FLAVOR Symmetries for Quarks and Leptons

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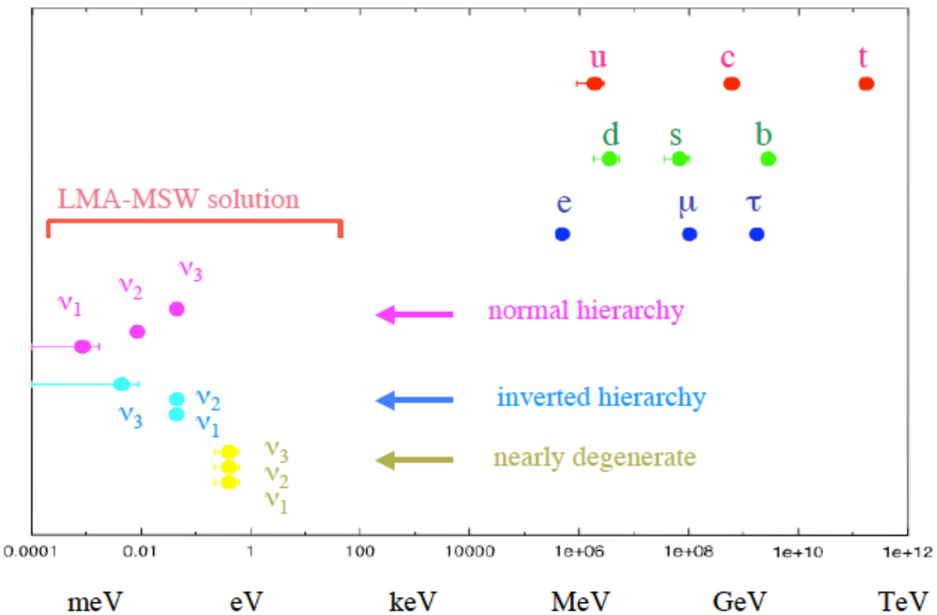
Origin of Mass Hierarchy and Mixing

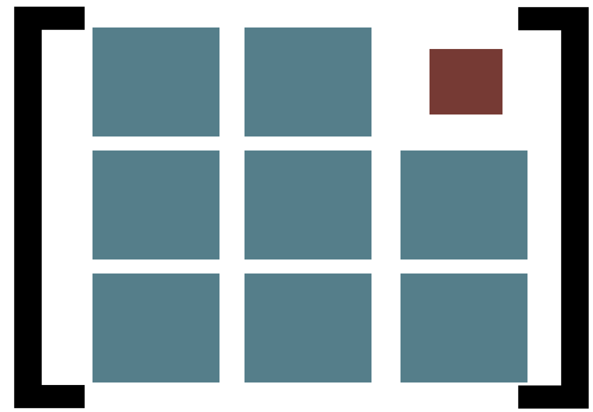
☞ Smallness of neutrino mass:

$$m_\nu \ll m_{e,\,u,\,d}$$

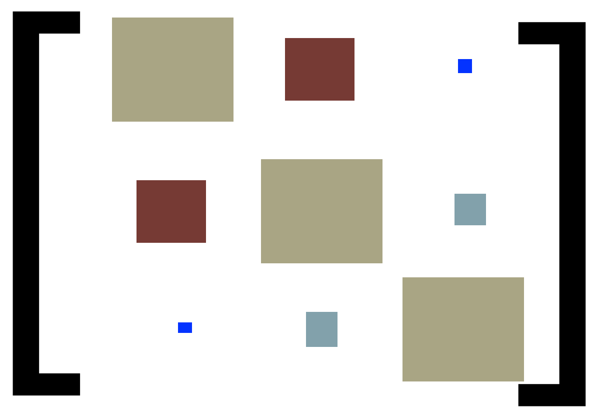


Fermion mass and hierarchy problem ➟ Many free parameters in the Yukawa sector of SM

☞ Flavor structure:



leptonic mixing



quark mixing

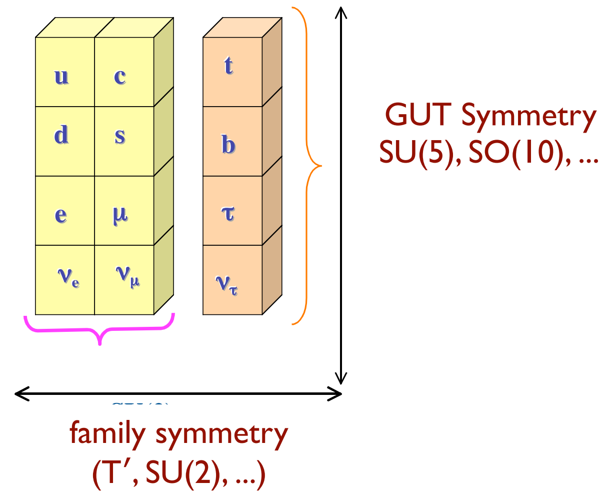
Origin of Mass Hierarchy and Mixing

- In the SM: 22 physical quantities which seem unrelated
- Question arises whether these quantities can be related
- No fundamental reason can be found in the framework of SM
- less ambitious aim ⇒ reduce the # of parameters by imposing symmetries
  - Grand Unified Gauge Symmetry
    - seesaw mechanism naturally implemented
    - GUT relates quarks and leptons: quarks & leptons in same GUT multiplets
      - one set of Yukawa coupling for a given GUT multiplet ⇒ intra-family relations
  - Family Symmetry
    - relate Yukawa couplings of different families
      - inter-family relations ⇒ further reduce the number of parameters

⇒ Experimentally testable correlations among physical observables

Origin of Flavor Mixing and Mass Hierarchy

- Several models have been constructed based on
  - GUT Symmetry [SU(5), SO(10)] ⊕ Family Symmetry G_F
- Family Symmetries G_F based on continuous groups:
  - U(1)
  - SU(2)
  - SU(3)



- Recently, models based on discrete family symmetry groups have been constructed
  - A_4 (tetrahedron)
  - T´ (double tetrahedron)
  - S_3 (equilateral triangle)
  - S_4 (octahedron, cube)
  - A_5 (icosahedron, dodecahedron)
  - Δ_{27}
  - Q_6

Tri-bimaximal Neutrino Mixing

Capozzi, Fogli, Lisi, Marrone, Montanino, Palazzo (March 2014)

- Latest Global Fit (3σ)

$$\sin^2\theta_{23} = 0.437\ (0.374 - 0.626)$$ $[\theta^{lep}_{23} \sim 41.2°]$

$$\sin^2\theta_{12} = 0.308\ (0.259 - 0.359)$$ $[\theta^{lep}_{12} \sim 33.7°]$

$$\sin^2\theta_{13} = 0.0234\ (0.0176 - 0.0295)$$ $[\theta^{lep}_{13} \sim 8.80°]$

Also NuFit: Bergström, Gonzalez-Garcia, Maltoni, Schwetz

- Tri-bimaximal Mixing Pattern

Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$$\sin^2\theta_{\mathrm{atm,\,TBM}} = 1/2 \qquad \sin^2\theta_{\odot,\mathrm{TBM}} = 1/3$$

$$\sin\theta_{13,\mathrm{TBM}} = 0.$$

- Leading Order: TBM (from symmetry) + higher order corrections/contributions
- More importantly, corrections to the kinetic terms

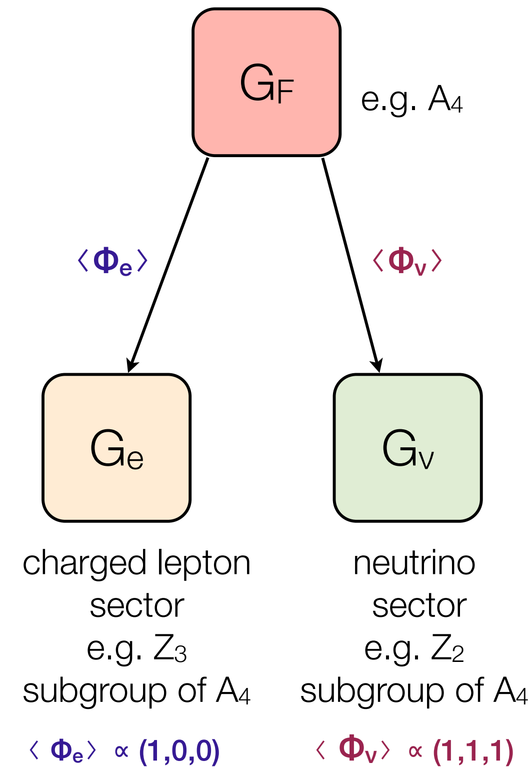
Leurer, Nir, Seiberg ('93);
Dudas, Pokorski, Savoy ('95)

  - sizable in discrete symmetry models for leptons M.-C.C, M. Fallbacher, M. Ratz, C. Staudt (2012)

SU(5) Compatibility ⇒ T′ Family Symmetry

M.-C.C, K.T. Mahanthappa (2007, 2009)

- Double Tetrahedral Group T′: double covering of A4
- Symmetries ⇒ 10 parameters in Yukawa sector ⇒ 22 physical observables
- neutrino mixing angles from group theory (CG coefficients)
- TBM: misalignment of symmetry breaking patterns



- GUT symmetry ⇒ **correlations among mixing parameters**

⇒ deviation from TBM related to Cabibbo angle θ_c

$$\theta_{13} \simeq \theta_c/3\sqrt{2}$$ ← CG's of SU(5) & T′

$\delta \approx 227^\circ$

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c \cos\delta$$

- large θ_{13} possible with one additional singlet flavon

M.-C. C., J. Huang, K.T. Mahanthappa, A. Wijiangco (2013)

Symmetry Relations

Quark Mixing

mixing parameters	best fit	3σ range
θ^q_{23}	2.36°	2.25° - 2.48°
θ^q_{12}	12.88°	12.75° - 13.01°
θ^q_{13}	0.21°	0.17° - 0.25°

Lepton Mixing

mixing parameters	best fit	3σ range
θ^e_{23}	41.2°	35.1° - 52.6°
θ^e_{12}	33.6°	30.6° - 36.8°
θ^e_{13}	8.9°	7.5° -10.2°

- QLC-I $\theta_C + \theta_{sol} \cong 45^o$ Raidal, '04; Smirnov, Minakata, '04

  (BM) $\theta^q_{23} + \theta^e_{23} \cong 45^o$ ☜ **slight inconsistent**

- QLC-II $\tan^2\theta_{sol} \cong \tan^2\theta_{sol,TBM} + (\theta_C / 2) * \cos\delta_e$ Ferrandis, Pakvasa; Dutta, Mimura; M.-C.C., Mahanthappa

  (TBM) $\theta^e_{13} \cong \theta_C / 3\sqrt{2}$ ☜ **Too small**

- testing symmetry relations: a *more* robust way to distinguish different classes of models

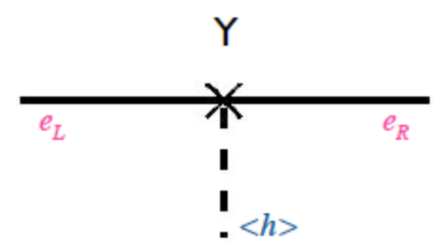
measuring leptonic mixing parameters to the precision of those in quark sector

Origin of CP Violation

- CP violation ⇔ complex mass matrices for quarks (and possibly) leptons

$$\overline{U}_{R,i}(M_u)_{ij}Q_{L,j} + \overline{Q}_{L,j}(M_u^\dagger)_{ji}U_{R,i} \xrightarrow{\mathcal{CP}} \overline{Q}_{L,j}(M_u)_{ij}U_{R,i} + \overline{U}_{R,i}(M_u)^*_{ij}Q_{L,j}$$

- Conventionally, CPV arises in two ways:
  - Explicit CP violation: complex Yukawa coupling constants Y
  - Spontaneous CP violation: complex scalar VEVs <h>



- **Complex CG coefficients in certain discrete groups ⇒ explicit CP violation**
  - CPV in quark and lepton sectors purely from complex CG coefficients

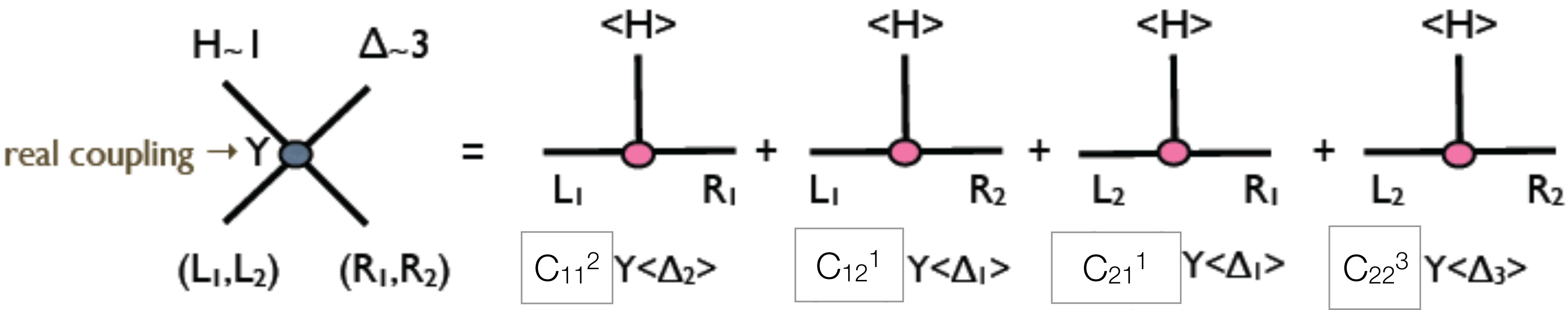
CG coefficients in non-Abelian discrete symmetries
➪ relative strengths and phases in entries of Yukawa matrices
➪ mixing angles and phases (and mass ordering)

Group Theoretical Origin of CP Violation

M.-C.C., K.T. Mahanthappa
Phys. Lett. B681, 444 (2009)

Basic idea

Discrete symmetry ***G***



- Scalar potential: if Z_3 symmetric $\Rightarrow$ $\langle \Delta_1 \rangle = \langle \Delta_2 \rangle = \langle \Delta_3 \rangle \equiv \langle \Delta \rangle$ real
- Complex effective mass matrix: **phases determined by group theory**

$C_{ij}{}^{k}$: complex CG coefficients of ***G***

$$\begin{pmatrix} L_1 & L_2 \end{pmatrix}$$

$$M = \begin{pmatrix} C_{11}{}^{2} & C_{21}{}^{1} \\ C_{12}{}^{1} & C_{22}{}^{3} \end{pmatrix} Y \langle \Delta \rangle \begin{pmatrix} R_1 \\ R_2 \end{pmatrix}$$

CP Transformation

- Canonical CP transformation

$$\phi(x) \stackrel{C\mathcal{P}}{\longmapsto} \eta_{C\mathcal{P}} \; \phi^*(\mathcal{P}x)$$

freedom of re–phasing fields

- Generalized CP transformation

Ecker, Grimus, Konetschny (1981); Ecker, Grimus, Neufeld (1987); Grimus, Rebelo (1995)

$$\Phi(x) \stackrel{\widetilde{C\mathcal{P}}}{\longmapsto} U_{\mathrm{CP}} \, \Phi^*(\mathcal{P} \; x)$$

unitary matrix

Generalized CP Transformation

- setting w/ discrete symmetry G

G and CP transformations do not commute

- generalized CP transformation

Feruglio, Hagedorn, Ziegler (2013); Holthausen, Lindner, Schmidt (2013)

- invariant contraction/coupling in A_4 or T'

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \propto \phi\,\left(x_1\,y_1 + \omega^2\,x_2\,y_2 + \omega\,x_3\,y_3\right)$$

$\omega = \mathrm{e}^{2\pi\,\mathrm{i}/3}$

- canonical CP transformation maps A_4/T' invariant contraction to something non–invariant

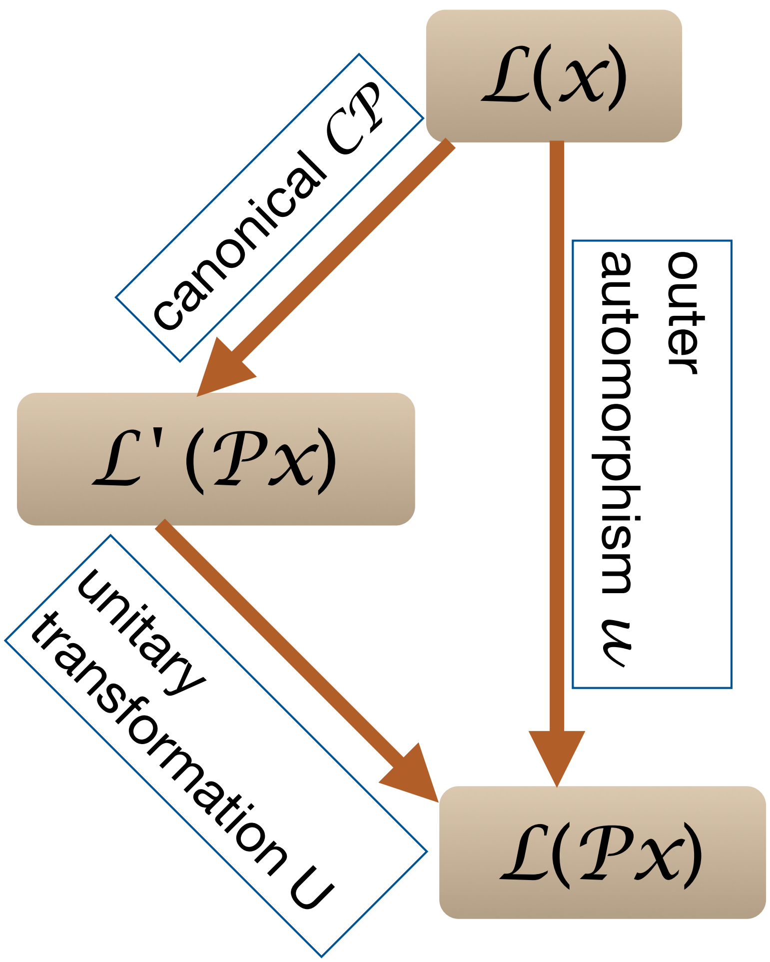
➥ need generalized CP transformation $\widetilde{\mathcal{CP}}$: $\phi \stackrel{\widetilde{\mathcal{CP}}}{\longmapsto} \phi^*$ as usual but

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \stackrel{\widetilde{\mathcal{CP}}}{\longmapsto} \begin{pmatrix} x_1^* \\ x_3^* \\ x_2^* \end{pmatrix} \quad \& \quad \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \stackrel{\widetilde{\mathcal{CP}}}{\longmapsto} \begin{pmatrix} y_1^* \\ y_3^* \\ y_2^* \end{pmatrix}$$

Group Theoretical Origin of CP Violation

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

complex CGs ➪ G and physical CP transformations do not commute



$$\Phi(x) \xmapsto{\widetilde{CP}} U_{\mathrm{CP}}\, \Phi^*(\mathcal{P}\ x)$$

$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) = U_{\boldsymbol{r}_i}\, \rho_{\boldsymbol{r}_i}(g)^*\, U_{\boldsymbol{r}_i}^\dagger \quad \forall\, g \in G \text{ and } \forall\, i$$

u has to be a **class-inverting**, **involuntory** automorphism of G

➪ such automorphism is NOT available in certain groups

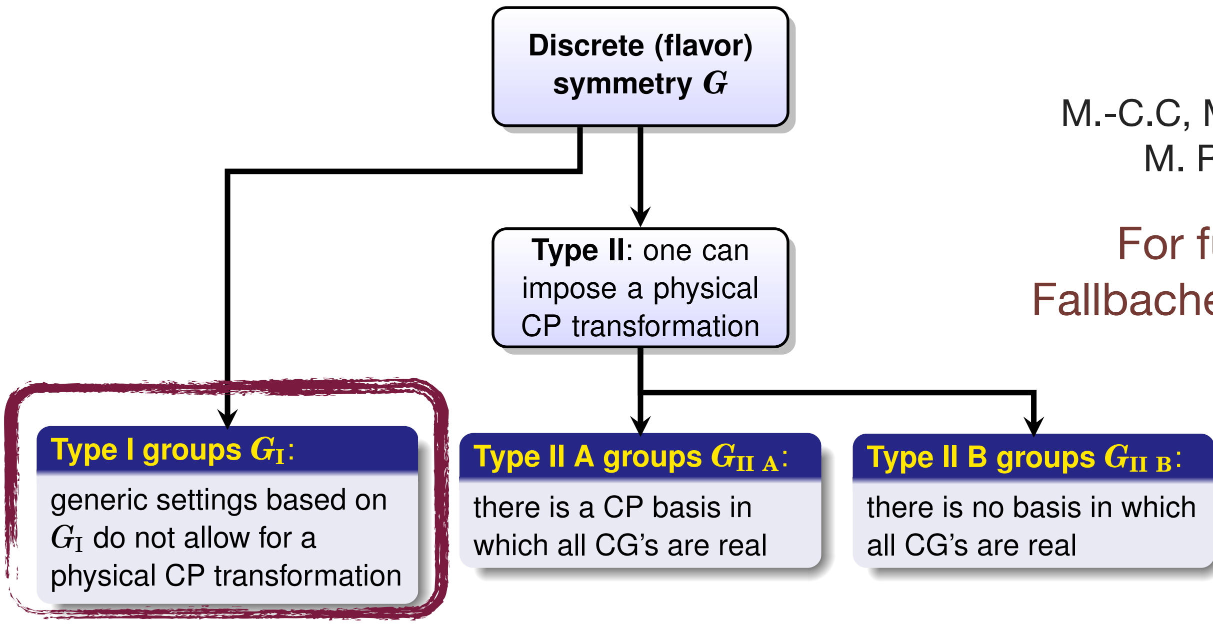
➪ explicit physical CP violation in generic setting

examples: T_7, $\Delta(27)$, …..

A Novel Origin of CP Violation

- **more generally, for discrete groups that do not have class-inverting, involutory automorphism, CP is generically broken by complex CG coefficients (Type I Group)**
- **Non-existence of such automorphism ⇔ physical CP violation**

CP Violation from Group Theory!



M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

For further insights, see M. Fallbacher, A. Trautner, NPB (2015)

- **Possible connection between leptogenesis and CPV in neutrino oscillation**

Example for a type I group:

$$\boldsymbol{\Delta(27)}$$

- decay asymmetry in a toy model
- prediction of CP violating phase from group theory

Toy Model based on Δ(27)

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Field content

fermions

field	S	X	Y	Ψ	Σ
$\Delta(27)$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{3}$	$\mathbf{3}$
U(1)	$q_\Psi - q_\Sigma$	$q_\Psi - q_\Sigma$	0	q_Ψ	q_Σ

$q_\Psi - q_\Sigma \neq 0$

- Interactions

$$\mathscr{L}_{\text{toy}} = F^{ij}\, S\, \overline{\Psi}_i \Sigma_j + G^{ij}\, X\, \overline{\Psi}_i \Sigma_j + H_\Psi^{ij}\, Y\, \overline{\Psi}_i \Psi_j + H_\Sigma^{ij}\, Y\, \overline{\Sigma}_i \Sigma_j + \text{h.c.}$$

$$F = f\, \mathbb{1}_3$$

$$G = g \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$H_{\Psi/\Sigma} = h_{\Psi/\Sigma} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}$$

with $\omega := e^{2\pi\, i/3}$

"flavor" structures determined by (complex) CG coefficients

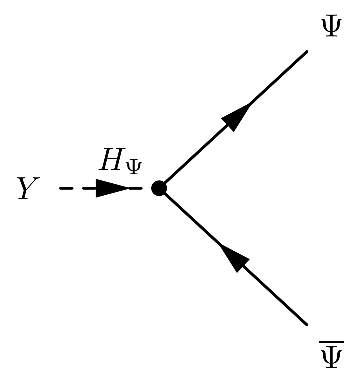
arbitrary coupling constants: f, g, h_Ψ, h_Σ

Toy Model based on Δ(27)

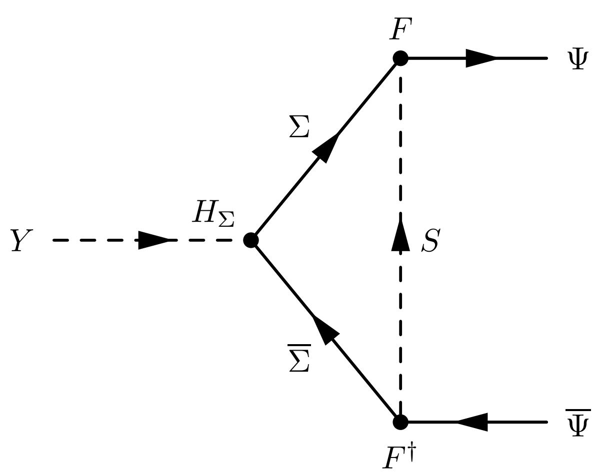
M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

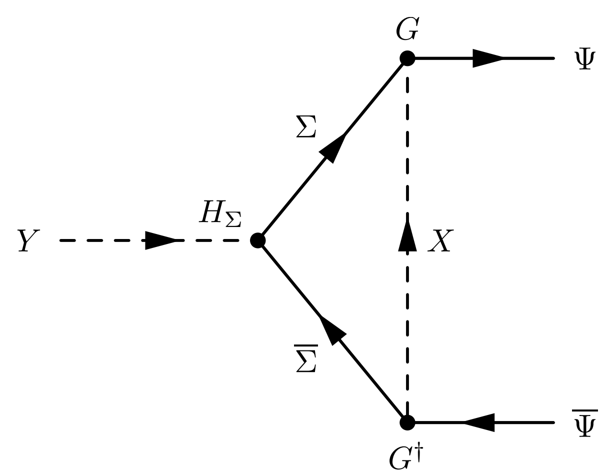
- Particle decay $Y \rightarrow \overline{\Psi}\Psi$

interference of



with





Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\varepsilon_{Y \to \overline{\Psi}\Psi} = \frac{\Gamma(Y \to \overline{\Psi}\Psi) - \Gamma(Y^* \to \overline{\Psi}\Psi)}{\Gamma(Y \to \overline{\Psi}\Psi) + \Gamma(Y^* \to \overline{\Psi}\Psi)}$$

$$\propto \;\; \mathrm{Im}\,[I_S]\;\mathrm{Im}\left[\mathrm{tr}\left(F^\dagger\, H_\Psi\, F\, H_\Sigma^\dagger\right)\right] + \mathrm{Im}\,[I_X]\;\mathrm{Im}\left[\mathrm{tr}\left(G^\dagger\, H_\Psi\, G\, H_\Sigma^\dagger\right)\right]$$

$$= \;\; |f|^2\;\mathrm{Im}\,[I_S]\;\mathrm{Im}\,[h_\Psi\, h_\Sigma^*] + |g|^2\;\mathrm{Im}\,[I_X]\;\mathrm{Im}\,[\omega\, h_\Psi\, h_\Sigma^*]\;\;.$$

one-loop integral $I_S = I(M_S, M_Y)$

one-loop integral $I_X = I(M_X, M_Y)$

- properties of ε
  - invariant under rephasing of fields
  - independent of phases of f and g
  - basis independent

Decay Asymmetry

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Decay asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = |f|^2 \ \mathrm{Im}\left[I_S\right] \ \mathrm{Im}\left[h_\Psi h_\Sigma^*\right] + |g|^2 \ \mathrm{Im}\left[I_X\right] \ \mathrm{Im}\left[\omega h_\Psi h_\Sigma^*\right]$$

  - cancellation requires delicate adjustment of relative phase $\varphi := \arg(h_\Psi h_\Sigma^*)$
  - for non-degenerate M_S and M_X: $\mathrm{Im}\left[I_S\right] \neq \mathrm{Im}\left[I_X\right]$
    - phase φ unstable under quantum corrections
  - for $\mathrm{Im}\left[I_S\right] = \mathrm{Im}\left[I_X\right]$ & $|f| = |g|$
    - phase φ stable under quantum corrections
    - relations cannot be ensured by outer automorphism of Δ(27)
    - require symmetry larger than Δ(27)

model based on Δ(27) violates CP!

Spontaneous CP Violation with Calculable CP Phase

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

field	X	Y	Z	Ψ	Σ	ϕ
$\Delta(27)$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{1}_8$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_0$
U(1)	$2q_\Psi$	0	$2q_\Psi$	q_Ψ	$-q_\Psi$	0

$\Delta(27) \subset \mathrm{SG}(54,5)$: $\begin{cases} (X, Z) & : \text{doublet} \\ (\Psi, \Sigma^C) & : \text{hexaplet} \\ \phi & : \text{non–trivial 1–dim. representation} \end{cases}$

- ☞ non–trivial $\langle\phi\rangle$ breaks $\mathrm{SG}(54,5) \to \Delta(27)$

- ☞ allowed coupling leads to mass splitting $\mathscr{L}^{\phi}_{\mathrm{toy}} \supset M^2\left(|X|^2 + |Z|^2\right) + \left[\frac{\mu}{\sqrt{2}}\langle\phi\rangle\left(|X|^2 - |Z|^2\right) + \text{h.c.}\right]$

CG coefficient of $\mathrm{SG}(54,5)$

- ➥ CP asymmetry with calculable phases

$$\varepsilon_{Y\to\overline{\Psi}\Psi} \propto |g|^2\,|h_\Psi|^2\ \mathrm{Im}\left[\ \omega\ \right]\ \left(\mathrm{Im}\left[I_X\right] - \mathrm{Im}\left[I_Z\right]\right)$$

phase predicted by group theory

Group theoretical origin of CP violation!

M.-C.C., K.T. Mahanthappa (2009)

Discrete R Symmetries in MSSM

- Minimal Supersymmetric Standard Model:
  - μ problem: why the parameter determining the Higgs mass << Planck scale?
  - dim-5 proton decay operators
- simultaneous solution possible with (generation dependent) discrete R symmetries (Abelian or even non-Abelian!) M.-C.C., M. Ratz, A. Trautner, JHEP 1309 (2013) 096
- Naturally small Dirac neutrino mass (no $\Delta L = 2$ violation) M.-C.C, M. Ratz, C. Staudt, P. Vaudrevange, Nucl. Phys. B866 (2013) 157
  - $\Delta L = 4$ violation possible ⇒ neutrinoless quadruple beta decay
- Evading current constraints on (non-observation of) SUSY:
  - R parity violation from discrete R symmetries ⇔ SUSY breaking

Poster by Volodymyr Takhistov

M.-C.C, M. Ratz, V. Takhistov, Nucl. Phys. B891 (2015) 322

- No-Go Theorem: no R-symmetries in 4D GUTs M. Fallbacher, M. Ratz, P. Vaudrevange, PLB705 (2011) 503
  - one way out ⇒ KK towers in extra dimensions M. W. Goodman, E. Witten, NPB 271 (1986) 21

A Giant Physicist and Human Being

Summary

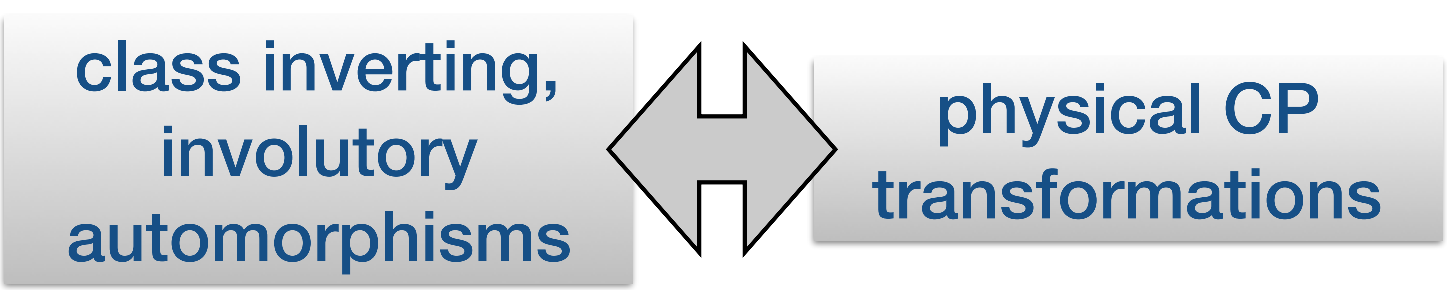
- Fundamental origin of fermion mass hierarchy and flavor mixing still not known
- Neutrino masses: evidence of physics beyond the SM
- Symmetries: can provide an understanding of the pattern of fermion masses and mixing
  - Grand unified symmetry + discrete family symmetry ⇒ predictive power
  - Symmetries ⇒ Correlations, Correlations, Correlations!!!
    - leading order sum rules between quark & lepton mixing parameters
    - among lepton flavor violating charged lepton decays
    - among proton (nucleon) decays, neutron-antineutron oscillation
  - corrections to kinetic terms need to be properly included
- Discrete R-symmetries:
  - naturally light Dirac neutrinos
  - suppressed nucleon decays and naturally small mu term

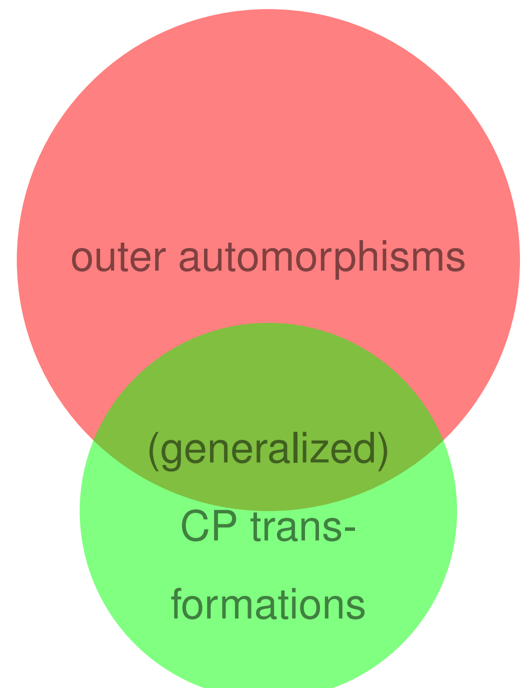
Summary

- Discrete Groups (of Type I) affords a Novel origin of CP violation:
  - Complex CGs ⇒ Group Theoretical Origin of CP Violation
- **NOT all outer automorphisms correspond to physical CP transformations**
- **Condition on automorphism for *physical* CP transformation**

$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) \;=\; U_{\boldsymbol{r}_i}\, \rho_{\boldsymbol{r}_i}(g)^*\, U_{\boldsymbol{r}_i}^{\dagger} \qquad \forall\, g \in G \text{ and } \forall\, i$$

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

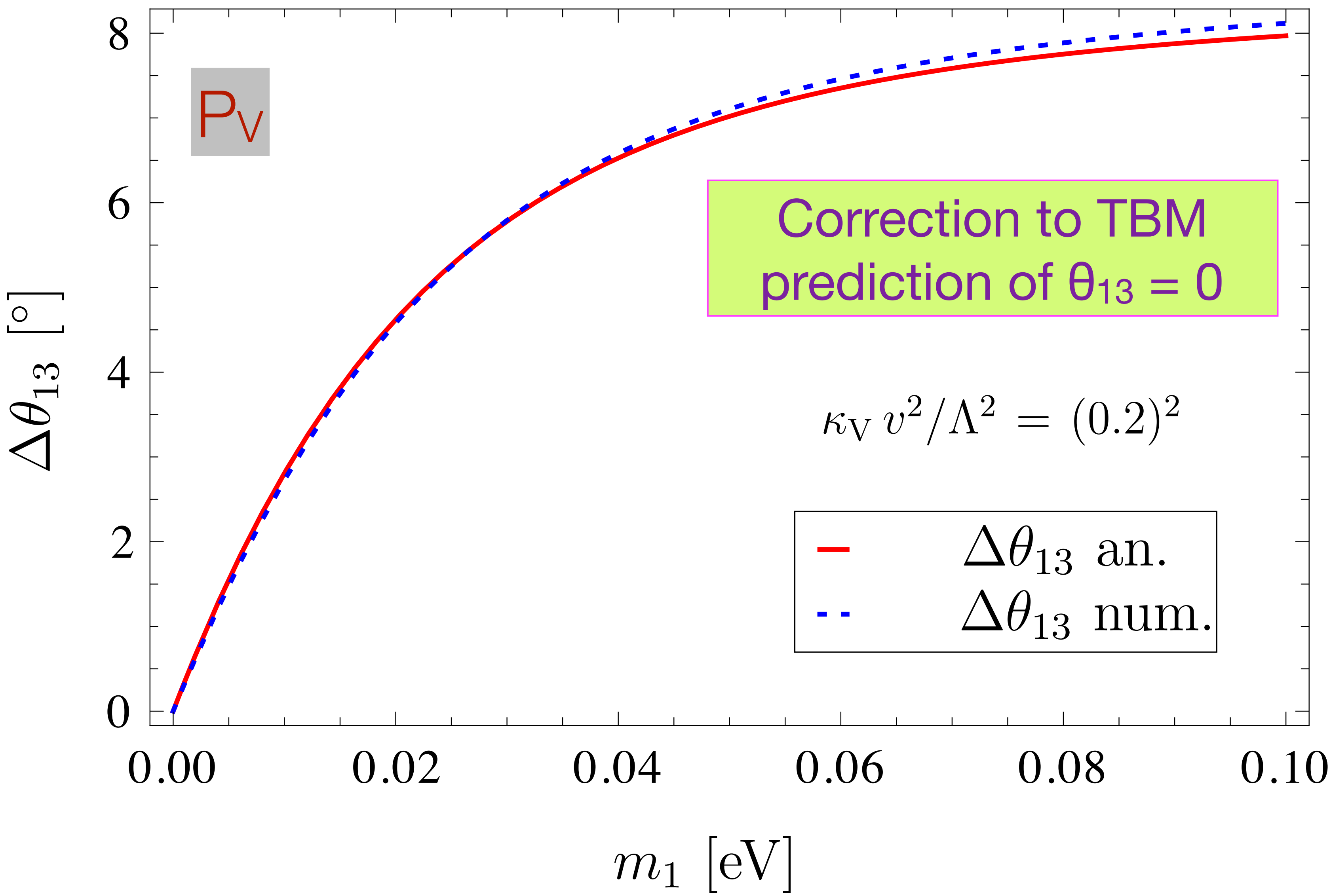




Backup Slides

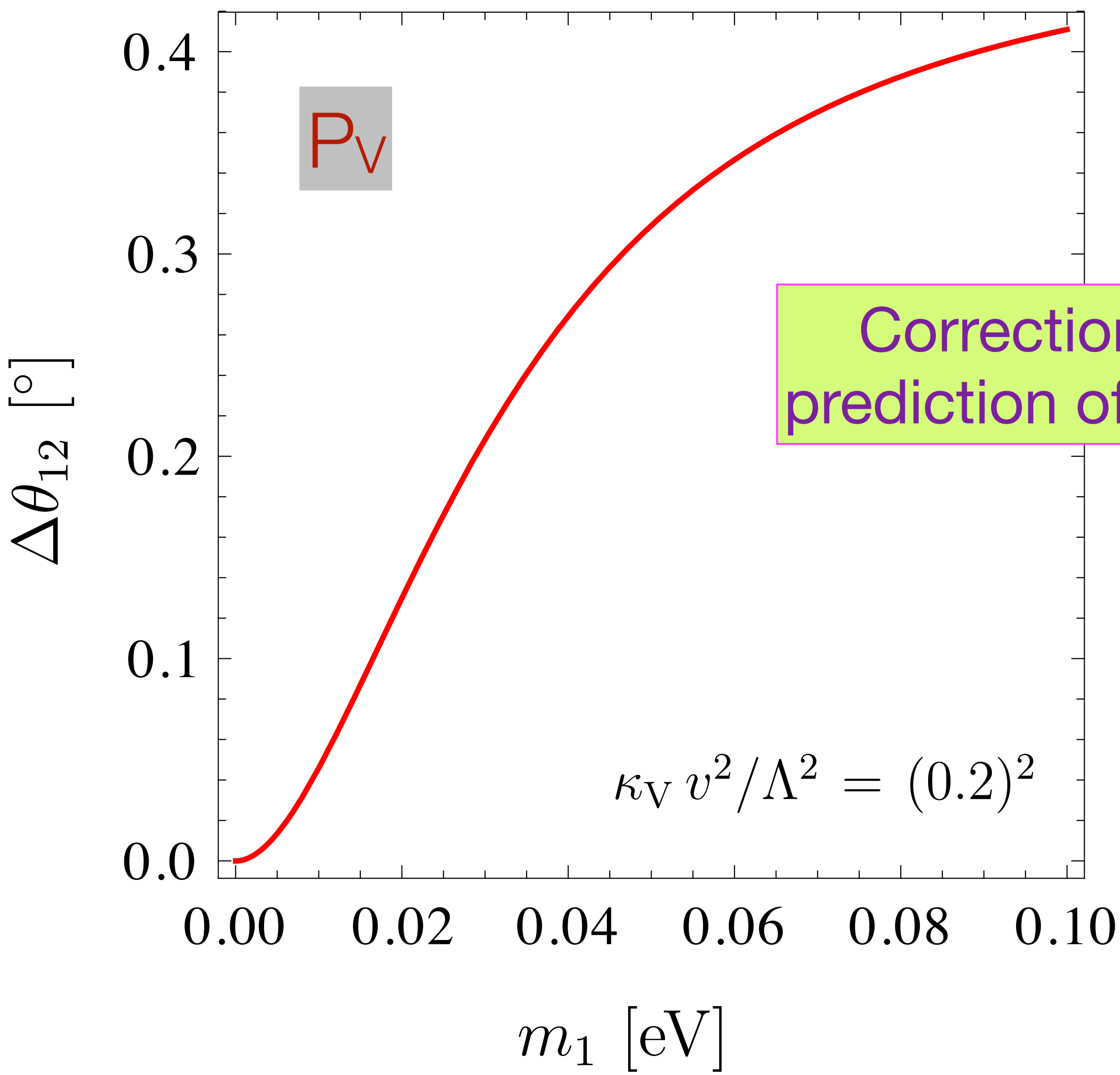
An Example: Enhanced θ_{13} in A_4

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



Corresponding Change in θ_{12}

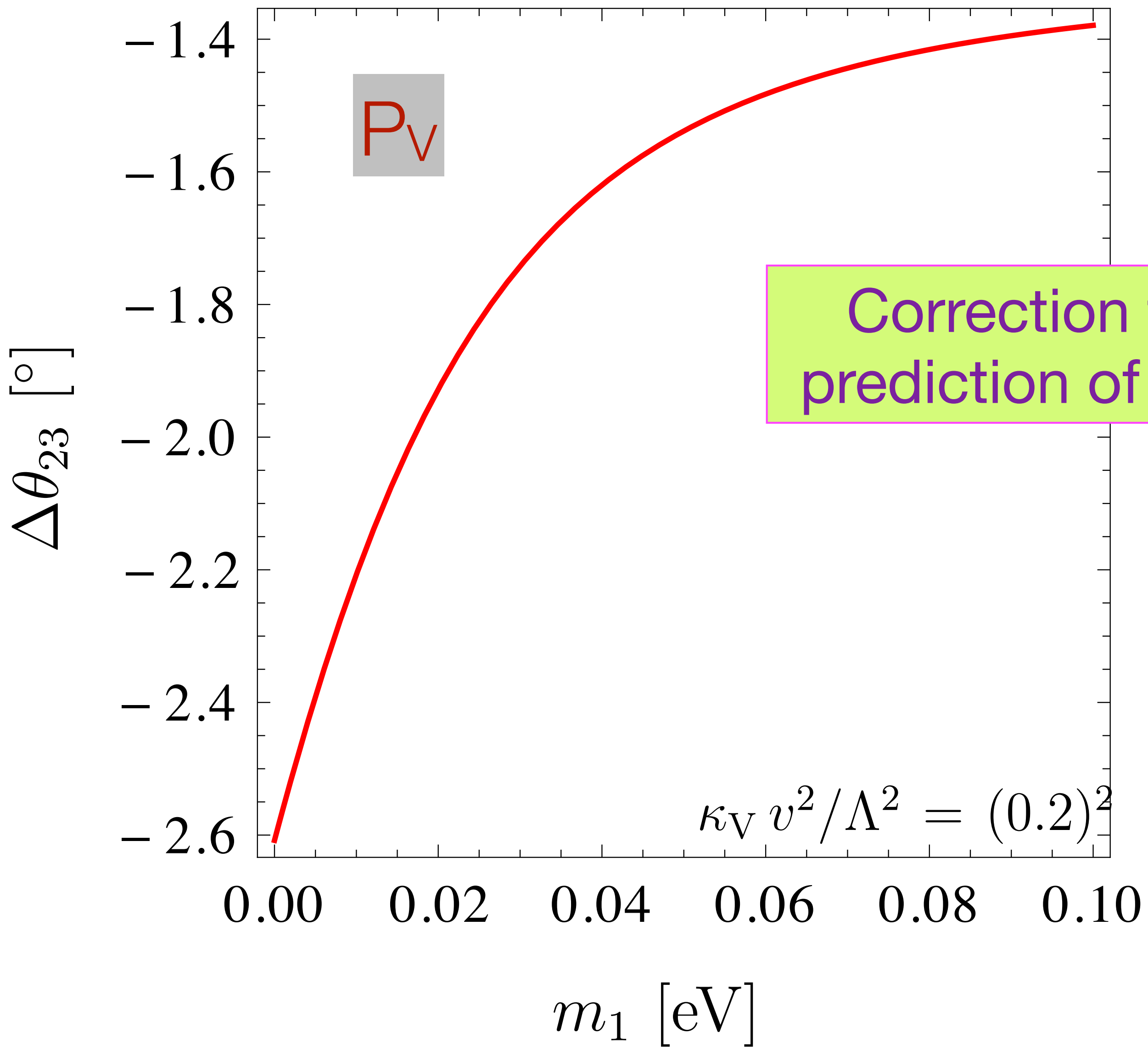
M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



Correction to TBM prediction of θ_{12} = 35.3°

Corresponding Change in θ_{23}

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



Correction to TBM prediction of $\theta_{23} = 45°$

Constraints on generalized CP transformations

☞ generalized CP transformation

$$\Phi(x) \quad \xrightarrow{\widetilde{CP}} \quad U_{\mathrm{CP}}\, \Phi^*(\,\mathcal{P}\;\, x)$$

☞ consistency condition

Holthausen, Lindner, and Schmidt (2013)

$$\rho\big(u(g)\big) \;=\; U_{\mathrm{CP}}\;\rho(g)^*\, U_{\mathrm{CP}}{}^{\dagger} \quad \forall\, g \in G$$

☞ further properties:

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) \;=\; U_{\boldsymbol{r}_i}\, \rho_{\boldsymbol{r}_i}(g)^*\, U_{\boldsymbol{r}_i}^{\dagger} \quad \forall\, g \in G \text{ and } \forall\, i$$

⟺ physical CP transformations

- u has to be class–inverting
- in all known cases, u is equivalent to an automorphism of order two

bottom–line:

u has to be a class–inverting (involutory) automorphism of G

The Bickerstaff-Damhus automorphism (BDA)

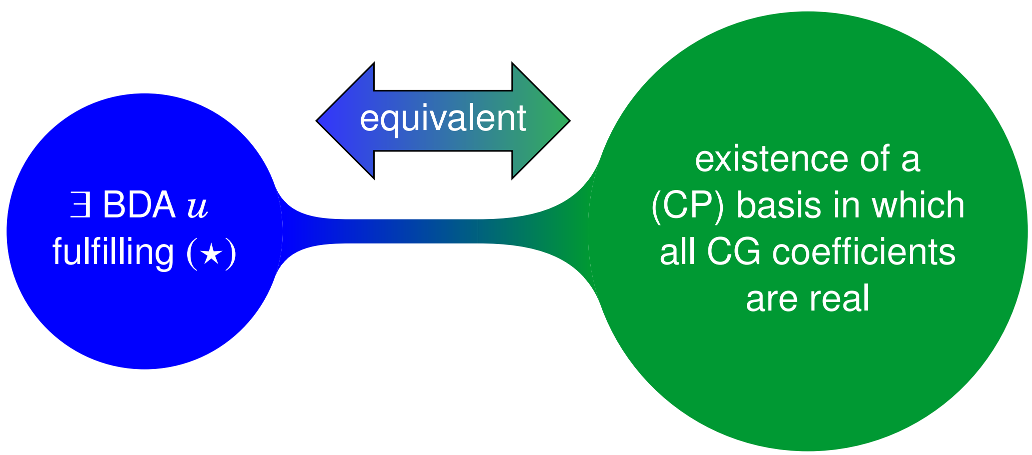
- Bickerstaff-Damhus automorphism (BDA) u

Bickerstaff, Damhus (1985)

$$\rho_{\boldsymbol{r}_i}(u(g)) \;=\; U_{\boldsymbol{r}_i}\;\rho_{\boldsymbol{r}_i}(g)^*\,U^\dagger_{\boldsymbol{r}_i} \quad \forall\, g \in G \text{ and } \forall\, i \qquad (\star)$$

unitary & symmetric

- BDA vs. Clebsch-Gordan (CG) coefficients



Twisted Frobenius-Schur Indicator

- How can one tell whether or not a given automorphism is a BDA?
- Frobenius-Schur indicator:

$$\mathrm{FS}(\boldsymbol{r}_i) := \frac{1}{|G|} \sum_{g \in G} \chi_{\boldsymbol{r}_i}(g^2) = \frac{1}{|G|} \sum_{g \in G} \mathrm{tr}\left[\rho_{\boldsymbol{r}_i}(g)^2\right]$$

$$\mathrm{FS}(\boldsymbol{r}_i) = \begin{cases} +1, & \text{if } \boldsymbol{r}_i \text{ is a real representation,} \\ 0, & \text{if } \boldsymbol{r}_i \text{ is a complex representation,} \\ -1, & \text{if } \boldsymbol{r}_i \text{ is a pseudo–real representation.} \end{cases}$$

- Twisted Frobenius-Schur indicator

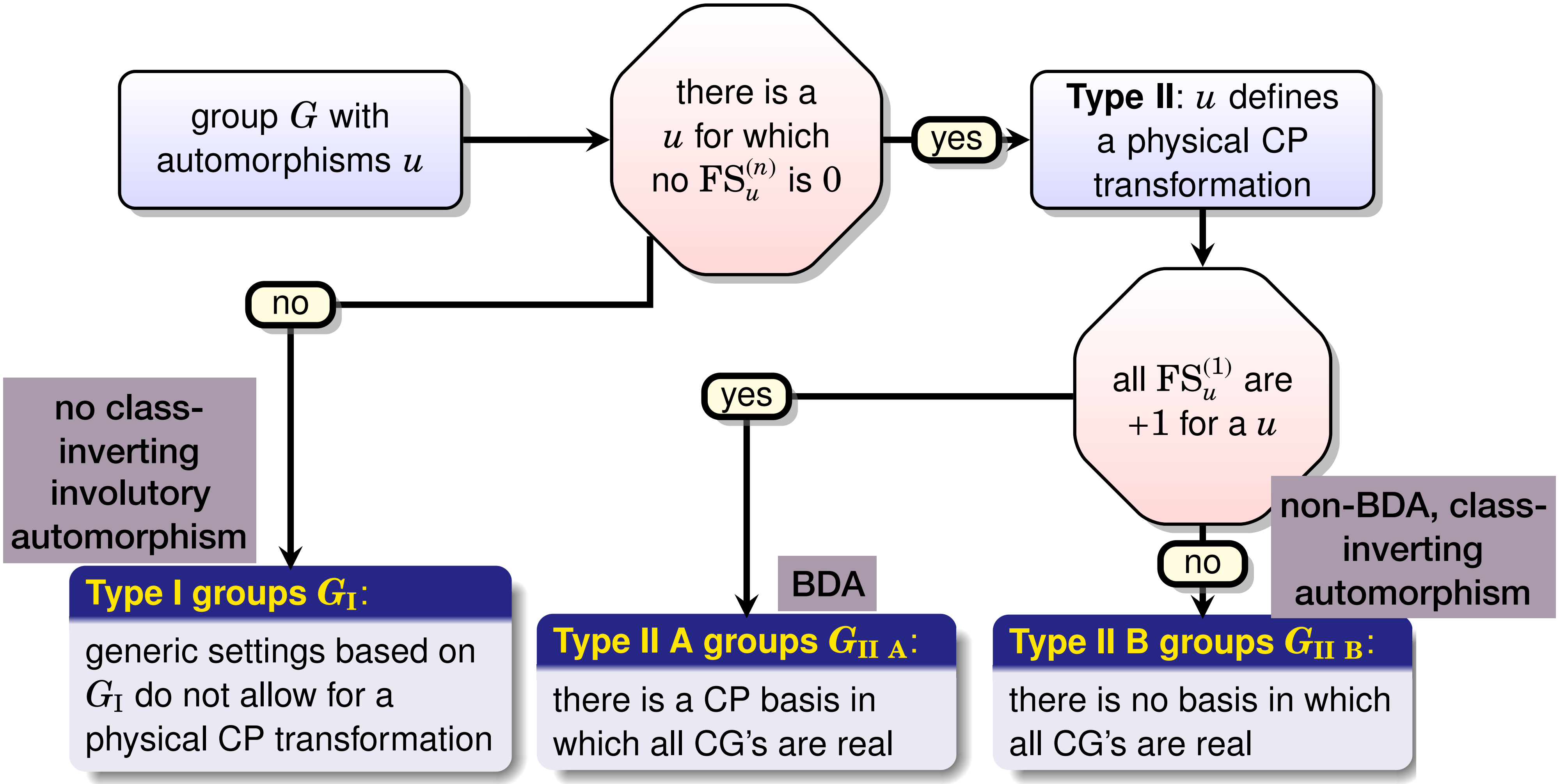
Bickerstaff, Damhus (1985); Kawanaka, Matsuyama (1990)

$$\mathrm{FS}_u(\boldsymbol{r}_i) = \frac{1}{|G|} \sum_{g \in G} \left[\rho_{\boldsymbol{r}_i}(g)\right]_{\alpha\beta} \left[\rho_{\boldsymbol{r}_i}(u(g))\right]_{\beta\alpha}$$

$$\mathrm{FS}_u(\boldsymbol{r}_i) = \begin{cases} +1 \quad \forall\, i, & \text{if } u \text{ is a BDA,} \\ +1 \text{ or } -1 \quad \forall\, i, & \text{if } u \text{ is class–inverting and involutory,} \\ \text{different from } \pm 1, & \text{otherwise.} \end{cases}$$

Three Types of Finite Groups

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)



Examples

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- Type I: all odd order non-Abelian groups

group	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$
SG	(20,3)	(21,1)	(27,3)	(27,4)

- Type IIA: dihedral and all Abelian groups

group	S_3	Q_8	A_4	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	T'	S_4	A_5
SG	(6,1)	(8,4)	(12,3)	(24,1)	(24,3)	(24,12)	(60,5)

- Type IIB

group	$\Sigma(72)$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG	(72,41)	(144,120)

CP Conservation vs Symmetry Enhancement

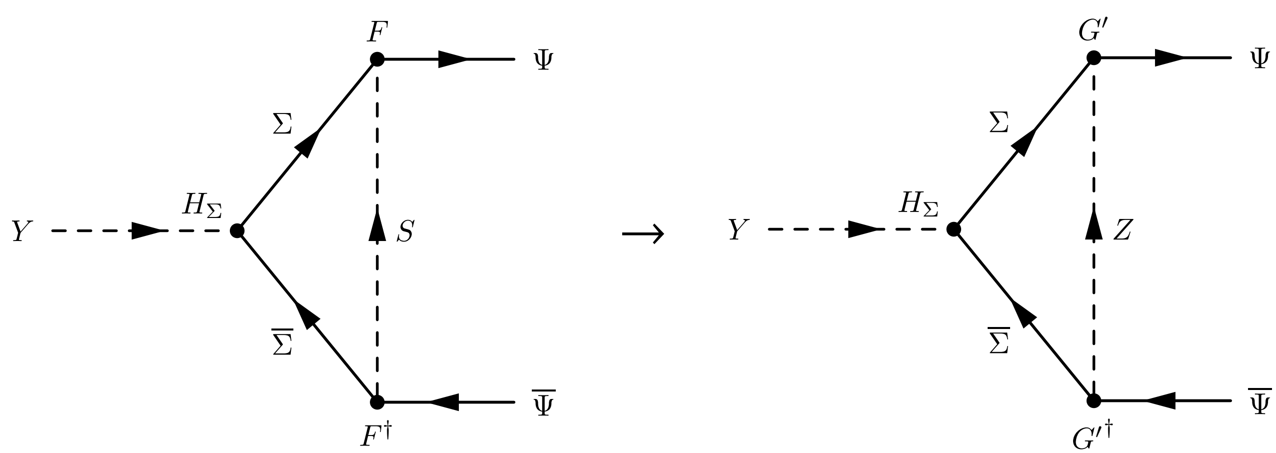
M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

- replace $S \sim \mathbf{1}_0$ by $Z \sim \mathbf{1}_8$ $\curvearrowright$ interaction

$$\mathscr{L}_{\text{toy}}^{Z} = g' \left[Z_{\mathbf{1}_8} \otimes \left(\overline{\Psi}\Sigma\right)_{\mathbf{1}_4} \right]_{\mathbf{1}_0} + \text{h.c.} = (G')^{ij}\; Z\, \overline{\Psi}_i \Sigma_j + \text{h.c.}$$

$$G' = g' \begin{pmatrix} 0 & 0 & \omega^2 \\ 1 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix}$$

and leads to new interference diagram



CP Conservation vs Symmetry Enhancement

M.-C.C., M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner (2014)

☞ replace $S \sim \mathbf{1}_0$ by $Z \sim \mathbf{1}_8$ ↷ interaction

$$\mathscr{L}^Z_{\text{toy}} = g' \left[Z_{\mathbf{1}_8} \otimes \left(\overline{\Psi} \Sigma \right)_{\mathbf{1}_4} \right]_{\mathbf{1}_0} + \text{h.c.} = (G')^{ij}\ Z\, \overline{\Psi}_i \Sigma_j + \text{h.c.}$$

➥ different contribution to decay asymmetry: $\varepsilon^S_{Y \to \overline{\Psi}\Psi} \ \to \ \varepsilon^Z_{Y \to \overline{\Psi}\Psi}$

☞ total CP asymmetry of the Y decay vanishes if

$$\begin{cases} \text{(i)} & M_Z = M_X \\ \text{(ii)} & |g| = |g'| \\ \text{(iii)} & \varphi = 0 \end{cases}$$

☞ relations (i)—(iii) can be due to an outer automorphism

$$X \overset{u_3}{\longleftrightarrow} Z\,, \quad Y \overset{u_3}{\longrightarrow} Y\,, \quad \Psi \overset{u_3}{\longrightarrow} U_{u_3}\, \Sigma^C \quad \& \quad \Sigma \overset{u_3}{\longrightarrow} U_{u_3}\, \Psi^C$$

requires $q_\Sigma = -q_\Psi$

$$U_{u_3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

. . . BUT this enlarges $\Delta(27) \to \mathrm{SG}(54,5) \simeq \Delta(27) \rtimes \mathbb{Z}_2^{u_3}$

SG(54, 5): group name from GAP library

Some Outer Automorphisms of Δ(27)

- sample outer automorphisms of Δ(27)

$$u_1 \;:\; \mathbf{1}_1 \leftrightarrow \mathbf{1}_2\;,\; \mathbf{1}_4 \leftrightarrow \mathbf{1}_5\;,\; \mathbf{1}_7 \leftrightarrow \mathbf{1}_8\;,\; \mathbf{3} \rightarrow U_{u_1}\,\mathbf{3}^*$$
$$u_2 \;:\; \mathbf{1}_1 \leftrightarrow \mathbf{1}_4\;,\; \mathbf{1}_2 \leftrightarrow \mathbf{1}_8\;,\; \mathbf{1}_3 \leftrightarrow \mathbf{1}_6\;,\; \mathbf{3} \rightarrow U_{u_2}\,\mathbf{3}^*$$
$$u_3 \;:\; \mathbf{1}_1 \leftrightarrow \mathbf{1}_8\;,\; \mathbf{1}_2 \leftrightarrow \mathbf{1}_4\;,\; \mathbf{1}_5 \leftrightarrow \mathbf{1}_7\;,\; \mathbf{3} \rightarrow U_{u_3}\,\mathbf{3}^*$$
$$u_4 \;:\; \mathbf{1}_1 \leftrightarrow \mathbf{1}_7\;,\; \mathbf{1}_2 \leftrightarrow \mathbf{1}_5\;,\; \mathbf{1}_3 \leftrightarrow \mathbf{1}_6\;,\; \mathbf{3} \rightarrow U_{u_4}\,\mathbf{3}^*$$
$$u_5 \;:\; \mathbf{1}_i \leftrightarrow \mathbf{1}_i{}^*\;,\; \mathbf{3} \rightarrow U_{u_5}\,\mathbf{3}$$

- twisted Frobenius-Schur indicators

$\boldsymbol{R}$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{1}_4$	$\mathbf{1}_5$	$\mathbf{1}_6$	$\mathbf{1}_7$	$\mathbf{1}_8$	$\mathbf{3}$	$\overline{\mathbf{3}}$
$\mathrm{FS}_{u_1}(\boldsymbol{R})$	1	1	1	0	0	0	0	0	0	1	1
$\mathrm{FS}_{u_2}(\boldsymbol{R})$	1	0	0	1	0	0	1	0	0	1	1
$\mathrm{FS}_{u_3}(\boldsymbol{R})$	1	0	0	0	0	1	0	1	0	1	1
$\mathrm{FS}_{u_4}(\boldsymbol{R})$	1	0	0	1	0	0	1	0	0	1	1
$\mathrm{FS}_{u_5}(\boldsymbol{R})$	1	1	1	1	1	1	1	1	1	0	0

  - none of the u_i maps all representations to their conjugates
  - however, it is possible to impose CP in (non-generic) models, where only a subset of representations are present, e.g. $\{\boldsymbol{r}_i\} \subset \{\mathbf{1}_0, \mathbf{1}_5, \mathbf{1}_7, \mathbf{3}, \overline{\mathbf{3}}\}$
  - CP conservation possible in non-generic models
    - e.g. some well-known multiple Higgs model Branco, Gerard, and Grimus (1984)

CP-like Symmetries

- outer automorphism u_5

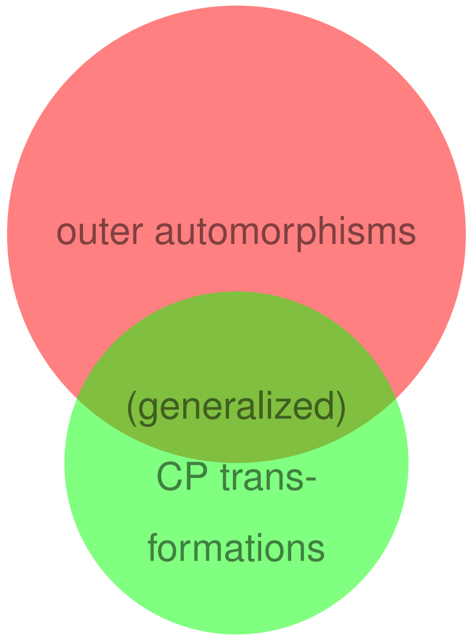
$$X \to X^* \,, \quad Z \to Z^* \,, \quad Y \to Y^* \,, \quad \Psi \to U_{u_5}\,\Sigma \quad \& \quad \Sigma \to U_{u_5}\,\Psi$$

$$U_{u_5} = \begin{pmatrix} 0 & 0 & \omega^2 \\ 0 & 1 & 0 \\ \omega & 0 & 0 \end{pmatrix}$$

- does **not** lead to a vanishing decay asymmetry

➥ in general, imposing an outer automorphism as a symmetry does not lead to physical CP conservation!

➥ CP–like symmetry



Summary

Three examples: M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

☞ Type I group: $\Delta(27)$

- generic settings based on $\Delta(27)$ violate CP!
- spontaneous breaking of type II A group $\mathrm{SG}(54, 5) \to \Delta(27)$
  $\curvearrowright$ prediction of CP violating phase from group theory!

☞ Type II A group: T'

- CP basis exists but has certain shortcomings
- advantageous to work in a different basis & impose generalized CP transformation
- CP constrains phases of coupling coefficients

☞ Type II B group: $\Sigma(72)$

- absence of CP basis but generalized CP transformation ensures physical CP conservation
- CP forbids couplings