Introduction to QCD and high energy spin physics

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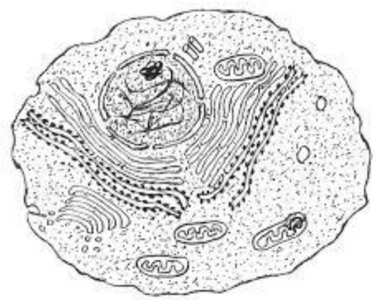
Outline

- Inside the microscopic world of hadrons
- The strong interaction and QCD
- Spin in quantum mechanics
- Physics of proton spin

The smallest unit in Nature

1m

human



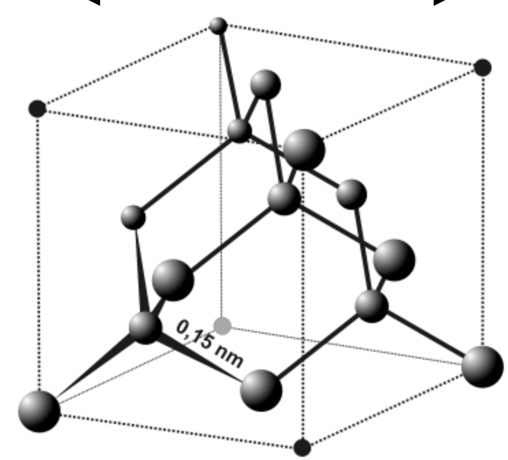
cell, bacteria,..

micrometer

$1\mu\text{m} = 10^{-6}\text{m}$

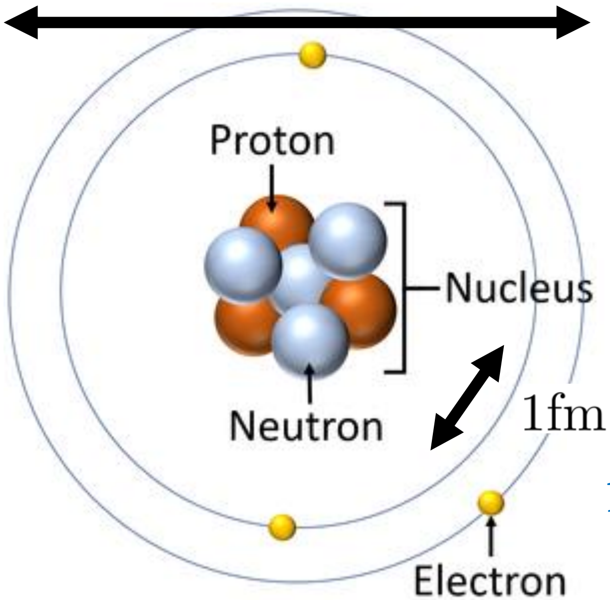
nanometer

$1\text{nm} = 10^{-9}\text{m}$



molecule

10^{-10} m



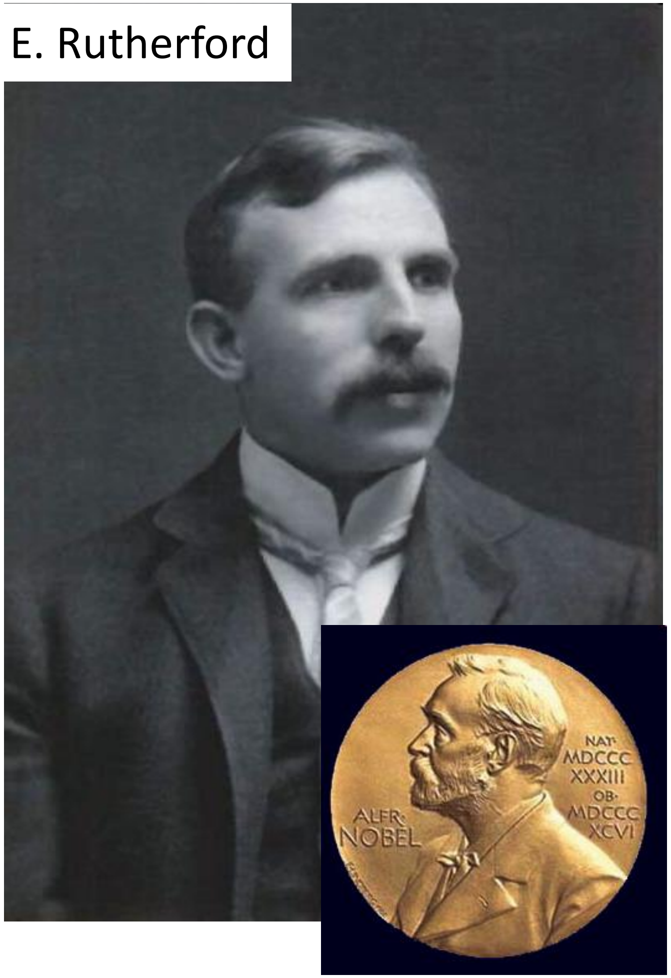
atom

$1\text{fm} = 10^{-15}\text{m}$

femtometer

cf. Planck length 10^{-35}m

Proton



Rutherford discovered the atomic nucleus in 1911.

Then in 1919, he discovered the proton.

In 1932, his student Chadwick discovered the neutron.

Protons and neutrons (`nucleons’) are the building block of all the known elements.

Mass $m_{proton} = 938\mathrm{MeV}/c^2$

Charge $+e$

Spin $\frac{1}{2}$

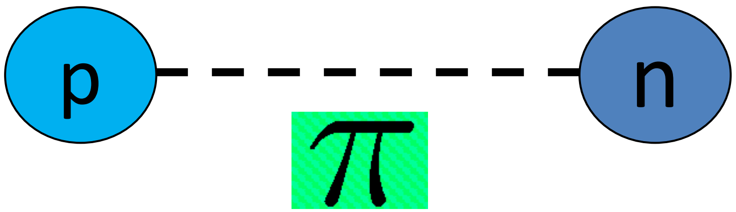
$$1\,\mathrm{MeV}/c^2 = 1.78 \times 10^{-30}\mathrm{kg}$$

$$E = mc^2$$

Pion

Force between particles arises from the exchange of a `mediator' particle, like `playing catch'.

Protons and neutrons are bound together to form a nucleus by exchanging pions.



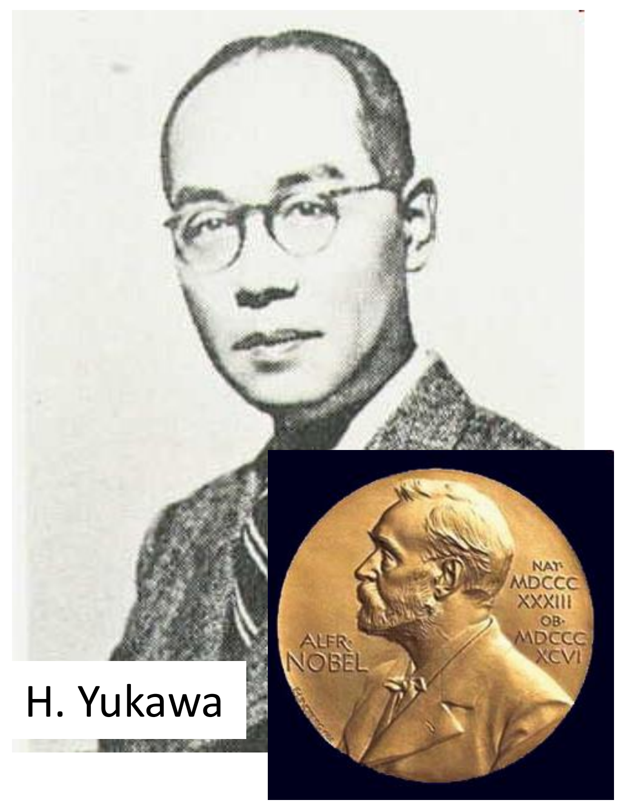
Much stronger than the electromagnetic interaction
Reach of the interaction
=inverse of the mass of the exchanged particle.

$$m_\pi \sim \frac{1}{1\mathrm{fm}} \sim 200\mathrm{MeV}/c^2$$

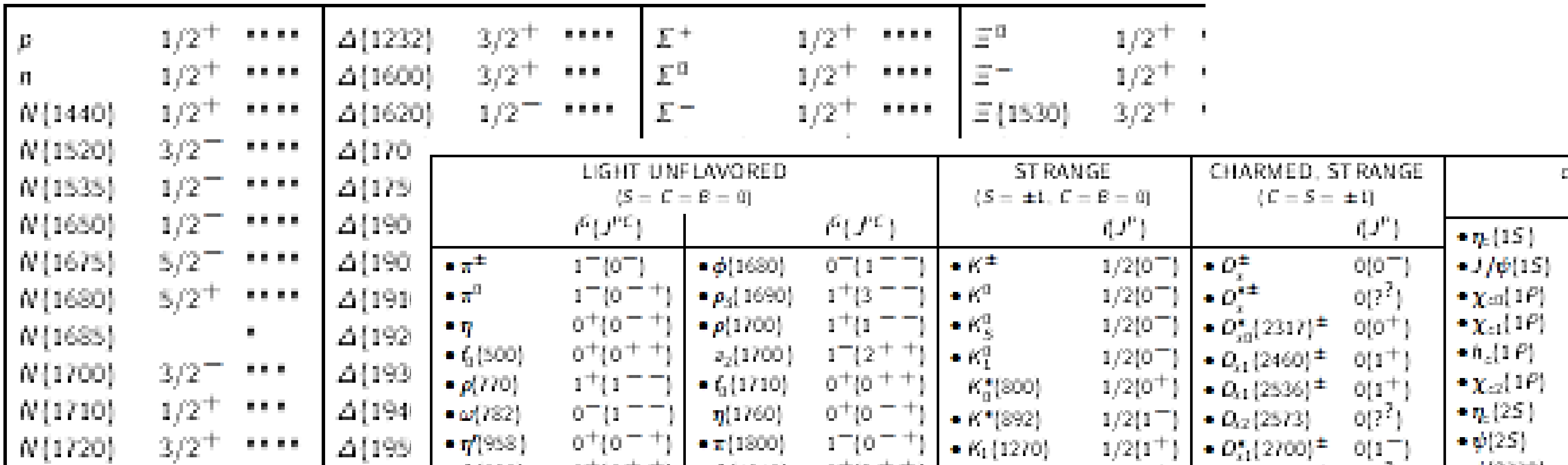
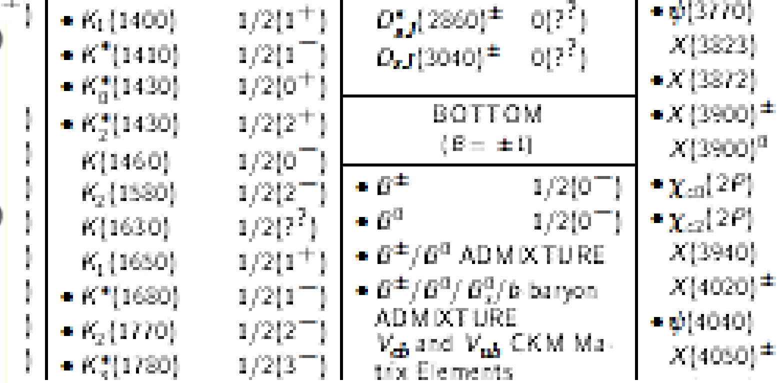
$$\text{distance} = (\text{mass or energy})^{-1}$$

Predicted in 1934, discovered in 1947

$$m_\pi \approx 140\mathrm{MeV}/c^2$$

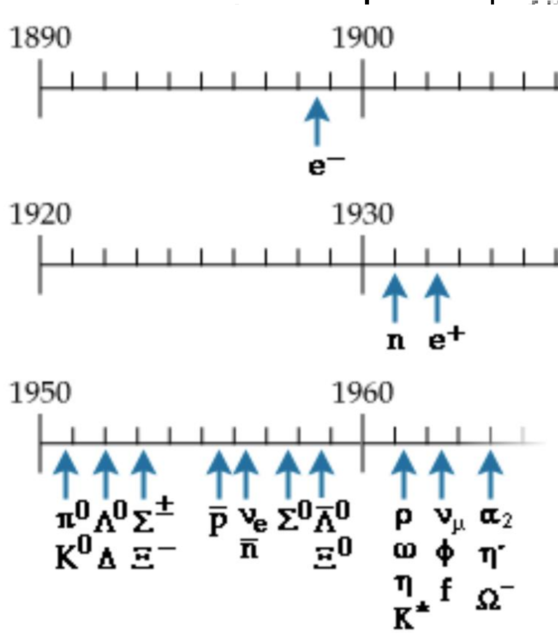


Hadrons (proton, neutron, pion, etc..)

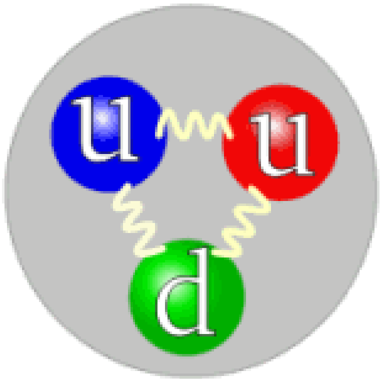
p	1/2+	****	Δ(1232)	3/2+	****	Σ+	1/2+	****	Ξ0	1/2+
n	1/2+	****	Δ(1600)	3/2+	***	Σ0	1/2+	****	Ξ−	1/2+
N(1440)	1/2+	****	Δ(1620)	1/2−	****	Σ−	1/2+	****	Ξ(1530)	3/2+
N(1520)	3/2−	****	Δ(170							
N(1535)	1/2−	****	Δ(175							
N(1650)	1/2−	****	Δ(190							
N(1675)	5/2−	****	Δ(190							
N(1680)	5/2+	****	Δ(191							
N(1685)		*	Δ(192							
N(1700)	3/2−	***	Δ(193							
N(1710)	1/2+	***	Δ(194							
N(1720)	3/2+	****	Δ(195							

LIGHT UNFLAVORED (S = C = B = 0)	$I^G(J^{PC})$		$I^G(J^{PC})$	STRANGE (S = ±1, C = B = 0)	$I(J^P)$	CHARMED, STRANGE (C = S = ±1)	$I(J^P)$	cc̄
• π±	1−(0−)	• φ(1680)	0−(1− −)	• K±	1/2(0−)	• $D_s^\pm$	0(0−)	• $\eta_c(1S)$
• π0	1−(0− +)	• $\rho_3(1690)$	1+(3− −)	• K0	1/2(0−)	• $D_s^{*\pm}$	0(??)	• J/ψ(1S)
• η	0+(0− +)	• ρ(1700)	1+(1− −)	• K_S^0	1/2(0−)	• $D_{s0}^*(2317)^\pm$	0(0+)	• $\chi_{c0}(1P)$
• $f_0(500)$	0+(0+ +)	$a_2(1700)$	1−(2+ +)	• K_L^0	1/2(0−)	• $D_{s1}(2460)^\pm$	0(1+)	• $\chi_{c1}(1P)$
• ρ(770)	1+(1− −)	• $f_0(1710)$	0+(0+ +)	$K_0^*(800)$	1/2(0+)	• $D_{s1}(2536)^\pm$	0(1+)	• $h_c(1P)$
• ω(782)	0−(1− −)	η(1760)	0+(0− +)	• K*(892)	1/2(1−)	• $D_{s2}(2573)$	0(??)	• $\chi_{c2}(1P)$
• η′(958)	0+(0− +)	• π(1800)	1−(0− +)	• $K_1(1270)$	1/2(1+)	• $D_{s1}^*(2700)^\pm$	0(1−)	• $\eta_c(2S)$
				• $K_1(1400)$	1/2(1+)	$D_{sJ}^*(2860)^\pm$	0(??)	• ψ(2S)
				• K*(1410)	1/2(1−)	$D_{sJ}(3040)^\pm$	0(??)	• ψ(3770)
				• $K_0^*(1430)$	1/2(0+)	BOTTOM (B = ±1)		X(3823)
				• $K_2^*(1430)$	1/2(2+)	• B±	1/2(0−)	• X(3872)
				K(1460)	1/2(0−)	• B0	1/2(0−)	• X(3900)±
				$K_2(1580)$	1/2(2−)	• B±/B0 ADMIXTURE		X(3900)0
				K(1630)	1/2(??)	• B±/B0/B_s^0/b-baryon ADMIXTURE		• $\chi_{c0}(2P)$
				$K_1(1650)$	1/2(1+)	V_{cb} and V_{ub} CKM Matrix Elements		• $\chi_{c2}(2P)$
				• K*(1680)	1/2(1−)			X(3940)
				• $K_2(1770)$	1/2(2−)			X(4020)±
				• $K_3^*(1780)$	1/2(3−)			• ψ(4040)
								X(4050)±

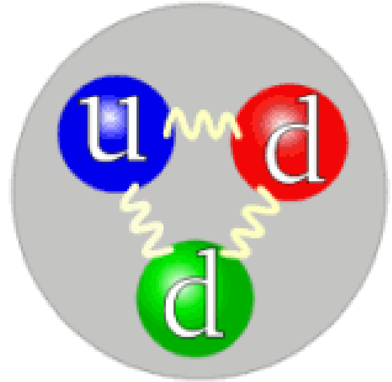


"Explosion" of new hadron species discovered in the 1950s.
They cannot be all elementary particles!

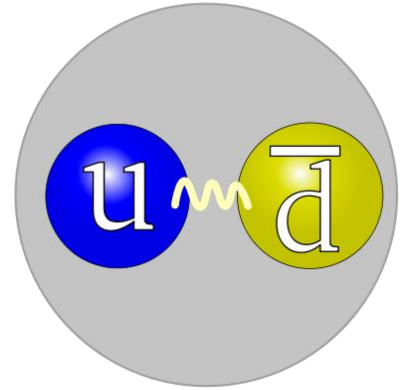
Quarks (1964)



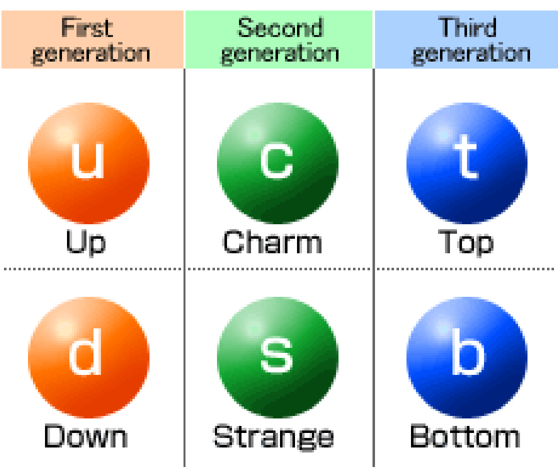
proton



neutron



pion

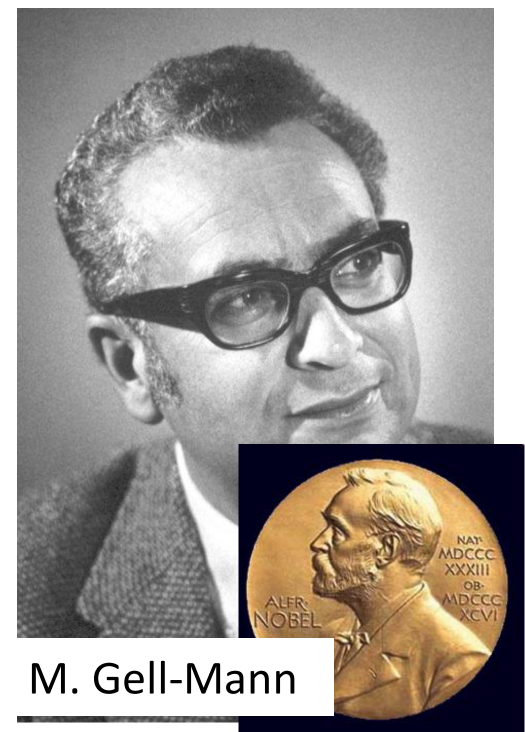


Quarks have fractional electric charges

$$1 = \frac{2}{3} + \frac{2}{3} - \frac{1}{3}$$

$$0 = \frac{2}{3} - \frac{1}{3} - \frac{1}{3}$$

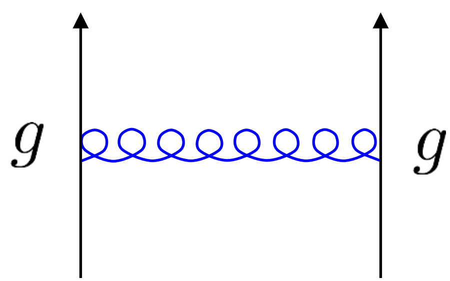
Each quark has its own antiquark.



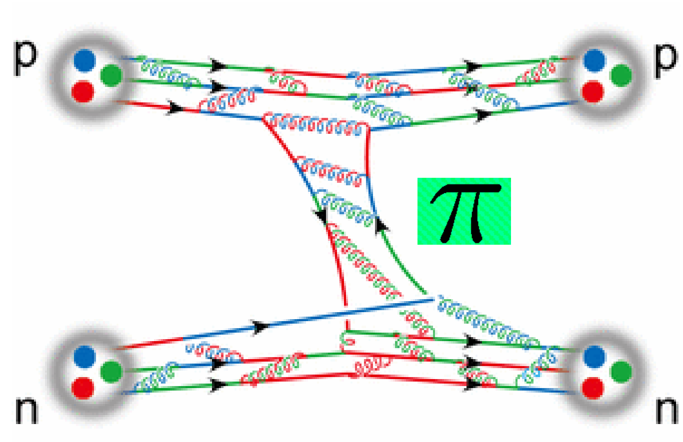
M. Gell-Mann

Gluons

Quarks interact with each other by exchanging massless particles called `gluons',
just like charged particles interact by exchanging photons.



Quarks have color charge
(in addition to electric charge)



pion exchange, more fundamental level

All the four known forces of Nature are explained by the exchange of gauge particles.

Electromagnetic interaction (QED) → photon
Strong interaction (QCD) → gluon
Weak intereaction → W,Z bosons
Gravity → graviton

Theory of the strong interaction

Quantum Chromodynamics

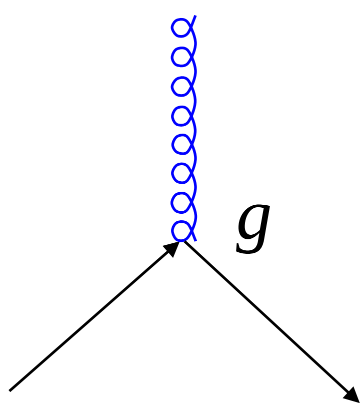
Lagrangian

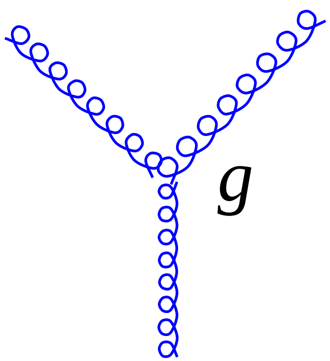
Gauge theory based on the color SU(3) group

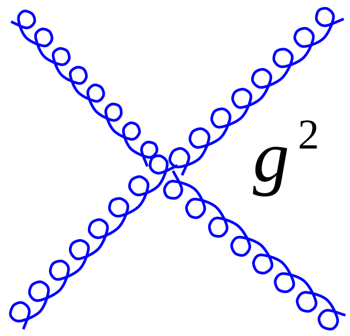
$$L = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \bar{q}(i\gamma^\mu \partial_\mu - m)q + g\bar{q}\gamma^\mu A_\mu q$$

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \qquad u = \begin{pmatrix} u \\ u \\ u \end{pmatrix}$$

red
green
blue

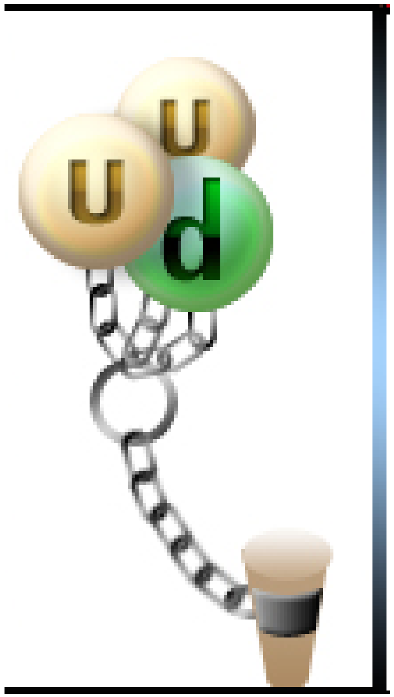


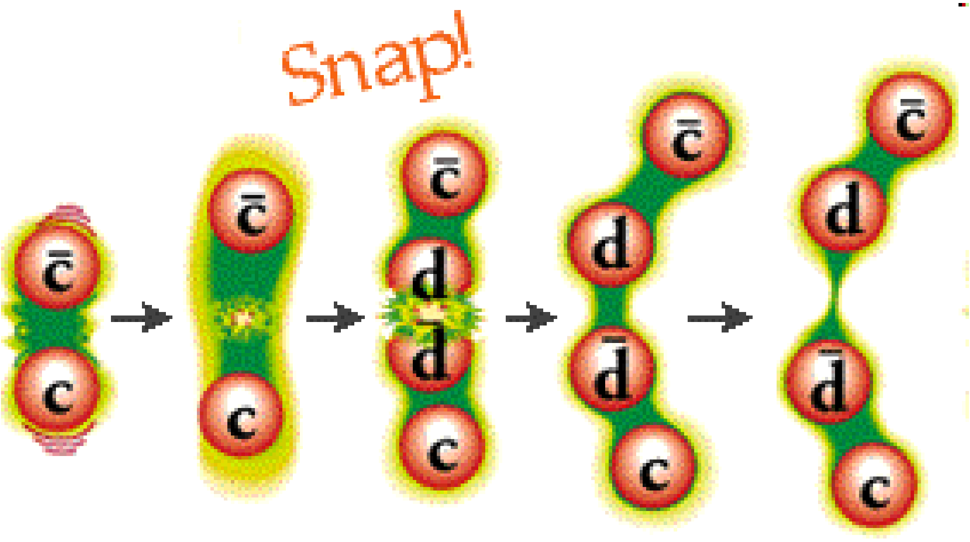




Confinement of color

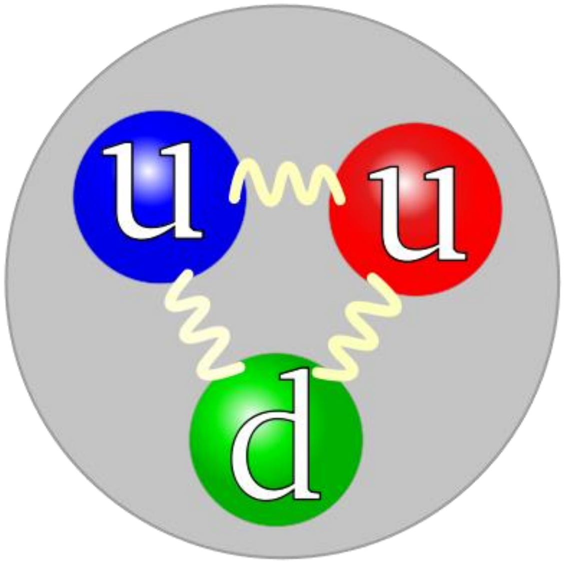
Quarks are permanently confined into hadrons.
Cannot be detected in isolation





Only color neutral particles (hadrons) can be directly observed.

The mystery of proton mass



$$m_u = 3\mathrm{MeV}/c^2$$
$$m_d = 5\mathrm{MeV}/c^2$$
$$m_g = 0$$
$$m_{proton} = 938\mathrm{MeV}/c^2$$

$$1\,\mathrm{MeV}/c^2 = 1.78 \times 10^{-30}\mathrm{kg}$$

$$3 + 3 + 5 + 0 = 938\ !?$$

ABOUT PROGRAMS MILLENNIUM PROBLEMS PEOPLE PUBLICATIONS EUCLID EVENTS

Millennium Problems

Yang–Mills and Mass Gap

Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known.

Riemann Hypothesis

The prime number theorem determines the average distribution of the primes. The Riemann hypothesis tells us about the deviation from the average. Formulated in Riemann's 1859 paper, it asserts that all the 'non-obvious' zeros of the zeta function are complex numbers with real part 1/2.

P vs NP Problem

If it is easy to check that a solution to a problem is correct, is it also easy to solve the problem? This is the essence of the P vs NP question. Typical of the NP problems is that of the Hamiltonian Path Problem: given N cities to visit, how can one do this without visiting a city twice? If you give me a solution, I can easily check that it is correct. But I cannot so easily find a solution.

Navier–Stokes Equation

This is the equation which governs the flow of fluids such as water and air. However, there is no proof for the most basic questions one can ask: do solutions exist, and are they unique? Why ask for a proof? Because a proof gives not only certitude, but also understanding.

Hodge Conjecture

The answer to this conjecture determines how much of the topology of the solution set of a system of algebraic equations can be defined in terms of further algebraic equations. The Hodge conjecture is known in certain special cases, e.g., when the solution set has dimension less than four. But in dimension four it is unknown.

Poincaré Conjecture

In 1904 the French mathematician Henri Poincaré asked if the three dimensional sphere is characterized as the unique simply connected three manifold. This question, the Poincaré conjecture, was a special case of Thurston's geometrization conjecture. Perelman's proof tells us that every three manifold is built from a set of standard pieces, each with one of eight well-understood geometries.

Birch and Swinnerton-Dyer Conjecture

Supported by much experimental evidence, this conjecture relates the number of points on an elliptic curve mod p to the rank of the group of rational points. Elliptic curves, defined by cubic equations in two variables, are fundamental mathematical objects that arise in many areas: Wiles' proof of the Fermat Conjecture, factorization of numbers into primes, and cryptography, to name three.

Millennium prize problems

Clay mathematics institute (2000)

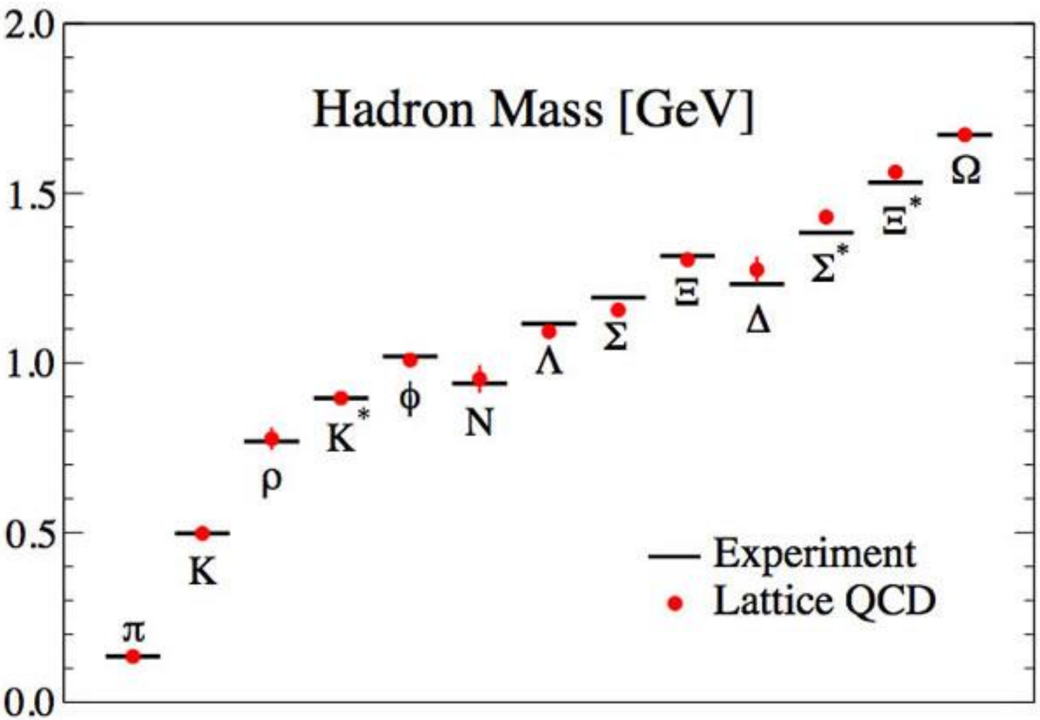
$1 million prize!

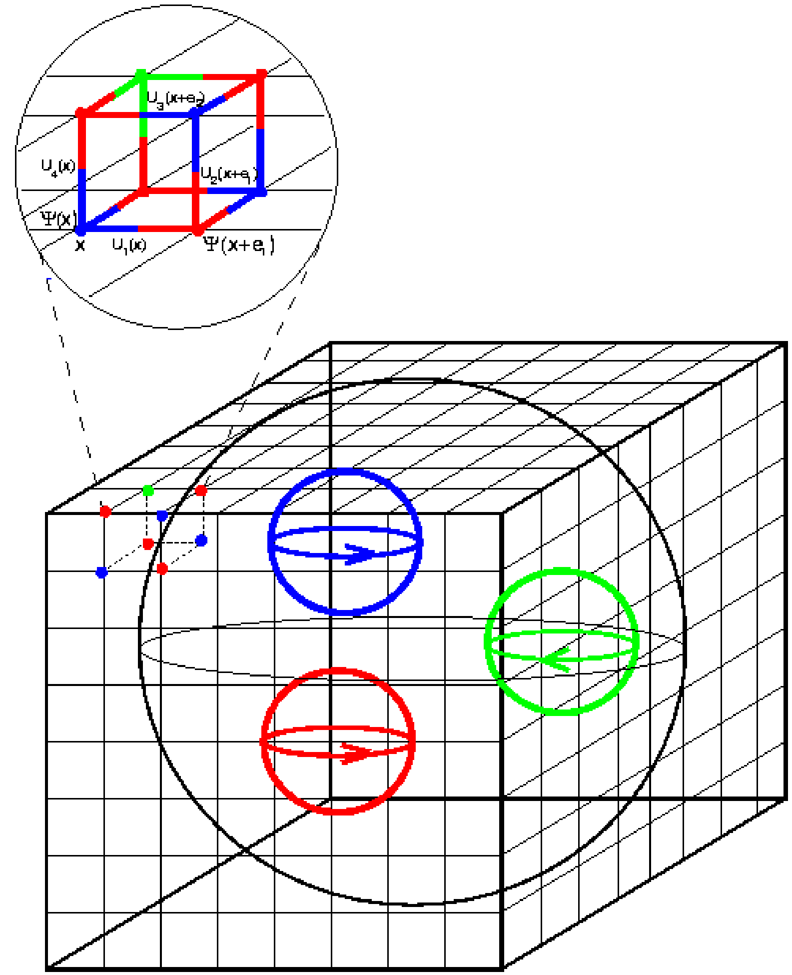
Solved by G. Perelman

"I'm not interested in money or fame; I don't want to be on display like an animal in a zoo."

Hadron mass from lattice QCD simulations

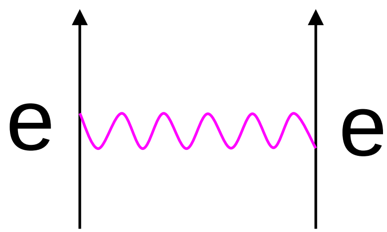
Solve QCD on a discrete lattice using a supercomputer.



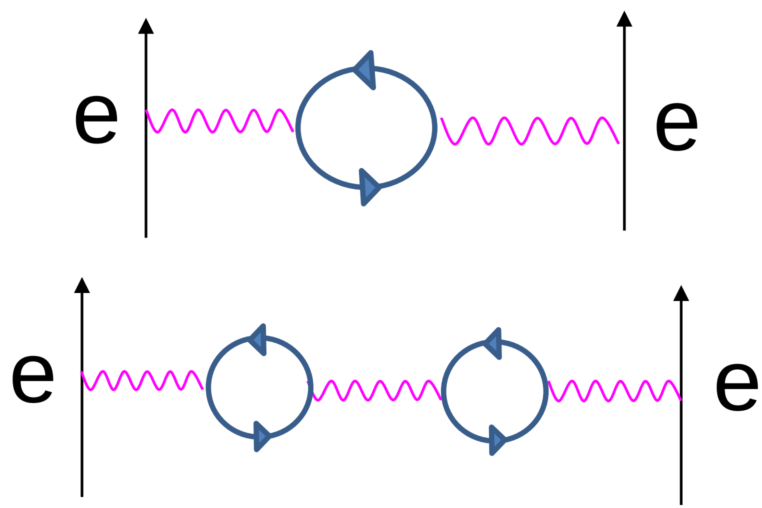


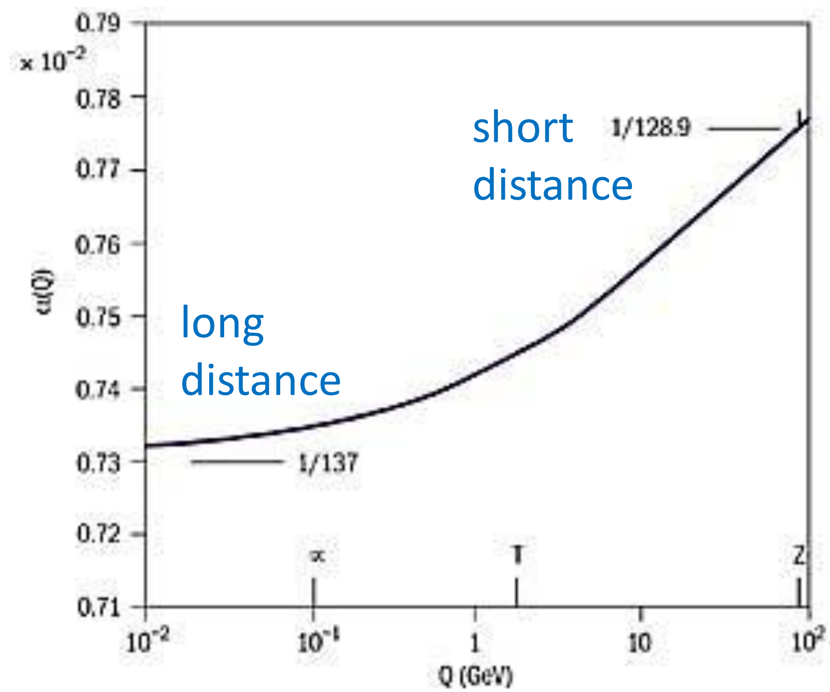
Coupling `constant' is not a constant

Coulomb interaction



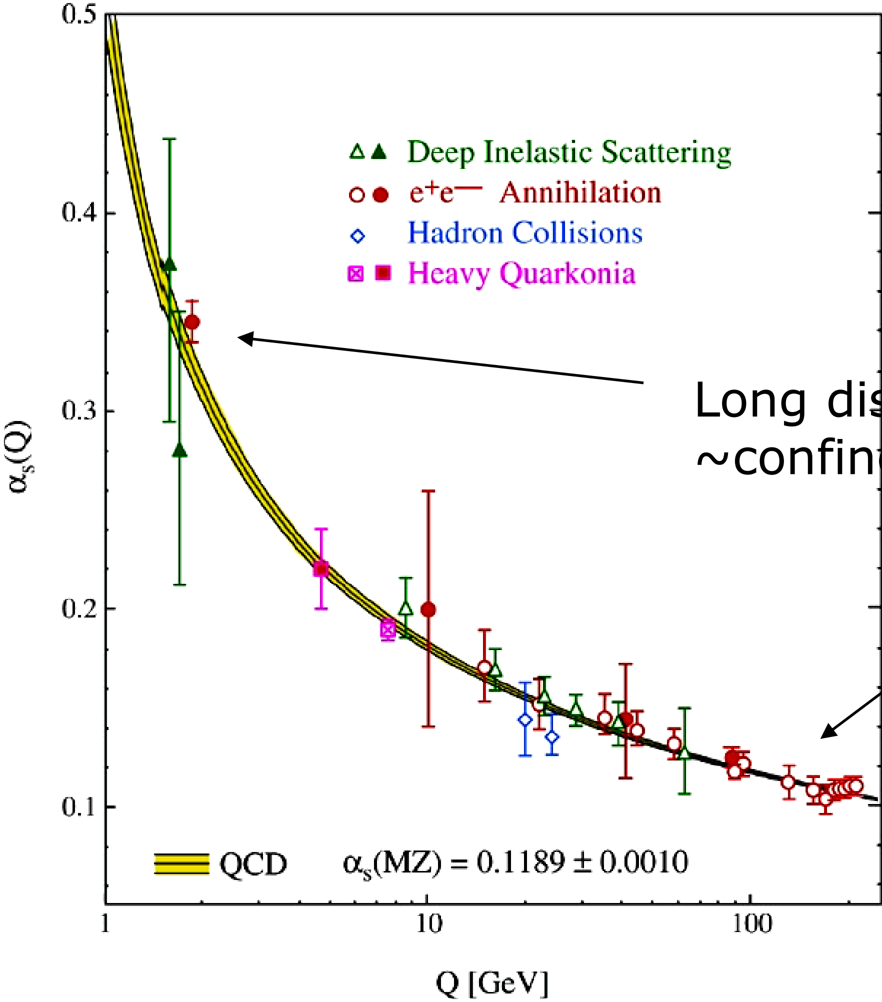
$$\vec{F} \sim \alpha_{em} \frac{\vec{r}}{r^3} \qquad \alpha_{em} = \frac{e^2}{4\pi} \approx \frac{1}{137}$$



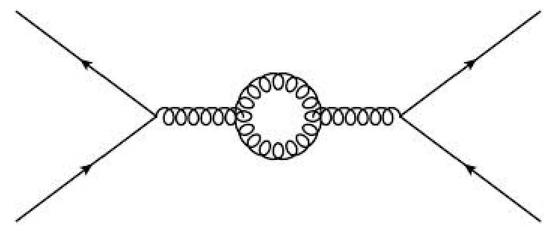


Coupling constant gets weaker at longer distances. → Charge screening

Asymptotic freedom

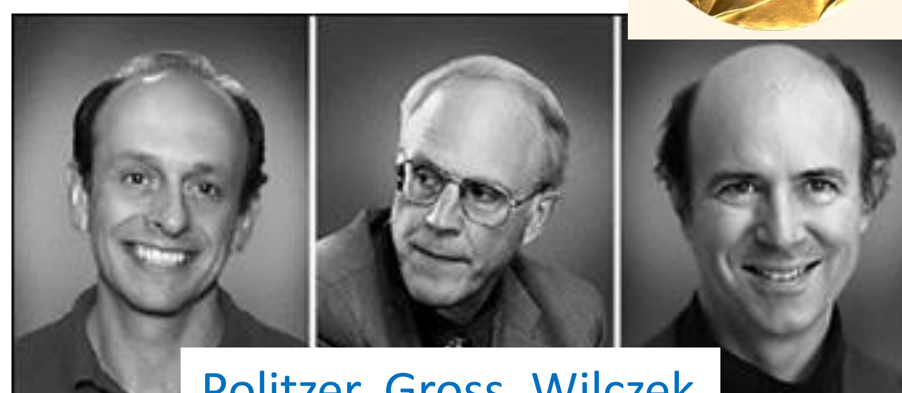


In QCD, the slope has an opposite sign due to a graph like this.



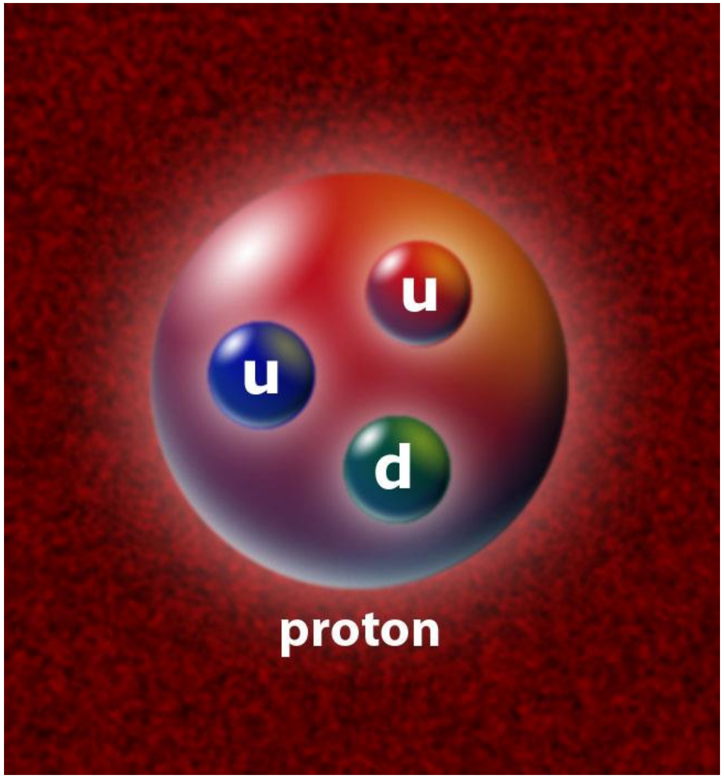
Long distance
~confinement of quarks

Short distance
~perturbation theory



Politzer, Gross, Wilczek

Probing the internal structure of proton



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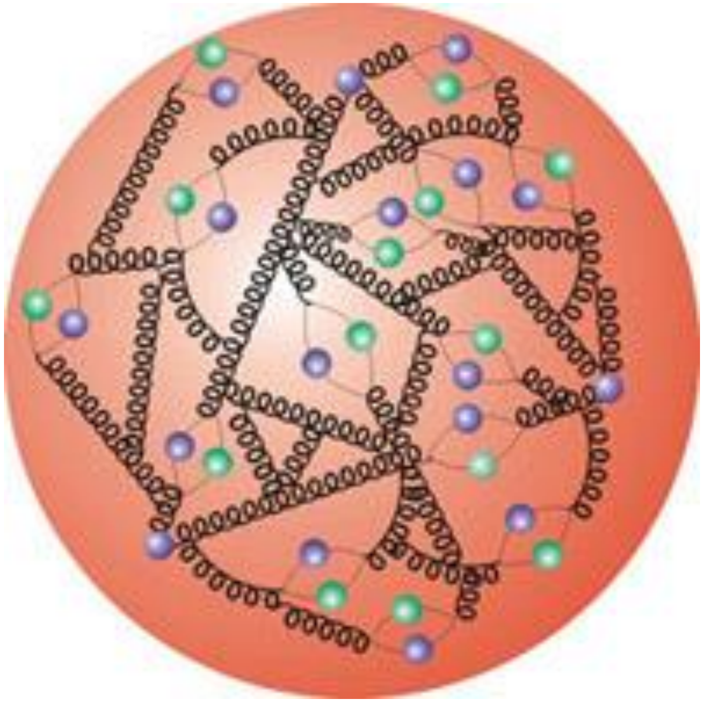


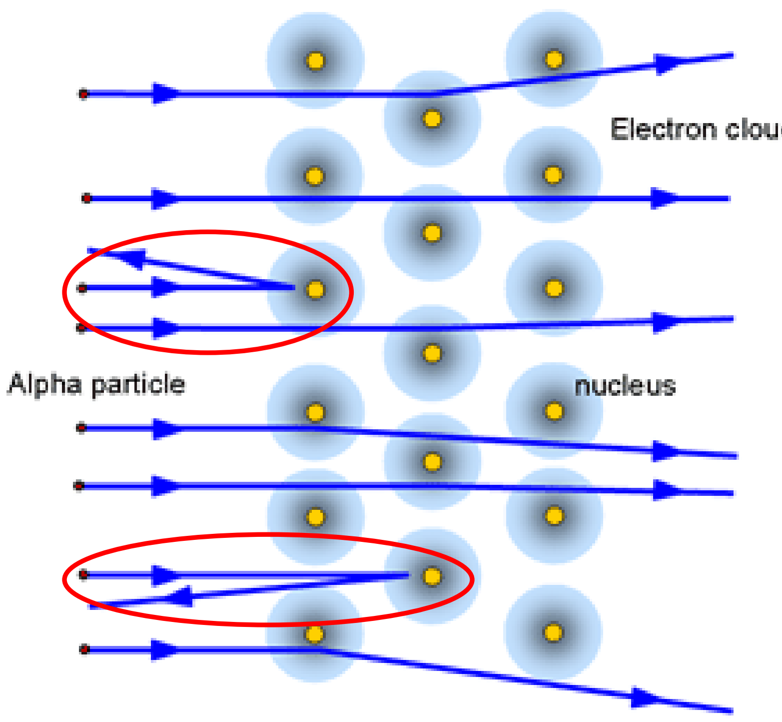
Quarks cannot be directly observed.
Can we probe them indirectly?

Discovery of nucleus (Rutherford, 1911)

Bombard a gold foil with a beam of alpha particles.

Most of the alpha particles pass through, but some bounced back 180 degrees.

There is a small and hard `core' inside an atom → gold nucleus



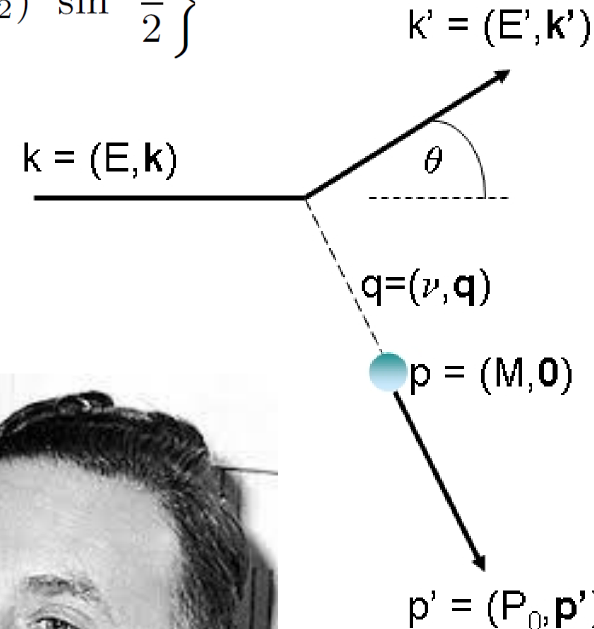
"It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you."

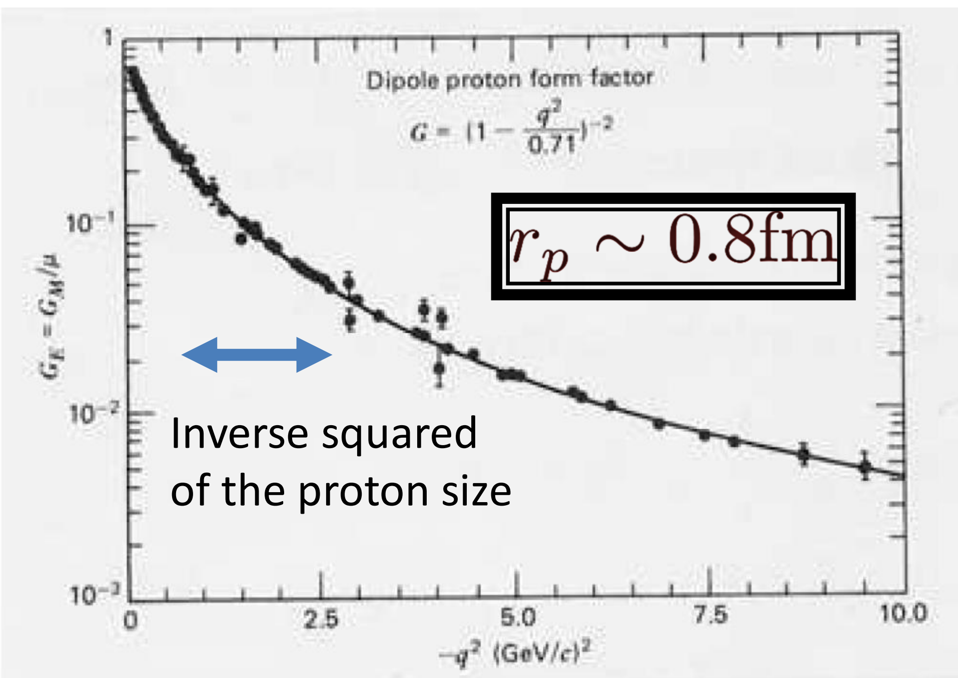
Measuring the proton size in electron scattering

Cross section (probability of scattering)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \frac{E_3}{E_1} \left\{ \left(F_1^2 - \frac{\kappa_p^2 q^2}{4m_p^2} F_2^2 \right) \cos^2 \frac{\theta}{2} - \frac{q^2}{2m_p^2} (F_1 + \kappa_p F_2)^2 \sin^2 \frac{\theta}{2} \right\}$$

$F_{1,2}(q = k - k')$ Form factors

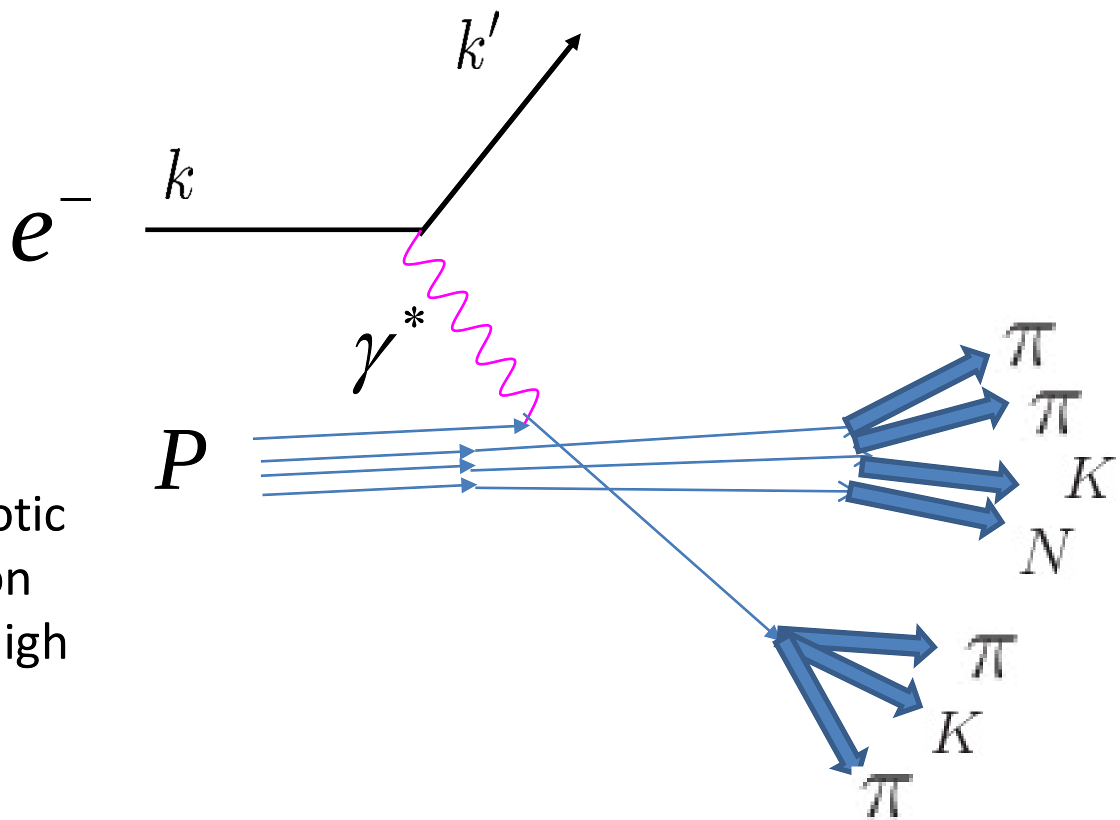




Deep inelastic scattering (DIS)

At higher energy, elastic scattering becomes inelastic.
In this regime, the incoming proton behaves like a bunch of pointlike, non-interacting particles, or `partons' (quarks and gluons).

R. Feynman



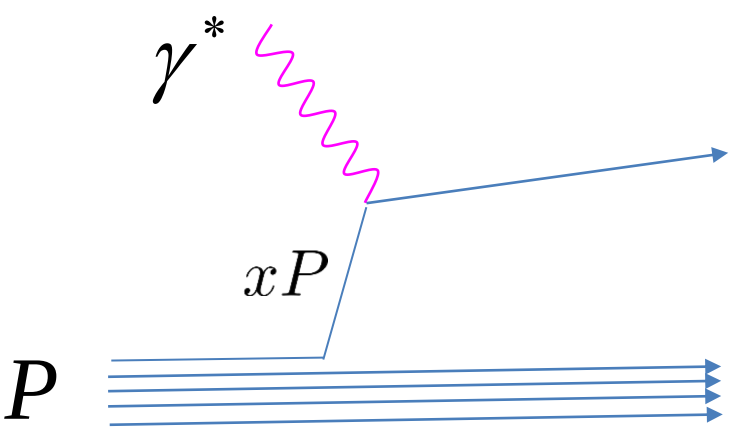
Due to the asymptotic freedom, interaction becomes weak at high energy.

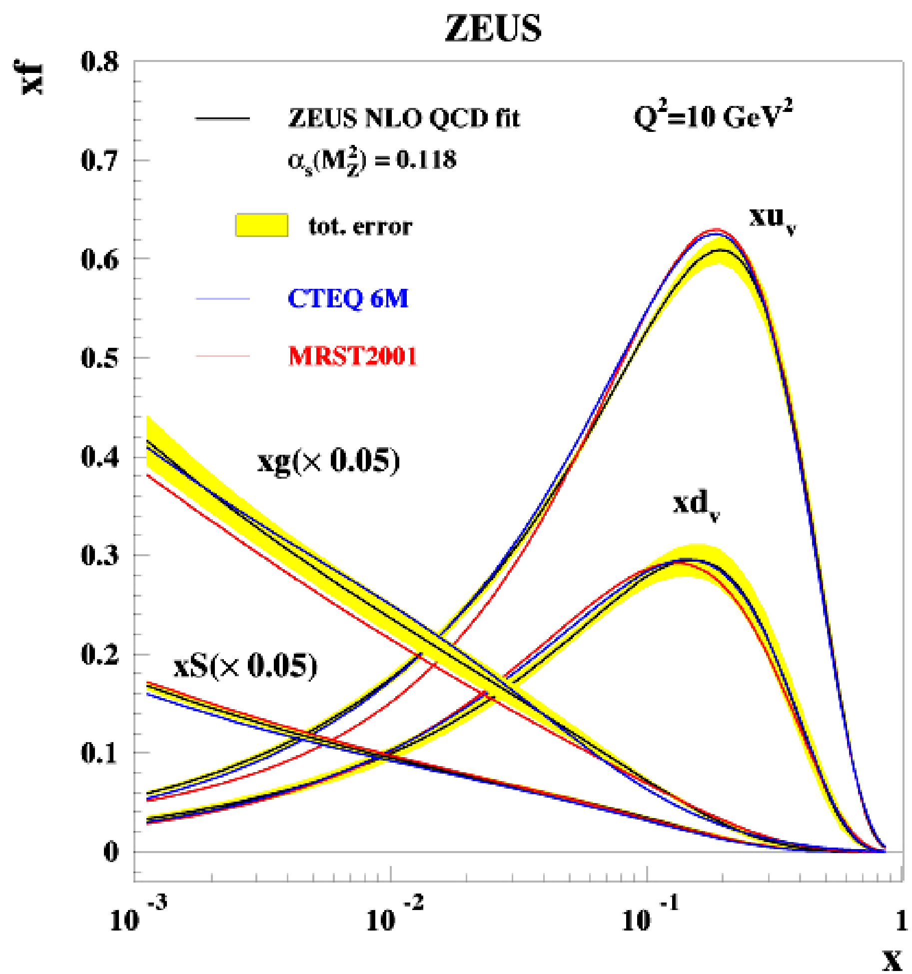
Parton distribution function (PDF)

$$\sigma = \sum_i^{u,d,s,..} \int dx f_i(x) \sigma_i(xP)$$

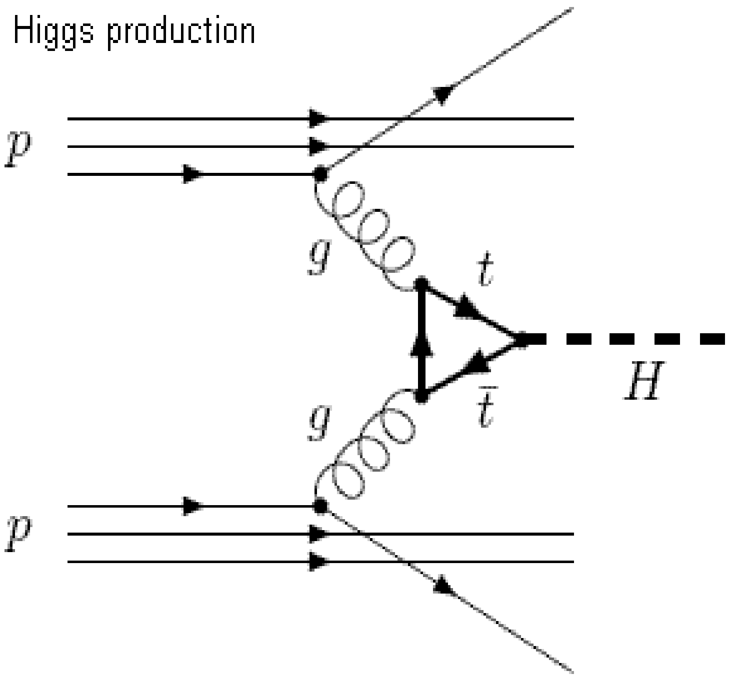
Number density of partons with energy fraction

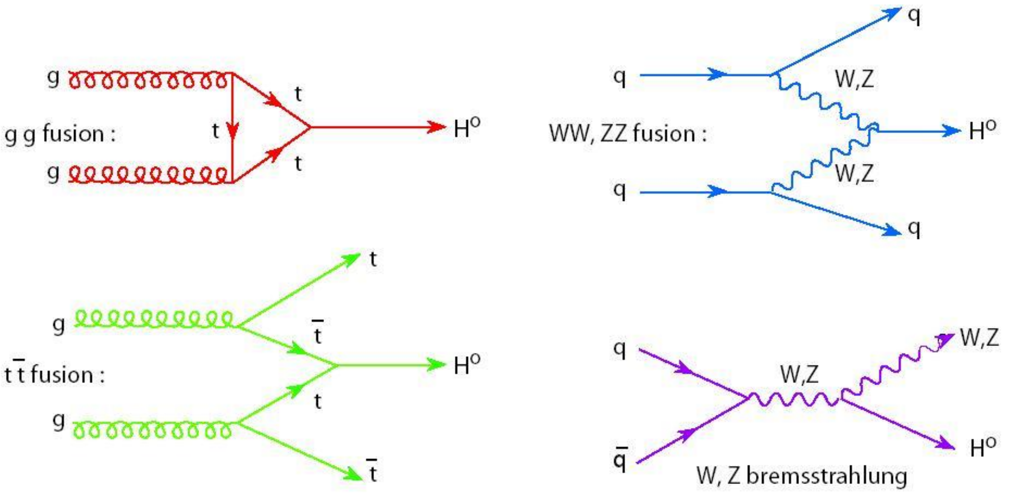
$$x = \frac{E_{parton}}{E_{proton}}$$





Higgs particle production at the Large Hadron Collider (LHC)





$$\sigma = \sum_{ij} \int dx_1 dx_2 f_i(x_1) f_j(x_2) \sigma_{ij}(x_1, x_2)$$

Spin physics

Angular momentum in classical mechanics

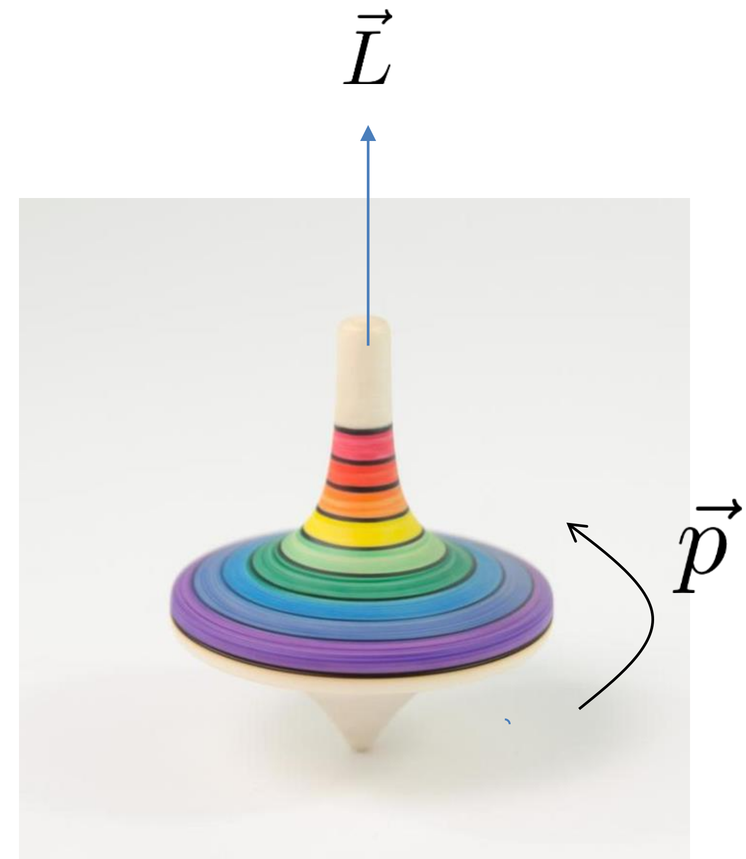
In classical mechanics, angular momentum is a vector perpendicular to the direction of rotation

$$\vec{L} = \vec{r} \times \vec{p}$$

It is conserved in the absence of external force (torque) ← rotational symmetry

$$\frac{d\vec{L}}{dt} = \dot{\vec{r}} \times \vec{p} + \vec{r} \times \dot{\vec{p}} = 0$$

and it can take any (continuous) value.



Angular momentum in quantum mechanics

In quantum mechanics, the (orbital) angular momentum $\vec{L} = \vec{r} \times \vec{p}$

is quantized and takes integer values $|\vec{L}| = 0, 1, 2, \ldots$ in units of $\hbar = \frac{h}{2\pi}$ (Planck constant)

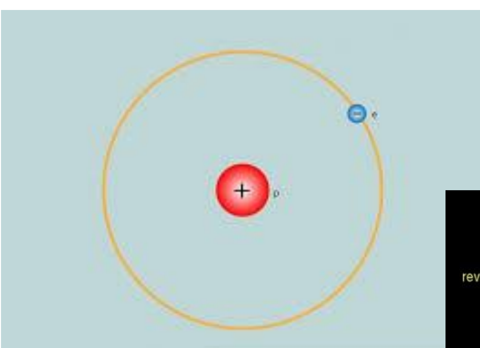
Separately to this, an electron has an intrinsic, half-integer $|\vec{s}| = \frac{1}{2}$ angular momentum called spin.

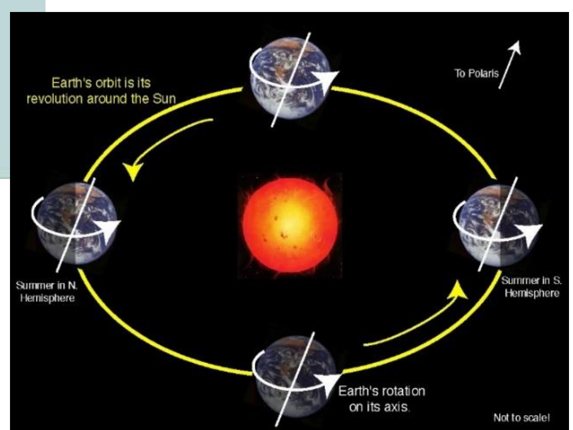
spin (`rotation')

Electron wavefunction in a hydrogen atom

$$\psi(\vec{r}, s) = R(r) Y_{\ell m}(\theta, \phi) \chi(s)$$

orbital (`revolution')





Beware, an electron is a pointlike particle, not really like a `ball' spinning around some axis.

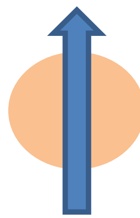
Still, the analogy to a spinning object is convenient for the sake of visualization.

Conception of spin

In 1925, G. Uhlenbeck and S. Goudsmit proposed the notion of spin to explain some puzzling phenomena. Completely ad hoc, but it worked.

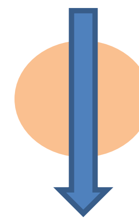
`spin-up'

$$s_z = \frac{1}{2}$$



`spin-down'

$$s_z = -\frac{1}{2}$$



"This is a good idea. Your idea may be wrong, but since both of you are so young without any reputation, you would not lose anything by making a stupid mistake." —— P. Ehrenfest

Earlier that same year, R. Kronig came up with the same idea, but he did not publish because

"It is indeed very clever but of course has nothing to do with reality."

—— W. Pauli

Today, spin is a fundamentally important concept in particle, nuclear and condensed matter physics.

Spin in particle physics

To really understand spin, you need to learn

1. special relativity
2. relativistic quantum mechanics
3. group theory (Lorentz group)

Every elementary particle has a unique value of spin

$$s = 0, \frac{1}{2}, 1, \ldots$$

Integer spin → bosons (photon, gluon, W,Z boson, Higgs,…)
Half-integer spin → fermions (quark, electron, neutrino…)

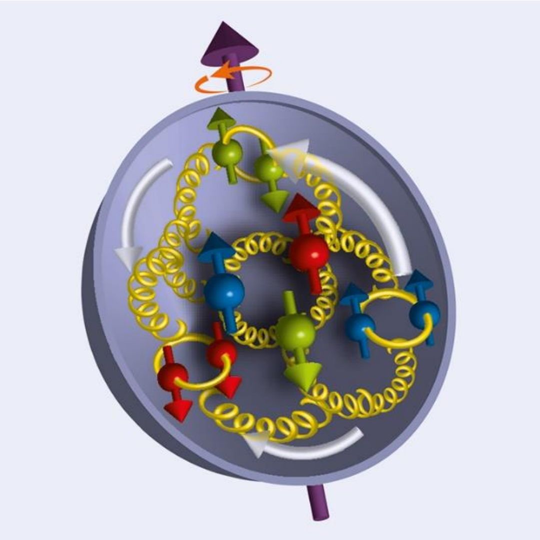
P. Dirac

Dirac equation

$$(i\gamma^\mu(\partial_\mu - igA_\mu) - m)q = 0$$

4x4 matrices

Understanding the origin of proton spin



The proton has spin-1/2

The proton is not an elementary particle.

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_z$$

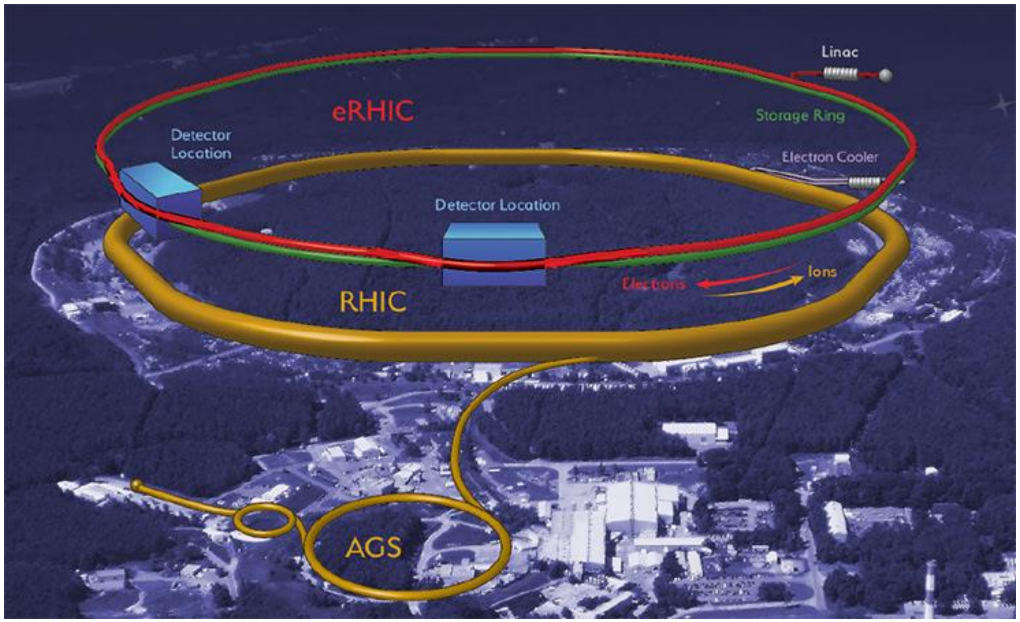
quark spin　　gluon spin　　orbital angular momentum

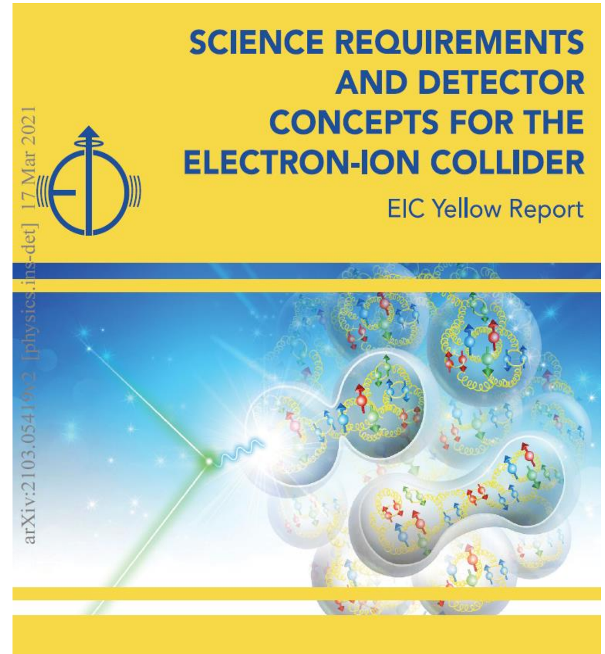
The spin of a composite particle can be ultimately understood by combining the angular momenta (spin+orbital) of elementary constituent particles.

Electron-Ion Collider (EIC)

→ Lecture by Elke Aschenauer

Next-generation nuclear physics facility to be built at BNL





EIC yellow report
(902 pages, 415 authors, 151 institutions)

National Science Academy report

Finding 1: An EIC can uniquely address three profound questions about nucleons-protons—and how they are assembled to form the nuclei of atoms:

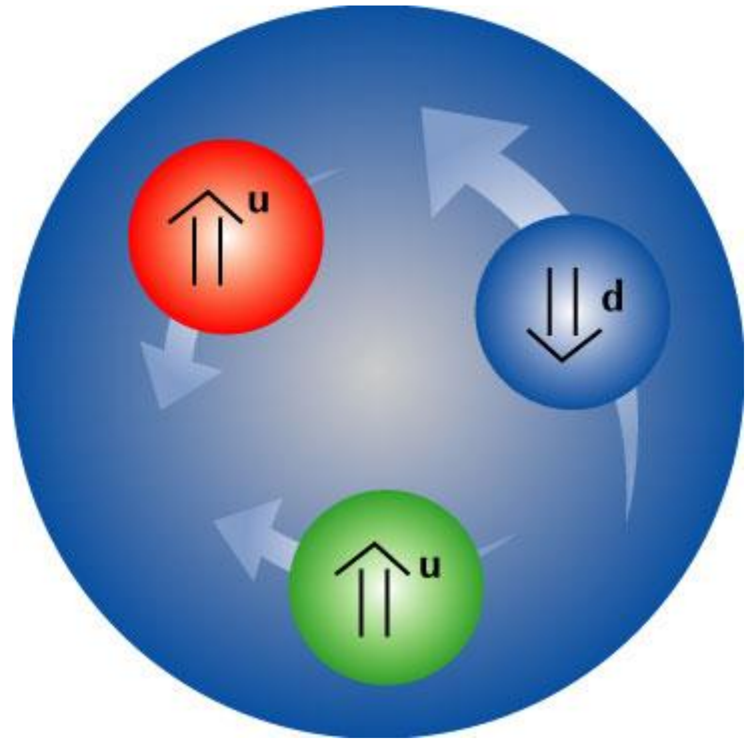
- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

A naive expectation

A proton consists of 2 up-quarks and 1 down-quark.

$$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} - \frac{1}{2}$$

$$\Rightarrow \quad \Delta\Sigma = 1$$



After taking into account the motion of quarks inside the proton,

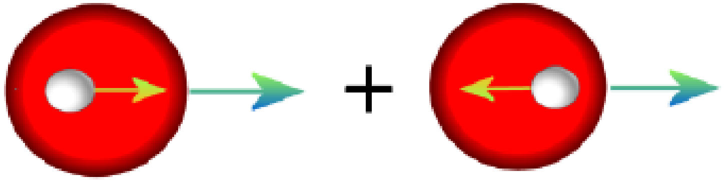
$$\Delta\Sigma \approx 0.7$$

Polarized parton distribution function

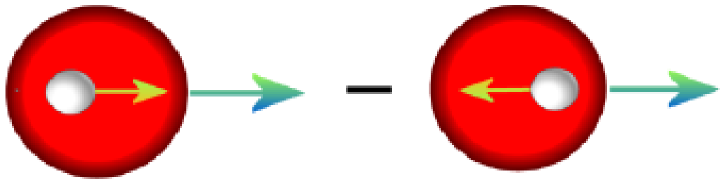
Polarize : orient spins in one particular direction

Parton distribution function (PDF)

$$f(x) = f_{\uparrow}(x) + f_{\downarrow}(x)$$



Polarized PDF for a proton
(distribution of net spin in the direction of motion)

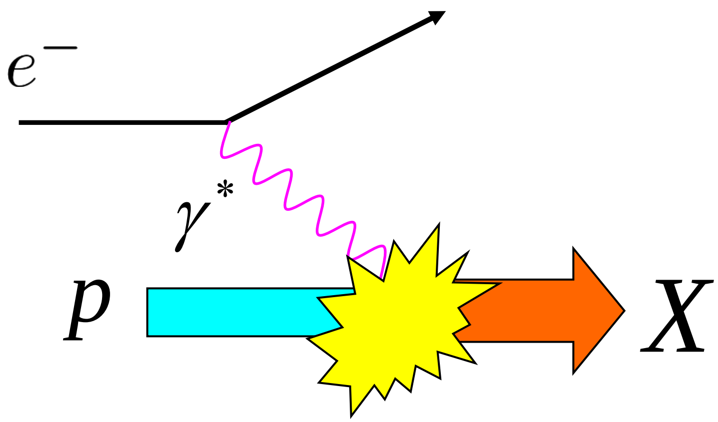


$$\Delta f(x) = f_{\uparrow}(x) - f_{\downarrow}(x)$$

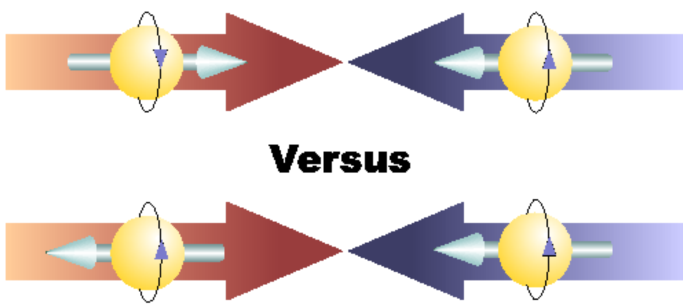
$$\Delta\Sigma = \sum_{f}^{u,d,s,..} \int_0^1 dx \Delta f(x)$$

Measurement of $\Delta\Sigma$ in polarized DIS

Both electron and proton are longitudinally polarized (direction of spin = direction of motion)



$$A_{LL} = \frac{e^{\uparrow}p^{\downarrow} - e^{\uparrow}p^{\uparrow}}{e^{\uparrow}p^{\downarrow} + e^{\uparrow}p^{\uparrow}} \sim \left(1 + \frac{\sigma_L}{\sigma_T}\right)\frac{2x g_1}{F_2}$$



Spin opposite direction

Spin same direction

$$\int_0^1 dx\, g_1(x) = \frac{1}{9}\Delta\Sigma + \cdots$$

`Spin crisis'

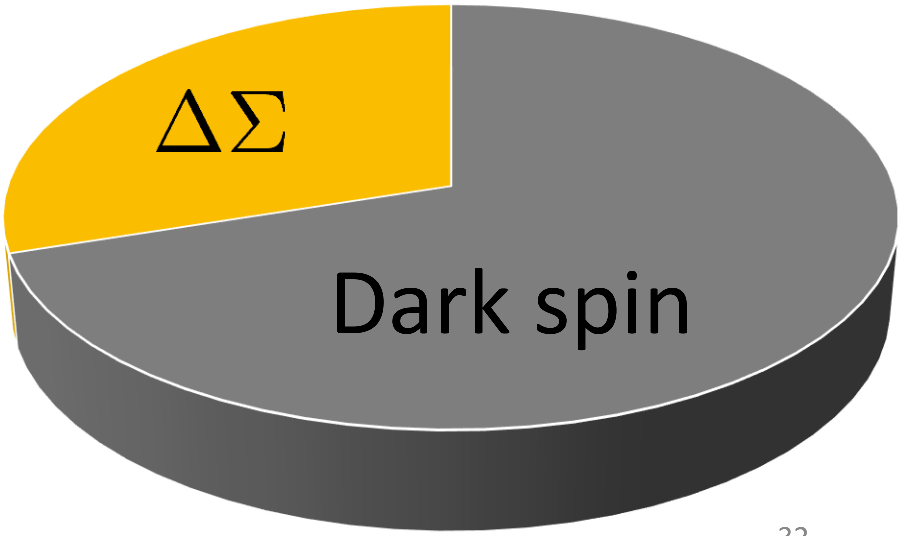
In 1987, the European Muon Collaboration at CERN announced a very small value of the quark spin contribution to proton spin

$$\Delta\Sigma = 0.12 \pm 0.09 \pm 0.14 \ !?$$

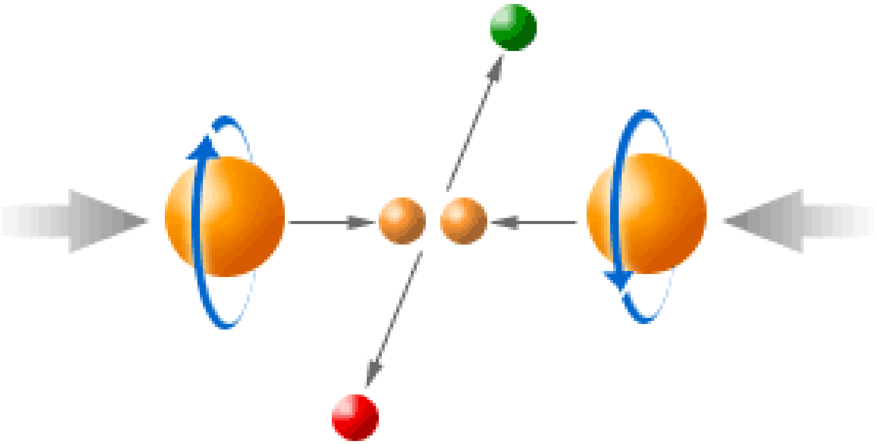
Recent value

$$\Delta\Sigma = 0.25 \sim 0.3$$

Still significantly less than 1.



RHIC spin project



For 20 years, RHIC (relativistic heavy-ion collider) has been colliding polarized protons to study the spin structure of the proton (in parallel with the heavy-ion program ← lecture by R. Pisarski).

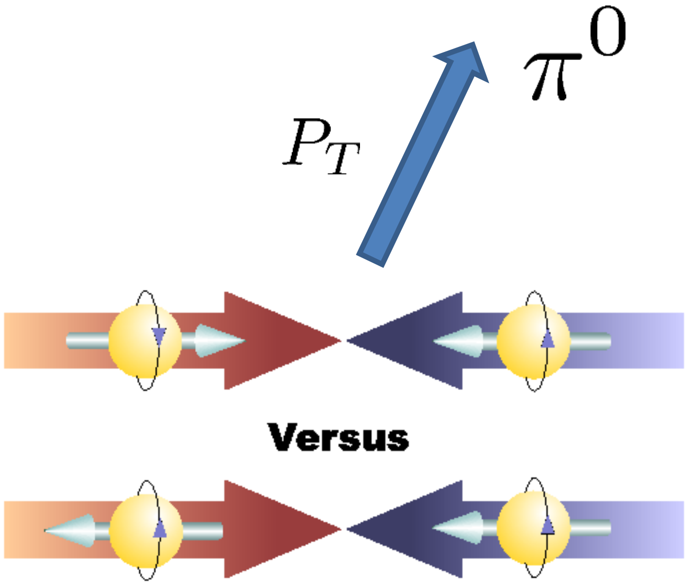
RHIC is the only machine in the world that can produce a high energy polarized proton beam.

The main physics goal is to pin down the gluon spin contribution

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_z$$

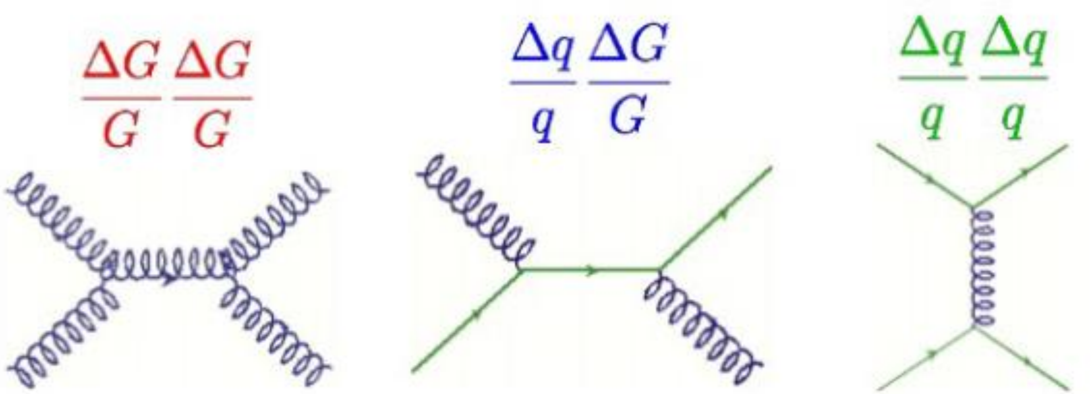
Determination of ΔG at RHIC

Pion production in polarized proton-proton collisions



$$A_{LL} = \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

$$\propto \sum_{a,b} \Delta f_a \otimes \Delta f_b(x) \otimes \Delta\sigma_{ab}$$



The result

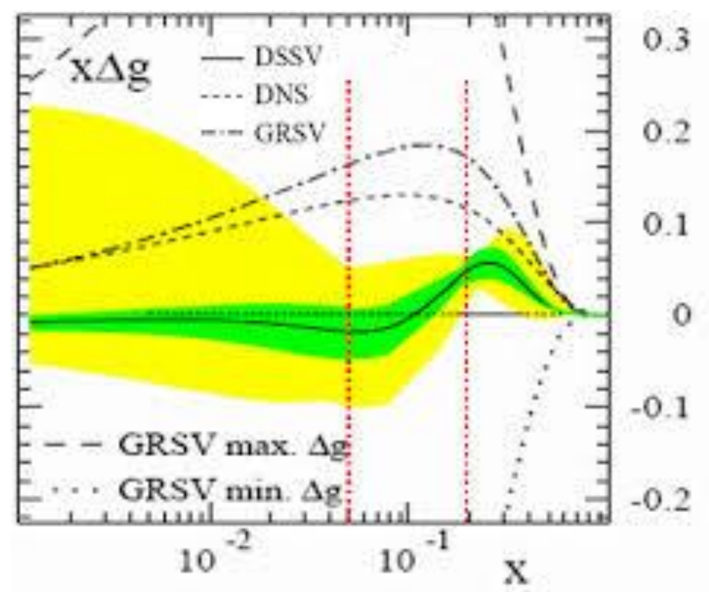
$$\int_{0.05}^{1} dx\Delta g(x,Q^2=10GeV^2) = 0.20^{+.06}_{-.07} \qquad \text{DSSV++}$$

$$\int_{0.05}^{0.2} dx\Delta g(x,Q^2=10GeV^2) = 0.17+-0.06 \qquad \text{NNPDFpol1.1}$$

$$\int_{0.001}^{0.8} dx\Delta g(x,Q^2 = 1\ GeV^2) = 0.5+-0.4 \qquad \text{JAM15}$$

Huge uncertainties from the small-x region. This will be settled at the EIC.

EIC should also address the orbital angular momentum contribution.



Computing (polarized) PDF at small-x

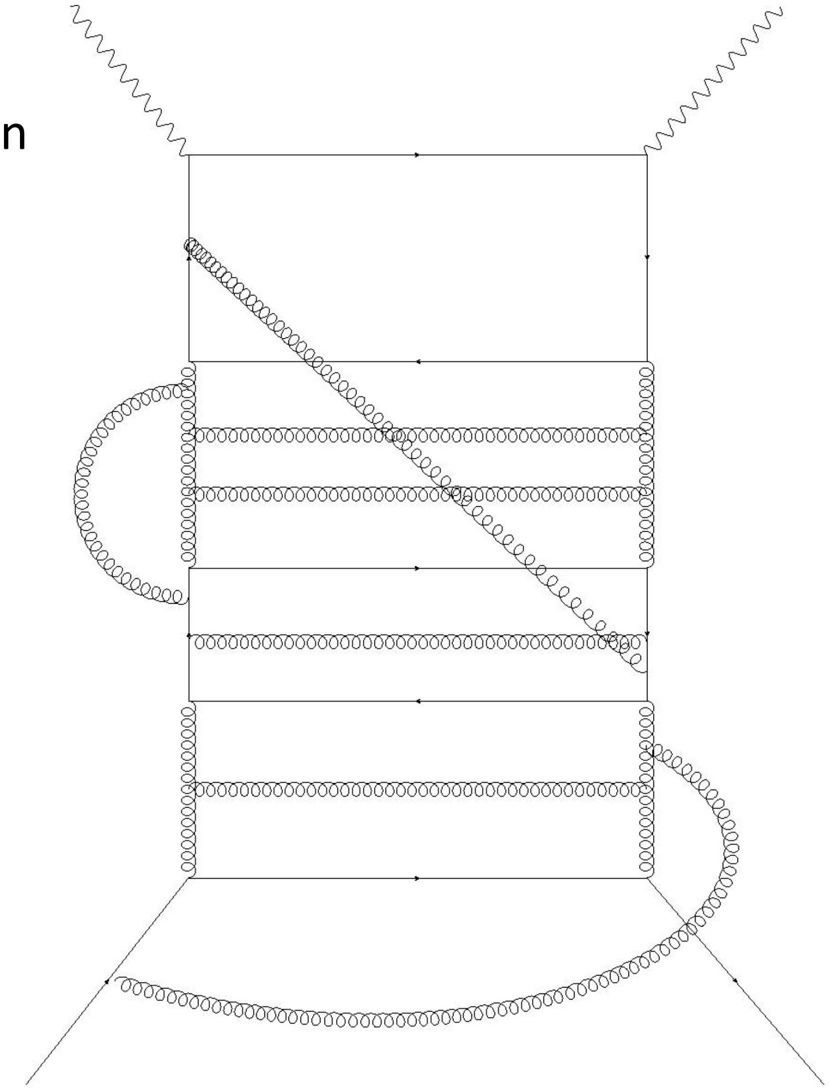
The behavior of $\Delta G(x)$ etc. in the small-x region can be calculated in perturbation theory (expansion in powers of $\alpha_s = \frac{g^2}{4\pi}$)

Add one gluon → add one power of α_s

One has to sum infinitely many gluons.

$$\Delta G(x) \sim \left(\frac{1}{x}\right)^{\#\sqrt{\alpha_s}}$$

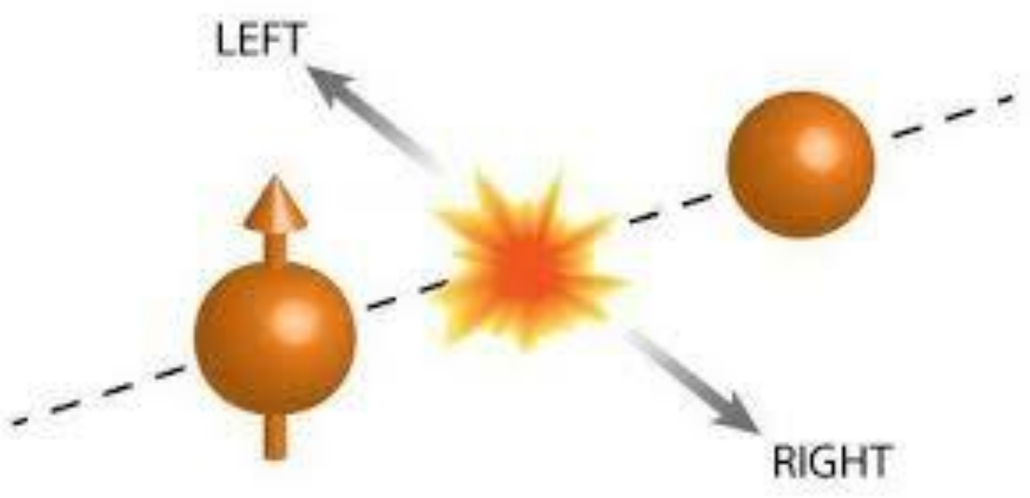
$$L_g(x) \approx -2\Delta G(x)$$



Boussarie, Hatta, Yuan (2019)

Physics of transverse spin

Polarize spin perpendicularly to the direction of motion and hit an unpolarized target.



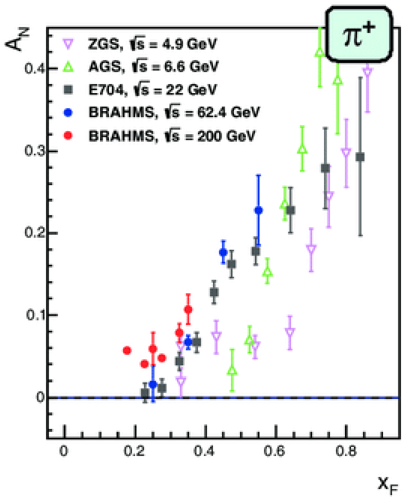
The number of particles produced on the left hand side and right hand side are different.

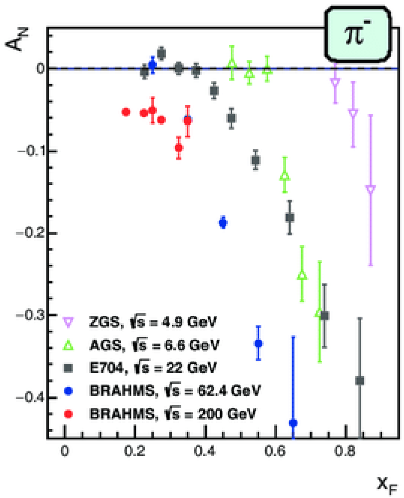
Single spin asymmetry

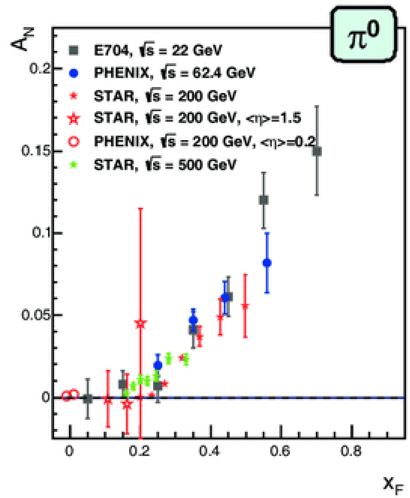
Single spin asymmetry at RHIC

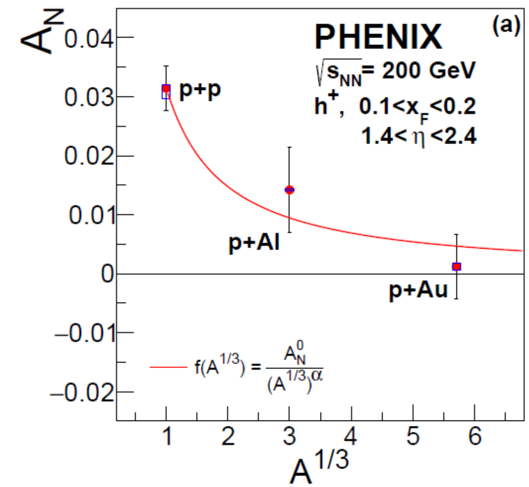
Up to 40% left-right asymmetry

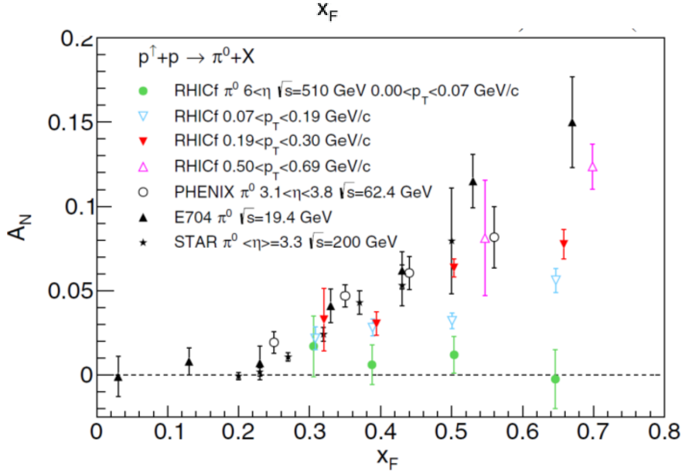
$$A_N = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

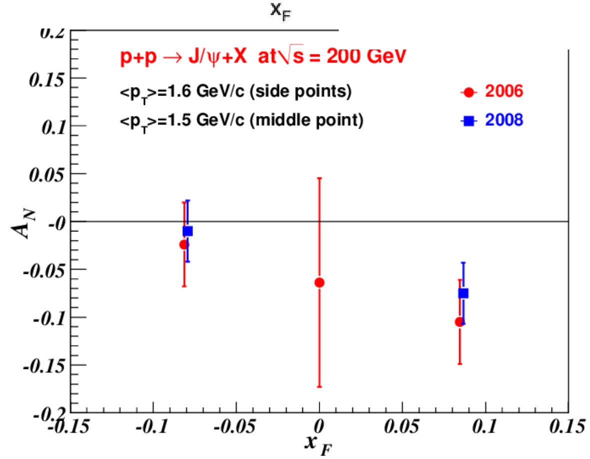












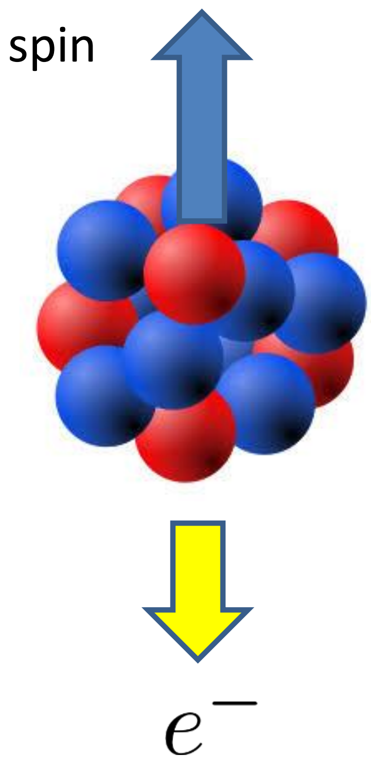
...but why is there asymmetry in the first place?

Parity

The laws of physics are unchanged under the inversion of coordinates $\vec{r} \rightarrow -\vec{r}$

The first experiment which demonstrated parity violation in weak interaction (Wu, 1956)

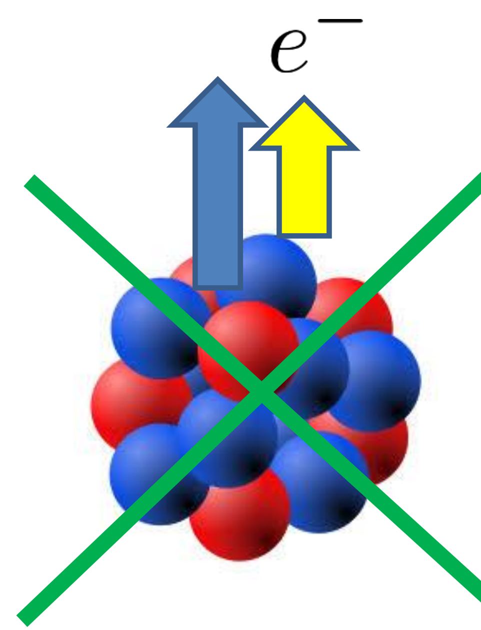
beta-decay $^{60}_{27}\mathrm{Co} \longrightarrow {}^{60}_{28}\mathrm{Ni} + e^{-} + \bar{\nu}_e$



Parity transformation

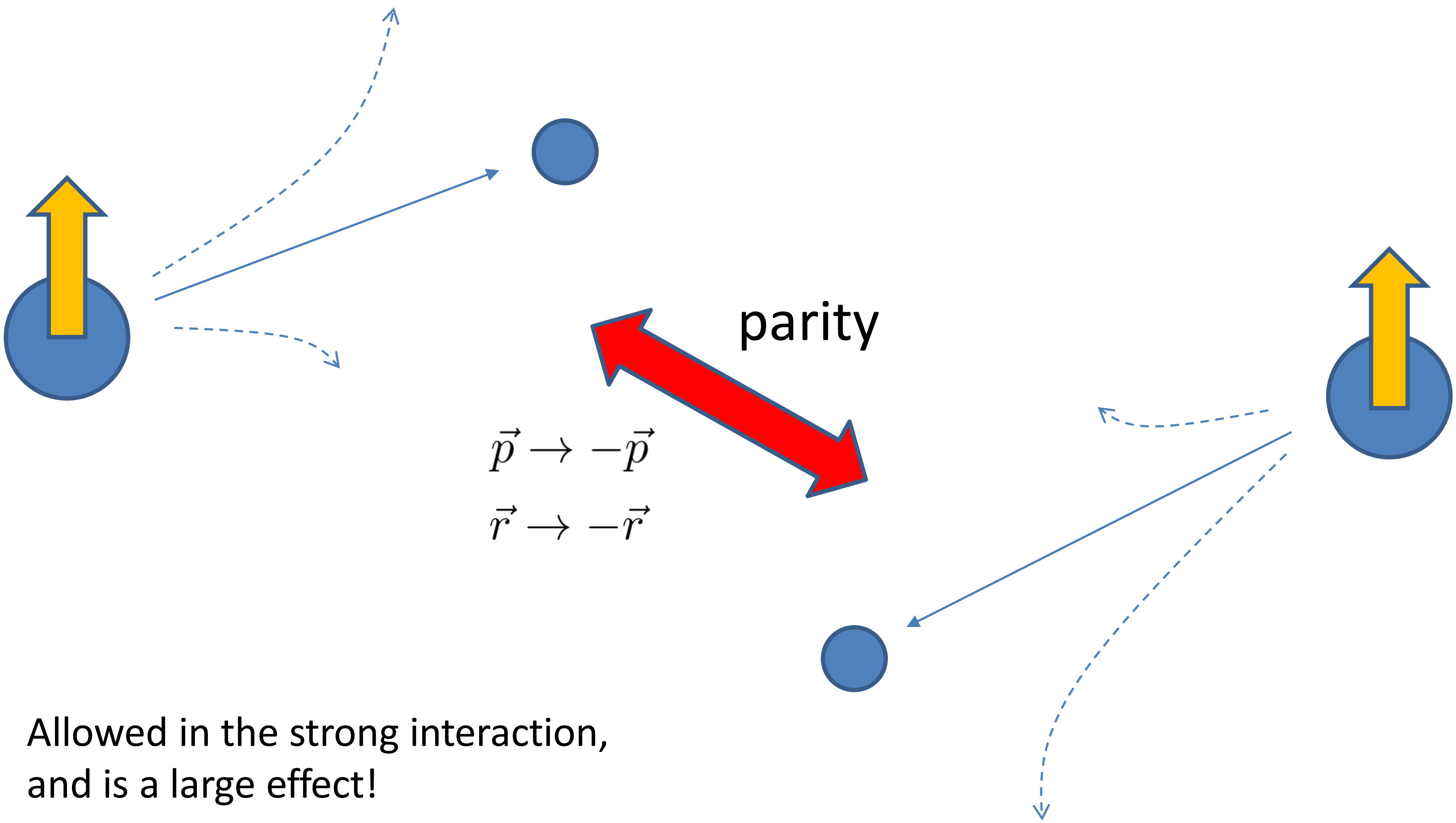
$$\vec{r} \rightarrow -\vec{r},\ \vec{p} \rightarrow -\vec{p}$$

$$\vec{L} = \vec{r} \times \vec{p} \rightarrow \vec{L}$$



Spin vector does not flip signs → `pseudovector'
Electron preferentially emitted opposite to the nucleus spin.
→ Violation of parity

Unlike up-down asymmetry,
left-right asymmetry does not violate parity.

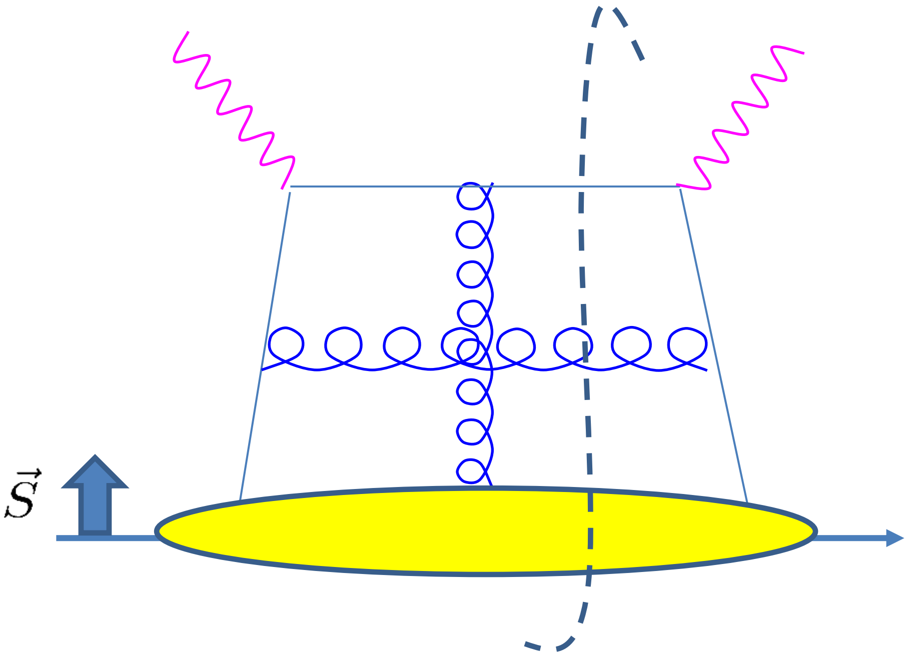


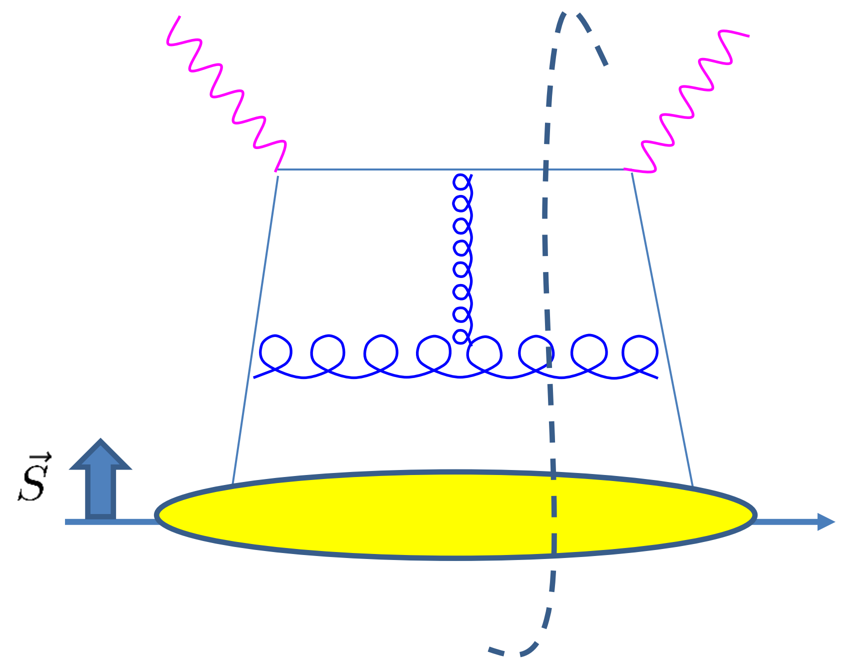
Allowed in the strong interaction,
and is a large effect!

Calculating single spin asymmetry

Several mechanisms for generating SSA are known.

Spin-dependent effects are usually subleading, challenging to compute





Benic, Hatta, Li, Yang (2019)

Conclusion

- QCD as the theory of the strong interaction, full of mysteries even 40 years after the discovery.

- EIC is the next-generation (YOUR generation) machine that will uncover the internal structure of the proton and nuclei. The spin of the proton is a vital part of the program.