Quarkonium structure in LQCD at zero and finite temperature

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Heavy quarkonium, non-relativistic potential picture and effective field theory approach (NRQCD, pNRQCD)

Lattice results on quarkonia at zero temperature
(relativistic approach, NRQCD, mass spectrum, potentials and wave functions)

Quarkonia properties at finite temperature
(masses, width and the complex potential)

Physics Opportunities with Heavy Quarkonia at the EIC, CFNS, Oct 25-27, 2021

Quarkonia and potential models

$m_b, m_c \gg \Lambda_{QCD}$ $\Rightarrow$ non-relativistic bound states, analogs QED positronium

1-gluon exchange, $\alpha_s \sim 0.4$

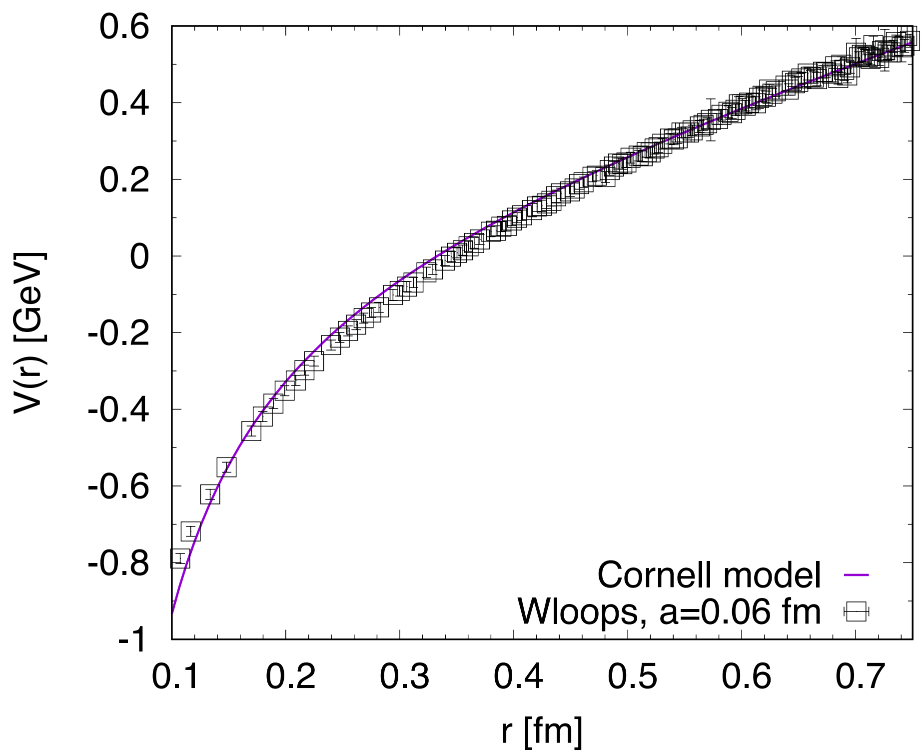
$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r + spin\ dep.$$

Confinement

Eichten et al, PRL 34 (75) 369, PRD 21 (80) 203

Very successful in describing charmonium and bottomonium spectrum below the the open charm and beauty threshold

Nevertheless nearly perfect agreement between the phenomenological and lattice potentials



Problems:

Running of α_s ?

Linear potential valid only for $r \gg 1$ fm,

$$V(r) = \sigma r - \frac{\pi}{12r} + \dots$$

Lüscher term

Soft gluon fields ??

Quarkonia in effective theory approach

$M \gg 1/r \sim Mv \gg Mv^2, \;\; M = m_{c,b}$ ➡ Effective theory (EFT) approach

Non-relativistic QCD (NRQCD) : EFT at scale $1/r$ (scale M is integrated out):

$$L_{NRQCD} = \psi^\dagger \left(iD_0 - \frac{D_i^2}{2M} \right) \psi + \chi^\dagger \left(iD_0 + \frac{D_i^2}{2M} \right) \chi + \ldots + \tfrac{1}{4} F_{\mu\nu}^2 + \bar{q} \gamma_\mu D_\mu q$$

Heavy quark fields are Pauli spinors, heavy pair creation is only present implicitly through higher dimension 4-fermion operators

Caswell, Lepage, PLB 167 (86) 437

potential NRQCD (pNRQCD): EFT at scale $E_{bin} \sim Mv^2$ (scale $1/r \sim Mv$ is integrated out):

$$L_{pNRQCD} = \int d^3\mathbf{r} \mathrm{Tr}\Big[S^\dagger \left[i\partial_0 - \left(\frac{-\nabla_r^2}{M} + V_s(r) + \ldots \right) \right] S + O^\dagger \left[iD_0 + \frac{-\nabla_r^2}{M} + V_o(r) + \ldots \right) \Big] O \Big]$$

$$+ V_A(r) \mathrm{Tr}\left[O^\dagger \mathbf{r} g \mathbf{E} S + S^\dagger \mathbf{r} g \mathbf{E} O \right] + V_B(r) \mathrm{Tr}\left[O^\dagger \mathbf{r} g \mathbf{E} O + O^\dagger O \mathbf{r} g \mathbf{E} \right] +$$

$$\mathcal{O}\left(r^2, \frac{1}{M} \right) + \frac{1}{4} F_{\mu\nu}^2 + \bar{q} \gamma_\mu D_\mu q$$

Brambilla, Pineda, Soto, Vairo, NPB 566 (00) 275

$S = S(\mathbf{r}, \mathbf{R}, t), \;\; O = O(\mathbf{r}, \mathbf{R}, t), \;\; E = E(\mathbf{R}, t)$

Potentials are parameters of the EFT Largangian

Tree level $\leftrightarrow$ potential model $\left(i\partial_0 + \frac{\nabla_r^2}{M} - V_s(r) \right) S(\mathbf{r}, \mathbf{R}, t) = 0$

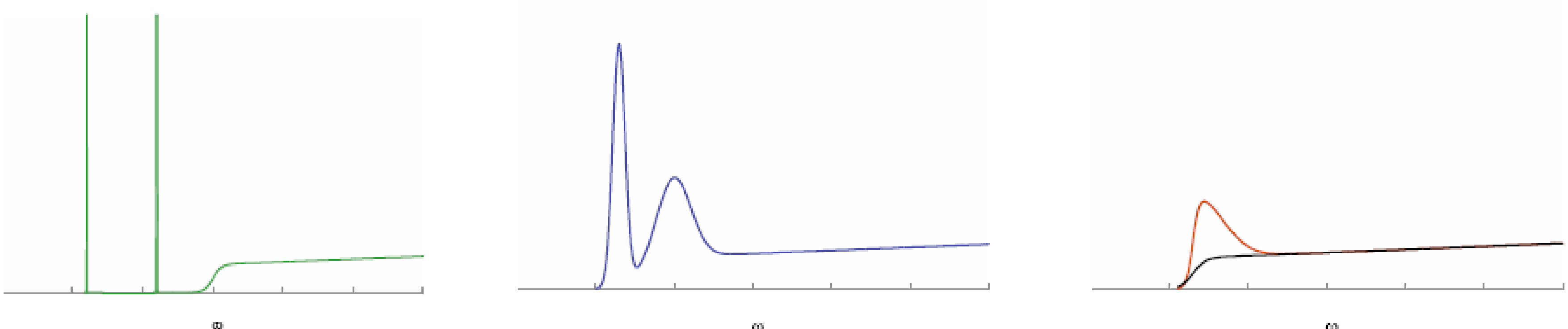
Meson correlators and spectral functions

Vacuum and in-medium properties as well as dissolution of mesons are encoded in the spectral functions:

$$\rho(\omega, p, T) = \frac{1}{2\pi} \mathrm{Im} \int_{-\infty}^{\infty} dt e^{i\omega t} \int d^3x e^{ipx} \langle [O(x,t), O(0,0)] \rangle_T, \quad O(x,t) \sim \bar{Q}(x,t) \Gamma Q(x,t)$$

Melting is seen as progressive broadening and disappearance of the bound state peaks

Modifications of quarkonium yields in heavy ion collisions Matsui and Satz, PLB 178 (1986) 416



$$C(\tau, T) = \sum_x \langle O(x,\tau) O(0,0) \rangle_T \quad \longleftrightarrow \quad C(\tau, T) = \int_{-\infty}^{+\infty} d\omega \rho(\omega, T) e^{-\tau\omega}$$

Consider large τ behavior of $C(\tau, T=0)$:

$$C(\tau, T) \sim \sum_n |\langle 0|O|n\rangle|^2 e^{-M_n \tau} \simeq f_1 e^{-M_1 \tau} + f_2 e^{-M_2 \tau} + \dots$$

$T > 0: \quad \tau < 1/T \Rightarrow \text{reconstruct } \rho(\omega, T)$

NRQCD on the Lattice

Advantages: No large cutoff effects $\sim aM_b$, large τ range for $T > 0$

Inverse lattice spacing provides a natural UV cutoff for NRQCD, provided $a^{-1} \leq 2M_Q$ (lattices cannot be too fine)

Quark propagators are obtained as initial value problem:

$$G_\psi(\mathbf{x}, t) = \langle \psi(\mathbf{x}, t)\, \psi^\dagger(\mathbf{0}, 0) \rangle \qquad G_\chi(\mathbf{x}, t) = -G_\psi^\dagger(\mathbf{x}, t)$$

$$G_\psi(t) = K(t) G_\psi(t-1),$$

$$K(t) = \left(1 - \frac{a\delta H|_t}{2}\right)\left(1 - \frac{aH_0|_t}{2n}\right)^n U_4^\dagger(t) \times \left(1 - \frac{aH_0|_{t-1}}{2n}\right)^n \left(1 - \frac{a\delta H|_{t-1}}{2}\right),$$

$$t = \tau/a,\ \ H_0 = \frac{-\Delta^{(2)}}{2M_b},\quad \delta H \sim v^4,\ v^6(\text{ spin} - \text{dep.})$$

@ Tree level

Meinel, PRD 82 (2010) 114502

masses are only defined up to *a*-dependent shift: $M_{\Upsilon(1S)} = E_{\Upsilon(1S)} + C_{\text{shift}}(a)$

Use kinetic mass instead: $E_{\Upsilon(1S)}(p) = E_{\Upsilon(1S)} + C_{\text{shift}}(a) + \dfrac{p^2}{2M^{kin}_{\Upsilon(1S)}}$

Tune M_b such that $M^{kin}_{\Upsilon(1S)} = M^{PDG}_{\Upsilon(1S)}$

NRQCD meson correlators

Point correlators:

Aarts et al (FASTUM) , Kim, PP, Rothkopf

$$C_p(t) = \sum_{\mathbf{x}} \langle O_p(t,\mathbf{x}) O_p(0,\mathbf{0}) \rangle,$$

$$O_p(t,\mathbf{x}) = \chi^\dagger(t,\mathbf{x}) \Gamma \psi(t,\mathbf{x}))$$

State	Irrep Λ^{PC}	Γ
η_b	A_1^{-+}	1
Υ	T_1^{--}	σ_j
h_b	T_1^{+-}	∇_j
χ_{b0}	A_1^{++}	$\boldsymbol{\sigma} \cdot \boldsymbol{\nabla}$
χ_{b1}	T_1^{++}	$(\boldsymbol{\sigma} \times \boldsymbol{\nabla})_j$
χ_{b2}	T_2^{++}	$\sigma_j \nabla_k + \sigma_k \nabla_j$

Extended correlators:

$$O_p(t,\mathbf{x}) \to O(t,\mathbf{x}) = \sum_{\mathbf{r}} \Psi(\mathbf{r}) \chi^\dagger(t,\mathbf{x}) \Gamma \psi(t,\mathbf{x}+\mathbf{r}))$$

$\Psi(\mathbf{r}) \sim e^{-|\mathbf{r}|^2/\sigma^2}$

or realistic wave-function

Optimized correlators: use several different extended meson operators with realistic wave functions and form orthogonal combinations

$$O_i \to \tilde{O}_\alpha = \Omega_{\alpha j} O_j, \ \langle \tilde{O}_\alpha(t) \tilde{O}_\beta^\dagger(0) \rangle \propto \delta_{\alpha,\beta}, \ i = 1, 2, 3, \dots$$

Mixed correlators (Bethe-Salpeter amplitudes):

Bethe-Salpeter amplitude

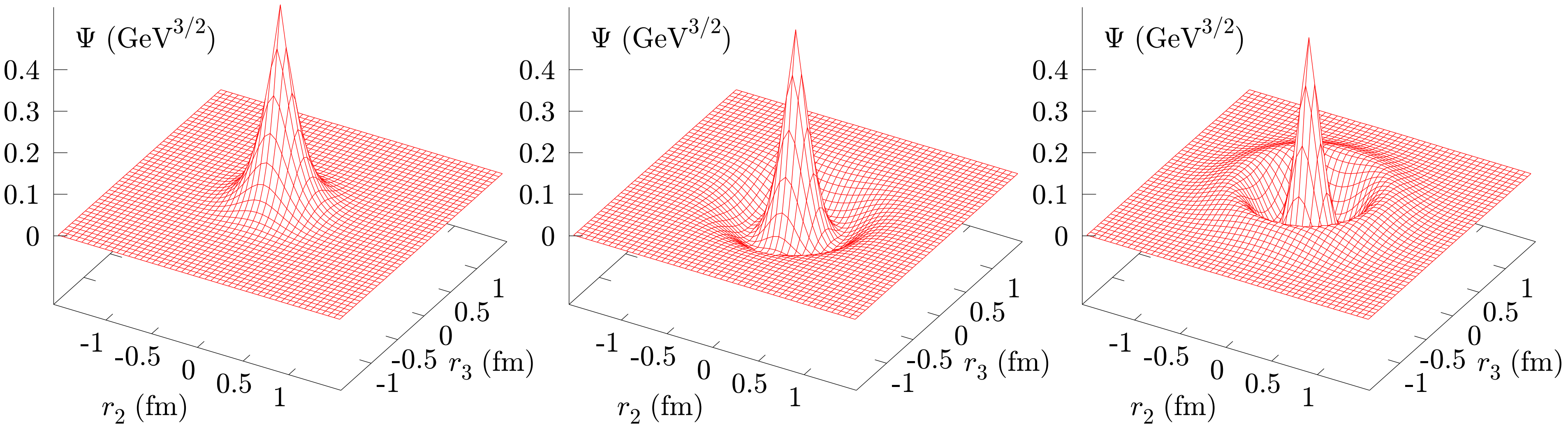
$$\tilde{C}_\alpha^r(t) = \sum_{\mathbf{x}} \langle O_{qq}^r(t,\mathbf{x}) \tilde{O}_\alpha(0,\mathbf{0}) \rangle \sim \phi_\alpha(r) e^{-E_\alpha t}, \ t \to \infty$$

$$O_{qq}^r(t,\mathbf{x}) = \chi^\dagger(t,\mathbf{x}) \Gamma \psi(t,\mathbf{x}+\mathbf{r}))$$

Optimized Meson Operators

$O_i(\mathbf{x},t) = \sum_{\mathbf{r}} \Psi_i(\mathbf{r})\chi^\dagger(\mathbf{x}+\mathbf{r},t)\Gamma\psi(\mathbf{x},t)$ $\Psi_i(\mathbf{r})$ from potential model with Cornell potential

Meinel, PRD 82 (2010) 114502



Good overlap with bottomonium states but

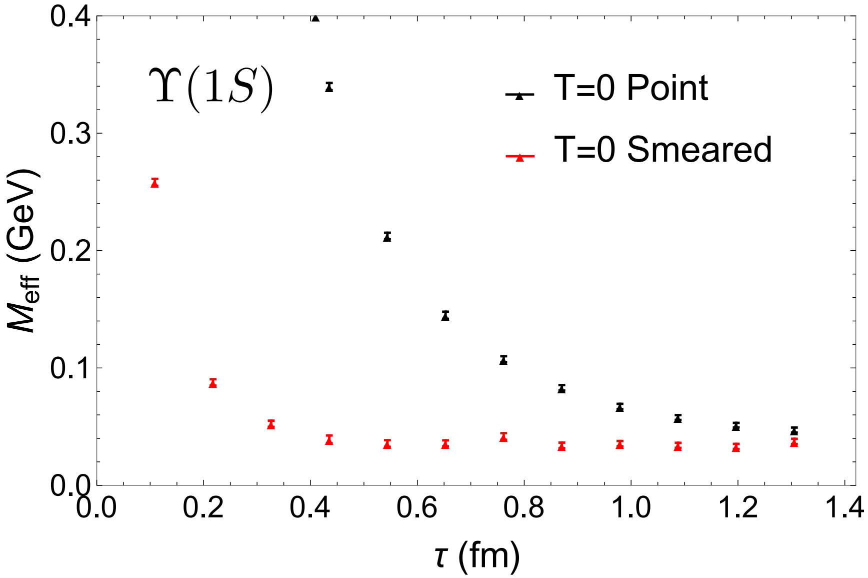
$C_{ij}(t) = \langle O_i(t) O_j^\dagger(0)\rangle \neq 0$ for $i \neq j$

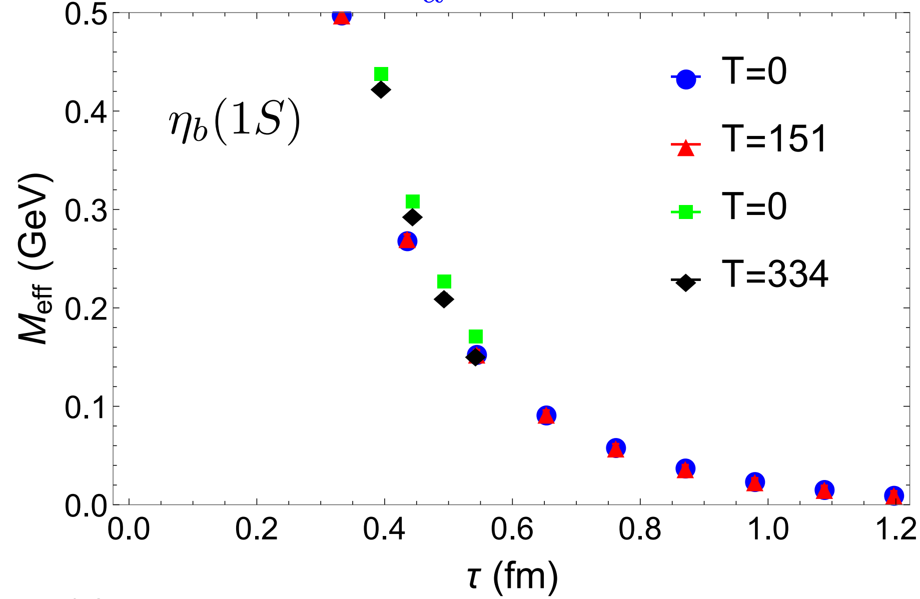
$O_i \to \tilde{O}_\alpha = \Omega_{\alpha j} O_j$ such that
$\langle \tilde{O}_\alpha(t)\tilde{O}_\beta^\dagger(0)\rangle \propto \delta_{\alpha,\beta}$
$\Omega_{\alpha j}$ can be obtained as
$G_{ij}(t)\Omega_{\alpha j} = \lambda_\alpha(t,t_0) G_{ij}(t_0)\Omega_{\alpha j}$.

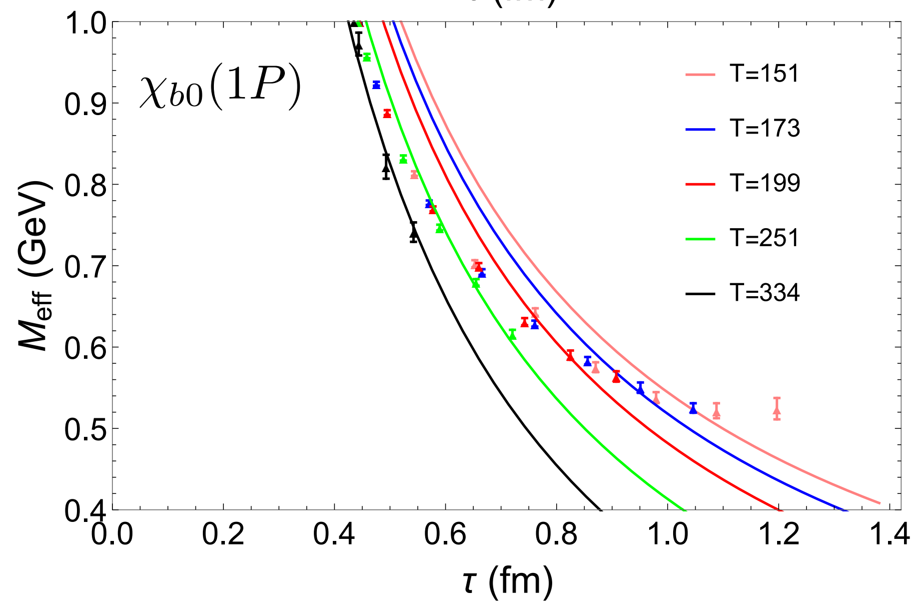
Point operators vs. extended operators

Larsen, Meinel, Mukherjee, PP, PRD100 (2019) 074506

$$M_{\rm eff}(\tau) = \frac{1}{a}\ln[C_\alpha(t)/C_\alpha(\tau+a)]$$



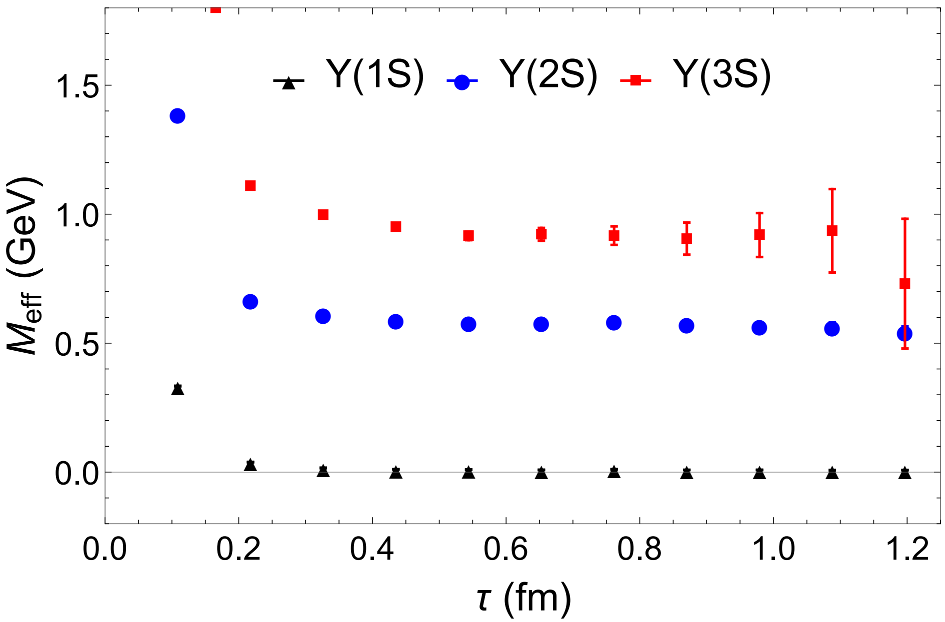


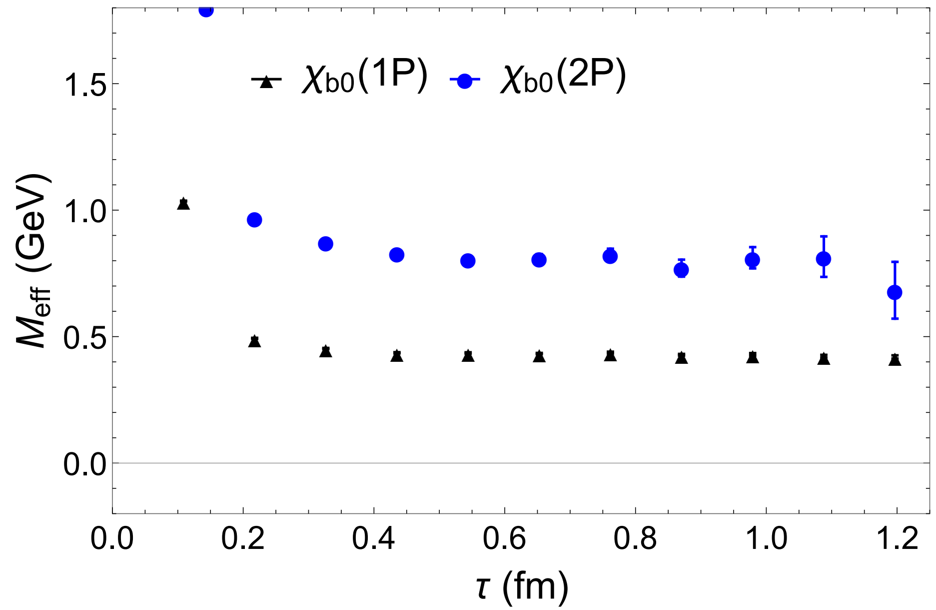


- The effective masses of point correlators do not show a plateau for $\tau < 1.2$ fm and have very small temperature dependence
- The small τ behavior of the effective masses is well described by perturbation theory for P-wave bottomonia
- The correlators of extended operators approach a plateau for $\tau < 1$ fm.

Correlators of Optimized Meson Operators at T=0

$$M_{\rm eff}(\tau) = \frac{1}{a}\ln[C_\alpha(t)/C_\alpha(\tau+a)]$$





$$C_\alpha(\tau,T) = \int_{-\infty}^{\infty} d\omega \rho_\alpha(\omega,T) e^{-\omega\tau} \qquad \rho_\alpha(\omega,T) = \rho_\alpha^{\rm med}(\omega,T) + \rho_\alpha^{\rm high}(\omega)$$

$$\rho_\alpha^{\rm med}(\omega,T=0) = A_\alpha \delta(\omega - M_\alpha) \Rightarrow C_\alpha(\tau,T=0) = A_\alpha e^{-M_\alpha\tau} + C_\alpha^{\rm high}(\tau)$$

Determine A_α,M_α from single exponential fit for $\tau > 0.6$fm and then $C_\alpha^{\rm high}(\tau)$

Lattice results on bottomonium spectrum

$$\Delta M = M - \overline{M}(1S), \quad \overline{M}(1S) = (M_{\eta_b(1S)} + 3M_{\Upsilon(1S)})/4$$

state	ΔM [MeV]	$\Delta M(PDG)$ [MeV]
$\Upsilon(3S)$	906.0(25.0)(5.2)	910.3(0.7)
$h_b(2P)$	804.4(35.8)(4.7)	814.9(1.3)
$\chi_{b2}(2P)$	809.2(36.2)(4.7)	823.8(0.9)
$\chi_{b1}(2P)$	802.2(34.9)(4.7)	810.6(0.7)
$\chi_{b0}(2P)$	786.8(32.7)(4.6)	787.6(0.8)
$\Upsilon(2S)$	582.7(9.8)(3.4)	578.4(0.6)
$h_b(1P)$	454.5(4.7)(2.6)	454.4(0.9)
$\chi_{b2}(1P)$	463.3(4.8)(2.7)	467.3(0.6)
$\chi_{b1}(1P)$	448.9(4.6)(2.6)	447.9(0.6)
$\chi_{b0}(1P)$	421.3(4.7)(2.4)	414.5(0.7)
hyperfine(3S)	13.4(6.2)(0.1)	NA
hyperfine(2S)	24.1(1.0)(0.1)	24.5(4.5)

Larsen, Meinel, Mukherjee, PP, PLB 800 (20) 135119

Prediction for $\eta_b(3S)$!

$1S$ hyperfine splitting, $M_{\Upsilon(1S)} - M_{\eta_b(1S)}$ is not reproduced within this approach because it is very sensitive to short distance physics $\Rightarrow$ need radiative correction in the NRQCD Lagrangian Dowdal et al (HPQCD), PRD 85 (12) 054509; PRD 89 (14) 031502(R)

or relativistic approach Hatton et al, PRD 103 (21) 054512

For charmonium it is better to use relativistic lattice formulation, Burch et al, PRD 81 (2010) 034508

Bottomonium Bethe-Salpeter amplitude at T=0

$$\tilde{C}^r_\alpha(\tau) = \sum_{\mathbf{x}} \langle O^r_{qq}(\tau,\mathbf{x}) \tilde{O}_\alpha(0,0)\rangle, \; O^r_{qq}(\tau,\mathbf{x}) = \chi^\dagger(\tau,\mathbf{x})\Gamma\psi(\tau,\mathbf{x}+r))$$

$$\tilde{C}^r_\alpha(\tau) = \sum_n \langle 0|O^r_{qq}(0)|n\rangle\langle n|\tilde{O}_\alpha(0)|0\rangle e^{-E_n\tau}|_{\tau\to\infty} \sim \langle 0|O^r_{qq}(0)|\alpha\rangle e^{-E_\alpha\tau}$$

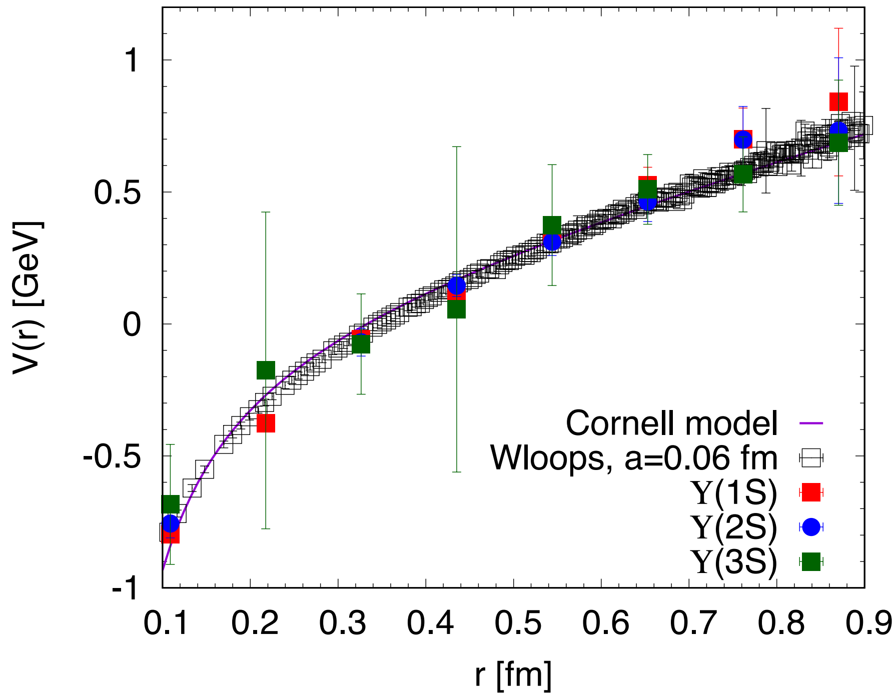
$\phi_\alpha(r)$- Bethe-Salpeter amplitude

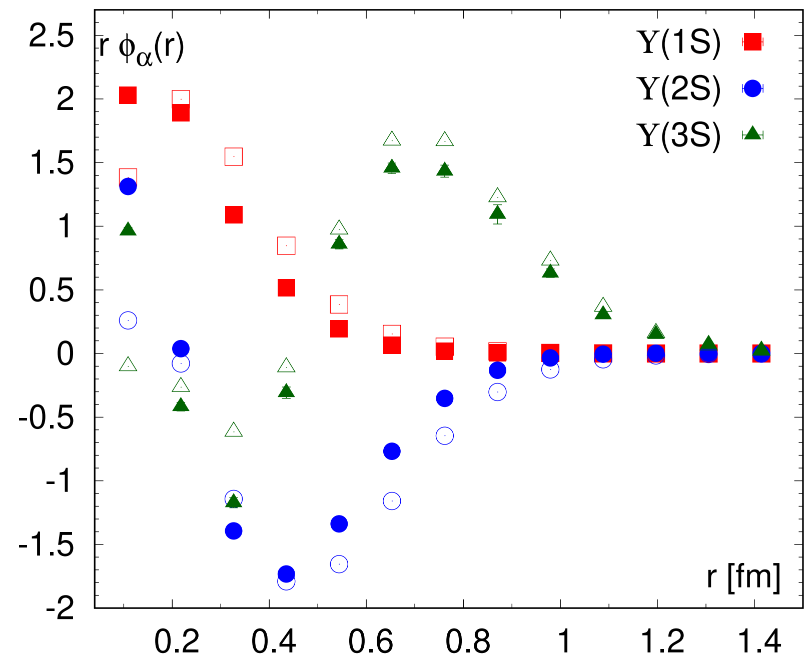
Kawanai, Sasaki, PRL 107 (11) 091601 (charmonium)

Larsen, Meinel, Mukherjee, PP, PRD 102 (20)114508

$$\left(\frac{-\nabla^2}{m_b} + V(r)\right)\phi_\alpha = E_\alpha\phi_\alpha$$

$$m_b = 5.52 \pm 0.33 \text{ GeV}$$

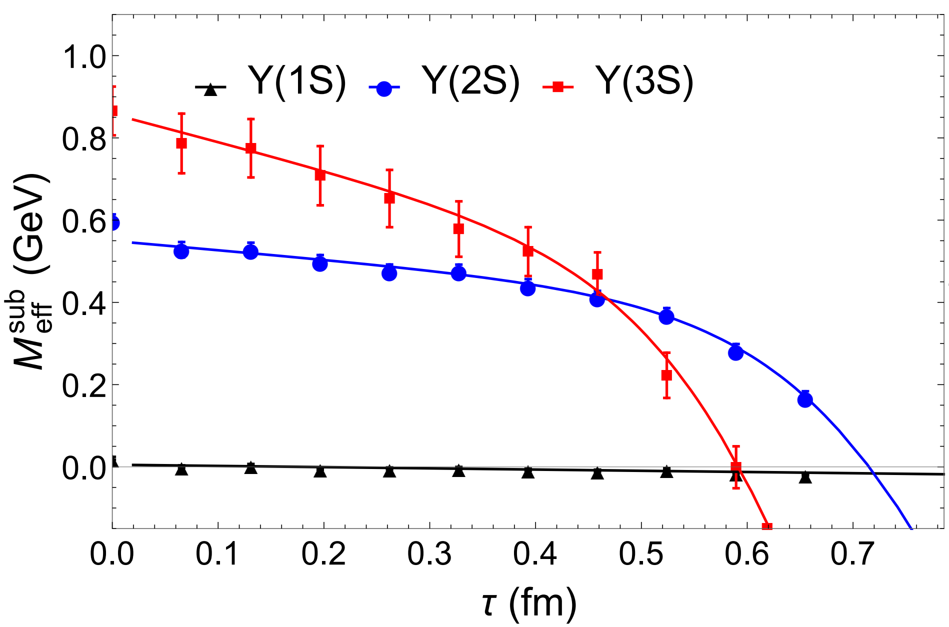


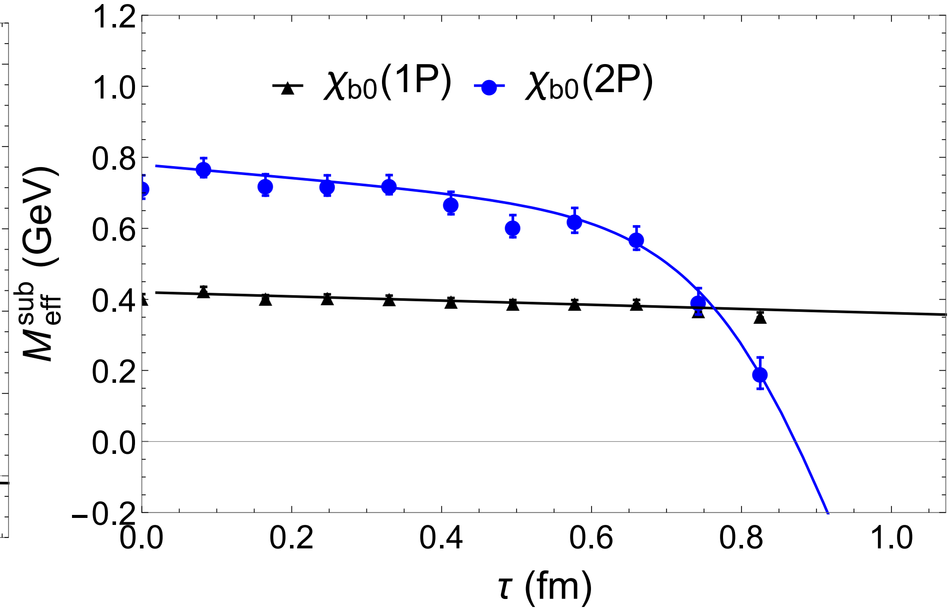


Correlators of Extended Meson Operators at T>0

HISQ, $N_\tau = 12$

$$C_\alpha^{\rm sub}(\tau,T) = C_\alpha(\tau,T) - C_\alpha^{\rm high}(\tau) \Rightarrow aM_{\rm eff}^{\rm sub}(\tau,T) = \ln\left(C_\alpha^{\rm sub}(\tau,T)/C_\alpha^{\rm sub}(\tau+a,T)\right)$$





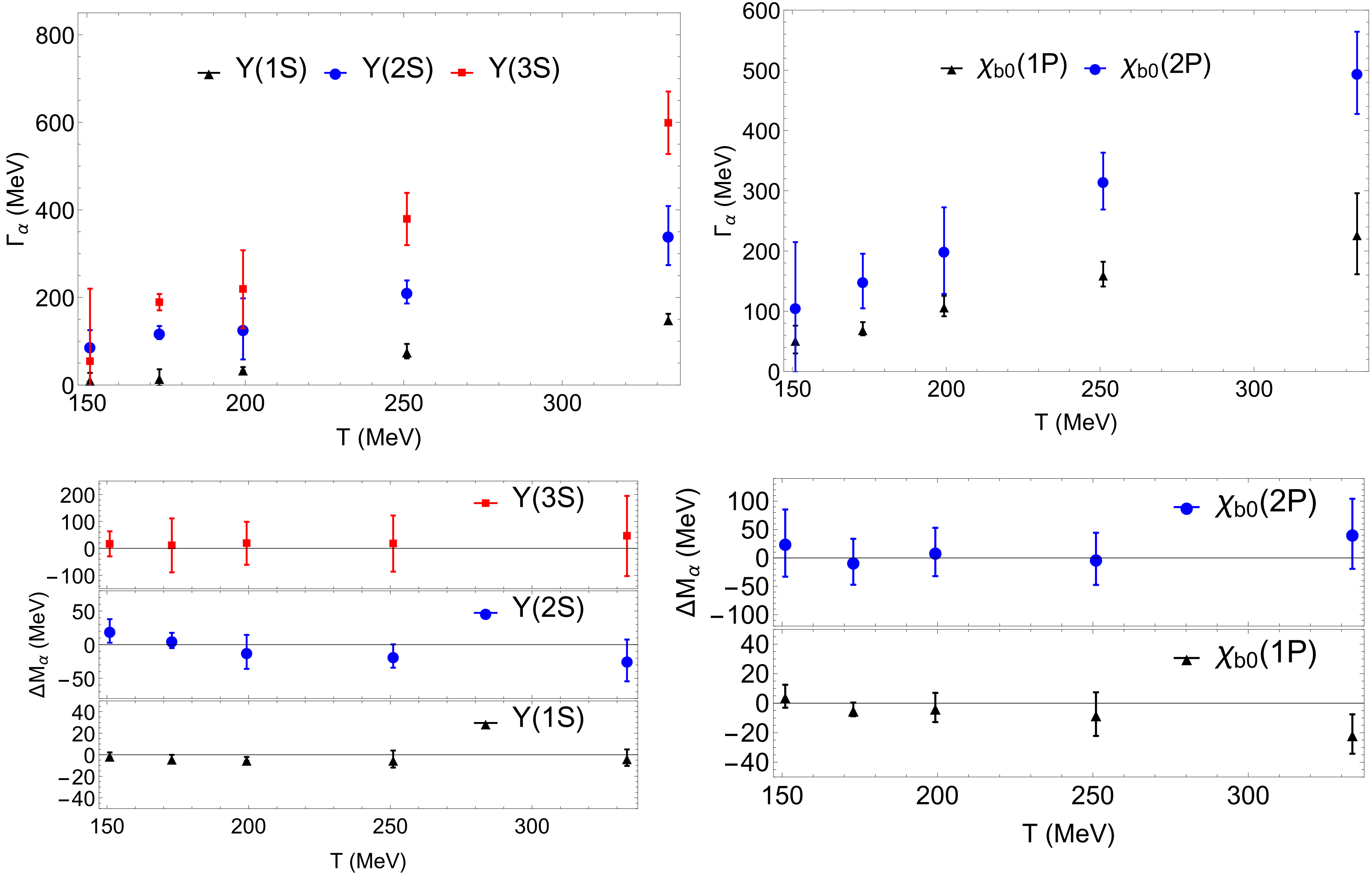
Fit $M_{\rm eff}^{\rm sub}(\tau,T)$ using a simple Ansatz:

$$\rho_\alpha^{\rm med}(\omega,T) = A_\alpha^{\rm cut}(T)\,\delta\left(\omega - \omega_\alpha^{\rm cut}(T)\right) + A_\alpha(T)\exp\left(-\frac{[\omega - M_\alpha(T)]^2}{2\Gamma_\alpha^2(T)}\right)$$

Low energy tail

$$\Rightarrow M_\alpha(T), \Gamma_\alpha(T)$$

Thermal width and mass shift of bottomonium



Quark anti-quark potential at T>0

Conjecture, Matsui and Satz, PLB 178 (86) 416 $-\frac{4}{3}\frac{\alpha_s}{r}+\sigma r \to -\frac{4}{3}\frac{\alpha_s}{r}e^{-m_D r}, T > T_c$

Extending pNRQCD to *T>0*: the potential is complex, the real part can have thermal correction but is not necessarily screened, except when $r \sim 1/m_D$

Based on weak coupling

Laine, Philipsen, Romatschke, Tassler, JHEP 03 (06) 054
Brambilla, Ghiglieri, PP, Vairo, PRD 78 (08) 014017

Calculate the potential non-perturbatively on the lattice by considering Wislon loops of size $r \times \tau$ at $T>0$

$$W(r,\tau,T)=\int_{-\infty}^{\infty}\rho_r(\omega,T)e^{-\omega\tau}$$

If potential at $T>0$ exists the $\rho_r(\omega,T)$ should have a well define peak at $\omega \simeq \mathrm{Re}V(r,T)$, and the width of the peak is $\mathrm{Im}V(r,T)$

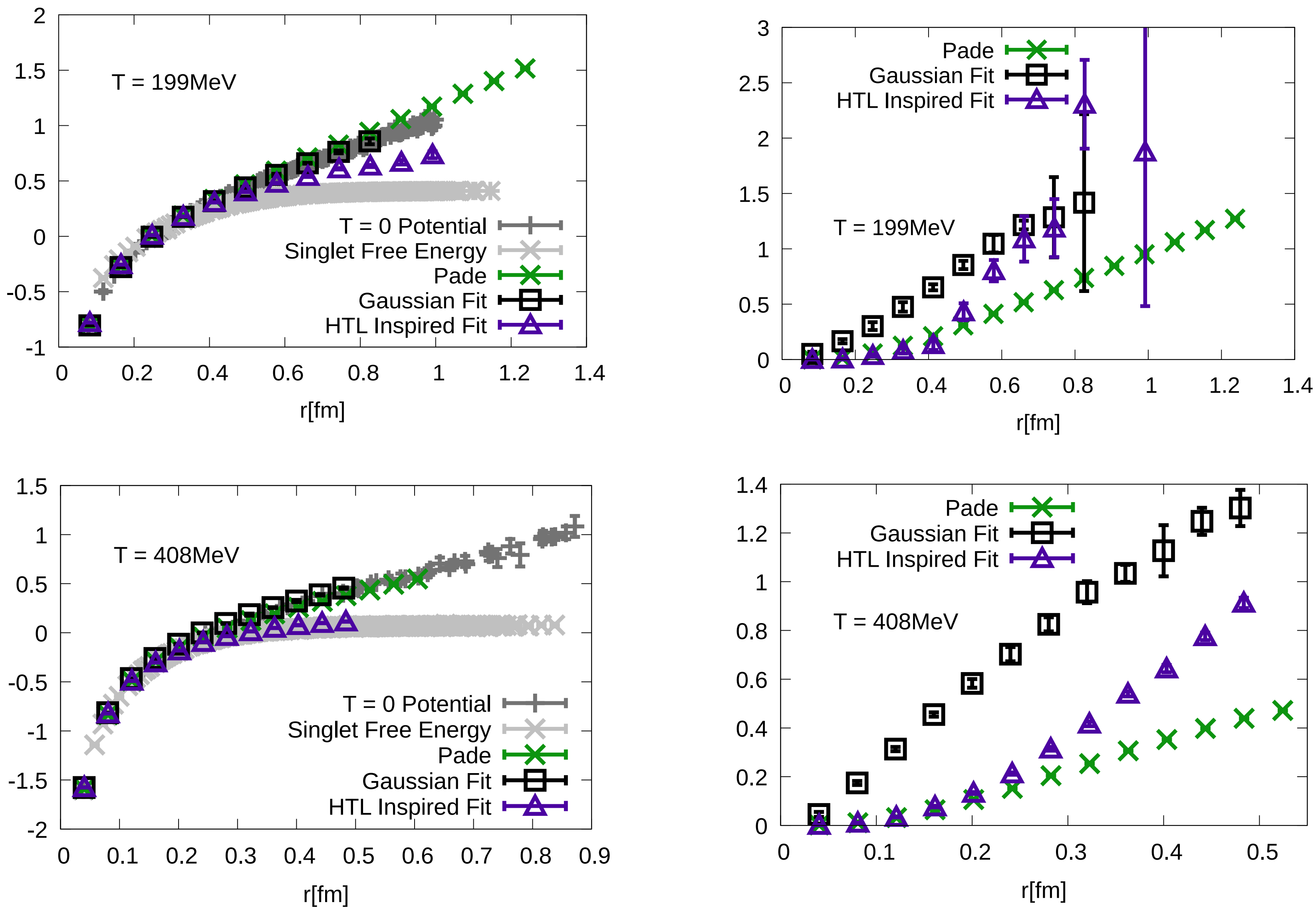
Rothkopf, Hatsuda, Sasaki, PRL 108 (2012) 162001

Challenge: reconstruct $\rho_r(\omega,T)$

Quark anti-quark potential at T>0 from the lattice

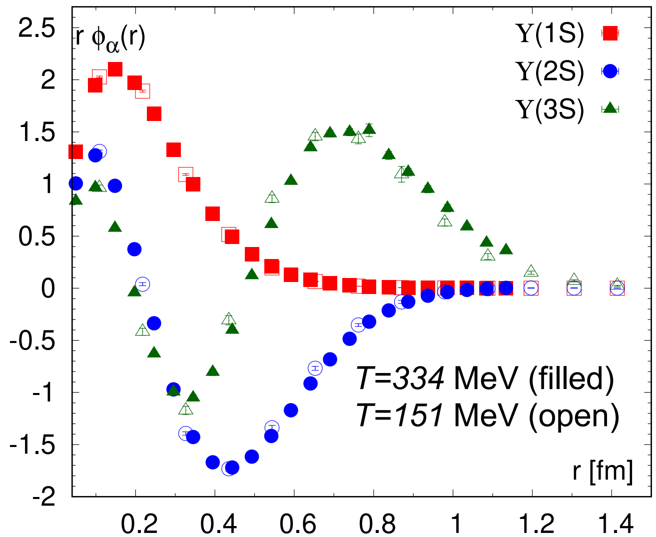
HISQ, $N_\tau = 12$

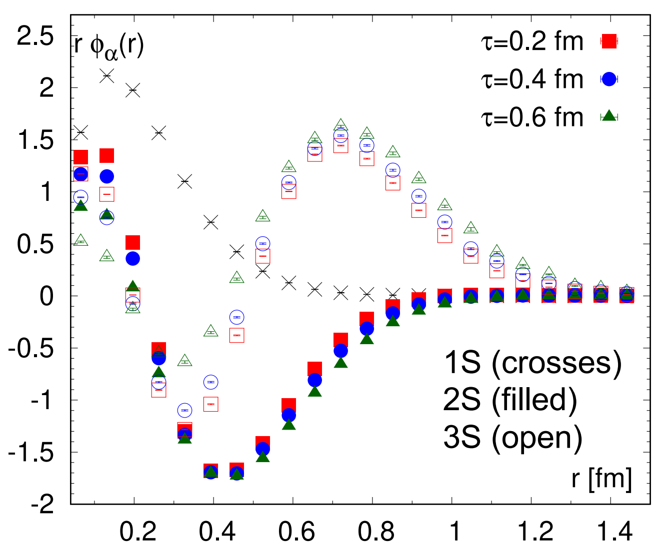
Bala et al (HotQCD), arXiv:2110.11659



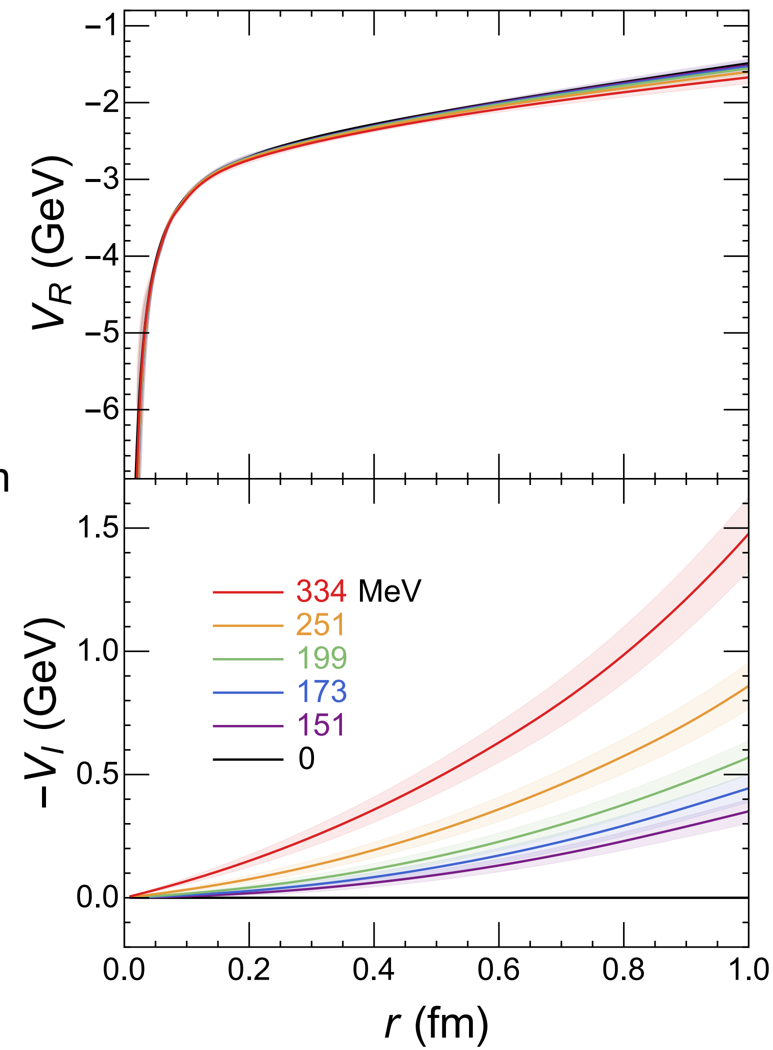
Bethe-Salpeter amplitude at $T>0$ and potential model

Shi et al, arXiv:2105.07862





potential model
with inverse problem
+
Thermal width
from lattice
+
Machine learning



Summary

- Using the correlators of extended operators (or combinations of extended operators) and NRQCD we can obtain a good description of bottomonium spectrum from NRQCD

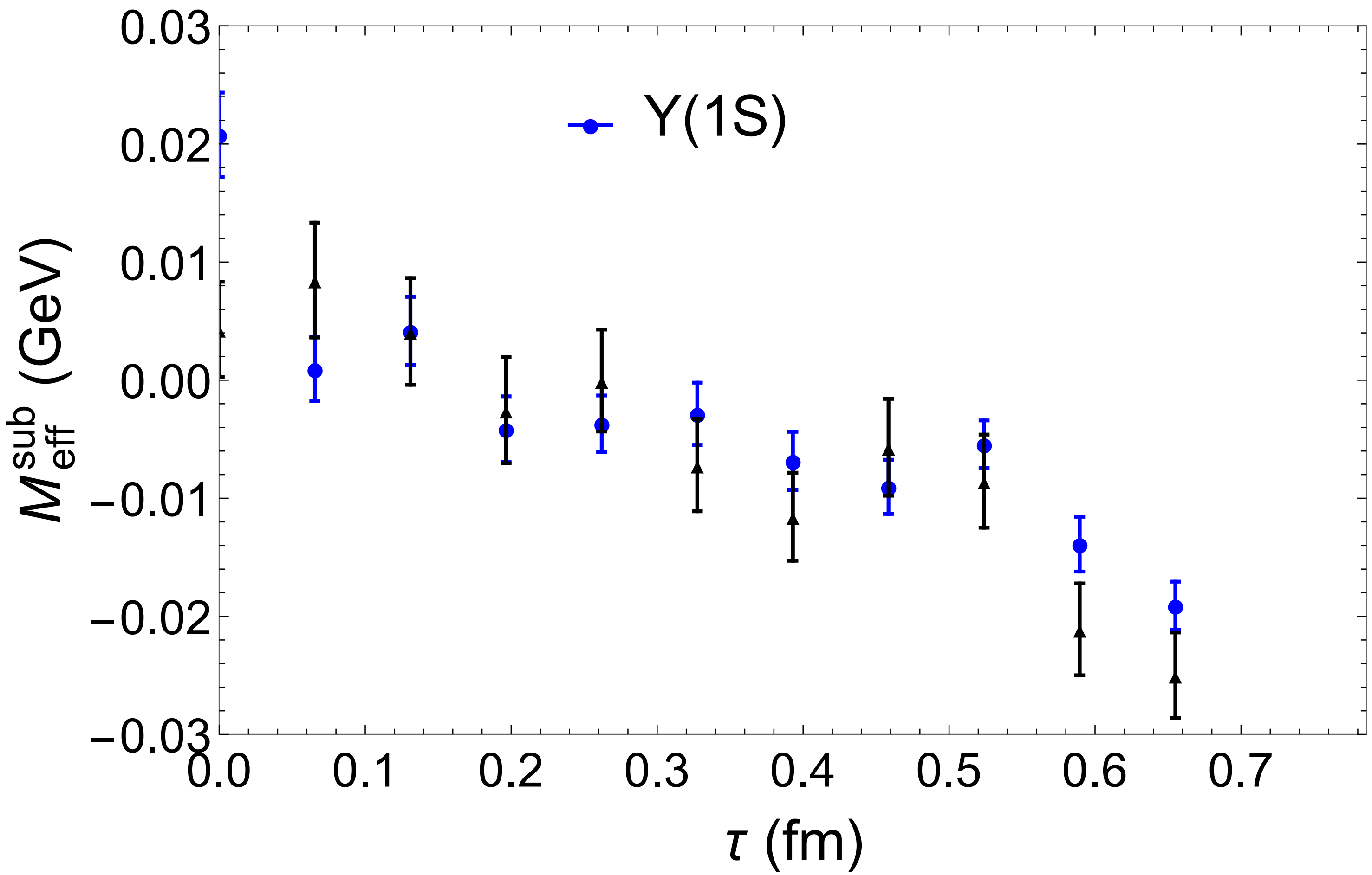
- The lattice study of Bethe-Salpeter amplitudes confirms the potential model description of bottomonium at $T = 0$, the potential obtain from Wilson loops on the lattice agrees very well with the Cornell parametrization

- Using a simple Ansatz for the spectral function we extracted the thermal width of $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\chi_{b1}(1P)$ and $\chi_{b2}(2P)$ states and found that the value of the thermal width follows the hierarchy of the bottomium sizes, as expected

- No significant thermal modification of bottomonium masses have been found in contrast with the expectations based on potential models with screened potential

- Lattice calculations confirm the existence of the imaginary part of the potential; There is no clear evidence for the screening of the real part of the potential

Back-up:Comparison of different Meson Operators

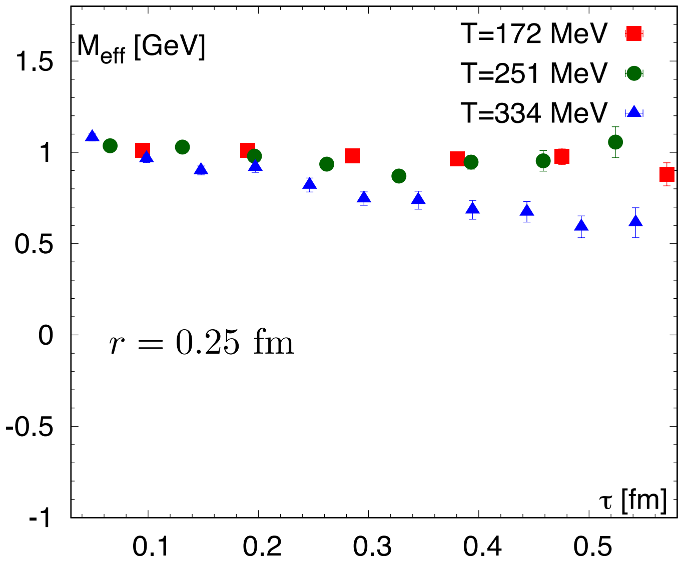


Blue circles: optimized operators

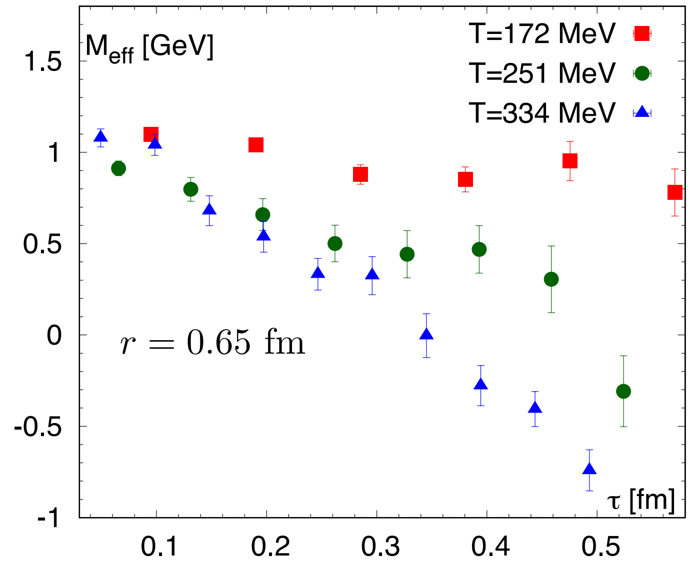
Black triangles: extended operators with Gaussian smearing

Back-up: Bottomonium Bethe-Salpeter amplitude at T>0

$\Upsilon(3S)$

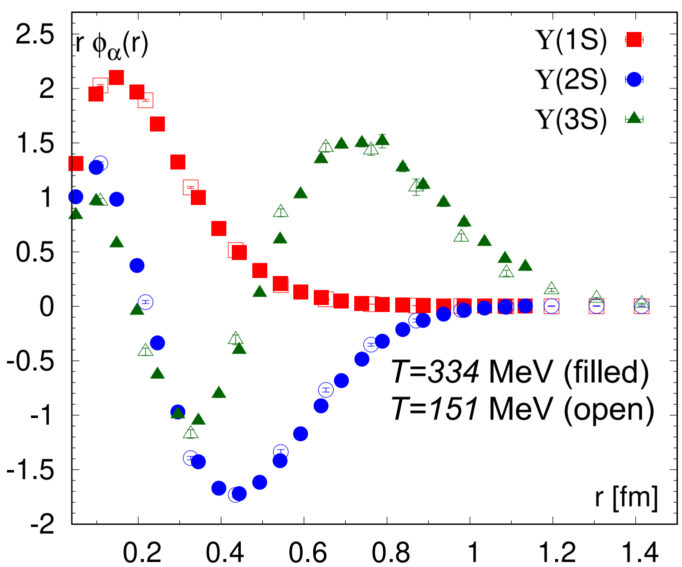


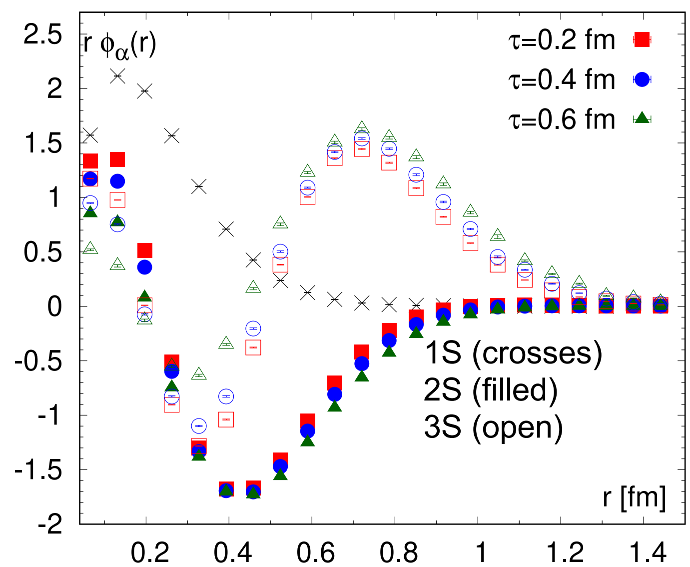
$\Upsilon(3S)$



M_{eff} shows similar thermal effects similar to one obtained from optimized correlators;

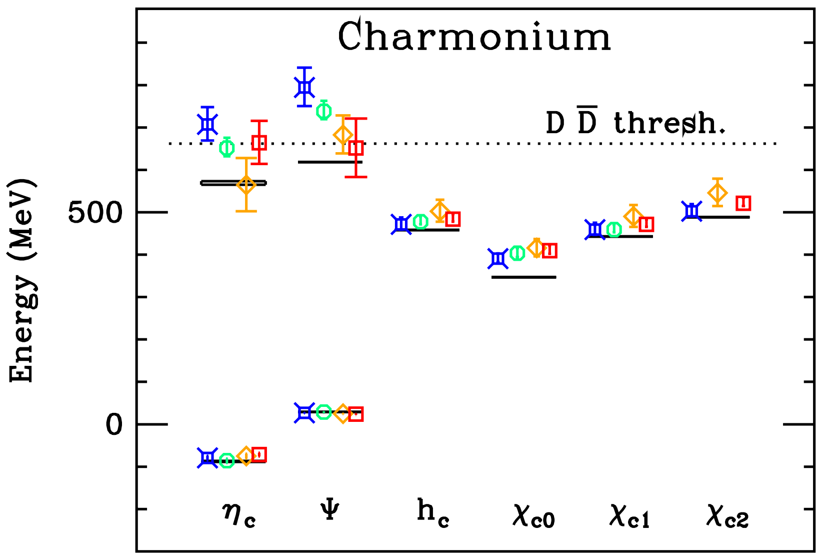
Thermal effects are larger for larger r

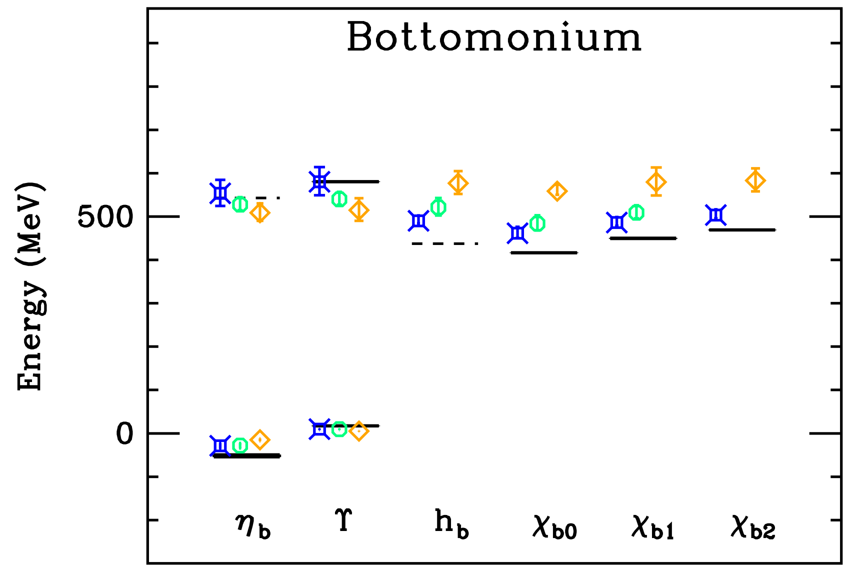




Thermal effects can be seen but ϕ_α is similar to the $T=0$ result at qualitative level

Back-up: relativistic approach





Splitting	Charmonium		Bottomonium	
	This work	Experiment	This work	Experiment
$\overline{1P}$-$\overline{1S}$	$473 \pm 12^{+10}_{-0}$	457.5 ± 0.3	$446 \pm 18^{+10}_{-0}$	456.9 ± 0.8
$^1P_1$-$\overline{1S}$	$469 \pm 11^{+10}_{-0}$	457.9 ± 0.4	$440 \pm 17^{+10}_{-0}$	—
$\overline{2S}$-$\overline{1S}$	$792 \pm 42^{+17}_{-0}$	606 ± 1	$599 \pm 36^{+13}_{-0}$	$(580.3 \pm 0.8)^1$
1^3S_1-1^1S_0	$116.0 \pm 7.4^{+2.6}_{-0}$	116.4 ± 1.2	$54.0 \pm 12.4^{+1.2}_{-0}$	69.4 ± 2.8
$1P$ tensor	$15.0 \pm 2.3^{+0.3}_{-0}$	16.25 ± 0.07	$4.5 \pm 2.2^{+0.1}_{-0}$	5.25 ± 0.13
$1P$ spin-orbit	$43.3 \pm 6.6^{+1.0}_{-0}$	46.61 ± 0.09	$16.9 \pm 7.0^{+0.4}_{-0}$	18.2 ± 0.2
$1S$ $\bar{s}Q$-$\bar{Q}Q$	$1058 \pm 13^{+24}_{-0}$	1084.8 ± 0.8	$1359 \pm 304^{+31}_{-0}$	1363.3 ± 2.2