

NNN15

International Workshop for the Next Generation Nucleon Decay and Neutrino Detector

Global Picture of Neutrino Oscillations

UD2

Unification Day 2 (UD2)

Simons Center for Geometry and Physics
Stony Brook University

October 28-31, 2015

Srubabati Goswami

Physical Research Laboratory
Ahmedabad



Neutrinos in the limelight

Nobelpriset i fysik 2015

The Nobel Prize in Physics 2015

Nobelpriset i fysik 2015



Takaaki Kajita
Super-Kamiokande Collaboration
University of Tokyo, Kashiwa, Japan

Arthur B. McDonald
Sudbury Neutrino Observatory Collaboration
Queen's University, Kingston, Canada

"för upptäckten av neutrinooscillationer, som visar att neutriner har massa"
the discovery of neutrino oscillations, which shows that neutrinos have mass

Golden Era of Neutrino Physics

- The neutrino has been the **show stopper** in high energy physics for the period **1998 - 2005**
- Results from **SuperKamiokande** experiment confirmed oscillation of **atmospheric neutrinos**
- Results from the **SNO solar neutrino** experiment established the **presence of ν_μ/ν_τ** in the solar ν_e flux
- Result from the **KamLAND** experiment confirmed solar neutrino oscillations using **reactor neutrinos**
- Result from **K2K, MINOS** confirmed atmospheric neutrino oscillations using **accelerator neutrinos**
- The results could be **explained** by dominant **two flavour** oscillations

Neutrino Oscillations

If neutrinos have mass then

Flavour states $\nu_\alpha = \sum_i U_{\alpha i} \nu_i$ Mass states

Two-Flavours

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i < j} \text{Re}(U_{\alpha i}^* U_{\beta j}^* U_{\alpha j} U_{\beta i}) \sin^2 \Delta_{ij} + 2 \sum_{i > j} \text{Im}(U_{\alpha i}^* U_{\beta j}^* U_{\alpha j} U_{\beta i}) \sin 2\Delta_{ij}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2(\Delta m^2 L / 4E)$$



Not sensitive to the **sign** of Δm^2
 Not sensitive to the **octant** of θ



$$\Delta_{ij} = \Delta m_{ij}^2 L / 4E$$

$$\Delta m_{ij}^2 = m_j^2 - m_i^2$$

$$\bar{\nu} : U \rightarrow U^*$$

Neutrino Oscillation requires
 Non-zero neutrino mass
 Non-zero mixing angles
 Oscillation effect $\Delta m^2 \sim E/L$

Not sensitive to the absolute masses

MSW effect

- In matter, only ν_e 's undergoes Charged current interaction \rightarrow an effective potential of $\sqrt{2}G_F N_e$

Effective mixing angle θ_M in matter

$$\tan 2\theta_M = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2\sqrt{2}G_F n_e E}$$

$$\Delta m^2 \cos 2\theta = 2\sqrt{2}G_F n_e E, \theta_M \rightarrow \pi/4 \quad \rightarrow \text{MSW Resonance}$$

L. Wolfenstein, PRD 17, 1978 S.P. Mikheyev, A.Yu. Smirnov, SJNP 42, 1985

For antineutrinos the potential changes sign

Resonance occur for $\Delta m^2 < 0$

Matter effect is sensitive to the ordering of the mass states

The mass squared difference in matter

$$\Delta m_M^2 = \sqrt{(\Delta m^2 \cos 2\theta - 2\sqrt{2}G_F n_e E)^2 + \sin^2 2\theta}$$

- Probabilities are obtained by solving propagation equation in matter
- Depends on density profile of matter

Three neutrino oscillation parameters

- 3 masses, 3 mixing angles, 1 phase

Atm + LBL

Sol+KL

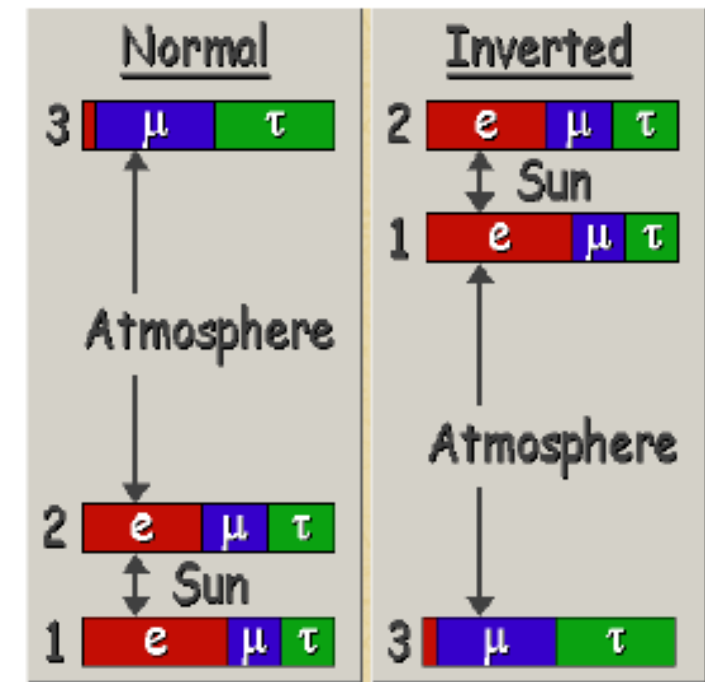
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} \\ & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & e^{-i\delta} s_{13} \\ & 1 & \\ -e^{i\delta} s_{13} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \\ & & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$c_{12} = \cos \theta_{12}$ etc., δ CP-violating phase

- $\Delta m_{21}^2, \theta_{12}, \theta_{13}$ Solar + KamLND
- $\Delta m_{31}^2, \theta_{13}$ Reactor
- $\Delta m_{31}^2, \theta_{23}, \theta_{13}, \delta_{CP}$ Atm + LBL



Interplay among different sectors because of θ_{13}



Status of oscillation parameters (2014)

NuFIT 2.0 (2014)

Parameter	Best-fit for NH(IH)	3σ Ranges for NH(IH)
θ_{12}°	33.48	31.29 - 35.91
θ_{23}°	42.3(49.5)	38.2 - 53.3(38.6 - 53.3)
θ_{13}°	8.50(8.51)	7.85 - 9.10(7.87 - 9.11)
δ_{CP}°	306(254)	0 - 360
$\frac{\Delta m_{21}^2}{10^{-5}\text{eV}^2}$	7.50	7.02-8.09
$\frac{\Delta m_{3l}^2}{10^{-3}\text{eV}^2}$	2.457(2.449)	2.317 - 2.607(2.590 - 2.307)
$\Delta m_{3l}^2 = \Delta m_{31}^2 > 0$ for NH and $\Delta m_{3l}^2 = \Delta m_{32}^2 < 0$ for IH		

No hierarchy sensitivity

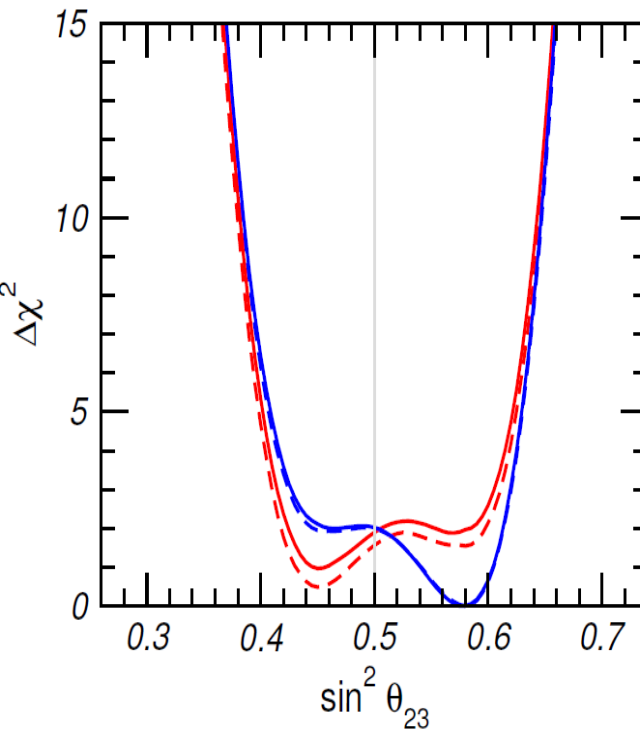
$$\chi^2(NH) - \chi^2(IH) < 1$$

Gonzalez-Garcia Maltoni Schwetz
[JHEP 11 \(2014\) 052](#)[arXiv:1409.5439]



Status of θ_{23} (2014)

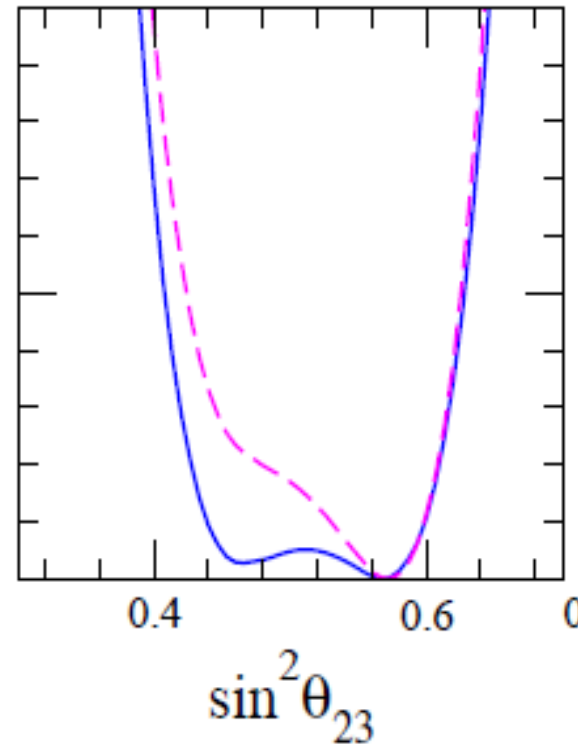
Nu-fit 2014



Global best-fit at 2nd octant and IH

NH : local best-fit at 1st octant

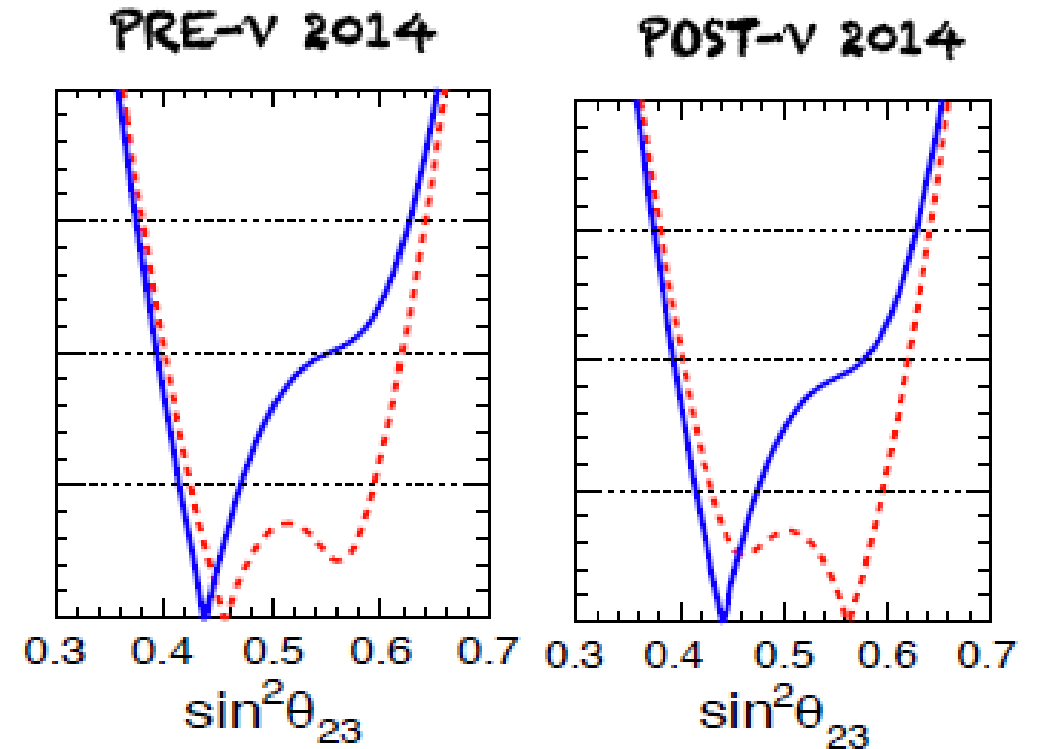
Forero et al 1405.7540



NH and IH separate fit

Best-fit at 2nd octant for both NH and IH

Capozzi et al. 1312.2878



NH and IH separate fit

Best-fit in 2nd octant for IH post Neutrino 2014

Data set included in Global Analysis

Solar experiments

- Chlorine total rate [3], 1 data point.
- Gallex & GNO total rates [4], 2 data points.
- SAGE total rate [5], 1 data point.
- SK1 full energy and zenith spectrum [6], 44 data points.
- SK2 full energy and day/night spectrum [7], 33 data points.
- SK3 full energy and day/night spectrum [8], 42 data points.
- SK4 1669-day energy spectrum and day/night asymmetry [9], 24 data points.
- SNO combined analysis [10], 7 data points.
- Borexino 740.7-day low-energy data [11], 33 data points.
- Borexino 246-day high-energy data [12], 6 data points.

Reactor experiments

- KamLAND combined DS1 & DS2 spectrum [14], 17 data points.
- CHOOZ energy spectrum [15], 14 data points.
- Palo-Verde total rate [16], 1 data point.
- Double-Chooz 227.9-day spectrum [17], 18 data points.
- Daya-Bay 621-day spectrum [18], 36 data points.
- Reno 800-day near & far total rates [19], 2 data points (with free normalization).
- SBL reactor data (including Daya-Bay total flux at near detector), 77 data points [18, 20].

Atmospheric experiments

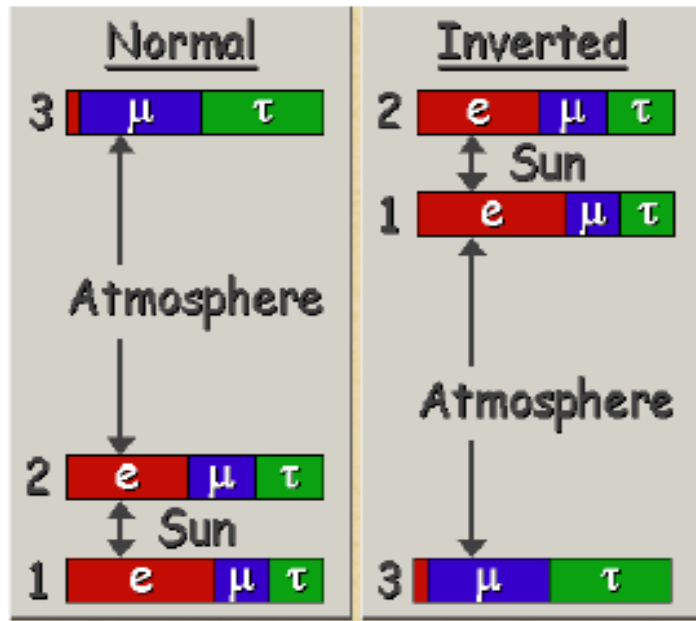
- SK1–4 (including SK4 1775-day) combined data [13], 70 data points.

Accelerator experiments

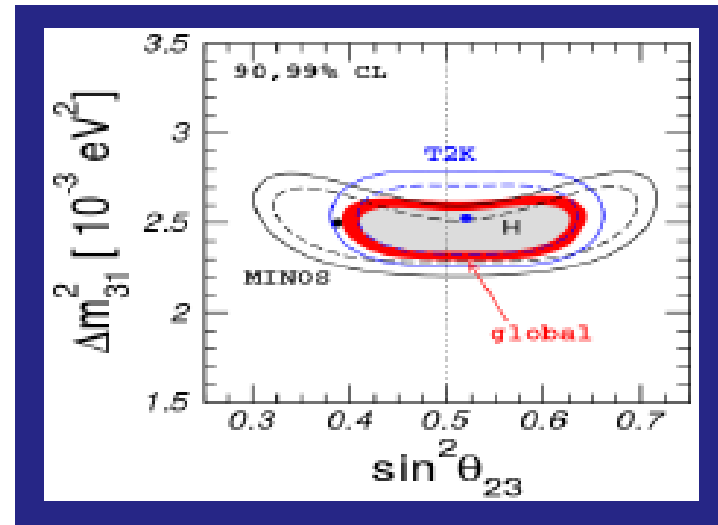
- MINOS 10.71×10^{20} pot ν_μ -disappearance data [21], 39 data points.
- MINOS 3.36×10^{20} pot $\bar{\nu}_\mu$ -disappearance data [21], 14 data points.
- MINOS 10.6×10^{20} pot ν_e -appearance data [22], 5 data points.
- MINOS 3.3×10^{20} pot $\bar{\nu}_e$ -appearance data [22], 5 data points.
- T2K 6.57×10^{20} pot ν_μ -disappearance data [23], 16 data points.
- T2K 6.57×10^{20} pot ν_e -appearance data [24], 5 data points.



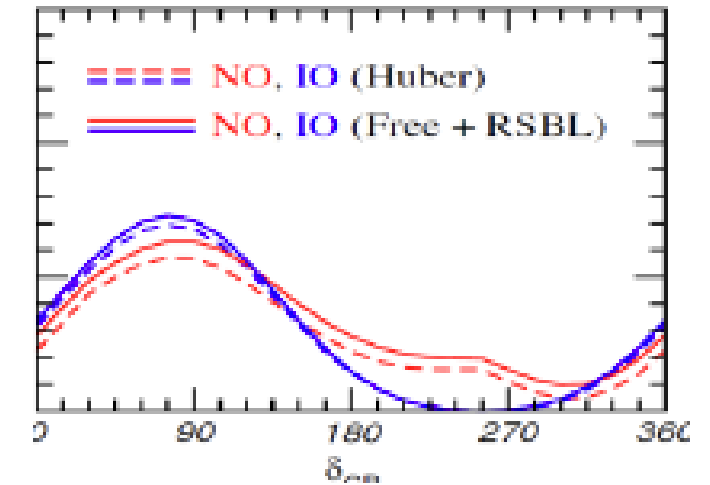
The unknowns



The sign of Δm_{31}^2 i.e.
 $\Delta m_{31}^2 > 0 \Rightarrow$ Normal Hierarchy (NH) or
 $\Delta m_{31}^2 < 0 \Rightarrow$ Inverted Hierarchy (IH).



The octant of θ_{23} i.e.
 $\theta_{23} > 45^\circ \Rightarrow$ Higher Octant (HO) or
 $\theta_{23} < 45^\circ \Rightarrow$ Lower Octant (LO).

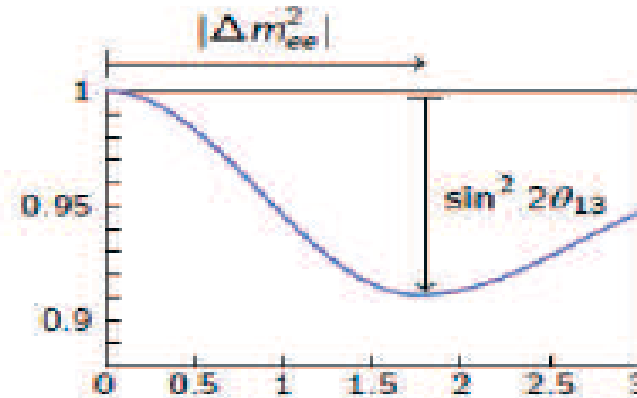
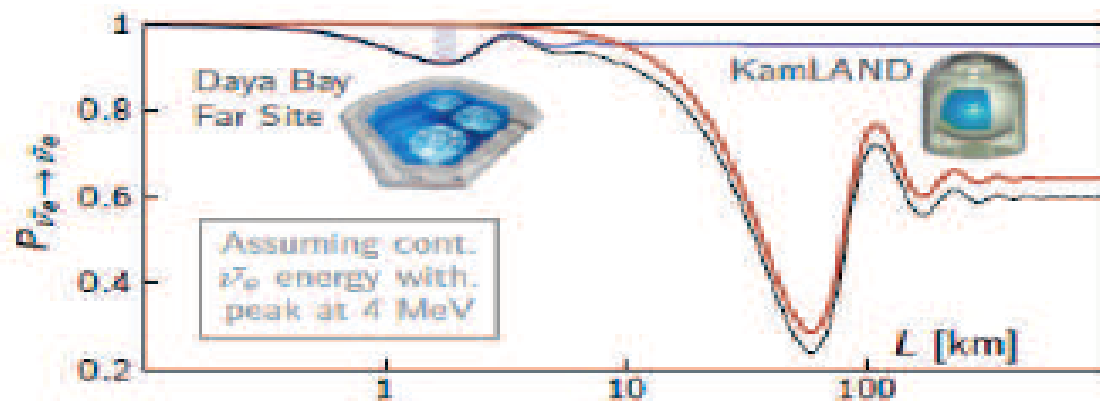


Leptonic CP phase δ_{CP}
 $\delta_{CP} = 0, \pm 180^\circ$ (CP conserving)
 $\delta_{CP} = \pm 90^\circ$ (maximal CP violation)
 $-180^\circ < \delta_{CP} < 0^\circ$ (LHP)
 $0^\circ < \delta_{CP} < 180^\circ$ (UHP)

A **non-zero** value of θ_{13} crucial for all three

Measurement of θ_{13}

- Till 2011 : $\theta_{13} < 13^\circ$
- Reactor experiments - Double Chooz, Daya-Bay and RENO has measured non-zero θ_{13}
- Long Baseline: distance ~ 1 km $L/E \sim$ Km/MeV
- $P_{ee} = 1 - \sin^2 2\theta_{13} \sin^2 1.27\Delta_{ee}L/E$



Latest Daya Bay Result

$$\sin^2 2\theta_{13} = 0.084 \pm 0.005$$

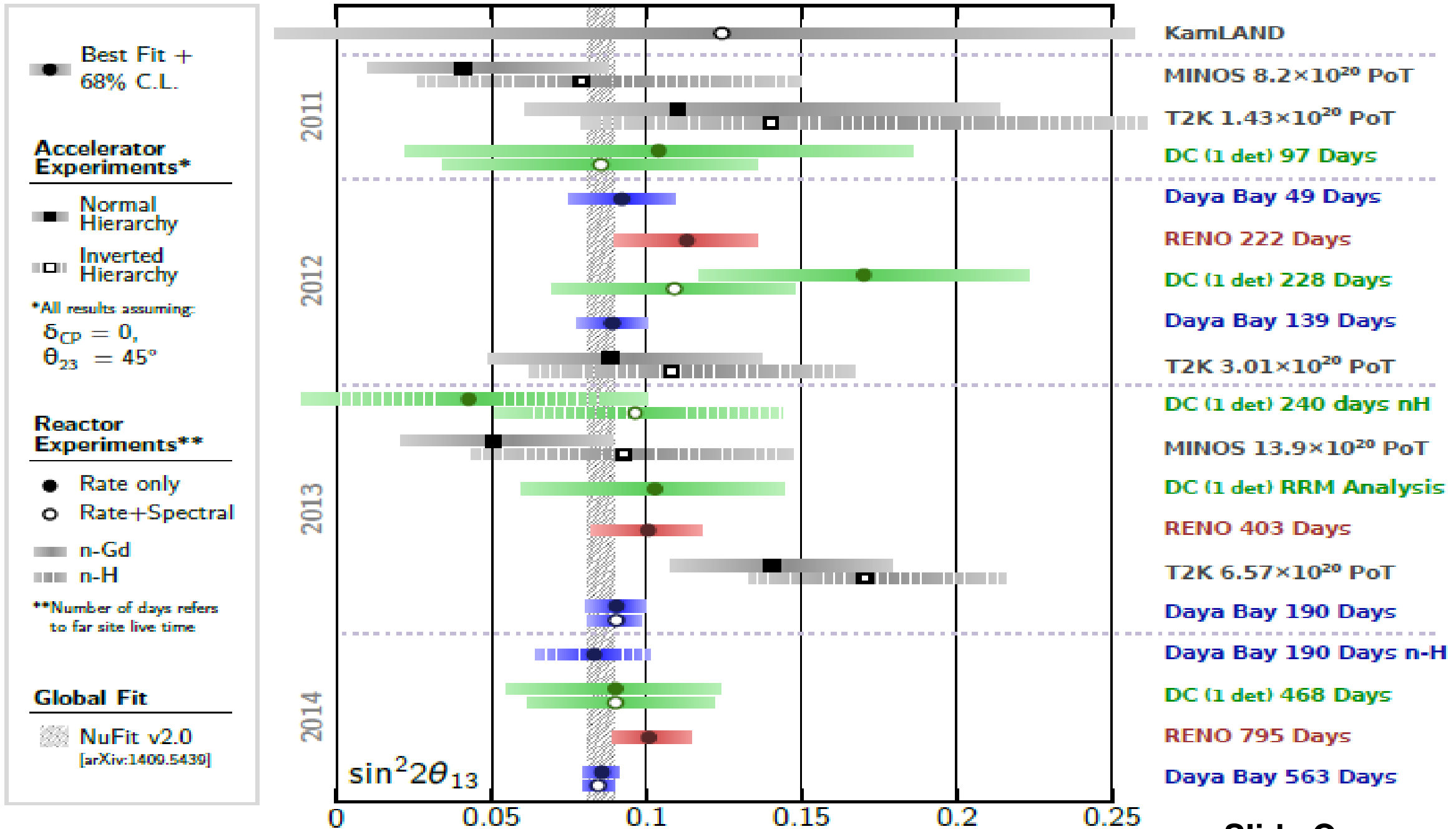
Phys. Rev. Lett. 115, 111802

- The deficit is governed by θ_{13}
- L/E dependence is sensitive to Δ_{ee}

See talk by S. Wagner NNN15

Global Picture of θ_{13}

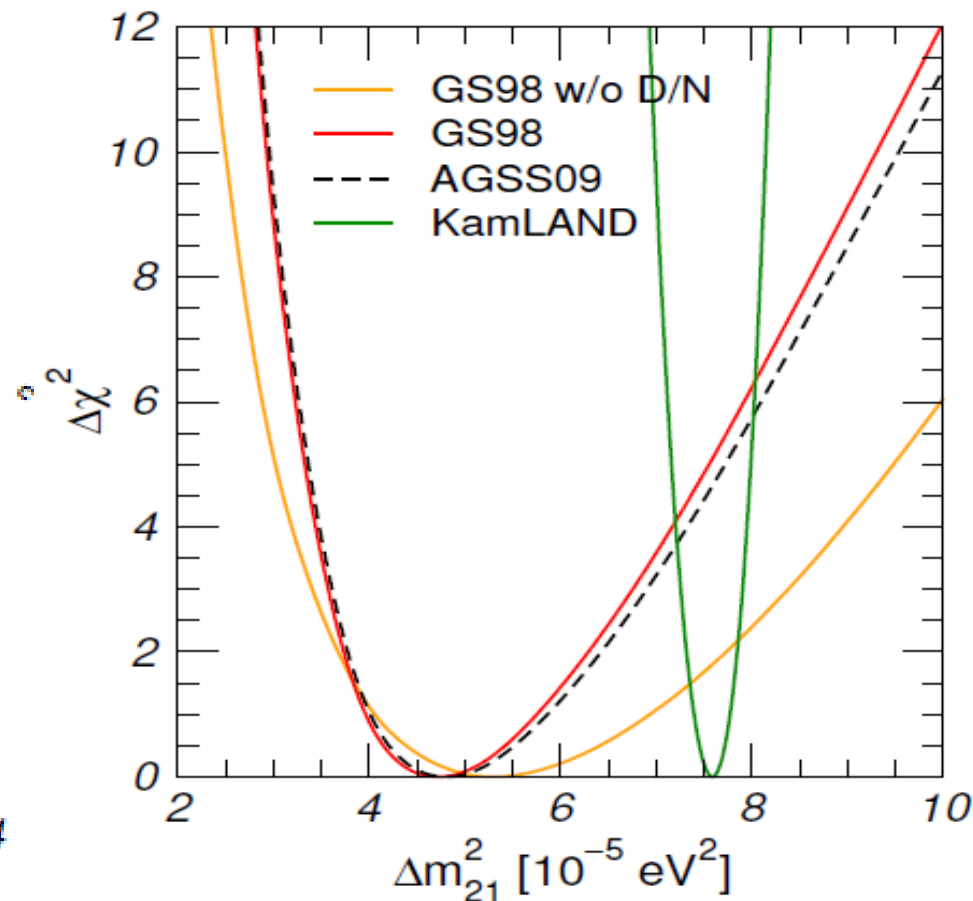
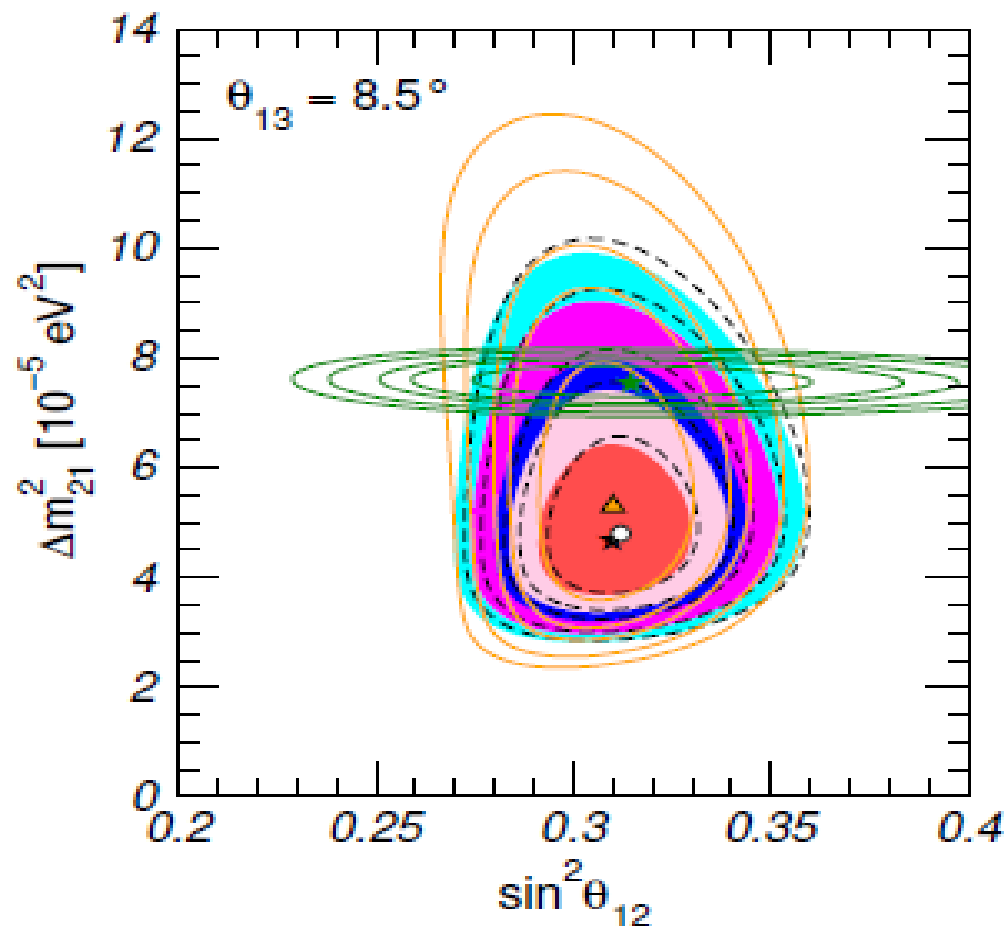
$$\theta_{13}$$



Slide Coursey: S. Jetter

Current Status of Solar Parameters

- The best-fit value of Δm_{21}^2 from KamLAND lies above that of solar $\rightarrow 2\sigma$ effect

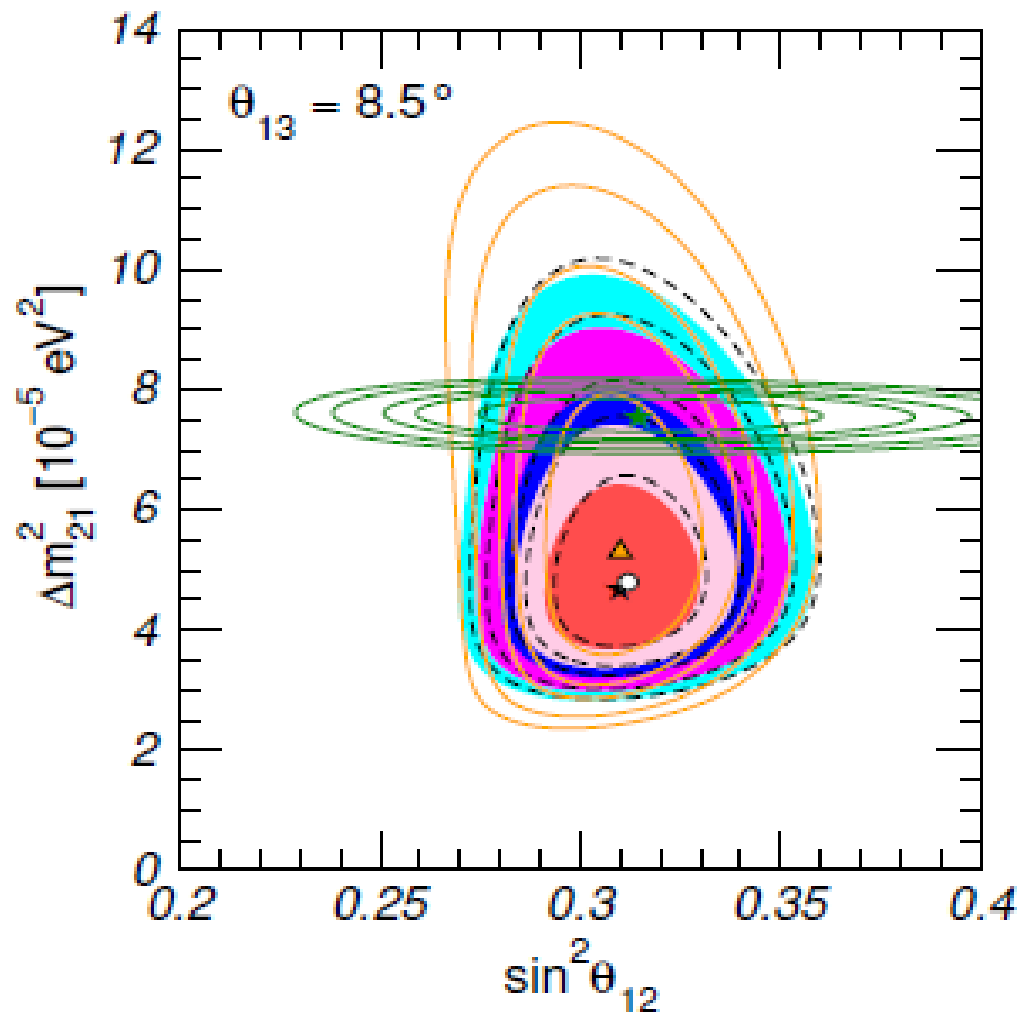


Δm_{21}^2 controlled by KamLAND
 θ_{12} controlled by solar

Gonzalez-Garcia
 Maltoni Schwetz

[JHEP 11 \(2014\) 052](#)

- Absence of low energy spectrum turn up of ^8B spectrum as expected in LMA MSW
- The indication of a non-zero day-night asymmetry in SuperKamiokande



The Probabilities for solar and KamLAND

$$P_{ee}^{\text{Solar}}(^8\text{B}) \approx c_{13}^4 \sin^2 \theta_{12}$$

$$P_{ee}^{\text{KL}} \approx c_{13}^4 (1 - \sin^2 2\theta_{12} \sin^2 \Delta m_{21}^2 L/4E)$$

- For $\theta_{13} = 0$ solar and KamLAND prefer different values of θ_{12}
- For $\theta_{13} > 0$ solar prefer higher θ_{12}
KamLAND prefer lower θ_{12}
- Impact of non-zero θ_{13} \rightarrow best-fit θ_{12} from solar and KamLAND lies at the same value

Gonzalez-Garcia
Maltoni Schwetz
[JHEP 11 \(2014\) 052](#)

Three generation effect in Atmospheric Neutrinos

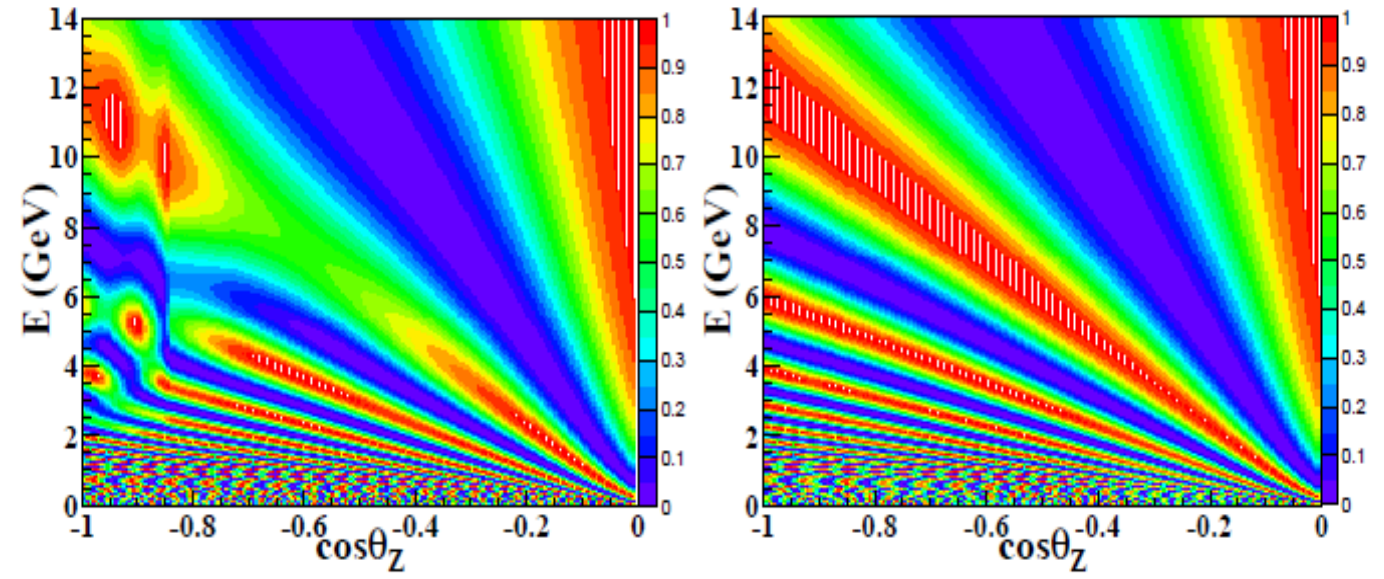
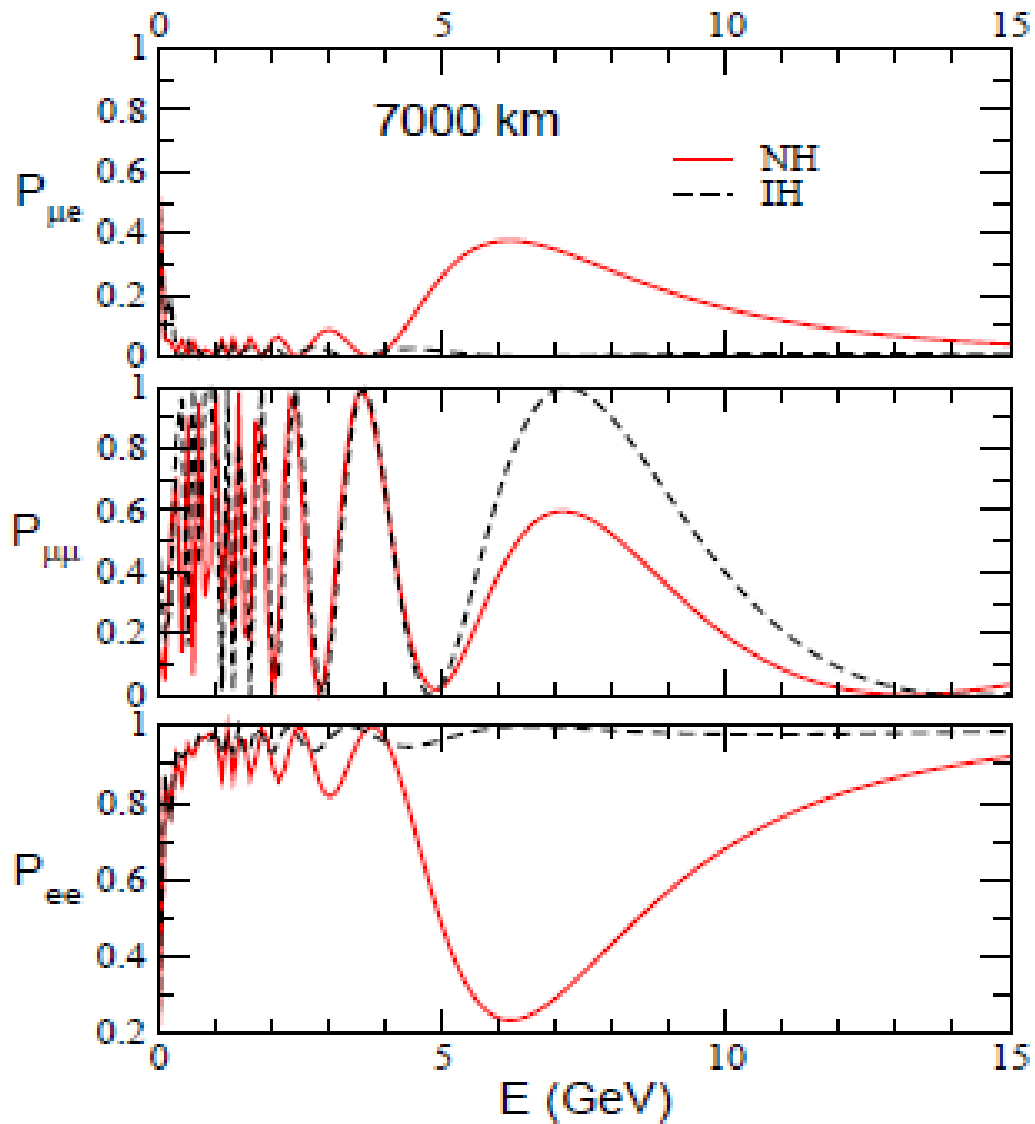
Matter effect starts becoming visible : Resonant enhancement of θ_{13}

Large matter effects for both ν_μ and ν_e

$$\tan 2\theta_{13}^m = \frac{\Delta m_{31}^2 \sin 2\theta_{13}}{\Delta m_{31}^2 \cos 2\theta_{13} \pm 2\sqrt{2}G_F n_e E}$$

- For $\Delta m_{31}^2 > 0$ resonance in neutrinos
- For $\Delta m_{31}^2 < 0$ resonance in antineutrinos

➔ Hierarchy Sensitivity



Three generation effect in atmospheric neutrinos

Excess of electron-like events:

$$\begin{aligned} \frac{N_e}{N_e^0} - 1 &\simeq (rs_{23}^2 - 1)P_{2\nu}(\Delta m_{31}^2, \theta_{13}) && \theta_{13} - \text{effects} \\ &+ (rc_{23}^2 - 1)P_{2\nu}(\Delta m_{21}^2, \theta_{12}) && \Delta m_{21}^2 - \text{effects} \\ &- 2s_{13}s_{23}c_{23}\text{Re}(A_{ee}^*A_{\mu e}) && \text{interference : } \delta_{CP} \end{aligned}$$

$$r = \frac{F_{\mu}^0}{F_e^0} \approx 2$$

● Resonant matter effect in $P_{2\nu}(\Delta m_{31}^2, \theta_{13})$ for multi-GeV neutrinos .

● All **three** terms important for **sub-GeV** neutrinos

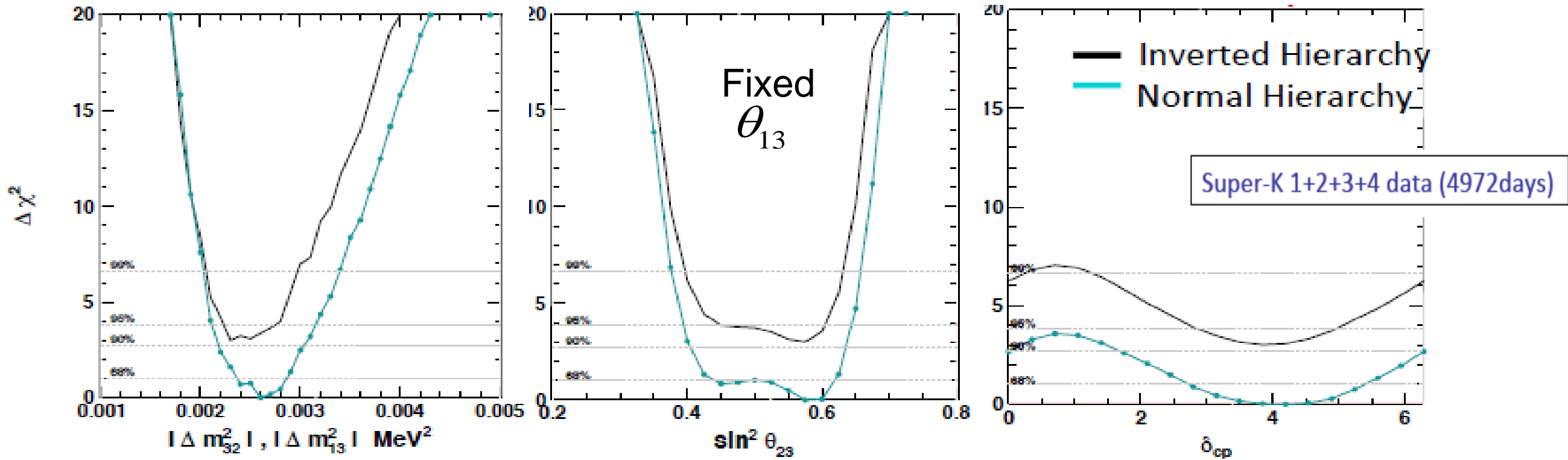
● Sensitivity to both **Hierarchy** and **Octant** and δ_{CP}

● For $s_{23}^2 < 0.5$ the 1-2 sector gives excess while 1-3 sector gives depletion

Peres and Smirnov , 2004

Gonzalez-Garcia, Maltoni, Smirnov, 2004

Three Generation analysis of SuperK data

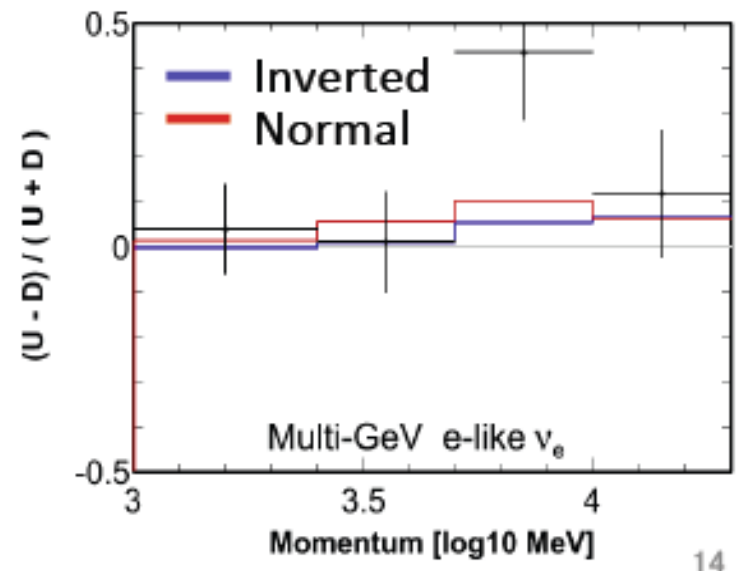


$\chi^2_{\text{NH}} - \chi^2_{\text{IH}} = -3.0$ (-0.9 at 2014) **NH-HO**

Driven by **excess** of upward going **electron** events

Mainly from **SK-IV** data

$\theta_{13} > 0$ weakly favoured



See talk by M. Ikeda NNN15

J. Kameda, NuFact 2015

Three generation effect in Accelerator Neutrinos

$$P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \Delta + \text{sub leading terms}$$

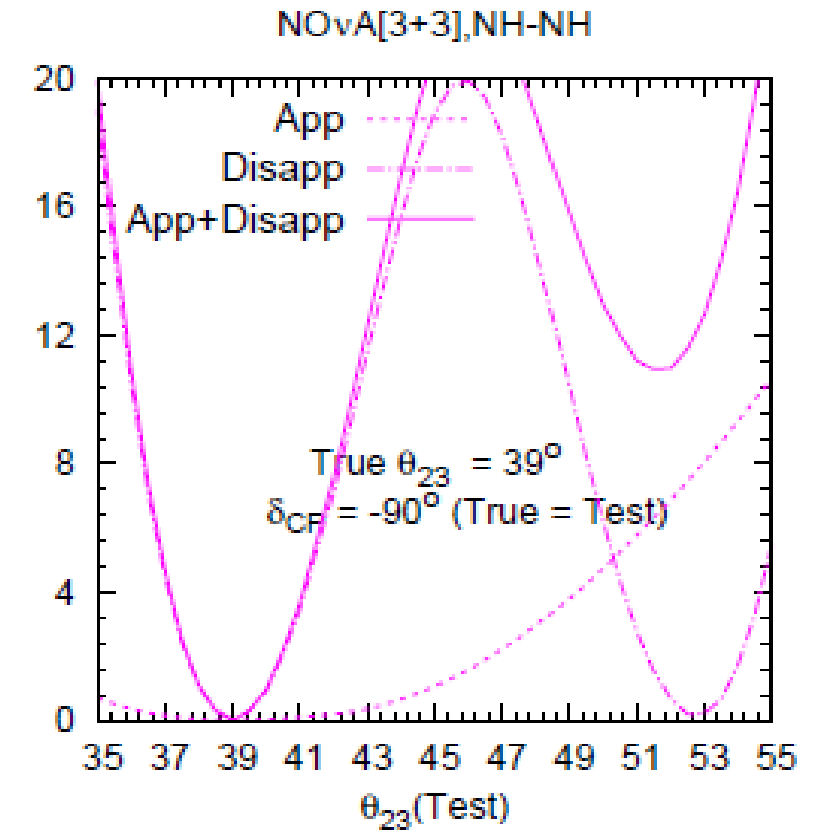
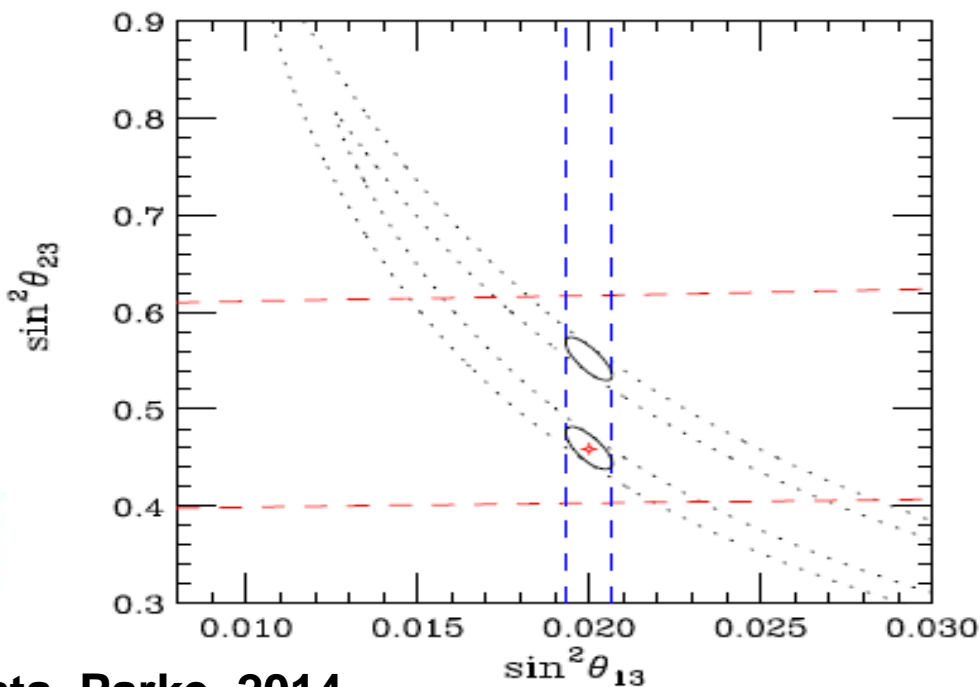
$$P_{\mu e} \simeq \sin^2 \theta_{23} \sin^2 2\theta_{13} \frac{\sin^2(\hat{A} - 1)\Delta}{(\hat{A} - 1)^2} + \alpha \sin 2\theta_{13} \sin 2\theta_{12} \cos(\Delta + \delta_{CP}) \frac{\sin(\hat{A} - 1)\Delta}{(\hat{A} - 1)} \frac{\sin \hat{A}\Delta}{\hat{A}}$$

$$\Delta = \Delta m_{31}^2 L / 4E, \alpha = \Delta m_{21}^2 / \Delta m_{31}^2, \hat{A} = \pm 2\sqrt{2}G_F n_e E / \Delta m_{31}^2$$

Octant sensitivity
 Correlated to θ_{13}
 Accelerator +
 Reactor data helpful

Huber, Lindner, Winter, 2002
 Hiraide et al., 2006

Coloma, Minakata, Parke, 2014



Synergy between appearance and disappearance channel

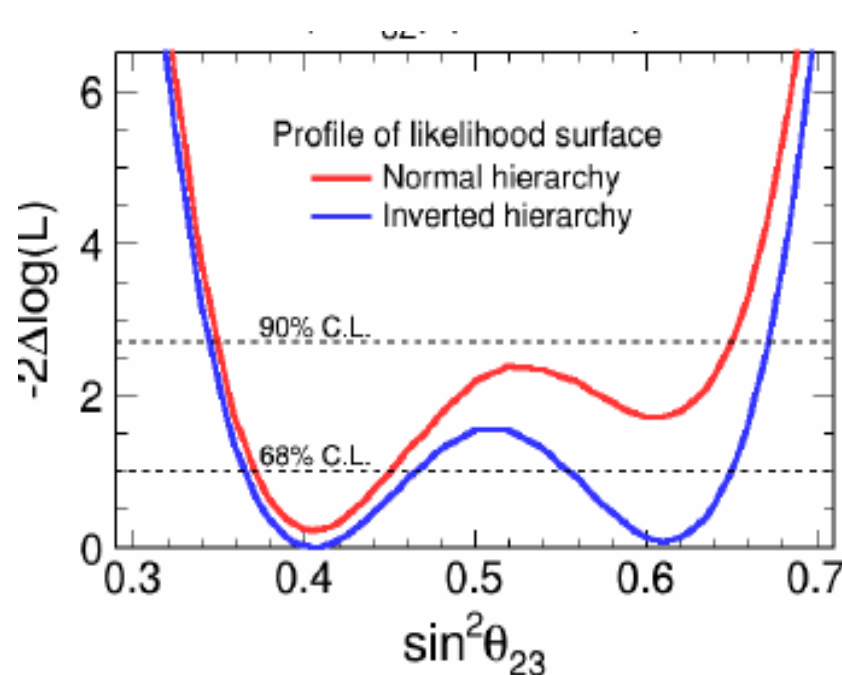
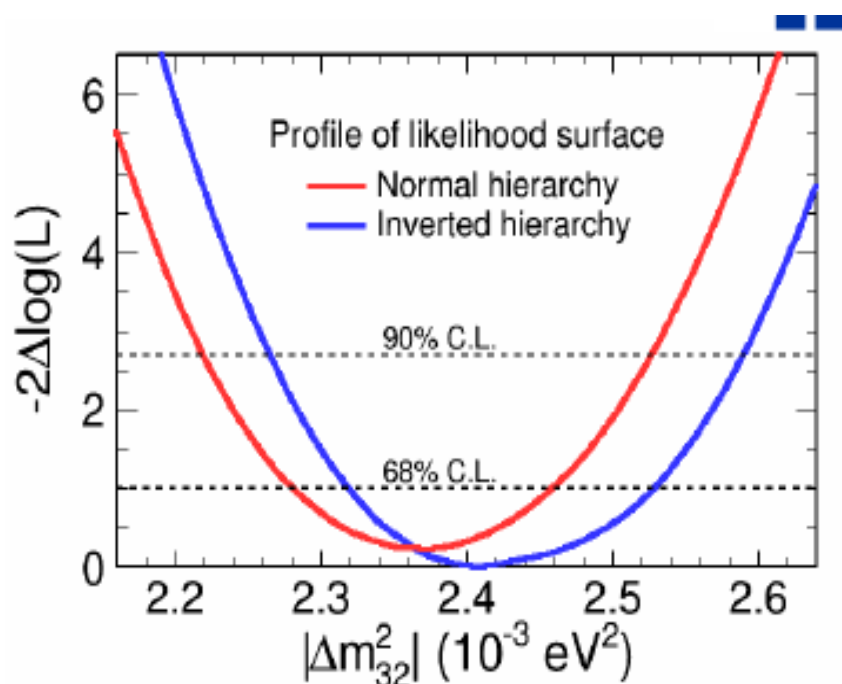
➡ Octant Sensitivity

Three flavour oscillation analysis : MINOS

Full three flavour analysis combining appearance and disappearance data

Also included atmospheric neutrino data

Phys. Rev. Lett. 112, 191801 (2014)



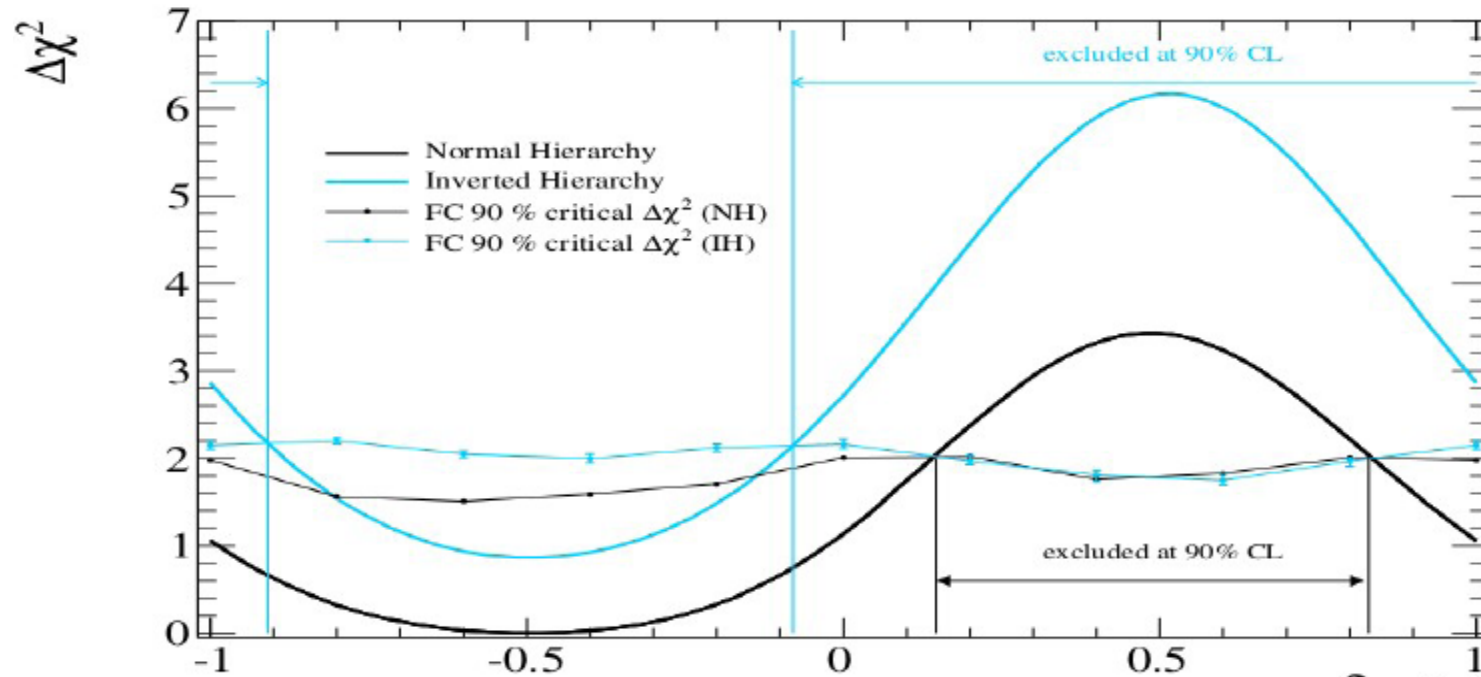
IH-LO

Combined analysis improves Previous constraints

See talk by L. Whitehead NNN15

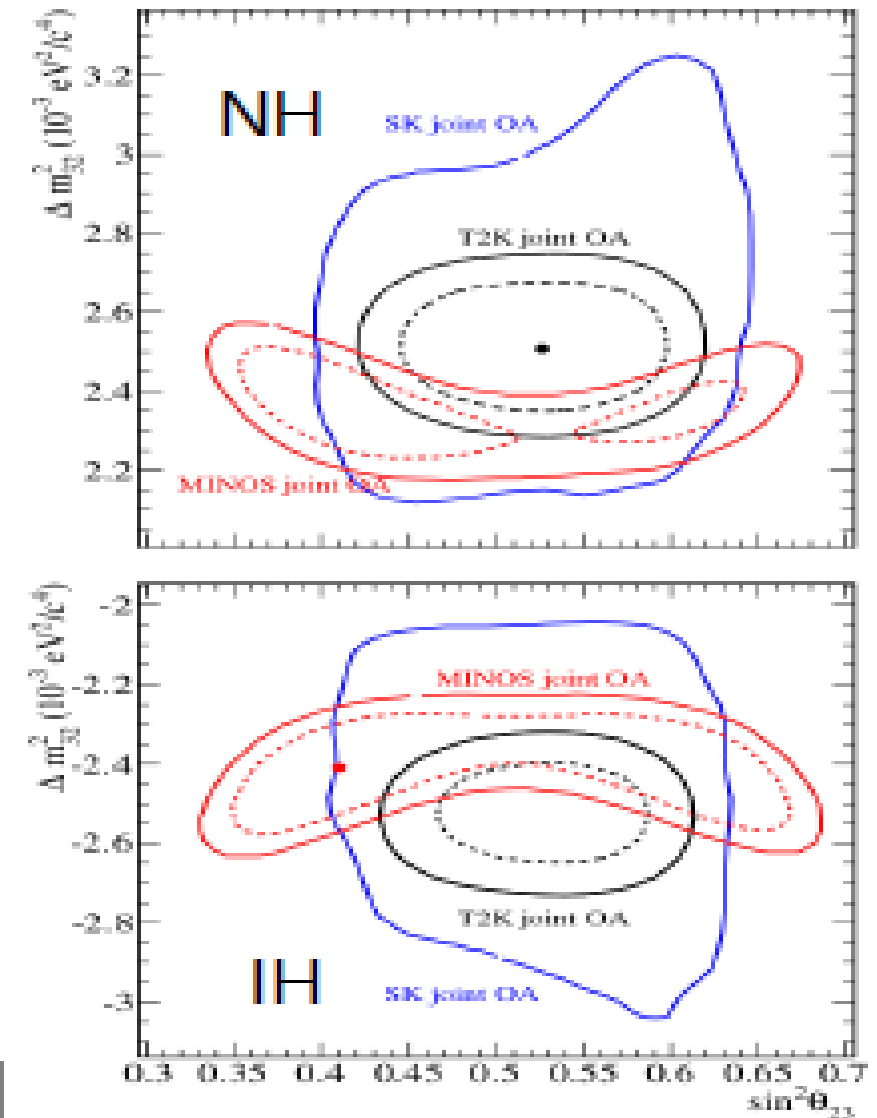
Hierarchy	Octant	Best fit parameters				$-2\Delta\log(L)$
		$\Delta m^2_{32}/10^{-3}\text{eV}^2$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$	δ_{CP}/π	
Inverted	Lower	-2.41	0.41	0.0243	0.62	-
Inverted	Higher	-2.41	0.61	0.0241	0.37	0.09
Normal	Lower	2.37	0.41	0.0242	0.44	0.23
Normal	Higher	2.35	0.61	0.0238	0.62	1.74

Three flavour oscillation analysis: T2K



Weak preference for Normal Hierarchy and HO

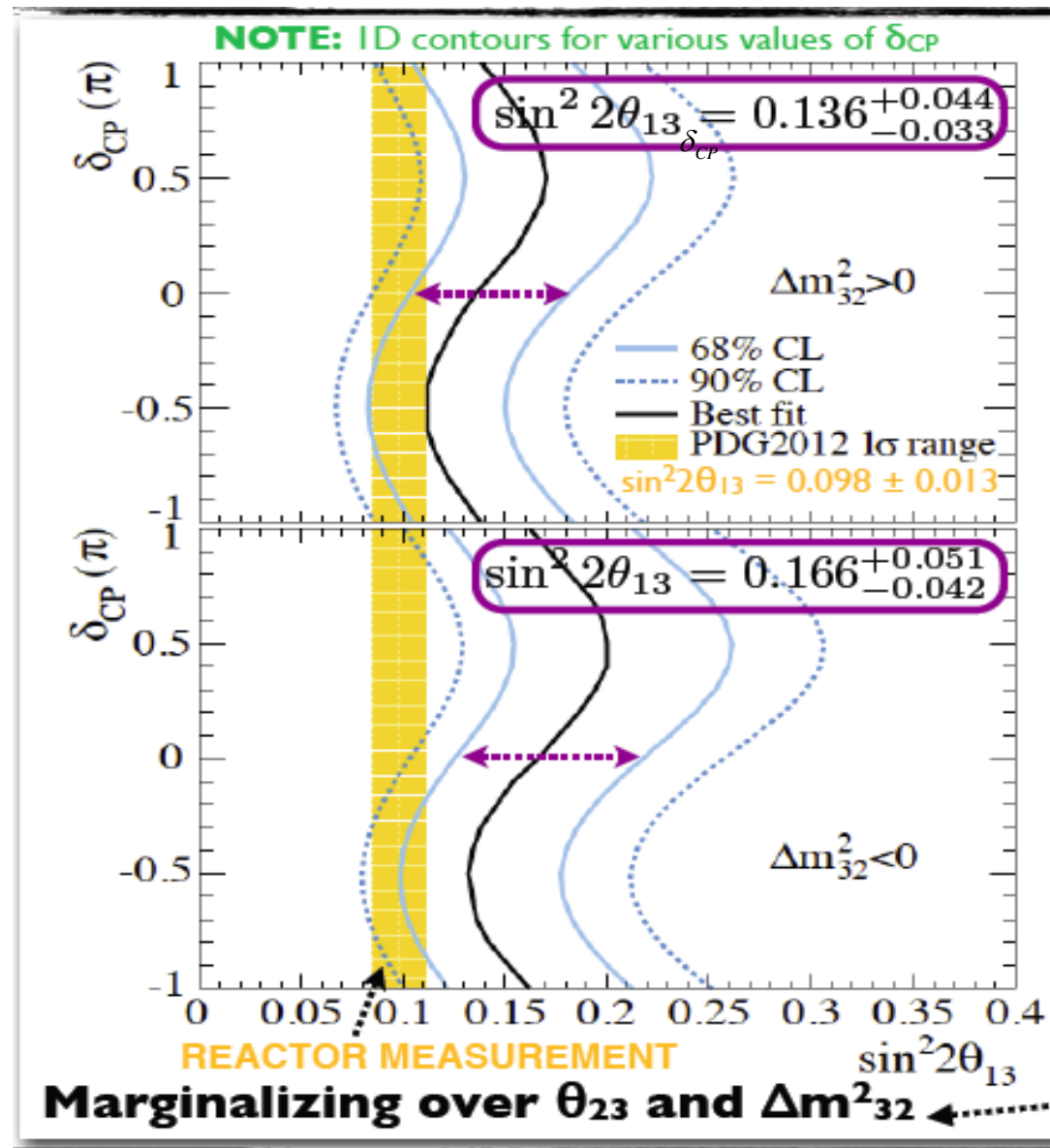
Best Precision for θ_{23}



Phys. Rev. D91, 072010,
(2015)

See talk by T. Hiraki NNN15

Hint of δ_{CP} from T2K data



First hint for $\delta_{CP} = -90^\circ$ from T2k data
Using only 8% of proton on targets

Best-fit value obtained for $\sin^2 2\theta_{13}$ is larger than the reactor value

When applying the reactor constraint, region where $\sin^2 2\theta_{13}$ is as small as possible is favored

Due to $\delta_{CP} - \sin^2 2\theta_{13}$ correlation, this favored region is for $\delta_{CP} \approx -\pi/2$

L. Escudero, ICHEP 2014

Global Analysis 2015

First anti- ν data from T2K, disapp.+app.
(included, preliminary)

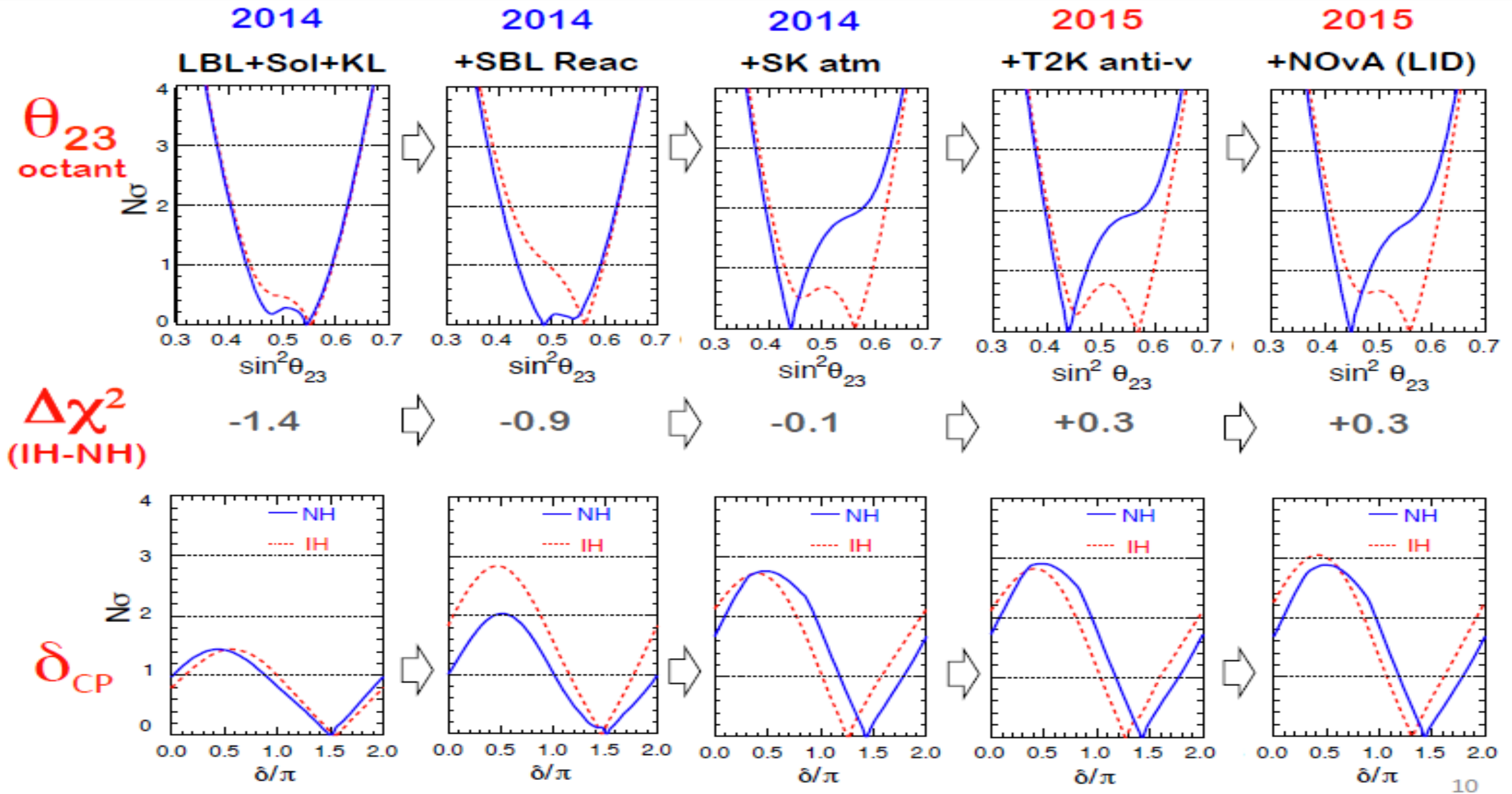
First data from NOvA, disapp.+app. (LID or LEM)*
(included, preliminary) See talk by A. Sousa NNN15

*LID (Likelihood Identification) = primary NOvA analysis
LEM (Library Event Matching) = alternative NOvA analysis

Frequentist Method using a χ^2 statistics

Capozzi, Lisi & Marrone (Univ. & INFN, Bari)

Global Analysis 2015



Capozzi, Lisi & Marrone (Univ. & INFN, Bari)

E. Lisi, VLVNT_2015

Global Analysis 2015

At $\sim 90\%$ C.L. (1.64σ)
we find that...

θ_{23}
octant

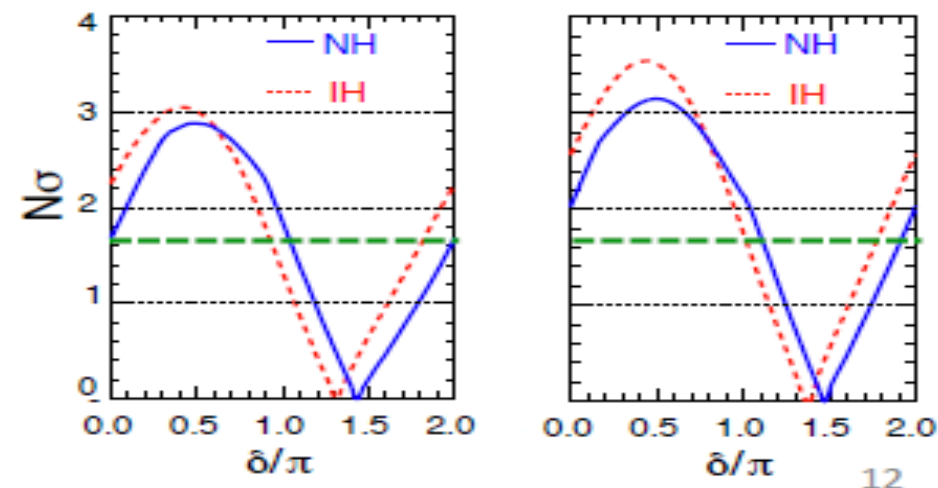
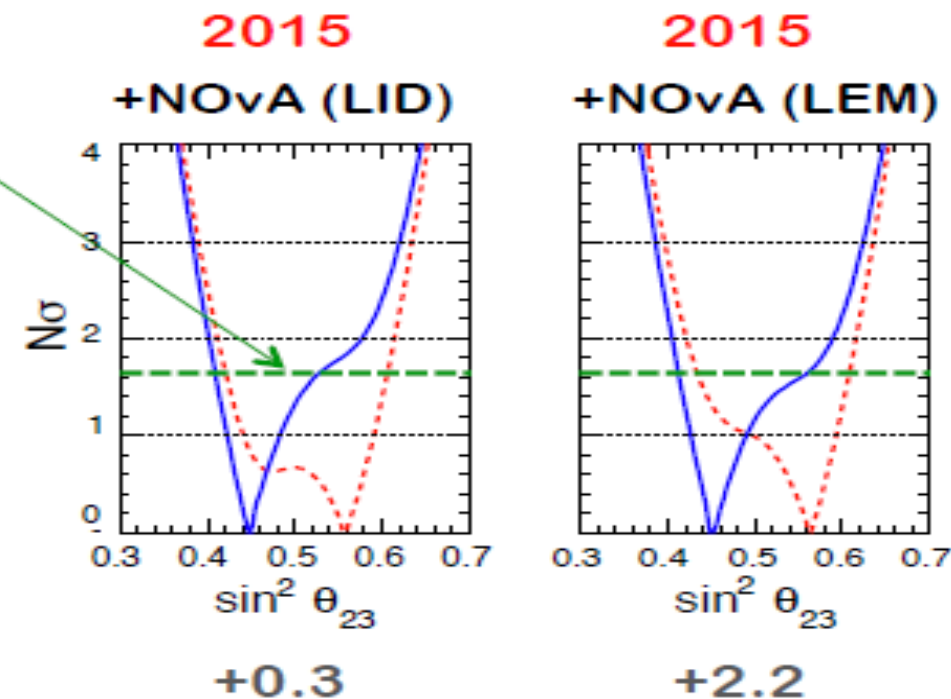
...both θ_{23} octants,
both NH and IH,
both CPV and no CPV
are still allowed...

$\Delta\chi^2$
(IH-NH)

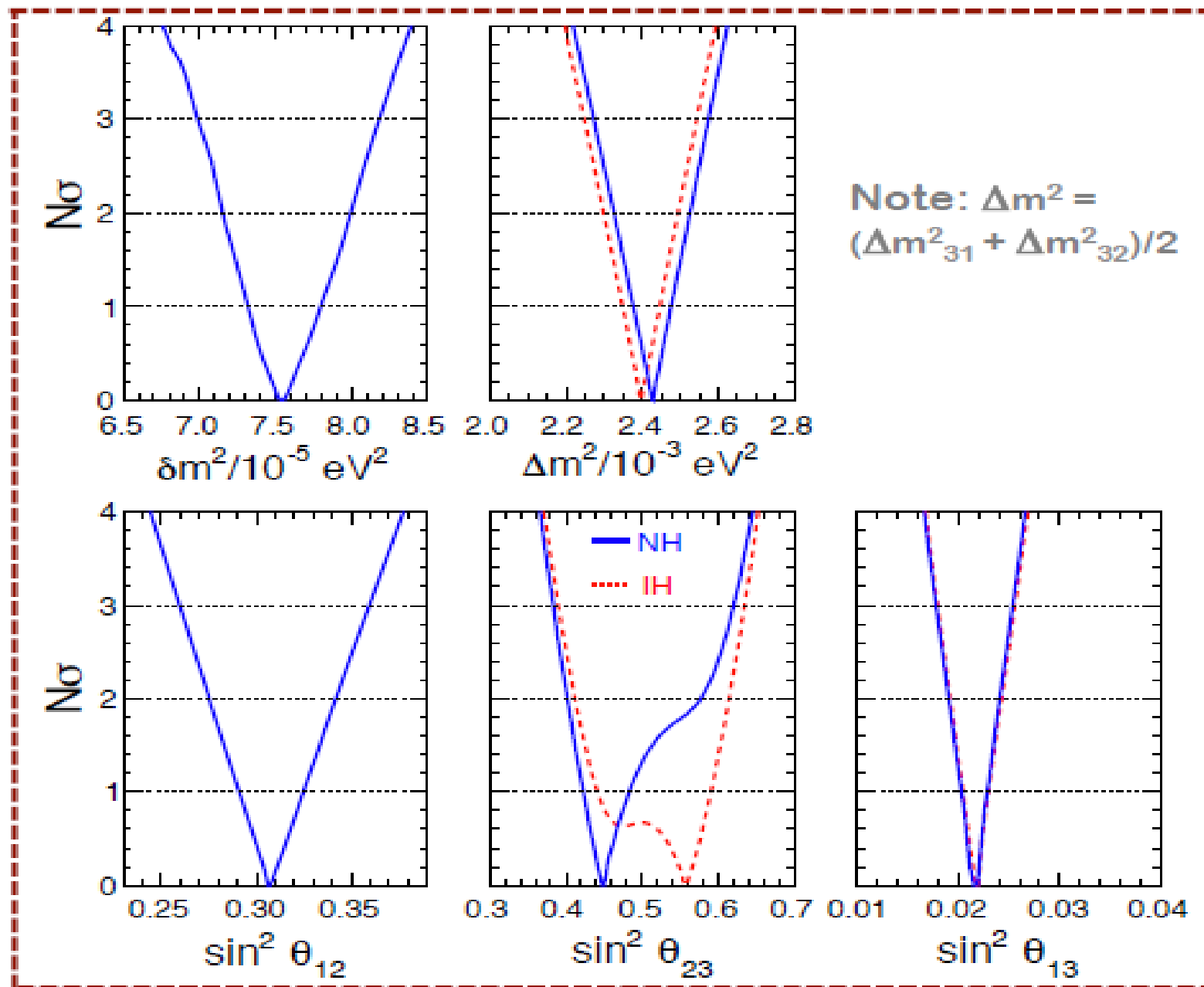
...but $\delta \sim 3\pi/2$ is clearly
favored over $\delta \sim \pi/2$
at the level of $\sim 3\sigma$

δ_{CP}

(Preliminary!)



Known Parameters (2015)

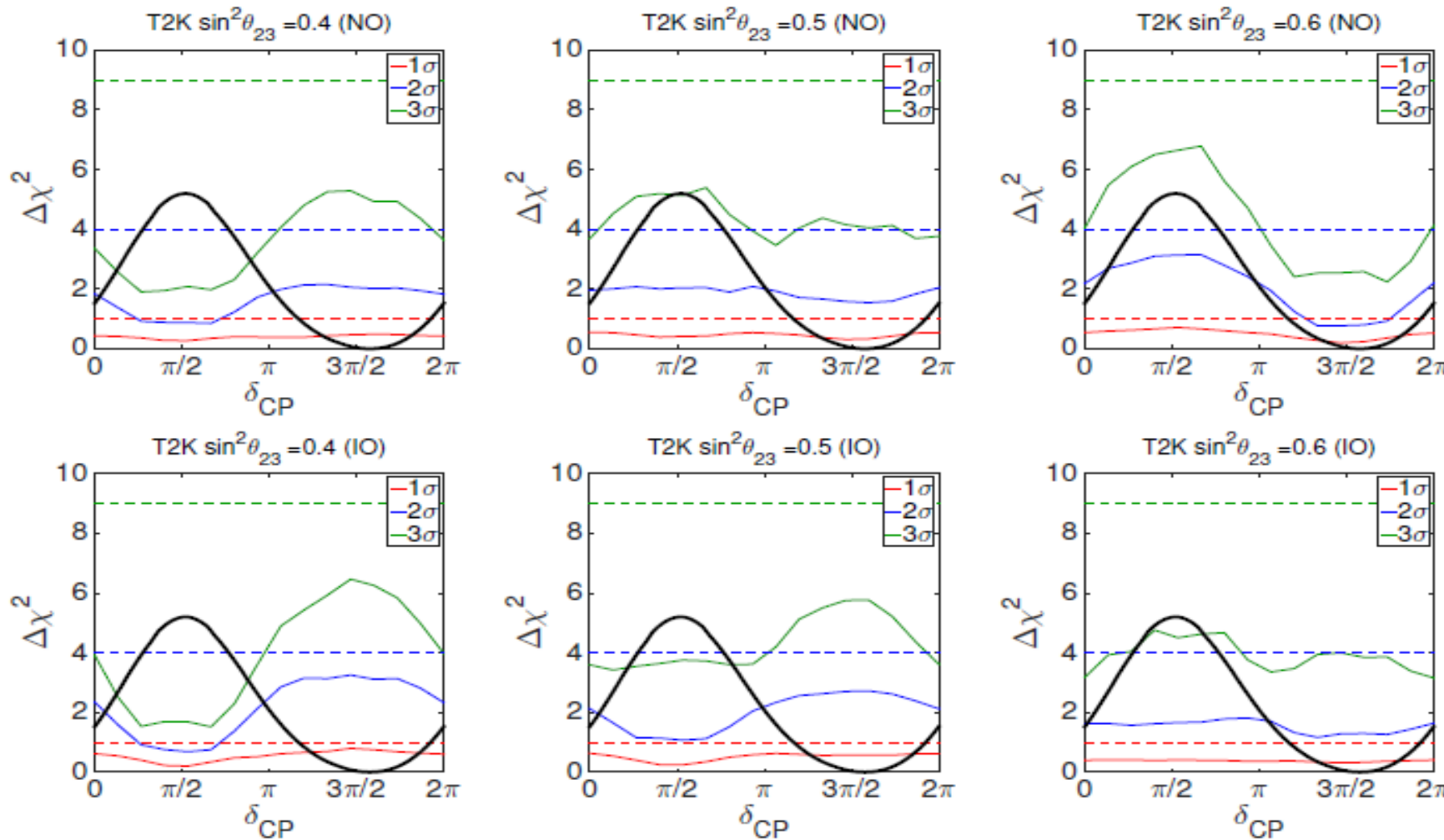


Note: $\Delta m^2 = (\Delta m^2_{31} + \Delta m^2_{32})/2$

Fractional 1σ error
(1/6 of $\pm 3\sigma$ range):

δm^2	2.6 %	
Δm^2	2.6 %	→ 2.2 %
$\sin^2 \theta_{12}$	5.4 %	
$\sin^2 \theta_{13}$	5.8 %	
$\sin^2 \theta_{23}$	~ 10 %	→ ~ 9 %

δ_{CP} sensitivity of T2K and Feldman-Cousin intervals



Wilkes theorem does not hold good for δ_{CP}

- Non-linear dependence of event rates on δ_{CP}
- Parameter degeneracy

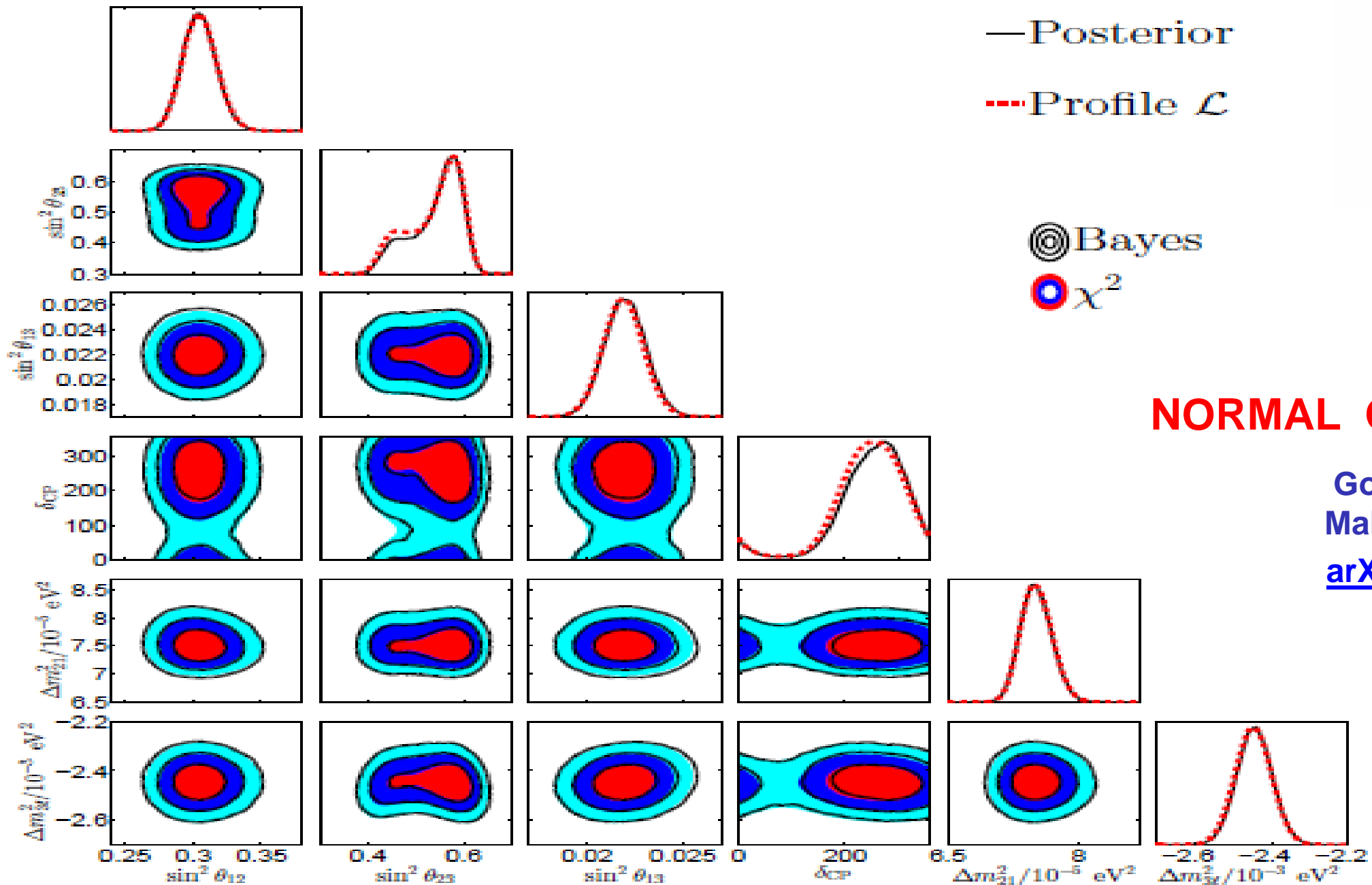
Standard $\Delta\chi^2$ confidence level definitions do not hold good

Monte Carlo simulation of data to determine the FC confidence intervals

- Large deviations from the Gaussian limit, depends on true hierarchy, θ_{23} , δ_{CP}
- $\delta_{CP} \approx \pi/2$ disfavoured at 2-3σ
- Stronger rejection of $\delta_{CP} \approx \pi/2$ as compared to Gaussian approximation

Elevant
Schwetz
1506.07685

Bayesian vs Frequentist



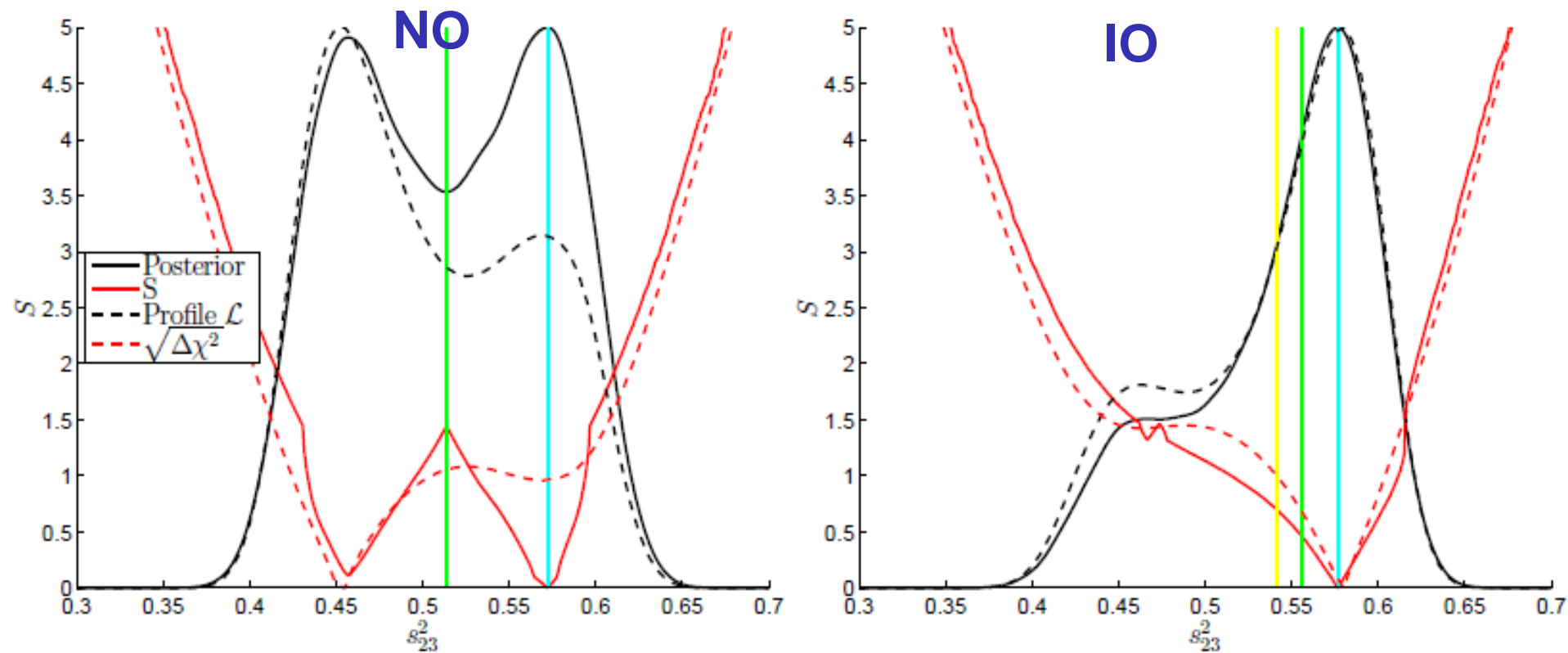
Results of Bayesian Analysis

Normal Ordering									
	Point Estimates				χ^2 Intervals		Bayes Credible Intervals		
	bfp	max of $\mathcal{L}_{\text{marg}}$	mean	median	1σ CI	3σ CI	1σ CI	3σ CI	
$\sin^2 \theta_{12}$	0.304	0.304	0.305	0.305	[0.292,0.317]	[0.270,0.344]	[0.292,0.317]	[0.269, 0.344]	
$\sin^2 \theta_{13}$	0.0218	0.0218	0.0218	0.0218	[0.0208,0.0228]	[0.0186,0.0250]	[0.0207,0.0228]	[0.0187,0.0250]	
$\frac{\Delta m_{21}^2}{10^{-5} \text{eV}^2}$	7.5	7.5	7.5	7.5	[7.33,7.69]	[7.02,8.07]	[7.33,7.69]	[7.03,8.09]	
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{eV}^2}$	2.457	2.460	2.459	2.459	[2.417,2.504]	[2.317,2.607]	[2.414,2.506]	[2.320,2.601]	
Inverted Ordering									
$\sin^2 \theta_{12}$	0.304	0.305	0.305	0.305	[0.292,0.317]	[0.270,0.344]	[0.292,0.317]	[0.269,0.344]	
$\sin^2 \theta_{13}$	0.0219	0.0219	0.0220	0.0220	[0.0209,0.0230]	[0.0188,0.0251]	[0.0209,0.0231]	[0.0189,0.0252]	
$\frac{\Delta m_{21}^2}{10^{-5} \text{eV}^2}$	7.5	7.5	7.5	7.5	[7.33,7.69]	[7.02,8.07]	[7.33,7.68]	[7.02,8.09]	
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{eV}^2}$	- 2.449 -	- 2.445 -	- 2.445 -	- 2.445	[-2.496,-2.401]	[-2.590,-2.307]	[-2.492,-2.400]	[-2.584,-2.308]	

Determination of four parameters robust against statistical analysis

Gonzalez-Garcia
Maltoni Schwetz
[arXiv:1507.04366](https://arxiv.org/abs/1507.04366)

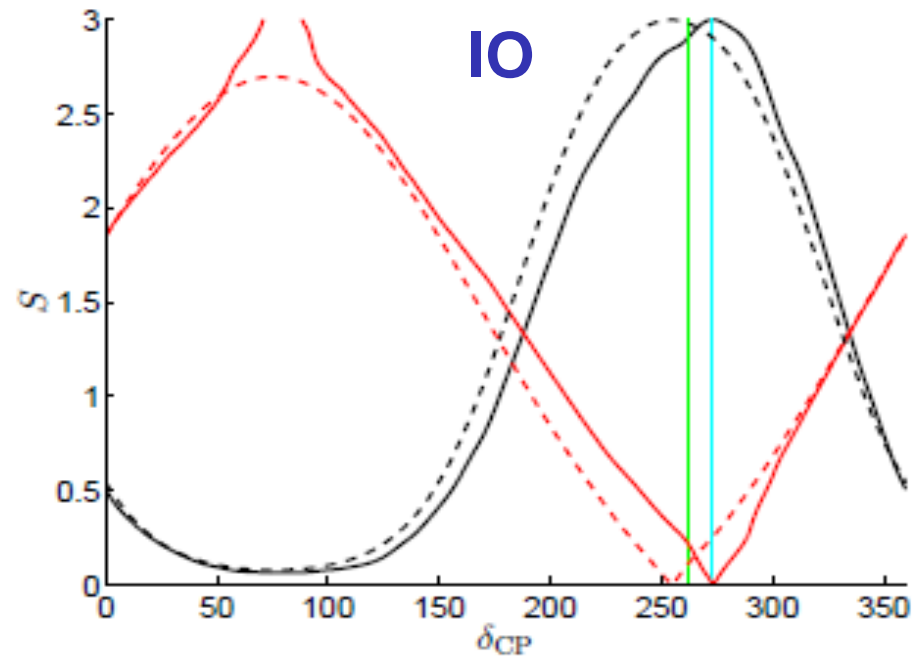
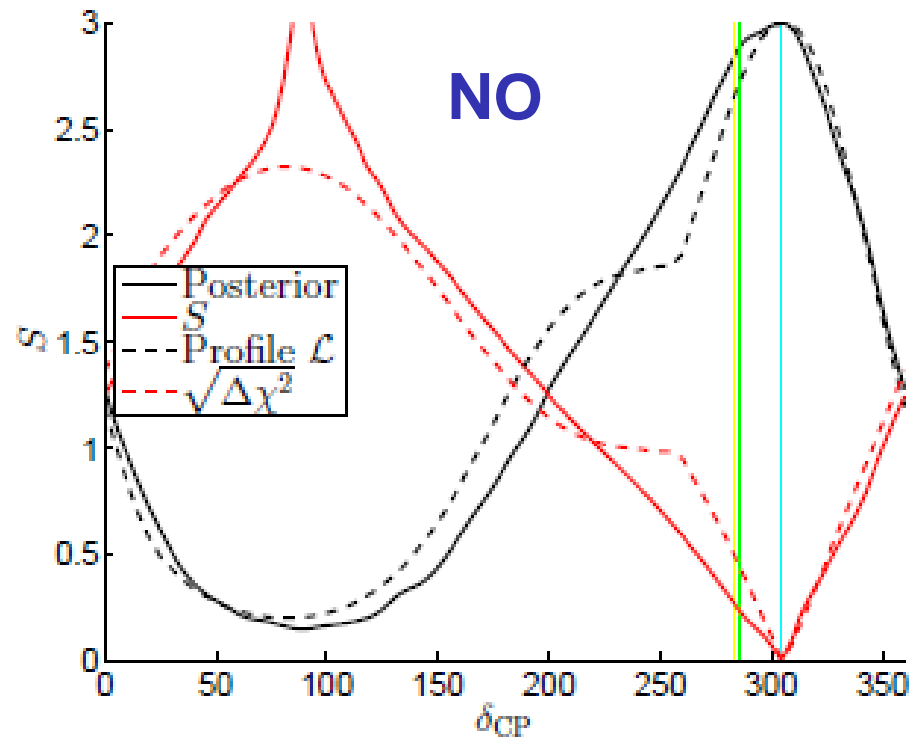
Bayesian Analysis of θ_{23}



Bayesian analysis prefers the second octant more for NO

Gonzalez-Garcia
Maltoni Schwetz
[arXiv:1507.04366](https://arxiv.org/abs/1507.04366)

Bayesian analysis and δ_{CP}



Marginal and Profile Likelihoods have maxima at the same value of δ_{CP}

Bayesian Analysis prefers larger δ_{CP}

Gonzalez-Garcia
Maltoni Schwetz
[arXiv:1507.04366](https://arxiv.org/abs/1507.04366)

Future Projections

Topics Not covered

Systematic Uncertainties : Review Talk by Pilar Coloma NNN15

Sterile Neutrinos : Review Talk by C. Giunti, NNN15

Future determination of hierarchy, octant and δ_{CP}

Current Generation Superbeam Experiments

T2K : Tokai to Kamioka, 295 km . 0.76 GeV , 0.75 MW , Detector: SuperK

NOvA : FNAL to Ash River , 810 km, 1.7 GeV, 0.7 MW, 14 kt T ASD detector

Next generation Superbeam experiments

T2HK: JPARC to Kamioka, detector: HyperK, 1.6 Mw

DUNE : FNAL-LEAD , 1300km, 0.7 MW, Detector 10 (34) kt LiqArTPC

ESS: European Spallation source Linac , configurations under study , 540 km, 2 GeV

Proton decay at rest experiments

DAE δ DALUS : low energy, low distance (50 MeV, 20 km)

Reactor Experiments

JUNO (China), RENO50 (Korea) , reactor neutrinos, 50 km

See talks by Z. Yu, J. Zhao, T. Kutter, NNN15

Future Atmospheric Neutrino Experiments

Magnetized iron detector

Talk by M. Naimuddin, NNN15

- Volume : 50 – 100 kton
- Excellent muon energy and direction reconstruction
- Charge identification
- Can determine the neutrino energy through hadron shower reconstruction
- Example : **INO**

Cerenkov Detectors Talks by H. Tanaka, T. Deyong , M. Jongen, Y. Highnight, Y. Nishimura NNN15

- Sensitive to both muon and electron events
- No charge id
- Mega ton Water detector (**HyperKamiokade, Memphys**)
- Multi Megaton ice detector (**PINGU of ICECUBE**)
- Multi Megaton under water detector (**ORCA**)

Liquid Argon TimeProjection Chambers Talks by S. Murphy, S. Dorota, NNN15

- Sensitive to both electrons and muons
- Excellent energy and direction reconstruction for both
- Charge id ?
- Example: **DUNE**

Degeneracy Menace

- Degeneracy \implies Different set of parameters giving same probability
 \implies Equally good fit to the data

θ_{13} unknown

Eightfold degeneracy : 

$$\begin{aligned}
 P_{\mu\mu}(\theta_{23}) &\simeq P_{\mu\mu}(\theta_{23} - \pi/2) \\
 P_{\mu e}(\theta_{13}, \delta_{CP}) &= P_{\mu e}(\theta'_{13}, \delta'_{CP}) \\
 P_{\mu e}(\Delta, \delta_{CP}) &= P_{\mu e}(\Delta', \delta'_{CP})
 \end{aligned}$$

θ_{13} Known

Hierarchy - δ_{CP} degeneracy

$$P_{\mu e}(\Delta, \delta_{CP}) = P_{\mu e}(-\Delta, \delta'_{CP})$$

Intrinsic octant degeneracy

$$P_{\mu\mu}(\theta_{23}) = P_{\mu\mu}(\theta_{23} - \pi/2 - \theta_{23})$$

Octant - δ_{CP} degeneracy

$$P_{\mu e}(\theta_{23}, \delta_{CP}) = P_{\mu e}(\theta'_{23}, \delta'_{CP})$$

General picture of degeneracy : post

 θ_{13}

$$P_{\mu e}(\theta_{23}, \theta_{13}, \delta_{CP}) = P_{\mu e}(\theta'_{23}, \theta'_{13}, \delta'_{CP}) \Rightarrow \text{generalized octant degeneracy.}$$

Coloma, Minakata, Parke, 1406.2551

$$P_{\mu e}(\theta_{23}, \Delta, \delta_{CP}) = P_{\mu e}(\theta'_{23}, -\Delta', \delta'_{CP}) \Rightarrow \text{generalized (hierarchy - } \theta_{23} - \delta_{CP} \text{) degeneracy.}$$

Ghosh, Ghoshal, Goswami, Nath, Raut, 2015

Solution with (right δ_{CP})	Solution with wrong δ_{CP})
I. RH-RO- $R\delta_{CP}$	V. WH-WO- $W\delta_{CP}$
x II. RH-WO- $R\delta_{CP}$	VI. RH-RO- $W\delta_{CP}$
III. WH-RO- $R\delta_{CP}$	VII. RH-WO- $W\delta_{CP}$
IV. WH-WO- $R\delta_{CP}$	x VIII. WH-RO- $W\delta_{CP}$

Eight possibilities
(different from the earlier eightfold degeneracy)

In literature mainly **discussed III, IV, VII**

V is often **part of IV** because of poor CP precision

VI intrinsic CP degeneracy also considered by **Minakata and Parke** arXiv 1303.6178

The diagnostics

Hierarchy Sensitivity

$$\Delta\chi^2 = \chi^2(NH) - \chi^2(IH)$$

Simulated Exptl
data (true)

Theory marginalized
over relevant
parameters (test)

Octant Sensitivity

$$\Delta\chi^2 = \chi^2(LO) - \chi^2(HO)$$

CP discovery

$$\chi^2 = \min \frac{(N_{ex}(\delta_{CP}^{tr}) - N_{th}(\delta_{CP}^{test}))^2}{N_{ex}(\delta_{CP}^{tr})}$$

$$\delta_{CP}^{test} = 0, \pi$$

$$\chi^2 = \chi_\nu^2 + \chi_{\bar{\nu}}^2$$

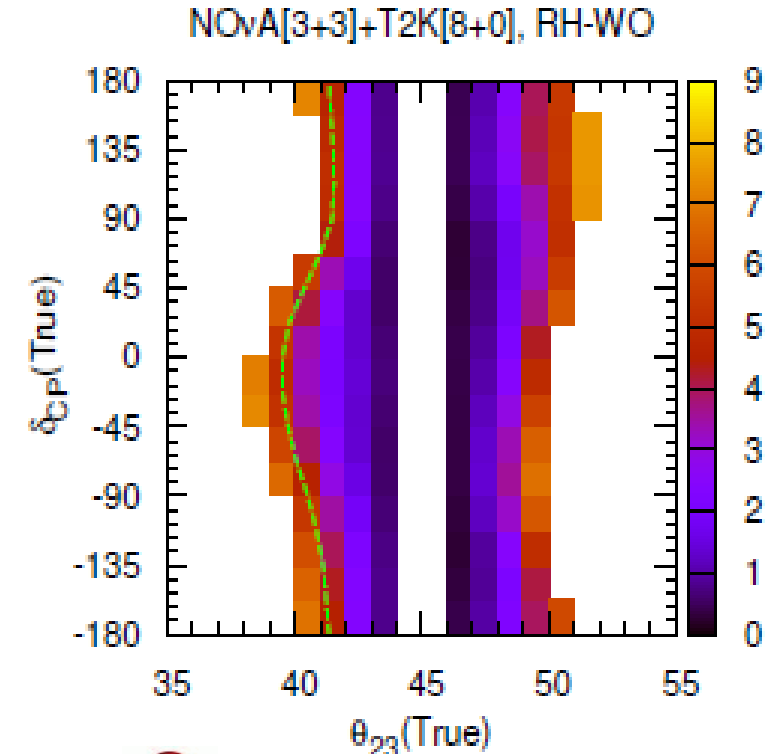
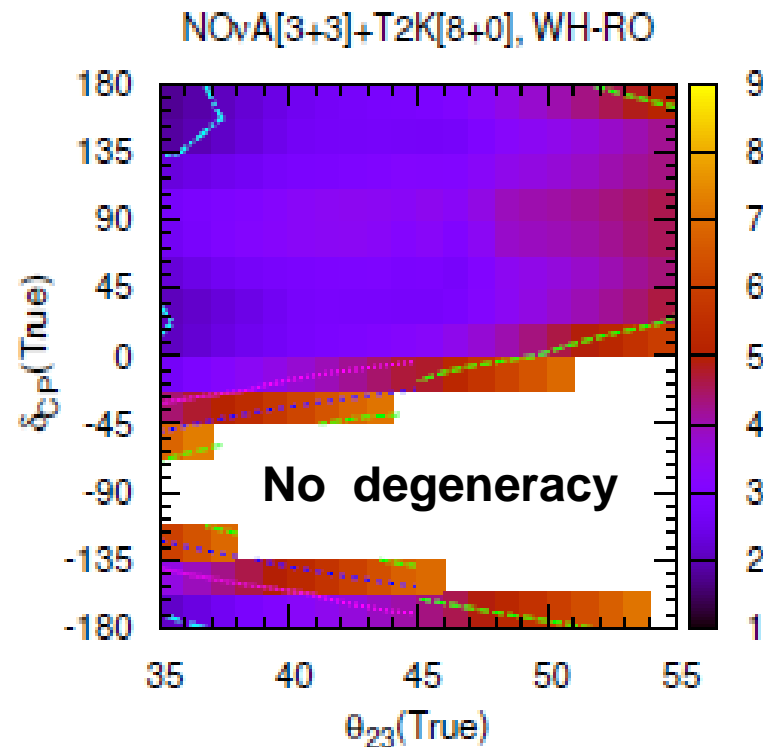
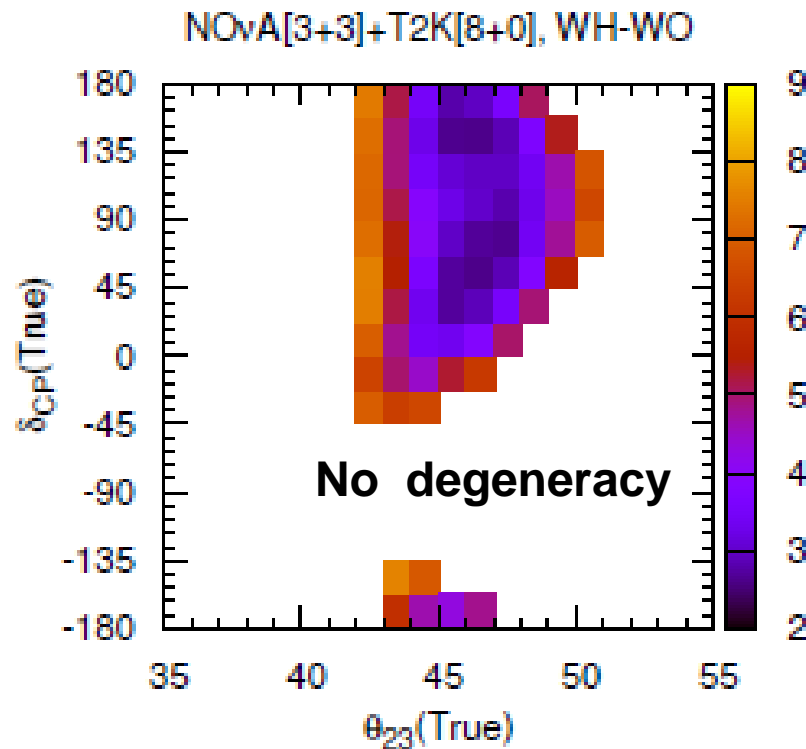
Degeneracies with T2K and NOVA

LHP

$$-180^\circ \leq \delta_{CP} \leq 0^\circ$$

Contours in true $\theta_{23}-\delta_{CP}$ plane (NH)

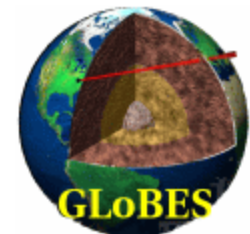
UHP $0^\circ \leq \delta_{CP} \leq 180^\circ$



- WH- WO solutions near $\theta_{23}=45^\circ$ and δ_{CP} in UHP.

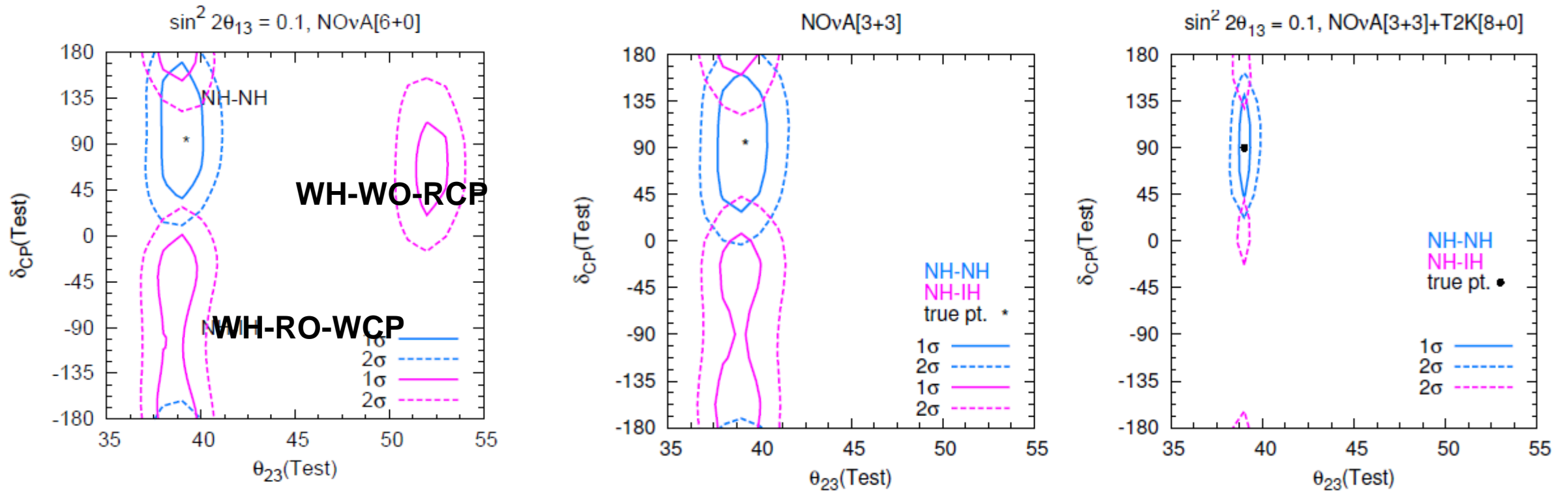
- WH solutions for δ_{CP} in UHP
- WH solution resolved at 3σ for all θ_{23} for $\delta_{CP}=-90^\circ$
- More CP coverage for higher θ_{23}

- WO solutions near $\theta_{23}=45^\circ$ for the whole range of δ_{CP}



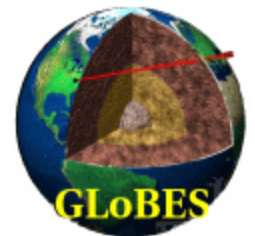
Removal of Degeneracy: Global Picture

Contours in test $\theta_{23} - \delta_{CP}$ plane (NH)



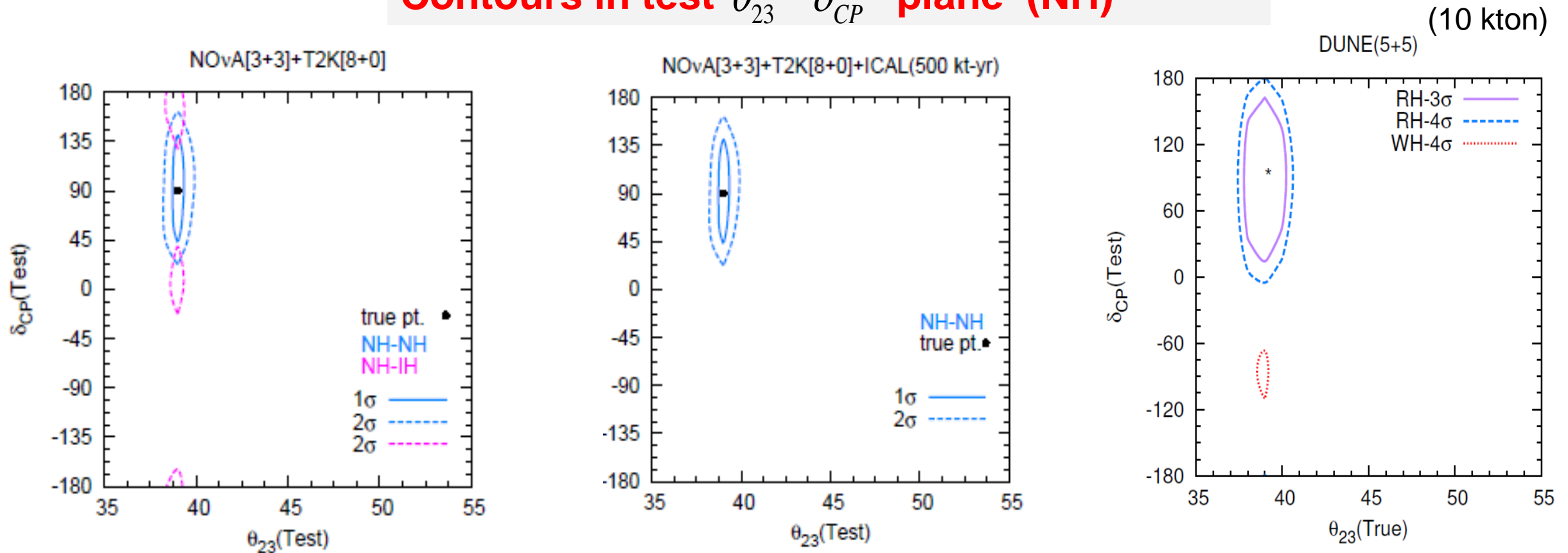
WO-WCP solution is removed by combining neutrinos and antineutrinos
(since the neutrinos and antineutrinos have degeneracy for opposite octants)

WH solution is removed by adding T2K and NOVA
(different dependence on δ_{CP} due to different baselines)



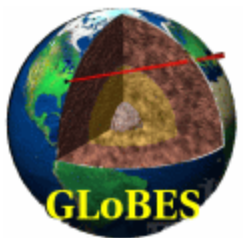
Removal of degeneracy : Global Picture

Contours in test $\theta_{23} - \delta_{CP}$ plane (NH)

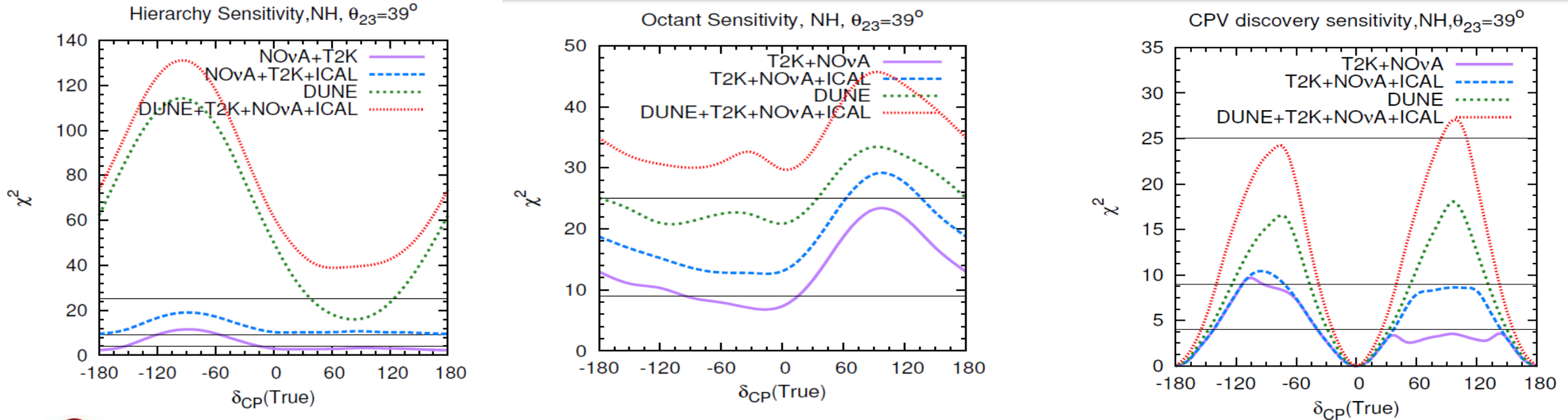


WH –WCP solution removed by adding atmospheric data from ICAL
 (matter effects break hierarchy degeneracies) (other atmospheric experiments/reactors)

Dune can remove WH and WO solutions at almost 4 σ
 (larger baseline and larger matter effects, pure ν_{μ} beam, known direction)



Global picture for hierarchy, octant and δ_{CP} (future)



- Addition of ICAL to T2K+NOVA increases the sensitivity in the unfavourable zones
- ICAL can enhance the CP sensitivity of T2K+NOVA by removing the WH solutions
- DUNE alone can reach more than 5σ sensitivity for hierarchy and octant for favourable δ_{CP} values and close to 4σ sensitivity for δ_{CP} in the whole range
- With addition of T2K+NOVA+ICAL to DUNE hierarchy and octant can reach more than 5σ and δ_{CP} close to 5σ over the full range

The Global Picture

- The parameters $\Delta m_{21}^2, |\Delta m_{31}^2|, \theta_{12}, \theta_{13}$ are **well determined** and stable across different statistical analysis
- With discovery of **non-zero θ_{13}** the three generation effects are beginning to be seen but not statistically significant yet
- The major undetermined quantities are **hierarchy, octant** and δ_{CP}
- The determination of these are interrelated and suffer from **degeneracies**
- For **favourable** values of parameters $2-3\sigma$ hint can come from T2K and NOVA
- In the **unfavourable zone improvement** possible exploiting **atmospheric neutrinos**
- DUNE can reach $4-5\sigma$ sensitivity
- Synergy between **different channels and experiments** plays an important role



Acknowledgements

- Monojit Ghosh, Newton Nath , Sushant Raut
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- Thomas Schwetz and Michele Maltoni for correspondence