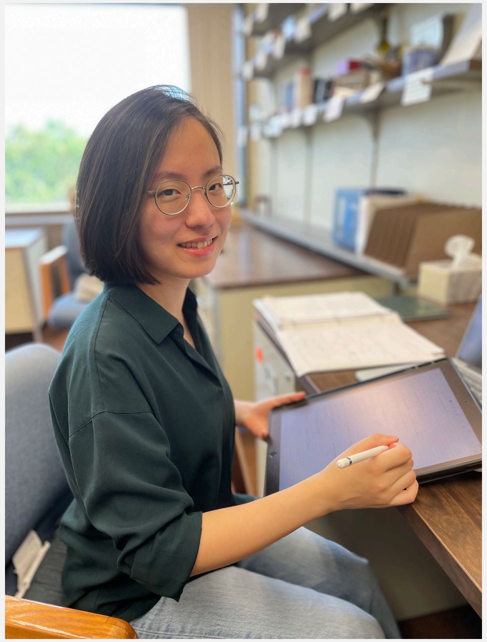

Congratulations Yanzhu Chen!

- “My initial interest in physics began in middle school, after reading *Black Holes & Time Warps: Einstein’s Outrageous Legacy* by Kip Thorne.”
- Under the guidance of Dr. Wei in the Quantum Information Science Group at SBU, Chen is interested in how topology in condensed matter physics can assist in quantum computing.
- **Paper with BNL collaborators:** Detector tomography on IBM quantum computers and mitigation of an imperfect measurement

YC, M. Farahzad, S. Yoo, and T.-C. Wei. Phys. Rev. A 100, 052315 (2019).

Characterization of near-term circuits in quantum computing

Yanzhu Chen

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Department of Physics and Astronomy

NSF

Stony Brook University

Brookhaven National Laboratory

July 29, 2021

Outline

- A glimpse of quantum computing

- A study of measurement: Detector tomography and readout error mitigation

- A ruler of performance: Scalable evaluation of quantum-circuit error loss using Clifford sampling

A *bit* of information

- Classical computing: **bit** (=binary digit, a number taking two possible values)

0

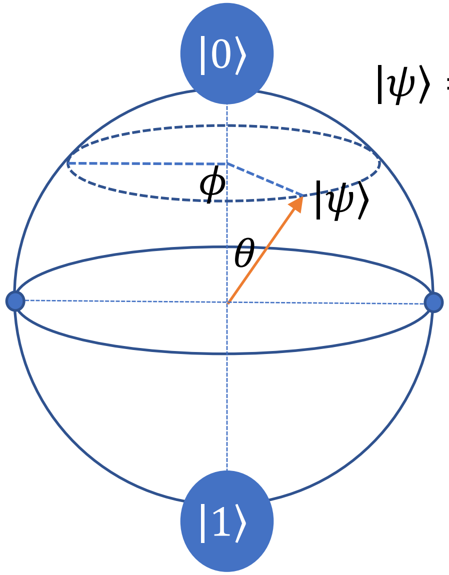
OR

1

- Quantum computing: **qubit** (=quantum bit, a two-level quantum system)

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$



$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

- Also: systems with more than two levels; systems with continuous variables

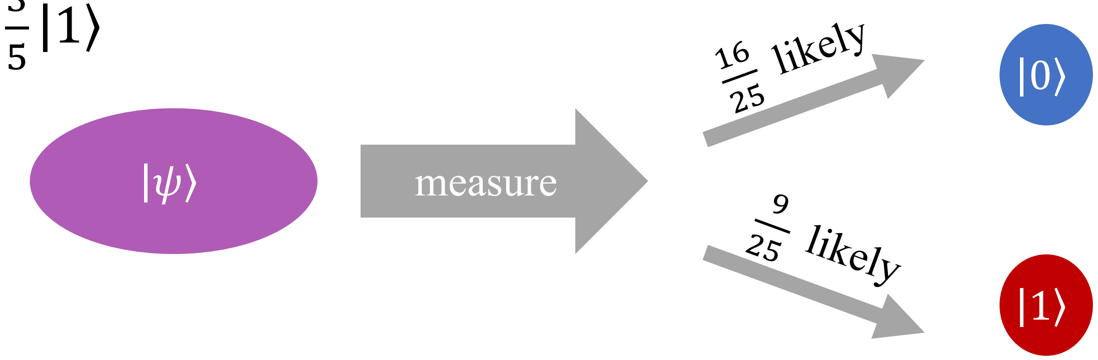
A *superposition* ≠ a probability distribution

- Classically: a probability distribution assigns one *probability* to each possible outcome; probabilities sum up to 1

  E.g. getting 0 and 1 with probabilities $\frac{4}{5}$ and $\frac{1}{5}$, respectively

- Quantum mechanically: a state vector with *amplitudes* (very loosely speaking, $\sim\sqrt{\text{probability}}$)

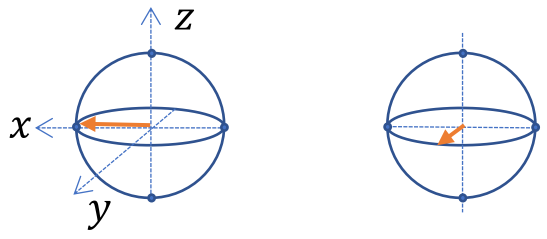
  E.g. $|\psi\rangle = \frac{4}{5}|0\rangle + i\frac{3}{5}|1\rangle$



An *entangled* system

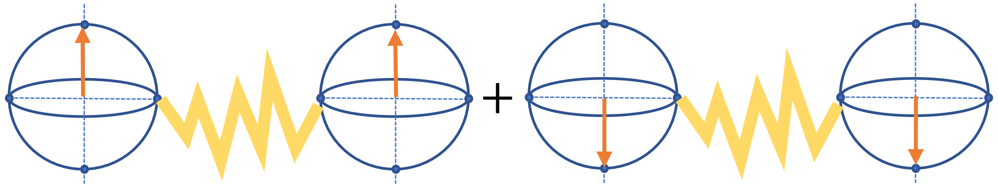
- Product state: in the form of $|\phi\rangle|\varphi\rangle$

  E.g. $|\psi\rangle = \frac{1}{2}(|0\rangle + |1\rangle)\otimes(|0\rangle + i|1\rangle)$

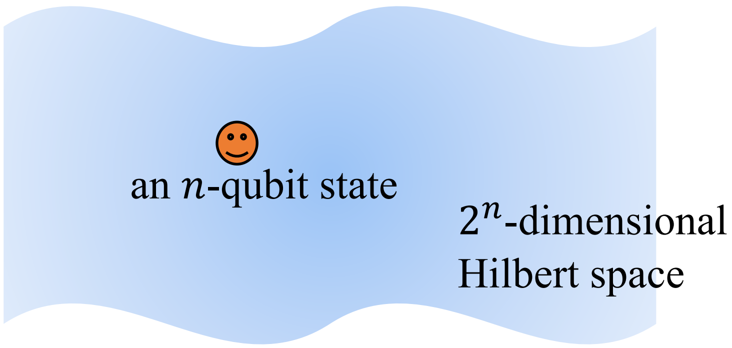


- Entangled state: fail to be written as a product state

  E.g. $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle\otimes|0\rangle + |1\rangle\otimes|1\rangle)$

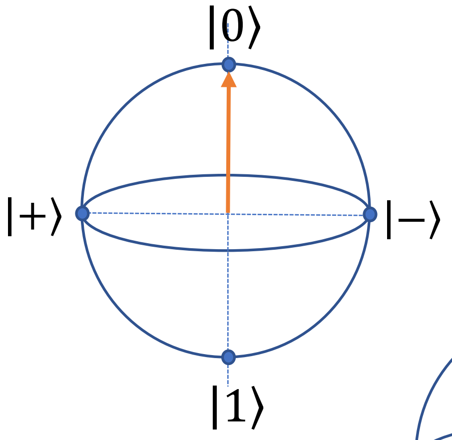


- For an n-qubit system, it allows the entire 2^n-dimensional Hilbert space to be explored

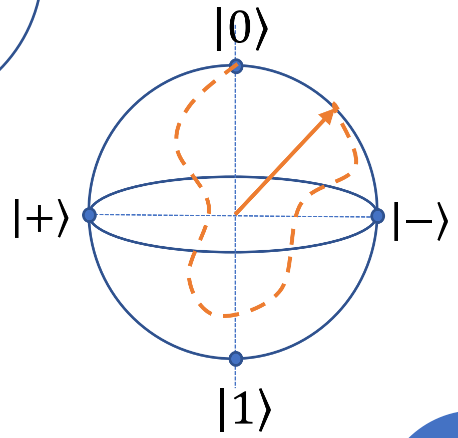


A way to manipulate information

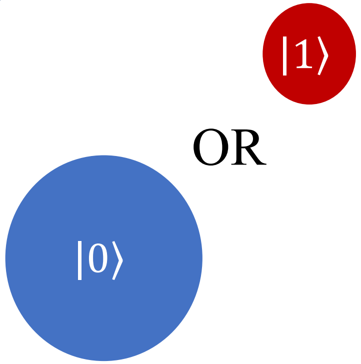
- Prepare the qubits in a desired state



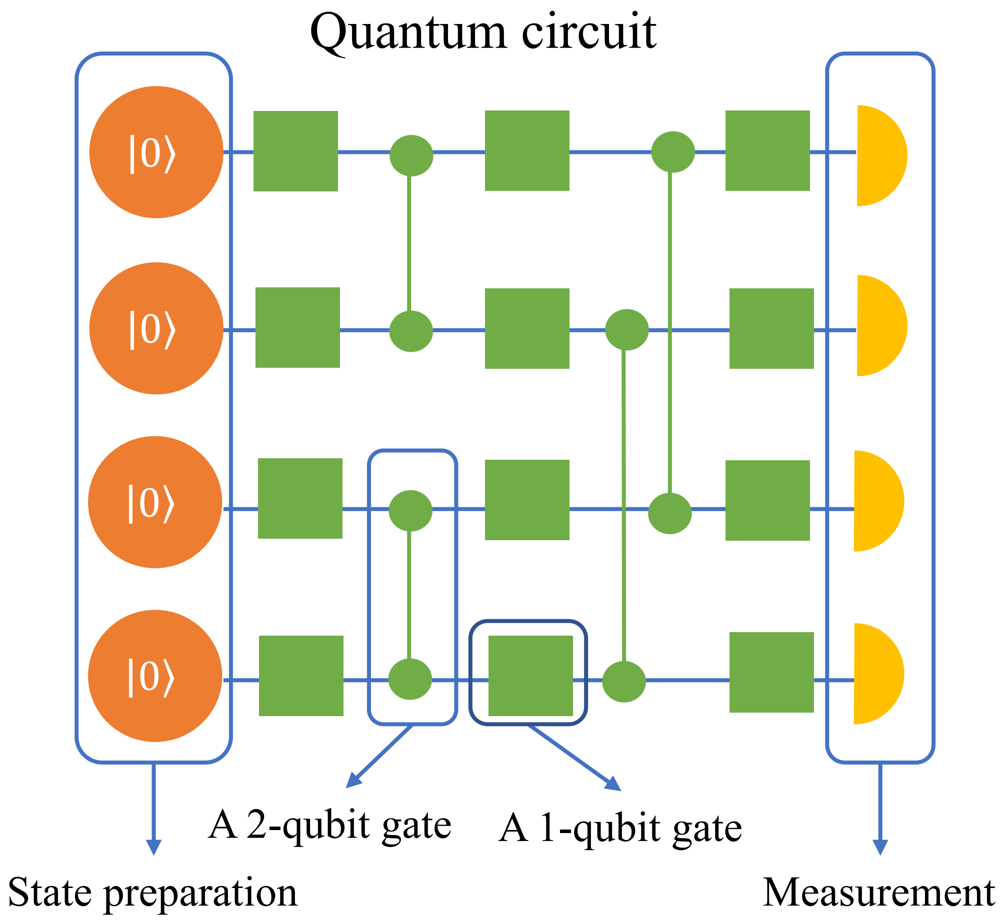
- Make the qubits evolve as desired



- Get classical information out of the quantum system



An idealization of quantum computer



- State preparation: state vector $|\psi\rangle$
- Gates: unitary maps $U_1, U_2, \dots, U_N$
- Measurement: projective measurement

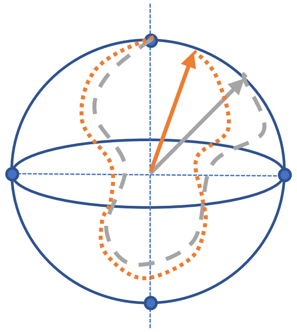
$$\{\langle\phi_a|\}, \quad \sum_a |\phi_a\rangle\langle\phi_a| = \mathbb{I}$$

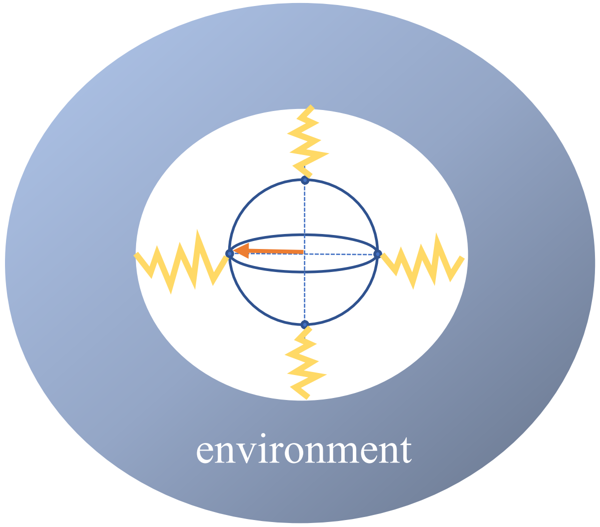
- Expectation value: probability of obtaining outcome a is

$$p_a = |\langle\phi_a|U_N \dots U_1|\psi\rangle|^2$$

A complication of *errors*

- Implementations: superconducting systems; trapped ion systems; photonic systems; ……
- Limit on the operations; unwanted interactions with the environment

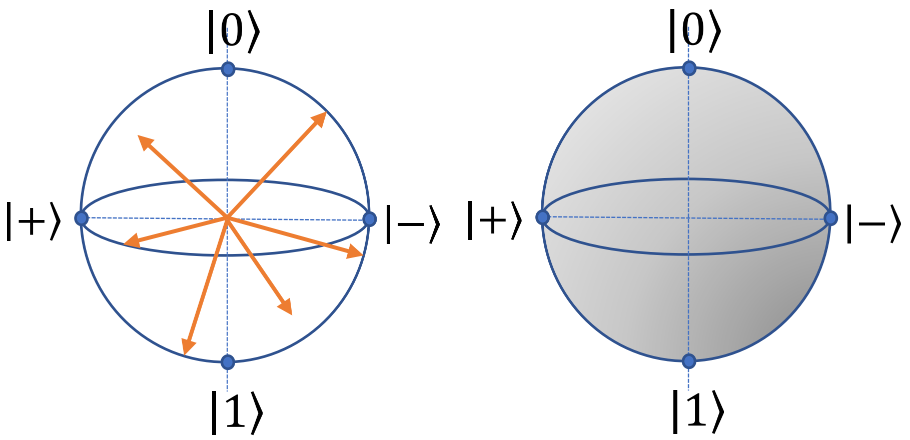




- Can end up in a state different from the desired
- Can end up in a *mixed* state: a classical mixture of *pure* quantum states

E.g. $\rho = \frac{4}{5}|0\rangle\langle 0| + \frac{1}{5}\frac{|0\rangle+|1\rangle}{\sqrt{2}}\frac{\langle 0|+\langle 1|}{\sqrt{2}}$

Classical probabilities   pure quantum states



Some remedies

Of course we should improve hardware, but we can also do:

- Quantum error correction

  D. A. Lidar and T. A. Brun. *Quantum Error Correction.* Cambridge University Press, 2013.

  E. T. Campbell, B. M. Terhal, and C. Vuillot. Nature, 549, 172-179 (2017).

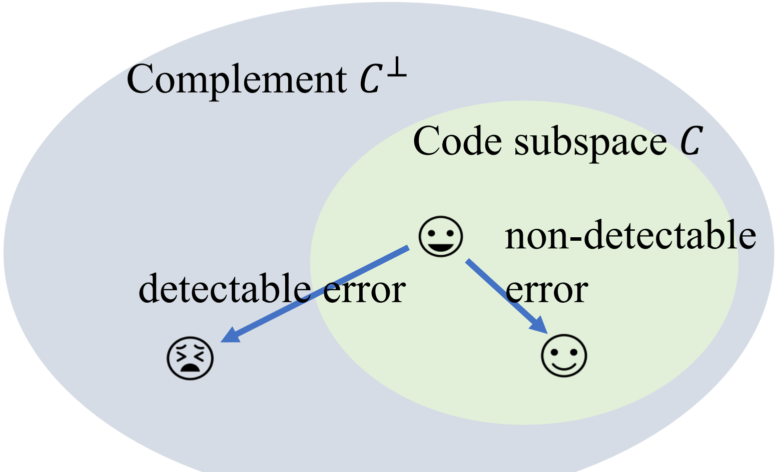
  T. Brun. *Quantum Error Correction.* Oxford Research Encyclopedia of Physics, 2021.

- Quantum error mitigation

  Zero-noise extrapolation, probabilistic error cancellation, ……

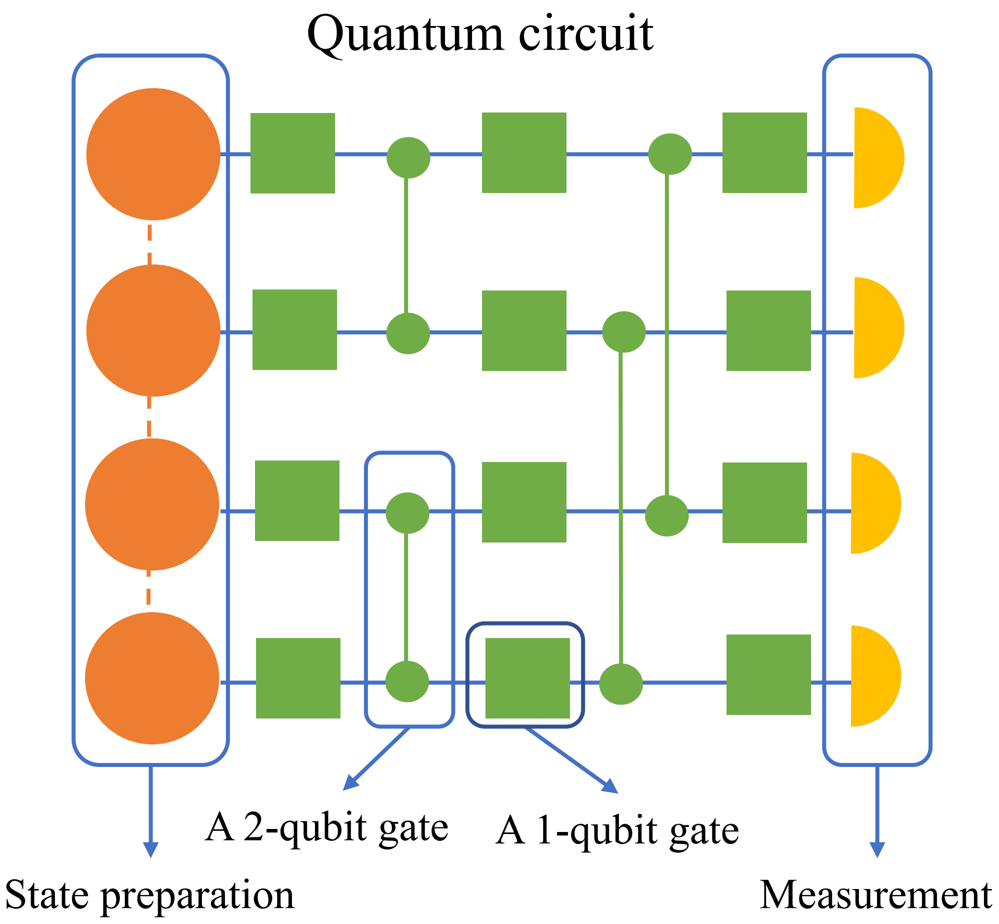
  K. Temme, S. Bravyi, and J. M. Gambetta. Phys. Rev. Lett., 119, 180509 (2017)

  Y. Li and S. C. Benjamin. Phys. Rev. X, 7, 021050 (2017)



Quantum error correction

A (slightly) more realistic circuit



- State preparation: density matrix ρ_0
- Gates: completely positive trace-preserving (CPTP) maps $G_1, G_2, \dots, G_N$
- Measurement: positive operator valued measure (POVM)

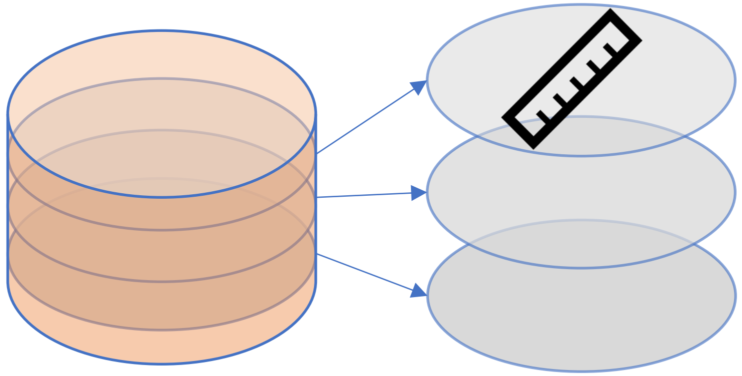
$$\{E_a\}, \quad \sum_a E_a = \mathbb{I}$$

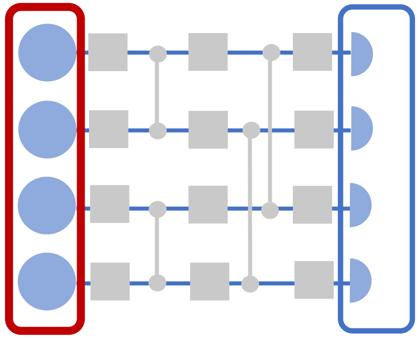
- Expectation value: probability of obtaining outcome a is

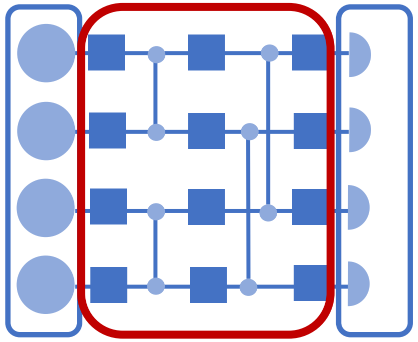
$$p_a = \mathrm{Tr}[E_a G_N \dots G_1(\rho_0)]$$

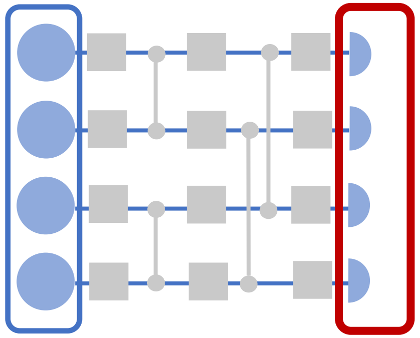
Quantum tomography tools

- Quantum state tomography
  Z. Hradil. Phys. Rev. A, 55, 1561-1564 (1997)
- Quantum process tomography
  J. F. Poyatos, J. I. Cirac, and P. Zoller. Phys. Rev. Lett., 78, 390–393 (1997)
  J. B. Altepeter et al. Phys. Rev. Lett., 90, 193601 (2003)
- Quantum detector tomography
  D. F. V. James et al. Phys. Rev. A, 64, 052312 (2001)
- All in one: Gate set tomography
  R. Blume-Kohout et al. Nat. Commun., 8, 14485 (2017)



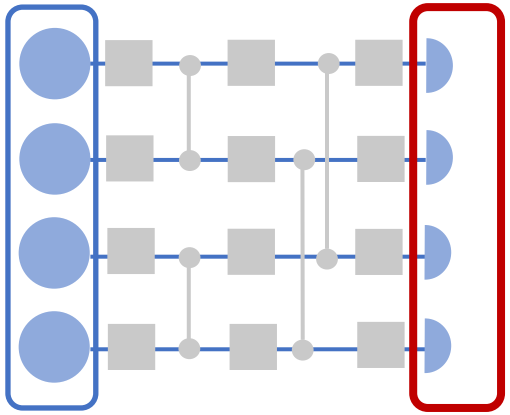






A study of measurement:
Detector tomography and readout error mitigation

YC, Maziar Farahzad, Shinjae Yoo*, and Tzu-Chieh Wei. Phys. Rev. A 100, 052315 (2019).



*Computational Science Initiative, Brookhaven National Laboratory

Detector tomography experiment

- To characterize a 2^N-outcome positive operator valued measure (POVM)
- Prepare test states that span the Hilbert-Schmidt space
- A 1-qubit detector:

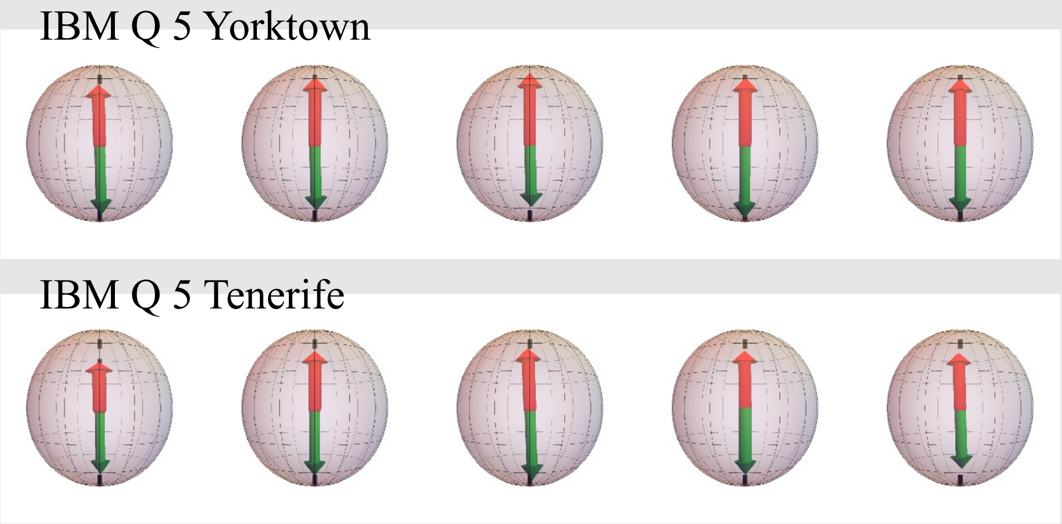
$$\pi^{(0)} = a^{(0)}\left(1 + \vec{r}^{(0)} \cdot \vec{\sigma}\right)$$

$$\pi^{(1)} = a^{(1)}\left(1 + \vec{r}^{(1)} \cdot \vec{\sigma}\right)$$

- An N-qubit detector:

$$\pi^{(\vec{n})} = \sum_{\vec{i}} c_{\vec{i}}^{(\vec{n})} \sigma_{i_1} \otimes \cdots \otimes \sigma_{i_N}$$

Not scalable!



1-qubit detector results. The arrow on the Bloch sphere indicates the vector $\vec{r}^{(0)}$ or $\vec{r}^{(1)}$; the width of the arrow represents magnitude $a^{(0)}$ or $a^{(1)}$.

Evidence of crosstalk

Use Frobenius norm distance to characterize difference between the same operators measured in different ways.

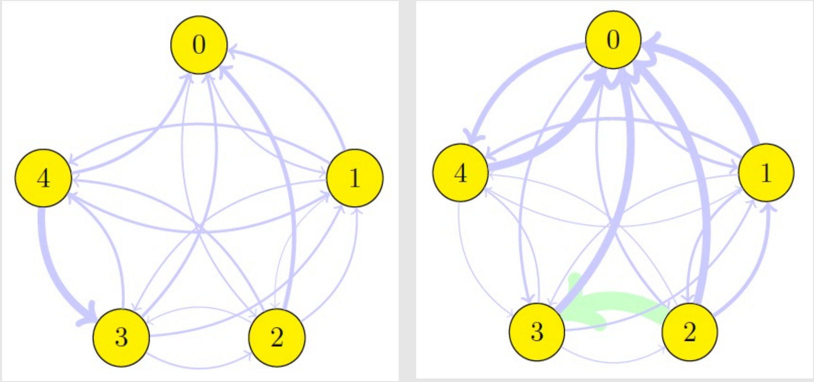
$$\Pi_1^{(0)} = \sum_{i=0}^{3} c_i^{(0)} \sigma_i$$

$$d(\Pi_1^{(0)}, \Pi_1'^{(0)}) = \sqrt{\sum_{i=0}^{3} (c_i^{(0)} - c_i'^{(0)})^2}$$

Distance between single-qubit detectors from individual measurement and those measured in parallel with other qubits

Qubit	0	1	2	3	4
IBMQ_5_Tenerife	0.011	0.010	0.023	0.087	0.025
IBMQ_5_Yorktown	0.042	0.017	0.044	0.031	0.024

Pairwise influence



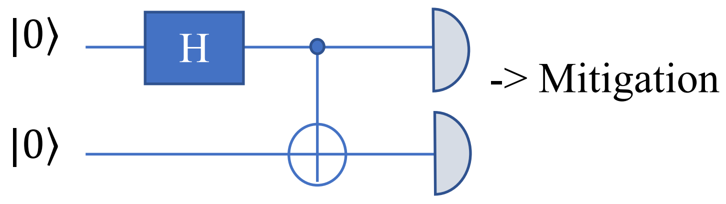
IBMQ_5_Yorktown   IBMQ_5_Tenerife

Distance between single-qubit detectors from individual measurement and those from two-qubit detectors

Readout error mitigation

- Mitigation according to: $P_{\text{measured}} = M P_{\text{mitigated}}$
  - Direct inversion with cut-off and normalization
  - Minimize with constraints of positivity and normalization $|P_{\text{measured}} - M P_{\text{mitigated}}|^2$
- Example: on qubits 0 and 1 of IBMQ_5_Yorktown

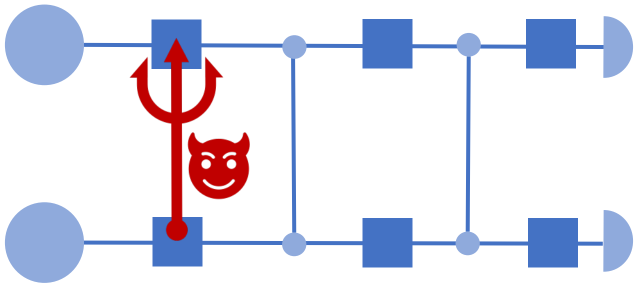
Probability	P_{00}	P_{01}	P_{10}	P_{11}
Ideal	0.5	0	0	0.5
Experiment	0.470	0.040	0.054	0.436
With inversion	0.489	0.003	0.000	0.509
With optimization	0.490	0.001	0.000	0.509
Uncertainty	0.0018	0.0014	0.0014	0.0016



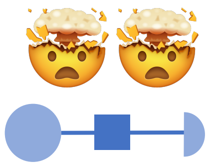
Now: qiskit.ignis.mitigation.measurement H. Abraham et al, 10.5281/zenodo.2562110 (2019)

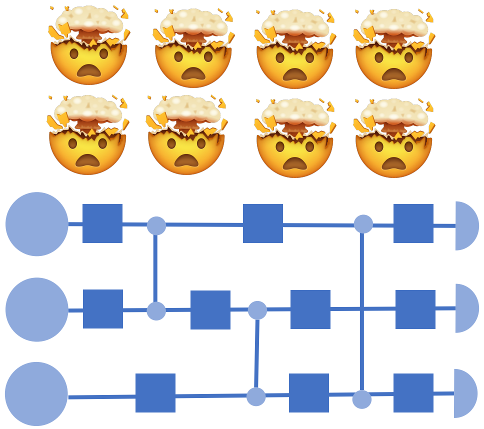
Obstacles in characterizing circuits

- Spatial correlations

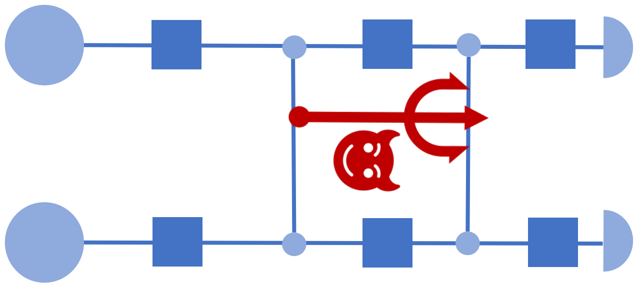


- Lead to the problem about scalability



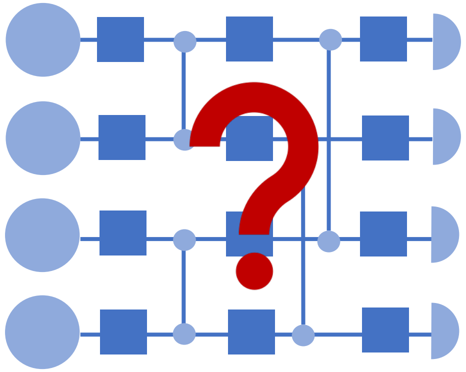


- Temporal correlations



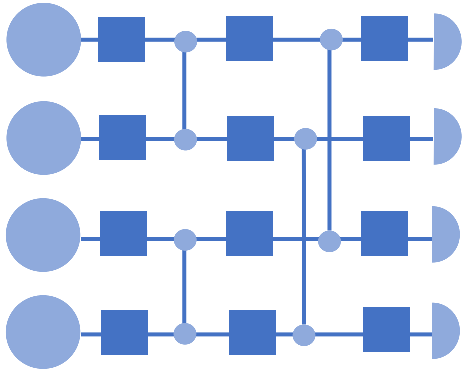
A ruler of performance: Scalable evaluation of quantum-circuit error loss using Clifford sampling

Z. Wang, **YC**, Z. Song, D. Qin, H. Li, Q. Guo, H. Wang, C. Song, and Y. Li. Phys. Rev. Lett. 126, 080501 (2021).



Characterizing circuit performance

- Loss function as a measure of quality; provide ground for optimization problems
- Consider a family of circuits with
  - the same **frame** operations: Clifford 2-qubit gates, state preparation and measurement in the computational basis
  - variable unitary **1-qubit gates**
- Address: noise from frame operations (that does not depend on 1-qubit gates)
- Ignore: gate-dependent noise from 1-qubit gates
- Use loss functions for characterization



Error-free result

Computational result: expectation value of observable

$$L = \int_{\boldsymbol{R}\in\mathbb{U}} d\boldsymbol{R}\, |\mathrm{com}^{\mathrm{ef}}(\boldsymbol{R}) - \mathrm{com}(\boldsymbol{R})|^2$$

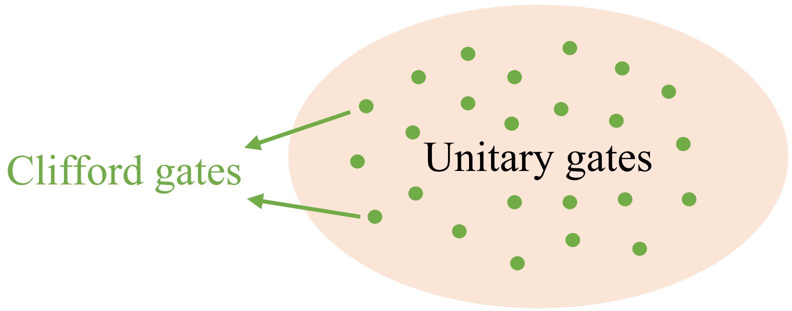
Evaluation by Clifford sampling

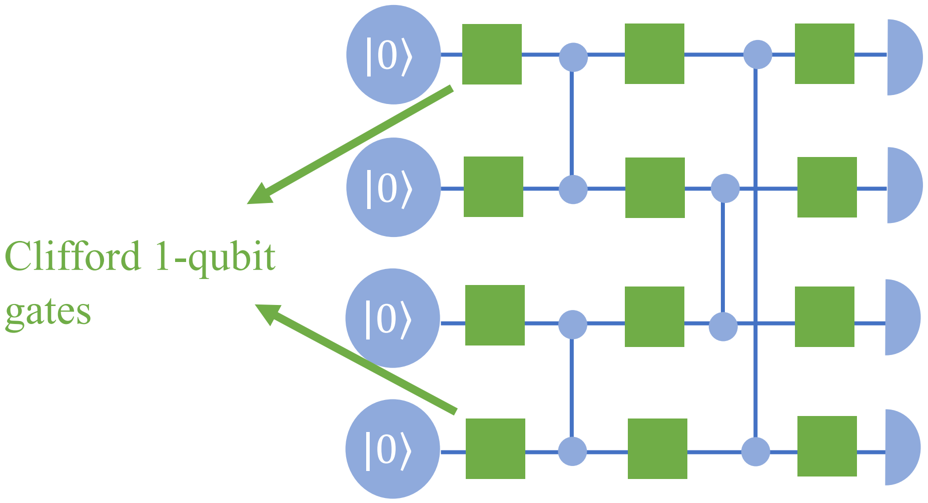
- Replacing the unitary 1-qubit gates with randomly selected **Clifford** 1-qubit gates

$$L = \frac{1}{|\mathbb{C}|} \sum_{\mathbf{R}\in\mathbb{C}} \left|\mathrm{com}^{\mathrm{ef}}(\mathbf{R}) - \mathrm{com}(\mathbf{R})\right|^2$$

- Ideal result is **classically simulable**: can efficiently estimate the loss function
- Clifford gates form a **unitary 2-design**: recovers sampling over unitary gates

$$L = \int_{\boldsymbol{R}\in\mathbb{U}} d\boldsymbol{R} \left|\mathrm{com}^{\mathrm{ef}}(\boldsymbol{R}) - \mathrm{com}(\boldsymbol{R})\right|^2$$



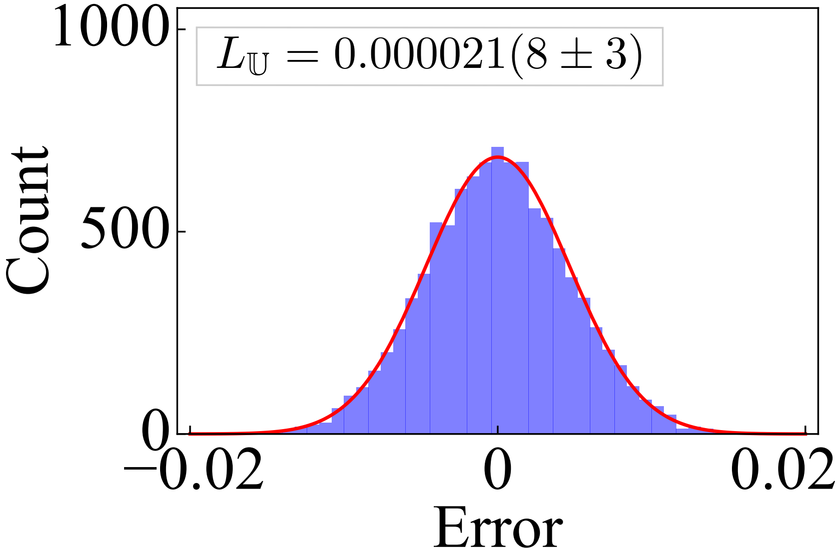


Clifford gates: map Pauli operators to Pauli operators.
Clifford 1-qubit gates = $\{G \in U(2) | G\sigma G^\dagger \in P_1 \ \forall \sigma \in P_1\}$ where P_1 is the 1-qubit Pauli group.

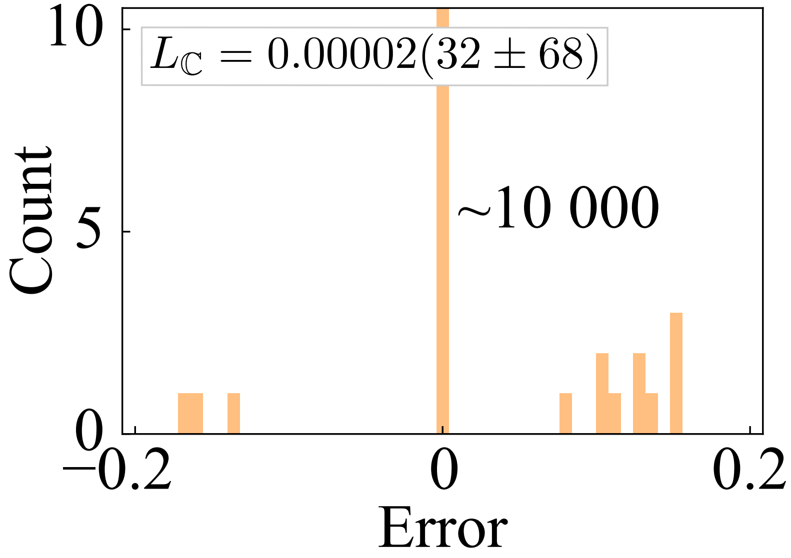
Numerical simulation

- We simulated a 10-qubit circuit with 100 Clifford 2-qubit gates
- Shown below: with depolarizing noise model
- Error = com - com^{ef}

(a) Unitary sampling simulation

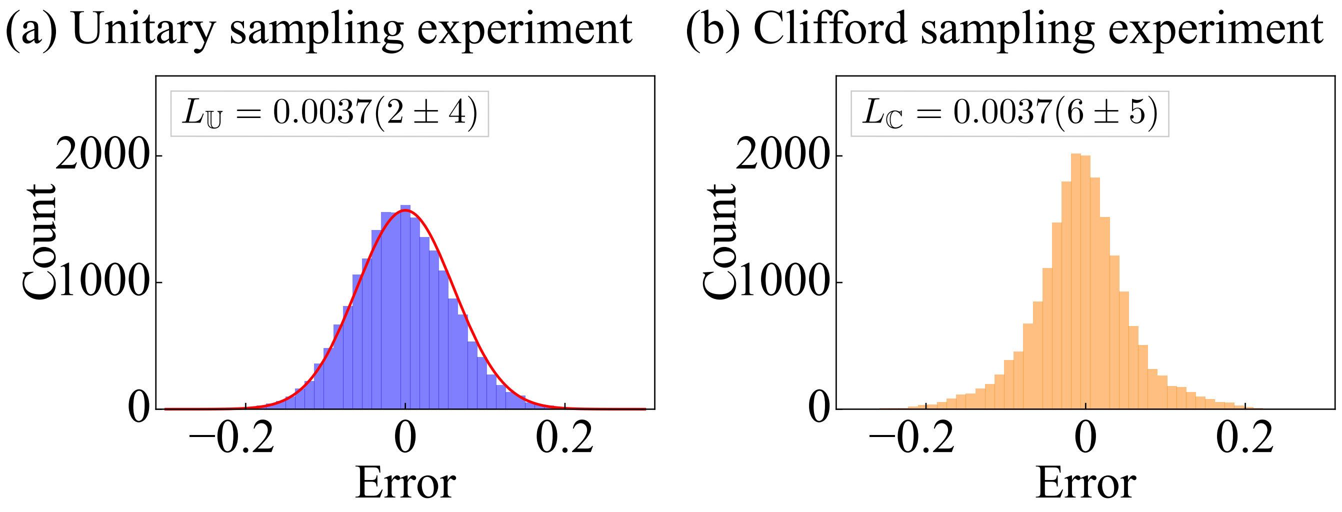


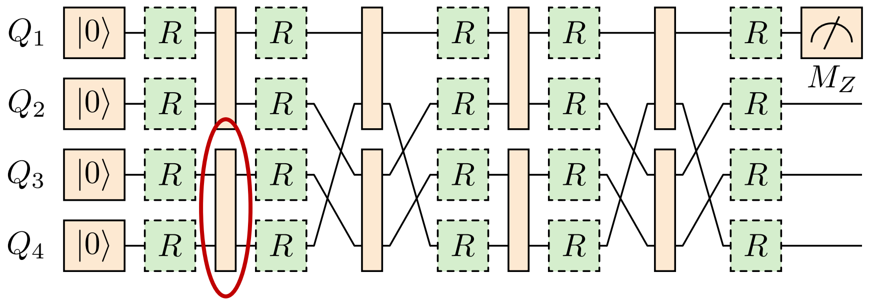
(b) Clifford sampling simulation



Experiment

- Superconducting device
- Done by experimentalists at Zhejiang University: Zhen Wang, Zixuan Song, Hekang Li, Qiujiang Guo, H. Wang, Chao Song





$$U_{\text{phase}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & i & 0 \\ 0 & i & 1 & 0 \\ i & 0 & 0 & 1 \end{pmatrix}$$

Summary

- Detector tomography and readout error mitigation
  - o Evidence of correlations
  - o Can use results for readout error mitigation
- Scalable evaluation of quantum-circuit error loss using Clifford sampling
  - o Loss function to characterize circuit performance
  - o Evaluated by Clifford sampling
  - o Can provide ground for optimization problems
- Current and future:
  - o Methods and measures suitable for specific physical systems and/or algorithms
  - o Implications for error mitigation and error correction?

Acknowledgements

- **Theory**

  Stony Brook University: Tzu-Chieh Wei, Maziar Farahzad (now at University of Toronto)

  Brookhaven National Laboratory: Shinjae Yoo

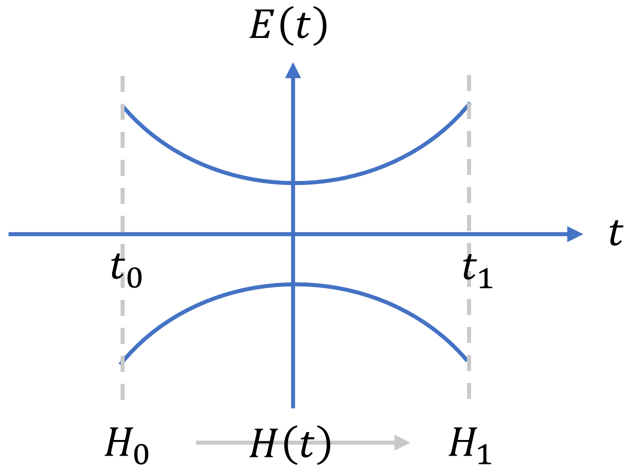
  Graduate School of China Academy of Engineering Physics: Ying Li, Dayue Qin

- **Experiment**

  Zhejiang Univeristy: Zhen Wang, Chao Song, Zixuan Song, Hekang Li, Qiujiang Guo, H. Wang

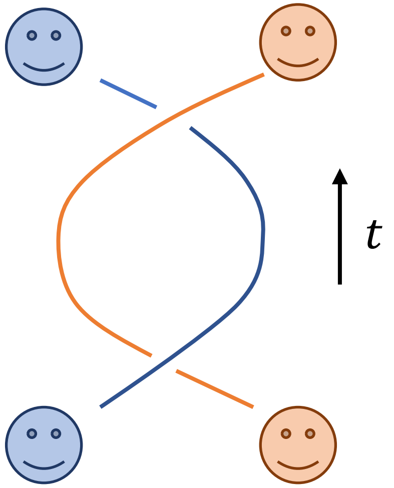
Appendix: examples of other schemes

- Adiabatic quantum computing



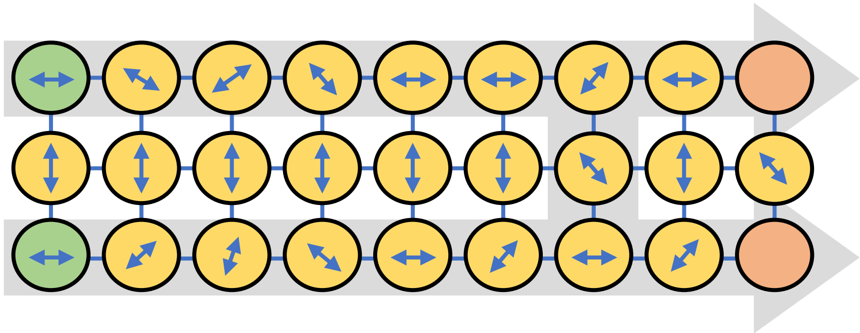
T. Albash and D.A. Lidar. Rev. Mod. Phys., 90, 015002 (2018).
E. Grant and T. Humble. *Adiabatic Quantum Computing and Quantum Annealing*. Oxford Research Encyclopedia of Physics, 2021.

- Topological quantum computing



M. H. Freedman, A. Yu. Kitaev, M. J. Larsen, Z. Wang. arXiv:quant-ph/0101025v2 (2002).
A.Yu. Kitaev. Ann. Phys., 303, 2-30, (2003).
Keisuke Fujii. *Quantum Computation with Topological Codes*. Springer, 2015.

- Measurement-based quantum computing



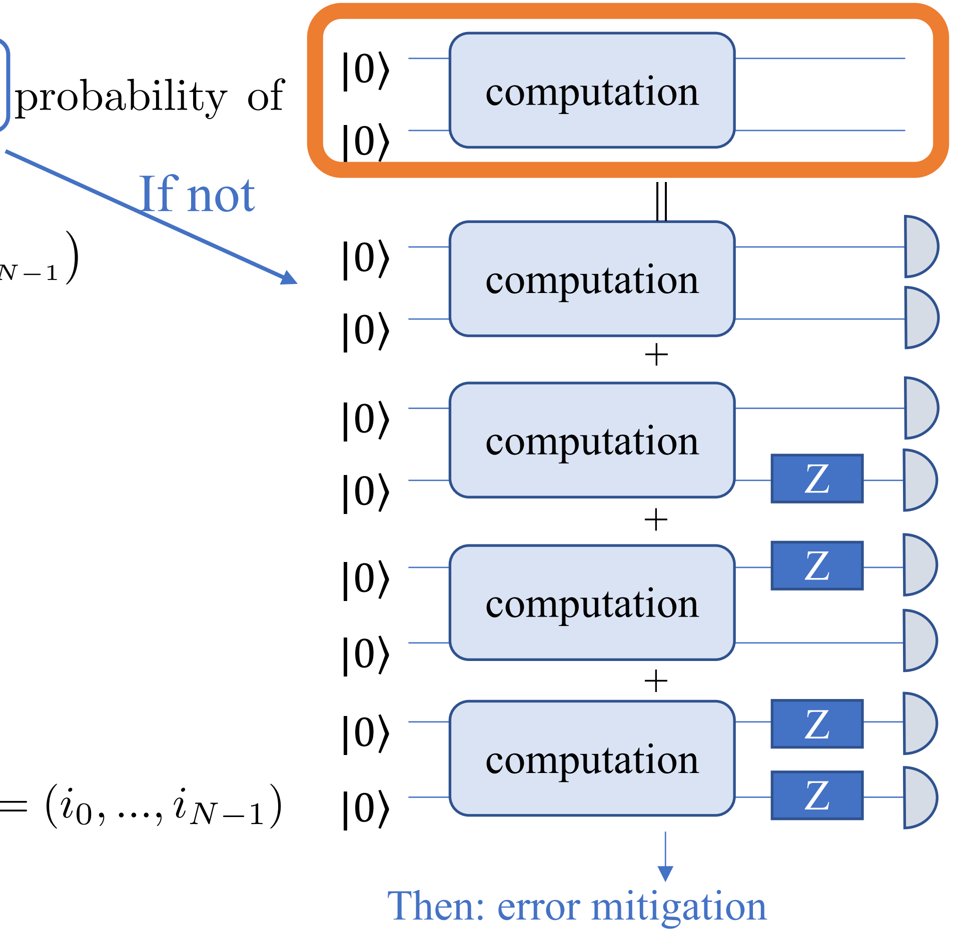
R. Raussendorf and H. J. Briegel. Phys. Rev. Lett., 86, 5188-5191 (2001).
R. Raussendorf, D. E. Browne, and H. J. Briegel. Phys. Rev. A, 68, 022312 (2003).
T.-C. Wei. *Measurement-Based Quantum Computation*. Oxford Research Encyclopedia of Physics, 2021.

Appendix: readout error mitigation

Neglecting coefficients $c_{\vec{i}}^{(\vec{n})}$ when entries of $\vec{i}$ involve 1 or 2, probability of obtaining outcome $\vec{n}$:

$$\begin{aligned} P_{\text{measured},\vec{n}} &= \sum_{i_0=0,3} \dots \sum_{i_{N-1}=0,3} c_{\vec{i}}^{(\vec{n})} \mathrm{Tr}\left(\rho \sigma_{i_0} \otimes \dots \otimes \sigma_{i_{N-1}}\right) \\ &= \sum_{i_0=0,3} \dots \sum_{i_{N-1}=0,3} c_{\vec{i}}^{(\vec{n})} \sum_{\vec{m}} (-1)^{\vec{m}\cdot\vec{i}/3} P_{\vec{m}}, \\ &= \sum_{\vec{m}} \hat{M}_{\vec{n};\vec{m}} P_{\text{ideal},\vec{m}}, \\ \hat{M}_{\vec{n};\vec{m}} &= \sum_{\vec{I}} c_{\vec{I}}^{(\vec{n})} (-1)^{\vec{m}\cdot\vec{I}/3} \end{aligned}$$

where $\vec{m}, \vec{n}$ are length-N binary strings, and entries of the index $\vec{I} = (i_0, \dots, i_{N-1})$ are either 0 or 3.



Appendix: definition for unitary 2-design

- Unitary 2-design:

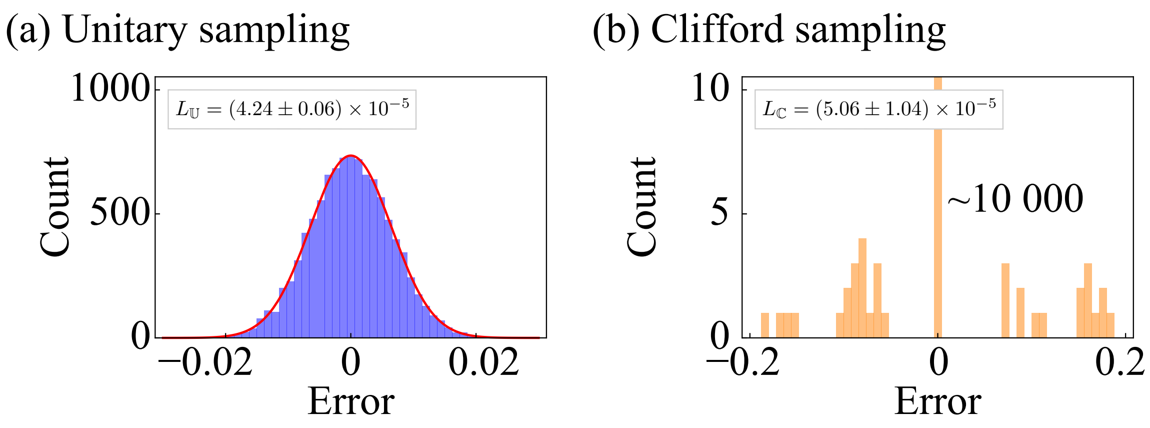
$$\frac{1}{|\mathbb{C}_n|}\sum_{U\in\mathbb{C}_n}(U^{\otimes 2})\otimes(U^{\otimes 2})^{\dagger}=\int_{U(2^n)}dU(U^{\otimes 2})\otimes(U^{\otimes 2})^{\dagger}$$

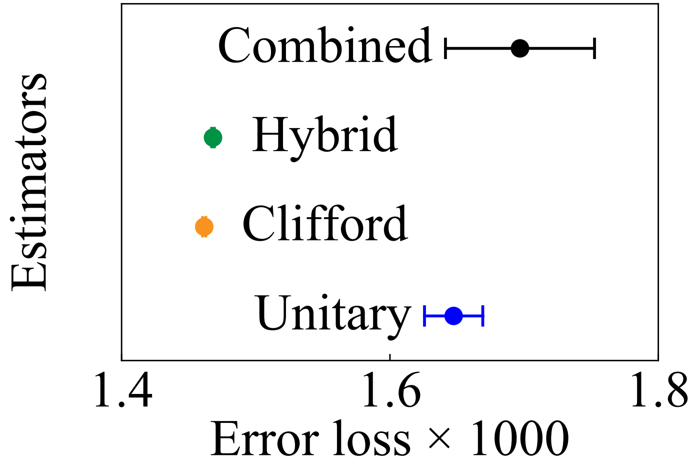
- Clifford group is a unitary 1- and 2-design
- For n-qubit systems, Clifford group is also a unitary 3-design (but not a 4-design)

Appendix: hybrid sampling

- Relax the assumption of negligible gate-dependent noise from 1-qubit gates
- Enlarge sampling set by allowing one unitary 1-qubit gate in the circuit -> One unitary 1-qubit gate + all others Clifford
- This accounts for weak (first-order) dependence on 1-qubit gates
- Still efficiently simulable
- Example: simulation with gate-dependent 1-qubit noise
- With *N_R* 1-qubit gates in the circuit, first-order hybrid sampling:

$$L_{\text{combined}} = N_R L_{\text{hybrid}} - (N_R - 1) L_{\mathbb{C}}$$





Appendix: Uphase gate in Clifford sampling experiment

Q. Guo et al, Phys. Rev. Lett. 121, 130501 (2018)

- Two on-resonantly coupled qubits, Q1 and Q2, each resonantly driven by a classical field
- Hamiltonian in the interaction picture (qubit reference frame) is

$$H = \hbar\left(\lambda\sigma_1^+\sigma_2^- + \sum_{j=1,2}\Omega_j e^{-i\phi_j}\sigma_j^+\right) + \text{h.c.}$$

Ω_j, ϕ_j: Rabi frequency and phase of the driving field on qubit j

- Under the condition, $\phi_1 = \phi_2 = \phi$, $|\Omega_1 - \Omega_2| \gg |\lambda|$, the effective Hamiltonian is

$$H_{\text{eff}} = \frac{1}{2}\hbar\lambda S_{z,\phi,1}S_{z,\phi,2} + \hbar\sum_{j=1,2}\Omega_j S_{z,\phi,j}$$

$$S_{z,\phi,j} = |+_{\phi,j}\rangle\langle+_{\phi,j}| - |-_{\phi,j}\rangle\langle-_{\phi,j}|$$

- The first term results in conditional phase shift in a dressed state basis; the second term is for single-qubit rotations, which produce null effect if the phases of both driving fields are reversed right in the middle of the interaction time

Appendix: error models in Clifford sampling simulation

- 2-qubit depolarizing channel
- 2-qubit dephasing channel
- 1-qubit amplitude damping channel
- Correlated coherent error as 1-qubit rotations with same angles
- In gate-dependent noise model: 1-qubit depolarizing channel with parameters dependent on 1-qubit gates