NNN15/UD2

Realistic F-theory Compactifications

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Outline:

I. Motivation: F-theory & Particle Physics

II. Key Ingredients of F-theory Compactification
(non-Abelian gauge symmetry, matter, Yukawas)

III. Particle Physics Model Building
(highlight concrete example of MSSM)

IV. Further Developments:
Abelian & Discrete Symmetries in F-theory

Emphasize geometric perspective

Apologies: Upenn-centric

F-theory?

F-theory = Type II String

F-theory
- Coupling g_s part of geometry (12dim)
- Torus fibered Calabi-Yau manifold

Type II String
- back-reacted D-branes
- regions with large g_s on non-CY space

g_s –string coupling

F-theory?

M-theory (11dim SG)

$\uparrow$ on S^1

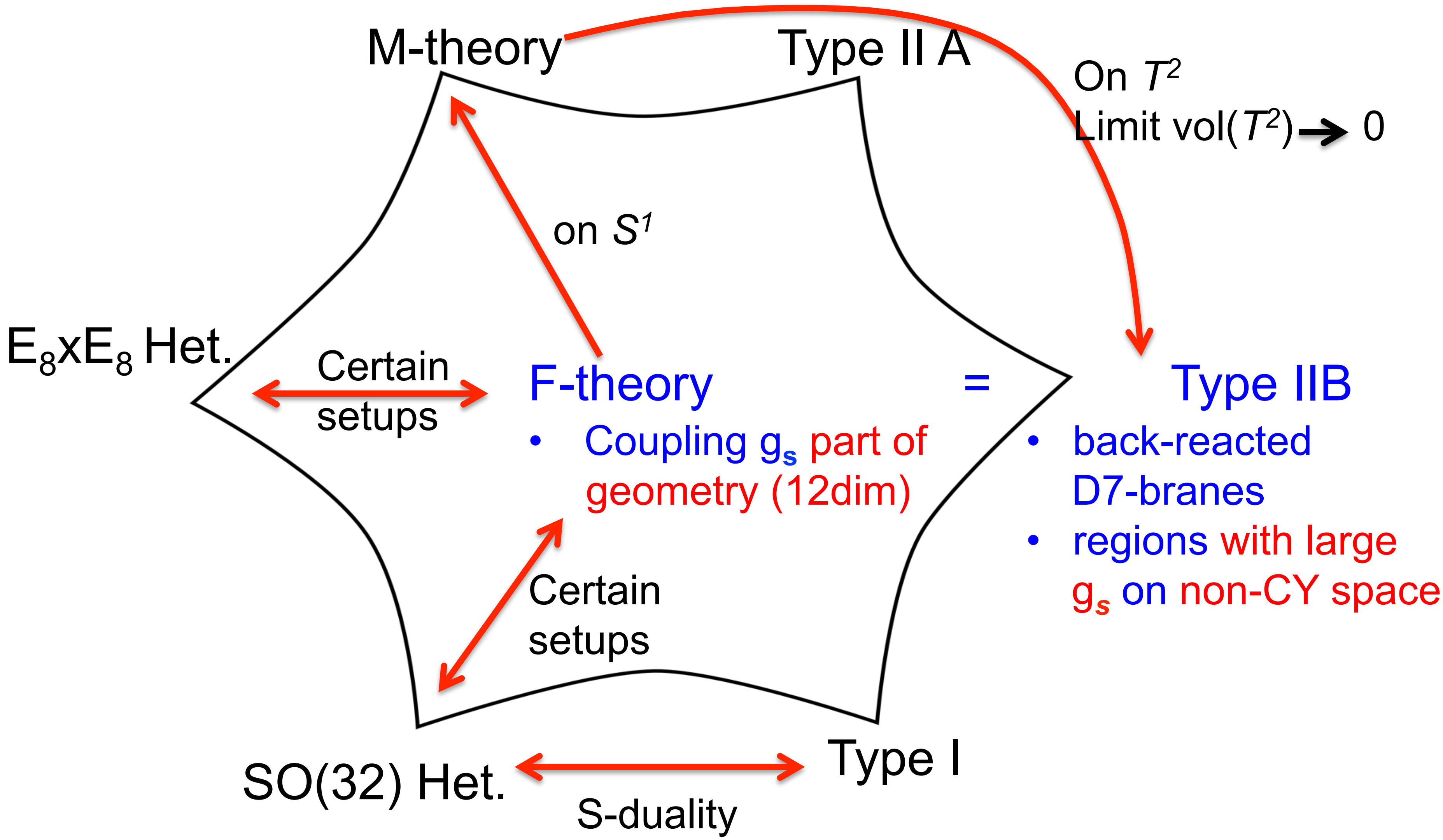
F-theory = Type IIB

F-theory
- Coupling g_s part of geometry (12dim)

Type IIB
- back-reacted D7-branes
- regions with large g_s on non-CY space

g_s –string coupling

F-theory?


M-theory
Type II A
On T^2
Limit vol(T^2) → 0
on S^1
E_8xE_8 Het.
Certain setups
F-theory
• Coupling g_s part of geometry (12dim)
=
Type IIB
• back-reacted D7-branes
• regions with large g_s on non-CY space
Certain setups
SO(32) Het.
Type I
S-duality

F-theory & Particle Physics

MOTIVATION

F-Theory Motivation

A broad domain of non-perturbative string theory landscape with new promising particle physics & ~~cosmology~~

(will not address moduli stabilization, though promising)

- SU(5) GUT couplings that are absent in perturbative string theory w/ D-branes, e.g.,10 10 5
- appearance of exceptional gauge symmetries (E_6)

[Donagi,Wijnholt'08]
[Beasley,Heckman,Vafa'08]....

Conceptual: geometric description at large string coupling

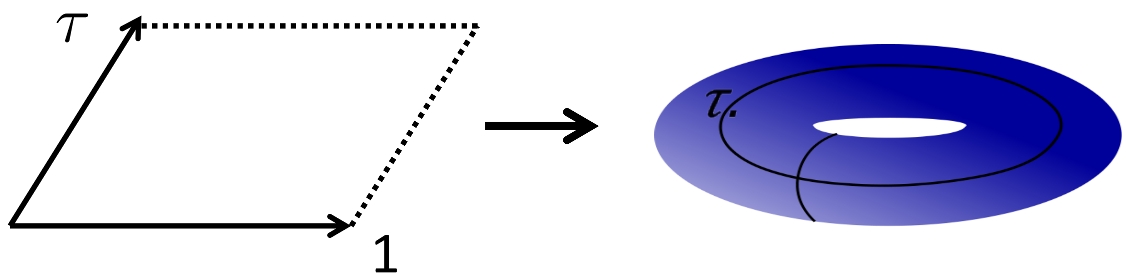
- Determine discrete data:
  gauge symmetry, matter reps. & multiplicities, Yukawa couplings

Type IIB perspective

F-THEORY BASIC INGREDIENTS

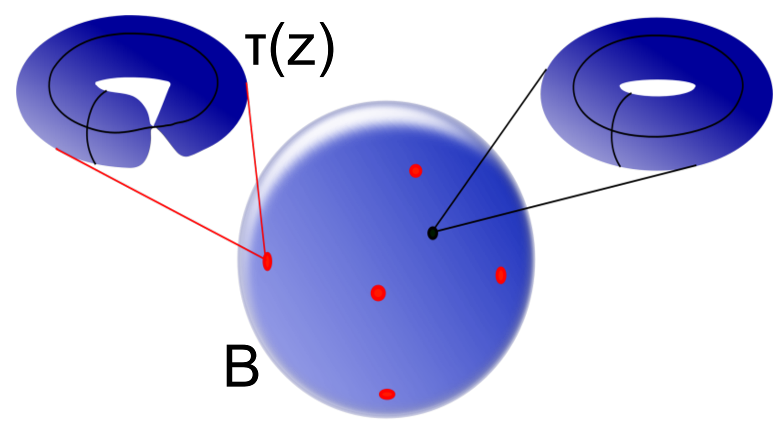
F-theory: basic ingredients

- F-theory is a geometric formulation of string theory w/D-branes, where one adds a geometric object: torus w/ SL(2,Z) symmetry



- τ- torus complex structure: $\tau \equiv C_0 + i g_s^{-1}$ - string coupling (axion-dilaton)
- Torus – fibered over a compactified (base) space B

i.e. torus coordinates depend on the base B



Torus= elliptic curve

Weierstrass form:

$$y^2 = x^3 + fxz^4 + gz^6$$

f, g- function fields on B

[z:x:y] coords on $\mathbf{P}^2(1,2,3)$

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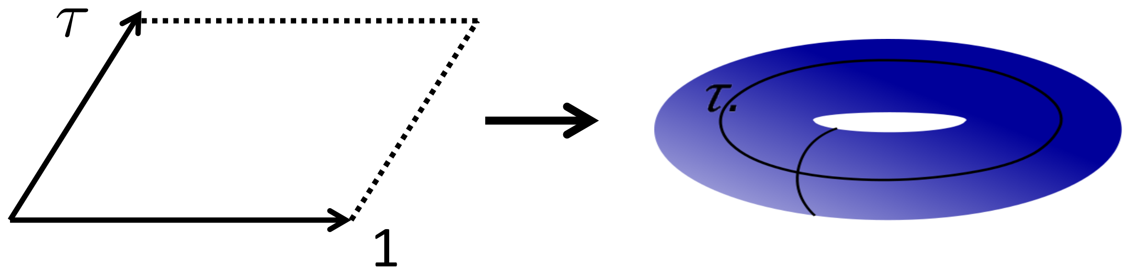

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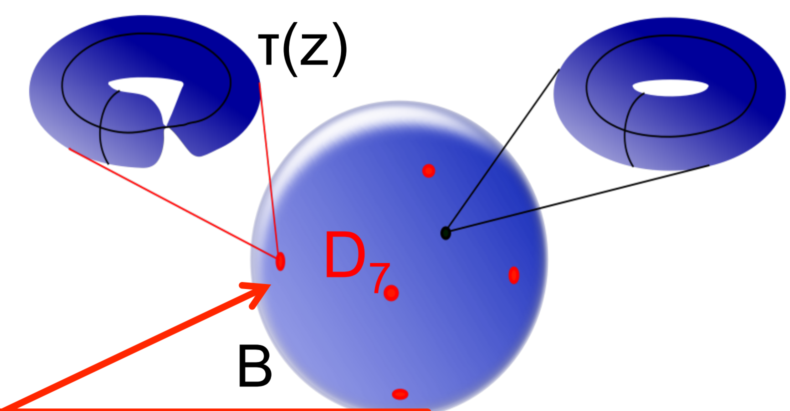


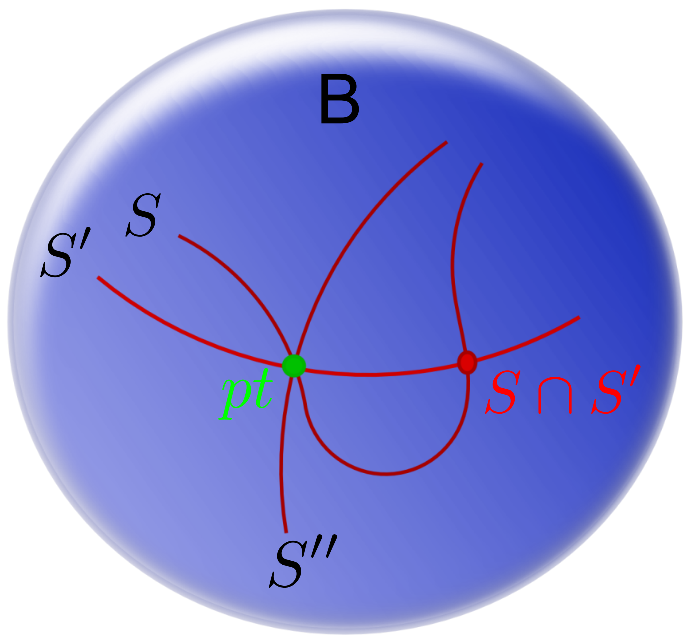
At brane location in B torus degenerates w/ $g_s \rightarrow \infty$ singular:
String Theory in non-perturbative regime

F-theory: basic ingredients

- Total space of torus-fibration: singular elliptic Calabi-Yau manifold X

  D=4, N=1 vacua: fourfold X_4 [all dimensions complex]

- Singularities encode complicated set-up of intersecting D-branes:



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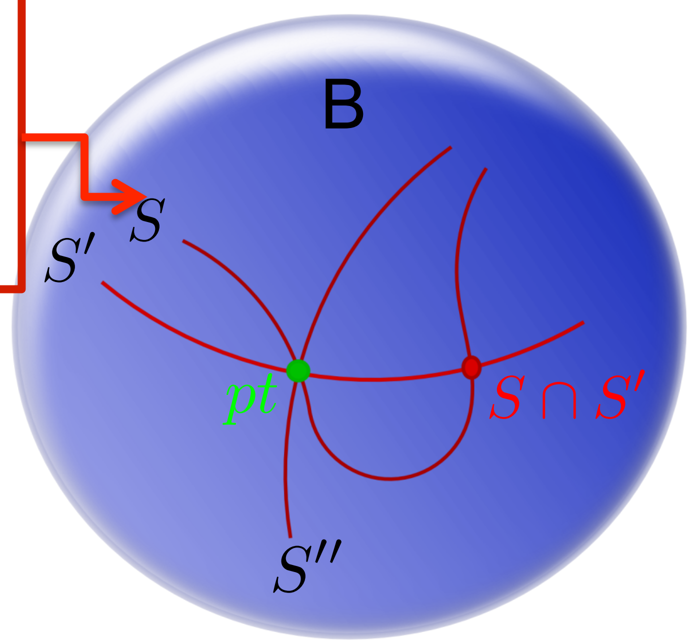
Non-Abelian gauge symmetry in codimension-one singul. in B (divisors wrapped by branes)



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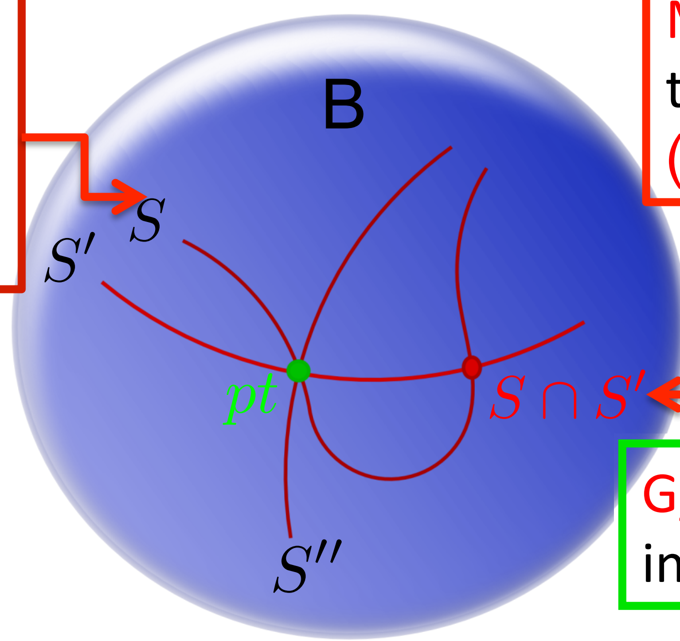

Matter in codimension-two singul. in B (intersecting branes)

G_4-flux at intersection induces chiral 4D matter

F-theory: basic ingredients

- Total space of torus-fibration: singular elliptic Calabi-Yau manifold X

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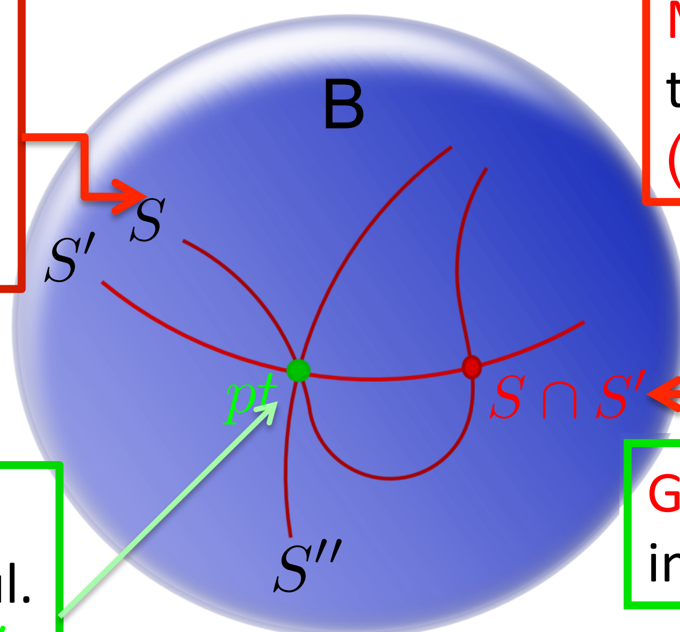


Yukawa couplings in codimension-three singul. in B $pt = S \cap S' \cap S''$

G_4-flux at intersection induces chiral 4D matter

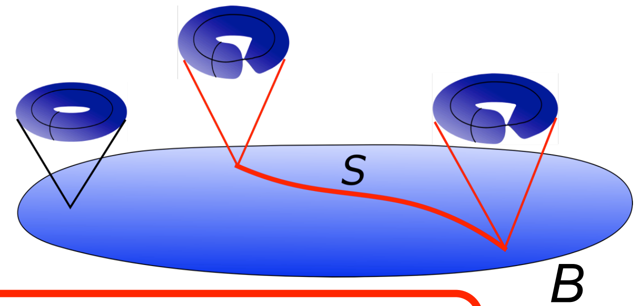
Highlights: Non-Abelian Gauge Symmetry

[Kodaira; Tate; Vafa; Morrison,Vafa;…]

1. Weierstrass form for elliptic fibration of *X*

$$y^2 = x^3 + fxz^4 + gz^6$$

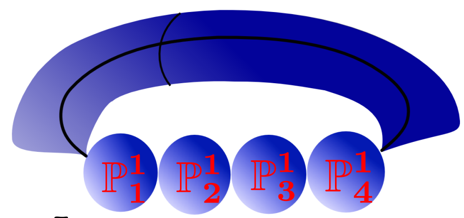
2. Severity of singularity along divisor S in B:



[$ord_S(f)$,$ord_S(g)$,$ord_S(\Delta)$] ←→ Singularity type of fibration of *X*

3. Resolution: singularity type ←→ structure of a tree of $\mathbb{P}^1$'s over *S*

I_n-singularity ←→ SU(n) Dynkin diagram



Deformation: [Grassi, Halverson, Shaneson]

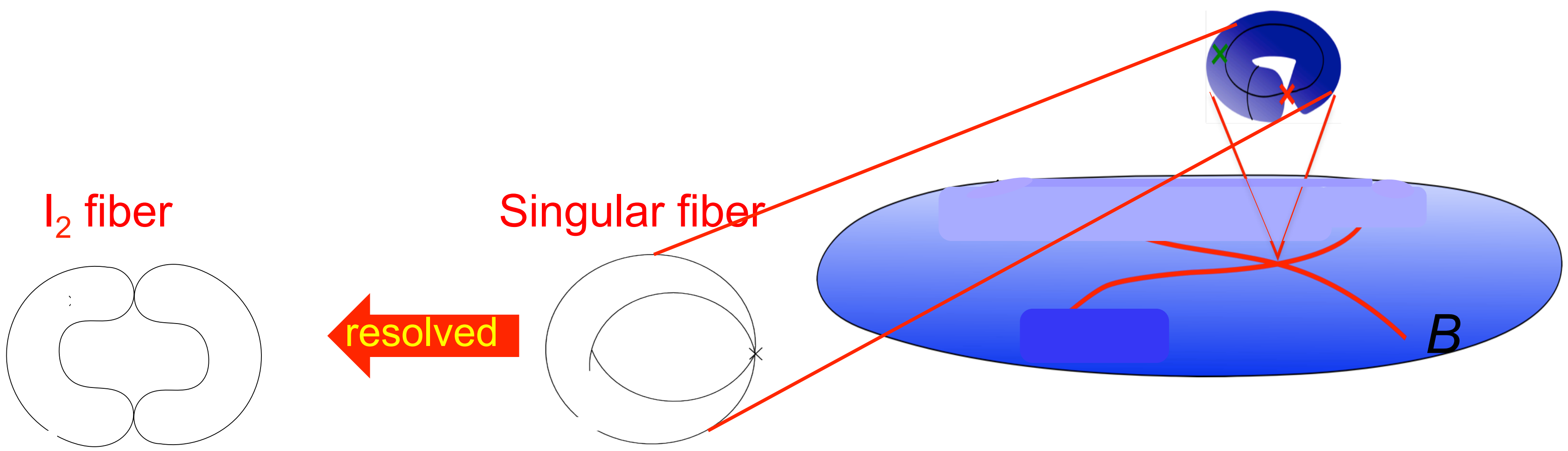
- Cartan generators for A^i gauge bosons: in M-theory via Kaluza-Klein (KK) reduction of C_3 potential along (1,1)-forms $\omega_i \leftrightarrow \mathbb{P}^1_i$ on X

$$C_3 \supset A^i \omega_i$$

- Non-Abelian generators: light M2-brane excitations on $\mathbb{P}^1$'s [Witten]

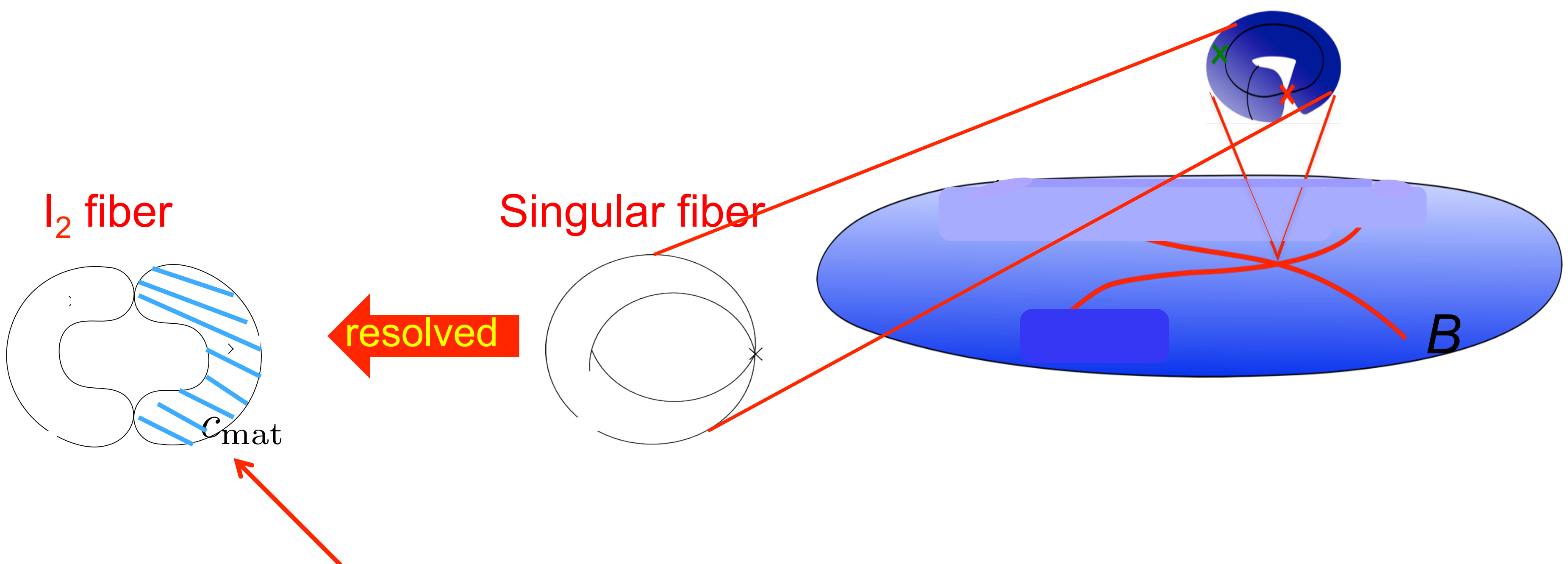
Highlights: Matter

Singularity at codimension-two in B:



Highlights: Matter

Singularity at codimension-two in B:



w/isolated (M2-matter) curve wrapping $\mathbb{P}^1$ → charged matter
(determine via intersection theory)

Initial focus: F-theory with SU(5) Grand Unification

[Donagi,Wijnholt'08][Beasley,Heckman,Vafa'08]...

Model Constructions:

[Donagi,Wijnholt'09-10]...[Marsano,Schäfer-Nameki,Saulina'09-11]...

Review: [Heckman]

[Blumehagen,Grimm,Jurke,Weigand'09][M.C., Garcia-Etxebarria,Halverson'10]...
[Marsano,Schäfer-Nameki'11-12]...[Clemens,Marsano,Pantev,Raby,Tseng '12]...

a bit more later

c.f., Hebecker's talk

Recent progress on other Particle Physics Models:

Standard Model building blocks (via tops of dP_2) [Lin,Weigand'14]

First Global 3-family Standard, Pati-Salam, Trinification Models
[M.C., Klevers, Peña, Oehlmann, Reuter 1503.02068]

⇒ highlights

I. Particle Physics & F-theory

concrete examples

Construction of Torus Fibrations

i. Torus = elliptic curv$\mathcal{C}$

Examples of constructions via toric techniques:

$\mathcal{C}_{F_i}$ as a Calabi-Yau hypersurface in the two-dimensional toric variety $\mathbb{P}_{F_i}$ (generalized projective spaces, associated with 16 reflexive polytops F_i):

[Klevers, Peña, Piragua, Oehlmann, Reuter'14]

$$\mathcal{C}_{F_i} = \{p_{F_i} = 0\} \text{ in } \mathbb{P}_{F_i}$$

ii. Elliptically fibered Calabi-Yau space: X_{F_i}

Impose Calabi-Yau condition:
coordinates in $\mathbb{P}_{F_i}$ and coeffs. of $\mathcal{C}_{F_i}$ lifted to sections on (specific functions of) B

$$\begin{array}{ccc} \mathcal{C}_{F_i} \subset \mathbb{P}_{F_i} & \longrightarrow & X_{F_i} \\ & & \downarrow \\ & & B \end{array}$$

Model Building Strategy:

i. Construction of X_{F_i} w/ Particle Physics

gauge symmetry (codim-1), matter reps. (codim-2) & Yukawas (codim-3)

F_{11} - Standard Model $\quad \mathbf{SU(3)\times SU(2)\times U(1)}$ focus

Representation	$(\mathbf{3},\mathbf{2})_{1/6}$	$(\bar{\mathbf{3}},\mathbf{1})_{-2/3}$	$(\bar{\mathbf{3}},\mathbf{1})_{1/3}$	$(\mathbf{1},\mathbf{2})_{-1/2}$	$(\mathbf{1},\mathbf{1})_{-1}$

$$p_{F_{11}} = s_1 e_1^2 e_2^2 e_3 e_4^4 u^3 + s_2 e_1 e_2^2 e_3^2 e_4^2 u^2 v + s_3 e_2^2 e_3^2 u v^2 + s_5 e_1^2 e_2 e_4^3 u^2 w + s_6 e_1 e_2 e_3 e_4 u v w + s_9 e_1 v w^2$$

[hypersurface constraint in dP_4 ($\mathbb{P}^2$[u:v:w] with four blow-ups [e_1:e_2:e_3:e_4])

F_{13} - Pati-Salam Model

F_{16} - Trinification Model

ii. Chiral index for D=4 matter:

$$\begin{tikzcd} c_{mat} \arrow[r] & \boxed{\mathcal{C}_{\mathbf{R}}} \arrow[d] \\ & \Sigma_{\mathbf{R}} \end{tikzcd}$$

$$\chi(\mathbf{R}) = \int_{\mathcal{C}_{\mathbf{R}}} G_4$$

a) construct G_4 flux by computing $H_V^{(2,2)}(\hat{X})$

b) determine matter surface $\mathcal{C}_{\mathbf{R}}$ (via resultant techniques)

iii. Global consistency – D3 tadpole cancellation:

$$\frac{\chi(X)}{24} = n_{\mathrm{D3}} + \frac{1}{2}\int_X G_4 \wedge G_4$$

a) satisfied for integer and positive n_{D3}

b) check, all anomalies are cancelled

Standard Model:

Base B = $\mathbb{P}^3$ Divisors in the base: $\mathcal{S}_7 = n_7 H_{\mathbb{P}^3}$

$\mathcal{S}_9 = n_9 H_{\mathbb{P}^3}$

Solutions (#(families);n_{D3}) for allowed (n_7,n_9):

$n_7 \backslash n_9$	1	2	3	4	5	6	7
7	–	$(27;16)$	–	–			
6	–	$(12;81)$	$(21;42)$	–	–		
5	–	–	$(12;57)$	$(30;8)$	–	$(3;46)$	
4	$(42;4)$	–	$(30;32)$	–	–	–	–
3	–	$(21;72)$	–	–	–	$(15;30)$	
2	$(45;16)$	$(24;79)$	$(21;66)$	$(24;44)$	$(3;64)$		
1	–	–	–	–			
0	–	–	$(12;112)$				
-1	$(36;91)$	$(33;74)$					
-2	–						

Pati-Salam Model

Solutions (#(families);n_{D3}) for allowed (n_7,n_9):

$n_7 \backslash n_9$	1	2	3	4	5	6	7
10	$(13;204)$						
9	–	$(11;140)$					
8	$(33;94)$	$(10;119)$	$(9;90)$				
7	–	$(9;100)$	$(6;77)$	$(14;48)$			
6	$(15;108)$	$(8;86)$	$(21;52)$	$(12;46)$	$(5;44)$		
5	$(6;106)$	$(35;44)$	–	$(30;16)$	–	$(3;44)$	
4	$(7;102)$	$(6;75)$	$(15;50)$	$(8;42)$	$(15;30)$	$(6;41)$	$(7;42)$
3	$(6;106)$	$(35;44)$	–	$(30;16)$	–	$(3;44)$	
2	$(15;108)$	$(8;86)$	$(21;52)$	$(12;46)$	$(5;44)$		
1	–	$(9;100)$	$(6;77)$	$(14;48)$			
0	$(33;94)$	$(10;119)$	$(9;90)$				
-1	–	$(11;140)$					
-2	$(13;204)$						

Trinification Model

Solutions (#(families);n_{D3}) for allowed (n_7,n_9):

$n_7 \backslash n_9$	1	2	3	4	5	6	7	8	9	10
10	$(5;120)$									
9	$(3;94)$	$(3;94)$								
8	$(4;72)$	$(8;69)$	$(4;72)$							
7	$(14;48)$	$(7;54)$	$(7;54)$	$(14;48)$						
6	$(5;50)$	$(8;44)$	$(3;44)$	$(8;44)$	$(5;50)$					
5	$(5;50)$	$(5;42)$	$(10;36)$	$(10;36)$	$(5;42)$	$(5;50)$				
4	$(14;48)$	$(8;44)$	$(10;36)$	$(16;30)$	$(10;36)$	$(8;44)$	$(14;48)$			
3	$(4;72)$	$(7;54)$	$(3;44)$	$(10;36)$	$(10;36)$	$(3;44)$	$(7;54)$	$(4;72)$		
2	$(3;94)$	$(8;69)$	$(7;54)$	$(8;44)$	$(5;42)$	$(8;44)$	$(7;54)$	$(8;69)$	$(3;94)$	
1	$(5;120)$	$(3;94)$	$(4;72)$	$(14;48)$	$(5;50)$	$(5;50)$	$(14;48)$	$(4;72)$	$(3;94)$	$(5;120)$

Yukawa Couplings (codimension-3 singularities)

Generically there for all gauge invariant couplings
(for MSSM example → μ-problem; R-parity violating terms)

Magnitudes?

Technology not developed, yet
- possibly tuned by adjusting complex structure moduli
- construction of MSSM w/ discrete symmetry

[work in progress, M.C., Klevers,Reuters]

Techniques in local models [Marchesano et al.]
Non-perturbative (D3-instanton) effects
[Martucci,Weigand]

Detailed Phenomenology → Long shot

II. U(1)-Symmetries in F-Theory

Abelian Symmetries in F-theory

Physics: important ingredient of the Standard Model and beyond

➡ Multiple U(1)'s desirable (F-theory applications [Antoniadis,Leontaris,King,..])

Formal developments: new CY elliptic fibrations with rational sections

While non-Abelian symmetries extensively studied ('96…)
[Kodaira; Tate;Morrison,Vafa; Bershadsky,Intriligator,Kachru,Morrison,Sadov,Vafa; Candelas,Font,...]

:

Abelian sector rather unexplored

A lot of recent progress '12-'15: [Grimm,Weigand;... Morrison,Park; M.C.,Grimm,Klevers;... Borchmann,Mayrhofer,Palti,Weigand; M.C.,Klevers,Piragua; MC,Grassi,Klevers,Piragua;... Braun,Grimm,Keitel; ... M.C.,Klevers,Piragua,Song;...Morrison,Taylor;... M.C.,Klevers,Piragua,Taylor]

U(1)'s-Abelian Symmetry

U(1) gauge bosons A^m should also arise via KK-reduction $C_3 \supset A^m \omega_m$

(1,1)-forms on X
dual to codimension-one divisors
only I_1-fibers

s

U(1)'s-Abelian Symmetry & Rational Torus Points

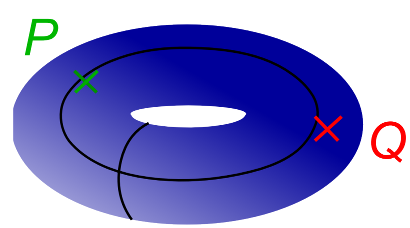
U(1) gauge bosons A^m should also arise via KK-reduction $C_3 \supset A^m \omega_m$

(1,1)-forms on X

[Morrison,Vafa]

$\boxed{\text{(1,1) - forms } \omega_m \Leftrightarrow \text{rational points}}$

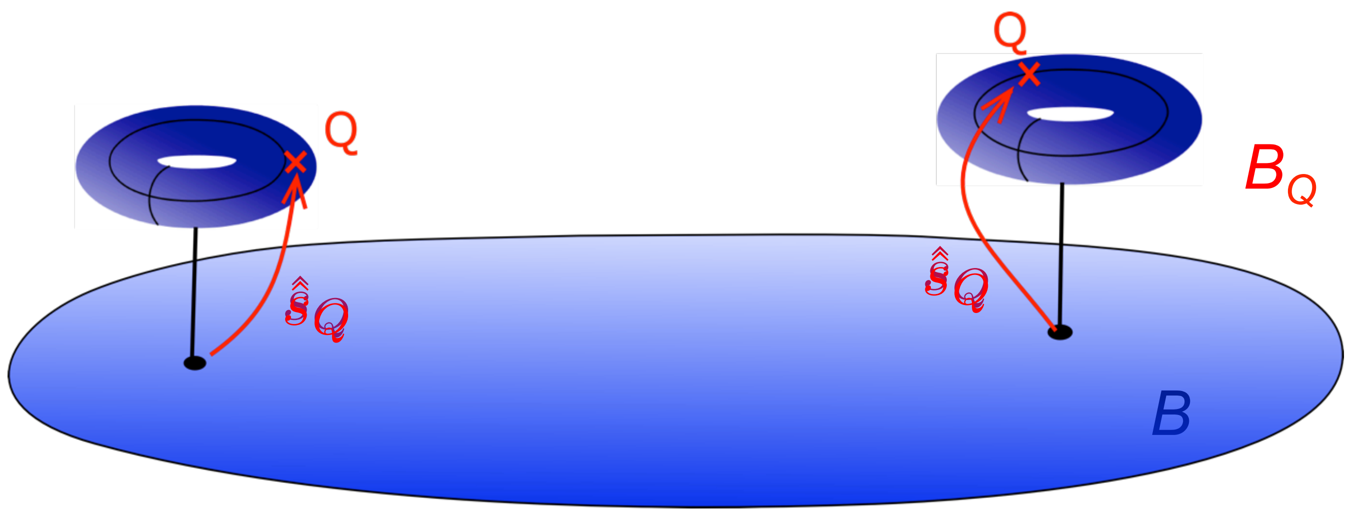
Torus=elliptic curve has a marked ``zero'' point P.
For a special shape there can be additional
marked ``rational'' points Q.
Rational points form a group under addition on torus.



➡ Mordell-Weil group of rational points

U(1)'s-Abelian Symmetry &Mordell-Weil Group

Point Q induces a rational section $\hat{s}_Q : B \rightarrow X$ of torus fibration



➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor B_Q in *X*

U(1)'s-Abelian Symmetry &Mordell-Weil Group

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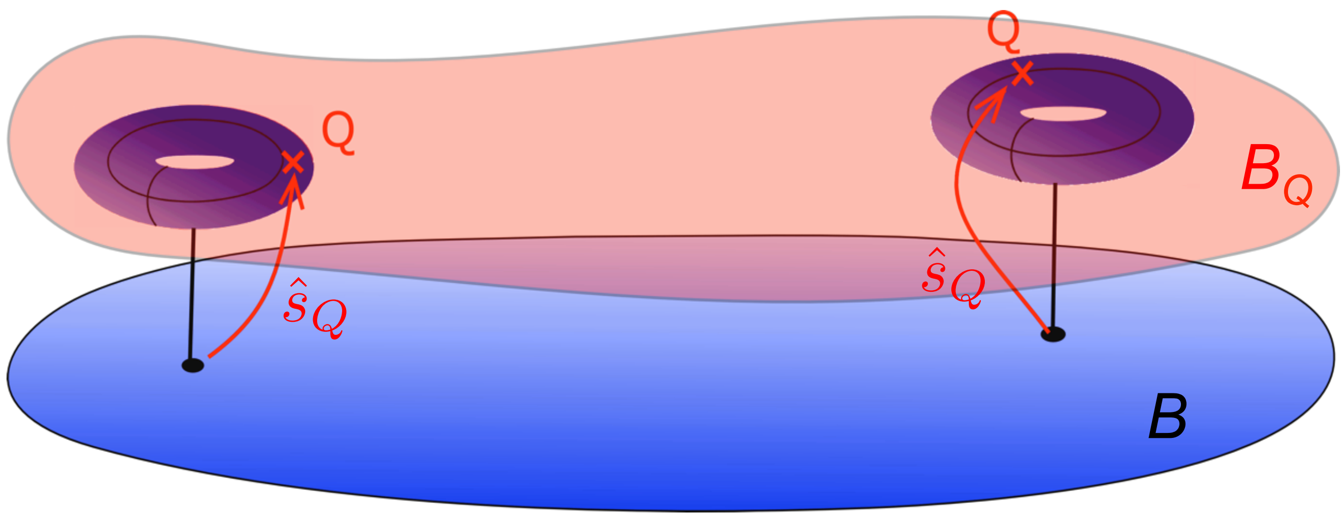


$\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor B_Q in *X*

(1,1)-form ω_m constructed from divisor B_Q (Shioda map)

indeed (1,1) - form ω_m ⟷ rational section

Explicit Examples with n-rational sections – $U(1)^n$

Torus=elliptic curve

n=0: with P - generic CY in $\mathbb{P}^2(1,2,3)$ (Tate form)

n=1: with *P, Q* - generic CY in $\mathrm{Bl}_1\mathbb{P}^2(1,1,2)$ [Morrison,Park'12]

n=2: with *P, Q, R* - specific example: generic CY in dP_2

[Borchmann,Mayerhofer,Palti,Weigand'13;
M.C.,Klevers,Piragua 1303.6970,1307.6425;
M.C.,Grassi,Klevers,Piragua 1306.0236]

- generalization: nongeneric cubic in $\mathbb{P}^2[u:v:w]$

[M.C.,Klevers,Piragua,Taylor 1507.05954]

n=3: with *P, Q, R, S* - CICY in $\mathrm{Bl}_3\mathbb{P}^3$ [M.C.,Klevers,Piragua,Song1310.0463]

n=4 determinantal variety in $\mathbb{P}^4$ ...

higher *n,* not clear…

$U(1)^2$: Concrete Example

[M.C., Klevers,Piragua]
[Borchmann,Mayrhofer,Palti,Weigand]

➡ representation as hypersurface in dP_2

$$p = u(s_1 u^2 e_1^2 e_2^2 + s_2 u v e_1 e_2^2 + s_3 v^2 e_2^2 + s_5 u w e_1^2 e_2 + s_6 v w e_1 e_2 + s_8 w^2 e_1^2) + s_7 v^2 w e_2 + s_9 v w^2 e_1$$

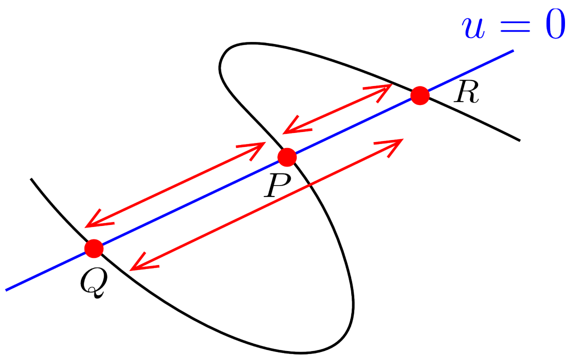
$[u:v:w:e_1:e_2]$ –homogeneous coordinates of dP_2

$$\boxed{\begin{array}{ll} & \quad\quad\quad\quad\quad\; u \quad\;\; v \quad\;\; w \quad e_1 \;\; e_2 \\ P: & E_2 \cap p = [-s_9 : s_8 : 1 : 1 : 0]\,, \\ Q: & E_1 \cap p = [-s_7 : 1 : s_3 : 0 : 1]\,, \\ R: & D_u \cap p = [0 : 1 : 1 : -s_7 : s_9]\,. \end{array}}$$

Sections represented by intersections of different divisors in dP_2 with p

$U(1)^2$: Further Developments

General $U(1)^2$ construction: [M.C., Klevers, Piragua, Taylor 1507.05954]



$$u f_2(u,v,w) + \prod_{i=1}^{3}(a_i v + b_i w) = 0$$

$f_2(u,v,w)$ degree two polynomial in $\mathbb{P}^2[u:v:w]$

Study of non-Abelian enhancement (unHiggsing) to $SU(3)xSU(2)^2$ by merging rational points P,Q,R → generalizations

Study of SU(5) w/ U(1)'s
[...M.C.,Grassi,Klevers,Piragua'13...]
via tops [Borchman,Mayrhofer,Weigand'13;Braun,Grimm,Keitel'13;...]
Systematic analysis
[Kuntzler,Schäfer-Nameki;Sacco,Lawrie; Lawrie,Schäfer-Nameki,Wong'14]

Study of SU(5)xU(1)xU(1) for Frogatt-Nielson flavor textures
[Krippendorf, Schäfer-Nameki,Wong 1507.0596]

III. Discrete Symmetries in F-Theory

Why Discrete Symmetries in F-theory?

Physics: important ingredient of beyond the Standard Model physics

⇒ forbid terms for fast proton decay and other R-parity violating terms, e.g., R-parity (Z_2), baryon triality (Z_3) and proton hexality (Z_6); family textures (F-theory implications [Leontaris,King,...])

Geometry: new Calabi-Yau geometries with genus-one fibrations

These geometries do not admit a section, but a multi-section

Earlier work: [Witten; deBoer, Dijkgraaf, Hori, Keurentjes, Morgan, Morrison, Sethi;...]

Recent extensive efforts'14-'15: [Braun, Morrison; Morrison, Taylor; Klevers, Mayorga-Pena, Oehlmann, Piragua, Reuter; Anderson,Garcia-Etxebarria, Grimm; Braun, Grimm, Keitel; Mayrhofer, Palti, Till, Weigand; M.C.,Donagi,Klevers,Piragua,Poretschkin; Grimm, Pugh, Regalado]

Abelian & Discrete Gauge Symmetry in F-theory

F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

[Morrison, Taylor;
Anderson, García-Etxebarria, Grimm, Keitel;
Braun, Grimm, Keitel]

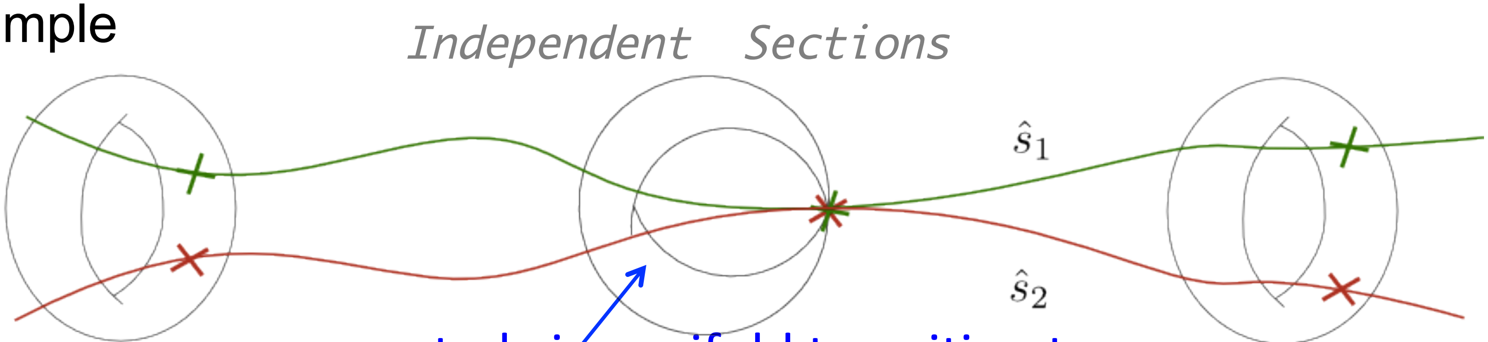
connected via conifold transition

F-theory compactification with an n-section (discrete Z_n symmetry)

Abelian & Discrete Gauge Symmetry in F-theory

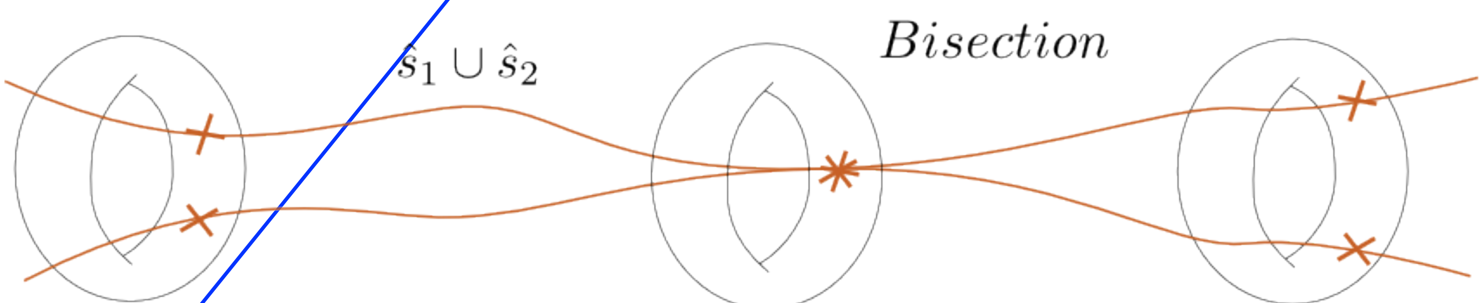
F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

n=2 example



connected via conifold transition to

F-theory compactification with an n-section (discrete Z_n symmetry)

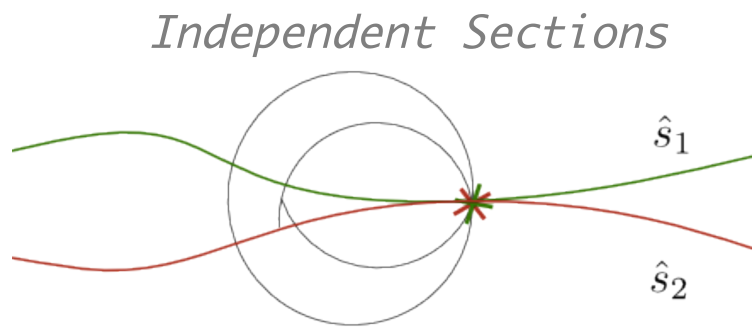


Torus fibration degenerates at
co-dimension two loci →matter

Abelian & Discrete Gauge Symmetry in F-theory

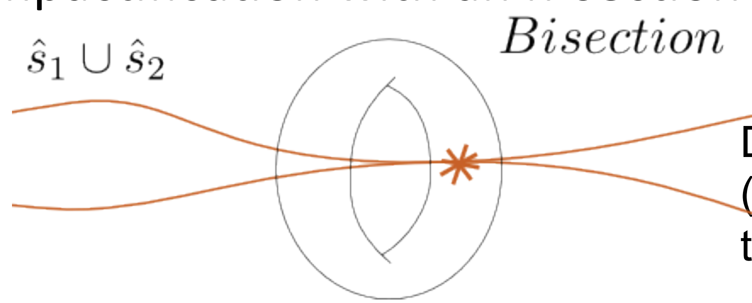
F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

n=2 example



blow-down
(P^1 in the geometry with
multiple sections collapses)

F-theory compactification with an n-section (discrete Z_n symmetry)



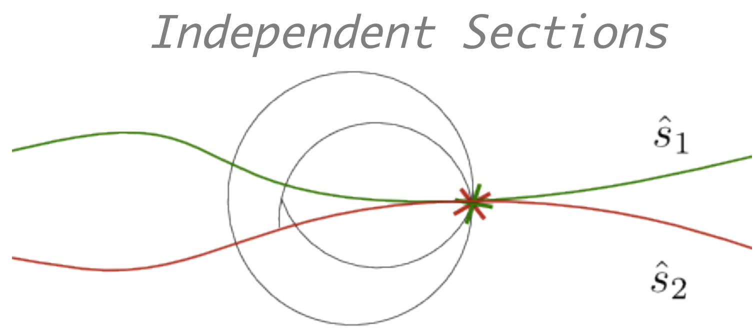
Deformation
(S^3 glues several sections
to a multi-section)

Conifold transition - Geometry

Abelian & Discrete Gauge Symmetry in F-theory

F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

n=2 example

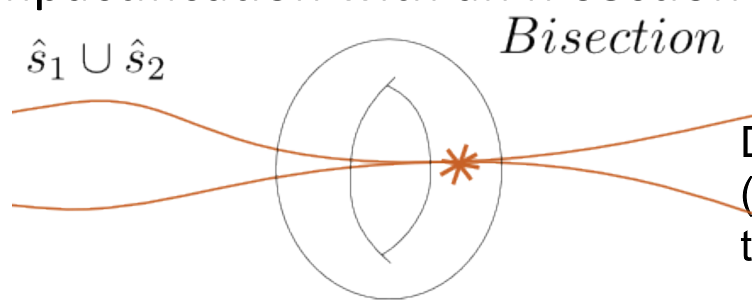


blow-down
(P^1 in the geometry with
multiple sections collapses)

appearance of massless field

ϕ with charge 2

F-theory compactification with an n-section (discrete Z_n symmetry)



Deformation
(S^3 glues several sections
to a multi-section)

massless field acquires VEV

$$U(1) \xrightarrow{\langle\phi\rangle\neq 0} Z_2$$

Conifold transition - Effective theory

Abelian & Discrete Gauge Symmetry in F-theory

F-theory compactification with n sections (Abelian $U(1)^{(n-1)}$)

$\downarrow$ blow-down Geometry

connected via conifold transition to

$\downarrow$ deformation Effective field theory

F-theory compactification with an n-section (discrete Z_n symmetry)

Since Abelian symmetries better understood (c.f., recent works)
most efforts focus on the geometry and spectrum of $U(1)^{(n-1)}$,
to deduce, primarily via effective field theory, implications for Z_n.

Z_2 [Anderson,Garcia-Etxebarria, Grimm; Braun,Grimm,Keitel; Mayrhofer, Palti, Till, Weigand]

Z_3 [M.C.,Donagi,Klevers,Piragua,Poretschkin] 1502.06953

Geometries with n-section $\Leftrightarrow$ Tate-Shafarevich Group Z_n

No time

Summary and Outlook

- Highlights of F-theory Compactification

  Geometric perspective - discrete data:

  gauge symmetry; matter reps and multiplicity;Yukawa couplings

- Construction of Particle Physics Models

  SU(5) GUT's & first examples of three family

  Standard, Pati-Salam and Trinification models (tip of the iceberg)

- Conceptual developments:

  Abelian & Discrete Symmetries (related to MW & TS groups, respectively)

  highlight $U(1)^2$ & applications

Issues: continuous data such as coupling magnitudes,…

moduli stabilization,…supersymmetry breaking,…

Further study