SIDIS - Working Group

June 16, 2021

Marco Radici (INFN - Pavia)
for the Pavia group

Standard TMD formula

$$f_1^q(x, b_T; \mu) = \sum_i (C_{qi} \otimes f_1^i)(x, b_*; \mu_b)\, e^{S(b_*;\mu_b,\mu)}\, e^{g_K(b_T)\log \mu/\mu_0} f_{\mathrm{NP}}^q(x, b_T)$$

Standard TMD formula

Perturbative part :

OPE matching coefficients C_{qi}

collinear PDFs at scale $\mu_b = \frac{2e^{-\gamma_E}}{\boxed{b_*(b_T)}}$

Sudakov factor → perturbative evolution

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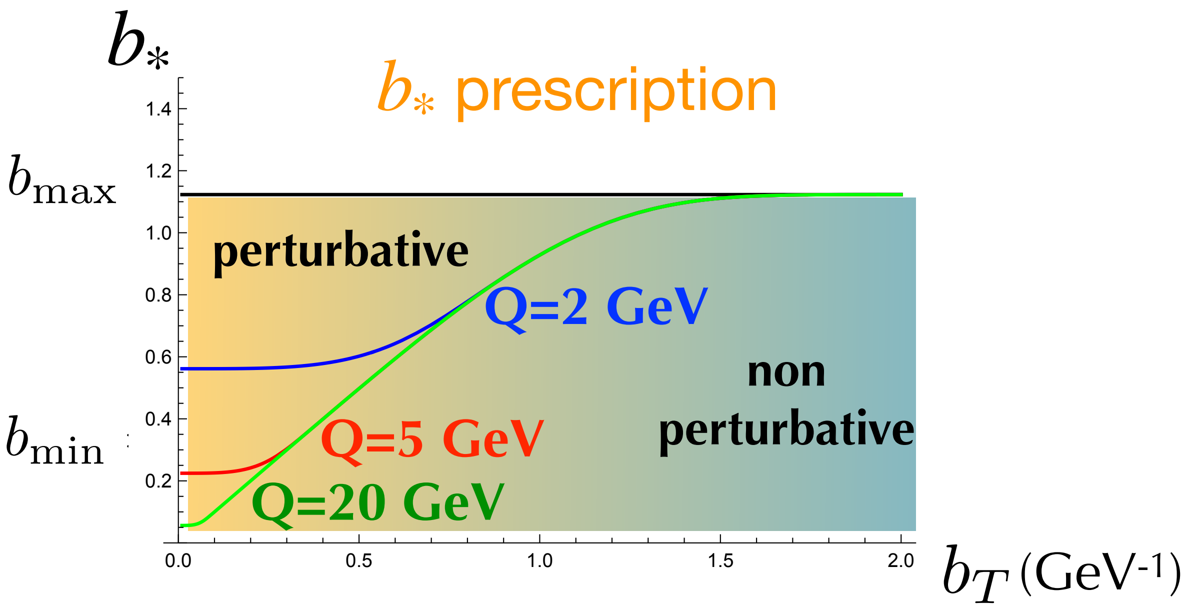

b_* prescription

$$\left.\begin{aligned} b_{\max} &= 2e^{-\gamma_E} \\ b_{\min} &= \frac{2e^{-\gamma_E}}{Q} \end{aligned}\right\} \quad 1 \le \mu_b \le Q$$

For μ=μ0=1 GeV $\mathrm{TMD}(x, b_T; \mu_0) = \mathrm{PDF}(x; \mu_0)\, f_{\rm NP}(x, b_T)$

other prescriptions possible..

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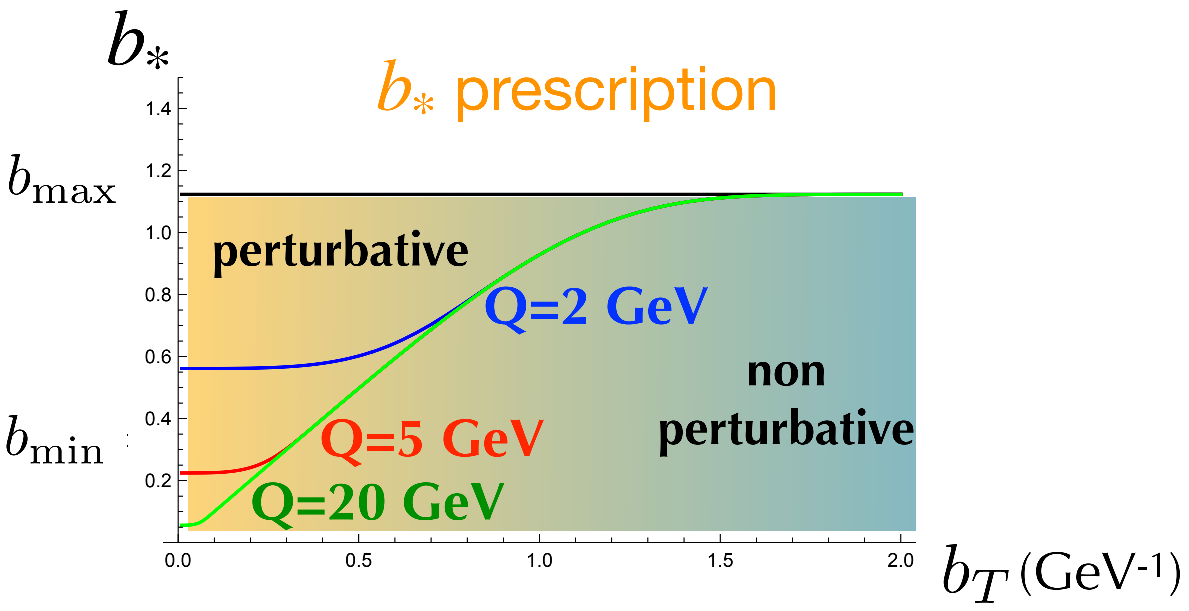
Focus on NonPerturbative part :

evolution

intrinsic wave function



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NonPerturbative functional form

PV17 fit

A. Bacchetta *et al.*, JHEP06 (2017) 081, arXiv:1703.10157

intrinsic wave function

$$f_{\mathrm{NP}}(x, \boldsymbol{k}_T^2) = \frac{1}{\pi} \frac{1 + \lambda \boldsymbol{k}_T^2}{g_1 + \lambda g_1^2} e^{-\boldsymbol{k}_T^2/g_1}$$

$$g_1(x) = N_1 \frac{(1-x)^\alpha x^\sigma}{(1-\hat{x})^\alpha \hat{x}^\sigma}$$

$$\hat{x} = 0.1$$

evolution

$$g_K(b_T) = -g_2 \frac{b_T^2}{4}$$

similar for TMD FF

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similar for TMD FF

g_2 → nonperturbative evolution

$N_1 = g_1(\hat{x})$ → mid-*x* width of TMD

σ → low-*x* width of TMD

α → high-*x* width of TMD

λ → weight of second Gaussian

$\}$ not much constrained by fit

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PV17 nonperturbative parameters

PV17 fit

A. Bacchetta *et al.*, JHEP06 (2017) 081, arXiv:1703.10157

Data from:	**SIDIS**	**Drell-Yan**	**Z-boson production**
	Hermes , Compass	FermiLab	Tevatron: CDF , D0

8059 pts , 11 parameters , χ^2/dof = 1.55 ± 0.05

g_2	N_1 [GeV^2]	σ	α	λ [GeV^{-2}]
0.13 ± 0.01	0.28 ± 0.06	0.17 ± 0.02	2.95 ± 0.05	0.86 ± 0.78

$$\underbrace{\bar{g}_2 \pm \Delta g_2 \qquad \bar{N}_1 \pm \Delta N_1 \qquad \bar{\sigma} \pm \Delta\sigma} \qquad \bar{\alpha} \pm \Delta\alpha \qquad \bar{\lambda} \pm \Delta\lambda$$

all standard deviations at 68% confidence level

Sensitivity coefficients of PV17 parameters

Yellow Report R. Abdul Khalek *et al.*, arXiv:2103.05419

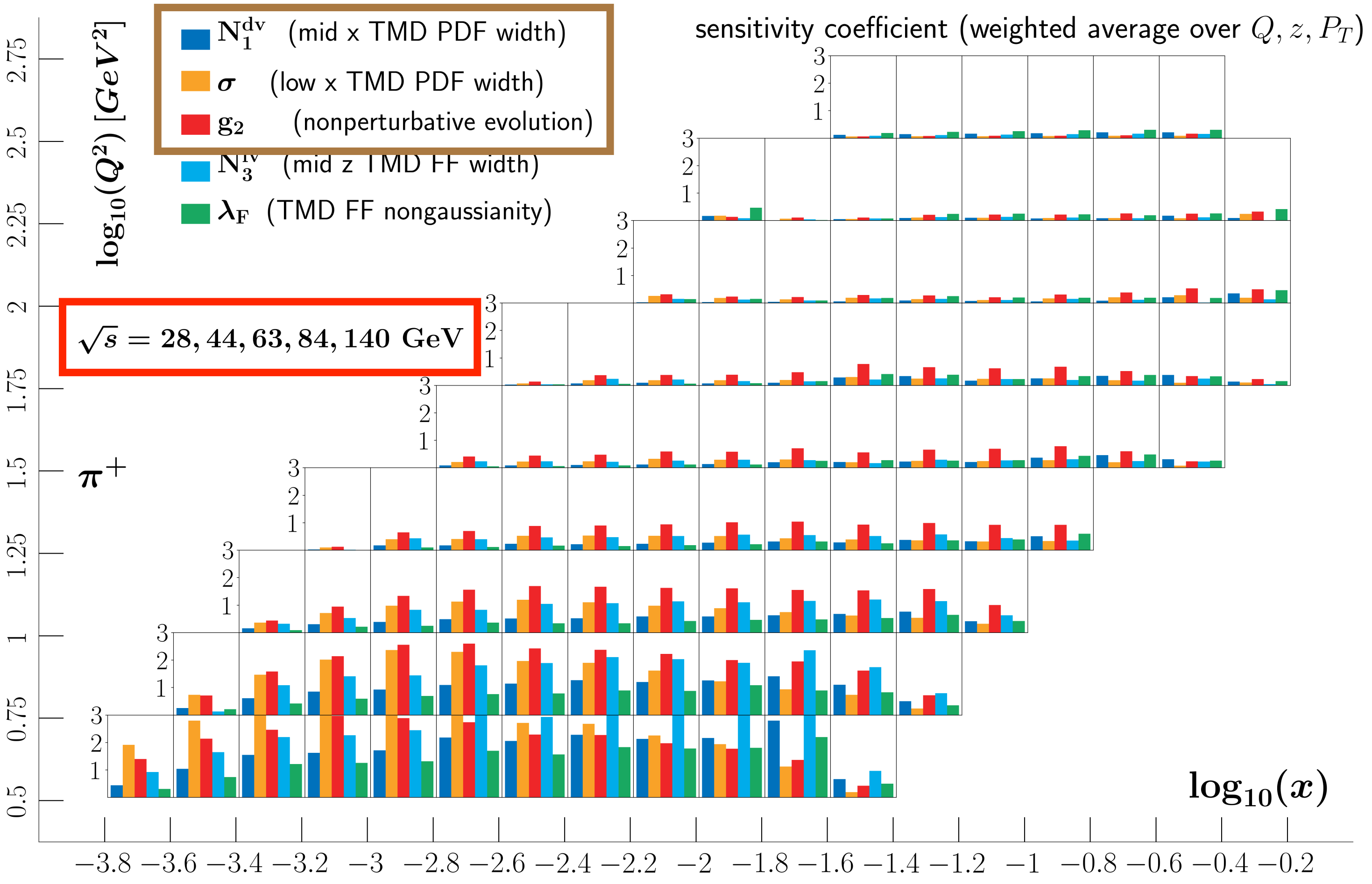

Figure 8.30: Expected sensitivities to various TMD PDF and FF parameters, as well as the TMD evolution as shown for the verious collision energy options and for detected final-state positive pions. The impact has been averaged over final state hadron transverse momentum and fractional energy for better visibility.

$S(\langle x\rangle_{\text{bin}}, \langle Q^2\rangle_{\text{bin}})$ with weighted average over $z, P_T, \sqrt{s}$

(weights proportional to 1/error of pseudodata)

Sensitivity coefficient and standard deviation

sensitivity coefficient *S* of object *f* w.r.t. observable O

$$S(\mathcal{O}, f) = \frac{\langle \mathcal{O} f \rangle - \langle \mathcal{O} \rangle \langle f \rangle}{\delta \mathcal{O} \, \Delta f}$$

experimental error for O

standard deviation for *f*

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standard deviation for *f*

1. from PV17, we know a parameter *A* with error Δ*A*
2. if we perform a new measurement that produces on *A* an error equal to its initial standard deviation, δ*A* = Δ*A*, we expect the error on *A* to scale as $1/\sqrt{2}$. We postulate that this corresponds to $S(A) = 1$

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3. in fact, if *A* can be ideally considered as parameter and observable, then

$$S(A, A) = \frac{\langle A \, A \rangle - \langle A \rangle \langle A \rangle}{\delta A \, \Delta A} = \frac{(\Delta A)^2}{\Delta A \, \Delta A} = 1$$

4. the error on *A* scales as $1/\sqrt{2} = 1/\sqrt{1 + (S = 1)}$. If the new measurement is more precise, then *S* >1 and the error is further reduced; viceversa, for *S*< 1

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experimental error for $\mathcal{O}$

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3. in fact, if A can be ideally considered as parameter and observable, then

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5. for *n* measurements, the error on *A* should scale as $1/\sqrt{1 + S_1 + .. + S_n}$

$S(\langle x\rangle_{\mathrm{bin}}, \langle Q^2\rangle_{\mathrm{bin}})$ with weighted average over z, P_T

🟦 $\mathbf{N_1^{dv}}$ (mid x TMD PDF width)
🟧 $\boldsymbol{\sigma}$ (low x TMD PDF width)
🟥 $\mathbf{g_2}$ (nonperturbative evolution)
🟦 $\mathbf{N_3^{fv}}$ (mid z TMD FF width)
🟩 $\boldsymbol{\lambda}_\mathbf{F}$ (TMD FF nongaussianity)

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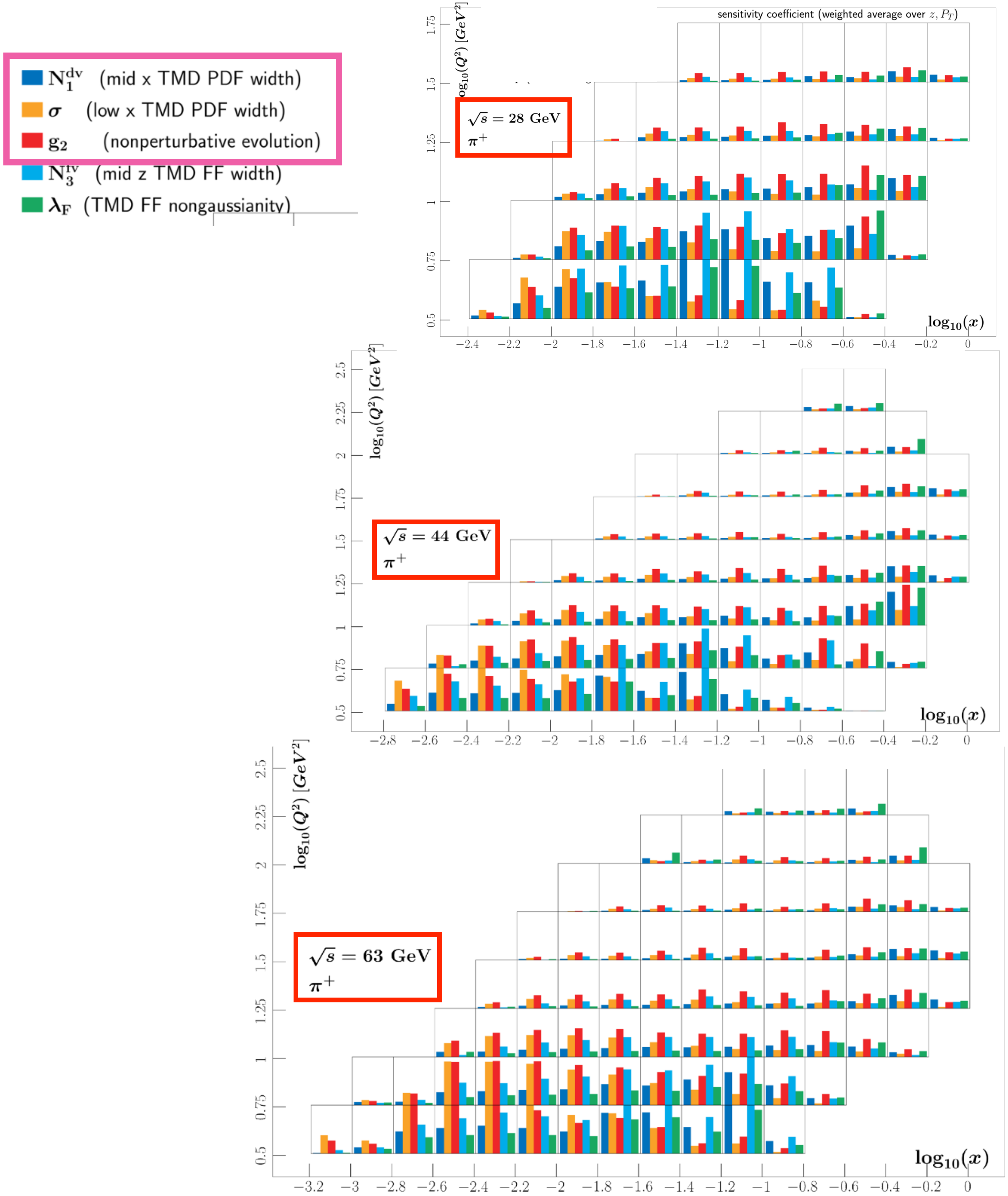

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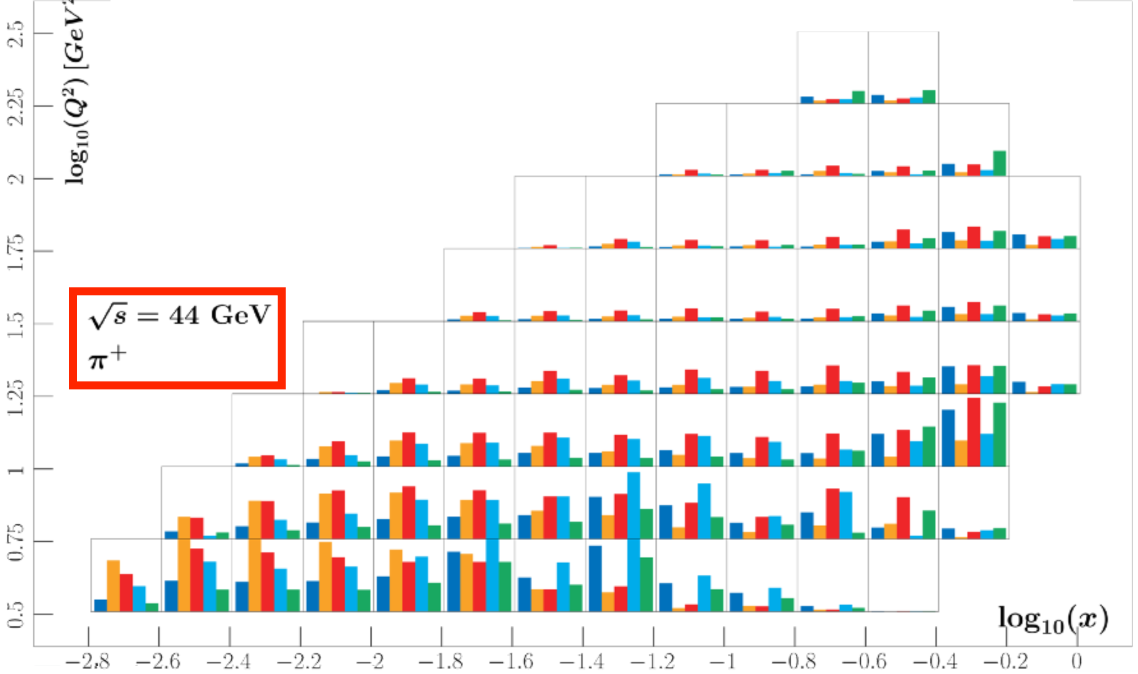
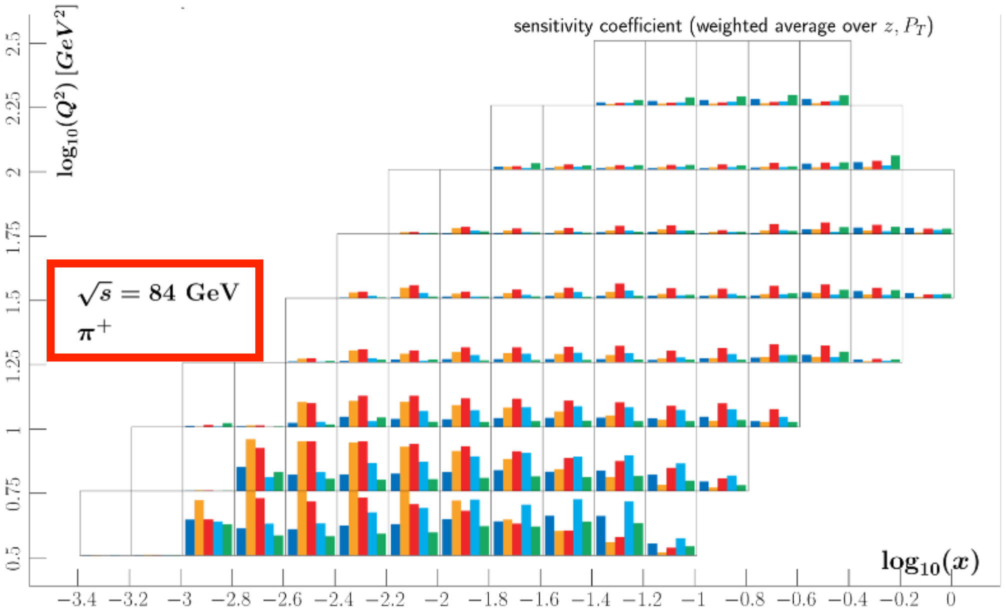
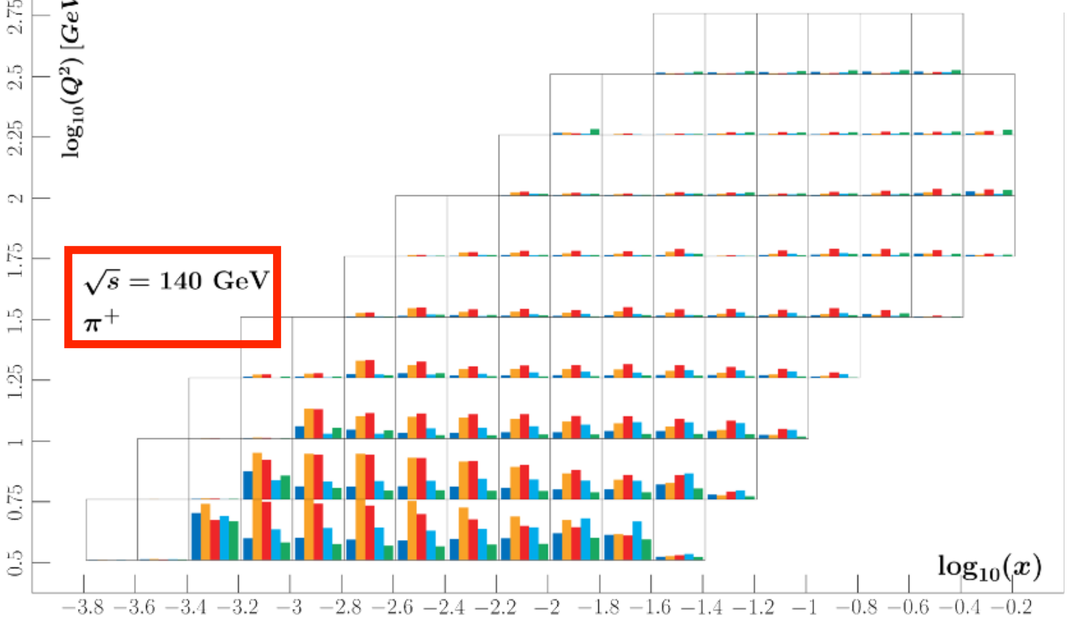
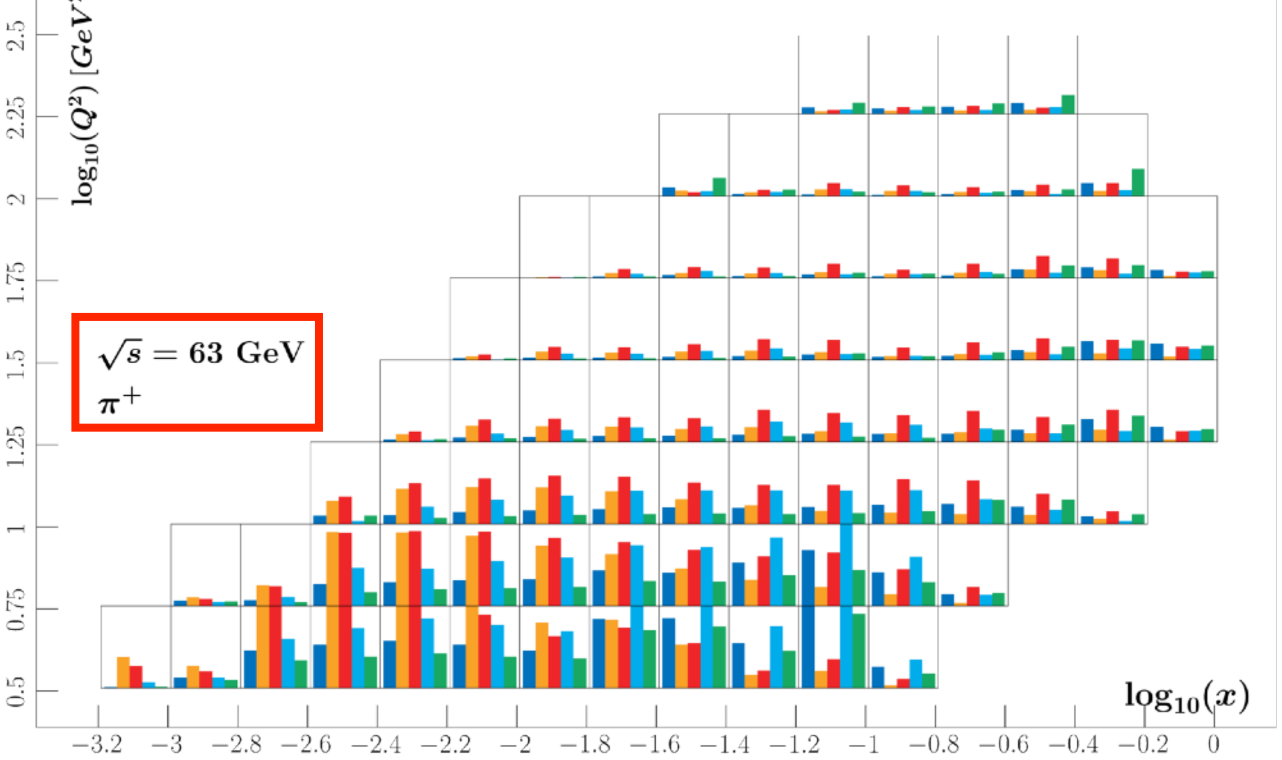
higher $\sqrt{s}$ ⟶ push to lower x, higher Q^2,
but larger errors ⟶ lower coefficients S









NonPerturbative evolution g_2

summing over all (x,Q^2) bins

PV17 fit: $\Delta g_2 = 0.01 \longrightarrow$ run at $\sqrt{s} = 28$ GeV, π+ $\longrightarrow 0.00155$ $R(g_2) = 6.45$

run at $\sqrt{s} = 44$ GeV, π+ $\longrightarrow 0.00120$ $R(g_2) = 8.33$

run at $\sqrt{s} = 63$ GeV, π+ $\longrightarrow 0.00108$ $R(g_2) = 9.26$

run at $\sqrt{s} = 84$ GeV, π+ $\longrightarrow 0.00105$ $R(g_2) = 9.52$

run at $\sqrt{s} = 140$ GeV, π+ $\longrightarrow 0.00096$ $R(g_2) = 10.36$

consistent trend:

larger $\sqrt{s} \longrightarrow$ larger covered (x,Q^2)

$\longrightarrow$ more stringent constraint

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Caveat

- optimistic upper limit in reduction coeff. *R* (no correlation between measurements in different bins)
- exercise biased by rigidity of PV17 functional form; future fits with (many) thousands points could demand more flexible forms

Max error reduction R for PV17 parameters

Energies Parameters	$\sqrt{s} = 28$	$\sqrt{s} = 44$	$\sqrt{s} = 63$	$\sqrt{s} = 84$	$\sqrt{s} = 140$
g_2 nonperturbative evolution	6.45	8.33	9.26	9.52	10.36
N_1 mid-x TMD width	5.94	6.52	6.96	6.80	6.73
σ low-x TMD width	5.05	6.85	8.00	8.55	10.00

Most sensitive (x,Q^2) bins: g_2

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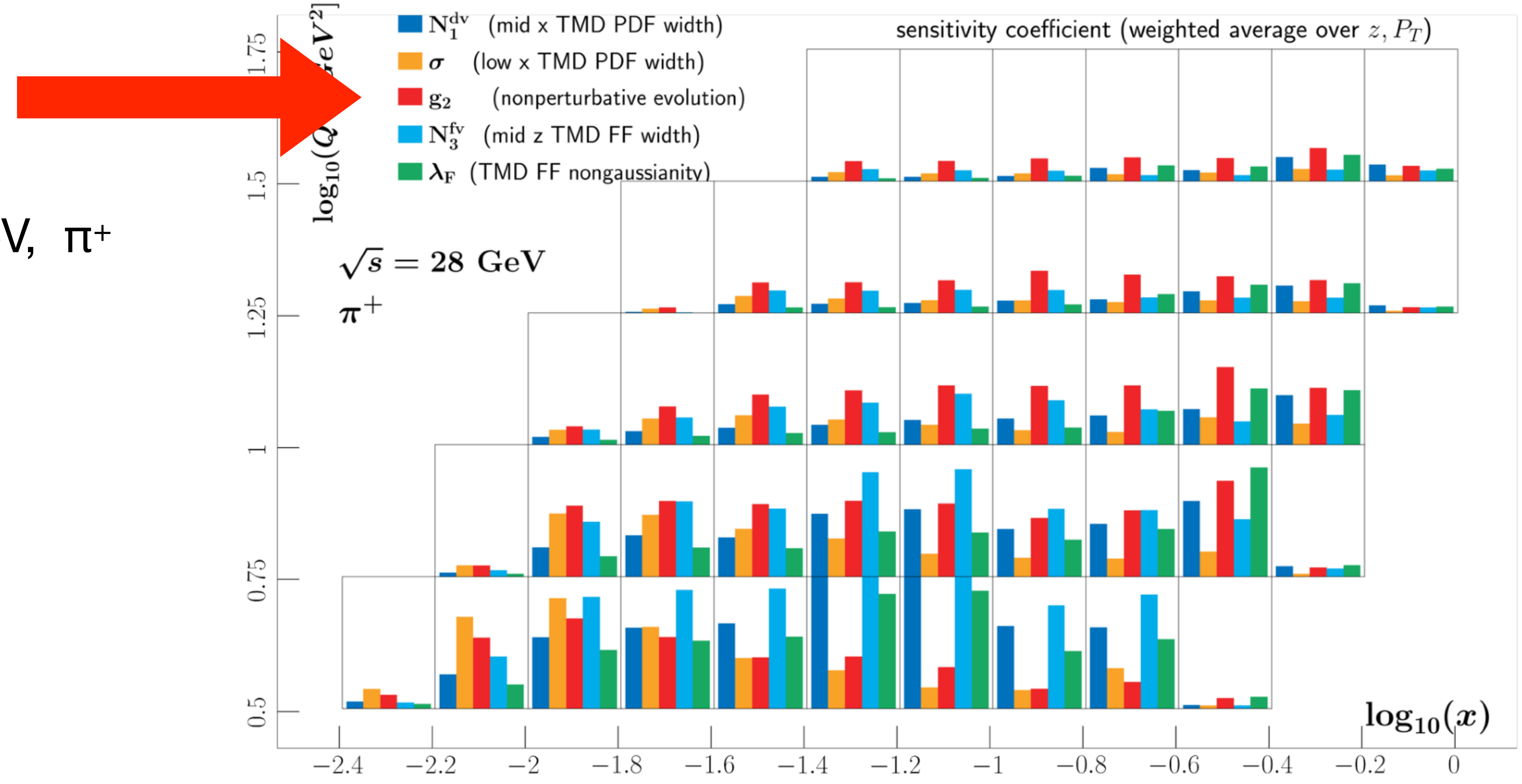

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summing over all $Q^2 \leq 10$ bins $\longrightarrow 0.00218$ 71% R

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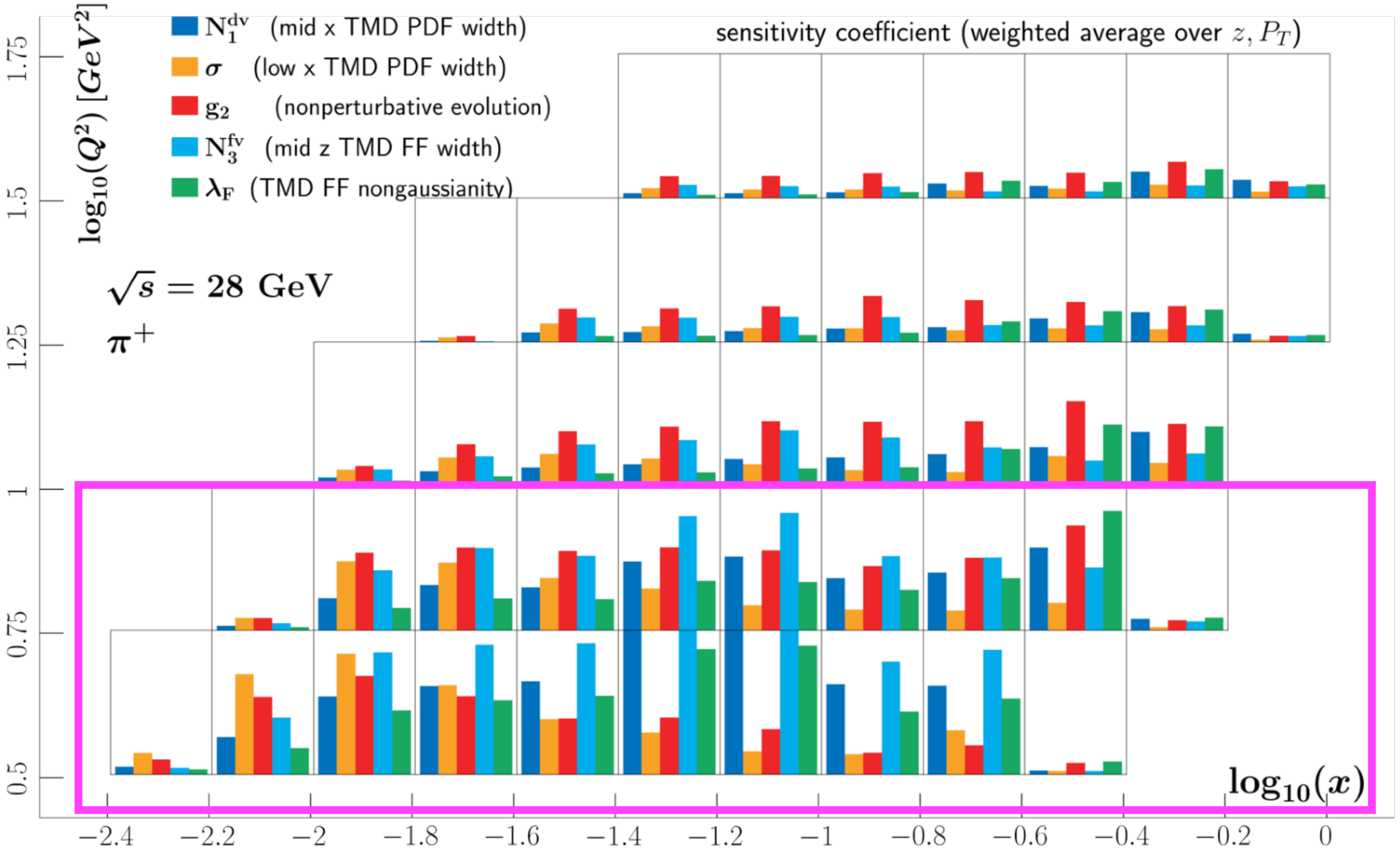

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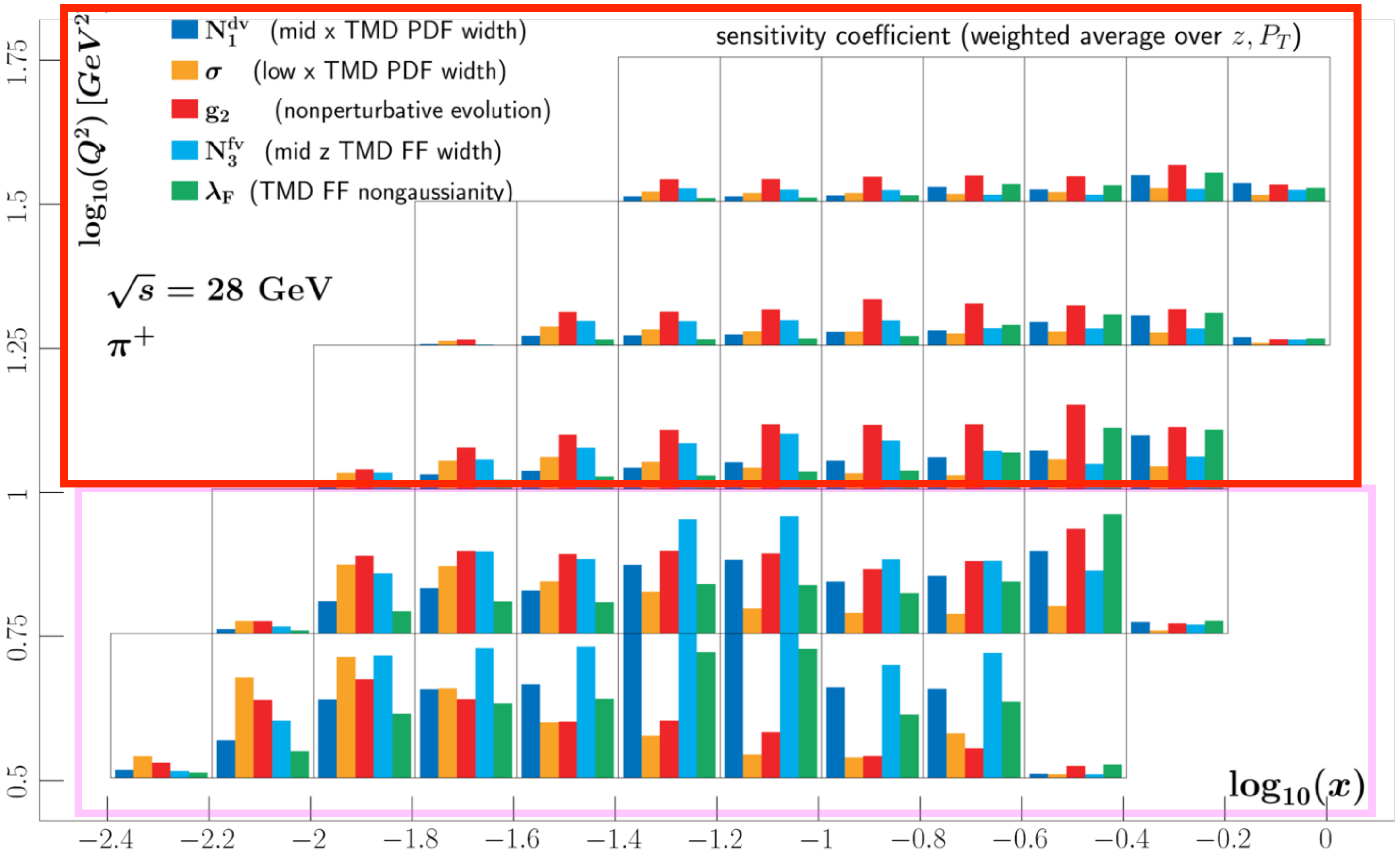
summing over all $Q^2 \leq 10$ bins $\longrightarrow 0.00218$ 71% R

summing over all $10 \leq Q^2 \leq 100$ bins $\longrightarrow 0.00216$ 72% R

$\sqrt{s} = 28$ GeV, π+



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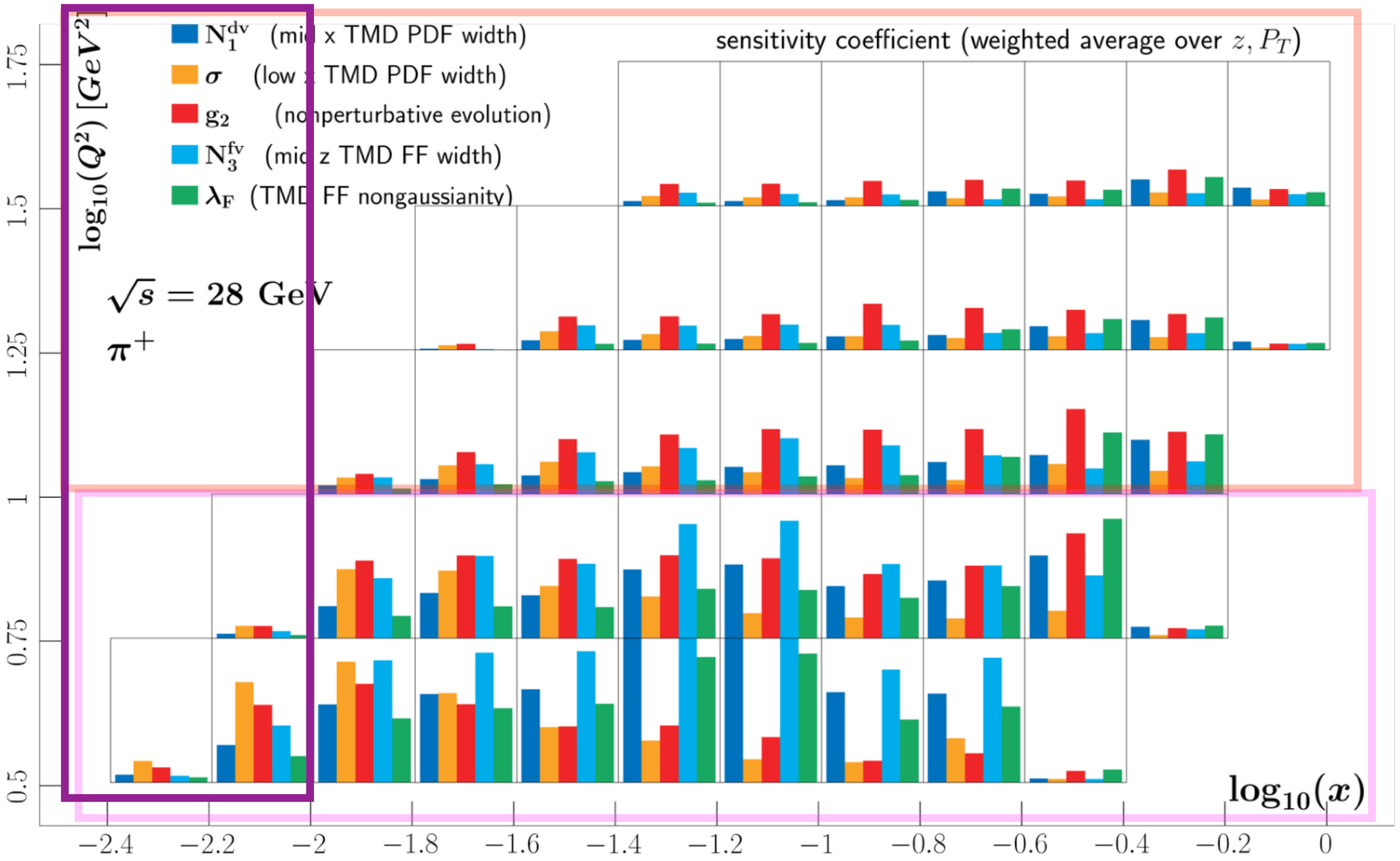
summing over all $Q^2 \leq 10$ bins $\longrightarrow 0.00218$ 71% *R*

summing over all $10 \leq Q^2 \leq 100$ bins $\longrightarrow 0.00216$ 72% *R*

summing over all $x \leq 0.01$ bins $\longrightarrow 0.00306$ 51% *R*

$\sqrt{s} = 28$ GeV, π⁺



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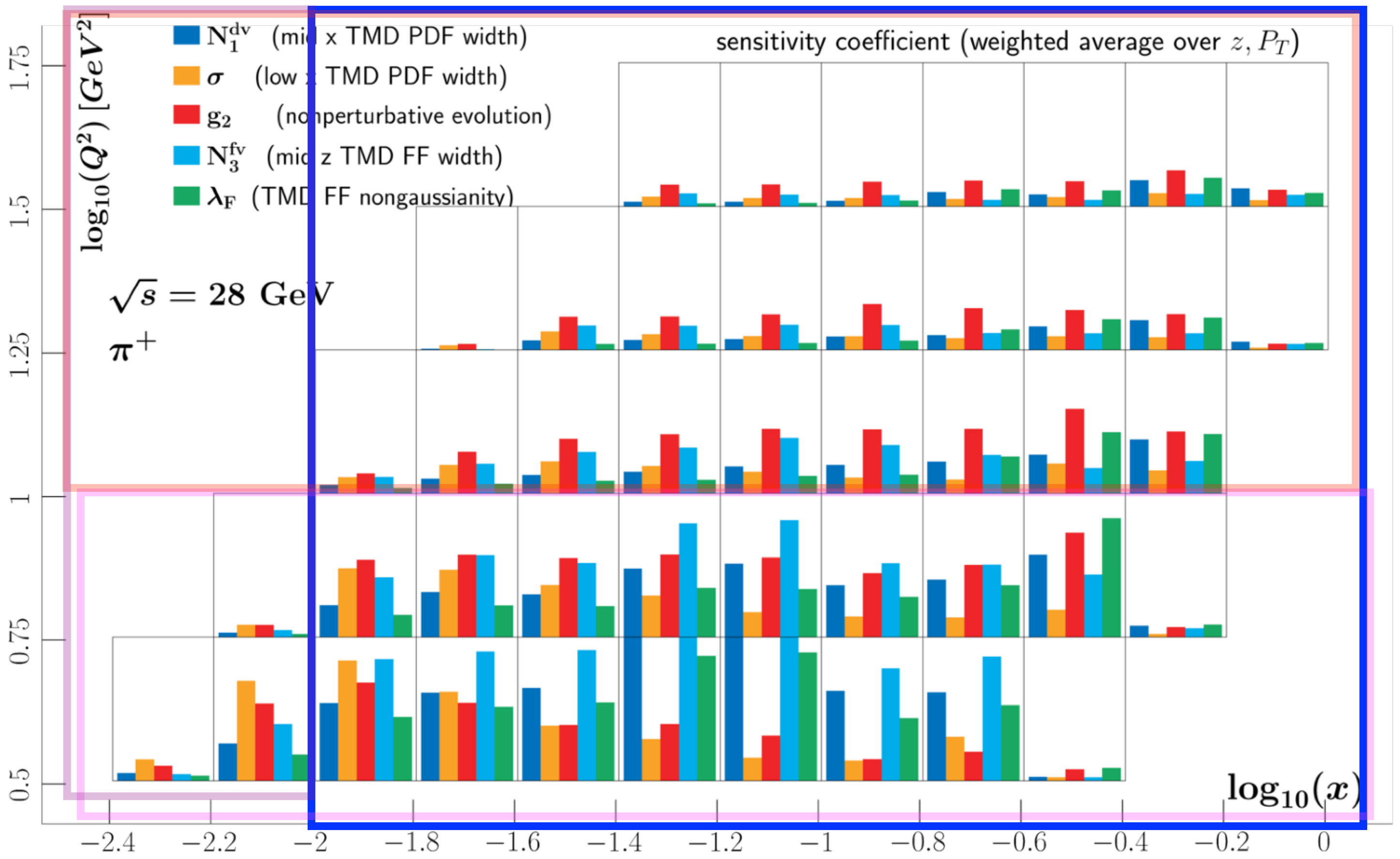
summing over all $10 \leq Q^2 \leq 100$ bins $\longrightarrow 0.00216$ 72% *R*

summing over all $x \leq 0.01$ bins $\longrightarrow 0.00306$ 51% *R*

summing over all $x \geq 0.01$ bins $\longrightarrow 0.00177$ 88% *R*

$\sqrt{s} = 28$ GeV, π^+



Most sensitive (x,Q^2) bins: g_2

bins Energies	All bins	$Q^2 \leq 10$	$10 \leq Q^2 \leq 100$	$x \leq 0.01$	$0.01 < x \leq 1$
$\sqrt{s} = 28$	**R = 6.45**	71% R	72% R	51% R	88% R
$\sqrt{s} = 44$	**R = 8.33**	71% R	69% R	48% R	89% R
$\sqrt{s} = 63$	**R = 9.26**	72% R	67% R	59% R	82% R
$\sqrt{s} = 84$	**R = 9.52**	73% R	66% R	64% R	77% R
$\sqrt{s} = 140$	**R = 10.36**	75% R	63% R	80% R	61% R

Most sensitive (x,Q^2) bins: g_2

good options at $\sqrt{s} = 140,\ x \lesssim 0.01$ **but also at** $\sqrt{s} = 44,\ 0.01 < x \lesssim 1$

bins Energies	All bins	$Q^2 \lesssim 10$	$10 \lesssim Q^2 \lesssim 100$	$x \lesssim 0.01$	$0.01 < x \lesssim 1$
$\sqrt{s} = 28$	***R* = 6.45**	71% *R*	72% *R*	51% *R*	88% *R*
$\sqrt{s} = 44$	***R* = 8.33**	71% *R*	69% *R*	48% *R*	89% *R*
$\sqrt{s} = 63$	***R* = 9.26**	72% *R*	67% *R*	59% *R*	82% *R*
$\sqrt{s} = 84$	***R* = 9.52**	73% *R*	66% *R*	64% *R*	77% *R*
$\sqrt{s} = 140$	***R* = 10.36**	75% *R*	63% *R*	80% *R*	61% *R*

Most sensitive (x,Q^2) bins: N_1

good options at medium-large x and/or at low Q^2

bins Energies	All bins	$Q^2 \lesssim 10$	$10 \lesssim Q^2 \lesssim 100$	$x \lesssim 0.01$	$0.01 < x \lesssim 1$
$\sqrt{s} = 28$	***R* = 5.94**	83% *R*	58% *R*	43% *R*	92% *R*
$\sqrt{s} = 44$	***R* = 6.52**	77% *R*	62% *R*	41% *R*	92% *R*
$\sqrt{s} = 63$	***R* = 6.96**	81% *R*	56% *R*	50% *R*	88% *R*
$\sqrt{s} = 84$	***R* = 6.80**	83% *R*	52% *R*	61% *R*	81% *R*
$\sqrt{s} = 140$	***R* = 6.73**	83% *R*	53% *R*	79% *R*	63% *R*

Most sensitive (x,Q^2) bins: σ

consistent trend: good option at $\sqrt{s} = 140,\ x \le 0.01$ but doesn't need large Q^2

bins Energies	All bins	$Q^2 \le 10$	$10 \le Q^2 \le 100$	$x \le 0.01$	$0.01 < x \le 1$
$\sqrt{s} = 28$	***R* = 5.05**	83% *R*	58% *R*	49% *R*	90% *R*
$\sqrt{s} = 44$	***R* = 6.85**	82% *R*	57% *R*	62% *R*	79% *R*
$\sqrt{s} = 63$	***R* = 8.00**	83% *R*	55% *R*	72% *R*	71% *R*
$\sqrt{s} = 84$	***R* = 8.55**	83% *R*	55% *R*	76% *R*	66% *R*
$\sqrt{s} = 140$	***R* = 10.00**	83% *R*	54% *R*	86% *R*	52% *R*