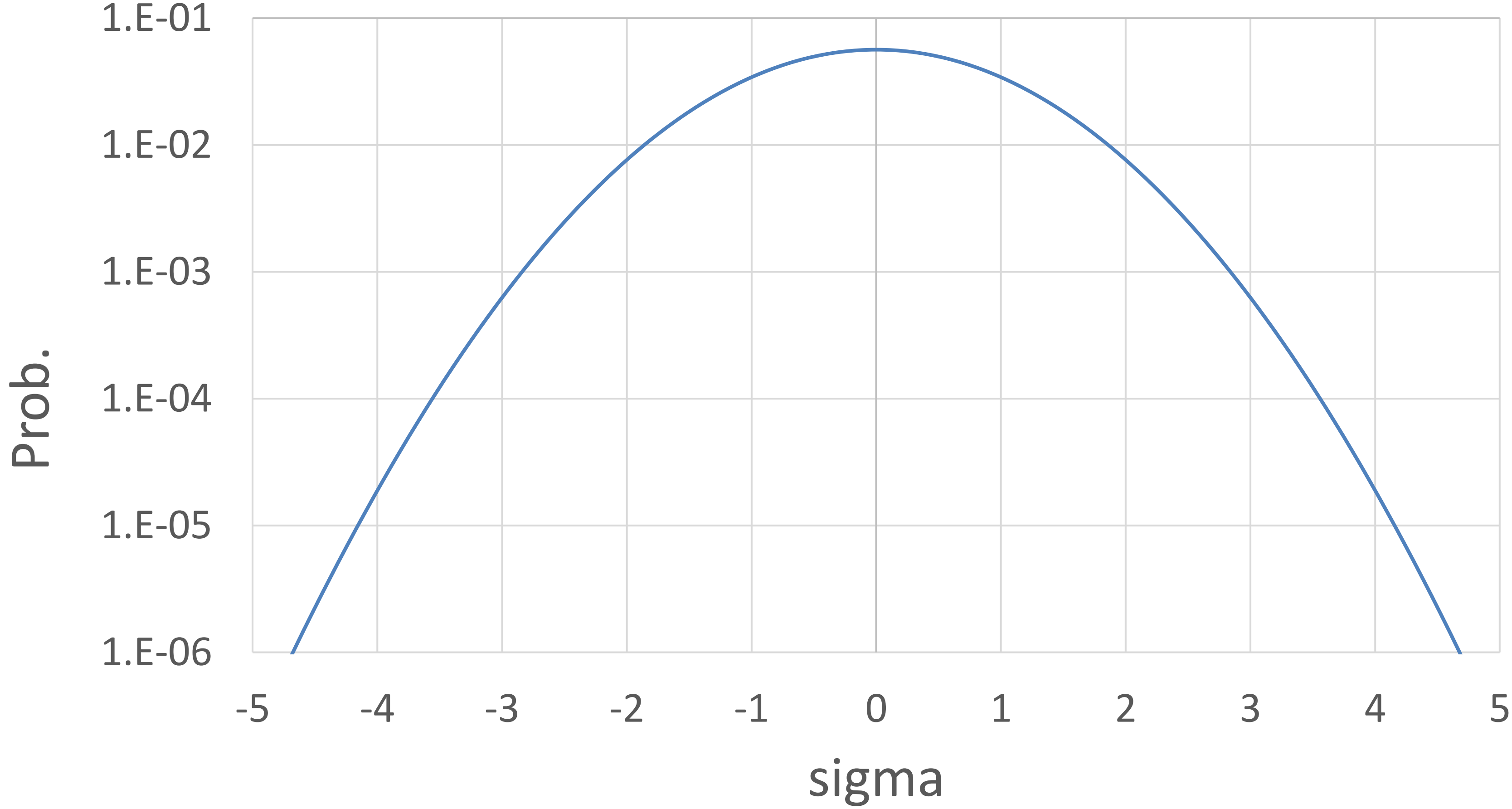
Last experimental result from BNL

What is 3σ?

- Probability $\approx e^{-\frac{(x-\overline{x})^2}{2\sigma^2}}$
- Toss a coin 100 times.
- How many heads? $\bar{x} = 50$.
- $\sigma = \sqrt{50}$

0	1	2	3	4
50	43	36	29	21

$$\text{Probability} \approx e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$$



FNAL

- FNAL is a National Lab,
- 1 hour west of Chicago, IL.
- They were on the "energy frontier".
- Center of mass energy 2 TeV.
- CERN in Geneva, Switzerland, LHC 13 TeV.
- FNAL went from the energy frontier to the intensity frontier.
- $20 \times$ more muons. Move the magnet to FNAL.

Move from BNL to FNAL

Smith Point Marina

Around FL, and up Miss. River

Up Illinois River

Entrance Ramp to Interstate 88

Interstate 88

Arriving at FNAL 2013

Ring reassembly at Fermilab

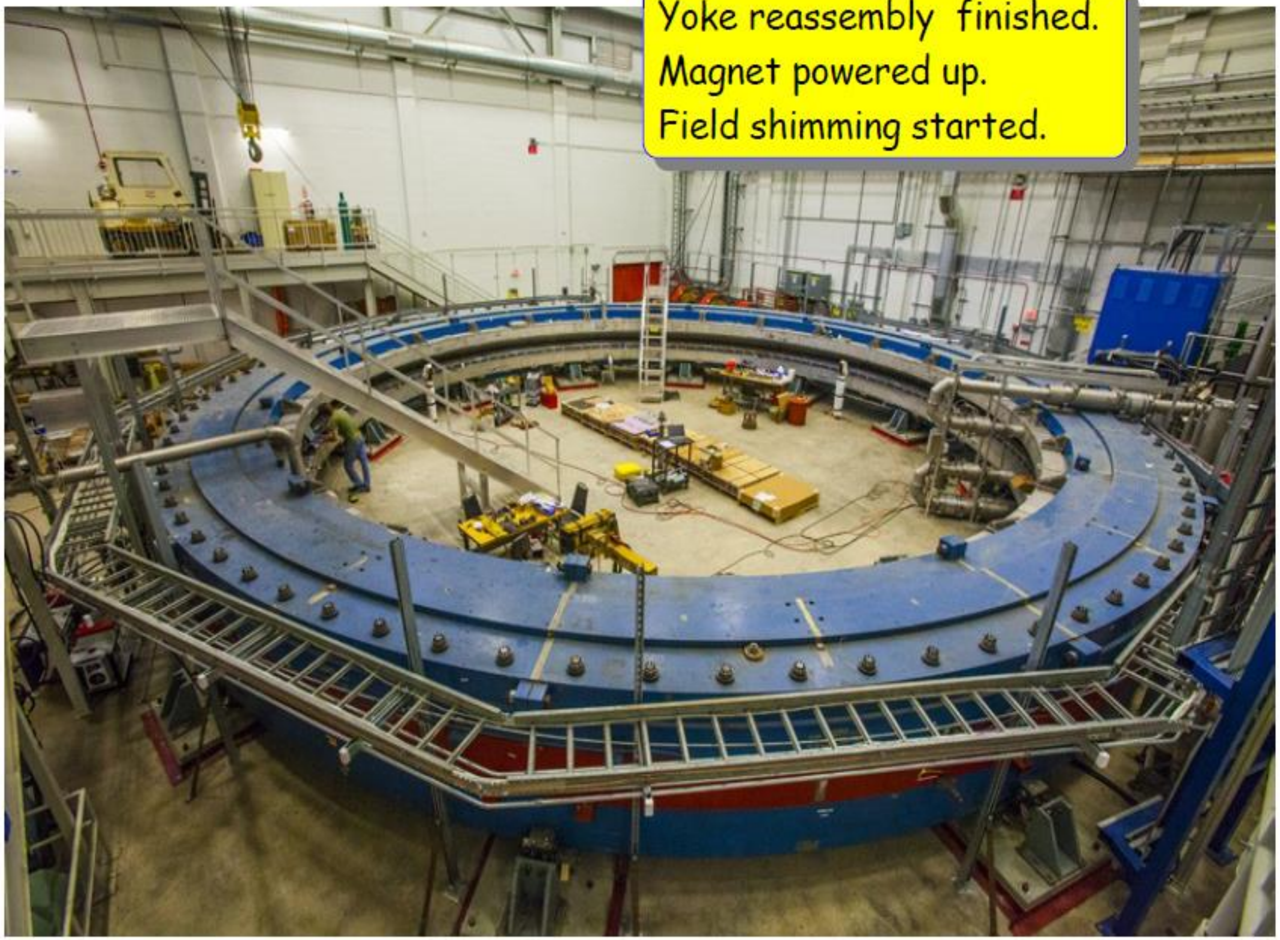
2009 | 2010 | 2011 | 2012 | 2013 | **2014** | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 | 2021

June 23, 2014. Bottom yoke. Reassembly progresses well. Superconducting coils will be moved into the experimental hall end of July 2014

g-2 storage magnet reassembled at FNAL

2009 › 2010 › 2011 › 2012 › 2013 › 2014 › **2015** › 2016 › 2017 › 2018 › 2019 › 2020 › 2021

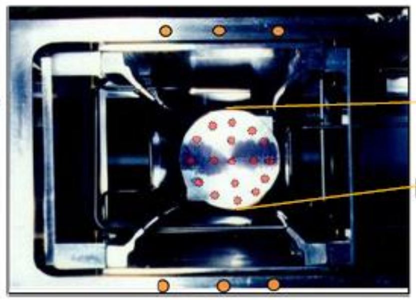
Yoke reassembly finished.
Magnet powered up.
Field shimming started.

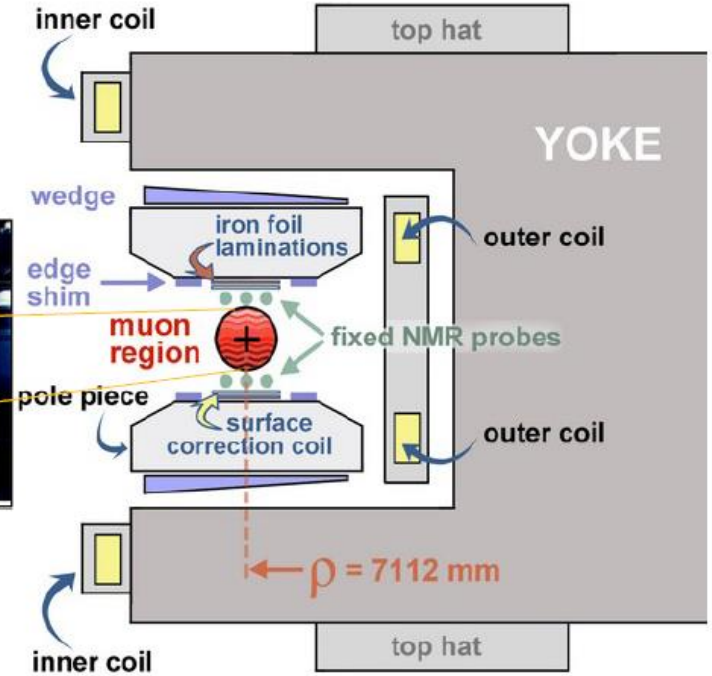


Magnet anatomy and $\omega'_p(T_r)$ measurement

Brilliant magnet design by Gordon Danby (BNL)

378 fixed NMR probes
17 NMR Trolley probes
A suit of shimming elements





12 C-shaped yokes
- 3 upper and 3 lower poles per yoke
- 72 total poles

Shimming knobs
- Pole separation determines field: pole tilts, non-flatness affect uniformity
- Top hats (dipole)
- Wedges (dipole, quadrupole)
- Edge shims (dipole, quadrupole, sextupole)
- Laminations (dipole, quadrupole, sextupole)
- Surface coils (quadrupole, sextupole,…)

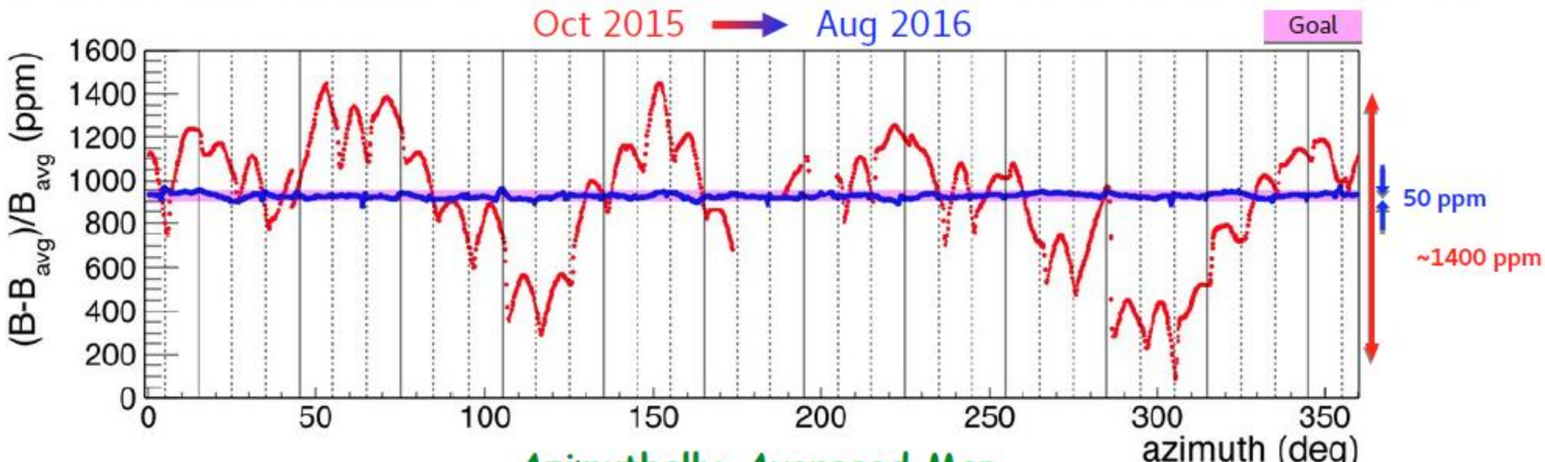
name	value
Magnetic field	1.45 T
Current	5200 A
Design orbit radius	7.112 m
Muon storoage region diameter	90 mm
Magnet gap	18 cm
Stored energy	6 MJ

NMR trolley maps the field at about 9000 locations over the entire azimuth every 3 days

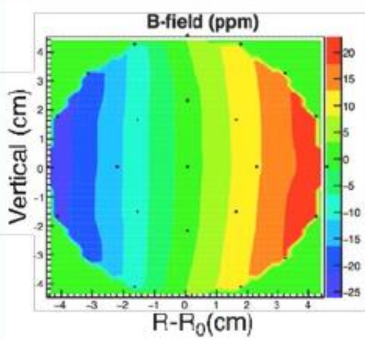
V. Tishchenko | Physics Colloquium BNL April 9, 2021

Rough Shimming Results

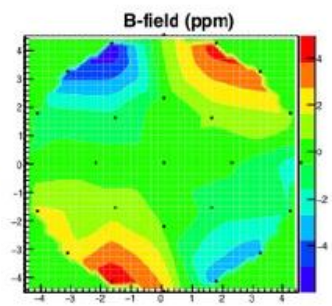
2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | **2016** | 2017 | 2018 | 2019 | 2020 | 2021



Azimuthally-Averaged Map



	Norm	Skew
Quad	25.13	-0.53
Sext	-1.99	-0.11
Octu	-1.16	-0.31
Decu	0.95	-0.07

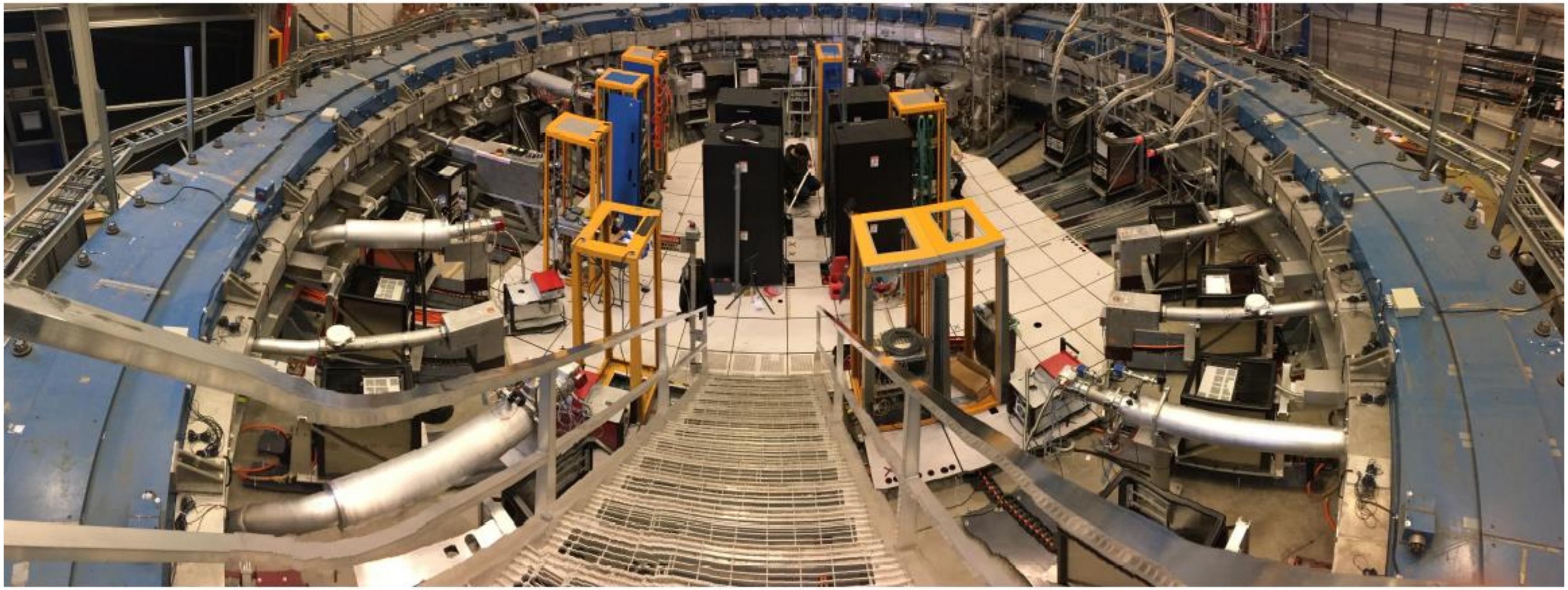


	Norm	Skew
Quad	-0.02	-0.57
Sext	-0.70	3.84
Octu	-0.76	0.56
Decu	0.44	-1.61

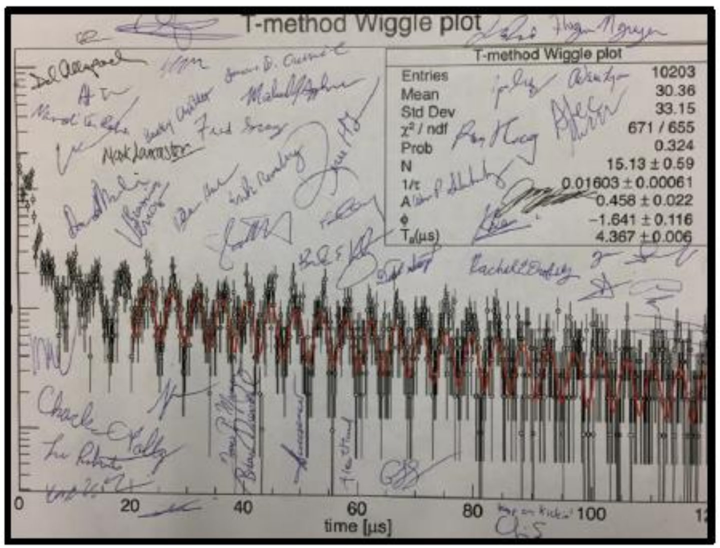
D. Flay

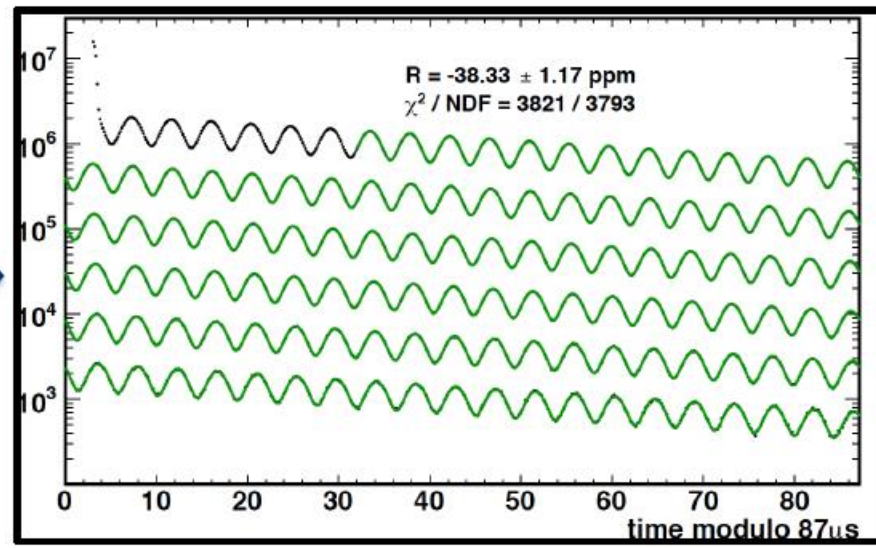
- BNL E821: 39 ppm RMS (dipole), 230 ppm peak-to-peak
- FNAL rough shimming: 10 ppm RMS (dipole), 75 ppm peak-to-peak

FNAL 2017



Data 2017 and 2018



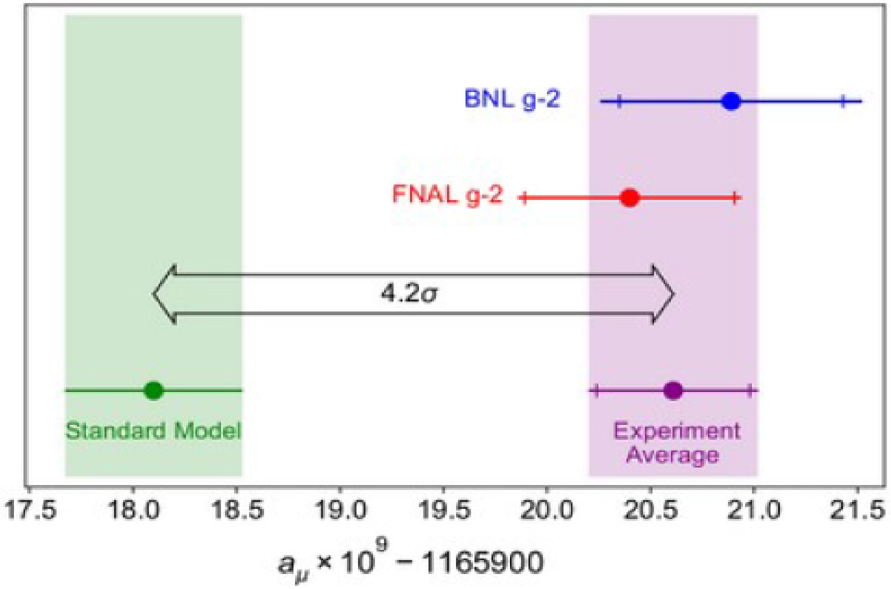


Data Taking and Analysis

- Data Taking 2018 – 2022.
- Analysis is finished for 2018 data.
- ≈10B muon decays. Same as BNL experiment.
- 2022, ≈200B muon decays.

FNAL E989 Run 1 vs. BNL E821

$$a_\mu(\text{FNAL}) = 116\,592\,040(54) \times 10^{-11} \quad (0.46\ \text{ppm}),$$
$$a_\mu(\text{Exp}) = 116\,592\,061(41) \times 10^{-11} \quad (0.35\ \text{ppm}).$$



From top to bottom: Experimental values of a_μ from BNL E821, FNAL E989 Run-1, and the combined average. The inner tick marks indicate the statistical contribution to the total uncertainties. The Muon $g-2$ Theory Initiative recommended value for the Standard Model is also shown.

$$a_\mu(\text{FNAL}) - a_\mu(\text{SM}) = (230 \pm 69) \times 10^{-11}\ (3.3\sigma)$$
$$a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (251 \pm 59) \times 10^{-11}\ (4.2\sigma)$$

Conclusion

- Getting closer to 5σ needed to declare new physics.
- Much more data being analyzed now.
- Theorists are still calculating with super-computers.
- E989 run 2-3 analyzed by mid 2022.