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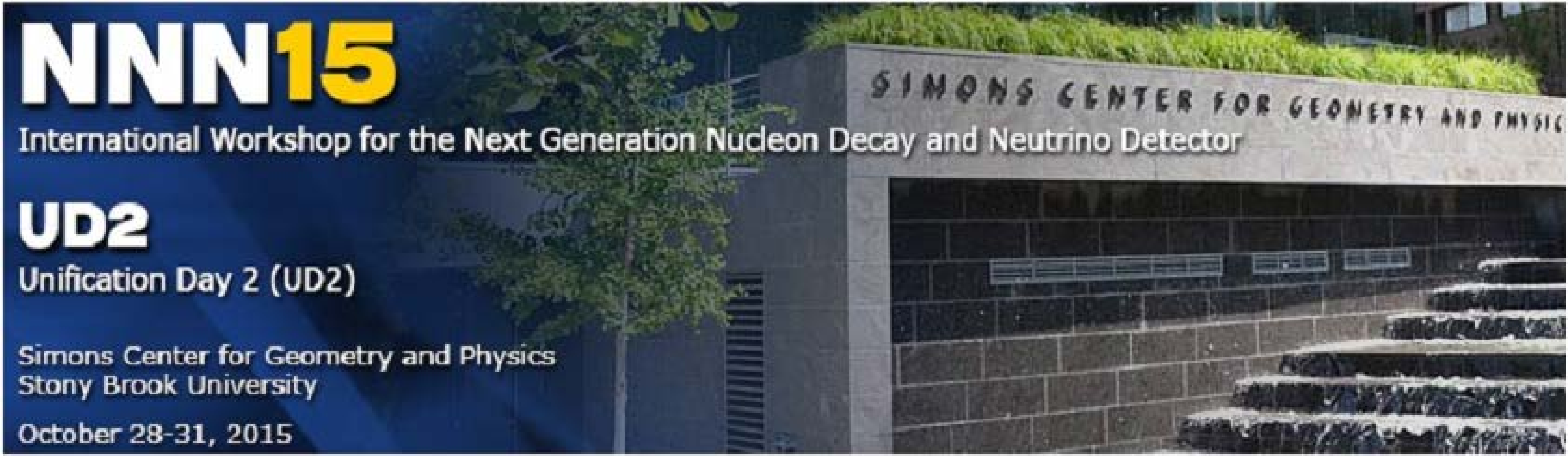
Aspects of a Realistic SUSY SO(10)

with K.S. Babu and J.C. Pati
JHEP 1006 (2010) 084
1511.???? [hep-ph]



Outline

- Some motivations for SUSY GUTs & Puzzles
- Realistic SUSY SO(10) Model →

  - Simple GUT breaking, -TD splitting
  - Calculable thresholds, improving $\alpha_3(M_Z)$
  - Correlation between d=5 & d=6 decays
  - Consistent & well defined fermion sector →
  - Upper bounds on lifetimes

- Summary

SUSY GUT →
udress questions & puzzles of SM/MSSM

- **Charge Quantization,Unification of multiplets** $\subset 16\, of\ SO(10)$

In $SO(10)$: $(q,\ u^c,\ e^c,\ d^c,\ l, \nu_R) = \mathbf{16}$

$\rightarrow$ Charge Quantization - $\frac{Y(q)}{Y(u^c)} = -\frac{1}{4},\ \frac{Y(q)}{Y(e^c)} = \frac{1}{6}, \cdots$

And interesting asymptotic relations [in SO(10)]:

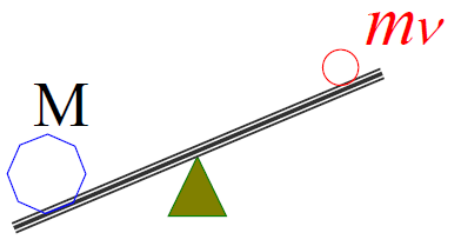
$$\lambda_t = \lambda_b = \lambda_\tau \ ,\ m_{\nu D} = m_t \ \ldots$$

- **Neutrino Masses : ν_R of SO(10) → see-saw**

→Neutrino masses, Oscillations

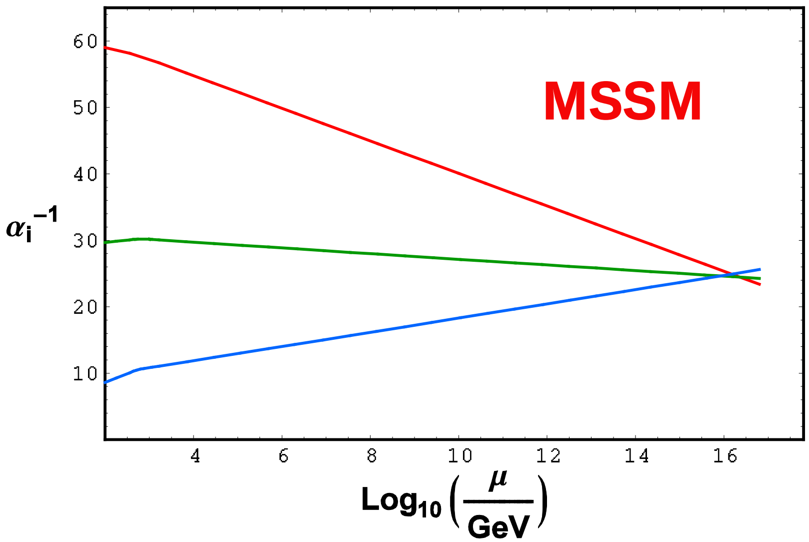
→ L-violation → leptogenesis

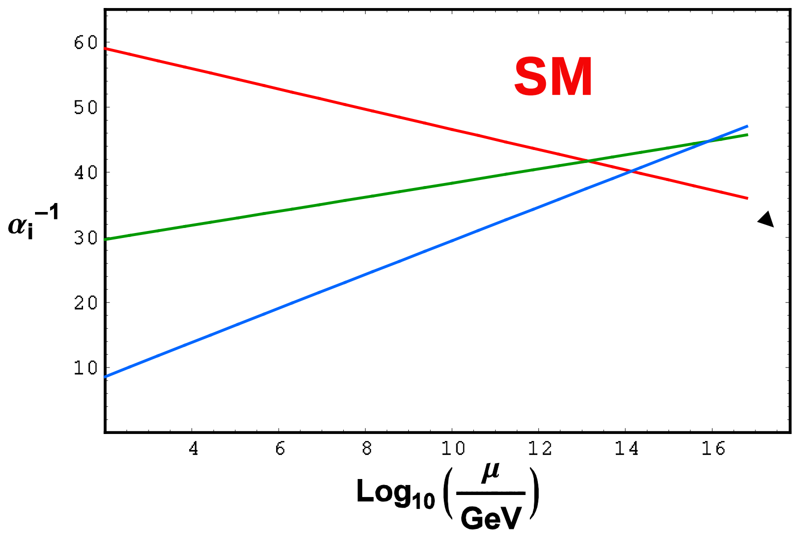
$$m_\nu \sim \frac{\langle H \rangle^2}{M_R}$$



SUSY GUT →

- Successful Coupling Unification





- Stab. Hierarchy (Light Higgs) ← low SUSY scale
- Dark Matter Candidate LSP (with R-parity)

GUT → Baryon Asymmetry

- Baryogenesis – GUT baryogenesis

K.S. Babu, R.N. Mohapatra
PRL 109 (2012) 091803
PR D86 (2012) 035018

via (see-saw) Leptogenesis

M. Fukugita, T. Yanagida
PLB 174 (1986) 45

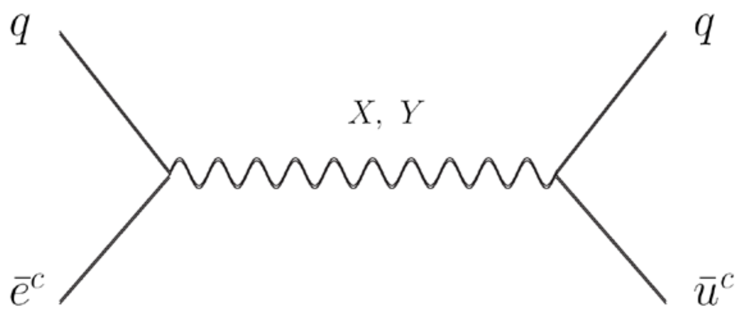
- Prediction: B-violation → proton decay

GUT Main Prediction:

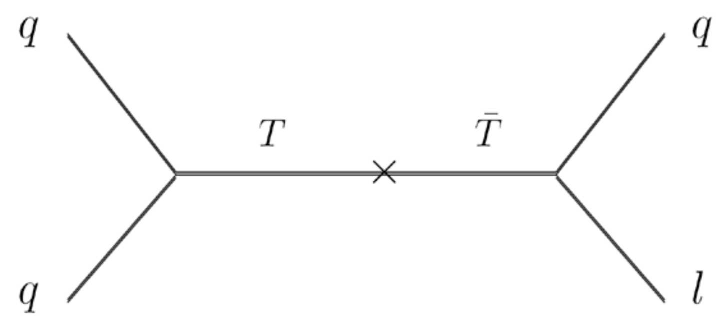
Baryon Number Violation: $\Delta B \neq 0$ → Proton Decay

- **Gauge Mediated d=6 Decay**

$$X, Y \subset \text{GUT/SM}$$



- **In SUSY: new d=5 Decays**



$$T, \bar{T} \subset \text{"Unified Higgses"}:$$

$$H(5) = (h_u, T), \; \bar{H}(\bar{5}) = (h_d, \bar{T})$$

Too Fast in SU(5)

SUSY GUT puzzles:

- **GUT Symmetry Breaking? (flat directions/goldstones)**
- **Doublet-Triplet Splitting**
- **How/why mu-Term ~ 100 GeV ? (harder in GUT)**
- **Proton Stability (especially d=5 decay)**
- **Fermion Masses & Mixings (flavor problem)**
- **SUSY FCNC (sflavor problem)**
- **Consistency of the Coupling Unification**
  - **Calculability of GUT Threshold Corrections**
  - **Perturbativity all the way up to M_{Planck}**

All these issues are closely related and

Unless *Unified* solution is found,
none of the predictions can be trusted..

Realistic SUSY SO(10)

SO(10) ->
Solution of DT hierarchy via missing VEV

Dimopoulos, Wilczek'81
Babu, Barr'93
Barr, Raby'97

Missing VEV Solution:

$$\langle A \rangle = \mathrm{i}\sigma_2 \otimes \mathrm{Diag}\,(a,\ a,\ a,\ 0,\ 0)$$

→ Massless Higgs doublets for unbroken SUSY

$$\langle A(45) \rangle \sim \begin{pmatrix} M_{\rm GUT} & & & & \\ & M_{\rm GUT} & & \text{O} & \\ & & M_{\rm GUT} & & \\ & \text{O} & & m_{\rm susy} & \\ & & & & m_{\rm susy} \end{pmatrix}$$

μ-Term Generation after SUSY breaking

$$H A H' \quad \Longrightarrow \quad \mu \sim m_{\rm susy}$$

Model: Realistic SUSY SO(10)

- \`Higgs' System: $H(10) \supset h_u + h_d + T_H + \bar{T}_H \longleftrightarrow 16_3 16_3 H$

  and H'(10) for HAH' coupling

$A(45)+C(16)+\bar{C}(16^*)$ for $SO(10) \xrightarrow{BR} SU(3)\times SU(2)\times U(1)$

additional plets: $C'(16)+\bar{C}'(16^*)$ and $Z(1)+S(1)$

(like by: Barr, Raby'97)

Insuring desirable sym. br. & NO flat dir. / pseudo-Golds.

- Additional Symmetries: $U(1)_A \times Z_2$

Symmetry Breaking, All order DT splitting,
mu –term, Nucleon stability (predictions)
Realistic & simple fermion pattern

Some More Studies of Missing VEV Mechanism

Several 45's →
Large GUT thresholds

Babu, Barr '93
Chacko, Mohapatra '99

Economical Higgs system
With 45+16+16*+10,10'
→ Small GUT thresholds

Babu, Pati, Wilczek '99
Barr, Raby '97

Anomalous U(1)→all order
Hierarchy, But Several 45's
→ Large GUT thresholds

Berezhiani, Tavartkiladze '97
Maekawa '01
Maekawa, Yamashita '02

Other possibility
(interesting, but not widely discussed):
Missing Partner mechanism in SO(10) GUT

Babu, Gogoladze, Tavartkiladze, PLB **650 (2007) 49;**

Babu, Gogoladze, Nath, Syed, **PRD85 (2012) 075002**

Interesting, but requires high representations.
→ Large GUT thresholds...

(not done complete analysis yet..)

$U(1)_A \times Z_2$ Transformations:

	$A(45)$	$H(10)$	$H'(10)$	$C(16)$	$\bar{C}(\overline{16})$	Z	S	$C'(16)$	$\bar{C}'(\overline{16})$	$16_{1,2}$	16_3
$\mathcal{U}(1)$	0	1	-1	$\frac{k+4}{2k}$	$-\frac{1}{2}$	$\frac{2}{k}$	$\frac{2}{k}$	$\frac{k-4}{2k}$	$-\frac{k+8}{2k}$	-1	$-\frac{1}{2}$
Z_2	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	$P_{1,2}$	$+$

`Scalar' Superpotential (fixed):

$$W(A) = M_A \mathrm{tr} A^2 + \frac{\lambda_A}{M_*}\left(\mathrm{tr} A^2\right)^2 + \frac{\lambda'_A}{M_*}\mathrm{tr} A^4 \,,$$

$$W(A,C,C') = C\left(\frac{a_1}{M_*} Z A + \frac{b_1}{M_*} C\bar{C} + c_1 S\right)\bar{C}' + C'\left(\frac{a_2}{M_*} Z A + \frac{b_2}{M_*} C\bar{C} + c_2 S\right)\bar{C}$$

$$W(DT) = \lambda_1 H A H' + \lambda_{H'} \frac{(S^k, Z^k)}{M_*^{k-1}} (H')^2 + \lambda_2 H \bar{C}\bar{C} + \frac{\lambda_3}{M_*} A H' C C' \,.$$

Missing VEV Solution: $\langle A \rangle = \mathrm{i}\sigma_2 \otimes \mathrm{Diag}\,(a,\ a,\ a,\ 0,\ 0)$

(incl. FI-term) Fixed VEVs: $\langle A \rangle, \langle C \rangle, \langle \bar{C} \rangle, \langle Z \rangle, \langle S \rangle \neq 0$

Symmetry breaking and VEVs:

Anom. D-term: $V_D = g^2{}_A(\xi + \Sigma q_\phi |\phi|^2)^2$

$$c^2 + |z|^2 + |s|^2 = -\frac{k}{2}\xi$$

At least one VEV is fixed. This trigger all remaining VEVs:

F=0 directions: $-\frac{3a_1}{M_*}za + \frac{b_1}{M_*}c^2 + c_1 s = 0 \ , \quad -\frac{3a_2}{M_*}za + \frac{b_2}{M_*}c^2 + c_2 s = 0$

$$\swarrow$$

$$c, \ \ s, \ z \neq 0$$

No flat directions, no pseudo-goldstones
Only Light MSSM states including doublets are massless

DT Splitting to All Orders

$$M_{D,T} = \begin{array}{c} \\ \bar{5}_H \\ \bar{5}_{H'} \\ \bar{5}_C \\ \bar{5}_{C'} \end{array} \begin{array}{c} \begin{array}{cccc} 5_H & 5_{H'} & 5_{\bar{C}} & 5_{\bar{C}'} \end{array} \\ \left(\begin{array}{cccc} 0 & \eta_{D,T}\lambda_1 a & \lambda_2 c & 0 \\ -\eta_{D,T}\lambda_1 a & M_{H'} & 0 & 0 \\ 0 & 0 & 0 & \kappa_{D,T} Y_1 \\ 0 & Y_{D,T} & \kappa_{D,T} Y_2 & M_{C'} \end{array} \right) \end{array}$$

with $\eta_D = 0\ ,\quad \eta_T = 1\ ,\quad \kappa_D = 3\ ,\quad \kappa_T = 2\ .$

$\Rightarrow$ massless doublet pair M(hu, hd)=0,

and *all* M(triplets)-heavy

In SUSY limit $\mu - \text{term} = 0$

After SUSY breaking $\mu \sim m_{\text{susy}}$

μ-Term Generation after SUSY breaking

Including soft A & B-terms, VEVs are shifted by $\sim m_{susy}$

$$\langle A(45)\rangle \sim \begin{pmatrix} M_{\rm GUT} & & & & \\ & M_{\rm GUT} & & & \\ & & M_{\rm GUT} & & \\ & & & m_{\rm susy} & \\ & & & & m_{\rm susy} \end{pmatrix}$$

$$H\,A\,H' \quad \Longrightarrow \quad \mu \sim m_{\rm susy}$$

This can always happen with
linear terms ($\sim m_{\rm susy}$) in potential

Dvali, Lazarides, Shafi PLB 424, 259;
Babu, Dutta, Mohapatra PRD65, 016005*; Kitano, Okada ph/0107084*
Hall, Nomura, Pierce PLB 538, 359

coupling SH_uH_d **with** $< S > \sim m_{susy}$ $\rightarrow \mu \sim m_{susy}$

GUT Spectrum (all heavy)

Well defined spectrum:

5-plets:

$$M_{D,T} = \begin{array}{c} \\ \bar{5}_H \\ \bar{5}_{H'} \\ \bar{5}_C \\ \bar{5}_{C'} \end{array} \begin{array}{c} \begin{array}{cccc} 5_H & 5_{H'} & 5_{\bar{C}} & 5_{\bar{C}'} \end{array} \\ \begin{pmatrix} 0 & \eta_{D,T}\lambda_1 a & \lambda_2 c & 0 \\ -\eta_{D,T}\lambda_1 a & M_{H'} & 0 & 0 \\ 0 & 0 & 0 & \kappa_{D,T} Y_1 \\ 0 & Y_{D,T} & \kappa_{D,T} Y_2 & M_{C'} \end{pmatrix} \end{array}$$

$$\text{with} \qquad \eta_D = 0\ , \quad \eta_T = 1\ , \quad \kappa_D = 3\ , \quad \kappa_T = 2\ .$$

$$\frac{M_{D_1} M_{D_2} M_{D_3}}{M_{T_1} M_{T_2} M_{T_3} M_{T_4}} = \frac{9}{4 M_{\text{eff}} \cos\gamma}\ , \qquad \text{with} \qquad \frac{1}{M_{\text{eff}}} = (M_T^{-1})_{11} = \frac{M_{H'}}{\lambda_1^2 a^2}$$

10-plets:

$$M(\Psi^{10}) = \begin{array}{c} \\ \Psi^{10}_A \\ \Psi^{10}_C \\ \Psi^{10}_{C'} \end{array} \begin{array}{c} \begin{array}{ccc} \overline{\Psi}^{\overline{10}}_A & \overline{\Psi}^{\overline{10}}_{\bar{C}} & \overline{\Psi}^{\overline{10}}_{\bar{C}'} \end{array} \\ \begin{pmatrix} M_\Psi & 0 & X_1 \\ 0 & 0 & \kappa_\Psi Y_1 \\ X_2 & \kappa_\Psi Y_2 & M_{C'} \end{pmatrix} \end{array}$$

$$\text{with} \qquad \Psi = (u^c, q, e^c)\ , \quad \kappa_\Psi = (2, 1, 0)\ , \quad M_\Psi = (0, 0, M_\Sigma/2)$$

10's fragments masses:

Matter: $$\mathcal{U}_1^c\mathcal{U}_2^c = Y_1Y_2(4+\tilde{p}^2)\ ,\quad \mathcal{Q}_1\mathcal{Q}_2 = Y_1Y_2(1+\tilde{p}^2)\ ,\quad \mathcal{E}_1^c\mathcal{E}_2^c = Y_1Y_2\hat{p}^2$$

$$\tilde{p} = p \qquad \tilde{p}^2 = \frac{|X_1|^2}{|Y_1|^2} = \frac{|X_2|^2}{|Y_2|^2}\ ,\quad \hat{p}^2 = \tilde{p}^2\left|1 - \frac{M_\Sigma M_{C'}}{2X_1X_2}\right|$$

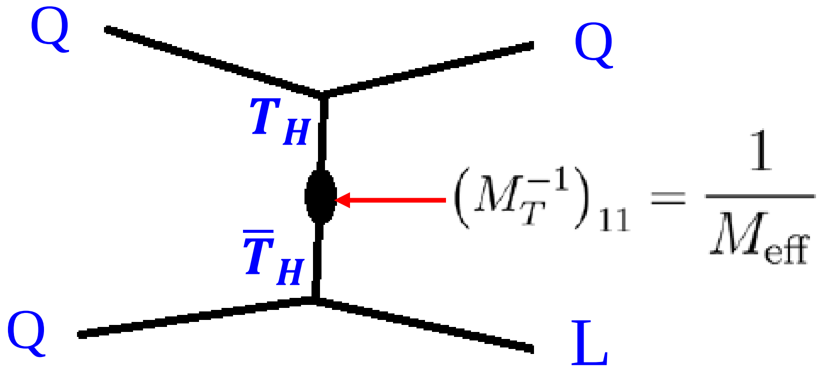
Gauge: $$M^2(X,Y) = g^2a^2 \equiv M_X^2\ ,\quad M^2(X',Y') = M_X^2(1+p^2)$$

$$M^2(V_{u^c,\bar{u}^c}) = M_X^2(4+p^2)\ ,\quad M^2(V_{e^c,\bar{e}^c}) = M_X^2 p^2 \qquad p^2 = \frac{4c^2}{a^2}$$

3-matter + 1 vector superfield ←→ N=4 SYM multiplet

Gauge β – function = 0 **←Great reduction of thresholds**

M_{eff}: In d=5 decay



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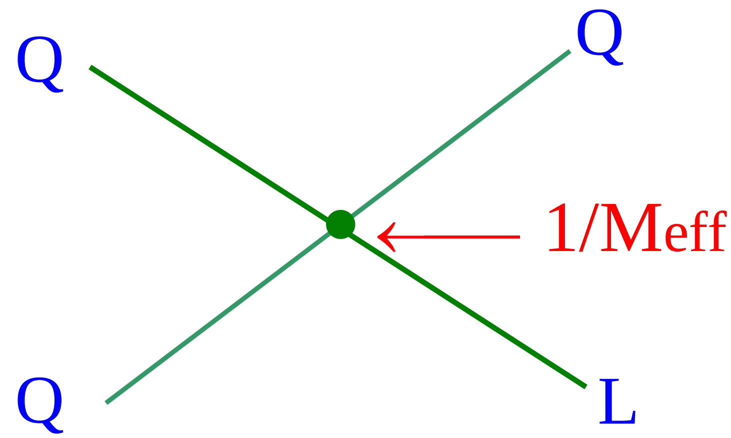


In RG Equations →

$$\frac{M_{D_1} M_{D_2} M_{D_3}}{M_{T_1} M_{T_2} M_{T_3} M_{T_4}} = \frac{9}{4 M_{\rm eff} \cos\gamma}$$

$\cos\gamma \sim \tan\beta/60$

$$\alpha_{\rm U}^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln\frac{\Lambda}{M_Z} + \Delta_{i,\,w}^{(2)} + \Delta_i^{\rm GUT}$$

Calculable Thresholds!

RG and Gauge Coupling Unification

$$\ln\frac{M_{\rm eff}\cos\gamma}{M_Z}=\frac{5\pi}{6}\left(3(\alpha_2^{-1}+\Delta^{(2)}_{2,\,w}-\frac{1}{6\pi})-2(\alpha_3^{-1}+\Delta^{(2)}_{3,\,w}-\frac{1}{4\pi})-(\alpha_1^{-1}+\Delta^{(2)}_{1,\,w})\right)-\ln\frac{4\kappa^{5/2}}{9}+\ln\frac{p}{\hat{p}}$$

$$\ln\frac{(M_X^2 M_\Sigma)^{1/3}}{M_Z}=\frac{\pi}{18}\left(5(\alpha_1^{-1}+\Delta^{(2)}_{1,\,w})-3(\alpha_2^{-1}+\Delta^{(2)}_{2,\,w}-\frac{1}{6\pi})-2(\alpha_3^{-1}+\Delta^{(2)}_{3,\,w}-\frac{1}{4\pi})\right)+\frac{1}{6}\ln\kappa-\frac{1}{3}\ln\frac{p}{\hat{p}}$$

$$\kappa\equiv M_8/M_3=2$$

SUSY spectrum → $\Delta^{(2)}_{i,\,w}$
Fermion sector → $\cos\gamma$

$$(\alpha_3(M_Z)^{-1})^{centr.}=1/0.1184$$

Can be obtained

By $\frac{\hat{p}}{p}\sim 10^{-4}$

$(M_X{}^2 M_\Sigma)^{1/3}\approx 10^{16}$ GeV

$M_{\rm eff}$ = few $\times 10^{19}$ GeV

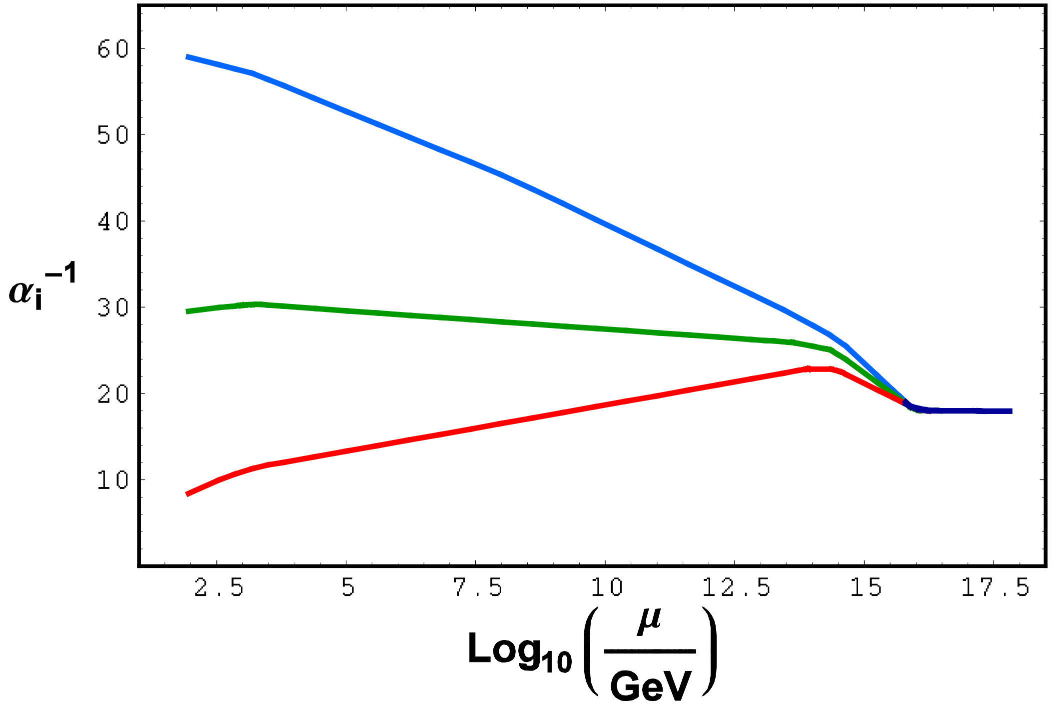
- **Eliminating $\frac{\hat{p}}{p}$ → correlation:** $M_{eff}\sim\frac{1}{{M_X}^3}$

[Compare with SU(5) - Hisano, Murayama, Yanagida'93]

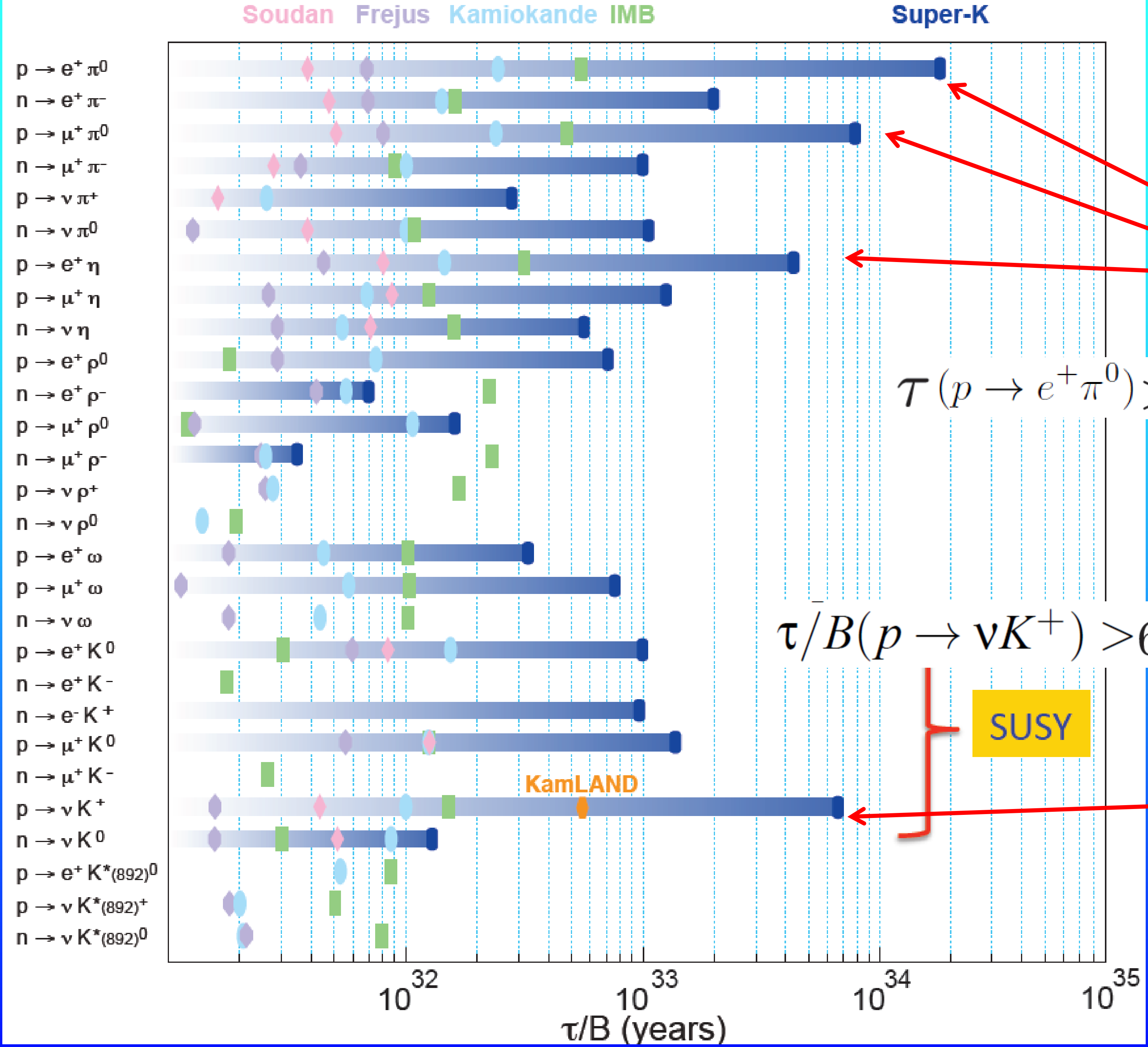
Large M_{eff} suppress Nucleon decay:

$$\Gamma^{-1}_{d=5}(p \to \bar{\nu} K^{+}) = 5.75 \cdot 10^{34} \text{yrs} \times$$
$$\times \left(\frac{0.012\text{GeV}^3}{|\beta_H|}\right)^2 \left(\frac{6.34}{\bar{A}^{\alpha}_{S}}\right)^2 \left(\frac{1.25}{R_L}\right)^2 \left(\frac{M_{\text{eff}}}{1.4 \times 10^{20}\text{GeV}}\right)^2 \left(\frac{835.7\text{GeV}}{m_{\tilde{W}}}\right)^2 .$$

And compatible with coupling unification..



Exp. limits on Nucleon lifetime [*from Kearns talk,* PDF2015]



In d=5 suppressed (e.g. by λu)

$$\tau\,(p \to e^+\pi^0) > 1.67 \times 10^{34}\ \mathrm{y}$$

$$\tau/B(p \to \nu K^+) > 6.61 \times 10^{33}\,\mathrm{years}$$

SUSY GUT's dominant d=5 mode

SUSY Spectrum

(Taking into account GUT thresholds)

Gauginos: $M_i = M_i^0 \frac{\alpha_G^0}{\alpha_G} \frac{m_{1/2}}{m_{1/2}^0}$

Squarks, sleptons: $m_{\tilde{f}}^2 = (m_{\tilde{f}}^0)^2 + \Delta m_{\tilde{f}}^2$

Higgses: $m_{h_{u,d}}^2 \simeq (m_{h_{u,d}}^0)^2 + \Delta m_{h_{u,d}}^2$

Changes due to GUT thresholds: e.g. $\Delta m_{\tilde{q}_i}^2 \simeq \frac{m_{1/2}^2}{4\pi} \left(12\alpha_X \ln \frac{M_G}{M_X} - C_{\tilde{q}}^i \Delta I_i \right) - \frac{\delta_{i3}}{4\pi^2} I_{\tilde{q}3}^{\lambda_t}$

Selecting Input: **Such that have**
rad. EWSB, Higgs mass=126 GeV
SUSY spectrum satisfy all LHC bounds

Several Examples with inverted squar/slepton masses:

(Inputs from Ref. **M. Badziak, E. Dudas, M. Olechowski, S. Pokorski, arXiv:1205.1675**

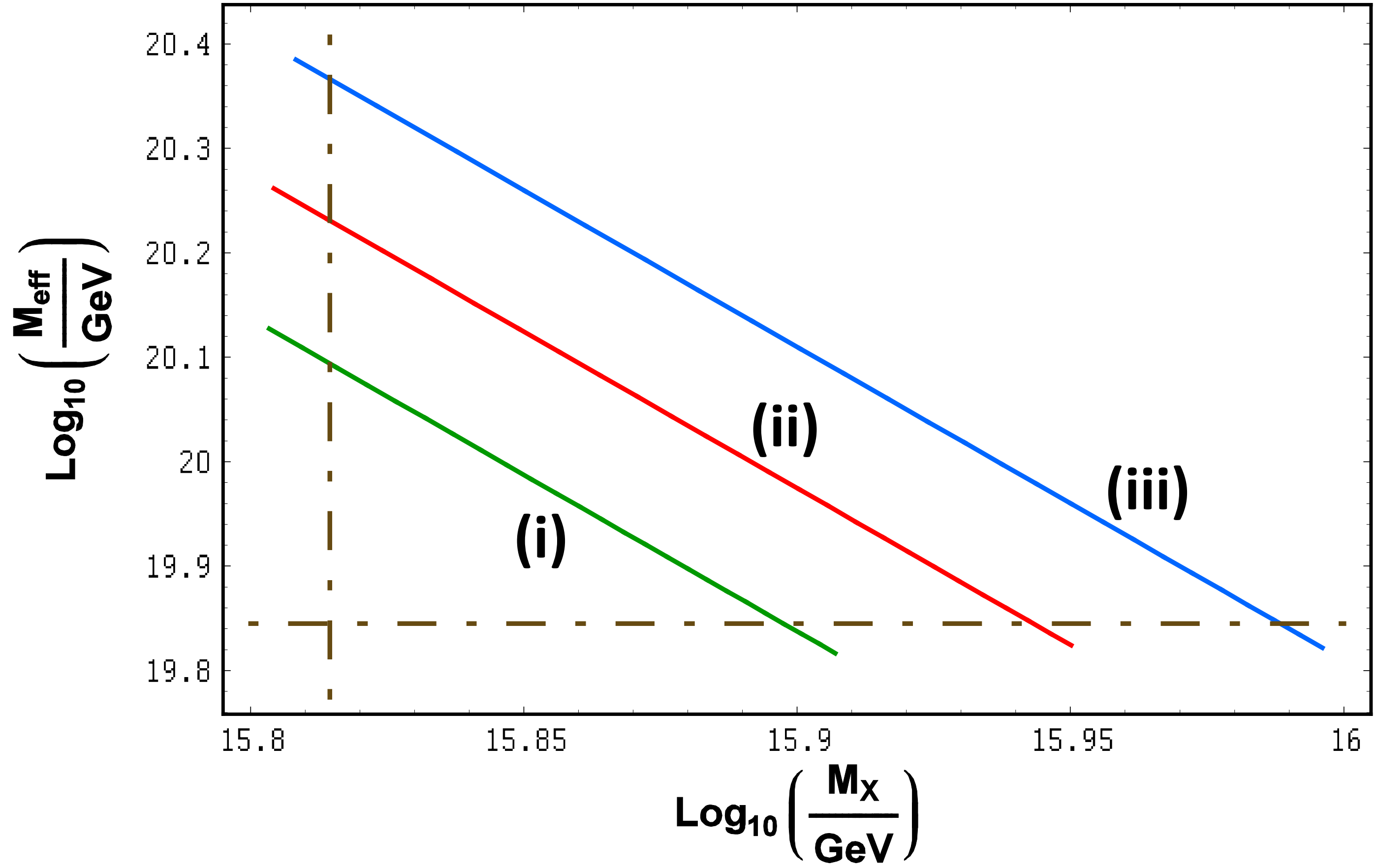
Our Output →

	Spectrum $A_{SO(10)}$	Spectrum $B_{SO(10)}$	Spectrum $C_{SO(10)}$
μ	1080.4	759.6	631.6
M_A	3648.3	3236.3	3569.3
A_t	-2298	-2392	-2393
$m_{\tilde{B}}$	457	652.7	685
$m_{\tilde{W}}$	835.73	1295.3	1301.25
$m_{\tilde{g}}$	2537.1	3561	3587.1
Neutralinos : $m(\tilde{\chi}_i^0) \simeq$	$(457-1081)$	$(653-1295)$	$(631-1301)$
Charginos : $m(\tilde{\chi}_i^\pm) \simeq$	$(836-1081)$	$(759-1295)$	$(631-1301)$
$m_{\tilde{t}_{1,2}}$	503.15, 1701	735.02, 1503.2	540.8, 1525
$m_{\tilde{q}_{1,2}}$	17682	21122	22532
$m_{\tilde{u}^c_{1,2}}$	17681	21125	22538
$m_{\tilde{b}_{1,2}}$	1847, 17688	1637.64, 21121	1696, 22534
$m_{\tilde{d}^c_{1,2}}$	17688	21121	22534
$m_{\tilde{l}_{1,2}}$	17686	21077	22504
$m_{\tilde{l}_3}$	3435	3115	3477
$m_{\tilde{e}^c_{1,2}}$	17693	21081	22510
$m_{\tilde{\tau}^c}$	3493.5	3227.6	3595.4
$m_h \simeq$	125	125	125

Correlation Between M_{eff} & M_X

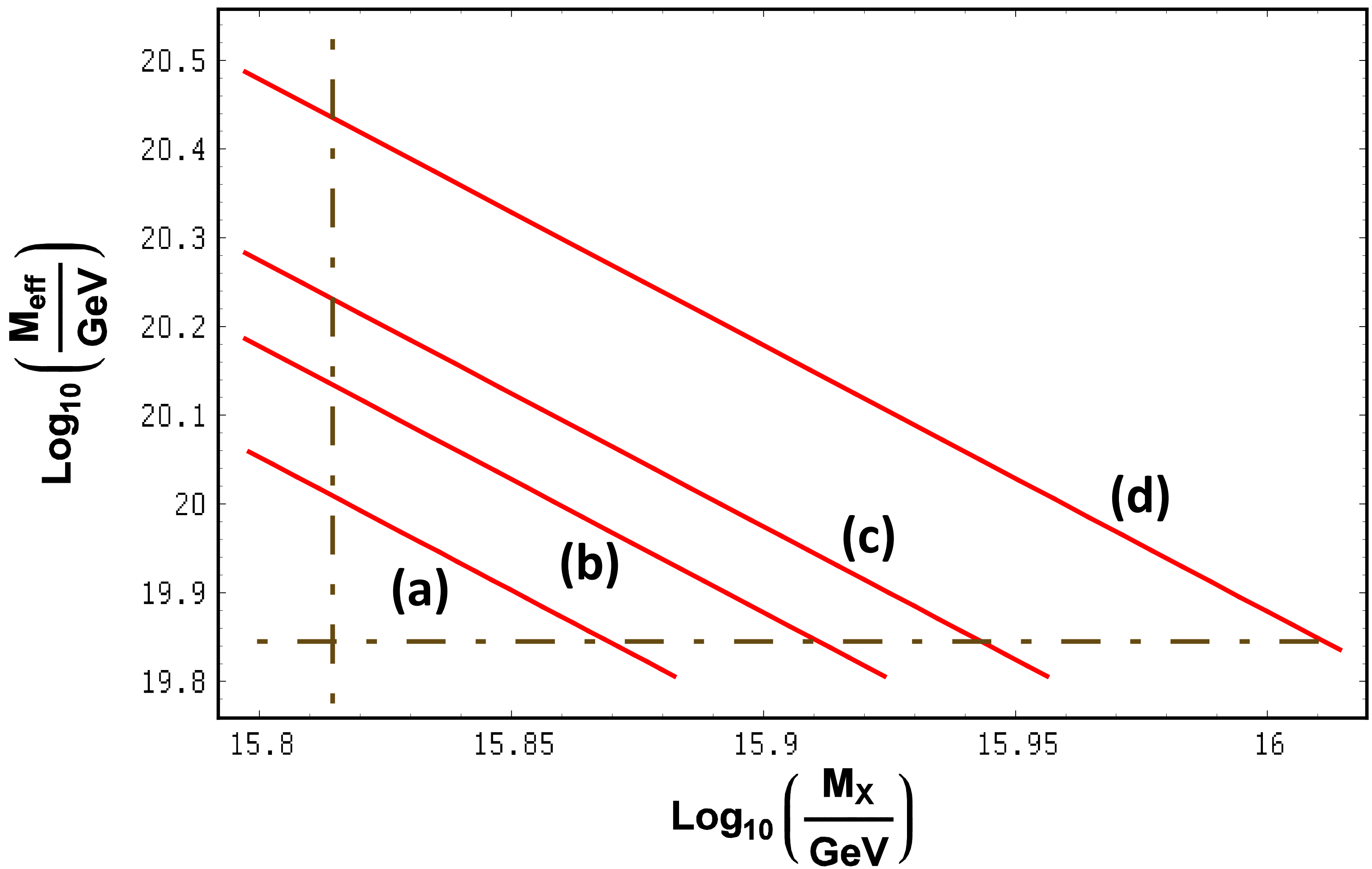
For spectrum $\mathrm{B}_{SO(10)}$

$$M_X r^{1/3} = (0.5644,\ 0.6269,\ 0.6957) \cdot 10^{15}\ \mathrm{GeV} \cdot \left(\frac{0.673 \times 10}{\eta_\gamma \tan\beta}\right)^{1/3} \left(\frac{4.5 \cdot 10^{19}\ \mathrm{GeV}}{M_{\mathrm{eff}}}\right)^{1/3}$$



Correlation for spectrum with $tan\beta$ =10 and r= $1/2500$

(i): $\alpha_3 = 0.1177$. **(ii)**: $\alpha_3 = 0.1184$. **(iii)**: $\alpha_3 = 0.1191$.



Correlation for spectrum with $tan\beta = 10$ and $\boldsymbol{\alpha}_3 = 0.1184$.

(a) r=1/1500. **(b)**: r=1/2000 **(c)** r=1/2500 **(d)** r=1/4000.

→Correlation Between d=5 & d=6 Proton Decays and Upper Bounds on Lifetimes

$$\Gamma^{-1}_{d=6}(p\to e^+\pi^0)\simeq 7.06\cdot 10^{34}\ \text{yrs}\left(\frac{0.012\text{GeV}^3}{|\alpha_H|}\right)^2\left(\frac{2.7}{A_R}\right)^2\left(\frac{5.12}{f(p)}\right)\left(\frac{1/20}{\alpha_X}\right)^2\left(\frac{M_X}{10^{16}\text{GeV}}\right)^4$$

$$\Gamma^{-1}_{d=5}(p\to \bar\nu K^+)=2.36\cdot 10^{34}\text{yrs}\left(\frac{0.012\text{GeV}^3}{|\beta_H|}\right)^2\left(\frac{6.33}{\bar A^\alpha_S}\right)^2\left(\frac{1.25}{R_L}\right)^2\left(\frac{M_{\text{eff}}}{1.4\cdot 10^{20}\text{GeV}}\right)^2\left(\frac{1295.3\text{GeV}}{m_{\tilde W}}\right)^2$$

$\mathrm{B}_{SO(10)}$

Naturalness suggest range: $\frac{1}{5000} < r\left(=\frac{M_\Sigma}{M_X}\right) < \frac{1}{1500}$

+exp. Bounds & correlation, for $r_{min}=\frac{1}{2500}$ **→**

Lead to: $$\Gamma^{-1}_{d=6}(p\to e^+\pi^0)\lesssim\begin{cases}1.66\cdot 10^{35}\ \text{yrs}, & \text{For}\ \mathrm{A}_{SO(10)}\\ 4.65\cdot 10^{34}\ \text{yrs}, & \text{For}\ \mathrm{B}_{SO(10)}\\ 4.76\cdot 10^{34}\ \text{yrs}, & \text{For}\ \mathrm{C}_{SO(10)}\end{cases}$$

Upper Bounds on d=5 Lifetimes:

$$\Gamma^{-1}(p \to \overline{\nu} K^+) \lesssim \begin{cases} 2.88 \cdot 10^{35} \text{ yrs} , & \text{For } \mathrm{A}_{SO(10)} \\ 4.26 \cdot 10^{34} \text{ yrs} , & \text{For } \mathrm{B}_{SO(10)} \\ 4.41 \cdot 10^{34} \text{ yrs} , & \text{For } \mathrm{C}_{SO(10)} \end{cases}$$

Potentially observable with improvement of exp. sensitivity by factor ~ 10

Comments:

1) For calculating d=5 Proton decay,
well defined Yukawa sector is important

We build Yukawa Sector with Q4 Flavor Symmetry (see blow)

2) Insure that Planck scale operators do not introduce large B-violation

Symmetries of the Yukawa sector suppress additional B-violation

Planck Scale induced d=5 p-decay is strongly suppressed

ANTICIPATING:

By model's symmetries: QQQL –type ops.

$$Z\vec{X}\vec{Y}^2\vec{16}^3 16_3,\ S\vec{Y}^2\vec{16}^2 16_3^2,\ Z^2 S\vec{Y}\,\vec{16}16_3^3$$

Emerged terms $q_1 q_2 q_2 l_i / M_*$ **have suppression** $\lesssim \mathrm{few} \times 10^{-9}$

Adequate suppression

Yukawa Sector with Q4 Flavor Symmetry

Q4 → Interesting (predictive) Textures,
Solves SUSY FCNC problem

Pouliot, Seiberg '93
(*) Babu, Kubo '05

$$\vec{q} \equiv (\hat{q}_1, \hat{q}_2) \qquad m^2_{\rm susy}\left(|\tilde{q}_1|^2 + |\tilde{q}_2|^2\right)$$

$$Q_4\colon\ \overrightarrow{16} \equiv (16_1, 16_2) \sim \mathbf{2}, \qquad 16_3 \sim \mathbf{1} \qquad \overrightarrow{X}, \overrightarrow{Y} \sim \mathbf{2} - \text{Flavons}$$

(*):

$$\mathbf{1}' \times \mathbf{1}' = \mathbf{1}'' \times \mathbf{1}'' = \mathbf{1}''' \times \mathbf{1}''' = \mathbf{1} \qquad \mathbf{1}' \times \mathbf{1}'' = \mathbf{1}'''$$

$$\mathbf{1}'' \times \mathbf{1}''' = \mathbf{1}' \qquad \mathbf{1}''' \times \mathbf{1}' = \mathbf{1}''$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \times \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_{\mathbf{2}} = (x_1 y_2 - x_2 y_1)_{\mathbf{1}} \oplus (x_1 y_1 + x_2 y_2)_{\mathbf{1}'} \oplus (x_1 y_2 + x_2 y_1)_{\mathbf{1}''} \oplus (x_1 y_1 - x_2 y_2)_{\mathbf{1}'''}$$

Effective Yukawa Interactions are fixed by symmetries → Predictive

$$W^{(D)}_{\rm Yukawa} = 16_3 16_3 H + \frac{\vec{X}}{M_*}\vec{16}16_3 H + \frac{SZ^2A}{M_*^4}\vec{16}\vec{16}H + \frac{Z^3C}{M_*^4}\vec{16}\vec{16}C' +$$

$$\frac{AC\vec{Y}}{M_*\langle Z\rangle^2}\left(\vec{16}\cdot 16_3 + 16_3\cdot\vec{16}\right)C' + \frac{AC}{M_*^2\langle Z\rangle^2}(\vec{X}\vec{16})(\vec{Y}\vec{16})C' \ .$$

$$\langle\vec{X}\rangle \simeq \left(\tfrac{M_*}{40}, 0\right) \qquad \langle\vec{Y}\rangle \simeq \left(\tfrac{M_*}{200}, 0\right)$$

Can be obtained by integrating heavy states

Mass Matrices

$$Y_u = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \end{array} \begin{array}{c} \begin{array}{ccc} u_1^c & u_2^c & u_3^c \end{array} \\ \left(\begin{array}{ccc} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \sigma \\ 0 & \sigma & 1 \end{array} \right) \end{array} \lambda_t$$

$$Y_d = \begin{array}{c} \\ d_1 \\ d_2 \\ d_3 \end{array} \begin{array}{c} \begin{array}{ccc} d_1^c & d_2^c & d_3^c \end{array} \\ \left(\begin{array}{ccc} 0 & \epsilon' + \eta' & 0 \\ -\epsilon' - \eta' & \xi_{22}^d & \sigma + \epsilon \\ 0 & \sigma + \overline{\epsilon} & 1 \end{array} \right) \end{array} \lambda_b$$

$$Y_e = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \end{array} \begin{array}{c} \begin{array}{ccc} e_1^c & e_2^c & e_3^c \end{array} \\ \left(\begin{array}{ccc} 0 & -3\epsilon' - \eta' & 0 \\ 3\epsilon' + \eta & 3\xi_{22}^d & \sigma + 3\overline{\epsilon} \\ 0 & \sigma + 3\epsilon & 1 \end{array} \right) \end{array} \lambda_b$$

$$Y_\nu = \begin{array}{c} \\ \nu_1 \\ \nu_2 \\ \nu_3 \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1^c & \nu_2^c & \nu_3^c \end{array} \\ \left(\begin{array}{ccc} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & \sigma \\ 0 & \sigma & 1 \end{array} \right) \end{array} \lambda_t$$

$$\lambda_b = \lambda_t \cos\gamma$$

$$\sigma = 0.0508, \epsilon = -0.0188 + 0.0333i \qquad \bar{\epsilon} = 0.106 + 0.0754i$$

Input:

$$\epsilon' = 1.56 \cdot 10^{-4}, \eta' = -0.00474 + 0.00177i, \xi^d_{22} = 0.014e^{4.1i}$$

At low energies: **(perform RG running)**

Input: $\tan\beta = 10 \qquad m_t(m_t) = 160 \text{ GeV} \qquad m_\tau(M_Z) = 1.746 \text{ GeV}$

Output→

- $m_e = 0.51 \text{ MeV} \qquad m_\mu = 105.66 \text{ MeV} \qquad m_\tau = 1.776 \text{ GeV}$
- $m_u(2 \text{ GeV}) = 3.55 \text{ MeV}, \; m_c(m_c) = 1.15 \text{ GeV}$
- $m_b(m_b) = 4.67 \text{ GeV}$
  $m_d(2 \text{ GeV}) = 6.45 \text{ MeV}, \; m_s(2 \text{ GeV}) = 137.6 \text{MeV}$

- **At M(Z) scale:**

$$|V_{us}| = 0.225 \ , \quad |V_{cb}| = 0.0414 \ , \quad |V_{ub}| = 0.0034 \ , \quad |V_{td}| = 0.00878$$

CP violation:

- $\overline{\eta} = 0.334 \ , \quad \overline{\rho} = 0.12$

$$\overline{\rho} + i\overline{\eta} = -V_{ud}V_{ub}^*/(V_{cd}V_{cb}^*)$$

$$(\ \sin 2\beta = 0.663\ )$$

All in good agreement with experiments

****See also Babu, Pati, Rastogi***
Phys. Rev. D71 (2005) 015005.
for similar results within SO(10)

Neutrino Sector

'Majorana' Couplings:

$$W^{(M)}_{\text{Yukawa}} = \frac{Z^{k-4}\overrightarrow{Y}^2}{M_*^{k-3}M_{N''}^2}\overrightarrow{16}^2\bar{C}^2 + \frac{Z^{k-2}\overrightarrow{Y}}{M_*^{k-2}M_{N'}^2}\overrightarrow{16}16_3\bar{C}^2 + \frac{Z^{k-1}S}{M_*^{k-1}M_N^2}16_3^2\bar{C}^2$$

$$M_R = \begin{array}{c} \\ \nu_1^c \\ \nu_2^c \\ \nu_3^c \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1^c & \nu_2^c & \nu_3^c \end{array} \\ \left(\begin{array}{ccc} c & 0 & 0 \\ 0 & b & a \\ 0 & a & 1 \end{array} \right) \end{array} M_0$$

← **4 parameters**

$$M_{\nu D} = \begin{array}{c} \\ \nu_1 \\ \nu_2 \\ \nu_3 \end{array} \begin{array}{c} \begin{array}{ccc} \nu_1^c & \nu_2^c & \nu_3^c \end{array} \\ \left(\begin{array}{ccc} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & \sigma \\ 0 & \sigma & 1 \end{array} \right) \end{array} m_N^0$$

$$m_N^0 = m_U^0$$

← **All fixed**

Neutrino Masses & Mixings

$$m_\nu = m_D \frac{1}{M_{\nu C}} m^T{}_D \rightarrow$$

- **With input:** $M_0 = 5.85 \times 10^{14}\ \mathrm{GeV} \quad c = 3.821 \cdot 10^{-8} e^{2.6857i}$

$$a = 0.0515366 e^{-0.017947i}\ , \quad b = 0.00265706 e^{-0.035831i}$$

- **Output:** $\theta_{12} = 33.6^o\ , \quad \theta_{23} = 38.4^o\ , \quad \theta_{13} = 8.93^o\ , \quad \delta \simeq \pi$

$$\Delta m^2_{\rm sol} = m_2^2 - m_1^2 = 7.5 \cdot 10^{-5}\ \mathrm{eV}^2\ , \qquad \Delta m^2_{\rm atm} = m_3^2 - m_2^2 = 2.41 \cdot 10^{-3}\ \mathrm{eV}^2$$

(Good agreement with data)

- **Normal Hierarchy →** $(m_1,\ m_2,\ m_3) = (0.0016,\ 0.00881,\ 0.0499)\ \mathrm{eV}$

Summary

- Presented minimal/economical SUSY SO(10)
- Minimal Higgs System, Simple GUT Breaking
  and Natural DT Splitting to all orders
- Natural mu-term Generation
- Calculable GUT Threshold Corrections
- Correlation Between d=5 & d=6 Proton Decay Modes
- Obtained Upper Limits on $\Gamma^{-1}_{d=6}(p \to e^+ \pi^0)$ and $\Gamma^{-1}(p \to \overline{\nu} K^+)$

Makes model testable by future experiments!

Thank You

Backup Slides

d=5 decay vs. Unification within SUSY SU(5)

$M_{eff} \cos\gamma \rightarrow M_T$ □ d=5 decay $(p \rightarrow \overline{\nu} K^+)$

- **Minimal renorm. SUSY SU(5) →**

$$\Gamma^{-1}_{d=5}(p \to \bar{\nu} K^+) \simeq 1.2 \cdot 10^{31}\ \text{yrs} \times \left(\frac{0.012\text{GeV}^3}{\beta_H}\right)^2 \left(\frac{7}{\bar{A}_S^\alpha}\right)^2 \left(\frac{1.25}{R_L}\right)^2 \times$$

$$\times \left(\frac{M_T}{2 \cdot 10^{16}\text{GeV}}\right)^2 \left(\frac{m_{\tilde{q}}}{1.5\text{TeV}}\right)^4 \left(\frac{190\text{GeV}}{M_{\tilde{W}}}\right)^2 ,$$

For $\tau_{exp}(p \to \overline{\nu} K^+) \gtrsim 4 \cdot 10^{33}$ yrs. **One needs** $M_T \gtrsim 3.6 \cdot 10^{17}$ GeV

How large can be M_T **?**

- **Minimal renorm. SUSY SU(5) →**

RG: $$\ln\frac{M_T}{M_Z}=\frac{5\pi}{6}\left(3(\alpha_2^{-1}+\Delta^{(2)}_{2,\,w}-\frac{1}{6\pi})-2(\alpha_3^{-1}+\Delta^{(2)}_{3,\,w}-\frac{1}{4\pi})-(\alpha_1^{-1}+\Delta^{(2)}_{1,\,w})\right)$$

$$\alpha_3(M_Z)=0.12 \quad\to\quad M_T\simeq 7\cdot 10^{14}\ \mathrm{GeV}\Longrightarrow \tau(p\to\overline{\nu}K^+)\simeq 1.5\cdot 10^{28}\ \mathrm{yrs}$$

$$M_T\gtrsim 2\cdot 10^{16}\ \mathrm{GeV}\quad\Longrightarrow\quad \alpha_3(M_Z)\gtrsim 0.132$$

[← also problem of unification]

In gross conflict with experiments:

$$\alpha_3^{exp}(M_Z)=0.1176\pm 0.002$$

$$\tau_{exp}(p\to\overline{\nu}K^+)\gtrsim 4\cdot 10^{33}\ \mathrm{yrs.}$$

- **One solution:**

SUSY SU(5) with high dim. Ops.→

Large threshold corrections->

(Bajc, Perez, Senjanovic ’02)

$$M_T\geq 10^{17}\ \mathrm{GeV}$$

Considered SO(10) has no these problems

μ -Term Generation by Triggered VEV $\sim m_{\rm susy}$

GUT VEV Fields: $\Phi_i^0 = v_i + \Phi_i \qquad \langle \Phi_i \rangle = 0$

$$W(\Phi_i) = \frac{1}{2}\Phi_i M_\Phi^{ij} \Phi_j + \cdots$$

$$\Phi^\dagger M_\Phi^\dagger M_\Phi \Phi$$

Soft SUSY br. terms

$$m_{susy} M_G^2 \phi_{C'}(\nu^c) \qquad m_{susy} M_G^2 \phi_{\bar{C}'}(\bar{\nu}^c)$$

Two categories of fields: $\Phi_i = (\hat{\Phi}_{\hat{i}}, \tilde{\Phi}_{\tilde{i}})$

$$V = \hat{\Phi}^\dagger \hat{M}_\Phi^\dagger \hat{M}_\Phi \hat{\Phi} + \tilde{\Phi}^\dagger \tilde{M}_\Phi^\dagger \tilde{M}_\Phi \tilde{\Phi} + \hat{\Phi}^\dagger \mathcal{M}_\Phi \tilde{\Phi} + \tilde{\Phi}^\dagger \mathcal{M}_\Phi^\dagger \hat{\Phi} + m_{susy}(\hat{\Phi}^\dagger \hat{\rho} + \hat{\rho}^\dagger \hat{\Phi})$$

$$\langle \hat{\Phi}_{\hat{i}} \rangle = -m_{susy} \left[\hat{M}_\Phi^\dagger \hat{M}_\Phi - \mathcal{M}_\Phi (\tilde{M}_\Phi^\dagger \tilde{M}_\Phi)^{-1} \mathcal{M}_\Phi^\dagger \right]^{-1}_{\hat{i}\hat{j}} \hat{\rho}_{\hat{j}}$$

$$\langle \tilde{\Phi}_{\tilde{i}} \rangle = -\left[(\tilde{M}_\Phi^\dagger \tilde{M}_\Phi)^{-1} \mathcal{M}_\Phi^\dagger \right]_{\tilde{i}\hat{j}} \langle \hat{\Phi}_{\hat{j}} \rangle$$

←Triggered VEV $\sim m_{\rm susy}$

Applying for present SO(10):

Gauge choice

$$Z = (z + Z_0)e^{\Omega}\ , \quad S = (s + S_0)e^{\Omega}\ , \quad \phi_C(\nu^c) = (c + \frac{\Delta}{\sqrt{2}})e^{\Delta' + \Omega}$$

$$\phi_{\bar{C}}(\bar{\nu}^c) = (c + \frac{\Delta}{\sqrt{2}})e^{-\Delta'}\ , \quad \phi_{C'}(\nu^c) = \phi' e^{\Delta' - \Omega}\ , \quad \phi_{\bar{C}'}(\bar{\nu}^c) = \bar{\phi}' e^{-\Delta' - 2\Omega}$$

$$A(45) = \langle A(45) \rangle \ \oplus\ \frac{1}{\sqrt{24}}\eta_a \ \oplus\ \frac{1}{\sqrt{8}}\eta_b\ ,$$

$$M_\Phi = \begin{array}{c} \\ \phi' \\ \bar{\phi}' \\ \eta_b \\ \eta_a \\ \Delta \\ S_0 \end{array} \begin{array}{c} \begin{array}{cccccc} \phi' & \bar{\phi}' & \eta_b & \eta_a & \Delta & S_0 \end{array} \\ \begin{pmatrix} 0 & 0 & \frac{a_2 zc}{M_* 2\sqrt{8}} & -\frac{3a_2 zc}{M_* 2\sqrt{6}} & \frac{\sqrt{2} b_2 c^2}{M_*} & c_2 c \\ 0 & 0 & \frac{a_1 zc}{M_* 2\sqrt{8}} & -\frac{3a_1 zc}{M_* 2\sqrt{6}} & \frac{\sqrt{2} b_1 c^2}{M_*} & c_1 c \\ \frac{a_2 zc}{M_* 2\sqrt{8}} & \frac{a_1 zc}{M_* 2\sqrt{8}} & M_{\eta_b} & 0 & 0 & 0 \\ -\frac{3a_2 zc}{M_* 2\sqrt{6}} & -\frac{3a_1 zc}{M_* 2\sqrt{6}} & 0 & M_{\eta_b} & 0 & 0 \\ \frac{\sqrt{2} b_2 c^2}{M_*} & \frac{\sqrt{2} b_1 c^2}{M_*} & 0 & 0 & 0 & 0 \\ c_2 c & c_1 c & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

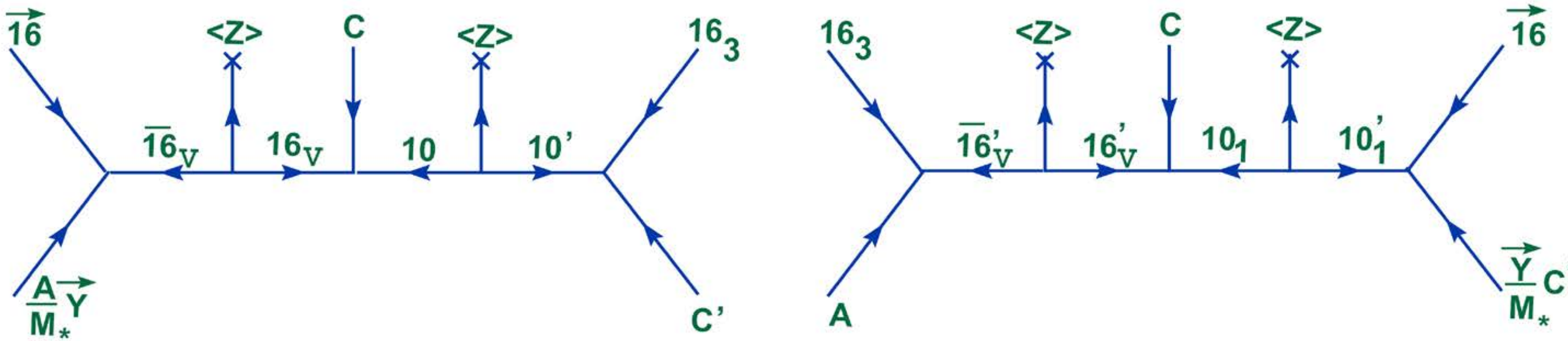
←Singlet mass matrix.
All massive!

State identification: $\hat{\Phi}^T = (\phi'\ ,\ \bar{\phi}')$ $\quad \tilde{\Phi}^T = (\eta_b,\ \eta_a,\ \Delta_0,\ S_0)$

VEV in B-L direction: $\langle \eta_b \rangle \simeq 3m_{susy}$

$$\lambda_1 HAH' \rightarrow \lambda_1 \frac{\langle \eta_b \rangle}{\sqrt{8}} h_u h_d \sim m_{susy} h_u h_d \qquad \mu \sim m_{\rm susy} \sim {\rm TeV}$$

Generation of Effective Yukawa Couplings



This 16, 10-plet exchange gives Clebsch 3 for lepton vs. quark → Good relation

Q4 Breaking: No flat directions and massless modes

$$\langle\vec{X}\rangle = (x_1, 0) \qquad \langle\vec{Y}\rangle = (y_1, 0) \qquad x_1, y_1 \neq 0$$

Additional Singlets: $S_1 \sim 1', \quad S_2 \sim 1, \quad \hat{X}, \hat{Y} \sim 1''$

For VEV fixing and mass generation

$$W_{\rm Flavon} = S_1\left(\frac{Z^6}{M_*^6}\vec{X}^2 + \frac{1}{M_*^4}\vec{X}^6\right) + S_2\left(\frac{1}{M_*^2}\vec{X}^2\vec{Y}^2 + \frac{1}{M_*^8}Z^{10}\right) + \hat{X}\vec{X}^2 + \hat{Y}\vec{Y}^2$$

$$F_Z = F_{S_{1,2}} = F_{\vec{X}} = F_{\vec{Y}} = 0$$

$$\langle S_1\rangle = \langle S_2\rangle = 0 \qquad x_1 \sim z\left(\frac{z}{M_*}\right)^{1/2} \sim \frac{M_*}{40} \qquad y_1 \sim z^5/(x_1 M_*^3) \sim \frac{M_*}{200}$$

$$\hat{X}\vec{X}^2 + \hat{Y}\vec{Y}^2 \to \hat{X}X_1X_2 + \hat{Y}Y_1Y_2$$

All states are heavy

Fermion mass/mixing RG from GUT scale down to low energies:

$$\left.\frac{m_{u,c}}{m_t}\right|_{m_t} = \eta_t^3 \frac{m^0_{u,c}}{m^0_t} \qquad m_u(2\ \mathrm{GeV}) = 1.8 m_u(m_t),\ m_c(m_c) = 2.11 m_c(m_t)$$

$$\eta_t = 1.1097 \qquad \eta_t = \exp\left[\frac{1}{16\pi^2}\int_{M_Z}^{M_G} \lambda_t^2(\mu)\, d\ln\mu\right]$$

$$\frac{m_b}{m_\tau} = \eta_t^{-1} R_{b\tau}\ \frac{m^0_b}{m^0_\tau} \qquad R_{b\tau} = \exp\left[\frac{1}{3\pi}\int_{M_Z}^{M_G} (4\alpha_3(\mu) - \alpha_1(\mu))\, d\ln\mu\right]$$

$$\left.\frac{m_{d,s}}{m_b}\right|_{M_Z} = \left.\frac{m_{d,s}}{m_b}\right|_{m_t} = 1.1159 \frac{m^0_{d,s}}{m^0_b}$$

$$V_{\alpha\beta} = \eta_t\ V^0_{\alpha\beta} \quad \text{for} \quad \alpha\beta = (ub,\ cb,\ td,\ ts) \qquad V_{\alpha\beta} = V^0_{\alpha\beta} \quad \alpha\beta = (ud,\ us,\ cd,\ cs,\ tb)$$

$$|V_{us}| = |V^0_{us}| \qquad |V_{cb}| = \eta_t |V^0_{cb}| \qquad |V_{ub}| = \eta_t |V^0_{ub}| \qquad \overline{\eta} = \overline{\eta}^0$$

GUT thresholds in soft terms' RG: 1.

$$\lambda_t 16_3 \cdot 16_3 H \to$$

$$\lambda_t \left((q_3 t^c + \nu_3^c l_3) H_u + (q_3 b^c + l_3 \tau^c) H_d + (\frac{1}{2} q_3 q_3 + t^c \tau^c + \nu_3^c b^c) T_H + (q_3 l_3 + t^c b^c) \bar{T}_H \right)$$

$$16\pi^2 \frac{d}{d \ln \mu} m^2_{\tilde{q}_3} = \beta^0_{\tilde{q}_3} + C^i_{\tilde{q}} \left((g^0_i M^0_i)^2 - (g_i M_i)^2 \right) - 12 \tilde{g}^2 \tilde{M}^2 \theta(\mu - M_X)$$

$$+ 4\lambda_t^2 (2 m^2_{\tilde{q}_3} + \tilde{m}^2_{T_H}) \theta(\mu - M_{T_H}) \,, \qquad C^i_{\tilde{q}} = \left(\frac{2}{15}, 6, \frac{32}{3} \right) \,,$$

$$16\pi^2 \frac{d}{d \ln \mu} m^2_{\tilde{t}^c} = \beta^0_{\tilde{t}^c} + C^i_{\tilde{u}^c} \left((g^0_i M^0_i)^2 - (g_i M_i)^2 \right) - 16 \tilde{g}^2 \tilde{M}^2 \theta(\mu - M_X)$$

$$+ 2\lambda_t^2 (m^2_{\tilde{t}^c} + m^2_{\tilde{\tau}^c} + \tilde{m}^2_{T_H}) \theta(\mu - M_{T_H}) \,, \qquad C^i_{\tilde{u}^c} = \left(\frac{32}{15}, 0, \frac{32}{3} \right) \,,$$

GUT thresholds in soft terms' RG: 2.

$$HAH' \to \frac{\lambda_1}{2}\left(h_u(\sigma_3 - \frac{3}{\sqrt{60}}S_\Sigma)h_d + T_H\sigma_Y h_d + h_u\sigma_X \bar{T}_{H'}\right)$$

$$\frac{\lambda_1}{\sqrt{20}}X_A h_u h_d + \frac{\lambda_1}{\sqrt{2}}h_u\left(\bar{e}^c_A {H_u}' + \bar{q}_A T_{H'}\right) \ .$$

$$\Delta\beta^{\lambda_1}_{h_u} = \frac{3}{8}\lambda_1^2\left(m^2_{h_u} + m^2_{h_d} + \tilde{m}^2_{\sigma_3}\right)\theta(\mu - M_{\sigma_3}) + \frac{3}{40}\lambda_1^2\left(m^2_{h_u} + m^2_{h_d} + \tilde{m}^2_{S_\Sigma}\right)\theta(\mu - M_{S_\Sigma})$$

$$+\frac{3}{4}\lambda_1^2\left(m^2_{h_u} + \tilde{m}^2_{\sigma_X} + \tilde{m}^2_{\bar{T}_{H'}}\right)\theta(\mu - \max(M_{\sigma_X}, M_{\bar{T}_{H'}})) + \frac{1}{10}\lambda_1^2\left(m^2_{h_u} + m^2_{h_d} + \tilde{m}^2_{X_A}\right)\theta(\mu - M_{X_A})$$

$$+\lambda_1^2\left(m^2_{h_u} + \tilde{m}^2_{{H_u}'} + \tilde{m}^2_{\bar{e}^c_A}\right)\theta(\mu - \max(M_{{H_u}'}, M_{\bar{e}^c_A})) + 3\lambda_1^2\left(m^2_{h_u} + \tilde{m}^2_{T_{H'}} + \tilde{m}^2_{\bar{q}_A}\right)\theta(\mu - \max(M_{T_{H'}}, M_{\bar{q}_A}))$$

$$\Delta\beta^{\lambda_1}_{h_d} = \frac{3}{8}\lambda_1^2\left(m^2_{h_u} + m^2_{h_d} + \tilde{m}^2_{\sigma_3}\right)\theta(\mu - M_{\sigma_3}) + \frac{3}{40}\lambda_1^2\left(m^2_{h_u} + m^2_{h_d} + \tilde{m}^2_{S_\Sigma}\right)\theta(\mu - M_{S_\Sigma})$$

$$+\frac{3}{4}\lambda_1^2\left(m^2_{h_d} + \tilde{m}^2_{\sigma_Y} + \tilde{m}^2_{T_H}\right)\theta(\mu - \max(M_{\sigma_Y}, M_{T_H})) + \frac{1}{10}\lambda_1^2\left(m^2_{h_u} + m^2_{h_d} + \tilde{m}^2_{X_A}\right)\theta(\mu - M_{X_A}) \ .$$