BNL seminar, 24th June 2021

The Branco Weiss Fellowship Society in Science

Self-Organised Localisation

Tevong You

CERN

UNIVERSITY OF CAMBRIDGE

Based on 2105.08617 G. Giudice, M. McCullough, **TY**

Outline

- **Motivation**
  - Criticality
  - Quantum phase transitions (QPT)
- **Fokker-Planck Volume (FPV) equation**
  - FPV dynamics
- **FPV + QPT = SOL**
  - Discontinuity
  - Flux conservation
- **SOL solutions**
  - Metastability
  - Higgs mass
  - Cosmological constant
- **Conclusion**
  - Measure problem

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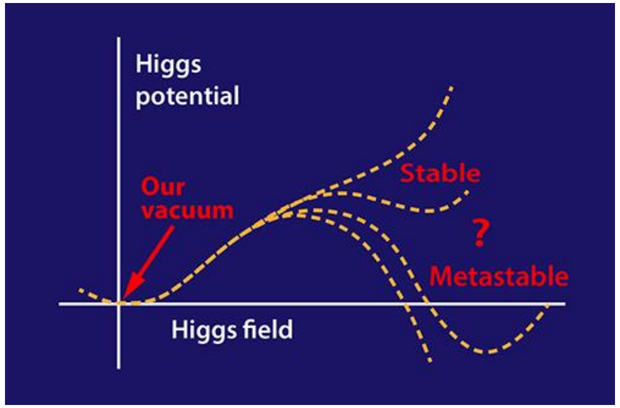
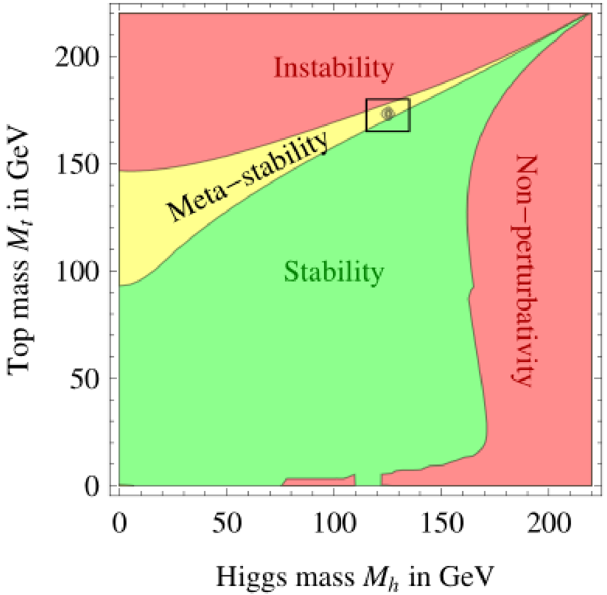
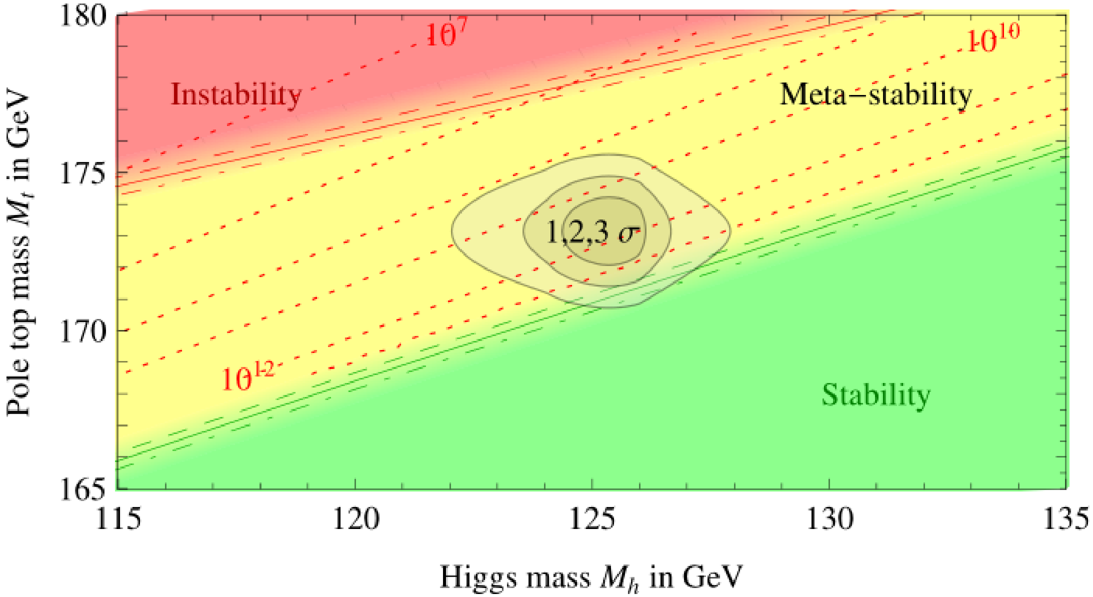
3 hints for **near-criticality** of our Universe

- 1) **Metastability**

1205.6497 Degrassi et al



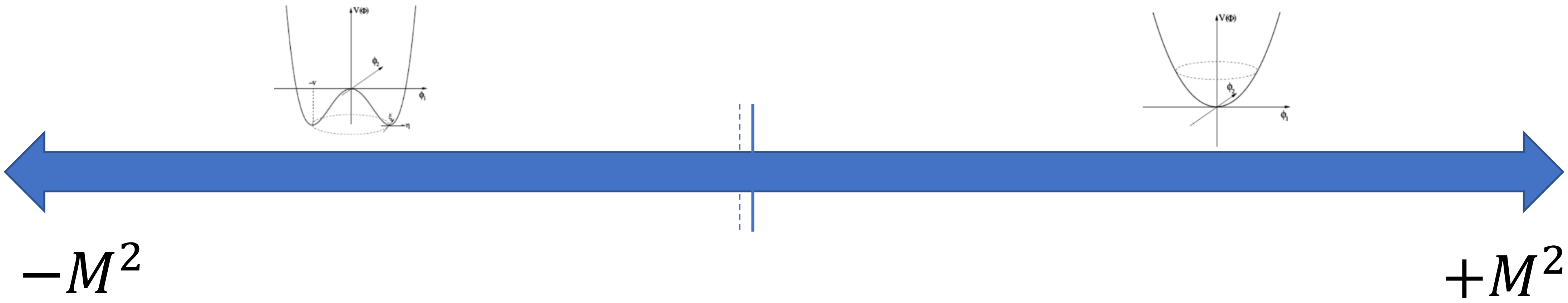




- Living on critical boundary of two phases coexisting

3 hints for **near-criticality** of our Universe

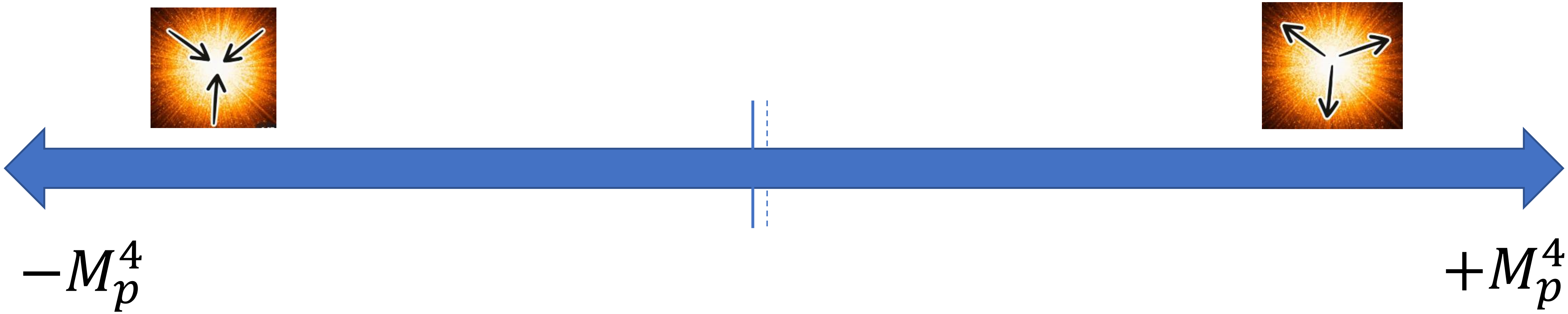
- 2) **Higgs mass**



- Tuned close to boundary between ordered and disordered phase

3 hints for **near-criticality** of our Universe

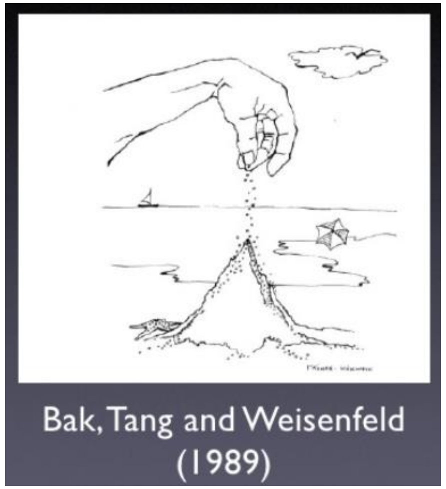
- 3) **Cosmological constant**

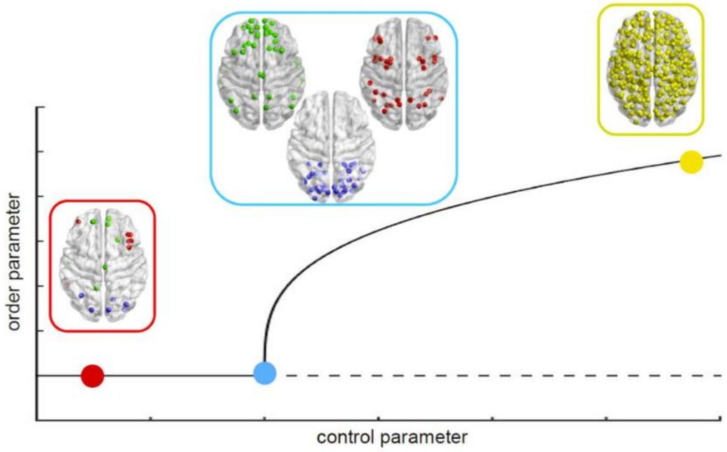


- Tuned close to boundary between implosion and explosion

Self-Organised Criticality?

- Many systems in nature **self-tuned** to live near criticality





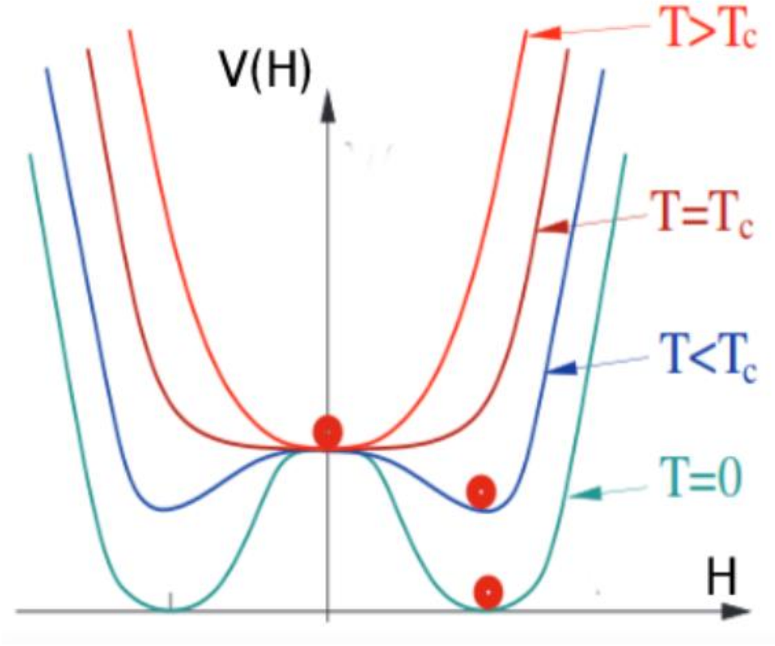
- Need a **mechanism** for self-organisation of **fundamental parameters**

  e.g. Self-Organized Criticality in eternal inflation landscape: J. Khoury et al 1907.07693, 1912.06706, 2003.12594

- Cosmological quantum phase transitions localise fluctuating scalar fields during inflation at critical points: **self-organised localisation (SOL)**

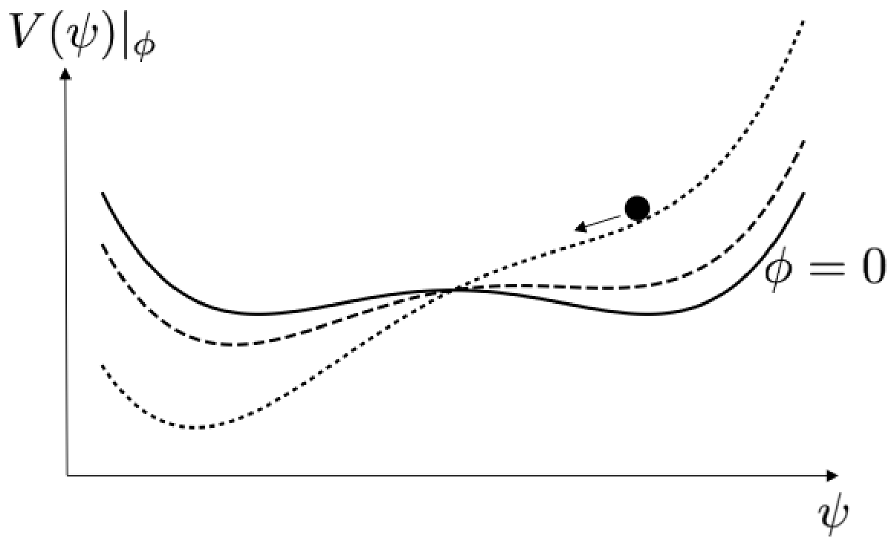
Phase Transitions (PT)

- **Classical** PT: varying background temperature



- **Quantum** PT: varying background field

$$V = \frac{\lambda}{4}\left(\psi^2 - \rho^2\right)^2 + \kappa\phi\psi$$



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Fokker-Planck Volume (**FPV**) equation

- Langevin equation: classical slow-roll + Hubble quantum fluctuations

$$\phi(t+\Delta t) = \phi(t) - \frac{V'}{3H}\Delta t + \eta_{\Delta t}(t)$$

- Volume-averaged Langevin trajectories: **FPV for volume distribution**

$$\frac{\partial}{\partial \phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial (H^3 P)}{\partial \phi} + \frac{V'P}{3H}\right] + 3HP = \frac{\partial P}{\partial t}$$

$$H(\phi) = \sqrt{\frac{V(\phi)}{3M_p^2}}$$

Quantum diffusion term

Classical drift term

Volume term

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- Volume-averaged Langevin trajectories: **FPV for volume distribution**

$$\frac{\partial}{\partial\phi}\left[\frac{\hbar}{8\pi^2}\frac{\partial(H^{2+\xi}P)}{\partial\phi} + \frac{V'P}{3H^{2-\xi}}\right] + 3H^{\xi}P = H_0^{\xi-1}\frac{\partial P}{\partial t_\xi} \qquad H(\phi) = \sqrt{\frac{V(\phi)}{3M_p^2}}$$

- **Ambiguity** in choosing time "gauge" $dt_\xi/dt = (H/H_0)^{1-\xi}$

FPV dynamics

- ϕ is *not* the inflaton: **apeiron** field scanning parameters
- Restrict to **EFT** field range f

$$\varphi \equiv \frac{\phi}{f} \qquad V = 3H_0^2 M_P^2 + g_\epsilon^2 f^4 \omega(\varphi) \ , \qquad \omega(\varphi) = \sum_{n=1}^{\infty} \frac{c_n}{n!} \varphi^n$$

- Assume sub-dominant energy density
- Expand around constant inflationary background H_0

$$H(\varphi) \simeq H_0 \left(1 + \frac{\epsilon^2 f^4 \omega(\varphi)}{6 M_p^2 H_0^2} \right)$$

- FPV becomes

$$\frac{\alpha}{2} \frac{\partial^2 P}{\partial \varphi^2} + \frac{\partial(\omega' P)}{\partial \varphi} + \beta \omega P = \frac{\partial P}{\partial T}$$

$$\alpha \equiv \frac{3\hbar H_0^4}{4\pi^2 \epsilon^2 f^4} \quad , \quad \beta \equiv \frac{3}{2} \frac{\xi f^2}{M_p^2} \quad , \quad T \equiv \frac{t}{t_R} \quad , \quad t_R \equiv \frac{3H_0}{g_\epsilon^2 f^2} = \frac{\alpha \beta S_{ds}}{3\xi H_0} \qquad S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$$

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- Maximum number of e-folds for non-eternal inflation: $N_{\text{e-folds}} < S_{ds} = \dfrac{8\pi^2 M_p^2}{\hbar H^2}$

FPV dynamics

- **Stationary** FPV distributions $P(\varphi, T) = \sum_\lambda e^{\lambda T} p(\varphi, \lambda)$

$$\frac{\alpha}{2} p'' + \omega' p' + (\omega'' + \beta\omega - \lambda)\, p = 0$$

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- Largest eigenvalue $\lambda = \lambda_{\max}$ inflates most
- Determines **peak location** of asymptotically stationary solution

- Note: **boundary conditions** necessary input for solution

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$$\lambda = \beta\omega(\bar{\varphi}) + \omega''(\bar{\varphi}) - \frac{\alpha}{2\sigma^2}$$

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$$\lambda = \beta\omega(\bar\varphi) + \omega''(\bar\varphi) - \frac{\alpha}{2\sigma^2} \longrightarrow \omega(\bar\varphi) = 1 - \frac{\omega_1'^2}{2\alpha\beta}$$

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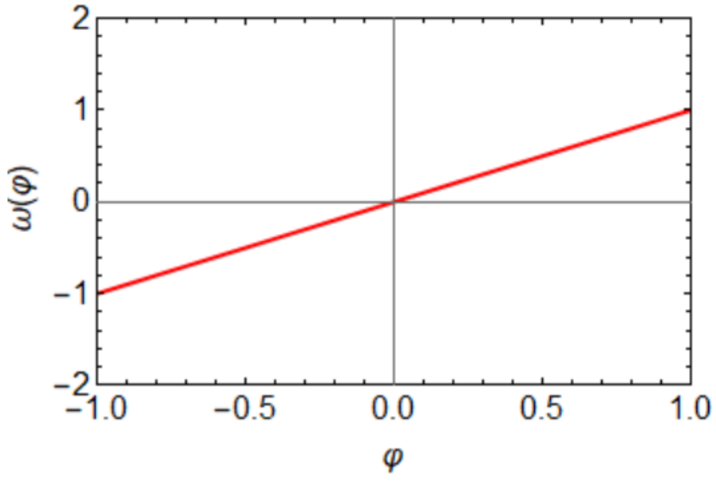

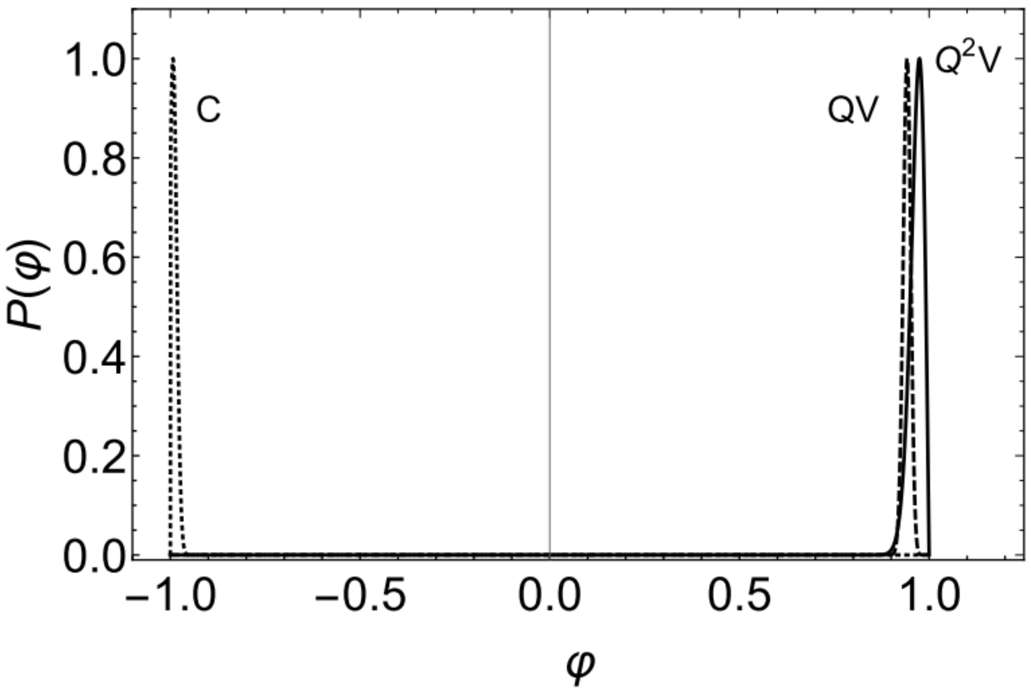

- C regime: $\alpha\beta \ll 1$. Peak is located as far down the potential as allowed by boundary condition.
- QV regime: $\alpha\beta \gg 1$, $\alpha^2\beta \ll 1$. Peak is a distance $1/(\alpha\beta)$ from the top with width $\sigma \simeq 1/\sqrt{\beta}$.
- Q^2V regime: $\alpha^2\beta \gg 1$. Peak as close to the top as possible, with a distance comparable to the width $\sigma \simeq (\alpha/\beta)^{1/3}$.

FPV dynamics

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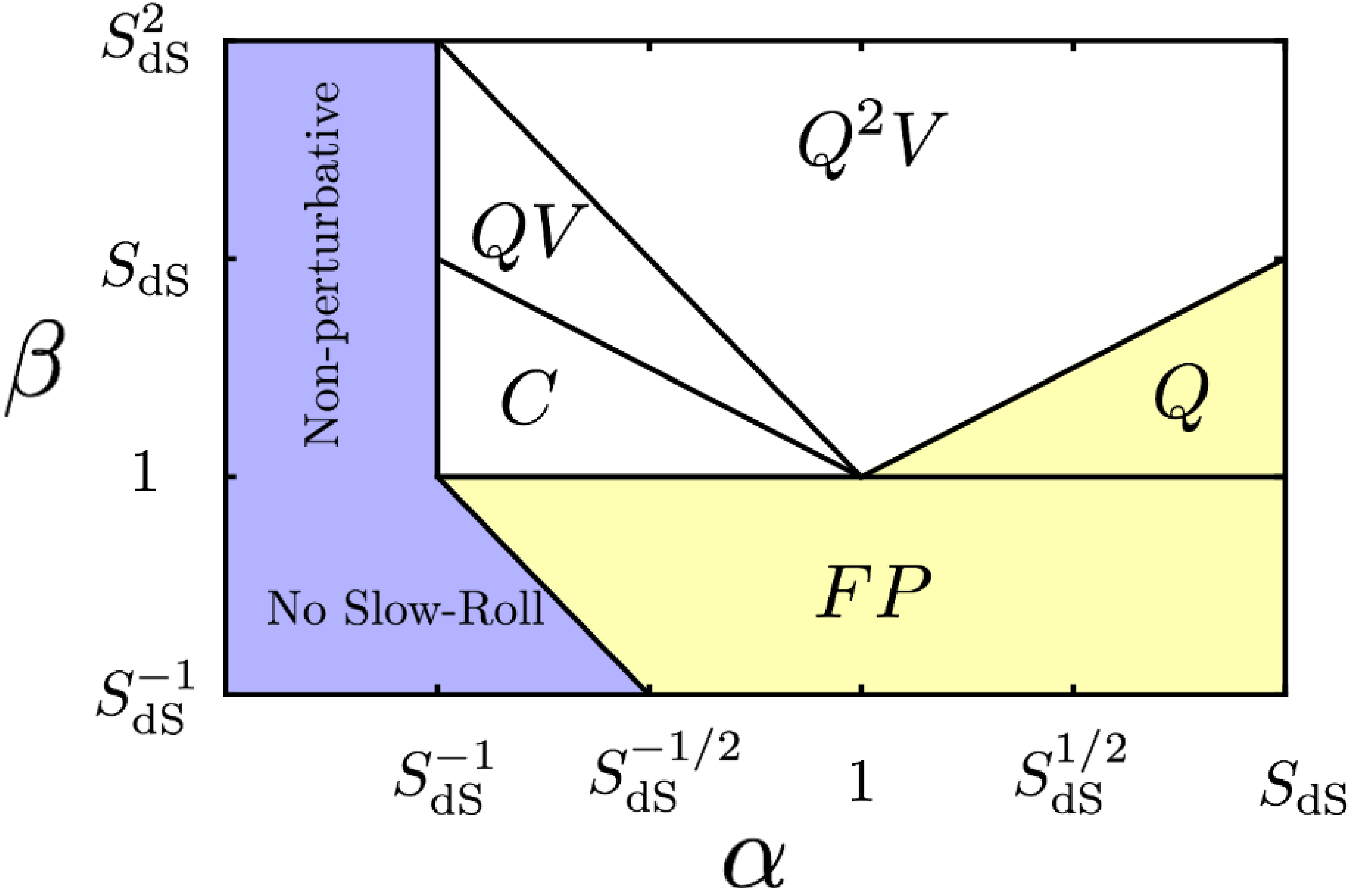
$S_{ds} = \frac{8\pi^2 M_p^2}{\hbar H^2}$



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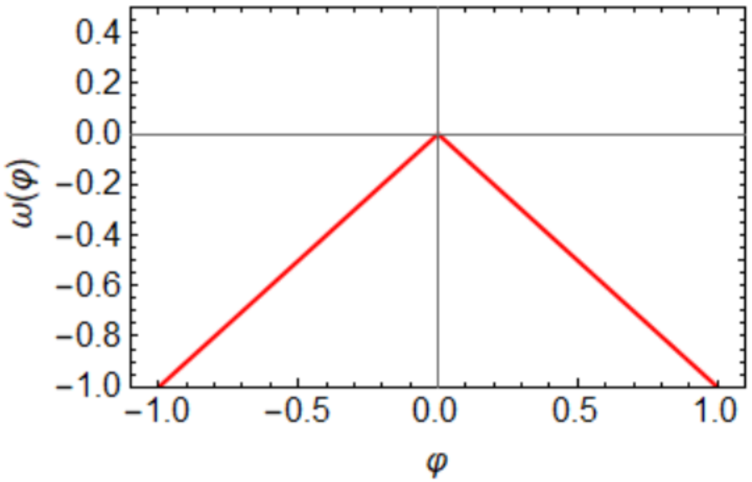
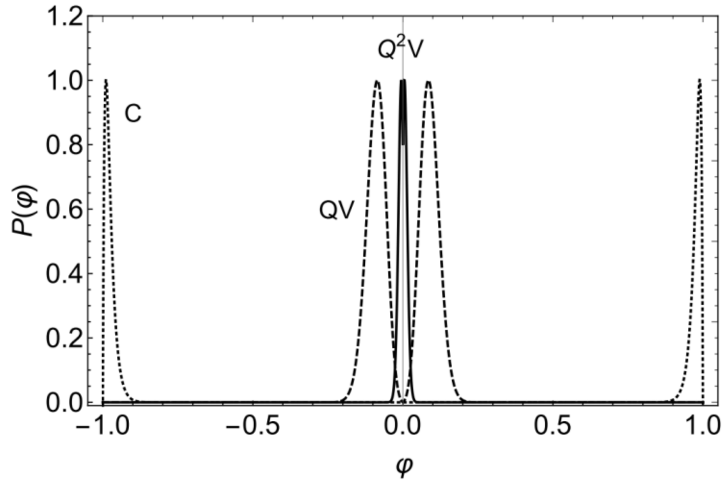
Junction conditions at phase transitions





- ϕ triggers 1st order quantum phase transition at ϕ_c
- Discontinuity in V' leads to discontinuous P'
- Requiring continuity of FPV across the critical point gives a junction condition to satisfy

$$\lim_{\epsilon\to 0}\int_{\phi_c-\epsilon}^{\phi_c+\epsilon} d\phi \frac{\partial}{\partial\phi}\left[\frac{V'P}{3H}+\frac{\hbar}{8\pi^2}\frac{\partial}{\partial\phi}\left(H^3P\right)\right]=0 \quad \Rightarrow \quad \frac{\Delta P'}{P(\varphi_c)}=-\frac{2\Delta\omega'}{\alpha}$$

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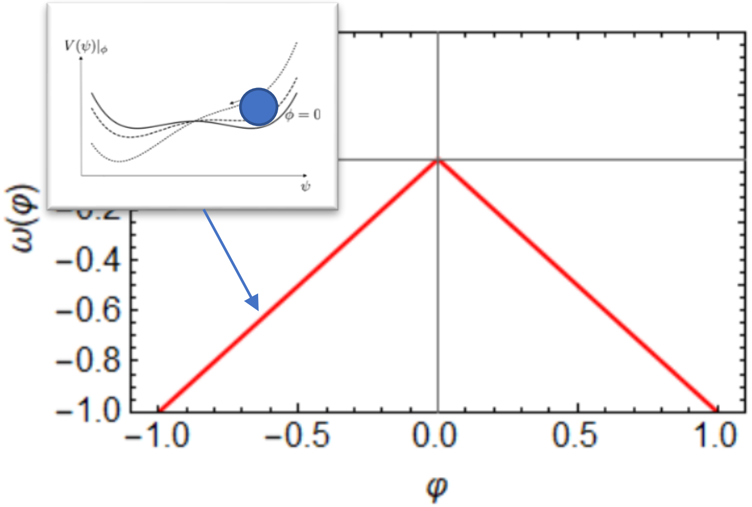

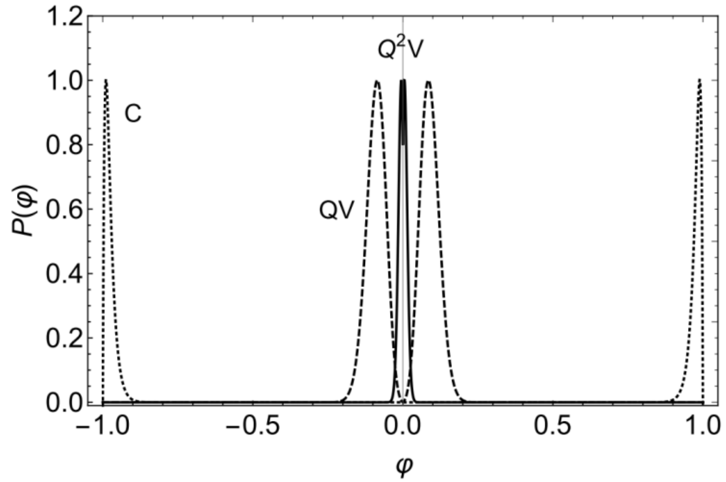

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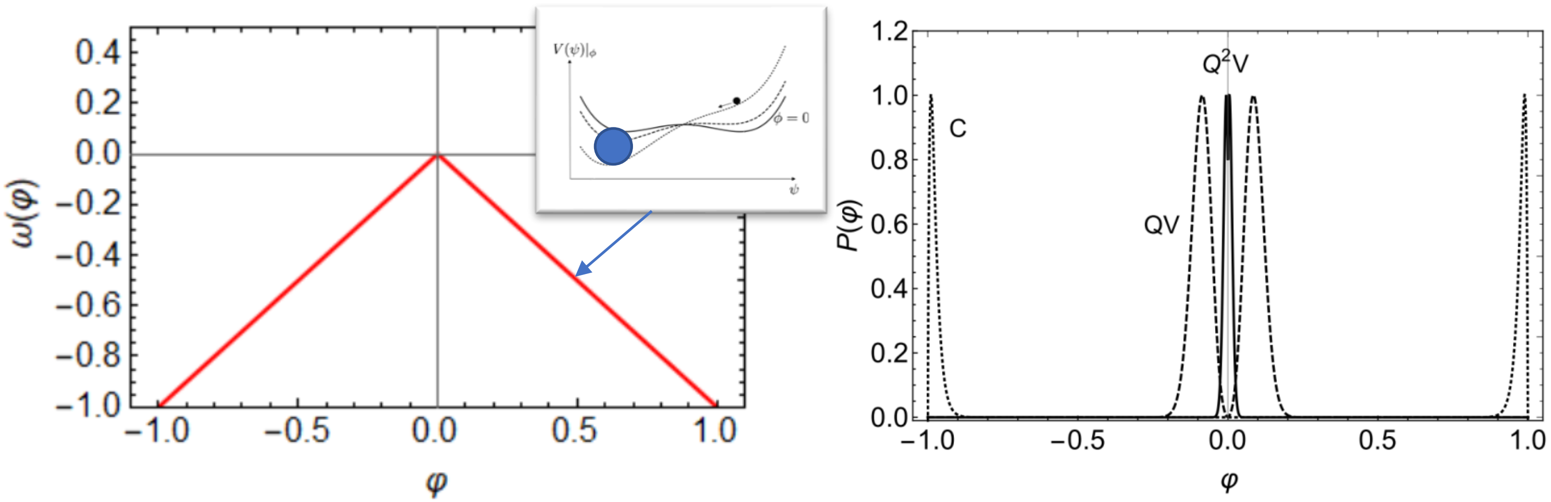

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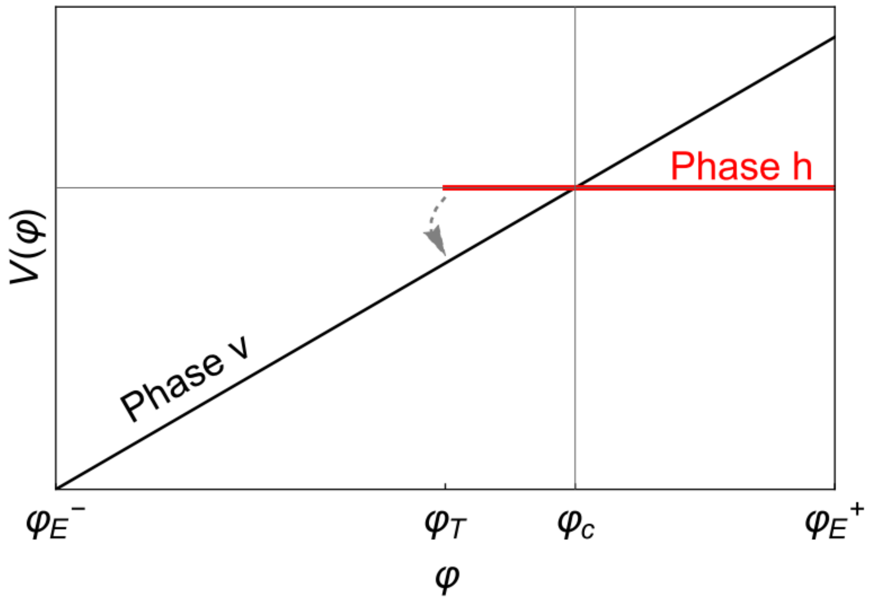

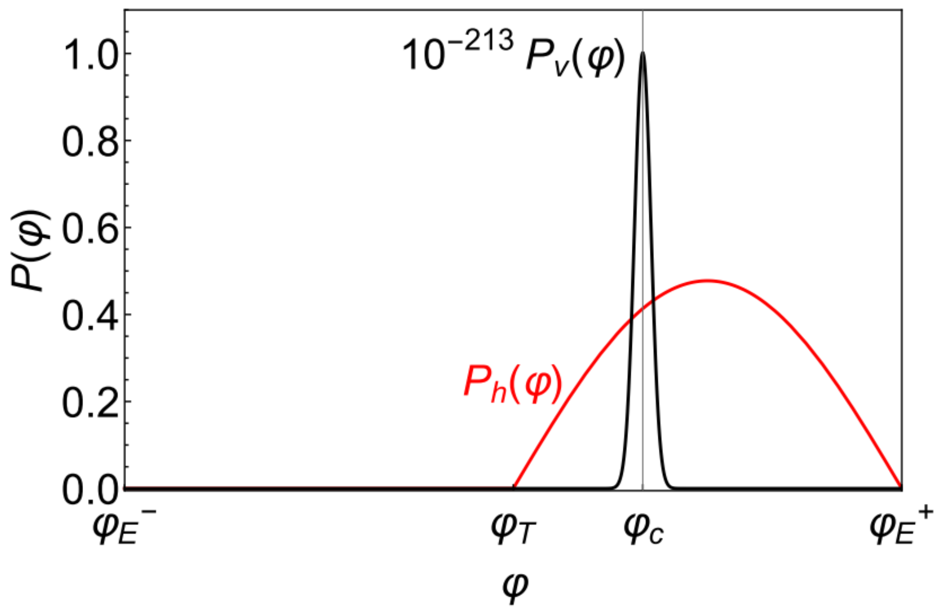

- Coexistence of branches of different phases, require continuity of P_V and $P_V + P_h$ in FPV at ϕ_T: flux conservation junction conditions

$$P_h(\phi_T) = 0 \qquad \Delta P_v' = -P_v'(\phi_T) \qquad \Delta P_v = 0$$

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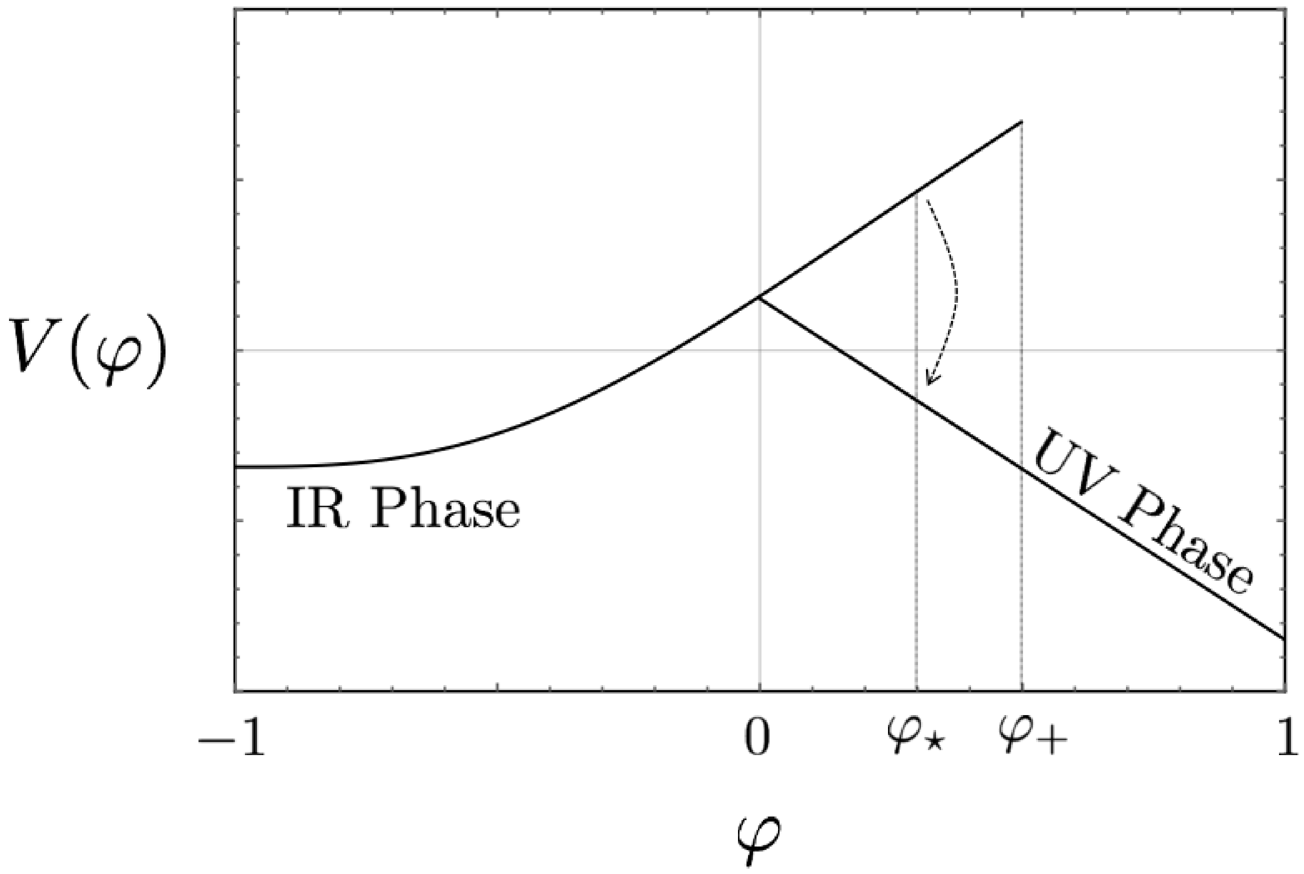
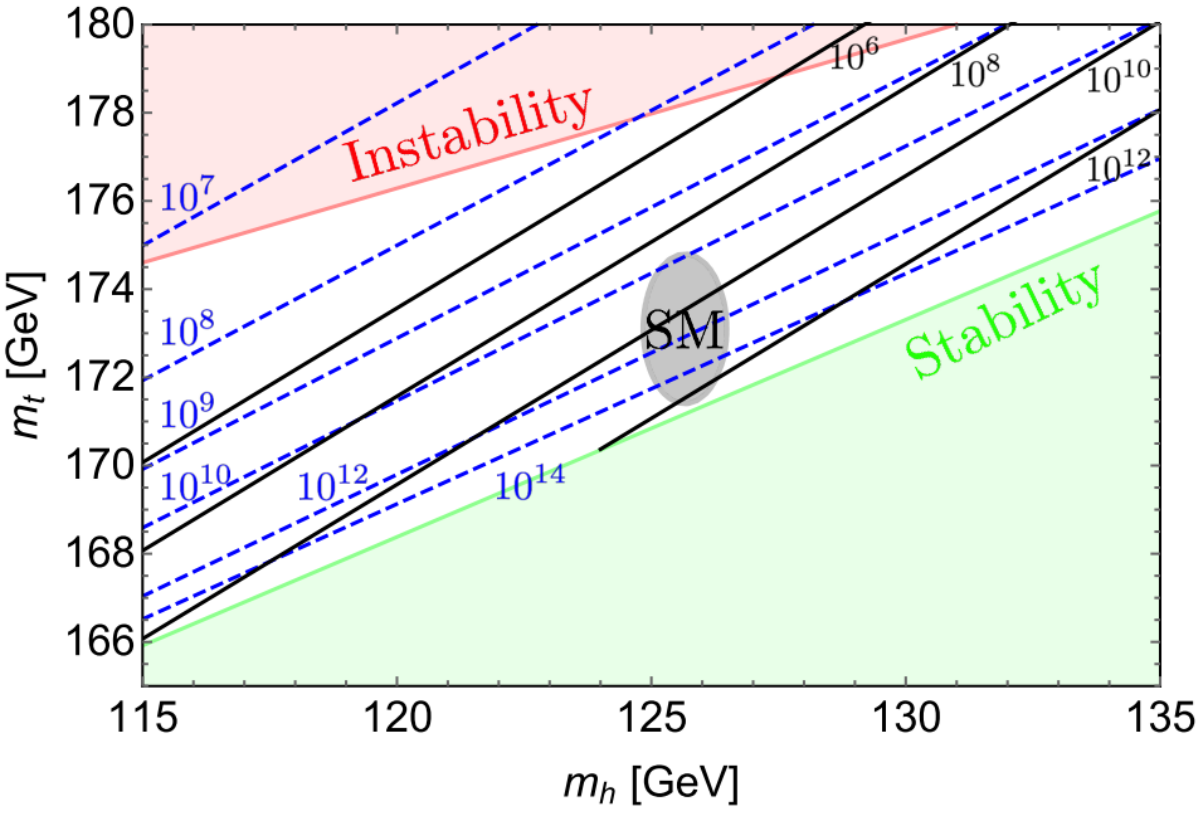
Higgs metastability

$$V(\varphi, h) = \frac{M^4}{g_*^2}\,\omega(\varphi) + \frac{\lambda(\varphi, h)}{4}\left(h^2 - v^2\right)^2$$

$$\lambda(\varphi, M/g_*) = -g_*^2\,\varphi$$



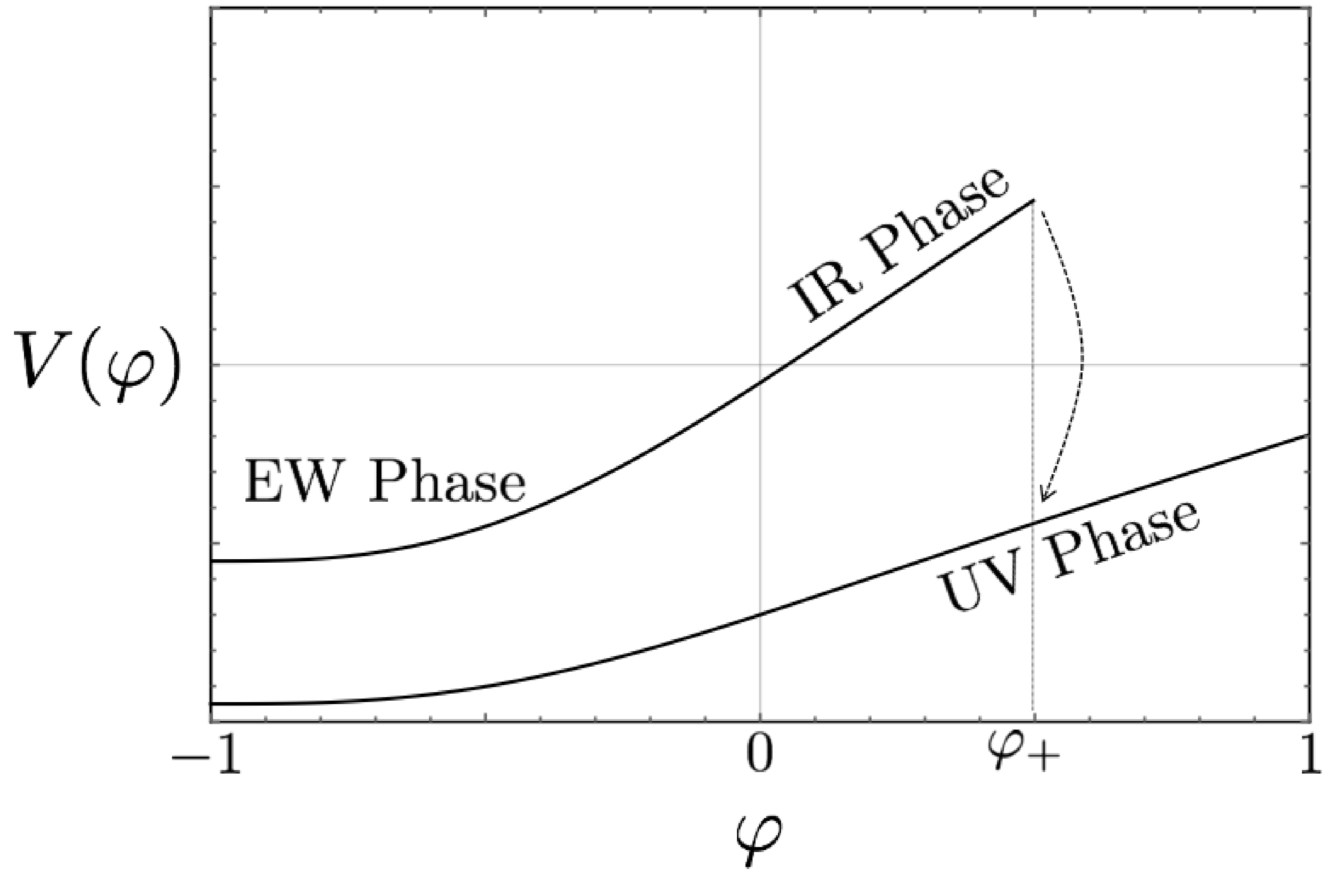


Higgs mass naturalness

$$V(\varphi, h) = \frac{M^4}{g_*^2}\,\omega(\varphi) - \frac{\varphi M^2 h^2}{2} + \frac{\lambda(h)\, h^4}{4}$$

$$\frac{V(\varphi,\langle h\rangle)}{M^4} = \begin{cases} \kappa_{\rm EW}\varphi + \kappa_2\varphi^2 + \ldots & \text{for } \varphi<0 \quad (\text{unbroken EW: } \langle h\rangle=0) \\ \kappa_{\rm EW}\varphi + \kappa_{\rm IR}\varphi^2 + \ldots & \text{for } 0<\varphi<\varphi_+ \quad (\text{IR phase: } \langle h\rangle=v) \\ -\kappa_0 + \kappa_{\rm UV}\varphi + \kappa_2\varphi^2 + \ldots & \text{for any } \varphi \quad (\text{UV phase: } \langle h\rangle=c_{\rm UV}M) \end{cases}$$

$$\kappa_{\rm EW} = \frac{\omega'(0)}{g_*^2}\,, \quad \kappa_2 = \frac{\omega''(0)}{2g_*^2}\,, \quad \kappa_{\rm IR} = \kappa_2 - \Delta\kappa\,, \quad \kappa_0 = \frac{-\lambda_{\rm UV} c_{\rm UV}^4}{4}\,, \quad \kappa_{\rm UV} = \kappa_{\rm EW} - \frac{c_{\rm UV}^2}{2}$$



- Unbroken to broken transition not sufficient

- Use broken IR to broken UV phase transition

$$\varphi_+ = \frac{-\beta_I\, e^{-\frac{3}{2}}\Lambda_I^2}{M^2} \quad \Longrightarrow \quad v = e^{-\frac{3}{4}}\,\Lambda_I$$

- Lower instability scale to ~TeV through VL fermions

- Naturalness motivation: scalars and vectors heavy

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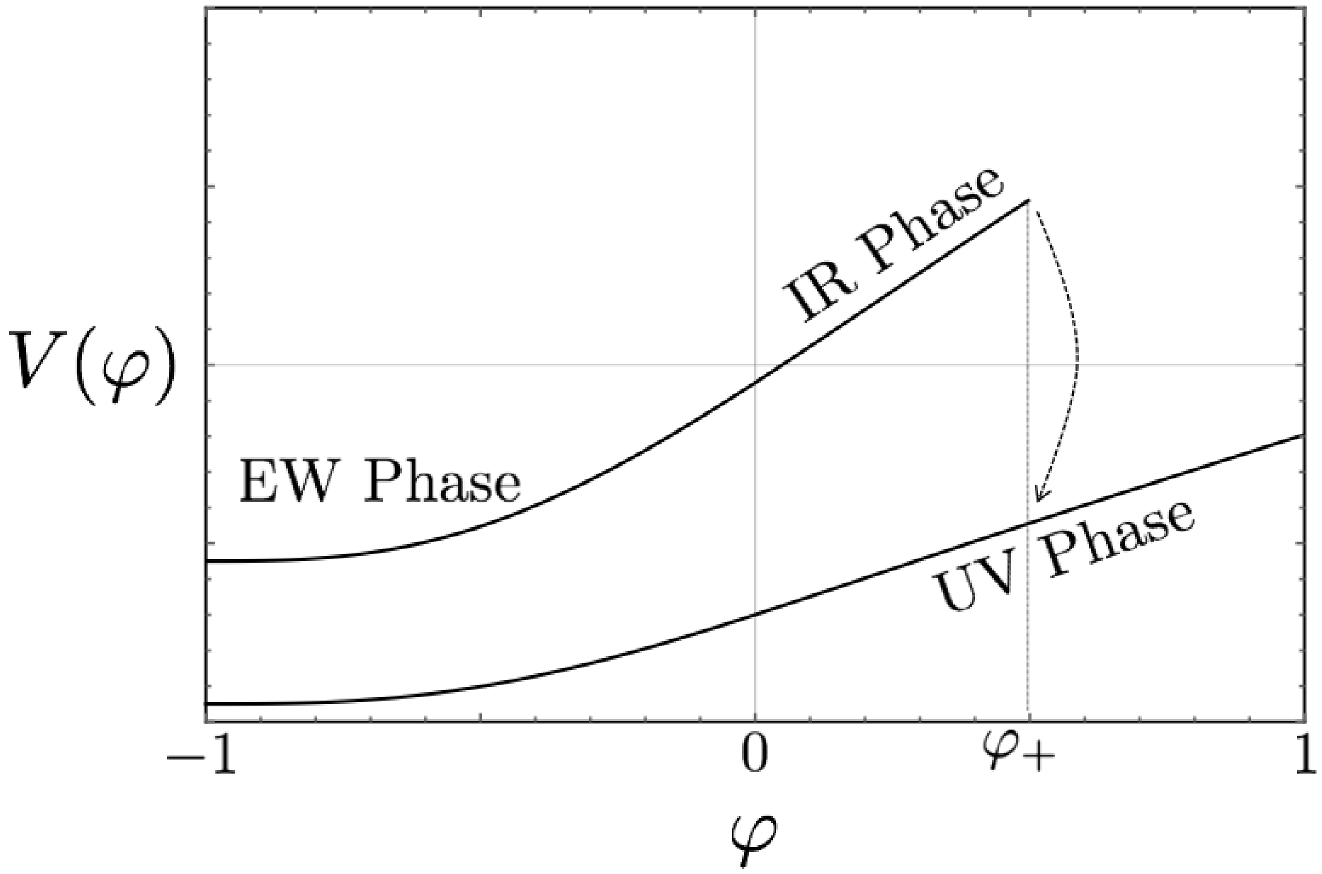

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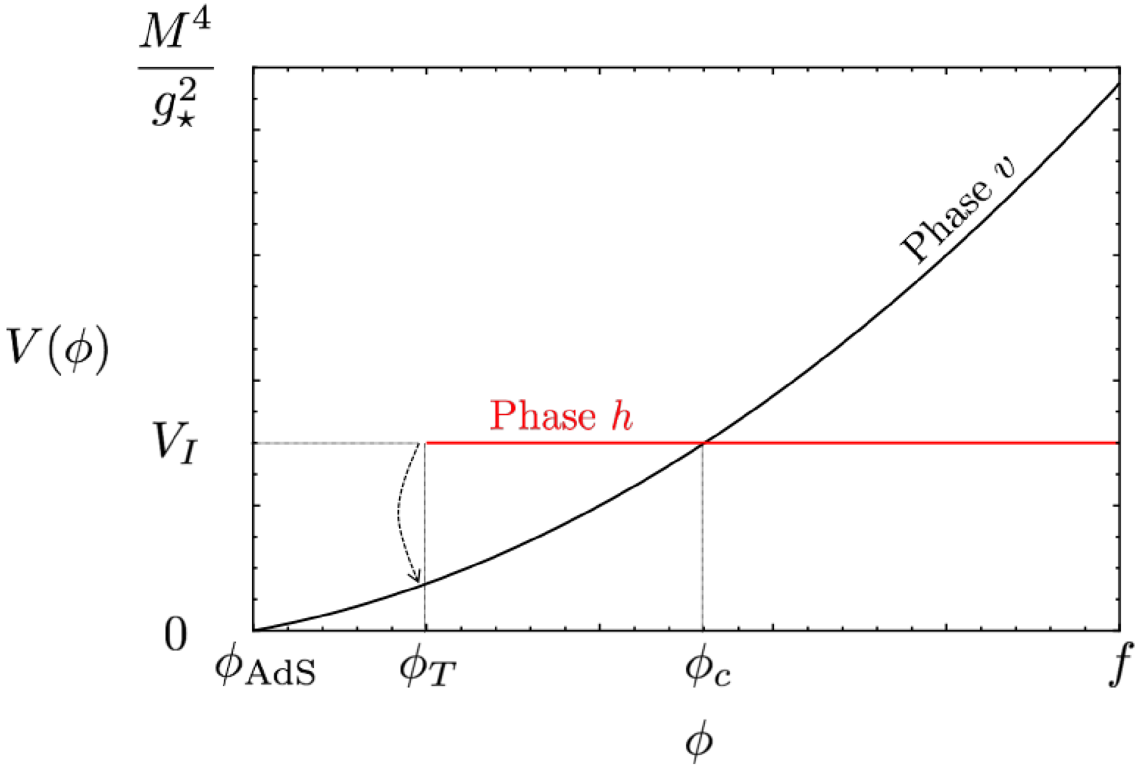
Cosmological constant

- Hidden phase: vanishing cosmological constant by R-symmetry
- Visible phase: SOL localises at vacuum degeneracy point



$$p_h(\phi) = \sin\left[\sqrt{\frac{6(1-\lambda_H)}{\hbar}}\,\frac{2\pi(\phi-\phi_T)}{H_I}\right]$$

$$\lambda_H = 1 - \frac{\hbar\, H_I^2}{24(f-\phi_T)^2}$$

$$V_v(\bar{\phi}) = V_I\, \lambda_H^{2/\xi}\ , \qquad \sigma = \sqrt{\frac{2}{3\xi}}\, M_P$$

$$\Rightarrow \quad V_v(\bar{\phi}) = V_I\left(1 - \frac{\hbar\, H_I^2}{12\xi f^2}\right)$$

- Solution must be in **C regime** with appropriate **boundary conditions**

Outline

- Motivation
  - Criticality
  - Quantum phase transitions (QPT)
- Fokker-Planck Volume (FPV) equation
  - FPV dynamics
- FPV + QPT = SOL
  - Discontinuity
  - Flux conservation
- SOL solutions
  - Metastability
  - Higgs mass
  - Cosmological constant
- **Conclusion**
  - Measure problem

Conclusion

- Quantum fluctuations of scalar fields during inflation can be localised at the critical points of quantum phase transitions: SOL
- SOL suggests our Universe lives at the critical boundary of coexistence of phases
- Measure problem is a major caveat $$\beta \equiv \frac{3}{2}\frac{\xi f^2}{M_p^2}$$
- Open problem -- motivates further study in context of SOL