

Yukawa Structure in Susy $SO(10)$

Charanjit S. Aulakh

Indian Institute of Science Education and Research, Mohali,
Punjab, India

Plan

- SO(10) Fermion Fitting : Generic/Specific ??
- Threshold effects and Calculable models.
- Threshold corrected Gauge and Yukawa Unification
- Threshold effects and Proton Decay
- Fits and Outlook

VIRTUES OF $SO(10)$ GUTs

- $\{(Q_L, L_L, u_L^c, d_L^c, l_L^c) \oplus \nu_L\} \equiv 16$: Tight and complete

- Simple Tri-band FM Higgs Channel Spectrum

$$16 \otimes 16 = 10 \oplus 120 \oplus 126 \Rightarrow (10 + 120 + \overline{126})_{FMHiggs}$$

$$\overline{126} = (15, 2, 2) + \Delta_R(10, 1, 3) + \Delta_L(\overline{10}, 3, 1) + (6, 1, 1)$$

- $(-)^{3(B-L)} \equiv M_p \subset U(1)_{B-L} \subset G_{LR} \subset G_{PS} \subset SO(10)$

- Only Even B-L vevs $\langle \Delta_{L,R} \rangle \Rightarrow R_p \sqrt{\nu} \Rightarrow$ Stable LSP

House with Two Seesaws

- **126** Natural home to Seesaws *and* Georgi-Jarlskog :

$$\overline{126} \supset \Delta_R(10, 1, 3)_{TypeI} \oplus \Delta(\overline{10}, 3, 1)_{TypeII} \oplus \Phi_{G-J}(15, 2, 2) \oplus + \dots$$

(SuperK $\Rightarrow M_{\beta} \simeq M_{\nu}$) :

- Type I : Right handed neutrino mass from Δ_R

$$M_{B-L} \sim \langle \vec{\Delta}_R \rangle_{SM=0} \Rightarrow M_{\nu^c} \Rightarrow M_{\nu}^I \sim \frac{v_W^2}{M_{B-L}}$$

- Type II : Tadpole in Δ_L : $v_{EW} \Rightarrow \Rightarrow$ small neutrino Majorana mass

$$\langle \vec{\Delta}_L \rangle_{Y=2, T_{3L}=-1} \Rightarrow \Rightarrow M_{\nu}^{II} \sim \frac{v_W^2}{M_{\Delta_L}}$$

MSSM Yukawas from Generic Renormalizable SO(10)

$$y^u = (H + F + G) \quad ; \quad y^d = r_1 H + r_2 F + r_3 G$$

$$y^\nu = H - 3F + r_4 G \quad ; \quad y^l = r_1 H - 3r_2 F + (r_5 - 3r_3)G$$

$$G^T = -G \quad ; \quad F^T = F \quad ; \quad H^T = H$$

$$M_{u/\nu} = Y_{u/\nu} v \sin \beta \quad ; \quad M_{d/l} = Y_{d/l} v \cos \beta$$

$$M_{AB}^{\bar{\nu}} = 8\sqrt{2}f_{AB} \langle \Delta_R \rangle \quad ; \quad M_{AB}^{\nu(I)} = 16if_{AB} \langle \Delta_L \rangle$$

$$M_{AB}^{\nu(I)} = -((y^\nu)^T (M^{\bar{\nu}})^{-1} y^\nu)_{AB} v_u^2$$

- Parameter Counting !:** Gauge(1) \oplus (h \oplus f)(15) \oplus g(6) \oplus r_{1..5} \oplus $\Delta_{L,R}$
 (GUM parameters ...) versus MSSM
 g_i(M_Z)(3) \oplus m_f(9) \oplus CKM(4) \oplus $\nu(3 + 3 + 1 + 2 \leq 25)$..

(Generic) Fermion Fitting Frenzy 1992-2005..

- Babu Mohapatra (1992) : Use only $10 + \overline{126}$ FM (minimal) : then Generic formulae + Charged Fermion Mass data predictive ?? $\implies \implies$ FFF
- Type I dominant shown feasible: Matsuda, Koide, Fukuyama, Nishiura (2001) .
- Bajc, Senjanovic Vissani (2003) : Type II dominant links $b - \tau$ unification to Large mixing angles.
- Generic fits: Quark sector good, some tension without 120-plet. Inclusion of 120-plet allows easy fits as long as coefficients $r_{1..4}, (H, F, G)_{AB}$ free.
- Achievement. Goh, Mohapatra, Ng; Babu, Macesanu.. (2004-5) Predicted $\theta_{13}^L > 6^\circ$:
- All threshold corrections ignored ("can only make it easier").
- Analysis in isolation from gauge unification ($y_i(M_X)$ used in fit) .

Futility of SO(10) Unification ?

- Effective MSSM by integrating out heavy supermultiplets at M_X .
- Top down: Superheavy particles modify MSSM gauge coupling starting values at M_X :

$$\frac{1}{\alpha_i(M_X)} = \frac{1}{\alpha_G(M_X)} - \Delta_i(\{M_\zeta\})$$
$$\Delta_i(\{M_\zeta\}) = \sum_{\zeta} \frac{b_\zeta}{2\pi} \log \frac{M_\zeta}{M_X}$$

- Dixit and Sher (1989) :Gauge threshold Corrections large in Renormalizable SO(10) models due to $N > 10^{2.5}$ imply $\Delta \sim 1/\alpha$: (Susy) SO(10) unification is futile !

Futility of SO(10) Unification ? : Not so fast !

- $\{M_\zeta\}$ spread around centroid near M_X can allow cancellation to give $\Delta_i \sim \alpha_G^0$ for specific $\{M_\zeta\}$ patterns !
- GUT SSB and spectra also determine matter Yukawa coefficients $(r_i, \Delta_{L,R..})$
- Will generic fits be compatible with gauge unification in any specific SO(10) model? CSA, Girdhar(2005)

Higgs Portal

- Bedrock MSSM Gauge unification only works for one light pair of light doublets $H[1, 2, 1] \oplus \overline{H}[1, 2, -1]$.
- Realistic (es. $SO(10)$) GUMs contain several pairs $(h, \overline{h})_i$
- 6 pairs of doublets from $\{\mathbf{10}, \overline{\mathbf{126}}, \mathbf{126}_H, \mathbf{210}_H, \mathbf{120}\}_H$ mix via mass matrix \mathcal{H} into the single pair of MSSM doublets H, \overline{H} :
- Light Higgs is PORTAL into guts of UV completion. Novel NMSGUT insights ALL flow from a focus on the implications of this crucial fact !!!

Higgs Portal

- Consistency Condition(a.k.a Fine tuning) :

$$\text{Det } \mathcal{H} = 0$$

- Bi-Unitary transformation diagonalizes: $\bar{U}^T \mathcal{H} U = \Lambda_H$

$$\begin{aligned} \alpha_j &= U_{j1} & ; & & \bar{\alpha}_j &= \bar{U}_{j1} \\ H &= \sum_i \alpha_i^* h_i & ; & & \bar{H} &= \sum_i \bar{\alpha}_i^* \bar{h}_i \\ L_{\text{eff}} &: h_j \rightarrow \alpha_j H & ; & & \bar{h}_j &\rightarrow \bar{\alpha}_j \bar{H} \end{aligned}$$

- Matter Yukawas, Masses determined by Higgs fractions :

$$\Psi_A \cdot (h_{AB} H + f_{AB} \Sigma + g_{AB} \Theta) \Psi_B \Rightarrow 3 + 12 + 6 = 21 \text{ parameters}$$

- Higgs Fractions $\alpha_j \equiv U_{j1}$, $\bar{\alpha}_j \equiv \bar{U}_{j1}$ are functions of GUM superpotential parameters and masses. They determine the Fermion mass formulae !

Specific Yukawa formulae: 1

$$r_1 = \frac{\bar{\alpha}_1}{\alpha_1} \quad ; \quad r_2 = \frac{\bar{\alpha}_2}{\alpha_2}$$

$$r_3 = \frac{\bar{\alpha}_6 + i\sqrt{3}\bar{\alpha}_5}{\alpha_6 + i\sqrt{3}\alpha_5} \quad ; \quad r_4 = \frac{4i\sqrt{3}\alpha_5}{\alpha_6 + i\sqrt{3}\alpha_5} + 3$$

$$r_5 = \frac{4i\sqrt{3}\bar{\alpha}_5}{\alpha_6 + i\sqrt{3}\alpha_5} \quad ; \quad H_{AB} = 2\sqrt{2}\alpha_1 h_{AB}$$

$$G_{AB} = 2i\sqrt{\frac{2}{3}}(\alpha_6 + i\sqrt{3}\alpha_5)g_{AB} \quad ; \quad F_{AB} = -4\sqrt{\frac{2}{3}}i\alpha_2 f_{AB}$$

$$\langle \Delta_L \rangle = \left(\frac{i\gamma}{\sqrt{2}}\alpha_1 + i\sqrt{6}\eta\alpha_2 - \sqrt{3}\zeta\alpha_6 + i\zeta\alpha_5 \right) \alpha_4 \frac{\sqrt{2}v_u^2}{M_O}$$

Specific Yukawa formulae: 2

- $SO(10)$ Gauge Unification delicately poised !
- Fermion Yukawas are intricately linked to and woven with GUT SSB and gauge unification through the Higgs Fractions $\alpha_j, \bar{\alpha}_j$
- $\alpha_j, \bar{\alpha}_j$ determined by the GUM parameters through the spectra determined by the SSB of specific GUMs.
- No alternative to a painstaking case by case GUM specific *joint* analysis!!.
- Feasible to date only in very limited number of models, in fact only in $SO(10)$ MSGUTs

MSO(10)GUT structure

- $3 \times 16_F, 45_V \{10_H, 120_H \overline{126}_H\}_{FM}, \{126_H, 210_H\}_{AM},$ ¹
- **AM Higgs** : $\langle 210, \overline{126}, 126 \rangle \Rightarrow$ Susy $SO(10) \rightarrow MSSM$
- **MSGUT(No 120) Superpotential**

$$\begin{aligned} W = & m 210^2 + \lambda 210^3 + M 126 \cdot \overline{126} + \eta 210 \cdot 126 \cdot \overline{126} \\ & + 10 \cdot 210(\gamma 126 + \bar{\gamma} \overline{126}) \\ & + M_H 10^2 + h_{AB} 16_A \cdot 16_B + f'_{AB} 16_A 16_B \end{aligned}$$

Superpotential Parameters : **Minimal** : $((2 \times 7 - 4) + 3 + 2 \times 6 = 25)$

² **New Minimal**(\equiv Old !!) : 38 (**still minimal !**)³

¹CSA, Mohapatra(1982), Clark, Kuo and Nakagawa (1982)

²CSA, Bajc, Melfo, Senjanovic, Vissani(2003)

³CSA, Garg(2006)

MSGUT SSB

- **GUT scale VEVs** : $SO(10) \rightarrow MSSM$

$$\begin{aligned}\langle (15, 1, 1) \rangle_{210} &: a & \langle (15, 1, 3) \rangle_{210} &: \omega & \langle (1, 1, 1) \rangle_{210} &: p \\ \langle (10, 1, 3) \rangle_{\overline{126}} &: \bar{\sigma} & ; & \langle (\overline{10}, 1, 3) \rangle_{126} &: \sigma\end{aligned}$$

- **D Terms conditions, preserve SUSY** : $|\sigma| = |\bar{\sigma}|$
- **F Terms**

$$\begin{aligned}F_a &= 0 = 2(m + \lambda a)a + 4\lambda\omega^2 + \eta\sigma\bar{\sigma} \\ F_p &= 0 = 2mp + 6\lambda\omega^2 + \eta\sigma\bar{\sigma} \\ F_\omega &= 0 = 2(m + \lambda p)\omega + 4a\omega - \eta\sigma\bar{\sigma} \\ F_{(\bar{\sigma})} &= 0 = (M + \eta(p + 3a - 6\omega))(\bar{\sigma})\end{aligned}$$

4 coupled cubic equations are analytically soluble !

MSGUT SSB Solution

- SSB completely analyzable 4 eqns (Units : $\frac{m}{\lambda}$, \Rightarrow)

$$\begin{aligned}x &\equiv -\lambda\omega/m \\ \tilde{a} &= \frac{(x^2 + 2x - 1)}{(1-x)} \quad ; \quad \tilde{p} = \frac{x(5x^2 - 1)}{(1-x)^2} \\ \tilde{\sigma}\tilde{\sigma} &= \frac{2\lambda x(1-3x)(1+x^2)}{\eta(1-x)^2}\end{aligned}$$

EOM reduce to single Cubic in x .

$$\begin{aligned}8x^3 - 15x^2 + 14x - 3 &= -\xi(1-x)^2 \\ \xi &= \frac{\lambda M}{\eta m}\end{aligned}$$

NMSGUT SPECTRA

- 592 Higgs Chiral and 33 Majorana gauge supermultiplets occur in 26 MSSM representation types(22 complex and 4 real). Explicit solution of SSB allows explicit determination of their mass matrices and eigenvalues.

(CSA, Girdhar, Garg; Bajc, Melfo, Senjanovic, Vissani; Fukuyama, Ilakovac, Kikuchi, Melajnac, Okada)(2003-2004)

- Explicit superheavy spectra allow computation of superheavy one loop threshold effects on gauge unification and allow constructive demonstration that $SO(10)$ realistic gauge unification is NOT necessarily futile.

(CSA, Girdhar 2005)

Bloom to Doom and the Necessity of NMSGUT

- **MSGUT specific formulae imply**
 - Type I seesaw always dominates Type II.
 - Type I masses are too small if charged fermion fits are ok and vice versa.
- $\overline{126}$ Yukawas f_{AB} overloaded ! Enter both the seesaw and charged fermion masses(enable Georgi-Jarlskog mechanism to split s, μ yukawas which are quite different at M_X).
- Obvious solution ? : Type I seesaw boosted by very small f_{AB} (Leptogenesis adapted conjugate neutrinos in $10^6 - 10^{12}$ GeV !) while 10, 120 Yukawas fit charged fermion masses (CSA, Garg 2006)

NMSGUT viability

- No go at Tree level(Grimus Lavoura) :
 - d, s from $10 \oplus 120$ *too small*
 - But well known (and FFF ignored) large $\tan \beta$ driven corrections at M_{Susy} affect (and increase) precisely these masses.
- If μ, A_t large enough then third generation gluino corrections cancel chargino driven by A_t and raise $m_{d,s}$!
- G Tree level effective Yukawas deived from $10 \oplus 120$ obey stringent constraints due to antisymmetry of 120-plet Yukawas e.g

$$|y_b - y_\tau| \simeq |y_s - y_\tau|$$

Achievements and Failure of NMSGUT

- Consistent threshold corrected gauge unification feasible. CSA, Girdhar(2004).
- Realistic fit of all fermion mass mixing data using Quantum corrected Mass Formulae C.S.A., S. K. Garg NPB 2008
- Susy Threshold corrections at $M_S \Rightarrow$ Prediction(2008) of distinctive mini-split Susy spectra (*Caveat Emptor* : Tree sfermions!)
 - Normal s-hierarchy ($m_{\tilde{q}_3, \tilde{l}_3} \gg m_{\tilde{q}_{1,2}, \tilde{l}_{1,2}}$)
 - A_0 and $\mu > 10$ TeV required for $y_{d,s}$ fit!(2008)
 - Large A_0 now (2012) necessary for $M_H^{Susy} \simeq 126$ GeV
 - Light smuon (muon g-2 and CDM co-annihilation) possible
 - But successful fits give $\tau_p^{d=5} = 10^{26} - 10^{27}$ years and are thus futile
 - What did we neglect ? GUT Yukawa threshold corrections !!

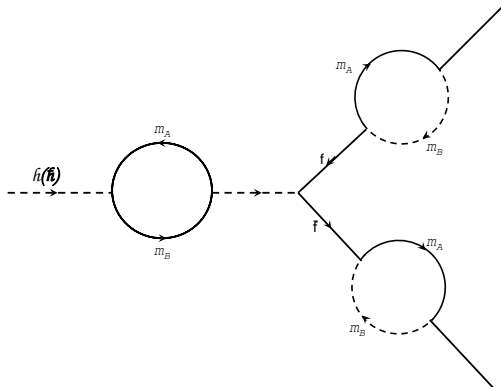


Figure: Loop corrections to fermion, antifermion and Higgs line

M_X^0 threshold : Quantum Naturopathy for $d = 5$ disease

- MSSM Higgs blend of 6 pairs from NMSGUT Higgs $\Rightarrow \Rightarrow$
 $\sim 10^3$ heavy fields renormalize light Higgs : *Generically* drive it to
“Higgs dissolution edge” :

$$Z_{H,\bar{H}} \simeq 0$$

- Rescaling to get Canonical Kinetic term $\Rightarrow \Rightarrow$ needed SO(10)
Yukawas are tiny !

$$h, g, f \sim Y_{GUT} \sim \sqrt{Z_H} Y^{MSSM}(M_X) \ll Y^{MSSM}(M_X) < 1$$

$$\mathcal{L} = \left[\sum_{A,B} (\bar{f}_A^\dagger (Z_{\bar{f}})_A^B \bar{f}_B + f_A^\dagger (Z_f)_A^B f_B) + H^\dagger Z_H H + \bar{H}^\dagger Z_{\bar{H}} \bar{H} \right]_D + ..$$

- Generic form Φ_i is ($Z = 1 - \mathcal{K}$) :

$$\mathcal{K}_i^j = -\frac{g_{10}^2}{8\pi^2} \sum_{\alpha} Q_{ik}^{\alpha*} Q_{kj}^{\alpha} F(m_{\alpha}, m_k) + \frac{1}{32\pi^2} \sum_{kl} Y_{ikl} Y_{jkl}^* F(m_k, m_l)$$

F : Passarino-Veltman 1-loop function.

- Precisely 26 different combinations (of the 26 MSSM representation types that occur in the (N)MSGUT multiplets) which can run in the loops on the Higgs lines in the MSSM matter fermion Yukawa vertices.

Threshold Effects On $\Gamma_{d=5}^{\Delta B \neq 0}$

$$W^{\Delta B} = L_{ABCD} Q_A Q_B Q_C L_D + R_{ABCD} \bar{U}_A \bar{U}_B \bar{D}_C \bar{L}_D$$

$$(L, R)_{ABCD} \sim \sum \frac{(h/f/g)_{AB}(h/f/g)_{CD}}{M_X}$$

- Canonical kinetic terms require rescaling by wavefunction renormalization matrices. Coefficients L_{ABCD} , R_{ABCD} of $d=5$, $\Delta B = \pm 1$ decay operators reduced by factors $\sim Z_H$
- *Unitarity and perturbativity via $Z > 0$ imply couplings are small $|Z_{H,\bar{H}}| \approx 0$ but $Z_{f,\bar{f}} \simeq 1$.*

- $1/\sqrt{Z_{H,\bar{H}}} \gg 1$ boost lowers SO(10) Yukawas required to match MSSM data. d=5 operators have no external Higgs line so lowered SO(10) couplings will suppress decay rate mediated by d=5 operators strongly.
- MSSM μ and B parameters larger by the factor of $(Z_H Z_{\bar{H}})^{-1/2}$ (consistent with EW SSB). Scalar soft masses and soft Higgs masses modified by a factor of Z_f^{-1} and $Z_{H/\bar{H}}^{-1}$ respectively. A_0 same. Y_ν and Higgs field redefinition modify the Type I seesaw formula.
- We constrained the B decay rates while searching :

$$\text{Max}(L_{ABCD}, R_{ABCD}) < 10^{-22} \text{ GeV}^{-1}$$

to get proton life time above 10^{34} Yrs. This constraint forces the search towards the regions of parameter space which produce

$$Z_{H,\bar{H}} \ll 1$$

Unification flows

- Improved one scale matching: Throw all hard SO(10) GUT and soft $(m_0, M_{1/2}, A_0, m_H^2, m_{\bar{H}}^2)$ at M_X .
- Calculate threshold corrections due to superheavy particles to tree level SO(10) Gauge and Yukawa couplings. Two loop RGEs to flow down to M_Z .
- Threshold corrections at $M_5 (\simeq M_Z)$ due to sparticle masses and large $\tan \beta$ corrections to (esp. down type) fermion Yukawas. Match corrected run down values to SM couplings (3 + 18) at M_Z using loop corrected sparticle masses.
- $d = 5, \Delta B \neq 0$ Operators renormalized down to M_Z and used to calculate the proton decay rates.

Achievement of NMSGUT : II

- Generic mechanism to raise $d = 5$ operator mediated proton lifetime from $\tau_p \sim 10^{27}$ yrs to $\tau_p > 10^{34}$ yrs .
(C.S.A(2011), C. S.A., I. Garg, C. K. Khosa, NPB882 (2014))
- (New !) Corrections at M_X weaken tree level 10 + 120 constraints and give less distinctive Susy spectra. Besides repairing τ_p can lift $m_{d,s}$ hugely.
- Programmatic shift : Quantum threshold corrections crucial at large N!

Effects on $Y_{d,s}(M_X)$

- Recent searches for "single throw at M_X " fits give much larger values of MSSM $Y_{d,s}(M_X)$ than ever possible before with $10 + 120$ tree level fits !
- Thus very large A_0, μ no longer required, though still large.
- S-Hierarchy still normal but not so glaring.

$$SO(10) \otimes O(N_g)_F$$

- $SO(10)$ matter kinetic terms $U(N_g)_H$ symmetry.
- Gauging symmetry avoids Goldstones
- $U(N_g)$ has anomalies $O(N_g)$ does not
- $U(N_g)$ enforces vanishing of half emergent fermion Yukawa
 $O(N_g)_F \subset U(N_g)_H$ does not.
- $10, \overline{126}$ (120) extend to (anti-)symmetric Yukawon representations of $O(N_g)$.

Yukawon Ultra Minimal GUTs(YUMGUTs)

- For flavour generation MSGUT ssb Higgs (**126, 210**) also symmetric.
- MSSM Yukawas are generated dynamically via **VEV of Yukawon** field determined by *renormalizable* MSGUT couplings !
- **Matter fermion Yukawa couplings are reduced from 15(21) to just 3(5) parameters in MSGUT(NMSGUT) with 3 generations.**
- YUMSGUT superpotential :

$$W = \text{Tr}(m\Phi^2 + \lambda\Phi^3 + M\bar{\Sigma}.\Sigma + \eta\Phi.\bar{\Sigma}.\Sigma + \Phi.H.(\gamma\Sigma + \bar{\gamma}.\bar{\Sigma}) + M_H H.H)$$

$$W_F = \Psi_A.(hH_{AB} + f\Sigma_{AB})\Psi_B$$

OUTLOOK

- NMSGUT could fit all fermion data with $\tau_p > 10^{34}$ years.
- Focus on Higgs portal highly rewarding : Susy spectra predicted , $d = 5$ baryon decay suppressed.
- $10 \oplus 120$ tree level constraints evadable by *both* M_S and M_X threshold corrections giving distinctive Susy Spectra.
- Complete predictive SM and BSM scenario. *fasifiable* !
- Basis for renormalizable dynamical flavour generation at M_X .
- Cosmology :
 - LSP Dark matter,
 - Type I Seesaw Leptogenesis
 - Inflation : Supersymmetric Seesaw Inflexion / GRIPI...

Parameter	Value
$\chi_{tot}^2, \chi^2(\sin^2 \theta^W), \chi^2(\alpha_{em}^{-1}), \chi^2(\alpha_s)$ $\chi_t^2, \chi_{d,s}^2, \chi^2(\sin^2(\theta_{23}^L))$ $\chi^2(\sin^2(\theta_{12,13}^L)), \chi^2(\sin^2(\theta_{12,13,23}^Q))$ $\chi_{c,b,e,\mu,\tau}^2$	13.655, 6.71, 1.05, 0.05 .15, {.82, .47}, 0.59 < 0.1, < .01 $< \simeq 10^{-2}$
m_t, m_b, m_c, m_s $\sin^2 \theta_W, \alpha_{em}^{-1}, \alpha_s$	174.3, 2.88, 0.62, .06 0.249, 126.6, 0.12
Sparticle (LoopCorr.)	Mass (TeV)
$\tilde{b}, \tilde{w}, \tilde{g},$ $M_{\tilde{\nu}}$ $M_{\tilde{u}}$ $M_{\tilde{e}}$ $M_{\tilde{d}}$ $M_{H^0}, M_A, M_{H^\pm}, M_{H^0}$ M_{χ^0} M_{χ^\pm}	0.6, 1.14, 3.24 8.38, 8.38, 8.42 5.0, 12.96, 5.0, 12.96, 12.18, 21.18 7.98, 17.72, , 7.98, 17.72, 8.42, 18.00 12.96, 13.93, 12.96, 13.93, 12.18, 21.18 0.124, 51.48, 51.48, 48.0 0.468, 0.91, 43.2, 43.2 1.14(\tilde{W}), 56.22(\tilde{H}_\pm)