ATHENA

SIDIS - Working Group

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for the Pavia group

Standard TMD formula

Perturbative part :

OPE matching coefficients C_{qi}

collinear PDFs at scale $\mu_b = \frac{2e^{-\gamma_E}}{b_*(b_T)}$

Sudakov factor → perturbative evolution

$$f_1^q(x, b_T; \mu) = \sum_i (C_{qi} \otimes f_1^i)(x, b_*; \mu_b)\, e^{S(b_*;\mu_b,\mu)}\, e^{g_K(b_T)\log \mu/\mu_0} f_{\rm NP}^q(x, b_T)$$

NonPerturbative part :

evolution

intrinsic wave function

$$b_{\rm min} = \frac{2e^{-\gamma_E}}{Q} \underset{b_T \to 0}{\longleftarrow} b_*(b_T) \underset{b_T \to \infty}{\longrightarrow} b_{\rm max} = 2e^{-\gamma_E}$$

$$Q \geq \mu_b \geq 1$$

For μ=μ0=1 GeV $\mathrm{TMD}(x, b_T; \mu_0) = \mathrm{PDF}(x; \mu_0)\; f_{\rm NP}(x, b_T)$

other prescriptions possible..

NonPerturbative functional form $f_{\rm NP}(x, b_T)$

PV17 fit

A. Bacchetta *et al.*, JHEP06 (2017) 081, arXiv:1703.10157

intrinsic wave function
(in k_T space)

$$f_{\rm NP}(x, \boldsymbol{k}_T^2) = \frac{1}{\pi} \, \frac{1 + \lambda \boldsymbol{k}_T^2}{g_1 + \lambda g_1^2} \, e^{-\boldsymbol{k}_T^2/g_1}$$

$$g_1(x) = N_1 \, \frac{(1-x)^\alpha \, x^\sigma}{(1-\hat{x})^\alpha \, \hat{x}^\sigma}$$

$$\hat{x} = 0.1$$

evolution

$$g_K(b_T) = -\, g_2 \, \frac{b_T^2}{4}$$

similar for TMD FF

- g_2 → nonperturbative evolution
- $N_1 = g_1(\hat{x})$ → mid-*x* width of TMD
- σ → low-*x* width of TMD

- α → high-*x* width of TMD
- λ → weight of second Gaussian

} not much constrained by fit

PV17 nonperturbative parameters

PV17 fit

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Data from:

- **SIDIS** Hermes , Compass
- **Drell-Yan** FermiLab
- **Z-boson production** Tevatron: CDF , D0

8059 pts , 11 parameters , χ^2/dof = 1.55 ± 0.05

g_2	N_1 [GeV^2]	σ	α	λ [GeV^{-2}]
0.13 ± 0.01	0.28 ± 0.06	0.17 ± 0.02	2.95 ± 0.05	0.86 ± 0.78
$\bar{g}_2 \pm \Delta g_2$	$\bar{N}_1 \pm \Delta N_1$	$\bar{\sigma} \pm \Delta\sigma$	$\bar{\alpha} \pm \Delta\alpha$	$\bar{\lambda} \pm \Delta\lambda$

all errors ΔX obtained by selecting the central 68% of all replicas

Sensitivity coefficient and standard deviation

sensitivity coefficient *S* of object *f* w.r.t. observable O

$$S(\mathcal{O}, f) = \frac{\langle \mathcal{O} f \rangle - \langle \mathcal{O} \rangle \langle f \rangle}{\delta \mathcal{O} \, \Delta f}$$

experimental error for O — standard deviation for *f*

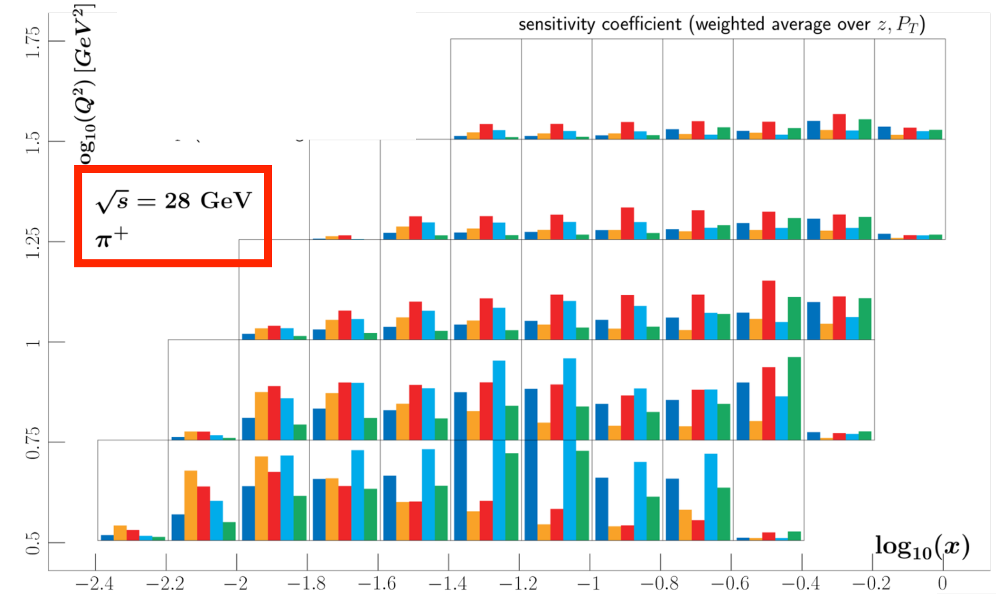
1. from PV17, we know a parameter *A* with error Δ*A*
2. if we perform a new measurement that produces on *A* an error equal to its initial standard deviation, δ*A* = Δ*A*, we expect the error on *A* to scale as $1/\sqrt{2}$. We postulate that this corresponds to $S(A) = 1$
3. in fact, if A can be ideally considered as parameter and observable, then

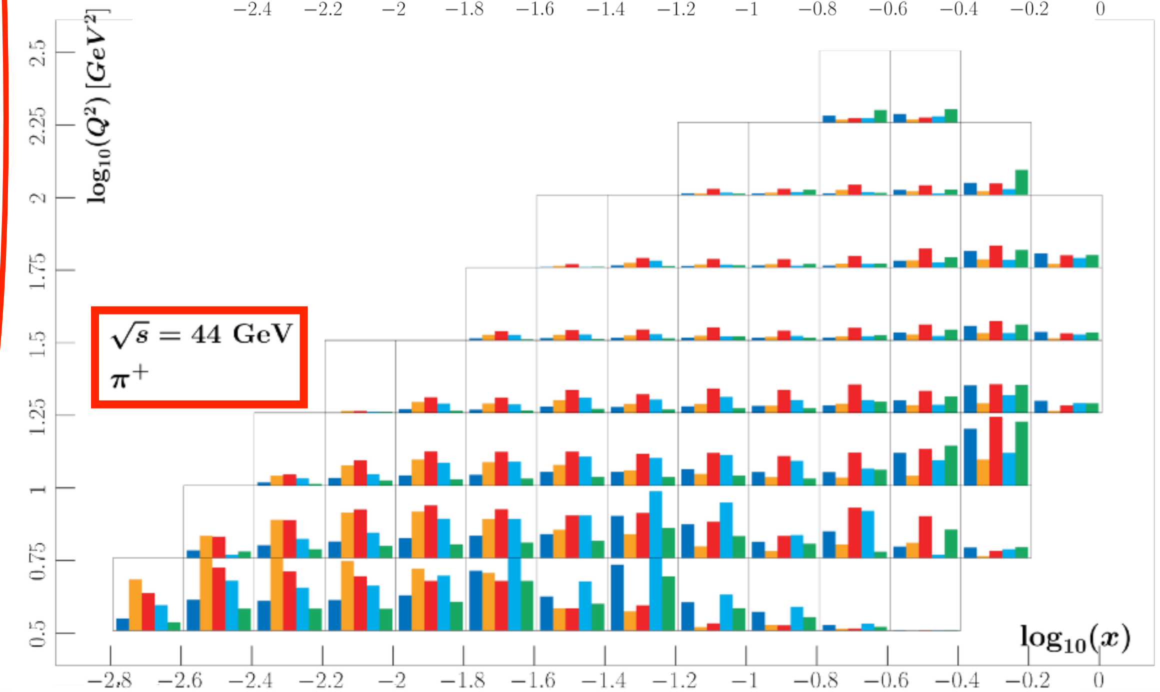
$$S(A, A) = \frac{\langle A \, A \rangle - \langle A \rangle \langle A \rangle}{\delta A \, \Delta A} = \frac{(\Delta A)^2}{\Delta A \, \Delta A} = 1$$

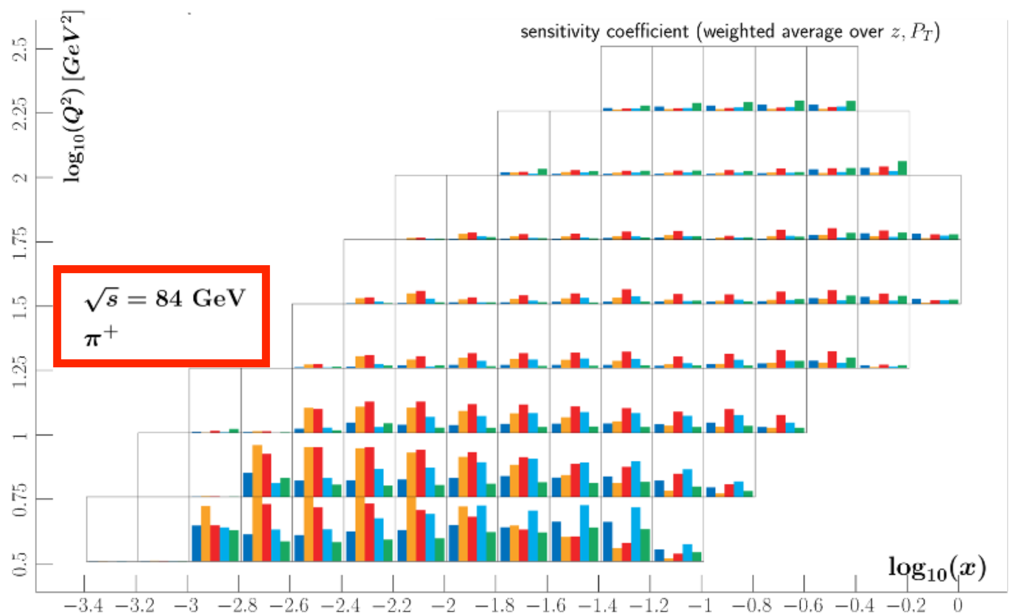
4. the error on *A* scales as $1/\sqrt{2} = 1/\sqrt{1 + (S = 1)}$. If the new measurement is more precise, then *S* >1 and the error is further reduced; viceversa, for *S*< 1
5. for *n* measurements, the error on *A* should scale as $1/\sqrt{1 + S_1 + .. + S_n}$

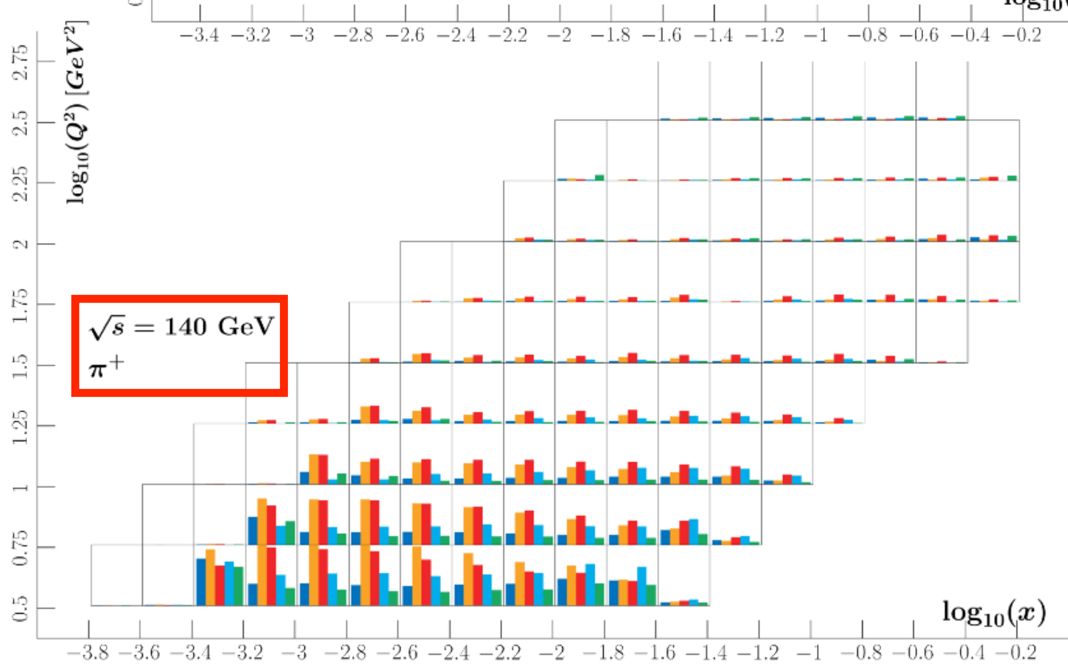
$S(\langle x\rangle_{\rm bin}, \langle Q^2\rangle_{\rm bin})$ with weighted average over z, P_T

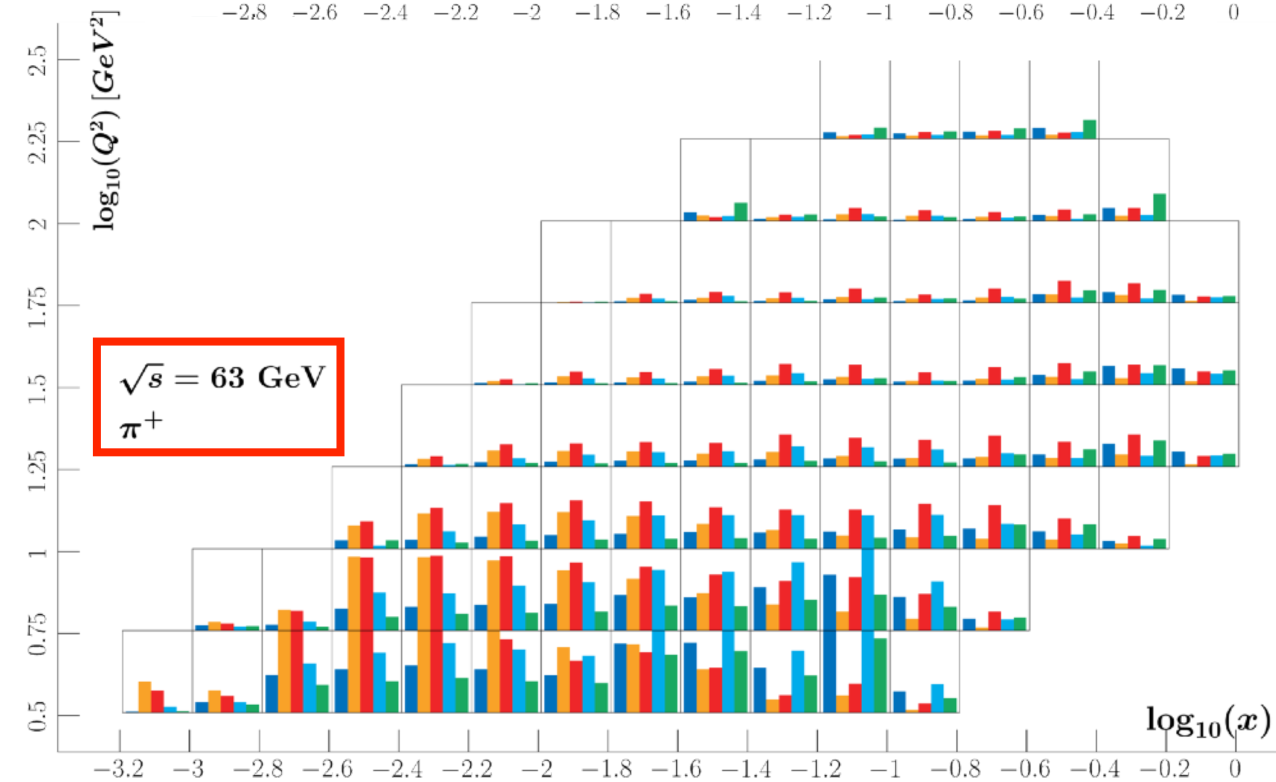
- N_1^{dv} (mid x TMD PDF width)
- σ (low x TMD PDF width)
- g_2 (nonperturbative evolution)
- N_3^{fv} (mid z TMD FF width)
- λ_F (TMD FF nongaussianity)











NonPerturbative evolution g_2

summing Sensitivity Coeffs. over all (x,Q^2) bins

PV17 fit: $\Delta g_2 = 0.01 \longrightarrow$

from best 68% replicas

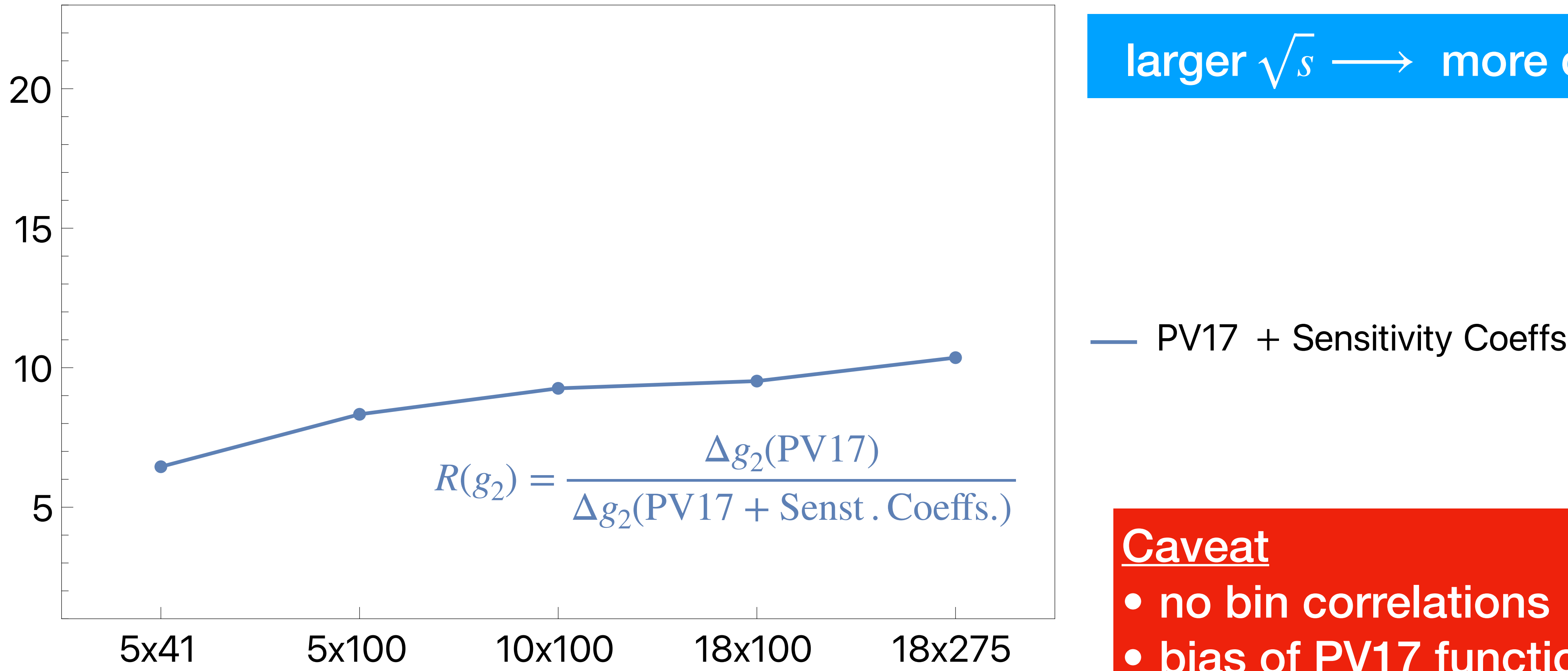
run at $\sqrt{s} = 28$ GeV, π^+ $\longrightarrow 0.00155$ $R(g_2) = 6.45$

run at $\sqrt{s} = 44$ GeV, π^+ $\longrightarrow 0.00120$ $R(g_2) = 8.33$

run at $\sqrt{s} = 63$ GeV, π^+ $\longrightarrow 0.00108$ $R(g_2) = 9.26$

run at $\sqrt{s} = 84$ GeV, π^+ $\longrightarrow 0.00105$ $R(g_2) = 9.52$

run at $\sqrt{s} = 140$ GeV, π^+ $\longrightarrow 0.00096$ $R(g_2) = 10.36$



larger $\sqrt{s} \longrightarrow$ more constraint

Caveat

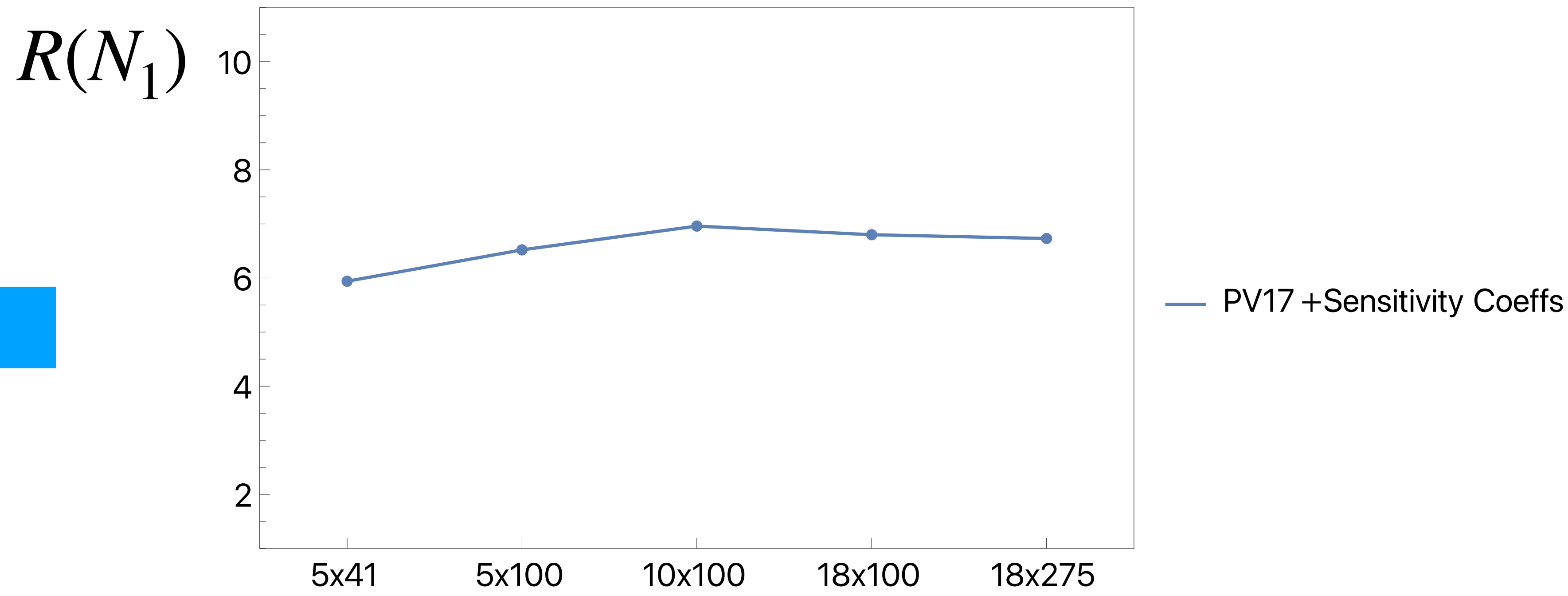
- no bin correlations
- bias of PV17 functional form

Mid-x width of TMD N_1

PV17 fit:

$\Delta N_1 = 0.06$

10x100 “best” option

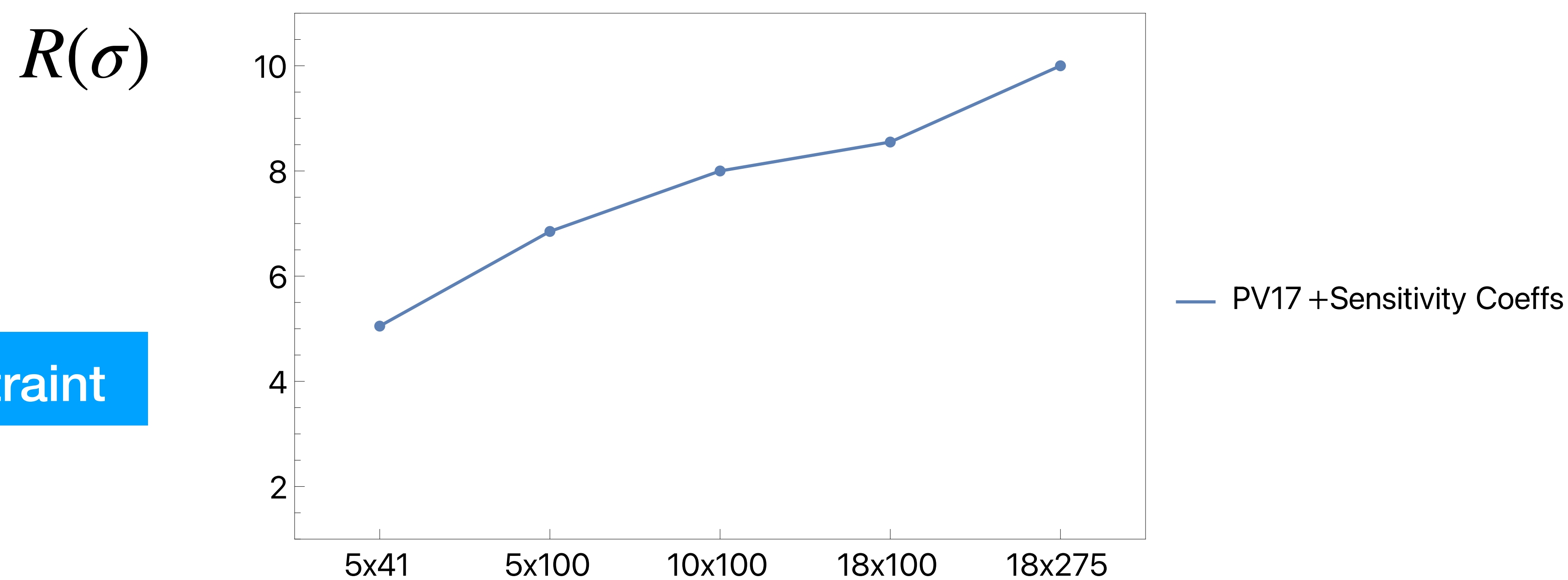


Low-x width of TMD σ

PV17 fit:

$\Delta\sigma = 0.02$

larger $\sqrt{s} \longrightarrow$ more constraint



Generate and fit EIC pseudodata

1. using the central replica from PV17 fit , generate pseudodata for unpolarized SIDIS cross section at all EIC kinematics: 5x41, 5x100, 10x100, 18x100, 18x275

A. Bacchetta *et al.*, JHEP06 (2017) 081, arXiv:1703.10157

$\sqrt{s} = 28$, 44 , 63 , 84 , 140 GeV

2. fit the EIC pseudodata with PV17 code, and compare the 68% band Δ for g_2, N_1, σ parameters with corresponding 68% band Δ in PV17 fit

$$R(X) = \frac{\Delta X(\text{PV17})}{\Delta X(\text{EIC})}$$

X =
- g_2 → nonperturbative evolution
- N_1 → mid-x width of TMD
- σ → low-x width of TMD

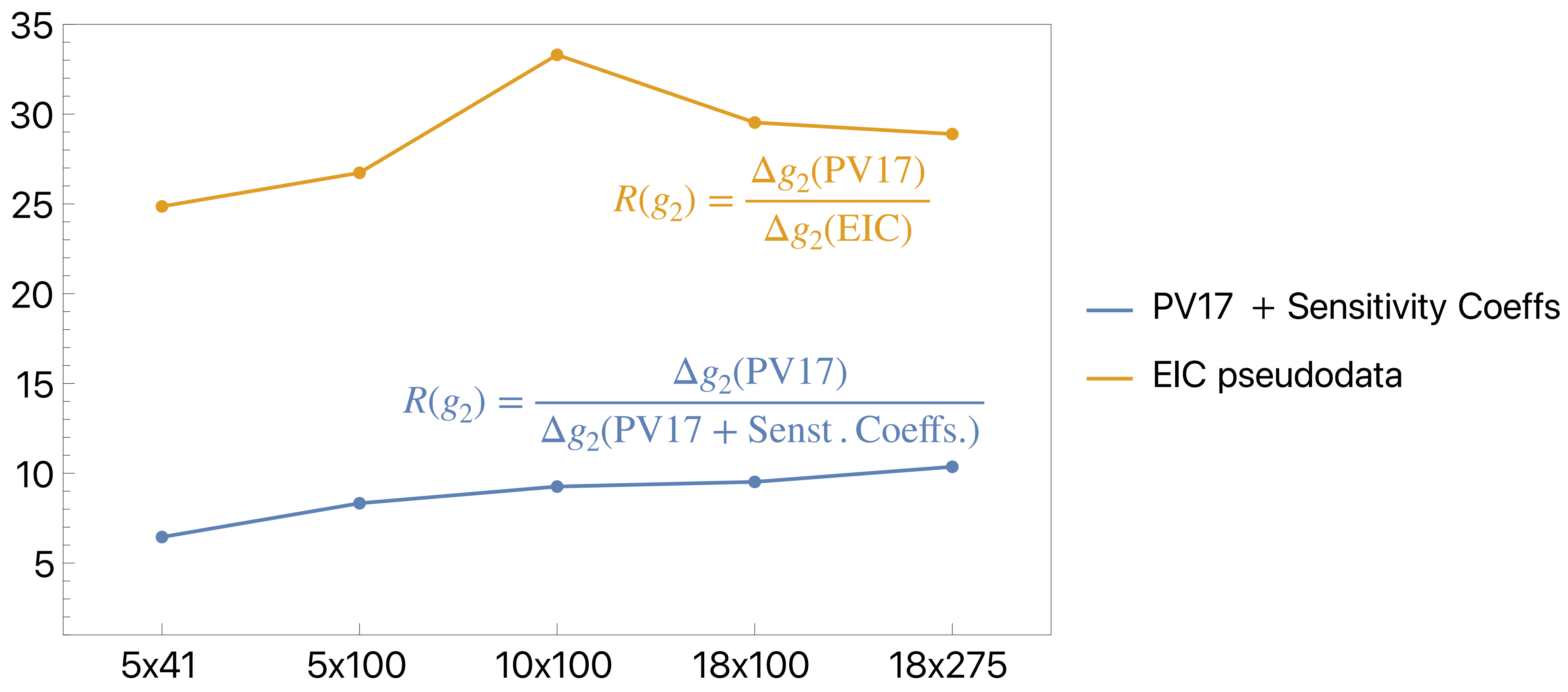
Caveat
- currently, ΔX from fit of only EIC pseudodata; including also data considered in PV17 fit is still in progress

3. χ^2 consistently very small:

	5x41	5x100	10x100	18x100	18x275
$\langle \chi^2 \rangle \pm \sigma_{\chi^2}$	0.0085 ± 0.004	0.0058 ± 0.0025	0.0056 ± 0.0022	0.0053 ± 0.0022	0.0045 ± 0.0018

NonPerturbative evolution g_2

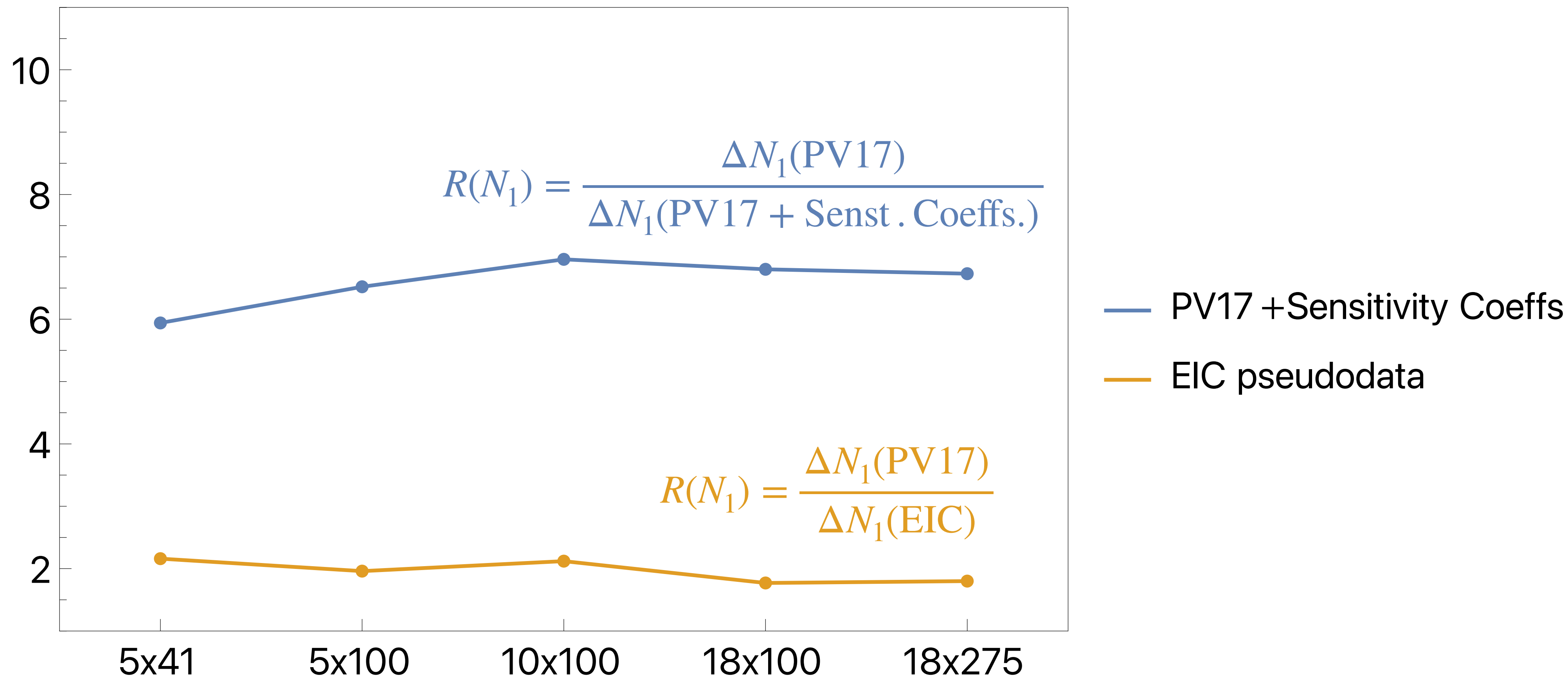
PV17 fit: 68% of replicas $\Delta g_2 = 0.01$



- increasing trend with $\sqrt{s}$ confirmed, but with $\times 4$ factor
- 10x100 kinematics seems the best option

Mid-x width of TMD N_1

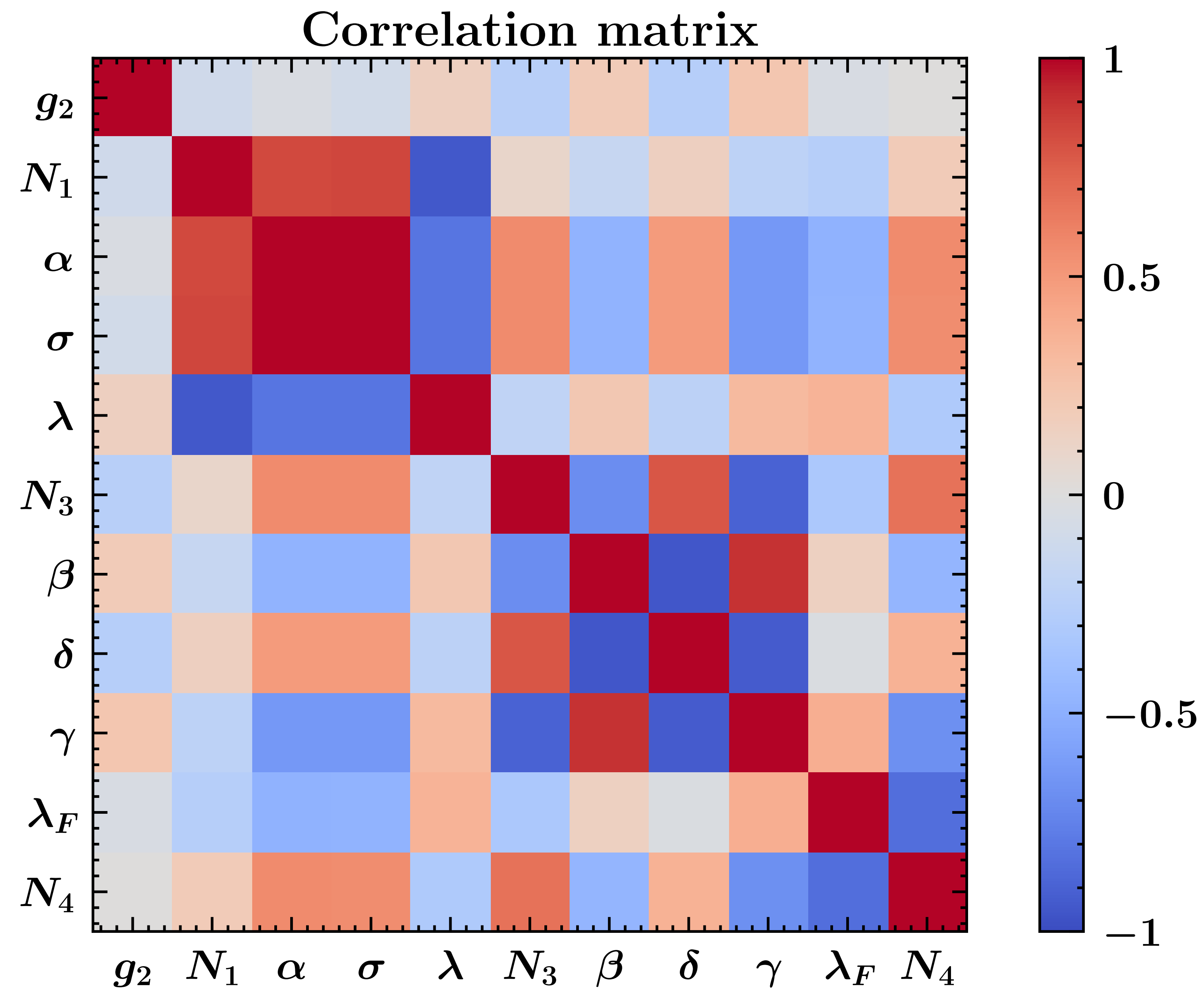
PV17 fit: 68% of replicas $\Delta N_1 = 0.06$



- increasing then decreasing trend with $\sqrt{s}$ confirmed, but with $\times 3$ factor smaller
- 10x100 kinematics seems again the best option

Correlation matrix

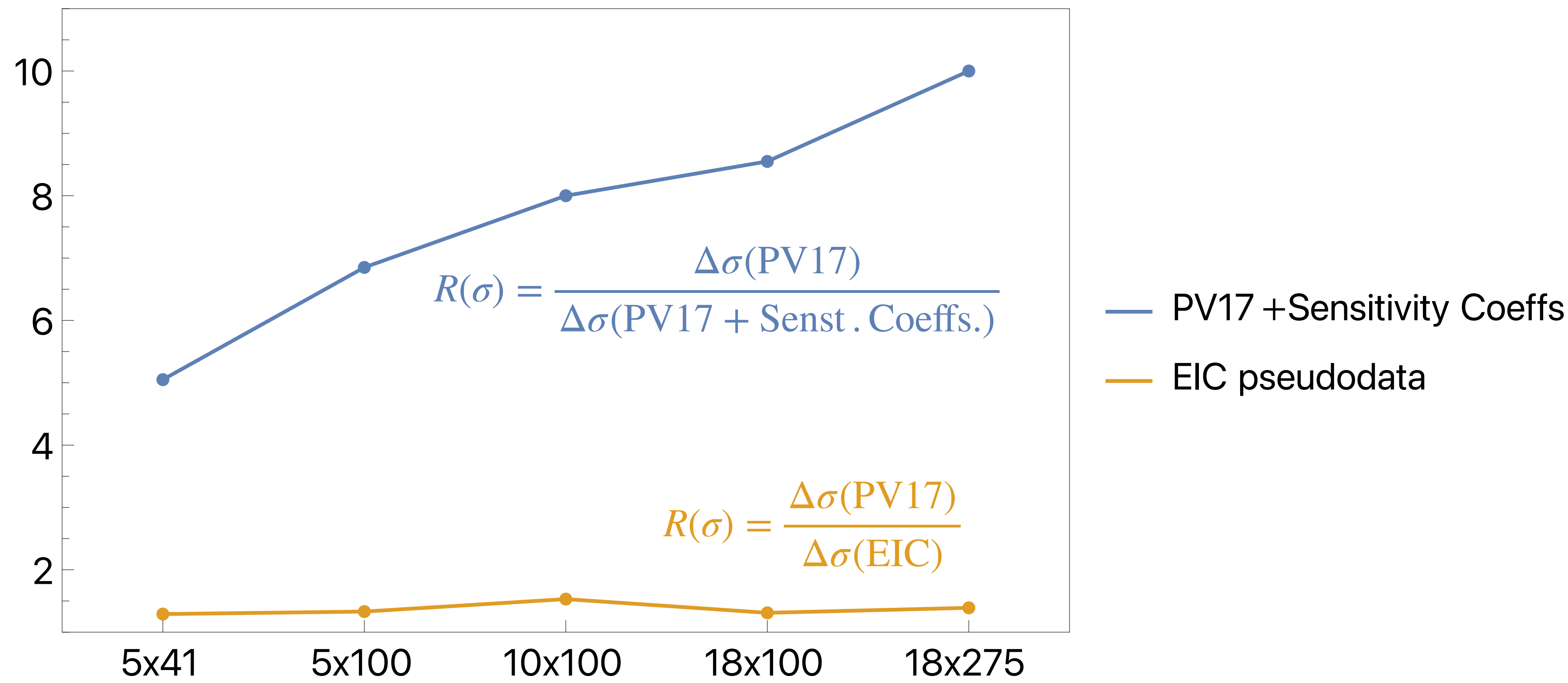
10x100
kinematics



large correlations between N_1 and σ

Low-x width of TMD σ

PV17 fit: 68% of replicas $\Delta\sigma = 0.02$



- increasing trend with $\sqrt{s}$ **NOT** confirmed, and **much smaller** size
- 10x100 seems again preferred

Comment

1. large (~ $\times 30$ factor) gain in precision for uncorrelated evolution parameter g_2, much smaller (~ $\times 2$) gain in correlated mid-/low- x TMD width parameters N_1, σ

2. with our functional form, EIC pseudodata seem to very well constrain evolution but not TMD "structure" at starting scale $Q_0 \Rightarrow$ need combined fit with other data

To-do List

1. fit EIC pseudodata (for all energies) + data included in PV17 fit, in order to make a consistent comparison with results using Sensitivity Coefficients

2. extend the analysis to fit parameters of TMD FF in PV17 functional form