BNL Virtual HET Seminar

Enhancing or Delaying Electroweak Phase Transitions via Simple Scalar Extensions

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Oct 28th, 2021

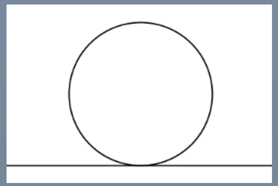
Higgs discovered and then what?

Electroweak Phase Transition

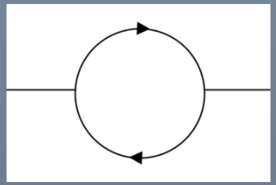
At finite temperatures, the Higgs potential receives thermal correction from degrees of freedom coupled to the Higgs field

$$V(H) = -(\mu_H^2 - c_H T^2)|H|^2 + \lambda_H |H|^4 + \cdots$$

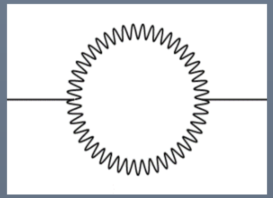
Higgs thermal mass



$\sim \lambda T^2$



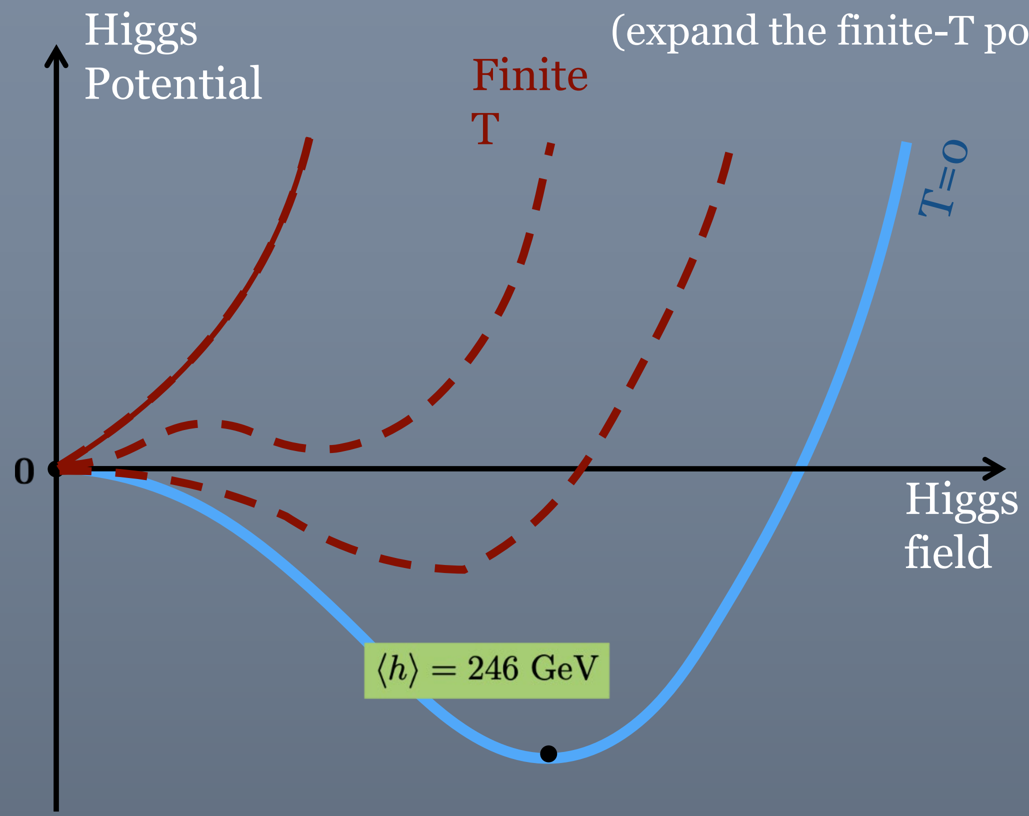
$\sim y^2 T^2$



$\sim g^2 T^2$

……

(expand the finite-T potential polynomially)

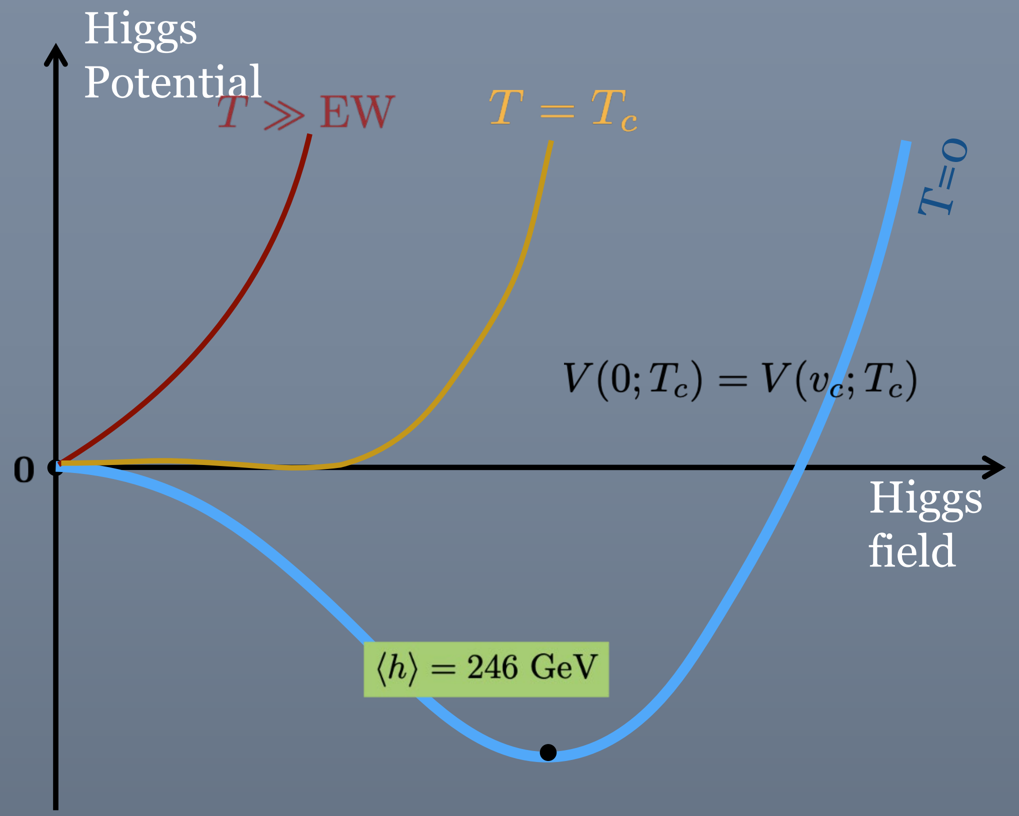


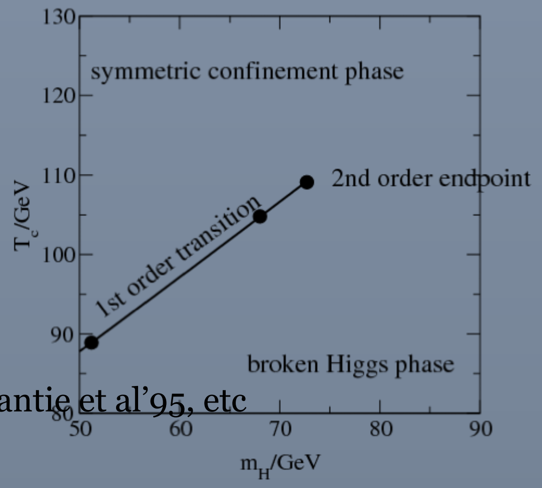
depends on particle content that couples to the Higgs

EW symmetry in the early universe: SM

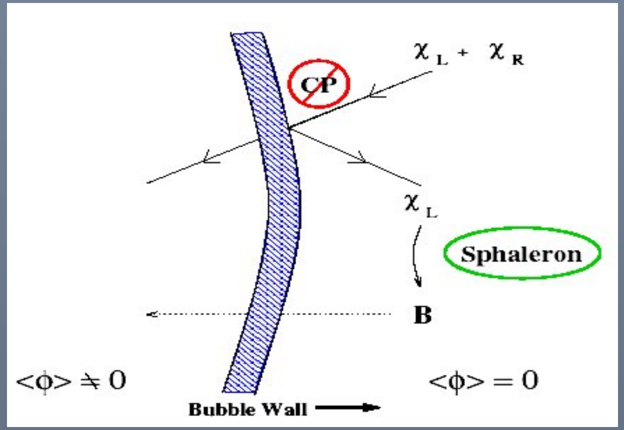
The Standard Model

The electroweak phase transition is a cross-over





Kajantie et al'95, etc



Sakharov's conditions

- Baryon number violation
- C and CP violation
- Out-of-equilibrium

Excluding the possibility of Electroweak baryogenesis (EWBG)

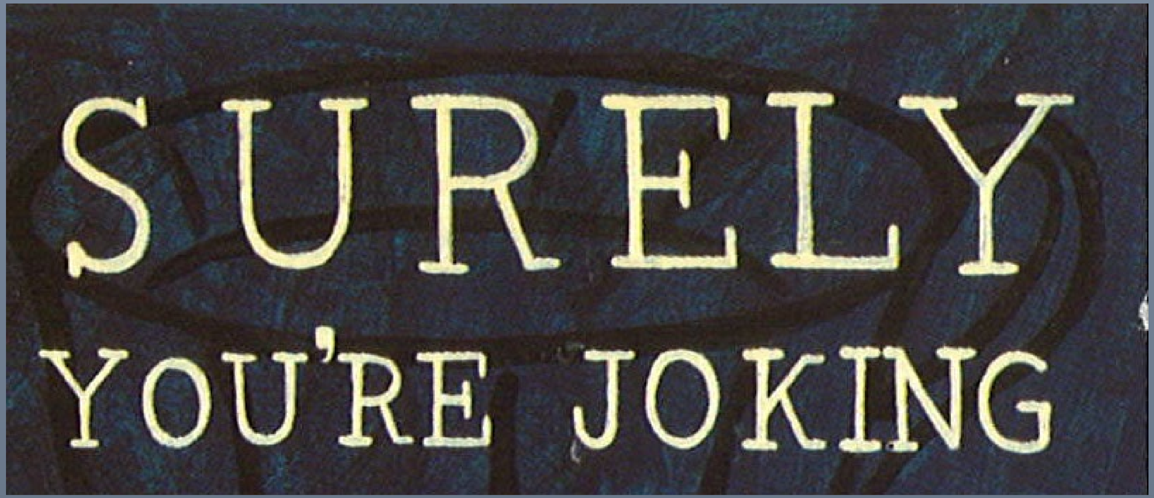
Outline

- A real singlet extension to Enhance EWPT
- A new approach to Delayed (or non-restoring) EW

Based upon work with
M. Carena and Y.-K. Wang, 1911.10206
M. Carena, C. Krause, Y.-K. Wang, 2104.00638

Outline

- A real singlet extension to Enhance EWPT



Based upon work with
M. Carena and Y.-K. Wang, 1911.10206

Enhancing EWPT through Singlet Extensions

One of the most generic extensions to Enhance EWPT;
An important benchmark to understand;

$$V_0(h,s) = -\frac{1}{2}\mu_h^2 h^2 + \frac{1}{4}\lambda_h h^4 + \frac{1}{2}\mu_s^2 s^2 + \frac{1}{4}\lambda_s s^4 + \frac{1}{4}\lambda_m h^2 s^2$$
$$+(\text{explicit Z2} - \text{breaking terms})$$

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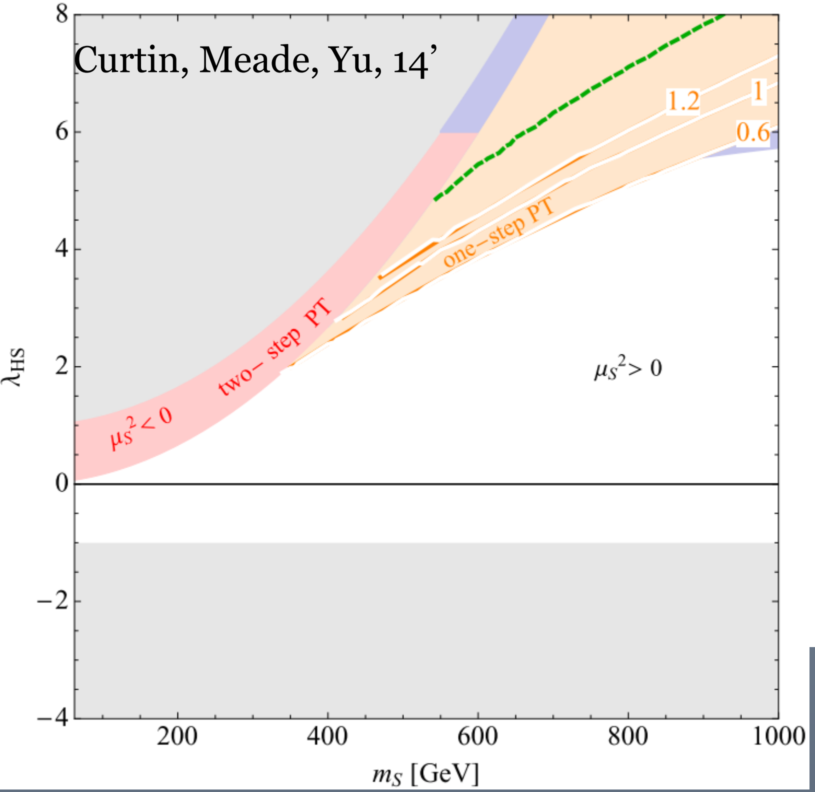

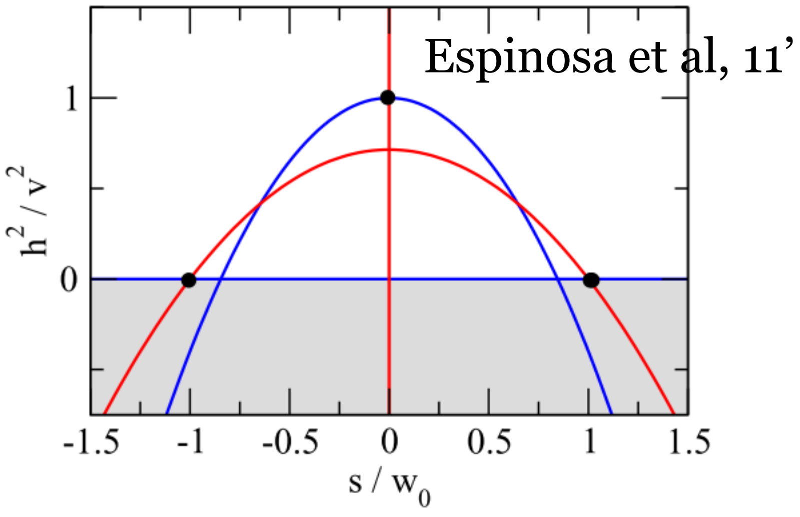

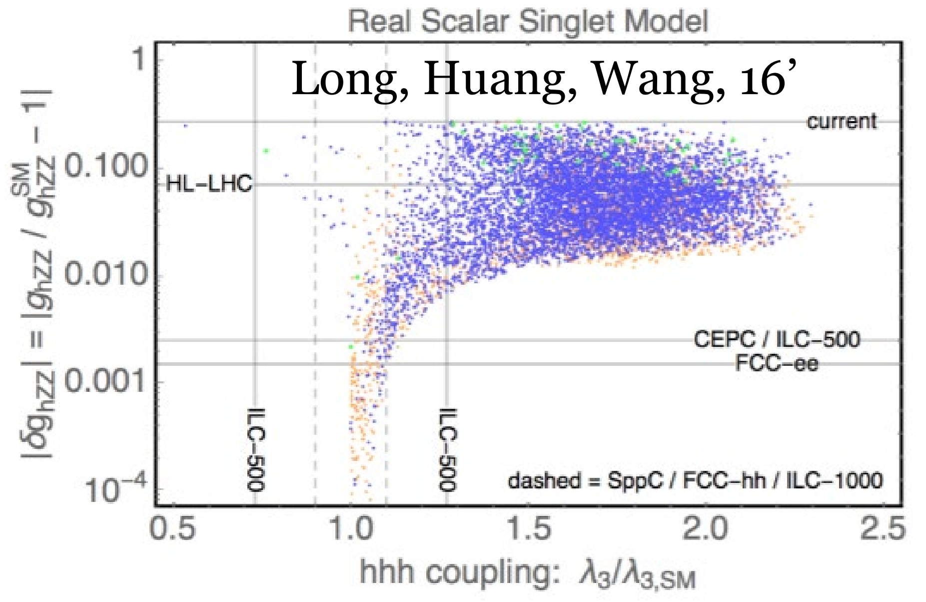

Spontaneous Z2 breaking Singlet Extension: a missing case

However, no clear studies on the Spontaneous Z2 breaking case

$$V_0(h,s) = -\frac{1}{2}\mu_h^2 h^2 + \frac{1}{4}\lambda_h h^4 + \frac{1}{2}\mu_s^2 s^2 + \frac{1}{4}\lambda_s s^4 + \frac{1}{4}\lambda_m h^2 s^2$$
$$+(\text{explicit Z2} - \text{breaking terms})$$

Well-Motivated:

- As a generic case possible for singlet extensions, which also leads to a rich thermal history;
- As a proxy (simplified discussion; with appropriate rescaling of couplings to match the d.o.f.) to evaluate:
  - Dark sector gauge theories needs to be Higgsed;
  - Dark Higgs talks to our sector through the $(H^+H)(H_d^+H_d)$ mixing quartic;

But Challenging to begin with!

Domain-wall problem solved by higher dimensional and highly suppressed operators that does not affect the phenomenology in this talk.

Spontaneous Z2 breaking Singlet Extension: a challenging case

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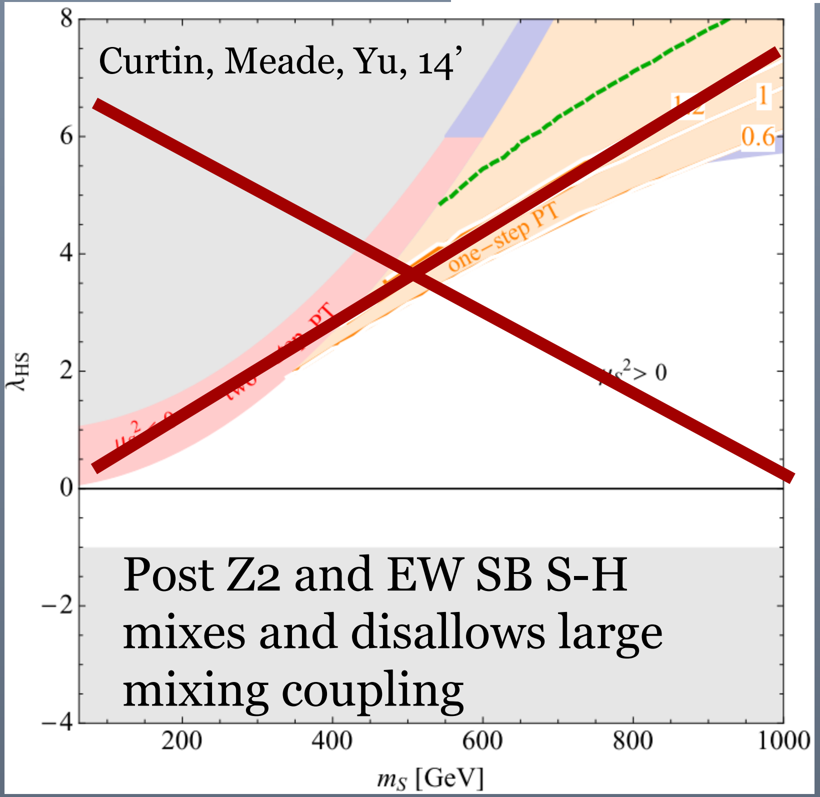
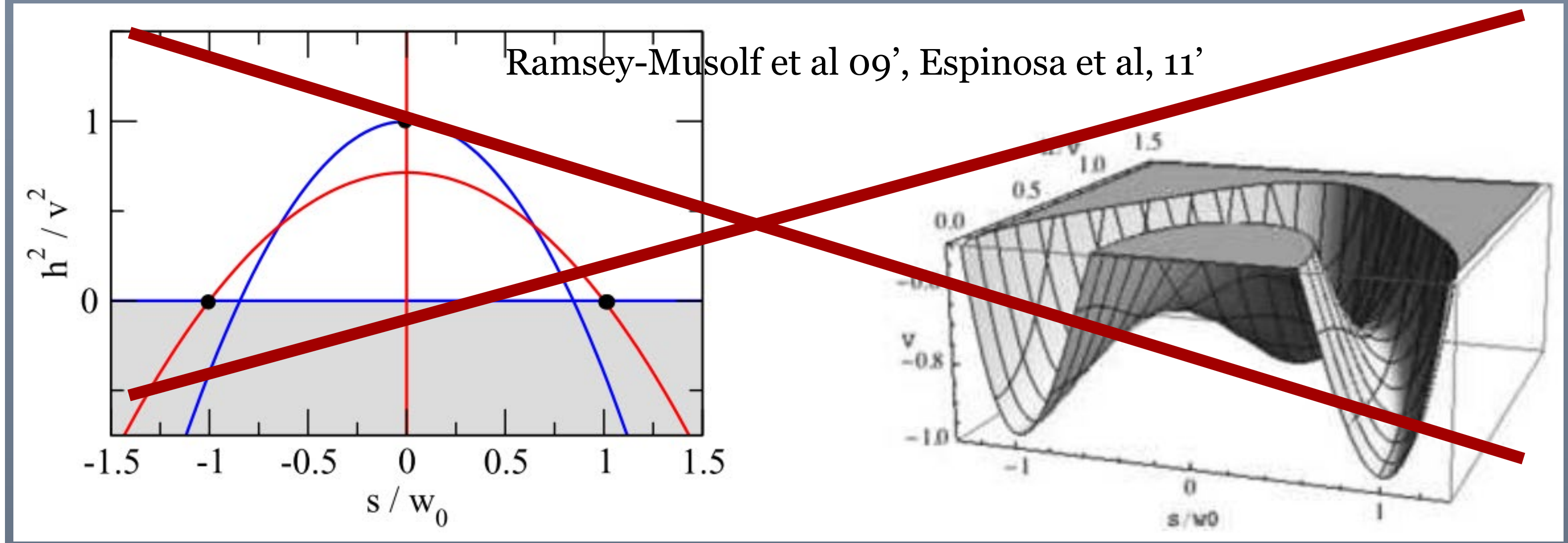
$$V_0(h,s) = -\frac{1}{2}\mu_h^2 h^2 + \frac{1}{4}\lambda_h h^4 + \frac{1}{2}\mu_s^2 s^2 + \frac{1}{4}\lambda_s s^4 + \frac{1}{4}\lambda_m h^2 s^2$$

$$+(\text{explicit Z2} - \text{breaking terms})$$





No enhanced tree-level barrier;
Always exists a saddle path to tunnel through, instead of a 2-step tunneling between two minima;

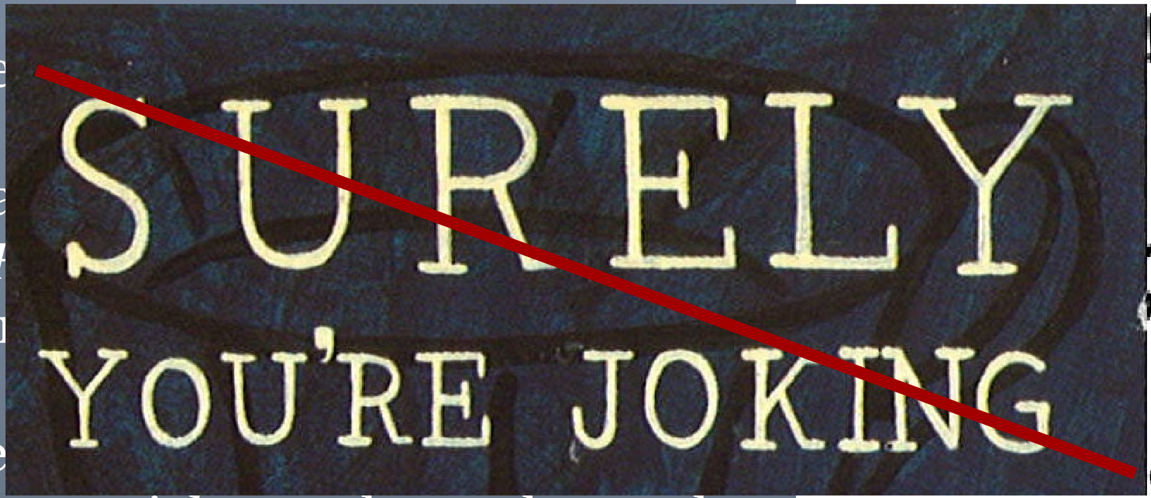
Spontaneous Z2 breaking Singlet Extension: a challenging case

However, no clear studies on the Spontaneous Z2 breaking case

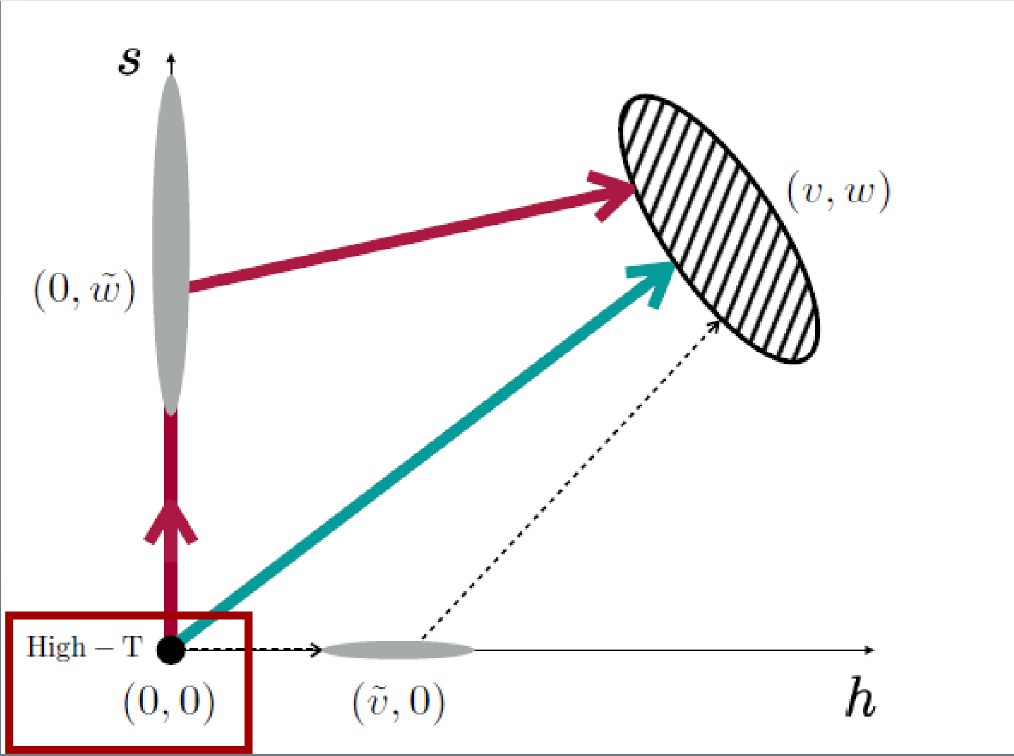
$$V_0(h,s) = -\frac{1}{2}\mu_h^2 h^2 + \frac{1}{4}\lambda_h h^4 + \frac{1}{2}\mu_s^2 s^2 + \frac{1}{4}\lambda_s s^4 + \frac{1}{4}\lambda_m h^2 s^2$$

$$+(\text{explicit Z2} - \text{breaking terms})$$

One can also get a fee
performing the usual
- **tree-level** integra
phase generates (
- The UV Z2 relatio
for $(H^+H)^n$;
- Operator generate
modify the Higgs potential enough to enhance the EWPT



Our study: A rich thermal History and enhanced EWPT are possible



Z2 and EW restoring High Temperature:

- Scenario A: two-step phase transition where last step is (0,w)->(v,w)
- Scenario B: one-step phase transition where last step is (0,0)->(v,w)

Other thermal histories are also possible, but hardly enhancing the EWPT

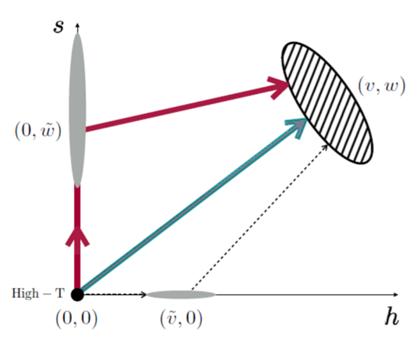
Near Criticality Condition

The calculations are carried out in a fixed Landau gauge. The gauge dependence of such calculation is long known since Jackiw and Dolan 74', Fukuda and T. Kugo, 76', etc., and also by Patel and Ramsey-Musolf 11', Konstantin et al 12'.

We perform our numerical study with a thermal potential including Coleman-Weinberg and daisy resummation, and further perform a nucleation calculation. The physical effects can be understood in most part via a high-temperature expansion approximation with the thermal potential alone.

$$V(h,s,T) \approx \frac{1}{2}(-\mu_h^2 + c_h T^2)h^2 - E^{\rm SM} T h^3 + \frac{1}{4}\lambda_h h^4 + \frac{1}{2}(\mu_s^2 + c_s T^2)s^2 + \frac{1}{4}\lambda_s s^4 + \frac{1}{4}\lambda_m s^2 h^2$$

For scenarios A, both in its Z2 restoring and non-restoring case, the VEVs between the last step of phase transition is:



$$v_c = 0 \qquad \tilde{w}(T_c) = \sqrt{\frac{-\mu_s^2 - c_s T_c^2}{\lambda_s}}$$

$$v_c \equiv v(T_c) = \frac{8E^{\rm SM}\lambda_s}{4\lambda_h\lambda_s - \lambda_m^2} T_c, \qquad w(T_c) = \sqrt{\frac{-\mu_s^2}{\lambda_s} - T_c^2\left[\frac{c_s}{\lambda_s} + 32\frac{(E^{\rm SM})^2\lambda_s\lambda_m}{4\lambda_h\lambda_s - \lambda_m^2}\right]}$$

Whose zero-temperature a vaccuum energy difference is:

$$\Delta V \equiv V(0, \tilde{w}|_{T=0}, T=0) - V(v_{\rm EW}, w_{\rm EW}, T=0) = \frac{v^4}{4}\left(\lambda_h - \frac{\lambda_m^2}{4\lambda_s}\right) = \frac{v^4}{4}\tilde{\lambda}_h$$

Recall that SM has:

$$\frac{v^4}{4}\lambda_h$$

and vacuum stability condition is:

$$\left(\lambda_h - \frac{\lambda_m^2}{4\lambda_s}\right) > 0$$

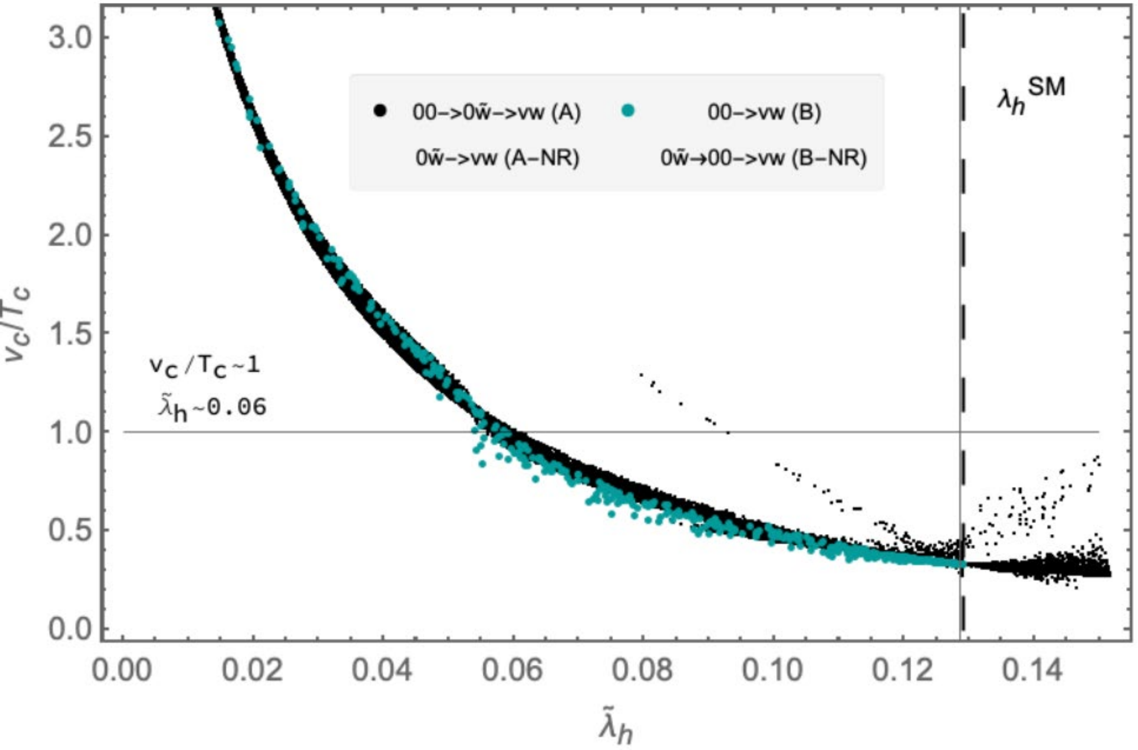
Near Criticality Condition

Smaller potential depth at zero temperature, less temperature it takes to be degenerate: lower T_c

$$T_c \approx \frac{v_{\rm EW}}{\sqrt{c_h - \frac{\lambda_m}{2\lambda_s} c_s}} \tilde{\lambda}_h^{\frac{1}{2}}$$

And also larger field value v_c at the critical temperature (closer to $v_{\rm EW}$):

$$v_c \approx v_{\rm EW} \frac{2E}{\sqrt{c_h - \frac{\lambda_m}{2\lambda_s} c_s}} \tilde{\lambda}_h^{-\frac{1}{2}}$$



Vacuum stability condition is:

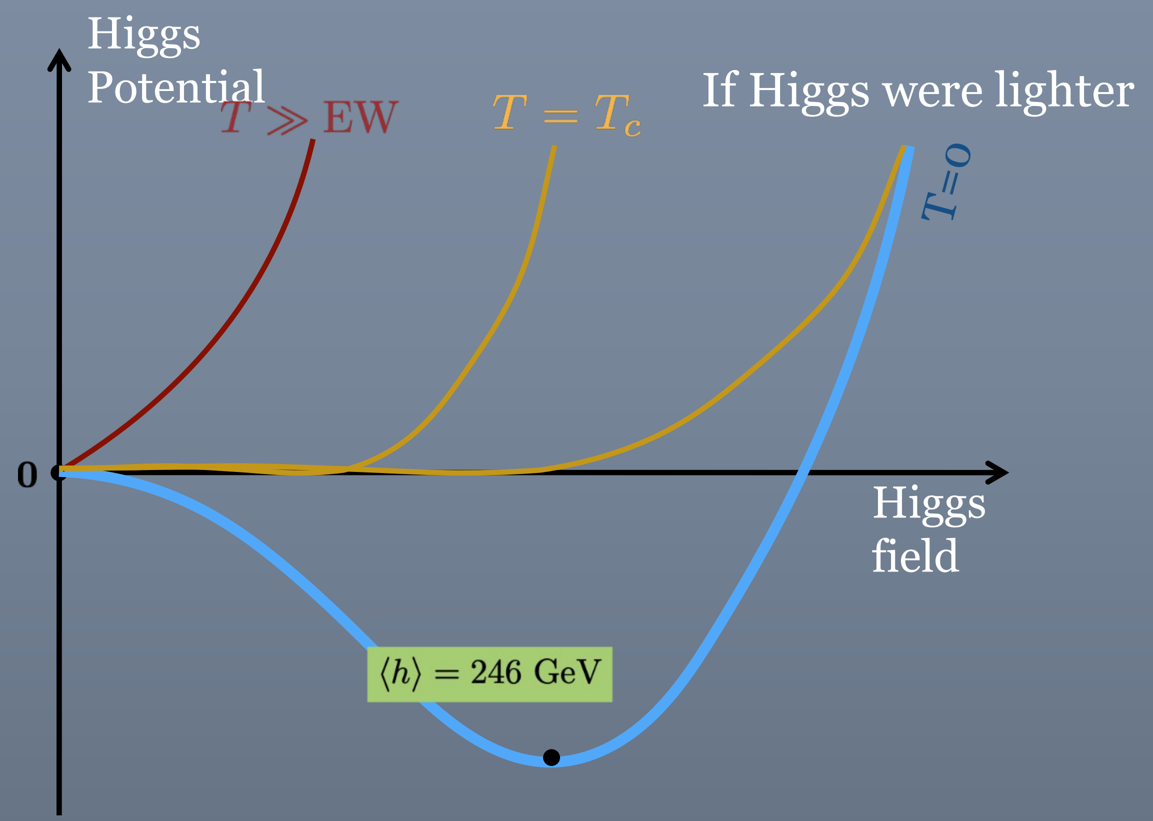
$$\left(\lambda_h - \frac{\lambda_m^2}{4\lambda_s}\right) > 0$$

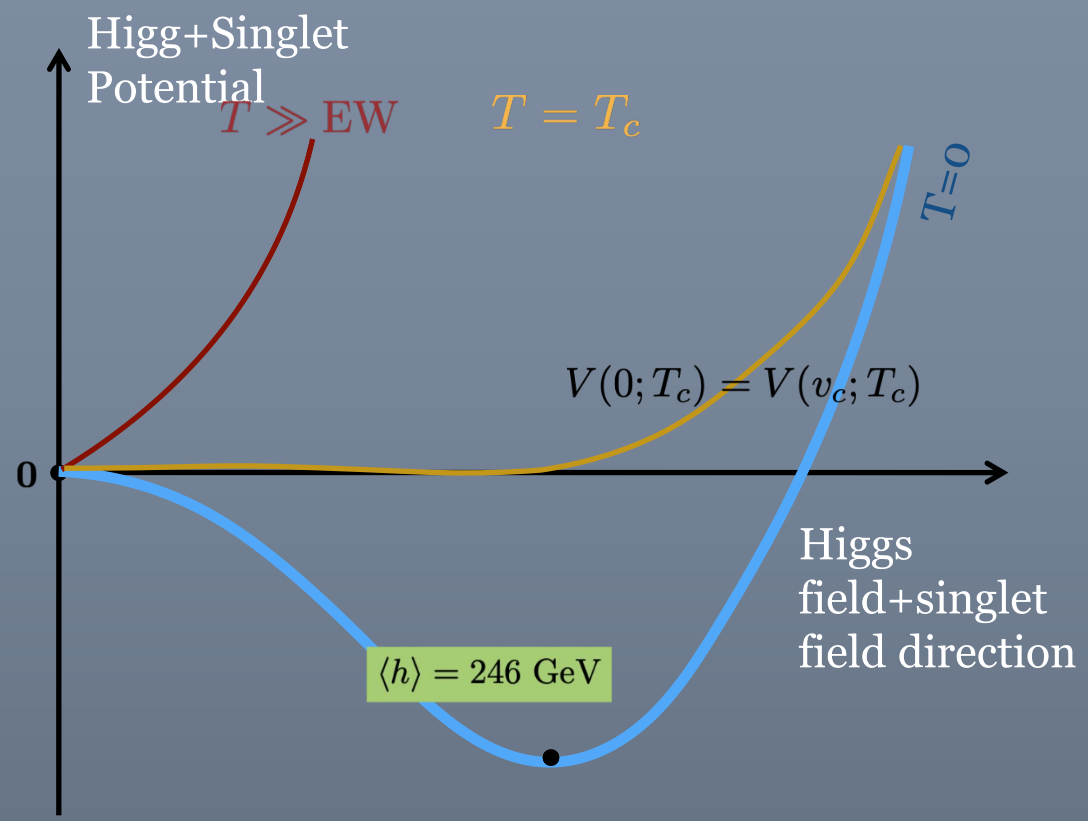
- When near criticality, a small T_c is expected as tiny amount of thermal correction is sufficient to make two vaccuum states equal in energy.
- This also implies a high value of V_c that is near 246 GeV.
- Implying large V_c/T_c

Figure also indicates how good a high temperature approximation is.

An interesting alternative by varying Higgs quartic coupling, Davoudiasl, 21'

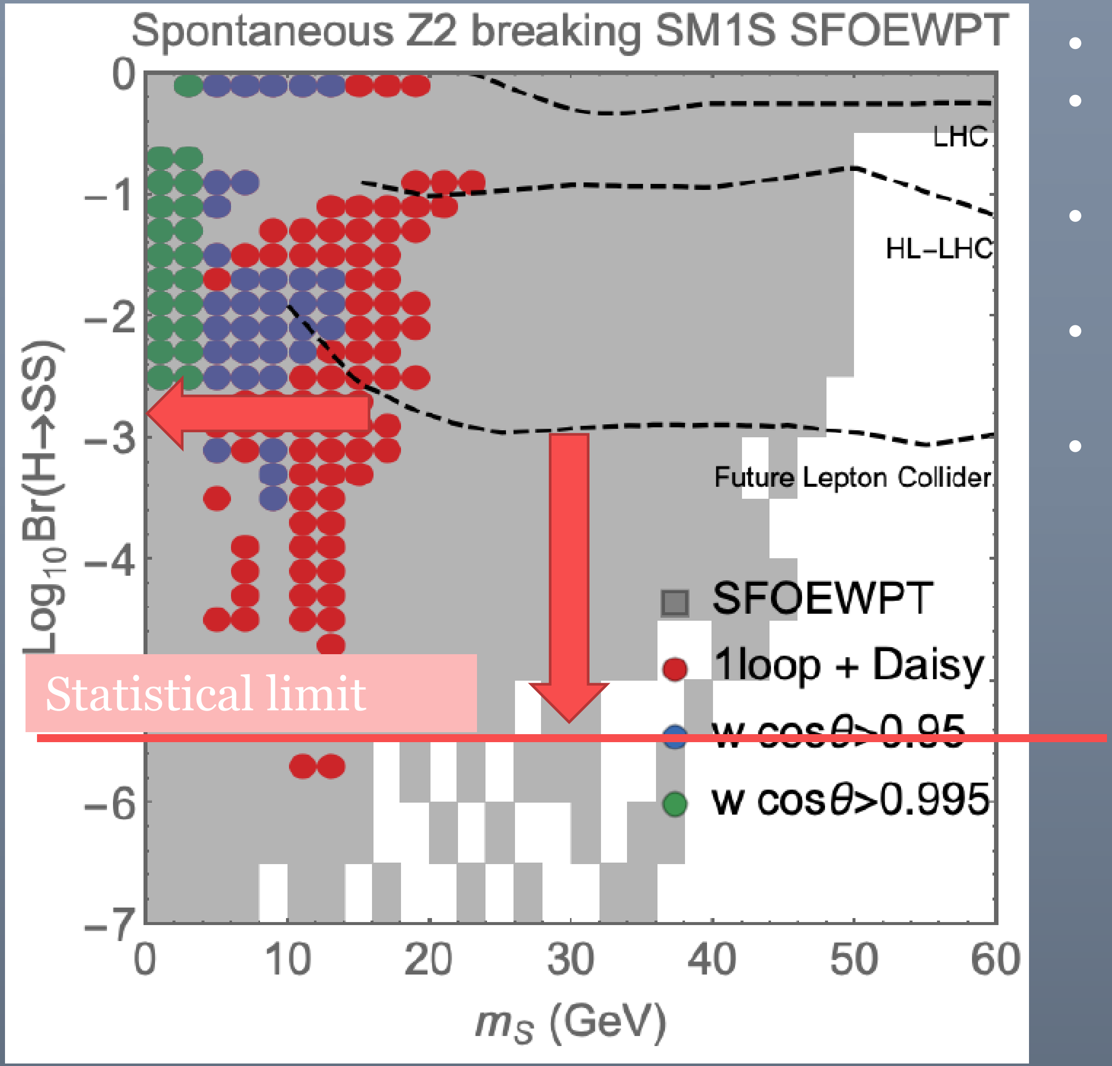
In other word…





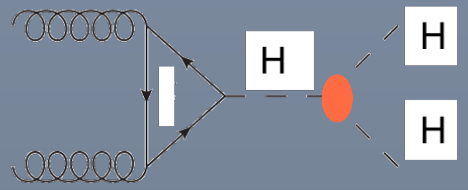
Higgs Exotic Decays

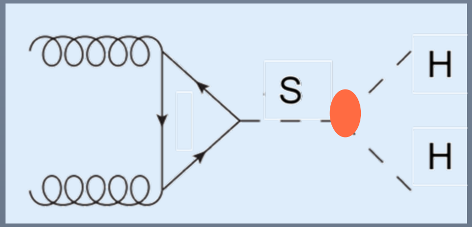
In the mass basis: $\frac{v_c}{T_c} = \left(\frac{v_c^{SM}}{T_c^{SM}}\right)\frac{\lambda_h^{SM}}{\widetilde{\lambda}_h} = \left(\frac{v_c^{SM}}{T_c^{SM}}\right)\left[1 + \sin^2\theta\,\frac{m_H^2 - m_S^2}{m_S^2}\right]$



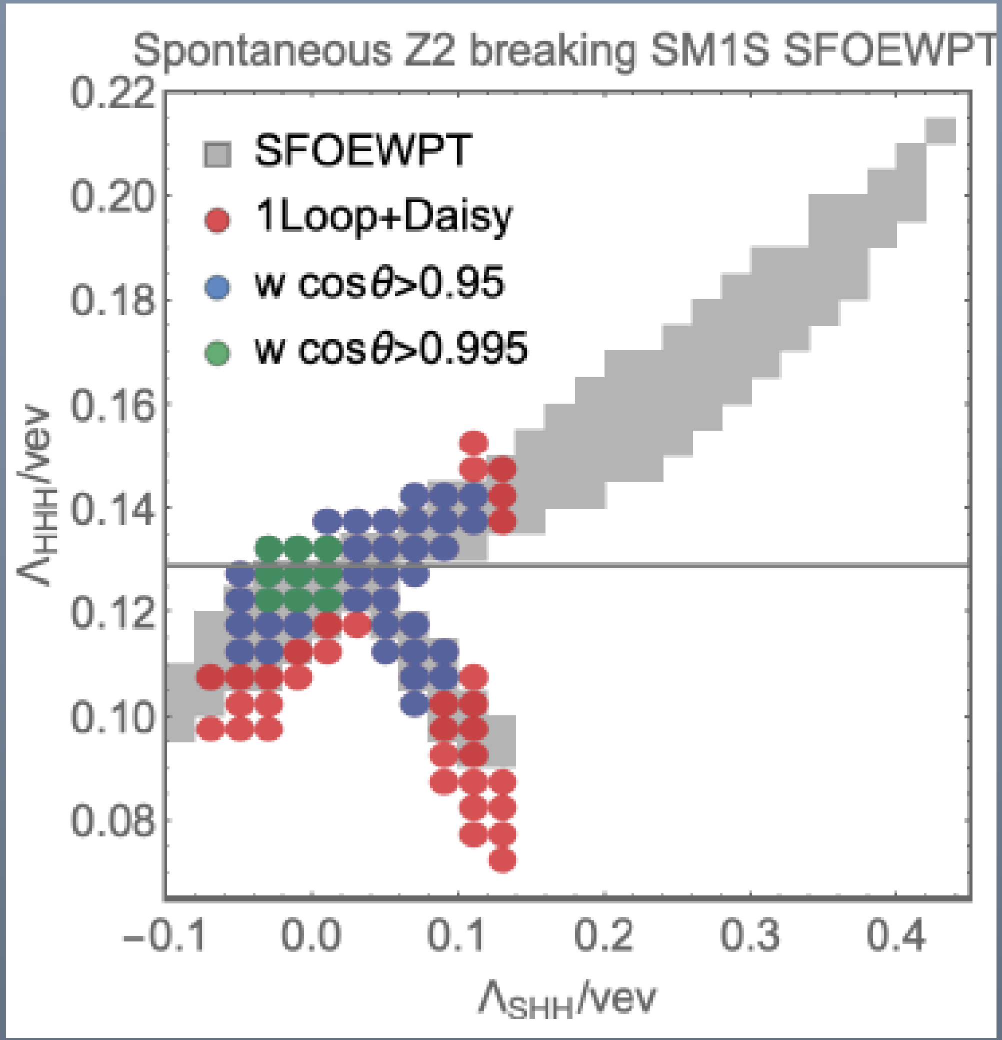
- A firm prediction of a light scalar in this model;
- Higgs exotic decay into a pair of light scalars is a crucial probe;
- Higgs exotic decays complements the Higgs precision program;
- Higgs exotic decays requires further studies of **merged jets** for lighter singlet masses(Jung, Liu, Wang, Xie, 21');
- Also possible to have long-lived Higgs exotic decays in certain parameter space; (e.g., Craig et al, 18'; Liu, Liu, Wang, 18')

Higgs Trilinears



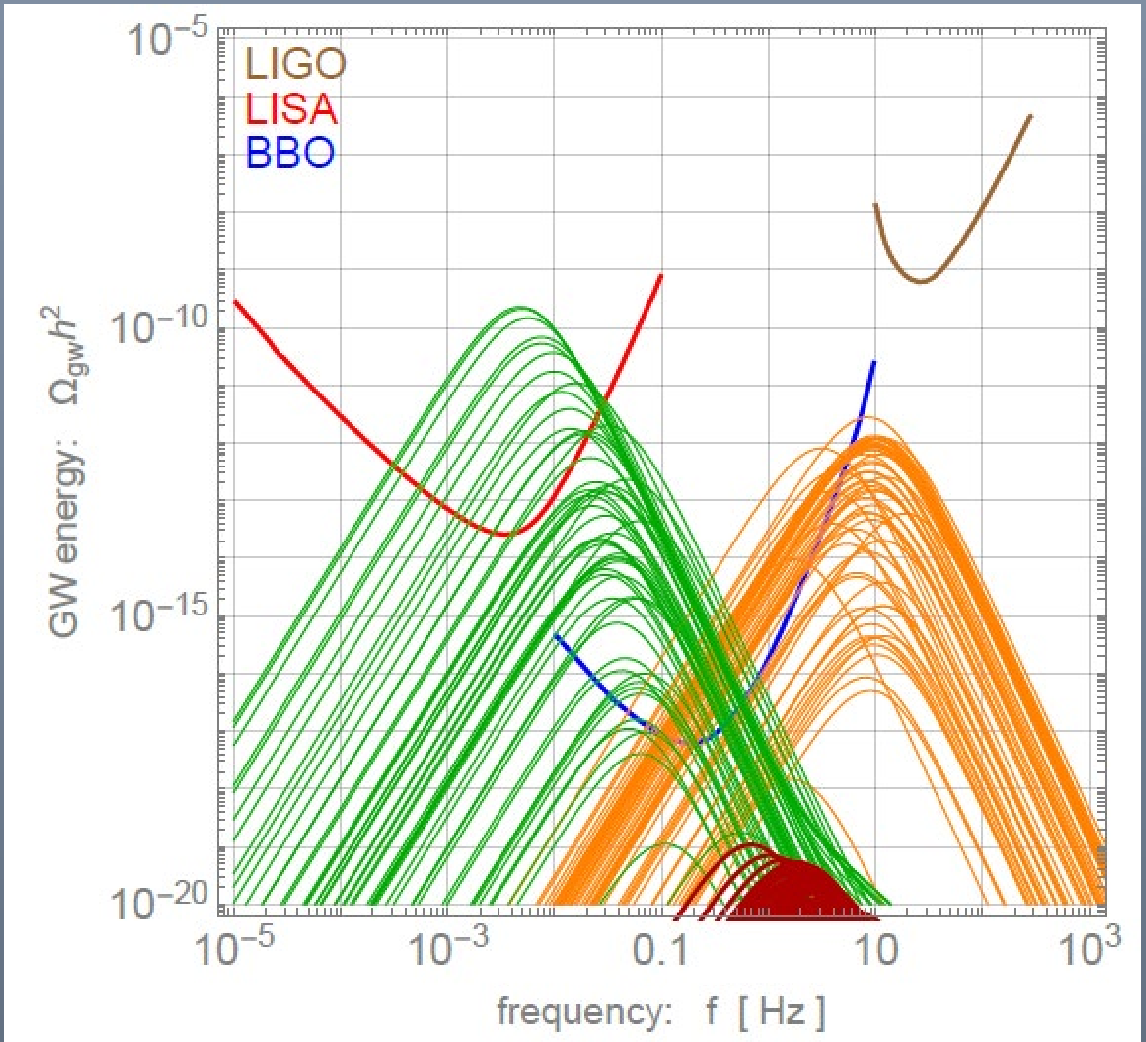


$$\left(\lambda_h - \frac{\lambda_m^2}{4\lambda_s}\right)$$



- Higgs trilinear coupling also modified;
- Can be either enhanced or suppressed by O(30%)
- Higgs precision program complementary (Grojean, Gu, Liu et al, 17');
- Double Higgs production at the HL-LHC and SPPC, FCC-hh provides additional insights into the mode (Carena, Riembau, Liu, 18'; Liu et al 19');

Gravitational Wave Signature



Gravitational wave provide complementary probe to the nature of first order EWPT in the mode.

Sound wave dominants the spectra.

Green: scenario A;
Orange: scenario B;
Red: full CW calculation

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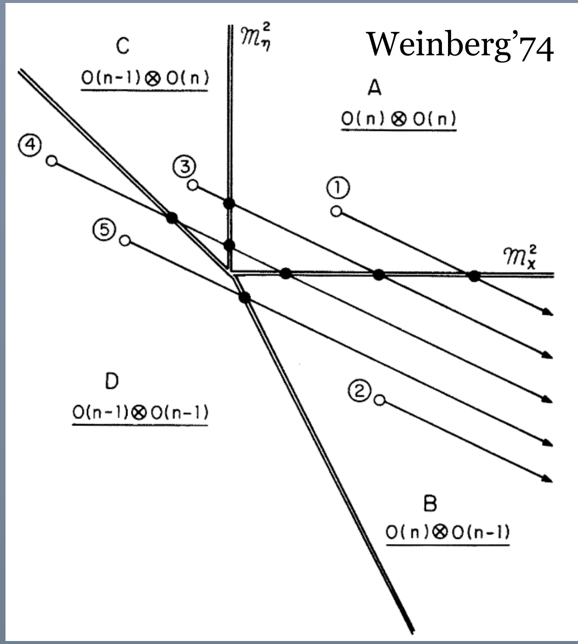
Symmetry non-restoration

$$P(\chi, \eta) = \tfrac{1}{2}\mathfrak{M}_\chi{}^2 \chi_A \chi_A + \tfrac{1}{2}\mathfrak{M}_\eta{}^2 \eta_a \eta_a + \tfrac{1}{4} e_{\chi\chi}{}^2 (\chi_A \chi_A)^2 - \tfrac{1}{2} e_{\chi\eta}{}^2 (\chi_A \chi_A)(\eta_a \eta_a) + \tfrac{1}{4} e_{\eta\eta}{}^2 (\eta_a \eta_a)^2 ,$$

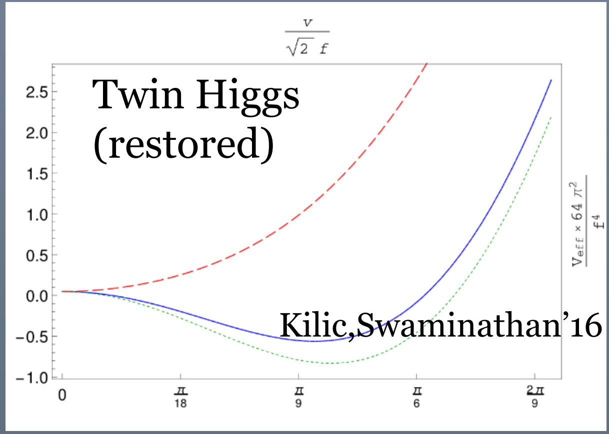


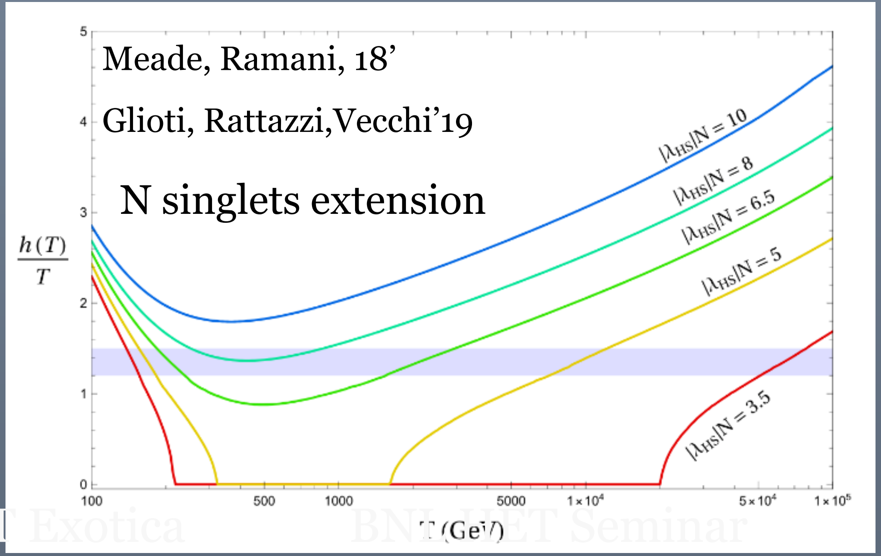
Symmetry non-restoration has be studied to discuss:

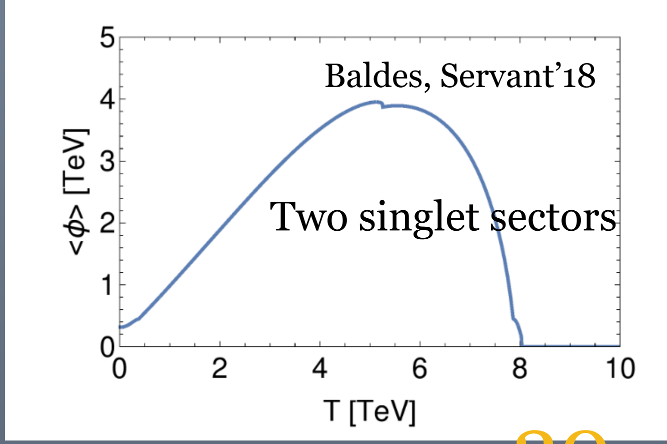
- High scale asymmetry creation;
- UV Model building has little dependence on EW scale physics;
- Avoid low scale constraints such as electron dipole moment on CP violation;
- …

More recent development of EWNR

Mohapatra, Senjanov'79, Dvali , Senjanov'95, etc.

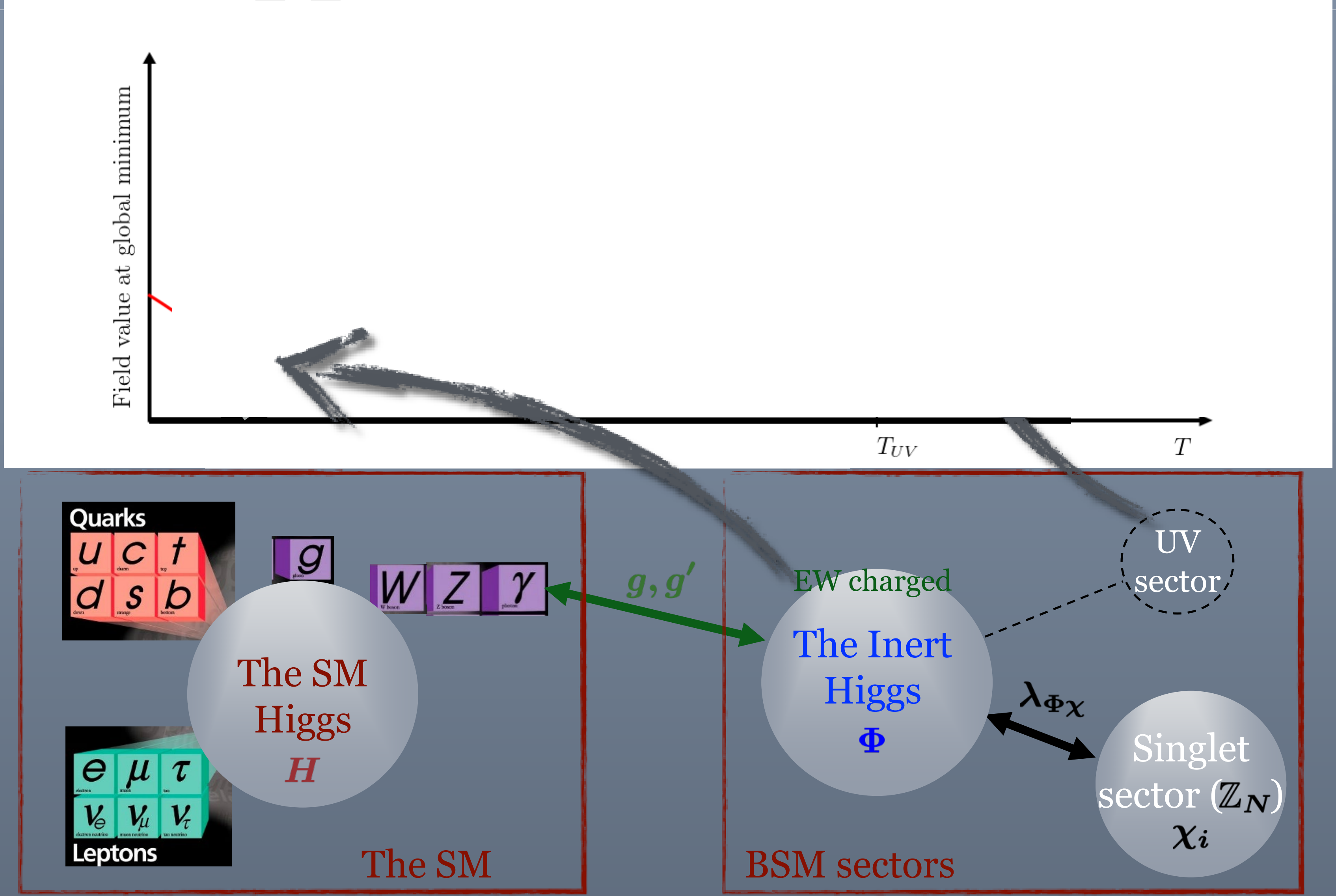






etc…

Our approach: broken EW relay



EW Symmetry Non-Restoration (EWNR)

- High scale asymmetry creation;
- UV Model building has little dependence on EW scale physics;
- Avoid low scale constraints such as electron dipole moment on CP violation;

The model and the effective potential

$$V_{\mathbb{Z}_N+\mathrm{I2HDM}} = -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + \mu_\Phi^2(\Phi^\dagger\Phi) + \lambda_\Phi(\Phi^\dagger\Phi)^2 + \lambda_{H\Phi}(H^\dagger H)(\Phi^\dagger\Phi) + \widetilde{\lambda}_{H\Phi}(H^\dagger\Phi)(\Phi^\dagger H)$$
$$+\frac{\mu_\chi^2}{2}\chi_i^2 + \frac{\widetilde{\lambda}_\chi}{4}\chi_i^4 + \frac{\lambda_\chi}{4}(\chi_i\chi_i)^2 + \frac{\lambda_{\Phi\chi}}{2}\chi_i^2(\Phi^\dagger\Phi) + \frac{\lambda_{H\chi}}{2}\chi_i^2(H^\dagger H)$$

- fixed parameters: $\{\mu_H^2,\ \lambda_H\}$,
- free parameters: $\{\mu_\Phi^2,\ \mu_\chi^2,\ \lambda_\Phi,\ \lambda_\chi,\ \lambda_{\Phi\chi},\ \lambda_{H\Phi},\ N\}$,
- free parameters set to zero: $\{\widetilde{\lambda}_{H\Phi},\ \lambda_{H\chi},\ \widetilde{\lambda}_\chi\}$,

Can be induced by RGE

Zero temperature constraints

$$\langle\{h,\varphi,\chi_1,\cdots,\chi_N\}\rangle = \{v_0,0,0,\cdots,0\}$$

Vacuum stability

Bounded from below (BFB)

$$\lambda_H > 0, \qquad \lambda_\Phi > 0, \qquad \lambda_\chi > 0,$$
$$\lambda_{H\Phi} > -\sqrt{4\lambda_H\lambda_\Phi}, \qquad \lambda_{\Phi\chi} > -\sqrt{4\lambda_\Phi\lambda_\chi}, \qquad \lambda_{H\chi} > -\sqrt{4\lambda_H\lambda_\chi},$$

$$\sqrt{4\lambda_H\lambda_\Phi\lambda_\chi} + \lambda_{H\Phi}\sqrt{\lambda_\chi} + \lambda_{\Phi\chi}\sqrt{\lambda_H} + \lambda_{H\chi}\sqrt{\lambda_\Phi} + \sqrt{\left(\lambda_{H\Phi}+\sqrt{4\lambda_H\lambda_\Phi}\right)\left(\lambda_{\Phi\chi}+\sqrt{4\lambda_\Phi\lambda_\chi}\right)\left(\lambda_{H\chi}+\sqrt{4\lambda_H\lambda_\chi}\right)} > 0$$

(tree level, copositivity of the quadratic potential)

Effective Potential

Zero temperature part
(Coleman-Weinberg potential)

$$V_{CW}\left(\{M_i^2(\hat\Phi)\};\mu_R\right)=\frac{1}{64\pi^2}\sum_{i=\mathrm{B,F}}(-1)^{2S_i}n_iM_i^4(\hat\Phi)\left[\log\frac{M_i^2(\hat\Phi)}{\mu_R^2}-a_i\right]$$

Finite temperature
part

$$V^T_{\text{1-loop}}(\{M_k^2(\hat\Phi)\},T)=\frac{T^4}{2\pi^2}\left[\sum_{i=B}n_iJ_B\left(\frac{M_i^2(\hat\Phi)}{T^2}\right)-\sum_{i=F}n_iJ_F\left(\frac{M_i^2(\hat\Phi)}{T^2}\right)\right],$$

$$\text{with}\quad J_{B/F}(y)=\int_0^\infty dx\, x^2\log\left(1\mp e^{-\sqrt{x^2+y}}\right)$$

Degrees for freedom in the plasma: $\{h,G_0,G^\pm,\varphi,\phi_0,\phi^\pm,\chi,\gamma,W^\pm,Z,t\}$

High temperature expansion and the thermal mass

"thermal mass"

$$V^{\mathrm{MF}}_{\mathbb{Z}_N+\mathrm{I2HDM}}=-\frac{1}{2}\left(\mu_H^2-c_hT^2\right)h^2+\frac{1}{2}\left(\mu_\Phi^2+c_\varphi T^2\right)\varphi^2+\frac{1}{2}\left(\mu_\chi^2+c_\chi T^2\right)\chi_i^2$$
$$+\frac{\lambda_H}{4}h^4+\frac{\lambda_\Phi}{4}\varphi^4+\frac{\widetilde\lambda_\chi}{4}\chi_i^4+\frac{\lambda_\chi}{4}(\chi_i\chi_i)^2+\frac{\Lambda_{H\Phi}}{4}\varphi^2h^2+\frac{\lambda_{\Phi\chi}}{4}\varphi^2\chi_i^2+\frac{\lambda_{H\chi}}{4}h^2\chi_i^2$$

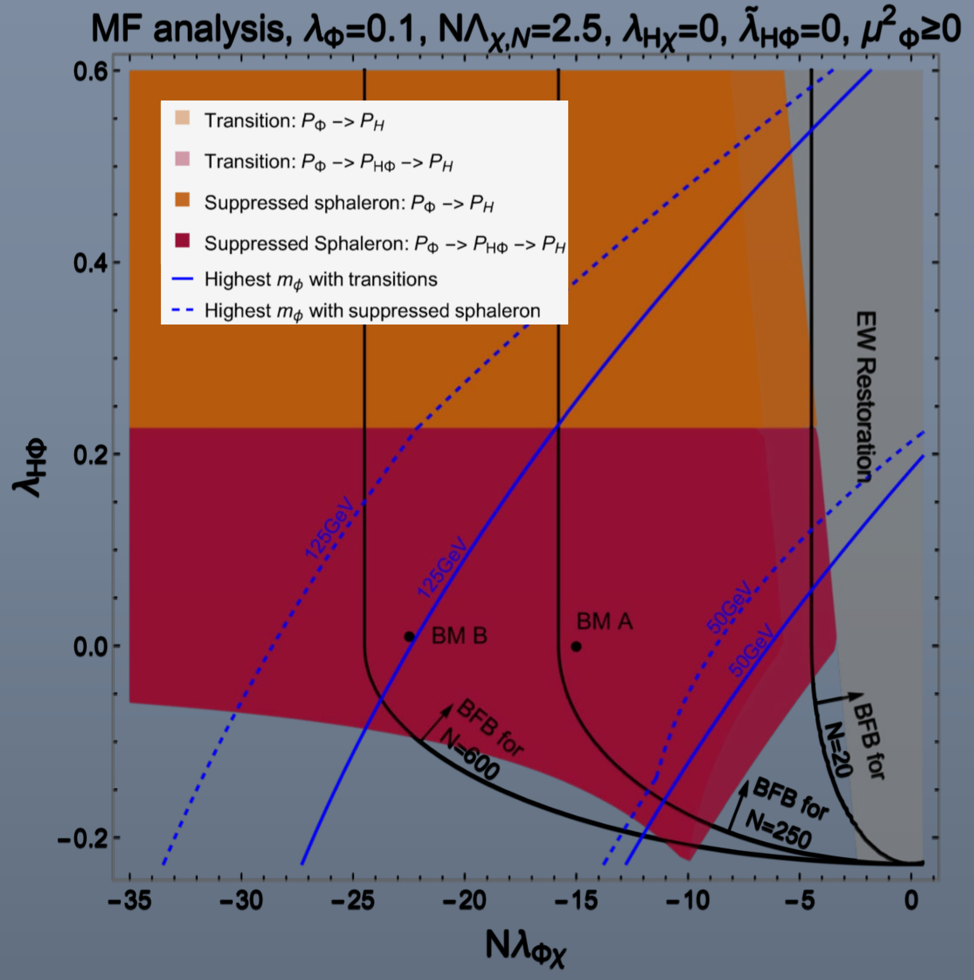
(leading order in the high-T expansion, no CW)

$$\text{e.g.,}\quad c_\varphi=\frac{\lambda_\Phi}{2}+\frac{\lambda_{H\Phi}+\widetilde\lambda_{H\Phi}/2}{6}+\frac{3g^2+g'^2}{16}+N\frac{\lambda_{\Phi\chi}}{24}$$

→ Being negative
results in a non-zero inert Higgs vev

A first look

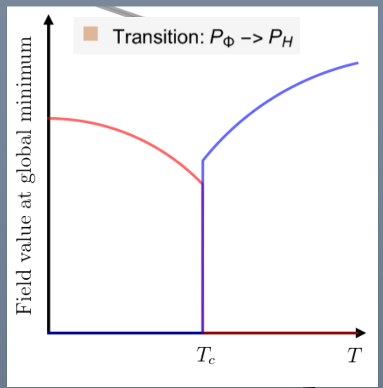
$$V^{\rm MF}_{\mathbb{Z}_N+{\rm I2HDM}} = -\frac{1}{2}\left(\mu_H^2 - c_h T^2\right) h^2 + \frac{1}{2}\left(\mu_\Phi^2 + c_\varphi T^2\right)\varphi^2 + \frac{1}{2}\left(\mu_\chi^2 + c_\chi T^2\right)\chi_i^2$$
$$+\frac{\lambda_H}{4}h^4 + \frac{\lambda_\Phi}{4}\varphi^4 + \frac{\widetilde{\lambda}_\chi}{4}\chi_i^4 + \frac{\lambda_\chi}{4}(\chi_i\chi_i)^2 + \frac{\Lambda_{H\Phi}}{4}\varphi^2 h^2 + \frac{\lambda_{\Phi\chi}}{4}\varphi^2\chi_i^2 + \frac{\lambda_{H\chi}}{4}h^2\chi_i^2$$

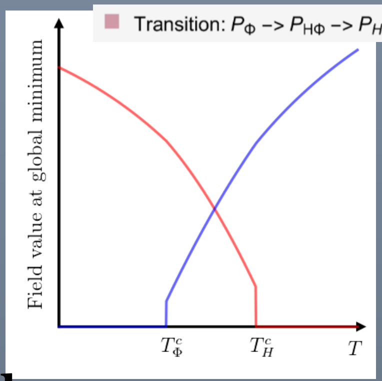


- **Non-restoration**

$$c_\varphi = \frac{\lambda_\Phi}{2} + \frac{\lambda_{H\Phi} + \widetilde{\lambda}_{H\Phi}/2}{6} + \frac{3g^2 + g'^2}{16} + N\frac{\lambda_{\Phi\chi}}{24} < 0$$

- **Transition**





- **Suppressed sphaleron rate**

$$\frac{\sqrt{\langle h\rangle^2 + \langle\varphi\rangle^2}}{T} \gtrsim 1$$

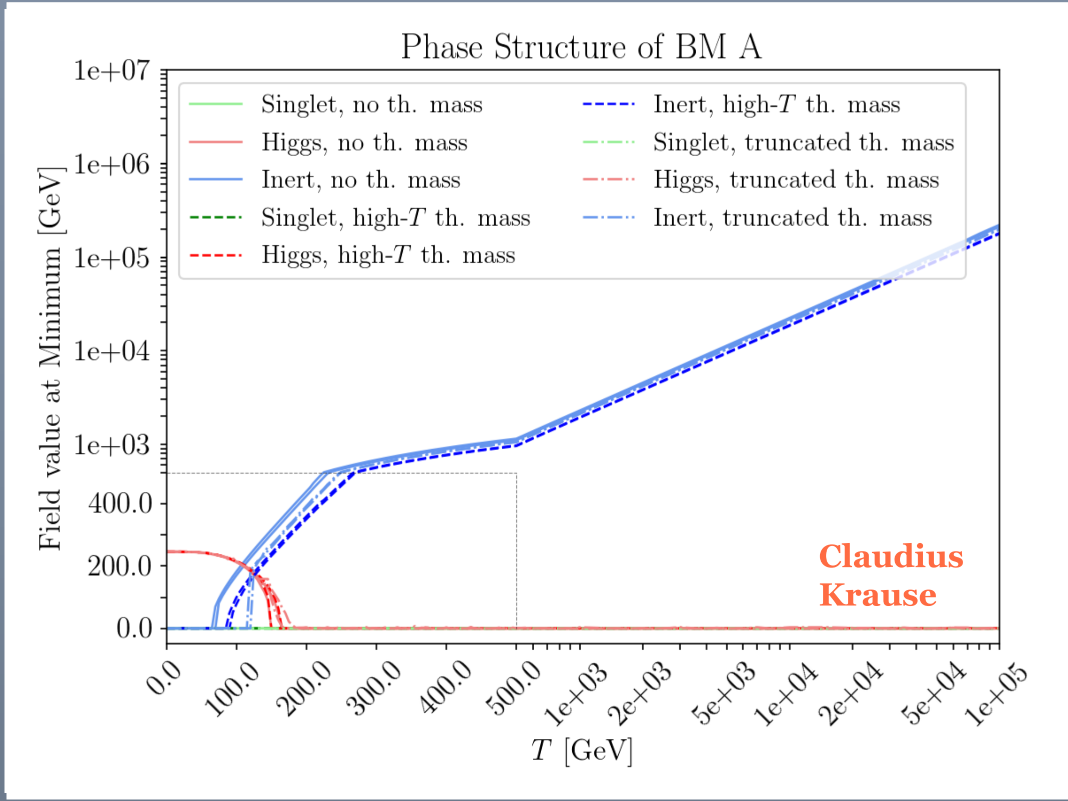
- **Bounded from below**

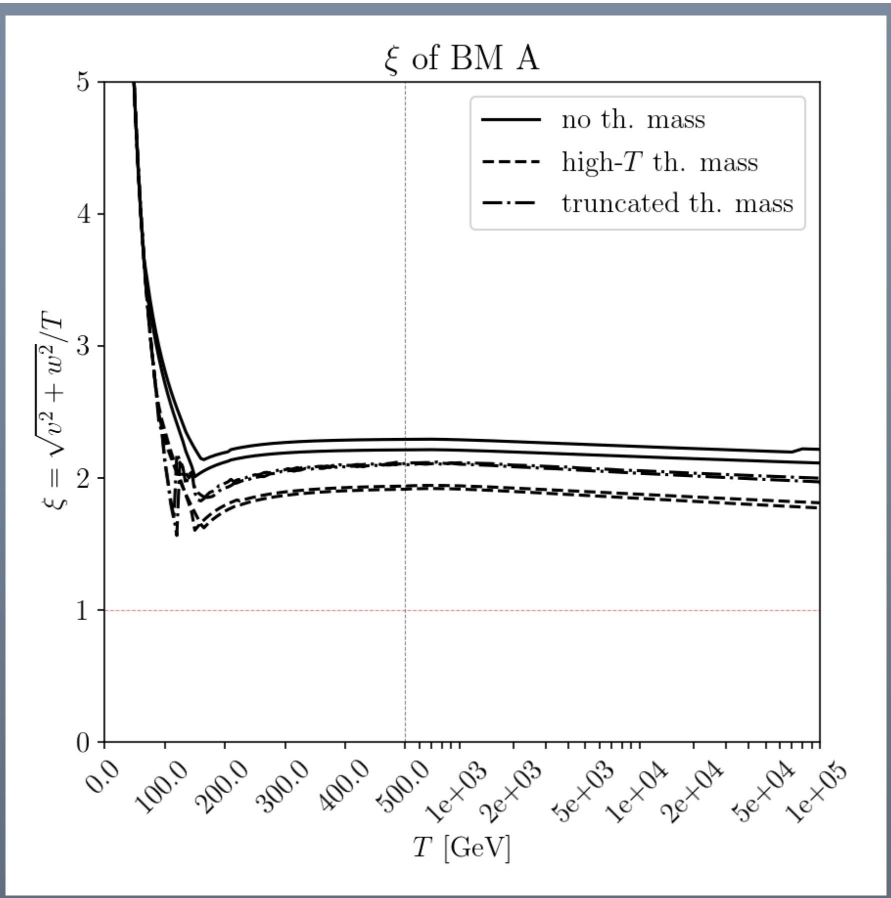
e.g. $\lambda_{\Phi\chi} > -\sqrt{4\lambda_\Phi\lambda_\chi}$

Benchmark A

Dilution factor $f_{w.o.} = 1 - \frac{n_B(t_{now})}{n_B(0)} = 1 - \exp\left[-\frac{13 n_f}{2}\int_0^{T_{\rm high}} dT\, \frac{\Gamma(T)}{VT^6} M_{Pl}\sqrt{\frac{90}{8\pi^3 g^*}}\right]$

	no th. mass	high-T th. mass	truncated th. mass
BM A	$< 10^{-16}$ / $\mathbf{10^{-16}}$ / 10^{-14}	10^{-11} / $\mathbf{10^{-9}}$ / 10^{-7}	$8 \cdot 10^{-11}$ / $\mathbf{8 \cdot 10^{-9}}$ / $8 \cdot 10^{-7}$
	$< 10^{-16}$ / $\mathbf{4 \cdot 10^{-15}}$ / $4 \cdot 10^{-13}$	$2 \cdot 10^{-11}$ / $\mathbf{2 \cdot 10^{-9}}$ / $2 \cdot 10^{-7}$	10^{-12} / $\mathbf{10^{-10}}$ / 10^{-8}





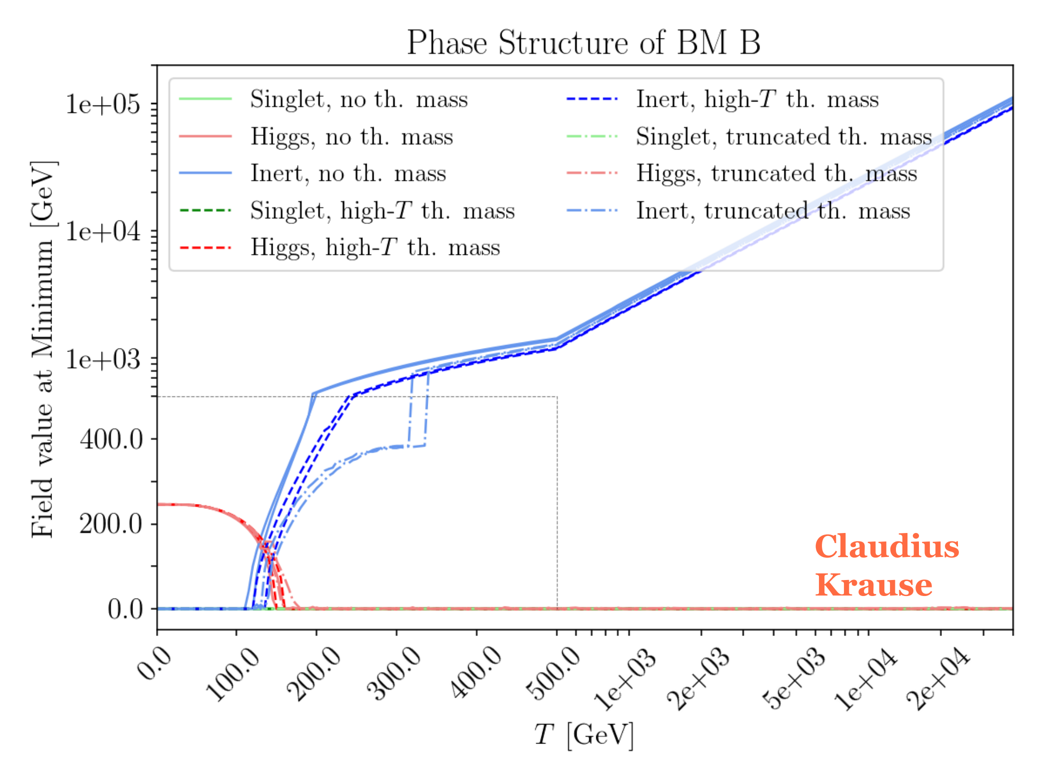
	μ_H^2	λ_H	μ_Φ^2	λ_Φ	μ_χ^2	λ_χ	$\lambda_{H\Phi}$	$\widetilde{\lambda}_{H\Phi}$
BM A	8994.45	0.119	2500	0.1	100	0.01	-0.001	0

	$\lambda_{\Phi\chi}$	$\widetilde{\lambda}_\chi$	$\lambda_{H\chi}$	N	m_h	m_ϕ	m_χ
BM A	-0.06	0	0	250	125	48.47	9.8

Benchmark B

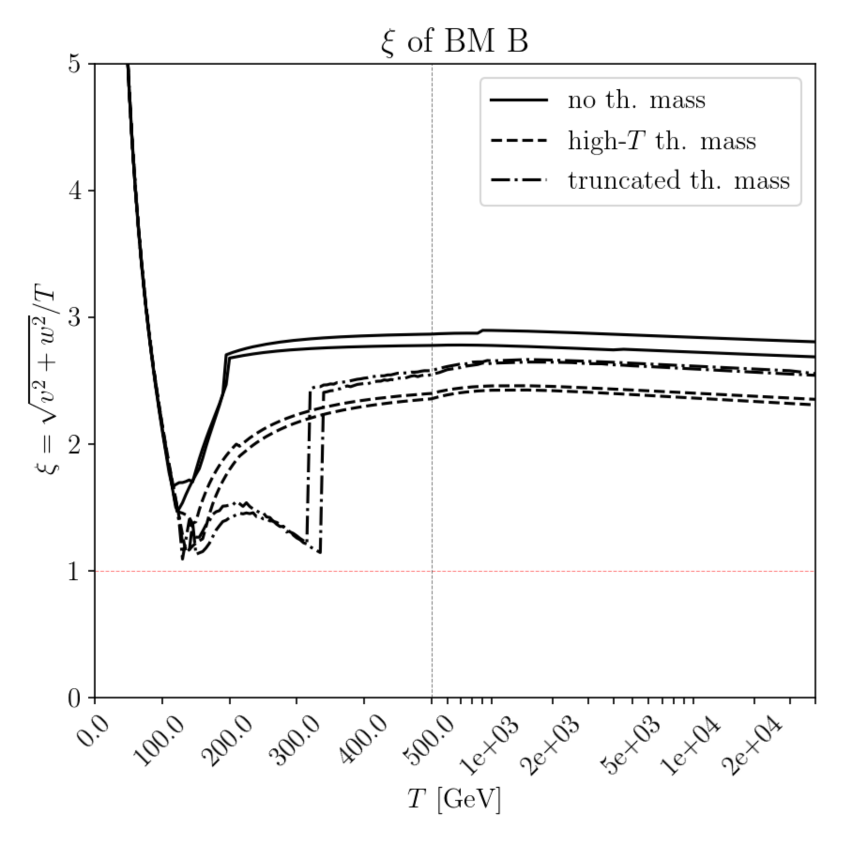
Dilution factor $f_{w.o.} = 1 - \frac{n_B(t_{now})}{n_B(0)} = 1 - \exp\left[-\frac{13 n_f}{2}\int_0^{T_{\text{high}}} dT \frac{\Gamma(T)}{V T^6} M_{Pl}\sqrt{\frac{90}{8\pi^3 g^*}}\right]$

	no th. mass	high-T th. mass	truncated th. mass
BM B	$9\cdot10^{-10}$ / $\mathbf{9\cdot10^{-8}}$ / $9\cdot10^{-6}$	$4\cdot10^{-5}$ / $\mathbf{4\cdot10^{-3}}$ / 0.296	$7\cdot10^{-5}$ / $\mathbf{7\cdot10^{-3}}$ / 0.498
	$4\cdot10^{-12}$ / $\mathbf{4\cdot10^{-10}}$ / $4\cdot10^{-8}$	$2\cdot10^{-8}$ / $\mathbf{2\cdot10^{-6}}$ / $2\cdot10^{-4}$	10^{-4} / **0.012** / 0.694



	μ_H^2	λ_H	μ_Φ^2	λ_Φ	μ_χ^2	λ_χ	$\lambda_{H\Phi}$	$\widetilde{\lambda}_{H\Phi}$
BM B	8991.84	0.119	5800	0.1	5000	0.004	0.01	0

	$\lambda_{\Phi\chi}$	$\widetilde{\lambda}_\chi$	$\lambda_{H\chi}$	N	m_h	m_ϕ	m_χ
BM B	-0.0375	0	0	600	125	84.58	68.87



Pheno Considerations

- **Higgs invisible decays**

$$\Gamma(h \to ss) = \frac{\lambda_{Hs}^2 v_0^2}{32\pi m_h}\sqrt{1-\frac{4m_s^2}{m_h^2}}$$

$$\sqrt{N\lambda_{H\chi}^2 + 2(\lambda_{H\Phi} + \widetilde{\lambda}_{H\Phi})^2 + 2\lambda_{H\Phi}^2} \leqslant 0.015\ (0.007)\ \text{for LHC(HL-LHC)}$$

- **Z boson invisible decays**

Excludes all inert masses below 45 GeV.

- **Electroweak precision observables (EWPO)**

$$\mathcal{O}_T = \frac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2,\ c_T = \frac{\widetilde{\lambda}_{H\Phi}^2}{192\pi^2\mu_\Phi^2}$$

$|\widetilde{\lambda}_{H\Phi}| < 0.36$ at 95% C.L

- **Higgs precision measurements**

Corrections to Higgs couplings, as well as Higgs to gauge boson couplings, are generated via loops. They provide less stringent constraints on Higgs-inert and Higgs-singlets cross quartics, than the Higgs invisible decay searches.

- **Disappearing tracks (charged states)**

Disappearing track searches exclude Higgsinos up to 78 GeV.
Charged inert Higgs has smaller (Drell-Yan) production rate compared to Higgsinos.
And the search can be avoided by turning on a tiny $\tilde{\lambda}_{H\Phi}$ that generates splitting between the charged and neutral inert states.

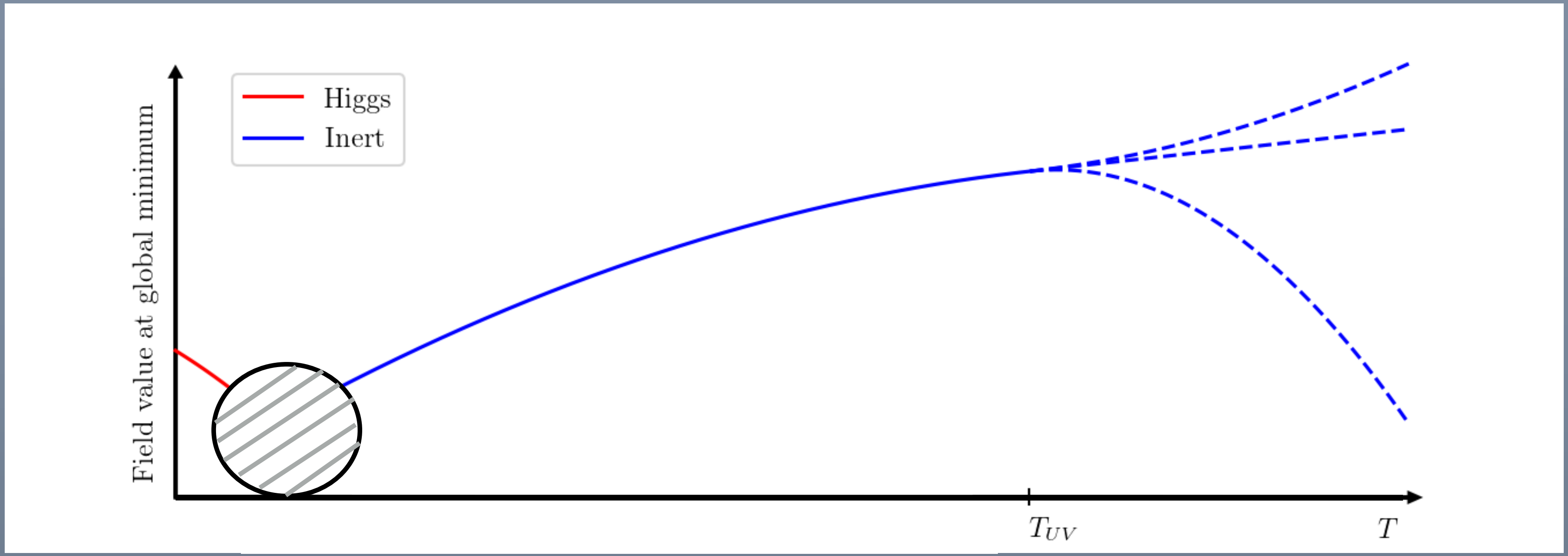
A decoupling behavior:
Put tuning aside, at zero temperature (deep IR), one can decouple all the new physics at tree-level (except for the gauge coupling of the inert doublet) and achieve huge modifications to thermal history.
Higgs doesn't need to couple to new physics (directly at tree-level, one can turn all mixing quartics to be zero).

Conclusion

- A real singlet extension to Enhance EWPT
  - Spontaneous Z2 extension are motived by its connection to dark sector physics, but its role in EWPT is was not clear;
  - We identify a rich thermal history is still possible with near criticality condition, and develop robust understanding of it;
  - Predicts light scalar that leads to new and important program of Higgs exotic decays; Modified Higgs couplings through mixing effects; Modified Higgs trilinear couplings; Certain model space can be probed by the Gravitation waves.
- A new approach to Delayed (or non-restoring) EW
  - Help enable EWBG
  - We provide a method where the EWNR is achieved by transmitting the SM broken electroweak symmetry to an inert Higgs sector at very high temperatures;
  - Pheno testable but also feature "decoupling".

Thank you!

Backup



$$
\begin{aligned}
\beta(g_s) &= -7g_s^3\\
\beta(g) &= -3g^3\\
\beta(g') &= 7g'^3\\
\beta(\mu_H^2) &= -4\lambda_{H\Phi}\mu_\Phi^2 - 2\widetilde{\lambda}_{H\Phi}\mu_\Phi^2 - \mu_H^2(-12\lambda_H + \tfrac{3}{2}(3g^2+g'^2) - 6y_t^2) - N\mu_\chi^2\lambda_{H\chi}\\
\beta(\mu_\Phi^2) &= -4\lambda_{H\Phi}\mu_H^2 - 2\widetilde{\lambda}_{H\Phi}\mu_H^2 - \mu_\Phi^2(-12\lambda_\Phi + \tfrac{3}{2}(3g^2+g'^2)) - N\mu_\chi^2\lambda_{\Phi\chi}\\
\beta(\mu_\chi^2) &= 4\mu_\Phi^2\lambda_{\Phi\chi} + 6\widetilde{\lambda}_\chi\mu_\chi^2 - 4\mu_H^2\lambda_{H\chi} + 2(N+2)\mu_\chi^2\lambda_\chi\\
\beta(\lambda_H) &= 2\lambda_{H\Phi}^2 + 2\lambda_{H\Phi}\widetilde{\lambda}_{H\Phi} + \widetilde{\lambda}_{H\Phi}^2 + 24\lambda_H^2 - 3\lambda_H(3g^2+g'^2)\\
&\quad + \frac{3}{8}(3g^4 + 2g^2g'^2 + g'^4) + 12\lambda_H y_t^2 - 6y_t^4 + \tfrac{N}{2}\lambda_{H\chi}^2\\
\beta(\lambda_\Phi) &= 2\lambda_{H\Phi}^2 + 2\lambda_{H\Phi}\widetilde{\lambda}_{H\Phi} + \widetilde{\lambda}_{H\Phi}^2 + 24\lambda_\Phi^2 - 3\lambda_\Phi(3g^2+g'^2)\\
&\quad + \frac{3}{8}(3g^4 + 2g^2g'^2 + g'^4) + \tfrac{N}{2}\lambda_{\Phi\chi}^2\\
\beta(\lambda_\chi) &= 2\lambda_{\Phi\chi}^2 + 2\lambda_{H\chi}^2 + 16\lambda_\chi^2 + 12\widetilde{\lambda}_\chi\lambda_\chi + 2N\lambda_\chi^2\\
\beta(\lambda_{H\Phi}) &= \frac{3}{4}(3g^4 - 2g^2g'^2 + g'^4) + 4\lambda_{H\Phi}^2 + 2\widetilde{\lambda}_{H\Phi}^2 + 4\widetilde{\lambda}_{H\Phi}(\lambda_H + \lambda_\Phi)\\
&\quad + \lambda_{H\Phi}(12\lambda_\Phi + 12\lambda_H - 3(3g^2+g'^2)) + 6\lambda_{H\Phi}y_t^2\\
\beta(\widetilde{\lambda}_\chi) &= 18\widetilde{\lambda}_\chi^2 + 24\widetilde{\lambda}_\chi\lambda_\chi\\
\beta(\lambda_{\Phi\chi}) &= (-\tfrac{3}{2}(3g^2+g'^2) + 12\lambda_\Phi + 6\widetilde{\lambda}_\chi + 4\lambda_{\Phi\chi} + 2N\lambda_\chi + 4\lambda_\chi)\lambda_{\Phi\chi}\\
&\quad + 4\lambda_{H\Phi}\lambda_{H\chi} + 2\widetilde{\lambda}_{H\Phi}\lambda_{H\chi}\\
\beta(\lambda_{H\chi}) &= (-\tfrac{3}{2}(3g^2+g'^2) + 12\lambda_H + 6\widetilde{\lambda}_\chi + 4\lambda_{H\chi} + 2N\lambda_\chi + 4\lambda_\chi + 6y_t^2)\lambda_{H\chi}\\
&\quad + 4\lambda_{H\Phi}\lambda_{\Phi\chi} + 2\widetilde{\lambda}_{H\Phi}\lambda_{\Phi\chi}\\
\beta(\widetilde{\lambda}_{H\Phi}) &= 3g^2g'^2 - 3\widetilde{\lambda}_{H\Phi}(3g^2+g'^2) + 6\widetilde{\lambda}_{H\Phi}y_t^2 + 4\widetilde{\lambda}_{H\Phi}(\lambda_H+\lambda_\Phi)\\
&\quad + 8\lambda_{H\Phi}\widetilde{\lambda}_{H\Phi} + 4\widetilde{\lambda}_{H\Phi}^2\\
\beta(y_t) &= -8y_tg_s^2 - \tfrac{9}{4}y_tg^2 - \tfrac{17}{12}y_tg'^2 + \tfrac{9}{2}y_t^3.
\end{aligned}
$$

Asymmetry washout - a model building consideration

The (EW) sphaleron process

Washout B + L; preserve B - L

If the sphalerons become active:

Net B-L generated at UV

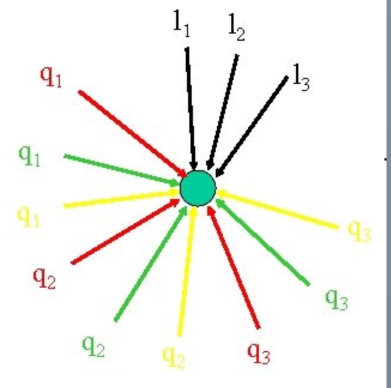
$$\mathbf{B_{now} = -L_{now} \neq 0}$$

e.g. leptogenesis

Net B+L generated at UV

$$\mathbf{B_{now} = L_{now} = 0}$$

e.g. baryogenesis



The sphaleron rate

$$\frac{\Gamma}{V} = 4\pi\omega_- \mathcal{N}_{tr}\mathcal{N}_{rot} T^3 \left(\frac{v_{EW}(T)}{T}\right)^6 \kappa \exp\left[-E_{sph}(T)/T\right]$$

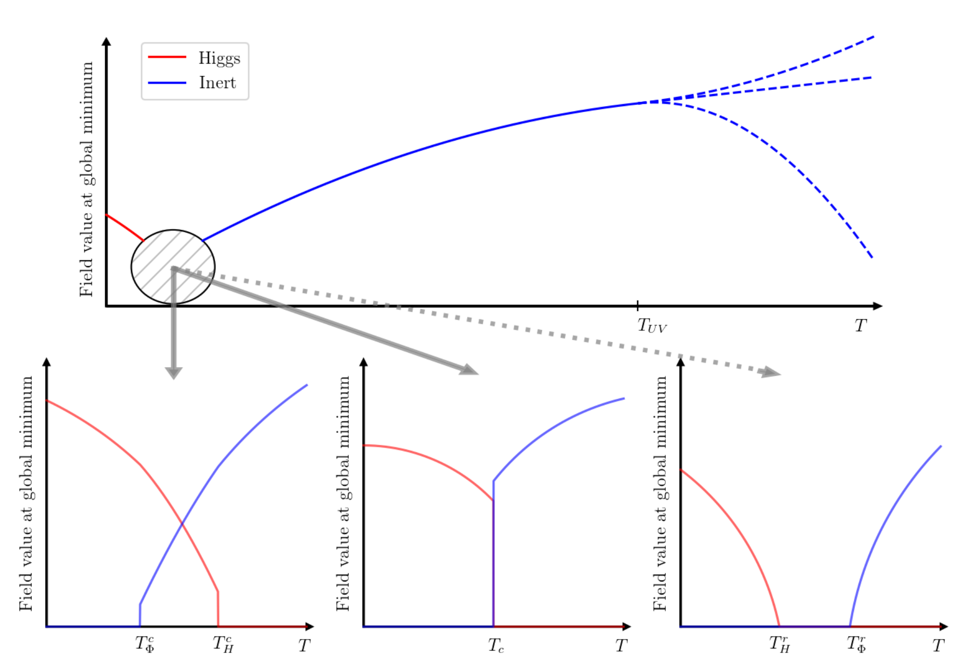
with

$$\frac{E_{sph}(T)}{T} = \frac{4\pi}{g} B \frac{\sqrt{\langle h\rangle^2 + \langle\varphi\rangle^2}}{T}$$

Inactive sphaleron process requires (naively)

$$\frac{\sqrt{\langle h\rangle^2+\langle\varphi\rangle^2}}{T} \gtrsim 1$$

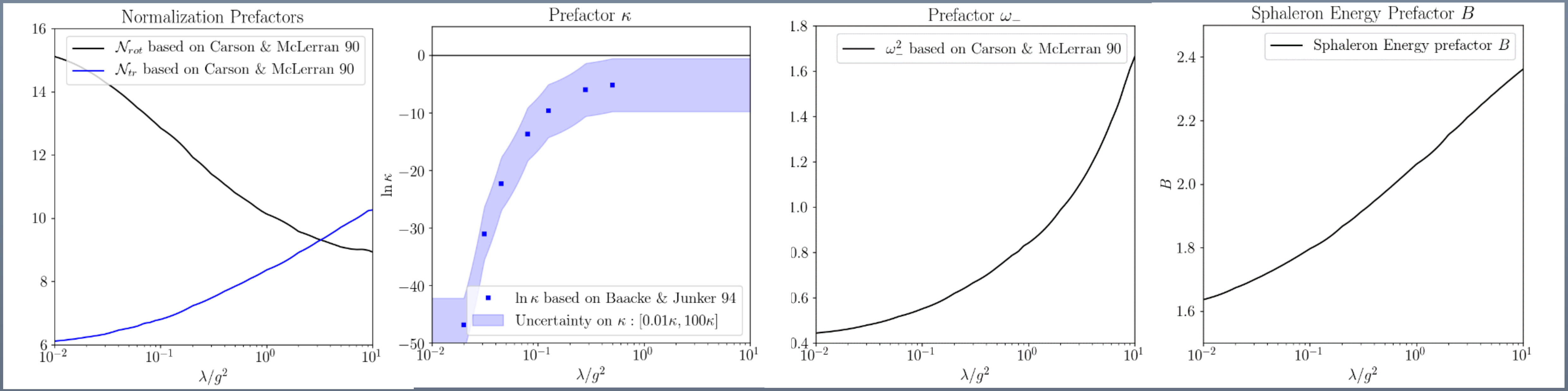
for all temperatures from UV to zero T.



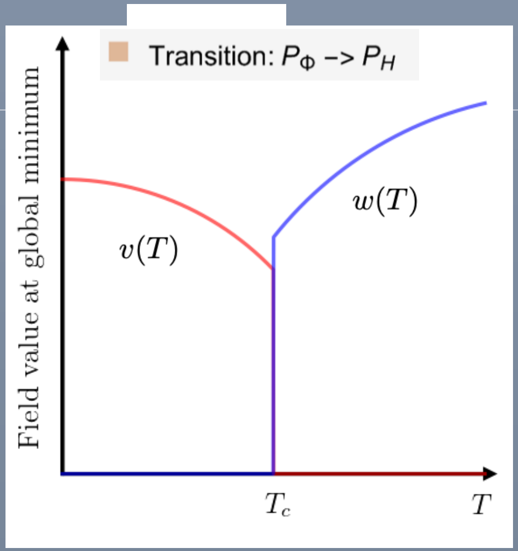
Supplementary: Sphaleron washout and dilution factor

$$\text{Dilution factor } f_{w.o.} = 1 - \frac{n_B(t_{now})}{n_B(0)} = 1 - \exp\left[-\frac{13n_f}{2}\int_0^{T_{\rm high}} dT \, \frac{\Gamma(T)}{VT^6} M_{Pl}\sqrt{\frac{90}{8\pi^3 g^*}}\right]$$

$$\frac{\Gamma}{V} = 4\pi\omega_- \mathcal{N}_{tr}\mathcal{N}_{rot}T^3\left(\frac{v_{\rm EW}(T)}{T}\right)^6 \kappa \exp\left[-E_{sph}(T)/T\right]$$

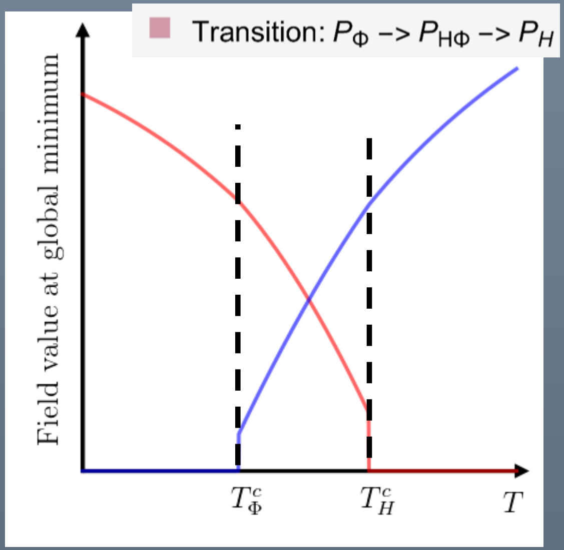


Supplementary - mean field analysis



$$\mathrm{P}_\Phi \text{ phase}: w(T) = \sqrt{-\frac{\mu_\Phi^2 + c_\varphi T^2}{\lambda_\Phi}} \qquad \mathrm{P}_H \text{ phase}: v(T) = \sqrt{\frac{\mu_H^2 - c_h T^2}{\lambda_H}}$$

$$\text{The critical temperature}: T_c = \sqrt{\frac{\mu_H^2 + \sqrt{\lambda_H/\lambda_\Phi}\,\mu_\Phi^2}{c_h - \sqrt{\lambda_H/\lambda_\Phi}\,c_\varphi}}$$



$$\mathrm{P}_{H\Phi} \text{ phase}: \widetilde{v}(T) = \sqrt{\frac{\widetilde{\mu}_H^2 - \widetilde{c}_h T^2}{\widetilde{\lambda}_H}}, \quad \widetilde{w}(T) = \sqrt{-\frac{\widetilde{\mu}_\Phi^2 + \widetilde{c}_\varphi T^2}{\widetilde{\lambda}_\Phi}}$$

which is the global minimum as long as existing if $4\lambda_\Phi\lambda_H - \lambda_{H\Phi}^2 \geq 0$

$$\text{The critical temperatures}: T_H^c = \sqrt{\frac{\widetilde{\mu}_H^2}{\widetilde{c}_h}}, \quad T_\Phi^c = \sqrt{\frac{\widetilde{\mu}_\Phi^2}{-\widetilde{c}_\varphi}}$$

Relevant parameters:

$$\widetilde{\mu}_H^2 \equiv \mu_H^2 + \frac{\Lambda_{H\Phi}}{2\lambda_\Phi}\mu_\Phi^2, \quad \widetilde{\mu}_\Phi^2 \equiv \mu_\Phi^2 + \frac{\Lambda_{H\Phi}}{2\lambda_H}\mu_H^2,$$

$$\widetilde{c}_h \equiv c_h - \frac{\Lambda_{H\Phi}}{2\lambda_\Phi}c_\varphi, \quad \widetilde{c}_\varphi \equiv c_\varphi - \frac{\Lambda_{H\Phi}}{2\lambda_H}c_h,$$

$$\widetilde{\lambda}_H \equiv \lambda_H - \frac{\Lambda_{H\Phi}^2}{4\lambda_\Phi}, \quad \widetilde{\lambda}_\Phi \equiv \lambda_\Phi - \frac{\Lambda_{H\Phi}^2}{4\lambda_H}$$