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Resummation of Super-Leading Logarithms

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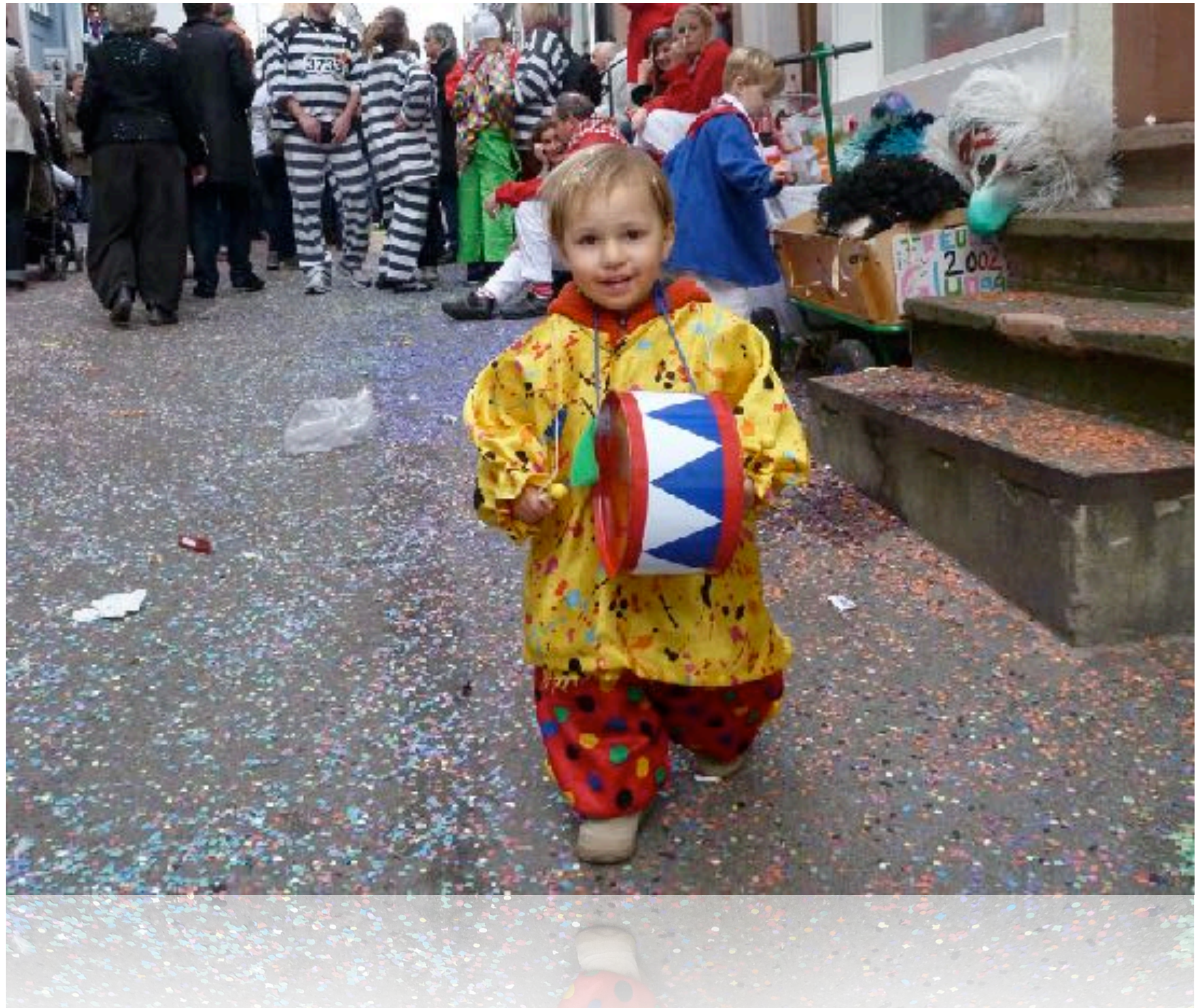
based on arXiv:2107.01212 + upcoming
with Matthias Neubert and Ding Yu Shao

Seminar at BNL, September 9, 2021

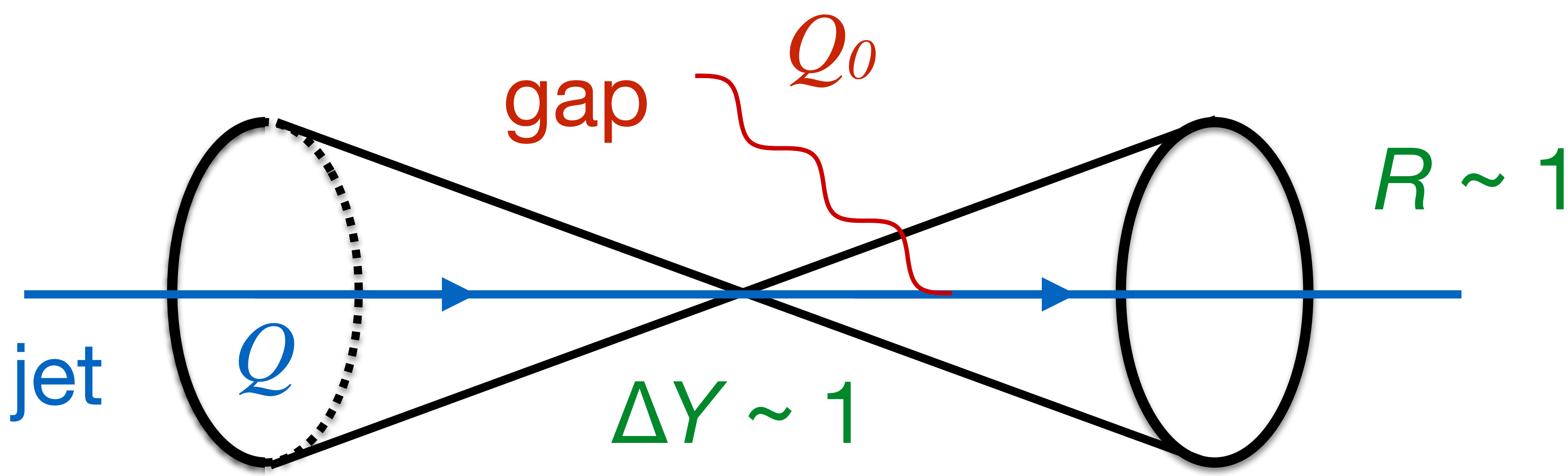


Super-leading and non-global logarithms

Super-Leading Logs (SLLs)?

Forshaw, Kyrieleis, Seymour '06 '08, …

Consider gaps between jets aka interjet energy flow observable:

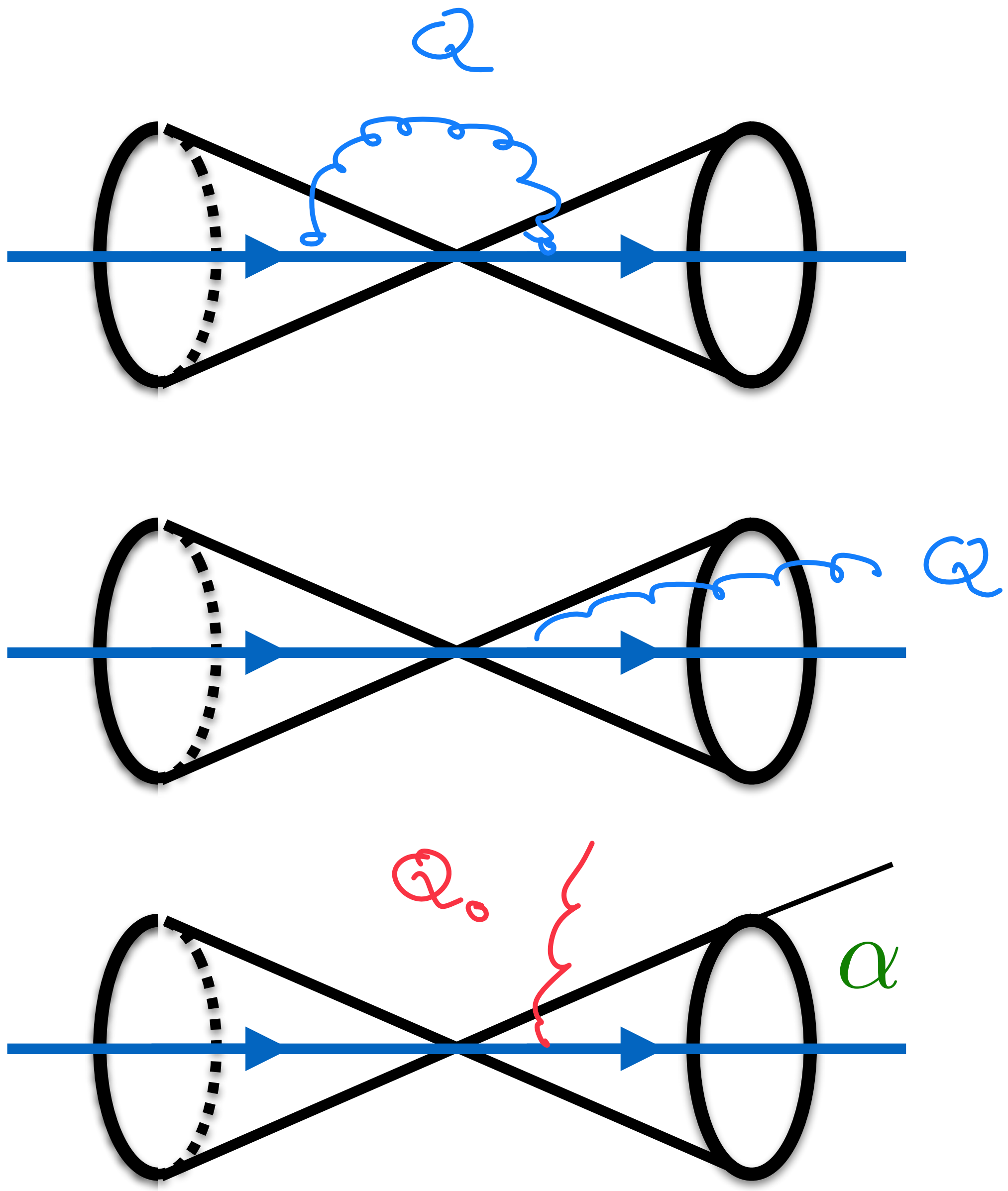


Large logarithms $\alpha_s^n L^m$ with $L = \ln(Q/Q_0)$

- e^+e^- : $m \leq n$, leading logs $m = n$
- pp : $\alpha_s L,\ \alpha_s^2 L^2,\ \alpha_s^3 L^3\ ,\alpha_s^4 L^5 \ldots\ ,\alpha_s^{3+n} L^{3+2n}$

Logarithms generally arise from "incomplete cancellation" of soft and collinear regions of diagrams. At NLO in e^+e^-

$$d = 4 - 2\epsilon$$



$$\sigma_0 \frac{\alpha_s C_F}{4\pi} \left(\frac{\mu^2}{Q^2}\right)^\epsilon \left[-\frac{4}{\epsilon^2} - \frac{6}{\epsilon} + \dots\right]$$

soft+collinear divergence

$$\sigma_0 \frac{\alpha_s C_F}{4\pi} \left(\frac{\mu^2}{Q^2}\right)^\epsilon \left[+\frac{4}{\epsilon^2} + \frac{6}{\epsilon} - \frac{4\ln(r)}{\epsilon} + \dots\right]$$

wide-angle soft

$$\sigma_0 \frac{\alpha_s C_F}{4\pi} \left(\frac{\mu^2}{Q_0^2}\right)^\epsilon \left[\frac{4\ln(r)}{\epsilon} + \dots\right]$$

$$r = \tan^2(\alpha/2)$$

$$\sigma_0 \frac{\alpha_s C_F}{4\pi} 4\ln(r) \ln\frac{Q^2}{Q_0^2} + \dots$$

- In e^+e^- collinear contributions cancel between real and virtual, only soft single logarithm (per order) remains.
- Using finiteness of σ, one can reconstruct log either from computation of loop + hard emission or soft emission
  - knowledge of divergences sufficient ↔ anomalous dimensions in SCET!
  - soft limit for hard emissions good enough
- Soft limit of amplitudes has simple eikonal form

$$|\mathcal{M}_{m+1}(\{\underline{p}, q\})\rangle = \varepsilon^{*\mu} \boldsymbol{J}_{\mu,a}(q) |\mathcal{M}_m(\{\underline{p}\})\rangle = g_s \sum_{i=1}^{m} \boldsymbol{T}_i^a \frac{n_i \cdot \varepsilon^*}{n_i \cdot q} |\mathcal{M}_m(\{\underline{p}\})\rangle$$

q momentum of soft gluon
ε polarization vector

$$n_i = \frac{p_i}{E_i}$$

$$(\boldsymbol{T}_i^a)_{bc} = \begin{cases} (t^a)_{bc} & \text{quark} \\ -(t^a)_{cb} & \text{anti-quark} \\ -if^{abc} & \text{gluon} \end{cases}$$

Jets from Quantum Chromodynamics

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(Received 26 July 1977)

The properties of hadronic jets in e^+e^- annihilation are examined in quantum chromodynamics, without using the assumptions of the parton model. We find that two-jet events dominate the cross section at high energy, and have the experimentally observed angular distribution. Estimates are given for the jet angular radius and its energy dependence. We argue that the detailed results of perturbation theory for production of arbitrary numbers of quarks and gluons can be reinterpreted in quantum chromodynamics as predictions for the production of jets.

μ: gluon mass

As expected, each separate contribution is singular for $\mu \to 0$, but cancellations[8] occur in the sum, and the final result is free of mass singularities:

$$\sigma(E, \theta, \Omega, \epsilon, \delta) = (d\sigma/d\Omega)_0\, \Omega[1 - (g_E{}^2/3\pi^2)(3 \ln\delta + 4 \ln\delta \ln 2\epsilon + \pi^2/3 - \tfrac{5}{2})]$$

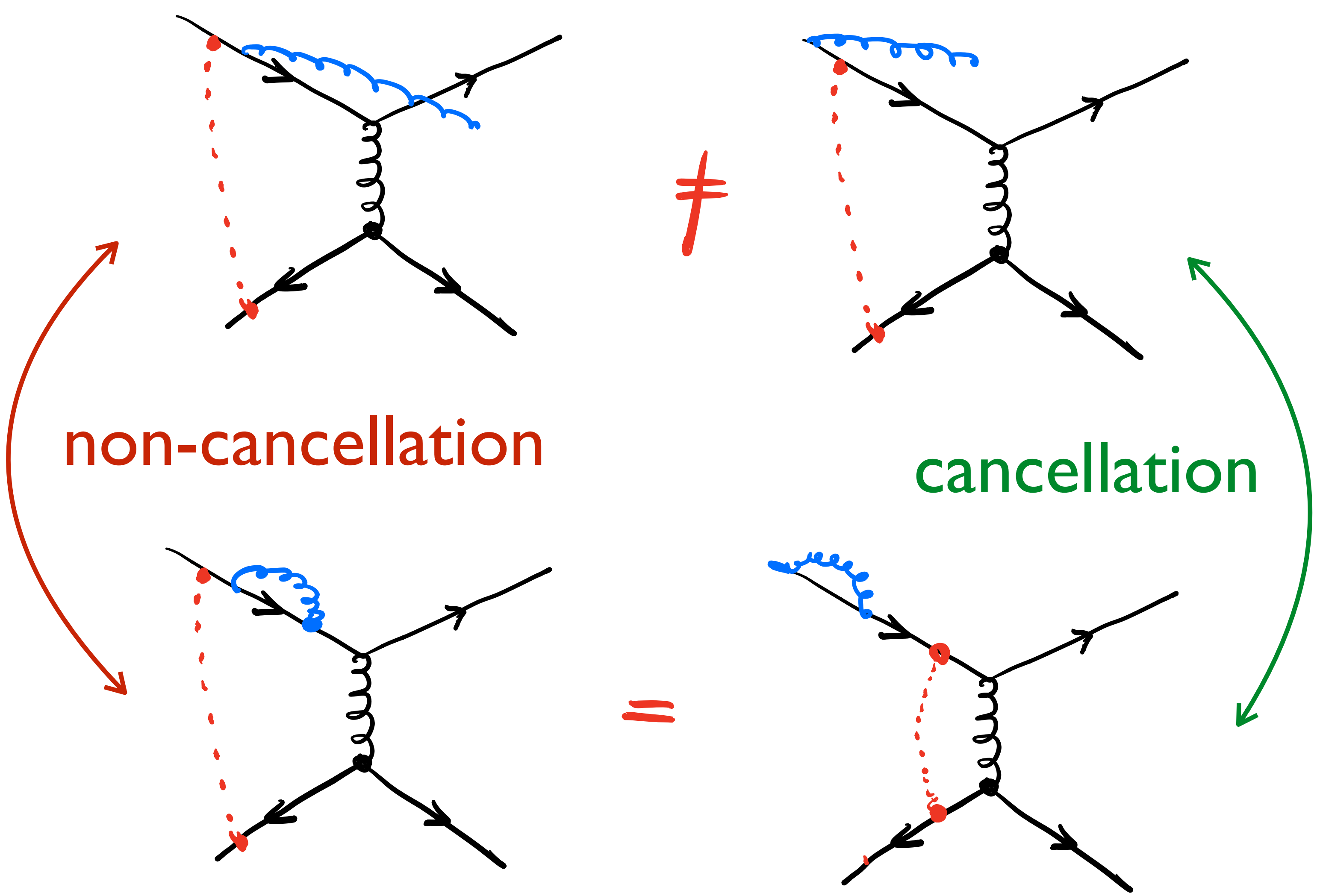
$r = \delta^2$

$2\epsilon = Q_0/Q$

Non-cancellation of collinear terms

Forshaw, Kyrieleis, Seymour ’06 ’08; Catani, de Florian, Rodrigo ’11, …

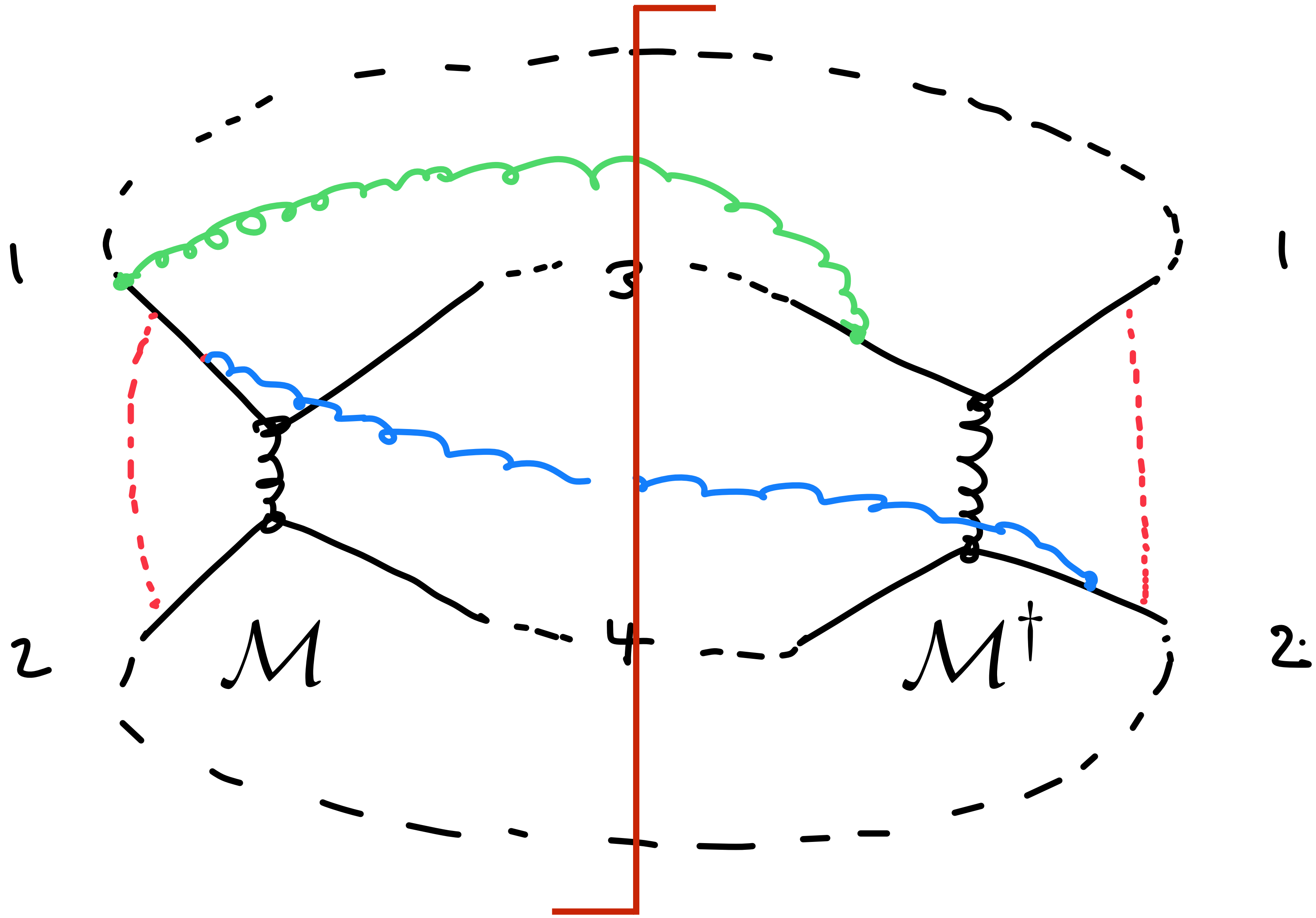
Double logarithms due to soft+collinear configurations.



Blue: collinear emission. Red: Glauber/Coulomb phase

Note: Glauber phases cancel in e^+e^- and in large-N_c limit

An SLL diagram



hard partons,

soft “Glauber” iπ,

soft emission, collinear to 1,

soft emission into gap region.

+ virtual corrections
+ many more, related diagrams

see Keates and Seymour ’09 for diagrammatic SLL calculation

Previous results on SLLs

Since effect first arises at $O(\alpha_s^4)$, only few results

- Discovery of effect, computation of first SLL in gaps between jets for $qq \to qq$ Forshaw, Kyrieleis, Seymour '06
- Colour space calculation of leading SLL Forshaw, Kyrieleis, Seymour '08
- Diagrammatic calculation, first *two* orders, different channels *qq, qg, gg* Keates and Seymour '09

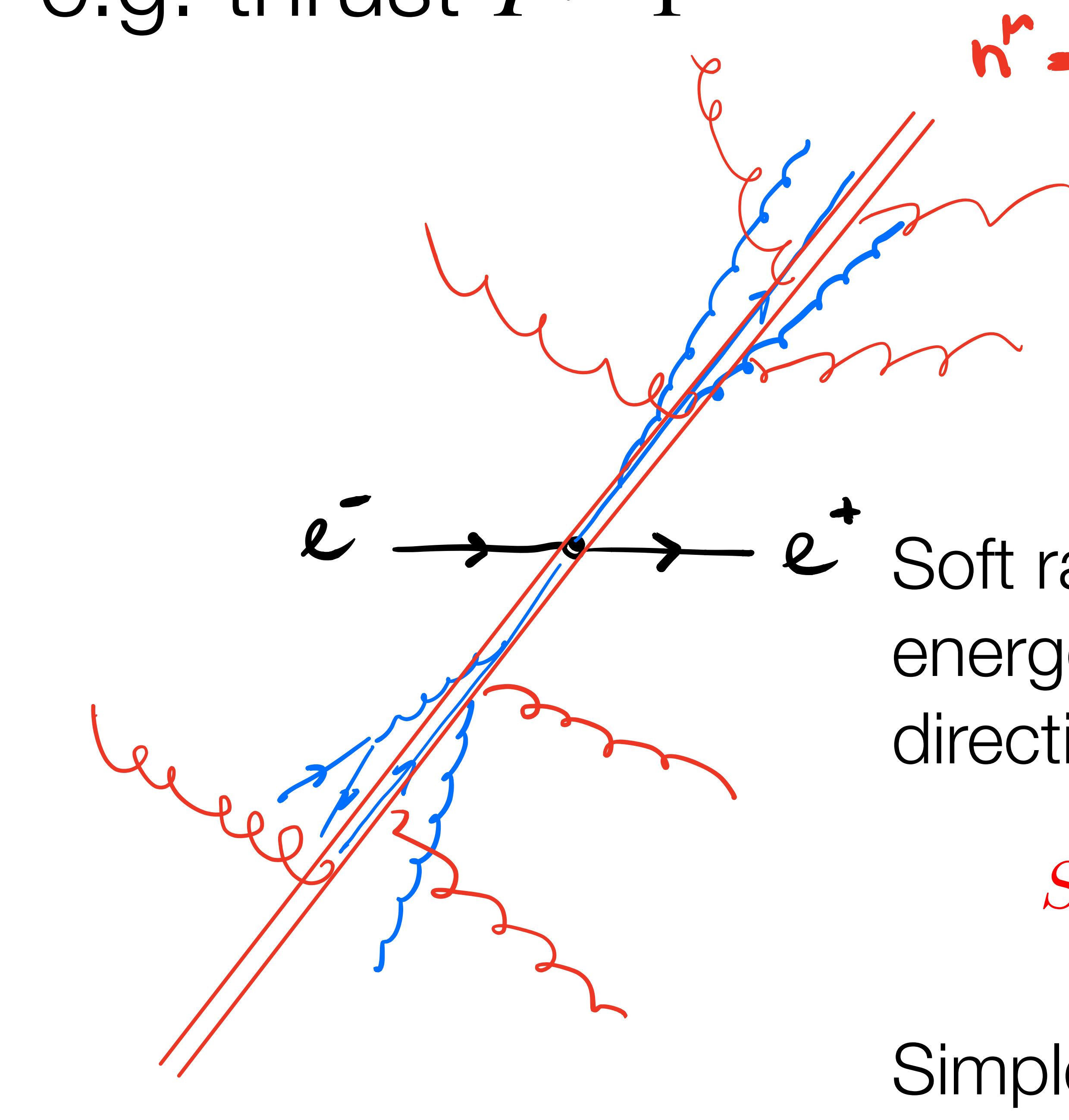
$$S_O^{(4)} = \left(\frac{\alpha_s}{4\pi}\right)^4 L_Q^5 \, \Delta Y \, \pi^2 \, \frac{8}{15} \left(3N_c^2 - 4\right) \sigma_0 \, ,$$

$$S_O^{(5)} = \left(\frac{\alpha_s}{4\pi}\right)^5 L_Q^7 \, \Delta Y \, \pi^2 \, \frac{4}{315} N_c \left(-27N_c^2 + 44\right) \sigma_0$$

Glauber $(i\pi)^2$

Soft radiation in global observables

e.g. thrust $T \sim 1$



$$\frac{d\sigma}{dT} = H \cdot J \otimes S$$

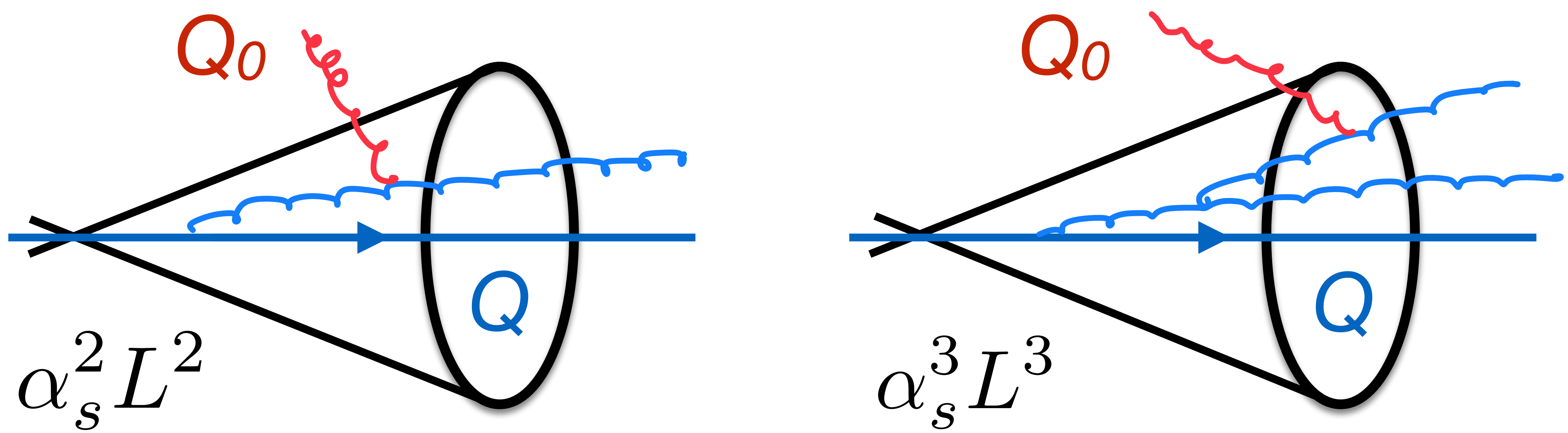
Soft radiation does not resolve individual energetic partons. Sensitive only to direction and total charge of the jets

$$S \sim \sum_{X_s} \left| \langle X_s | \boldsymbol{S}(n) \boldsymbol{S}(\bar{n}) \, |0\rangle \right|^2$$

Simple structure → N^{3}LL resummation

arXiv today: paper by Forshaw and Holguin on Glauber effects in pp event shapes, e.g. transverse thrust.

Non-global logarithms (NGLs)



Dasgupta, Salam ’02: soft gluons from secondary emissions inside the jets lead to complicated pattern of logs $(\alpha_s L)^n$, with $L = \ln(Q/Q_0)$

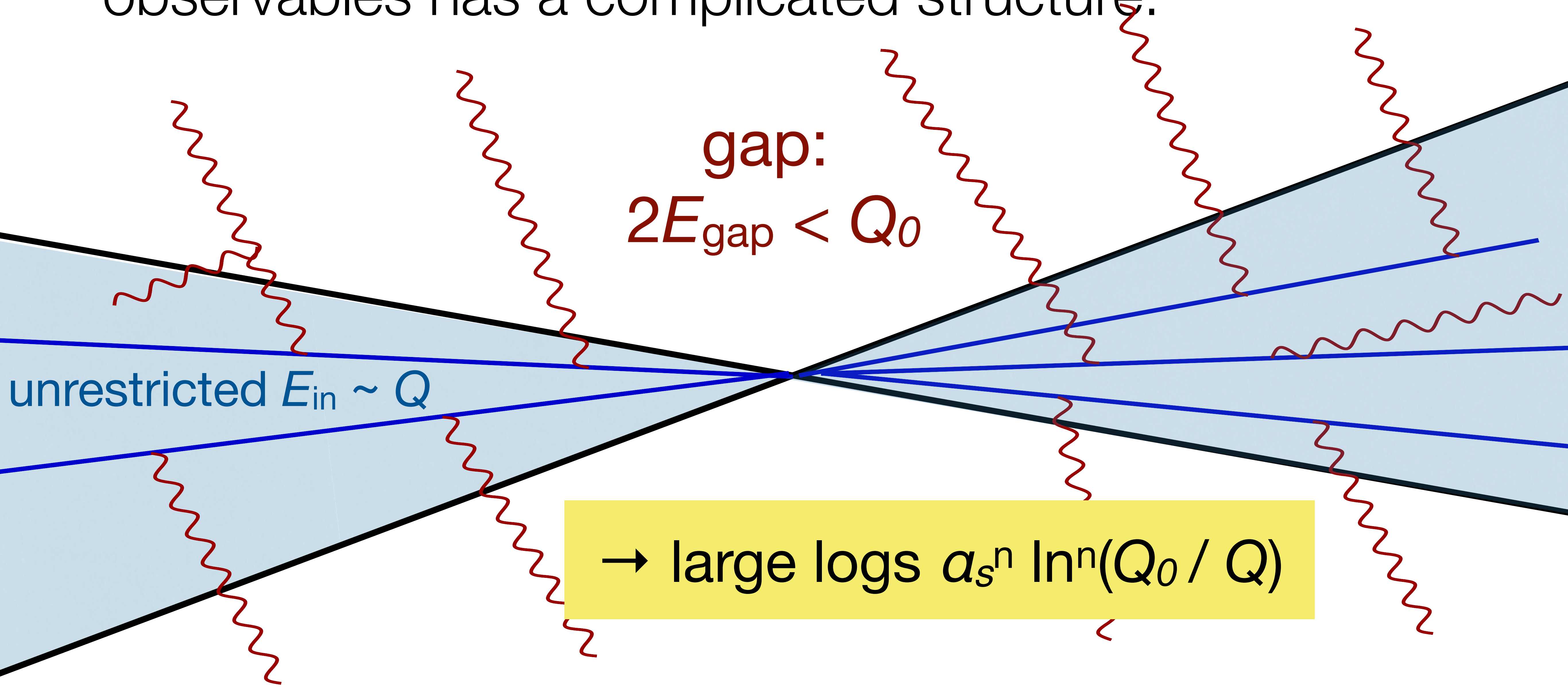
- Even leading NGLs do not simply exponentiate!
- At large N_c logs can be obtained with parton shower Dasgupta, Salam ‘02 or by solving a non-linear integral equation Banfi, Marchesini, Smye ’02. First finite-N_c results Hatta, Ueda ’13 + Hagiwara ‘15 based on Weigert ’03; De Angelis, Forshaw and Plätzer ’20

Resummation of SLLs

Outline

- Factorization theorem for non-global observables
- Resummation by RG evolution
  - One-loop anomalous dimension
  - Glauber/Coulomb phases
  - Collinear logarithms
- Super-leading logarithms
  - Order-by-order computation and resummation

As discussed, soft radiation in non-global observables has a complicated structure:



Hard partons (quarks and gluons) inside jets act as sources: soft radiation pattern depends on color-charges and directions of all hard partons!

Factorization for gap between jets in e^+e^-

TB, Neubert, Rothen, Shao '15 '16, see also Caron-Huot '15

Hard function
m hard partons along
fixed directions {n_1, …, n_m}
$\boldsymbol{\mathcal{H}}_m \propto |\mathcal{M}_m\rangle\langle\mathcal{M}_m|$

Soft function
squared amplitude
with *m* Wilson lines

$$\sigma(Q,Q_0) = \sum_{m=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\},Q,\mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\},Q_0,\mu) \rangle$$

color trace

integration over directions

Have factorization formulas for a variety of non-global observables.

Soft emissions in process with *m* energetic particles are obtained from the matrix elements of the operator

$$\boldsymbol{S}_1(n_1)\,\boldsymbol{S}_2(n_2)\,\ldots\,\boldsymbol{S}_m(n_m)|\mathcal{M}_m(\{\underline{p}\})\rangle$$

soft Wilson lines along the directions of the energetic particles / jets (color matrices)

hard scattering amplitude with *m* particles (vector in color space)

To get the amplitudes with additional soft partons, one takes the matrix element of the multi-Wilson-line operators:

$$\langle X_s|\,\boldsymbol{S}_1(n_1)\,\ldots\,\boldsymbol{S}_m(n_m)\,|0\rangle$$

$$\sigma(Q, Q_0) = \sum_{m=2}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{S}}_m(\{\underline{n}\}, Q_0, \mu) \rangle$$

Factorization theorem:

- Separates contributions from scales Q and Q_0
- Valid in the soft limit $Q_0/Q \to 0$ up to power corrections
- Operator definitions of ingredients
- Provides a natural way to perform resummation via renormalization group (RG) evolution
- Not limited to leading logarithms or leading color

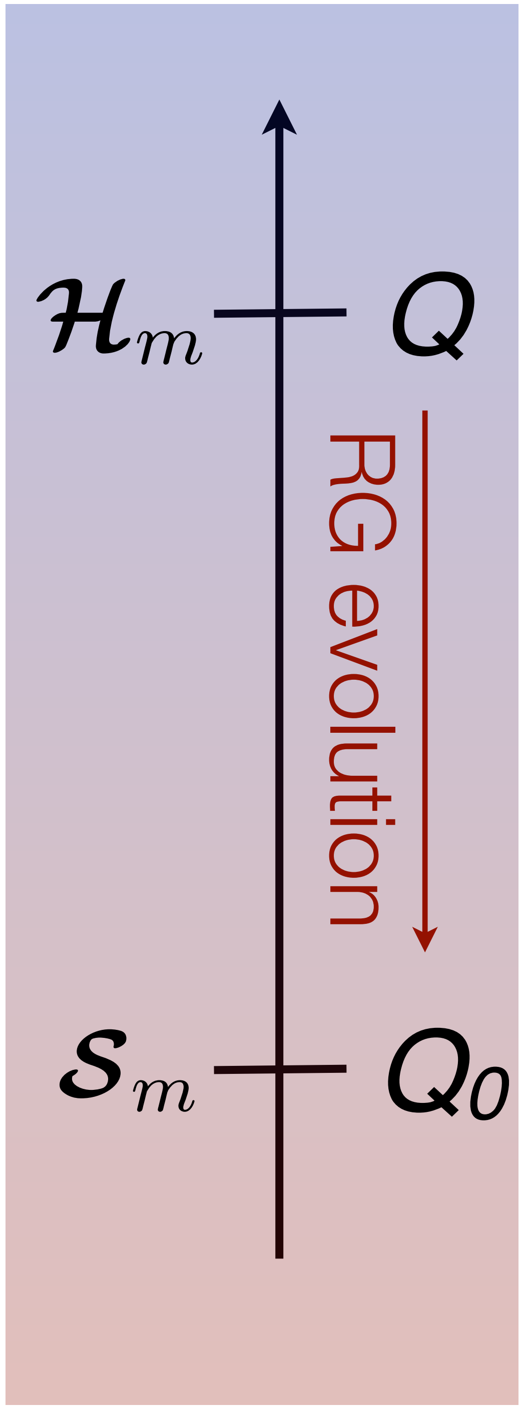
Resummation by RG evolution

Wilson coefficients fulfill RG equations

$$\frac{d}{d\ln\mu}\boldsymbol{\mathcal{H}}_m(Q,\mu) = -\sum_{l=2}^{m}\boldsymbol{\mathcal{H}}_l(Q,\mu)\,\boldsymbol{\Gamma}^H_{lm}(Q,\mu)$$

1. Compute $\mathcal{H}_m$ at a characteristic high scale $\mu_h \sim Q$
2. Evolve $\mathcal{H}_m$ to the scale of low energy physics $\mu_s \sim Q_0$
3. Evaluate $\mathcal{S}_m$ at low scale $\mu_s \sim Q_0$

Avoids large logarithms $\alpha_s^n \ln^n(Q/Q_0)$ of scale ratios which spoil convergence of perturbation theory.



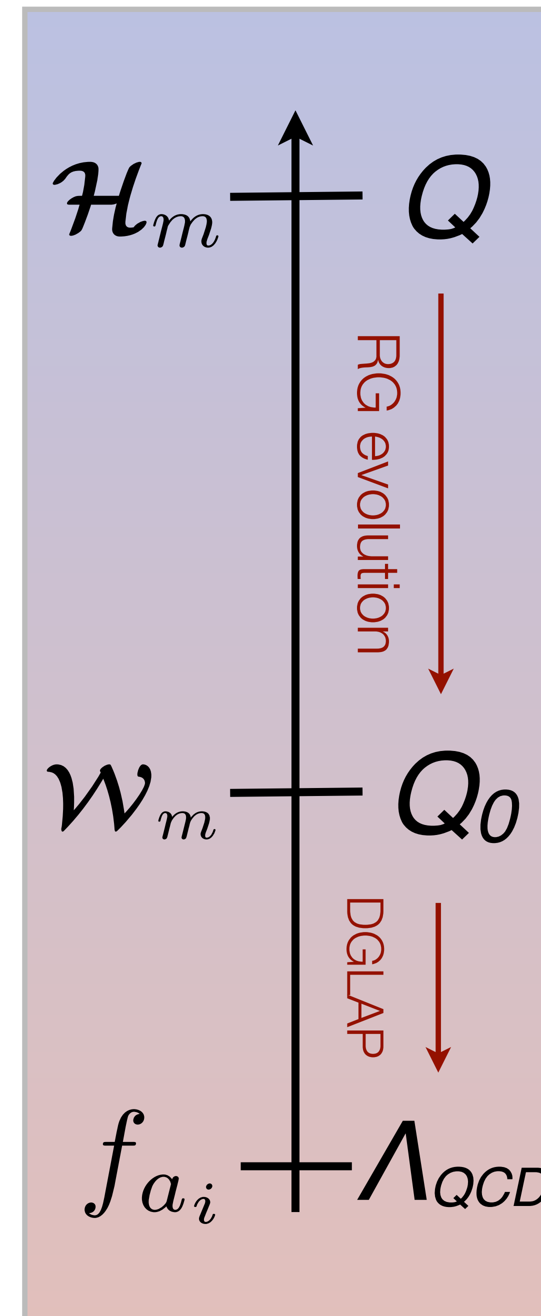
Hadronic collisions

$$\sigma(Q_0) = \sum_{a_1,a_2=q,\bar{q},g} \int dx_1 dx_2 \sum_{m=4}^{\infty} \langle \boldsymbol{\mathcal{H}}_m(\{\underline{n}\}, Q, \mu) \otimes \boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, x_1, x_2, \mu) \rangle$$

- $\boldsymbol{\mathcal{H}}_m$ analogous to e^+e^- but low energy matrix element $\boldsymbol{\mathcal{W}}_m$ in SCET contains soft Wilson lines + collinear fields for incoming partons
- Leading order matrix element

$$\boldsymbol{\mathcal{W}}_m(\{\underline{n}\}, Q_0, x_1, x_2, \mu_s) = f_{a_1}(x_1)\, f_{a_2}(x_2)\, \mathbf{1}$$

- Low-E theory at $\mu \sim Q_0$ involves Glauber gluons, which mediate soft-collinear interactions. Rothstein, Stewart '16
  - Rapidity divs. generate single logs of Q



Formal solution

$$\sigma(Q,Q_0) = \sum_{l=4}^{\infty} \Big\langle \boldsymbol{\mathcal{H}}_l(\{\underline{n}'\},Q,\mu_h) \otimes \sum_{m\geq l}^{\infty} \boldsymbol{U}_{lm}(\{\underline{n}\},\mu_s,\mu_h)\,\hat{\otimes}\,\boldsymbol{\mathcal{W}}_m(\{\underline{n}\},Q_0,\mu_s)\Big\rangle$$

$$\boldsymbol{U}(\{\underline{n}\},\mu_s,\mu_h) = \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\boldsymbol{\Gamma}(\{\underline{n}\},\mu)\right]$$

A way to generate (super)-leading logs order-by-order

- Use lowest order $\mathcal{H}_2 = \sigma_0$ and $\mathcal{W}_m = \mathbf{1} \times$ PDFs
- Expand

$$\boldsymbol{U}(\{\underline{n}\},\mu_s,\mu_h) = \mathbf{1} + \int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\boldsymbol{\Gamma} + \int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\int_{\mu}^{\mu_h}\frac{d\mu'}{\mu'}\boldsymbol{\Gamma}\,\boldsymbol{\Gamma} + \dots$$

RG = Parton Shower

- Ingredients for NLL

$$\boldsymbol{\mathcal{H}}_2(\mu = Q) = \sigma_0$$
$$\boldsymbol{\mathcal{H}}_m(\mu = Q) = 0 \text{ for } m > 2$$
$$\boldsymbol{\mathcal{S}}_m(\mu = Q_0) = 1$$

$$\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_2 & \boldsymbol{R}_2 & 0 & 0 & \dots \\ 0 & \boldsymbol{V}_3 & \boldsymbol{R}_3 & 0 & \dots \\ 0 & 0 & \boldsymbol{V}_4 & \boldsymbol{R}_4 & \dots \\ 0 & 0 & 0 & \boldsymbol{V}_5 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

- RG

$$\frac{d}{dt}\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t)\boldsymbol{V}_m + \boldsymbol{\mathcal{H}}_{m-1}(t)\boldsymbol{R}_{m-1}\,.$$

shower evolution time

$$t = \int_{\alpha(\mu)}^{\alpha(Q)} \frac{d\alpha}{\beta(\alpha)}\,\frac{\alpha}{4\pi}$$

- Equivalent to parton shower equation

$$\boldsymbol{\mathcal{H}}_m(t) = \boldsymbol{\mathcal{H}}_m(t_1)e^{(t-t_1)V_n} + \int_{t_1}^{t} dt'\boldsymbol{\mathcal{H}}_{m-1}(t')\boldsymbol{R}_{m-1}e^{(t-t')\boldsymbol{V}_n}$$

A full-color, amplitude-level shower as envisioned by Nagy, Soper '07, …

1-loop soft anomalous dimension

$$\boldsymbol{R}_m = -4 \sum_{(ij)} \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{j,R}\, W_{ij}^{m+1}\, \Theta_{\text{hard}}(n_{m+1})$$

$i \neq j$

$$\boldsymbol{V}_m = 2 \sum_{(ij)} \int \frac{d\Omega(n_k)}{4\pi} (\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R})\, W_{ij}^k$$

$$- 2\, i\pi \sum_{(ij)} (\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R})\, \Pi_{ij}$$

$$\boldsymbol{\Gamma}^{(1)} = \begin{pmatrix} \boldsymbol{V}_4 & \boldsymbol{R}_4 & 0 & 0 & \cdots \\ 0 & \boldsymbol{V}_5 & \boldsymbol{R}_5 & 0 & \cdots \\ 0 & 0 & \boldsymbol{V}_6 & \boldsymbol{R}_6 & \cdots \\ 0 & 0 & 0 & \boldsymbol{V}_7 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Glauber/Coulomb

Π_{ij} = 1 if both inc./out.

Arises from taking product of soft currents

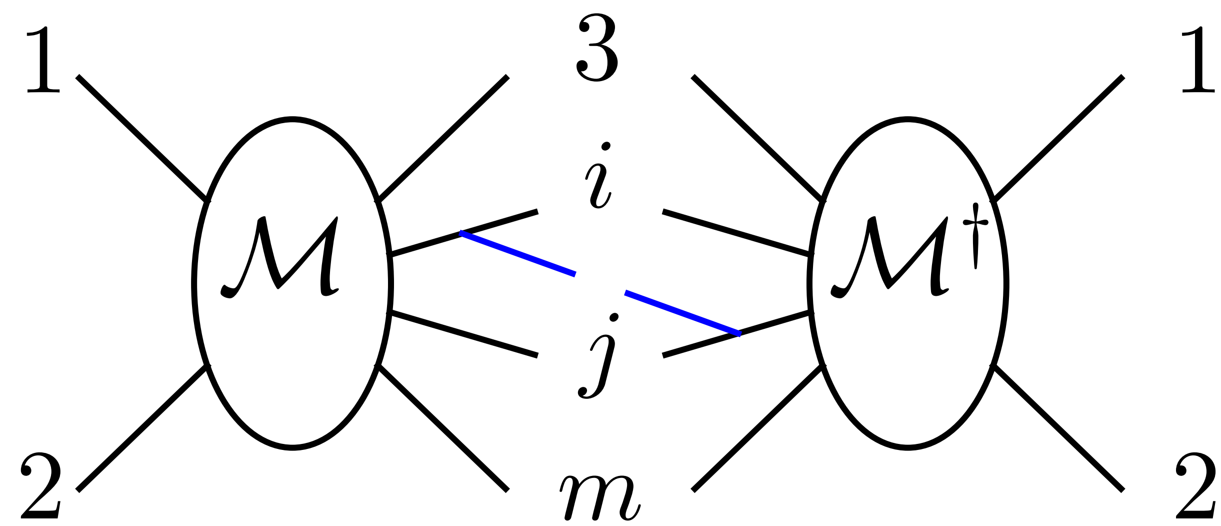
soft dipole

$$W_{ij}^k = \frac{n_i \cdot n_j}{n_i \cdot n_k\, n_j \cdot n_k}$$

← attachment to legs *i, j*
new parton along direction *k*

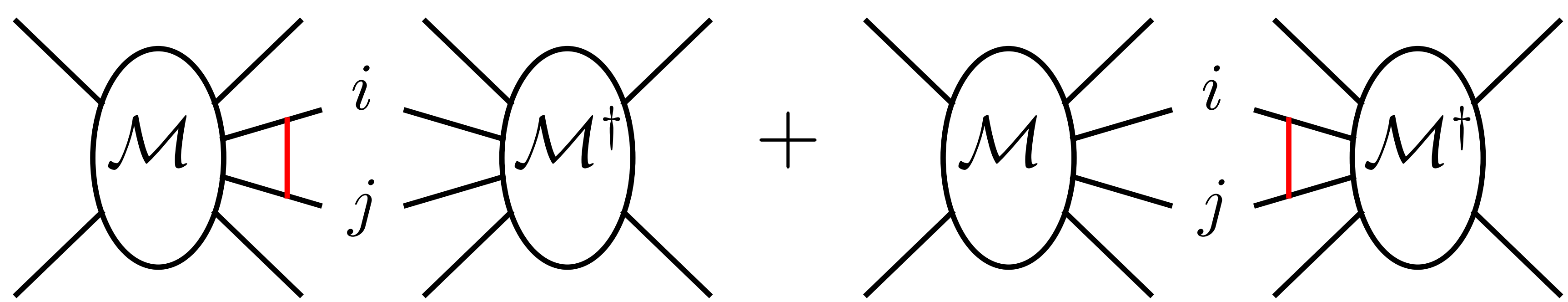
and performing energy integral to extract soft divergence.

$$\boldsymbol{\mathcal{H}}_m \boldsymbol{R}_m = \sum_{(ij)}$$



Structure $\boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{j,R}$ produces extra parton!

$$\boldsymbol{\mathcal{H}}_m \boldsymbol{V}_m = \sum_{(ij)}$$



Loop in amplitude $\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L}$ or conjugate $\boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}$

$\boldsymbol{T}_i \cdot \boldsymbol{T}_j = \sum_a \boldsymbol{T}_i^a \boldsymbol{T}_j^a$ becomes trivial in large-N_c limit.

Simplification of imaginary part

Consider process 1 + 2 → 3 + … + *m* and use color conservation $\sum_i \boldsymbol{T}_i = 0$

$$\begin{aligned}\sum_{(ij)} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \, \Pi_{ij} &= 2\,\boldsymbol{T}_1 \cdot \boldsymbol{T}_2 + \sum_{i=3}^{m} \boldsymbol{T}_i \cdot (-\boldsymbol{T}_1 - \boldsymbol{T}_2 - \boldsymbol{T}_i) \\ &= 2\,\boldsymbol{T}_1 \cdot \boldsymbol{T}_2 + (\boldsymbol{T}_1 + \boldsymbol{T}_2) \cdot (\boldsymbol{T}_1 + \boldsymbol{T}_2) - \sum_{i=3}^{m} C_i \\ &= 4\,\boldsymbol{T}_1 \cdot \boldsymbol{T}_2 + C_1 + C_2 - \sum_{i=3}^{m} C_i\end{aligned}$$

Π_{ij} = 1 if both inc./out.

$$\sum_{(ij)} (\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} - \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}) \, \Pi_{ij} = 4\,(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R})$$

Phase terms only present in pp, not in $e^- p$ or $e^+ e^-$

Collinear singularities

$$\boldsymbol{R}_m = -4 \sum_{(ij)} \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{j,R}\, W_{ij}^{m+1}\, \Theta_{\text{hard}}(n_{m+1})$$

$$\boldsymbol{V}_m = 2 \sum_{(ij)} \int \frac{d\Omega(n_k)}{4\pi} \left(\boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} + \boldsymbol{T}_{i,R} \cdot \boldsymbol{T}_{j,R}\right) W_{ij}^{k}$$

$$- 8\, i\pi \left(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R}\right)$$

$$W_{ij}^{k} = \frac{n_i \cdot n_j}{n_i \cdot n_k\, n_j \cdot n_k}$$

$\boldsymbol{R}_\mathrm{m}$ and $\boldsymbol{V}_\mathrm{m}$ contain singularities when emitted gluon k gets collinear to partons i or j

- Expect cancellation in inclusive observables (showers put collinear cutoff on $\boldsymbol{R}_\mathrm{m}$ and $\boldsymbol{V}_\mathrm{m}$)
- Glauber phases spoil this cancellation: soft+collinear double logarithms! ``Super-leading logarithms”

If angular integrals involve divergences we must make them explicit! In pure dim. reg. the soft anomalous dimension reads

$$\boldsymbol{R}_m = 4\sum_{(ij)} \boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,R}\left\{\left[\delta(n_k-n_i)\ln\frac{\mu}{2E_i}+\delta(n_k-n_j)\ln\frac{\mu}{2E_j}\right]-\overline{W}_{ij}^{m+1}\,\Theta_{\text{hard}}(n_{m+1})\right\}$$

$$\boldsymbol{V}_m = 2\sum_{(ij)}(\boldsymbol{T}_{i,L}\cdot\boldsymbol{T}_{j,L}+\boldsymbol{T}_{i,R}\cdot\boldsymbol{T}_{j,R})\left\{-\ln\frac{\mu^2}{2E_i 2E_j}+\int\frac{d\Omega(n_k)}{4\pi}\overline{W}_{ij}^{k}\right\}$$

$$-\,8\,i\pi\sum_{(ij)}(\boldsymbol{T}_{1,L}\cdot\boldsymbol{T}_{2,L}-\boldsymbol{T}_{1,R}\cdot\boldsymbol{T}_{2,R})\,\Pi_{ij}$$

→ superleading logs

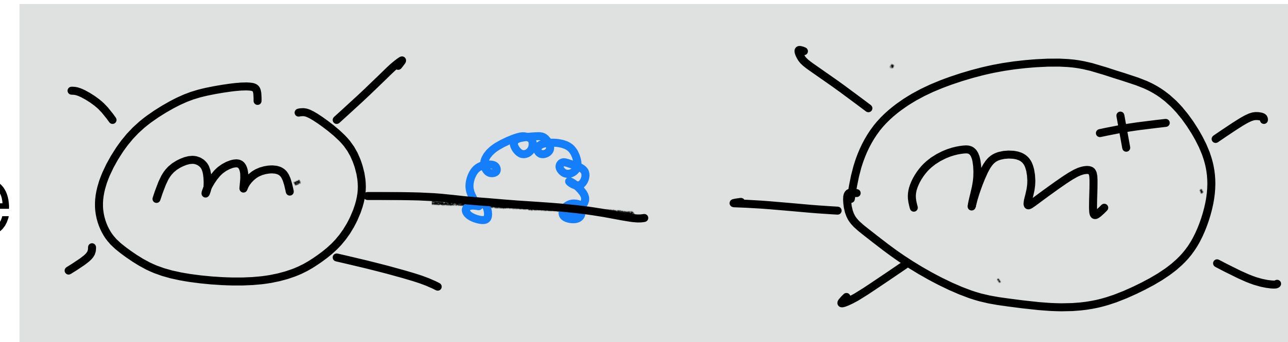
and involves subtracted dipole distribution

$$\overline{W}_{ij}^{k} = \frac{n_i\cdot n_j}{n_i\cdot n_k\, n_j\cdot n_k}-\frac{\delta(n_k-n_i)}{n_i\cdot n_k}-\frac{\delta(n_k-n_j)}{n_j\cdot n_k}$$

← angular δ-dist

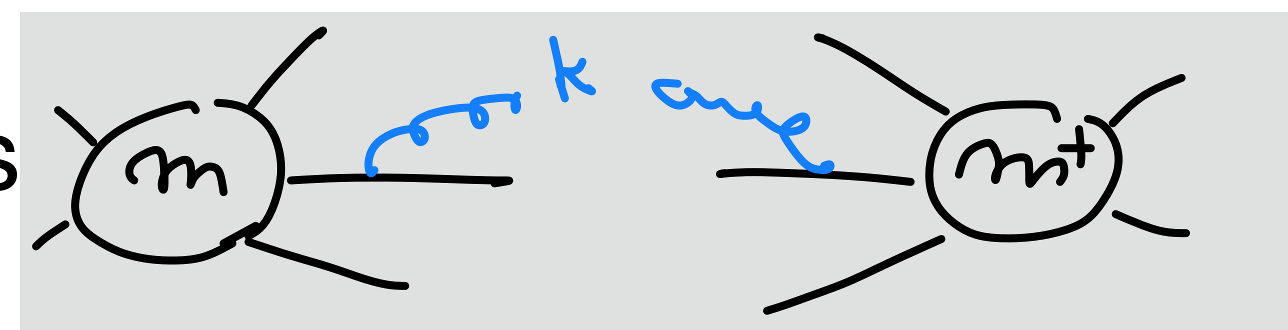
Collinear terms can be simplified using color conservation.

Virtual pieces become



$$\sum_{(ij)} \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{j,L} \ln \frac{\mu}{2E_i} = -\sum_i \boldsymbol{T}_{i,L} \cdot \boldsymbol{T}_{i,L} \ln \frac{\mu}{2E_i} = -\sum_i C_i \ln \frac{\mu}{2E_i}$$

and for **real-emissions**



$$-\sum_{(ij)} \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{j,R}\, \delta(n_k - n_i) \ln \frac{\mu}{2E_i} = +\sum_i \boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R}\, \delta(n_k - n_i) \ln \frac{\mu}{2E_i}$$

Final-state terms will cancel between real and virtual. Can restrict sums to *i* = 1, 2.

Final form of anomalous dimension (partonic CMS, $2E_1 = 2E_2 = \sqrt{\hat{s}}$)

$$\left.\begin{aligned} \boldsymbol{V}_m &= \overline{\boldsymbol{V}}_m + \boldsymbol{V}^G + \sum_{i=1,2} \boldsymbol{V}_i^c \ln \frac{\mu^2}{\hat{s}} \,, \\ \boldsymbol{R}_m &= \overline{\boldsymbol{R}}_m + \sum_{i=1,2} \boldsymbol{R}_i^c \ln \frac{\mu^2}{\hat{s}} \,, \end{aligned}\right\} \boldsymbol{\Gamma} = \overline{\boldsymbol{\Gamma}} + \boldsymbol{\Gamma}^G + \sum_i \boldsymbol{\Gamma}_i^c \ln \frac{\mu^2}{\hat{s}}$$

with collinear subtracted $\overline{\boldsymbol{V}}_m$ and $\overline{\boldsymbol{R}}_m$ and

$$\boldsymbol{V}^G = -8i\pi \left(\boldsymbol{T}_{1,L} \cdot \boldsymbol{T}_{2,L} - \boldsymbol{T}_{1,R} \cdot \boldsymbol{T}_{2,R}\right)$$

Glauber terms

$$\boldsymbol{V}_i^c = 4C_i \, \mathbf{1}$$

Collinear terms

$$\boldsymbol{R}_i^c = -4\boldsymbol{T}_{i,L} \circ \boldsymbol{T}_{i,R} \, \delta(n_k - n_i)$$

Computation of SLLs

Now compute order by order

$$\Big\langle \boldsymbol{\mathcal{H}}_4\, \boldsymbol{U}(\{\underline{n}\},\mu_s,\mu_h)\,\hat{\otimes}\,\mathbf{1}\Big\rangle = \Big\langle \boldsymbol{\mathcal{H}}_4\, \mathbf{P}\exp\left[\int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\boldsymbol{\Gamma}(\{\underline{n}\},\mu)\right]\hat{\otimes}\,\mathbf{1}\Big\rangle$$

$$= \langle \boldsymbol{\mathcal{H}}_4\rangle + \int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\,\langle \boldsymbol{\mathcal{H}}_4\,\boldsymbol{\Gamma}(Q,\mu)\hat{\otimes}\mathbf{1}\rangle + \int_{\mu_s}^{\mu_h}\frac{d\mu}{\mu}\int_{\mu}^{\mu_h}\frac{d\mu'}{\mu'}\langle \boldsymbol{\mathcal{H}}_4\boldsymbol{\Gamma}(Q,\mu)\,\boldsymbol{\Gamma}(Q,\mu')\hat{\otimes}\,\mathbf{1}\rangle + \dots$$

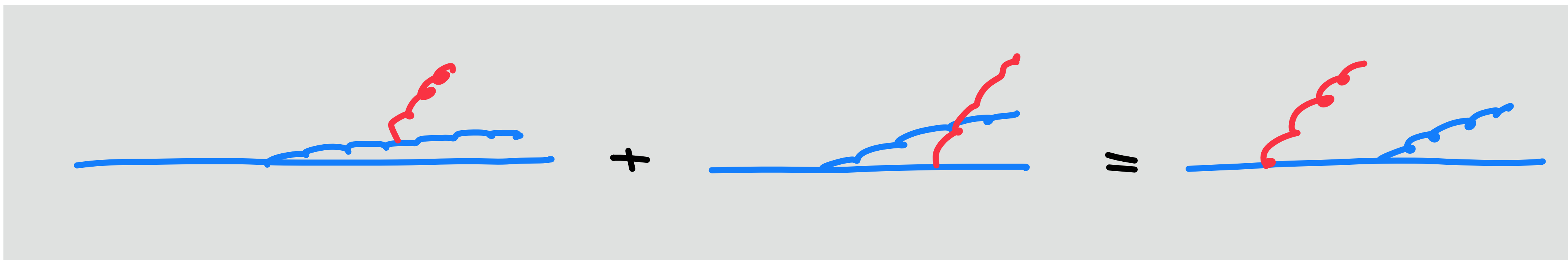
$\hat{\sigma}_{\rm LO}$ $\qquad$ $\alpha_s L$

Need products of anomalous dimensions. Each μ integral produces single log ($\overline{\boldsymbol{\Gamma}}$, $\boldsymbol{\Gamma}^G$) or double logs ($\boldsymbol{\Gamma}_i^c$), i.e. SLLs!

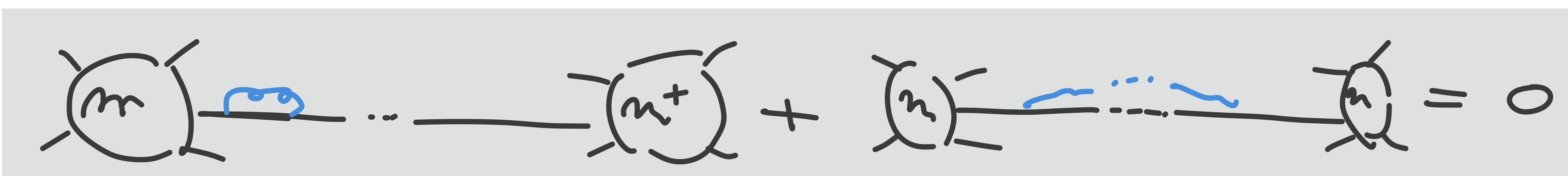
Will set μ_h=Q and μ_s=Q_0 and ignore running of α_s.

Basic properties of Γ

- Color coherence: $\boldsymbol{\mathcal{H}}_m\, \boldsymbol{\Gamma}^c\, \overline{\boldsymbol{\Gamma}} = \boldsymbol{\mathcal{H}}_m\, \overline{\boldsymbol{\Gamma}}\, \boldsymbol{\Gamma}^c$



- Collinear safety $\langle \boldsymbol{\mathcal{H}}_m\, \boldsymbol{\Gamma}^c \otimes \mathbf{1} \rangle = 0$



$\langle \boldsymbol{\mathcal{H}}_m\, (\boldsymbol{R}_i^c + \boldsymbol{V}_i^c) \otimes \mathbf{1} \rangle \propto \langle \boldsymbol{T}_i^a\, \boldsymbol{\mathcal{H}}_m\, \boldsymbol{T}_i^a - C_i\, \boldsymbol{\mathcal{H}}_m \rangle = 0$ cyclicity of trace

- $\langle \boldsymbol{\mathcal{H}}_m\, \boldsymbol{V}^G \otimes \mathbf{1} \rangle = 0$ cyclicity of trace

Leading SLLs

1. Want maximum number of $\mathbf{\Gamma}^c$'s at given order.
2. Need $\mathbf{\Gamma}^G$ to prevent $\mathbf{\Gamma}^c$ from commuting to the right and vanishing. Two insertions of $\mathbf{\Gamma}^G$ since cross section is real.
3. Need one emission $\overline{\mathbf{\Gamma}}$ at the end to prevent $\mathbf{\Gamma}^G$ from vanishing

Taken together, this implies that the leading SLLs at *n*-th order arise from matrix elements

$$C_{rn} = \left\langle \boldsymbol{\mathcal{H}}_4 \left(\mathbf{\Gamma}^c\right)^r \boldsymbol{V}^G \left(\mathbf{\Gamma}^c\right)^{n-r} \boldsymbol{V}^G \,\overline{\mathbf{\Gamma}} \otimes \mathbf{1} \right\rangle \qquad 0 \le r \le n$$

Evaluation of C_{rn} : Step 1

Basic strategy is to commute $\boldsymbol{\Gamma}^G$ and $\boldsymbol{\Gamma}^c$ to the right:

- Need $[\boldsymbol{V}^G, \overline{\boldsymbol{\Gamma}}]$ and $[\boldsymbol{\Gamma}^c, [\boldsymbol{V}^G, \overline{\boldsymbol{\Gamma}}]]$

Both commutators lead to the same structure

$$C_{rn} = -64\pi\,(4N_c)^{n-r}\,f_{abc}\sum_{j>2}\left\langle \boldsymbol{\mathcal{H}}_4\,(\boldsymbol{\Gamma}^c)^r\,\boldsymbol{V}^G\,\boldsymbol{T}_1^a\,\boldsymbol{T}_2^b\,\boldsymbol{T}_j^c\right\rangle J_i$$

with angular integrals

$\Theta_{\text{veto}}(n_k) = 1 - \Theta_{\text{hard}}(n_k)$

virtual + real

$$J_j = \int \frac{d\Omega(n_k)}{4\pi}\,\left(W_{1j}^k - W_{2j}^k\right)\Theta_{\text{veto}}(n_k)$$

Evaluation of C_{rn} : Step 2

Now commute remaining $\mathbf{\Gamma}^G$ and $\mathbf{\Gamma}^c$'s to the right. Leads to anti-commutators of color matrices.

Many independent structures but for incoming (anti-)quarks we can simplify

σ = +1 for *q*, σ = -1 anti-*q*

$$\{\boldsymbol{T}_i^a, \boldsymbol{T}_i^b\} = \frac{1}{N_c}\,\delta_{ab}\mathbf{1} + \sigma_i\, d_{abc}\,\boldsymbol{T}_i^c\,; \quad i = 1, 2$$

which leads to a result with only three structures

1. 2. 3.

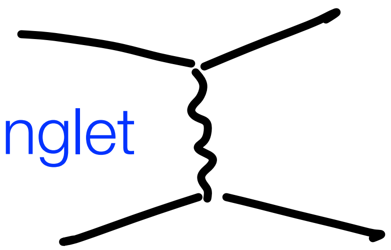
$$C_{rn} = 2^{8-r}\pi^2\,(4N_c)^n \Big\{ \sum_{j>2} J_j \,\big\langle \boldsymbol{\mathcal{H}}_4 \big[(\boldsymbol{T}_2 - \boldsymbol{T}_1)\cdot \boldsymbol{T}_j + 2^{r-1} N_c\,(\sigma_1 - \sigma_2)\, d_{abc}\,\boldsymbol{T}_1^a\,\boldsymbol{T}_2^b\,\boldsymbol{T}_j^c\big]\big\rangle$$

$$+\, 2\,(1-\delta_{r0})\, J_2 \,\big\langle \boldsymbol{\mathcal{H}}_4 \big[C_F + (2^r - 1)\,\boldsymbol{T}_1 \cdot \boldsymbol{T}_2\big]\big\rangle \Big\}\,.$$

Evaluation of C_{rn} : Step 3

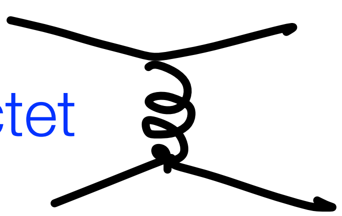
Evaluate remaining color structures explicitly for given partonic channel, e.g. $(J_{43} = J_4 - J_3)$

singlet



$$C_{rn}^{(S)} = \hat{\sigma}_B \, 2^{8-r} \pi^2 \, (4N_c)^n \, C_F \Big[- J_{43} + 2 J_2 (1 - \delta_{r0}) \Big]$$

octet



$$C_{rn}^{(O)} = \hat{\sigma}_B \, 2^{8-r} \pi^2 \, (4N_c)^n \left[C_F J_{43} + \frac{J_2}{N_c} \left(N_c^2 - 2^{r+1} + 1 \right) (1 - \delta_{r0}) \right]$$

and evaluate associated μ integrals and sum

$$\Delta\hat{\sigma} = \sum_{n=0}^{\infty} \hat{\sigma}_n^{\mathrm{SLL}} \quad \text{with} \quad \hat{\sigma}_n^{\mathrm{SLL}} = \left(\frac{\alpha_s}{4\pi} \right)^{n+3} L^{2n+3} \, \frac{(-4)^n \, n!}{(2n+3)!} \sum_{r=0}^{n} \frac{(2r)!}{4^r \, (r!)^2} \, C_{rn}$$

Resummed result

The dependence of C_{rn} on *n* and *r* is power-like and it is possible to carry out the sums.

Simplest case is singlet $\quad J_i = \pm\Delta Y$

$$\Delta\hat{\sigma}^{(S)} = -\hat{\sigma}_B \frac{4C_F}{3\pi} \alpha_s^3 L^3 \Delta Y \; {}_2F_2\left(1, 1; 2, \tfrac{5}{2}; -w\right)$$

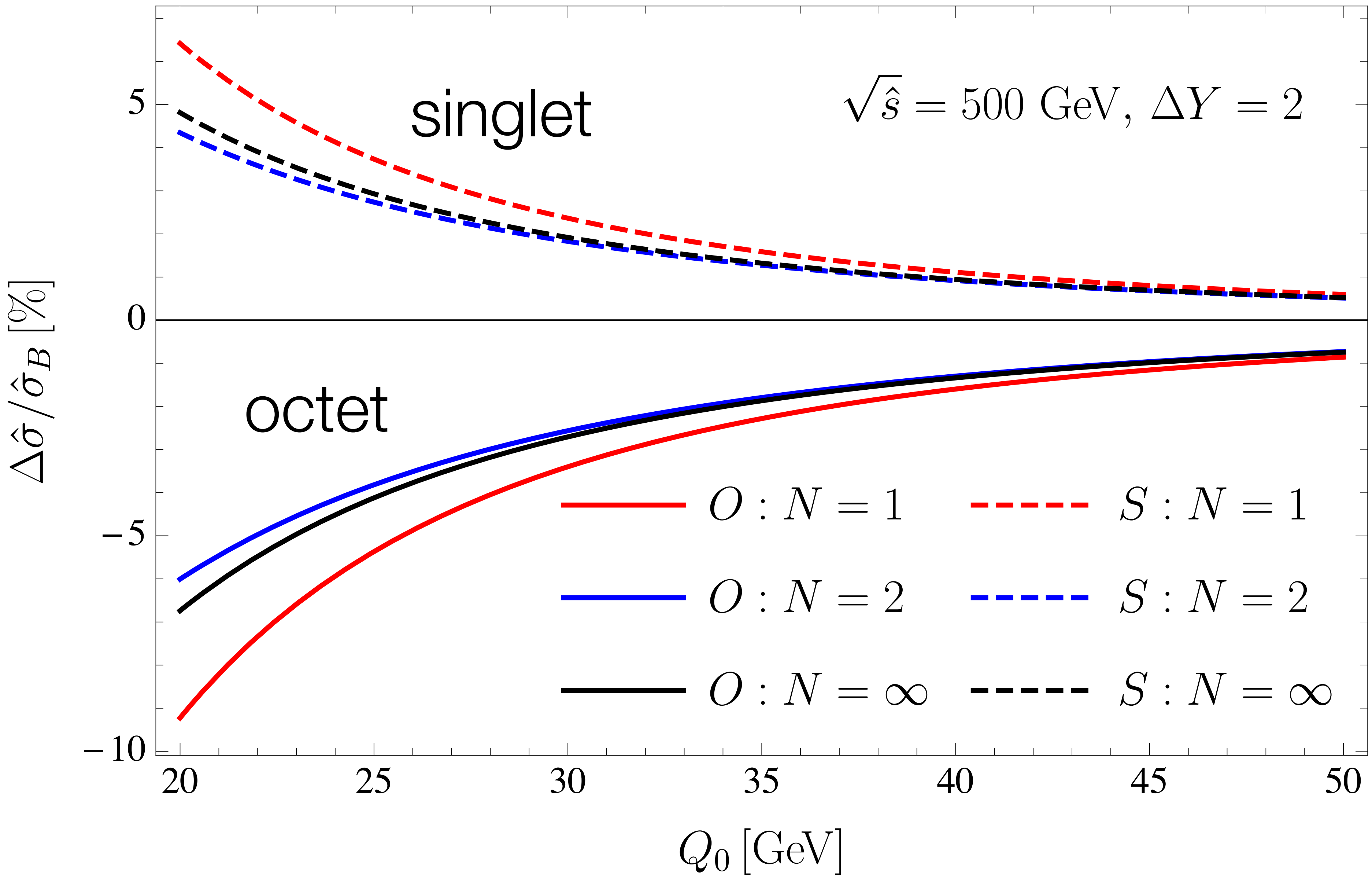
with $w = \dfrac{N_c \alpha_s}{\pi} L^2.$ $\qquad \sim \dfrac{\ln w}{w}$ for large w

Note: Standard Sudakov has form e^{-cw}.

Checks

We have automated the application of the anomalous dimension and then evaluated resulting color traces using ColorMath Sjödahl '12.

- reproduce 4 and 5 loops results of Forshaw, Kyrieleis, Seymour '06 '08, Keates, Seymour '09
- for $qq \to qq$ our code runs up to 8 loop order, for other channels 6 or 7 loops
  - agrees with expanded analytic result
  - this is how we first noticed the simple higher-order structure in this channel…



- Effect is only significant if logs are large.
- *Very sensitive* to choice of μ in α_s: should include running!

Other hard processes

Our analysis of the SLL matrix elements

$$C_{rn} = \langle \boldsymbol{\mathcal{H}}_4 \, (\boldsymbol{\Gamma}^c)^r \, \boldsymbol{V}^G \, (\boldsymbol{\Gamma}^c)^{n-r} \, \boldsymbol{V}^G \, \overline{\boldsymbol{\Gamma}} \otimes \mathbf{1} \rangle \qquad 0 \le r \le n$$

does not rely on any properties of $\boldsymbol{\mathcal{H}}_4$ and hold for *any* hard matrix element $\boldsymbol{\mathcal{H}}_m$

- Process dependence only from the 3 final color matrix elements!
- SLLs also arise in $qq \to Z + j$ at four loops and $qq \to Z$ and $gg \to H$ at five loops. In Higgs production with large color factor.

Summary and Outlook

Summary and Outlook

- First resummation of (super-)leading logs for non-global observable in hadronic collisions
    - presented quark channel in my talk, paper with general result is in preparation TB, Neubert, Shao
    - should perform full analysis of hadronic cross section
    - important to go beyond crude LL limit (running coupling, higher powers of $\overline{\mathbf{\Gamma}}, \mathbf{\Gamma}^{G} \ldots$)
- Interesting to analyze low-E matrix element in $\mathcal{L}_{\text{SCET, Glauber}}$
    - Rapidity logs, RG invariance, PDF factorization, …
- Important ongoing work toward a full-color shower which includes these effects. Angeles Martinez, De Angelis, Forshaw, Plätzer, Seymour ’18; De Angelis, Forshaw, Plätzer ‘20

Extra slides

Result for V and V+j

- $q\bar{q} \to V$, vanishes for n=1

$$C_{rn} = -\hat{\sigma}_B\, 2^{9-r} \pi^2 C_F\, (4N_c)^n (2^r - 2)(1 - \delta_{r0})\, J_2$$

- $q\bar{q} \to V + j$

$$C_{rn} = \hat{\sigma}_B\, 2^{10-r} \pi^2\, (4N_c)^{n-1} \left(N_c^2 + 2^r - 2\right) (1 - \delta_{r0})\, J_2$$

Resummed result for $qq \to qq$

Perform μ integrals and sum series and get

$$S_O = \left(\frac{\alpha_s}{\pi}\right)^3 \pi^2 \ln^3 \frac{Q}{\mu_s} \frac{1}{N_c} \left\{ J_1 \left(N_c^2 - 1\right) f_1(w) \right.$$
$$\left. + J_2 \left[2N_c^2 (f_1(w) - f_\delta(w)) + 2(f_1(w) - 2f_2(w) + f_\delta(w)) \right] \right\} \sigma_0$$

octet $\mathcal{H}_4$

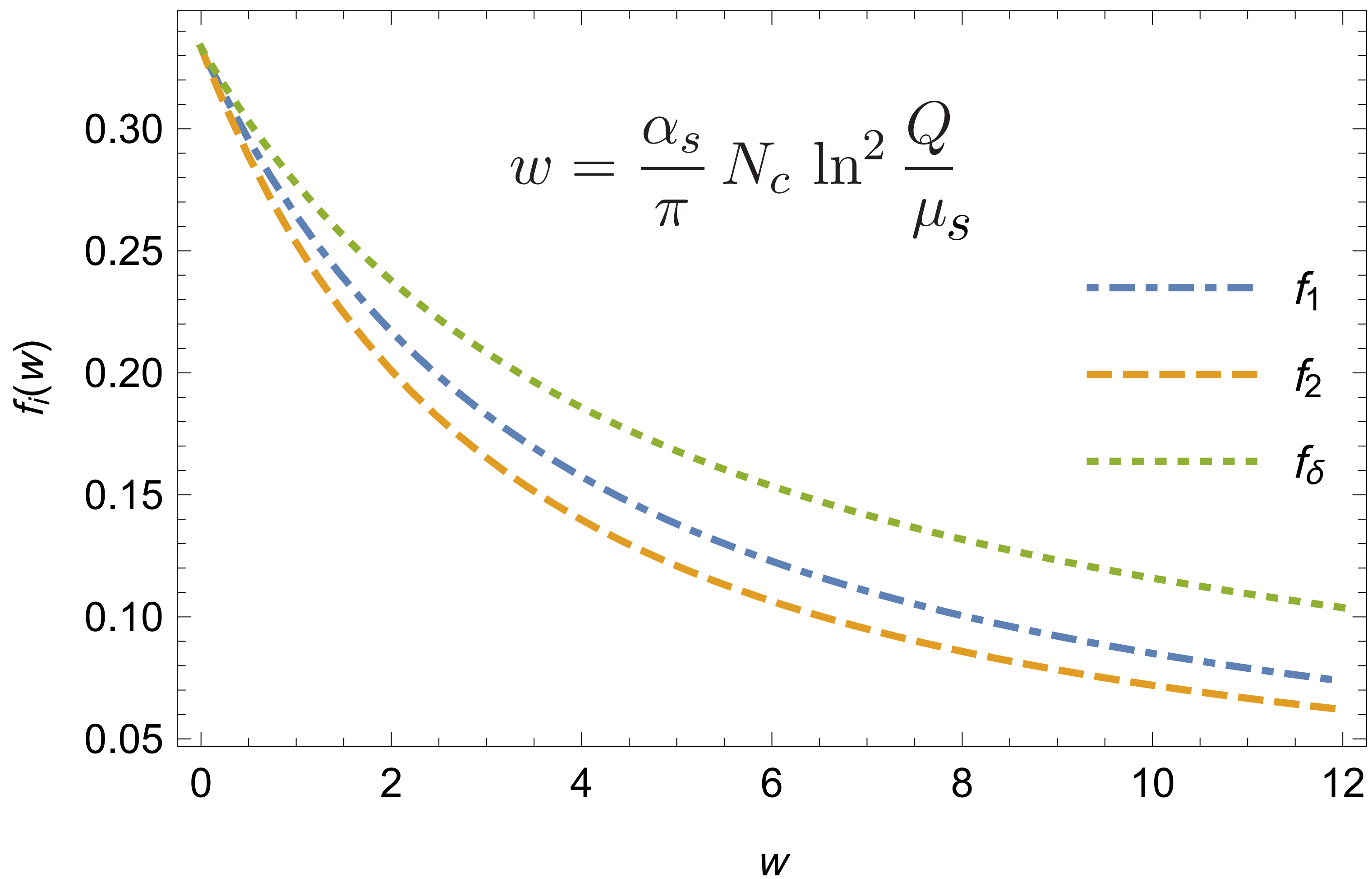
with $w = \frac{\alpha_s}{\pi} N_c \ln^2 \frac{Q}{\mu_s}$ and coefficient functions

$$f_\delta(w) = \frac{1}{3}\, {}_2F_2\left(1, 1; 2, \frac{5}{2}; -w\right) ,$$

$$f_2(w) = \frac{1}{w} - \frac{\sqrt{\pi}}{2w^{3/2}} \operatorname{erf}(\sqrt{w}) ,$$

$$f_1(w) = \frac{\sqrt{\pi}}{2w} \int_0^{\sqrt{\frac{w}{2}}} \frac{dz}{z^2} \left[\operatorname{erf}(z) - \frac{e^{-2z^2}}{i} \operatorname{erf}(iz) \right]$$

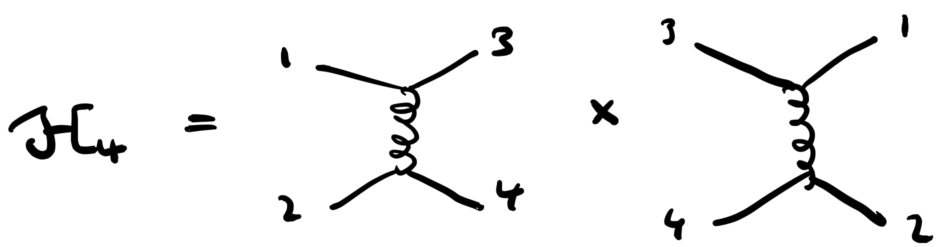
Coefficient functions



Observe "Sudakov" suppression at large w, also for super-leading logs.

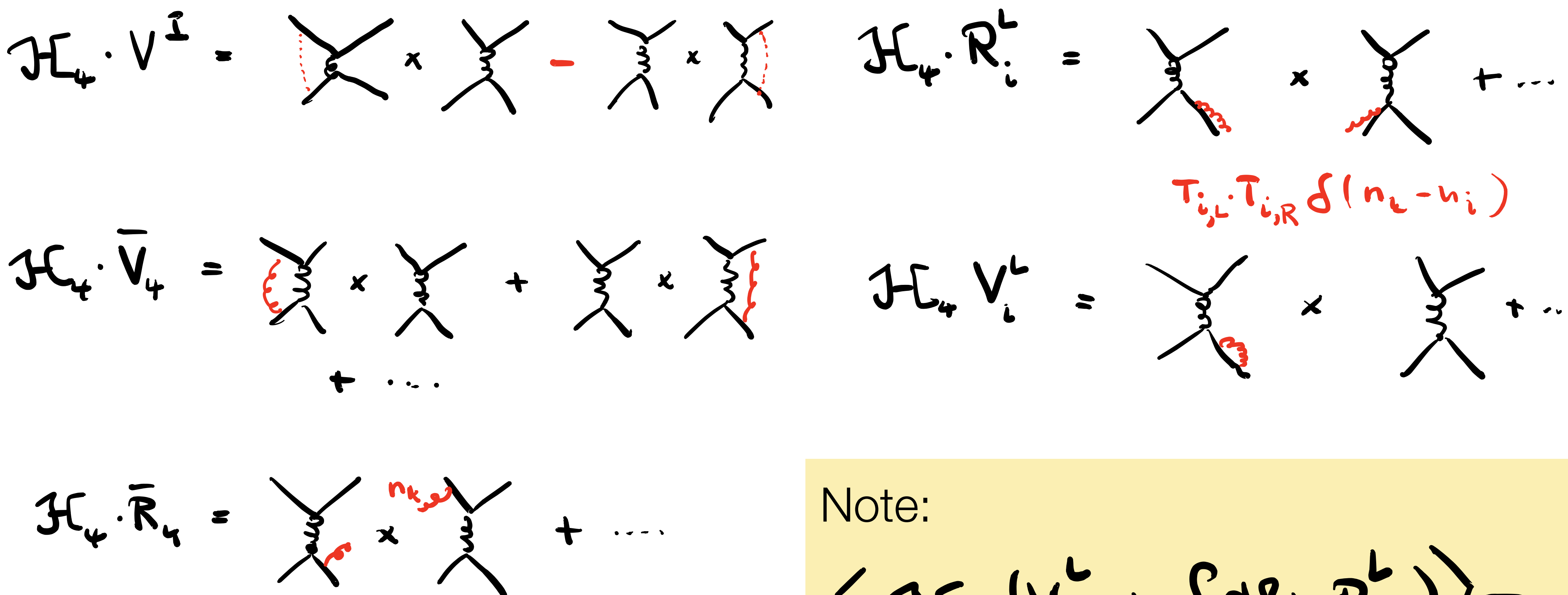
Note: regular LLs behave as e^{-w}

Hard function for octet exchange



$\sim t^a_{\alpha_3\alpha_1} t^a_{\alpha_4\alpha_2} t^b_{\beta_1\beta_3} t^b_{\beta_2\beta_4} \sigma_0$

Action of $\mathbf{\Gamma}$



Note:

$$\left\langle \mathcal{H}_4 \left(V_i^L + \int \frac{d\Omega_L}{4\pi} R_i^L \right) \right\rangle = 0$$

collinear sing. cancel