LDMEs extraction, relativistic correction, soft gluon resummation

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Physics Opportunities with Heavy Quarkonia at the EIC
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Outline

I. NRQCD factorization with dominant LDMEs

II. Relativistic corrections

II. Soft gluon factorization

IV. Summary and outlook

NRQCD: factorization

➢ Factorization formula

Bodwin, Braaten, Lepage, 9407339

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} = \sum_n d\hat{\sigma}_n(P_H)\langle\mathcal{O}_n^H\rangle$$

Production of a heavy quark pair

Hadronization (LDMEs)

- n: quantum numbers of the pair: color, spin, orbital angular momentum, total angular momentum, spectroscopic notation ${}^{2S+1}L_J^{[c]}$

➢ A glory history

- Solved IR divergences in P-wave quarkonium decay
- Explained ψ' surplus
- Explained χ_{c2}/χ_{c1} production ratio
- ….

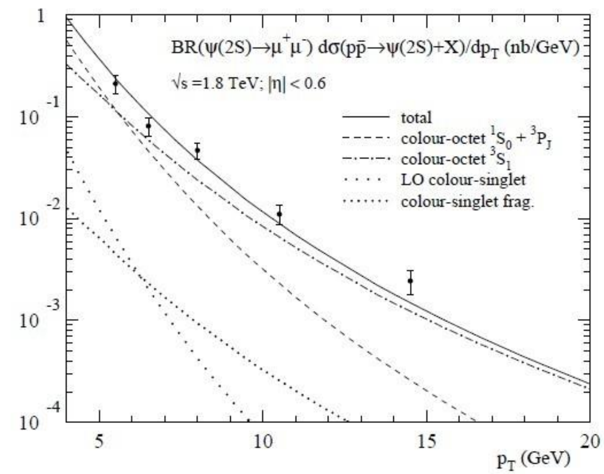
Thanks to color-octet mechanism

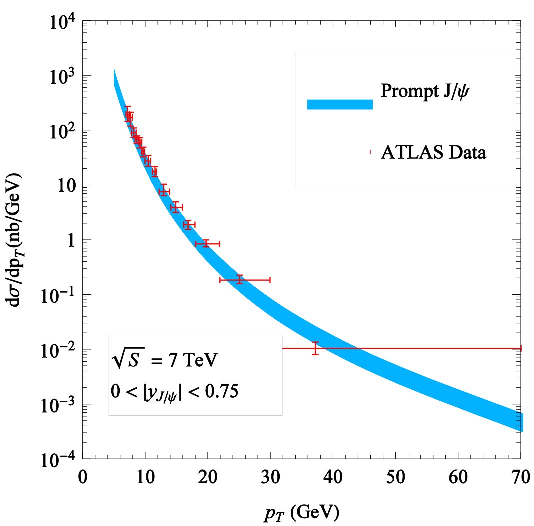
Achievement: explain $\psi(nS)$ surplus

- $\psi(nS)$ production in NRQCD

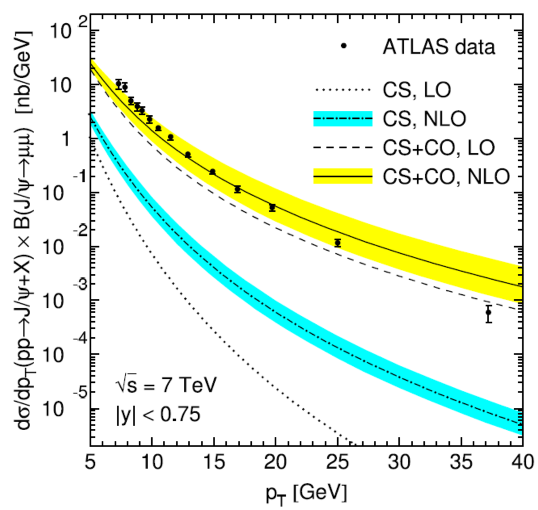
States	Power in v	p_T behavior at LO	p_T behavior at NLO
$^3S_1^{[1]}$	v^0	p_T^{-8}	p_T^{-6}
$^3S_1^{[8]}$	v^4	p_T^{-4}	p_T^{-4}
$^1S_0^{[8]}$	v^3	p_T^{-6}	p_T^{-4}
$^3P_J^{[8]}$	v^4	p_T^{-6}	p_T^{-4}

Kramer, 0106120

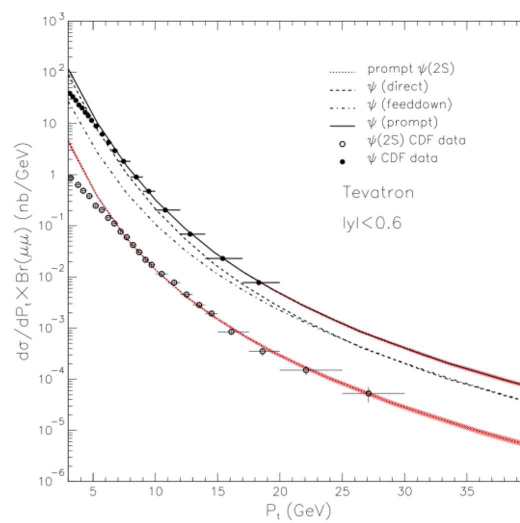




YQM, Wang, Chao, 1012.1030



Butenschoen, Kniehl, 1105.0820



Gong, Wan, Wang, Zhang, 1205.6682

NLO fit by PKU group

➢ Fit J/ψ yield data at Tevatron with $p_T > 7$ GeV

YQM, Wang, Chao, 1009.3655

- Due to p_T^{-4} and p_T^{-6} behaviors, constrain two combinations
- $\mathrm{M}_0 = \langle O\left({}^1\mathrm{S}_0^{[8]}\right)\rangle + 3.9\,\langle O\left({}^3\boldsymbol{P}_0^{[8]}\right)\rangle/m_c^2 \approx 0.074 \pm 0.019$ $\mathbf{GeV}^3$
- $\mathrm{M}_1 = \langle O\left({}^3\mathrm{S}_1^{[8]}\right)\rangle - 0.56\,\langle O\left({}^3\boldsymbol{P}_0^{[8]}\right)\rangle/m_c^2 \approx 0.0005 \pm 0.0002$$\mathbf{GeV}^3$

➢ Upper bound from Belle total cross section

Zhang, YQM, Wang, Chao, 0911.2166

$$M_0 < 0.02\mathrm{GeV}^3$$

- No universality of NRQCD LDMEs!

NLO fit by Hamburg group

➢ NLO NRQCD V.S. Global data

Butenschoen, Kniehl, 1105.0820

- Including Belle, LEP, HERA, RHIC, Tevatron, LHC
- Total of 194 data points from 26 data sets
- Exclude $p_T < 3\ GeV$ pp data and $p_T < 1\ GeV$ ep data

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]}) \rangle = 1.32\ \mathrm{GeV}^3 \qquad \chi^2_{\mathrm{d.o.f.}} = 725/194 = 3.74$$

$\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]}) \rangle$	$(4.97 \pm 0.44) \times 10^{-2}\ \mathrm{GeV}^3$
$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]}) \rangle$	$(2.24 \pm 0.59) \times 10^{-3}\ \mathrm{GeV}^3$
$\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]}) \rangle$	$(-1.61 \pm 0.20) \times 10^{-2}\ \mathrm{GeV}^5$

- Data are not well described by NLO NRQCD, especially Belle data

NLO fit by IHEP group

➢ Fit J/ψ yield data at Tevatron and LHC

Gong, Wan, Wang, Zhang, 1205.6682

- Exclude $p_T < 7\ GeV$ pp data

$$(\langle\mathcal{O}(^1S_0^{[8]})\rangle, \langle\mathcal{O}(^3S_1^{[8]})\rangle, \frac{\langle\mathcal{O}(^3P_0^{[8]})\rangle}{m_c^2}) \equiv \frac{\mathcal{O}}{100}\ \mathrm{GeV}^3$$

$$\mathcal{O} = (9.7 \pm 0.9, -0.46 \pm 0.13, -0.95 \pm 0.25)$$

➢ Results of the three groups: disagree

- $\mathrm{M}_0 \approx 0.074 \pm 0.019$ **GeV**^{3}
- $\mathrm{M}_1 \approx 0.0005 \pm 0.0002$**GeV**^{3}

PKU

- $\mathrm{M}_0 \approx 0.021$ **GeV**^{3}
- $\mathrm{M}_1 \approx 0.026$**GeV**^{3}

Hamburg

- $\mathrm{M}_0 \approx 0.081$ **GeV**^{3}
- $\mathrm{M}_1 \approx -0.0022$**GeV**^{3}

IHEP

NLO fit by ANL-Korea group

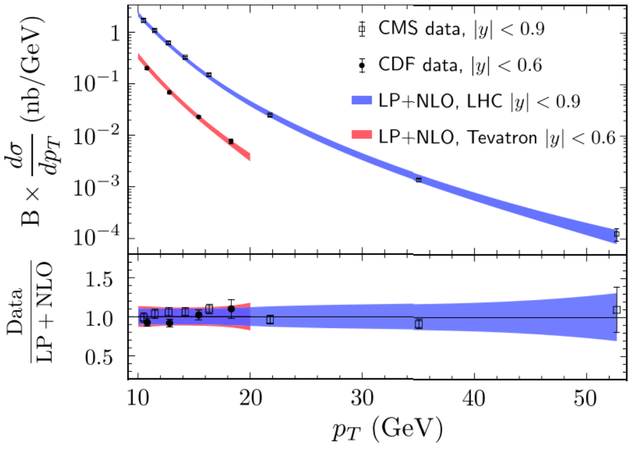
➢ Fit J/ψ yield data at Tevatron and LHC

Bodwin, Chung, Kim, Lee, 1403.3612

- Exclude $p_T < 10\ GeV$ data
- Large logs at LP in $1/p_T^2$ expansion are resumed

$$\frac{d\sigma^{\rm LP+NLO}}{dp_T} = \frac{d\sigma^{\rm LP}}{dp_T} - \frac{d\sigma^{\rm LP}_{\rm NLO}}{dp_T} + \frac{d\sigma_{\rm NLO}}{dp_T}$$

$$\langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle = -0.030 \pm 0.381\ \ {\rm GeV}^3$$
$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\rangle = 0.023 \pm 0.057\ {\rm GeV}^3$$
$$\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]})\rangle = 0.043 \pm 0.106\ {\rm GeV}^5$$



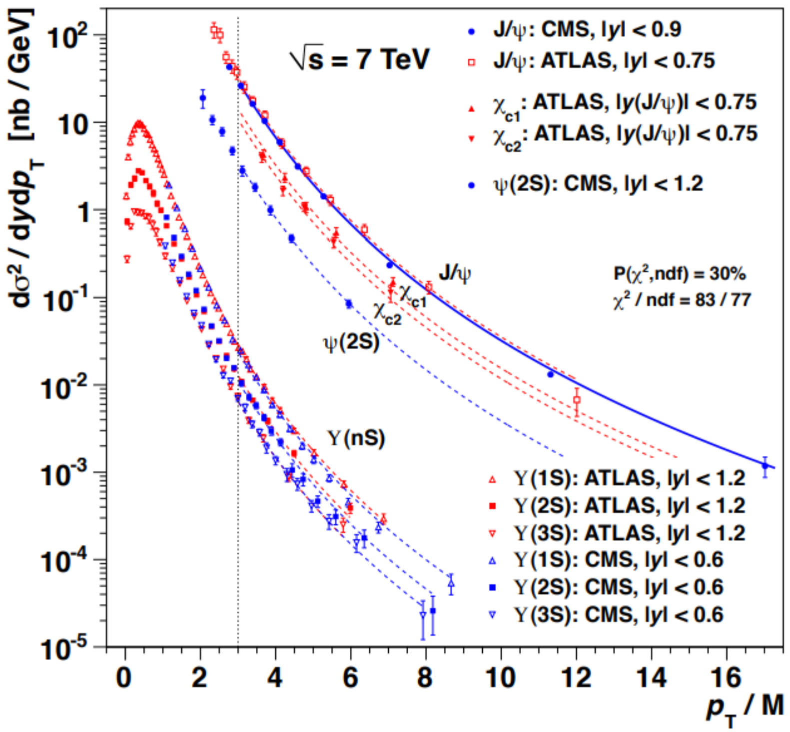
➢ Good fit, yet another different set of LDMEs

Data driven by CERN et. al. group

- Fit J/ψ yield data at Tevatron and LHC

Faccioli, Knunz, Lourenco, Seixas, Wohri, 1403.3970

- Similar shape as functions of p_T/M



- Ignoring ${}^3P_J^{[8]}$ contributions, ${}^1S_0^{[8]}$ dominance

Summary of current status

➢ Agreement among different groups:

- NLO NRQCD theory has difficulty to describe global data

(Warning: perfect agreement for χ_{cJ} production! Problems with η_c production not discussed)

➢ Rigorousness of NRQCD

- Based on EFT of QCD: NRQCD
- Factorization has been tested to NNLO

Nayak, Qiu, Sterman, 0509021
Bodwin, Chung, Ee, Kim, Lee, 1910.05497
Zhang, Meng, YQM, Chao, 2011.04905

➢ What is missing?

- Remember: summation over all possible n in NRQCD formula
- Corrections at high powers in v (relativistic corrections)!

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Relativistic corrections in NRQCD

➢ Relativistic (power) corrections

- Equations of motion of NRQCD EFT: $\left(iD_0 - \frac{\boldsymbol{D}^2}{2m} + \cdots\right)\psi = 0$
- NRQCD factorization: use EOM to remove ∇_0, leaving operators like:

(Warning: here D replaced by ∇, needs proper gluon fields to make them gauge invariant)

$\chi^\dagger \psi$

Leading term

$\chi^\dagger \overleftrightarrow{\nabla}^2 \psi$

Type I: Relative momentum

$\nabla^2 (\chi^\dagger \psi)$

Type II: Total momentum

$\chi^\dagger \left(\overleftrightarrow{\nabla}^i \overleftrightarrow{\nabla}^j - \overleftrightarrow{\nabla}^2 \delta^{ij}/3 \right) \psi$

Type III: Orbital excited states, like D-wave

$\chi^\dagger (g\boldsymbol{E} \cdot \boldsymbol{\sigma}) \psi$

Type IV: Intrinsic gluon insertion

(For pp collision, only type-I have been considered, about 30%-50% corrections for charmonium)

CS-channel:
Fan, YQM, Chao, 0904.4025

CO-channel:
Xu, Li, Liu, Zhang,1203.0207

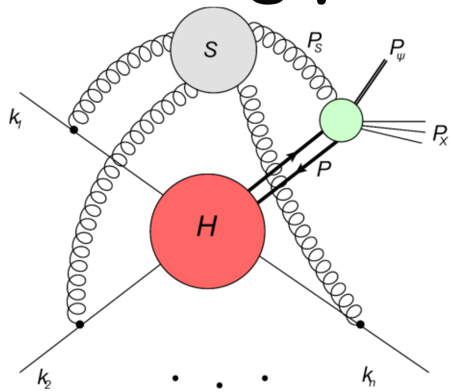
S-D mixing-channel (including ep collision):
He, Kniehl, 1507.03882

LP in p_T, all order in v:
Li, Chen, Huang, YQM, 1909.03554

However, more relativistic-correction terms may be needed!

Soft gluon emission

- **Soft gluon emission in color-bleaching process**
  - P_ψ **is different from** P, $P = P_\psi[1+O(\lambda)]$
  - **NRQCD expand** P **around** P_ψ



YQM, Vogt, 1609.06042

- **Bad convergence of NRQCD expansion**
  - **Cross section approximately** $\propto P^{-4} = P_\psi^{-4}[1+O(\lambda)]^{-4}$

$$\int_{-1}^{1} \frac{d\cos\theta}{2(1+\lambda+\lambda\cos\theta)^4} = 0.42$$

$$= 1 - 4\lambda + 40/3\lambda^2 - 40\lambda^3 + \cdots$$

$$= 1 - 1.2 + 1.2 - 1.08 + 0.91 - 0.73 + \cdots$$

With $\lambda \approx v^2 \approx 0.3$

Mangano, Petrelli, 9610364

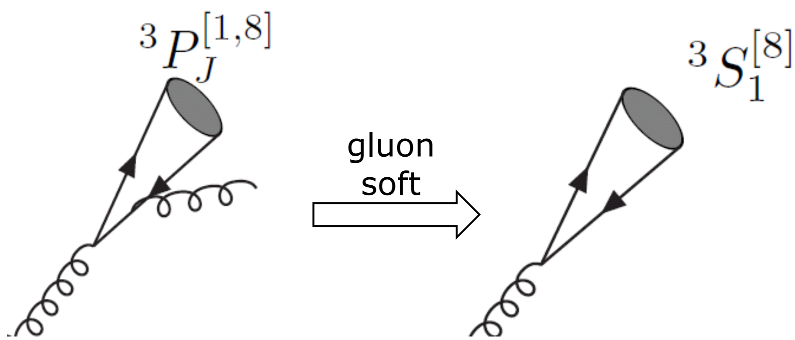
- **Solution: soft gluon momentum should be kept but not expanded, which means to resum relativistic corrections (due to kinematic effects) to all powers in** v**!**

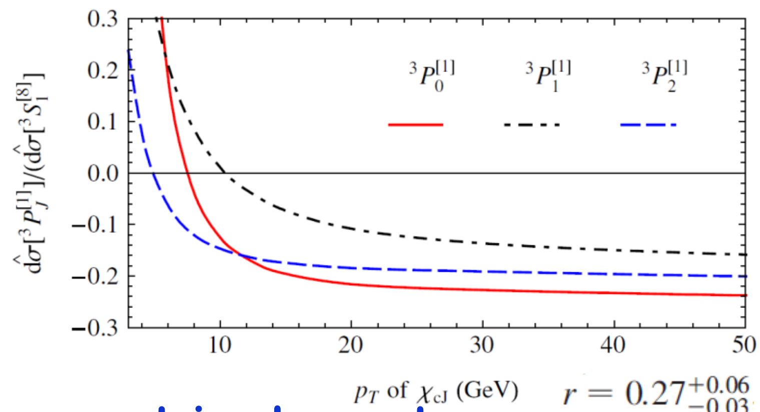
Over subtraction

Braaten, Chen, 9610401
YQM, Wang, Chao, 1002.3987

➢ Eg. χ_{cJ} production:

$$d\sigma_{\chi_{cJ}}/(2J+1) \approx d\hat{\sigma}_{^3P_J^{[1]}} \langle O\left(^3P_0^{[1]}\right)\rangle + d\hat{\sigma}_{^3S_1^{[8]}} \langle O\left(^3S_1^{[8]}\right)\rangle$$





- Soft gluon in P-wave: factorized to S-wave matrix element
- Subtraction scheme: at *zero momentum*, which contributes the largest production rate. Over subtracted! P-wave negative!
- Big cancellation between S-wave and P-wave! Perturbation unstable
- Solution: soft gluon momentum should be kept during subtraction process, or resum kinematic effects to all powers in v.

Threshold region

➢ At threshold region

- Large logarithms appear: can be resummed by introducing shape functions

  Beneke, Rothstein, Wise, 9705286
  Fleming, Leibovich, Mehen, 0306139
  Leibovich, Liu, 0705.3230

- Soft gluon momentum: has leading contribution for quarkonium momentum distribution, cannot be ignored

➢ Combination of logs resummation and powers resummation is needed

  - Keep soft gluon momentum unexpanded is the first step.

Comments

- Relativistic corrections with fixed power in v
  - Bad convergence, too many terms are needed
  - Involves too many LDMEs, very hard to fix them; Tentative values are chosen in literature
  - Solution: sum all LDMEs to obtain a function!

- What do we need to resum?
  - Type I and II ($\chi^\dagger \overleftrightarrow{\nabla}^2 \psi, \ \nabla^2(\chi^\dagger\psi)$) : kinematic effects, enhanced if the observable has a steep distribution. E.g., p_T distribution in pp collision, momentum distribution in endpoint region
  - Type III and IV: usually not enhanced, less important (can also be studied)

$$\chi^\dagger \left(\overleftrightarrow{\nabla}^i \overleftrightarrow{\nabla}^j - \overleftrightarrow{\nabla}^2 \delta^{ij}/3 \right) \psi \ \text{✗} \qquad \chi^\dagger (g\boldsymbol{E}\cdot\boldsymbol{\sigma})\psi \ \text{✗}$$

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SGF: exclusive processes

Li, Feng, YQM, 1911.05886
Chen, YQM, 2005.08786

➢ Different way to use EOM

- NRQCD factorization: use EOM to remove ∇_0
- SGF: use EOM to remove relative derivatives, leaving only total derivatives:

$$\langle 0|\nabla_0^{n_1}\nabla^{2n_2}(\chi^\dagger\psi)|H\rangle$$

the same degrees of freedom as NRQCD factorization

➢ Using integration by parts

- Remove operators unless $n_1 = n_2 = 0$

➢ Factorization

$$\mathcal{A}^Q = \sum_n \hat{\mathcal{A}}^n \overline{R}_Q^{n*} \qquad \overline{R}_Q^{n*} = \langle 0|[\overline{\Psi}\mathcal{K}_n\Psi](0)|\mathcal{Q}\rangle_S$$

- Matching coefficients are functions of quarkonium mass

"S": field operators are in small momentum regions

SGF: inclusive processes

- Use EOM to remove relative derivatives

$$\langle H+X|\nabla_0^{n_1}\nabla^{2n_2}(\chi^\dagger\psi)|0\rangle$$

YQM, Chao, 1703.08402
Chen, YQM, 2005.08786

- Using integration by parts
  - Remove operators unless $n_1 = n_2 = 0$
  - Matching coefficients are functions of: $P_H^2, P_H \cdot P_X, P_X^2$
- Factorization

$$(2\pi)^3 2P_H^0 \frac{d\sigma_H}{d^3P_H} \approx \sum_n \int \frac{d^4P}{(2\pi)^4} \mathcal{H}_n(P) F_{n\to H}(P, P_H)$$

- $n = {}^{2S+1}L_J^{[c]}$
- P: momentum of $Q\bar{Q}$
- $\mathcal{H}_n$: perturbatively calculable hard parts
- $F_{n\to H}$: nonperturbative soft gluon distributions (SGDs)
- UV renormalization scale is suppressed

Soft gluon distributions (SGDs)

Operator definition

- **Expectation values of bilocal operators in QCD vacuum**

$$F_{n\to H}(P,P_H)=\int d^4b e^{-iP\cdot b}\langle 0|[\overline{\Psi}\mathcal{K}_n\Psi]^\dagger(0)\left(a_H^\dagger a_H\right)[\overline{\Psi}\mathcal{K}_n\Psi](b)|0\rangle_{\rm S}$$

with

$$a_H^\dagger a_H=\sum_X\sum_{J_z^H}|H+X\rangle\langle H+X|$$

$$\mathcal{K}_n(rb)=\frac{\sqrt{M_H}}{M_H+2m}\frac{M_H+\not{P}_H}{2M_H}\Gamma_n\frac{M_H-\not{P}_H}{2M_H}\mathcal{C}^{[c]}$$

Spin project operators: $\Gamma_n=\sum_{L_z,S_z}\langle L,L_z;S,S_z|J,J_z\rangle\Gamma^o_{LL_z}\Gamma^s_{SS_z}$

Color project operators:

$$\mathcal{C}^{[1]}=\frac{\mathbf{1}_c}{\sqrt{N_c}}\qquad\qquad \mathcal{C}^{[8]}=\sqrt{2}t^{\bar{a}}\,\Phi^{(A)}_{a\bar{a}}(rb)$$

Soft gluon distributions (SGDs)

➢ Gauge link

$$\Phi^{(A)}(rb) = \mathcal{P}\exp\left\{-ig_s\int_0^\infty d\lambda\, b_\ell\cdot A^{(A)}(r\,b+\lambda\,b_\ell)\right\}$$

$$b_\ell^\mu = b^\mu + \varepsilon\ell^\mu \qquad\qquad 0 < \varepsilon \ll 1$$

- **When b is finite, gauge link along b direction (avoid gauge-link-collinear divergence)**
- **When $b\to 0$, gauge link unambiguously along l direction (agree with gauge-completed NRQCD matrix elements)**

Nayak, Qiu, Sterman, 0509021

➢ Evaluated in ***small*** region

- **Subscript "S": evaluate the matrix element in the region where off-shellness of all particles is much smaller than heavy quark mass**

RGEs for SGDs

➢ RGEs

Chen, Jin, YQM, Meng, 2103.15121

$$\frac{d}{d\ln\mu_f}F_{[L'\tilde{L}',\lambda']\to H}(z,M_H,m_Q,\mu_f)=\sum_{L,\tilde{L},\lambda}\int_z^1\frac{dx}{x}\boldsymbol{K}^{[L\tilde{L},\lambda]}_{[L'\tilde{L}',\lambda']}(\hat{z},M_H/x,m_Q,\mu_f)$$
$$\times F_{[L\tilde{L},\lambda]\to H}(x,M_H,m_Q,\mu_f),$$

- **Evolution kernels**

$$\boldsymbol{K}^{[L\tilde{L},\lambda],LO}_{[L'\tilde{L}',\lambda']}(\hat{z},M_H/x,m_Q,\mu_f)=\frac{d}{d\ln\mu_f}F^{NLO}_{[L'\tilde{L}',\lambda']\to Q\bar{Q}[L\tilde{L},\lambda]}(\hat{z},M_H/x,m_Q,\mu_f).$$

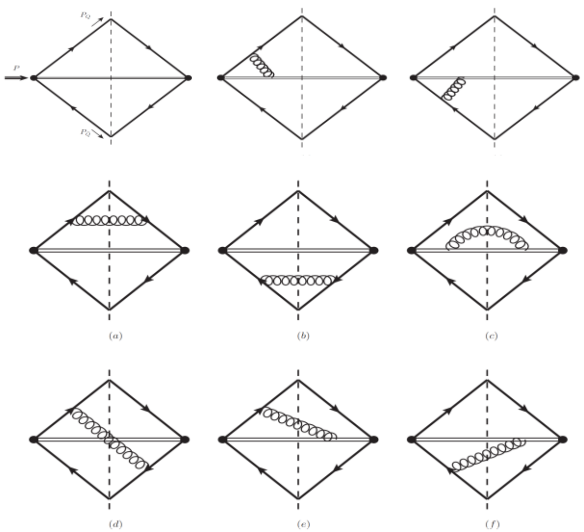
$$\boldsymbol{K}^{[SS],LO}_{[SS]}(z,M_H,m_Q,\mu_f)=\frac{\alpha_s}{\pi}\left\{N_c\left[\frac{2z}{(1-z)_+}-\ln\frac{\mu^2e^{-1}}{M_H^2}\delta(1-z)\right.\right.$$
$$\left.\left.-2\delta(1-z)\left(\frac{1}{2\Delta}\ln\frac{1+\Delta}{1-\Delta}-1\right)\right]+\frac{1}{N_c}\left(\frac{1+\Delta^2}{2\Delta}\ln\frac{1+\Delta}{1-\Delta}-1\right)\delta(1-z)\right\}.$$

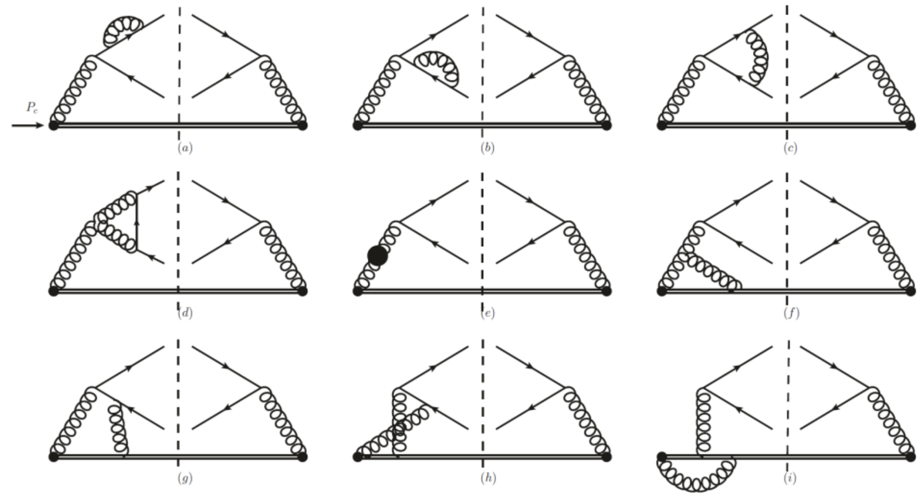
$$\Delta=\frac{\sqrt{M_H^2-4m_Q^2}}{M_H}$$

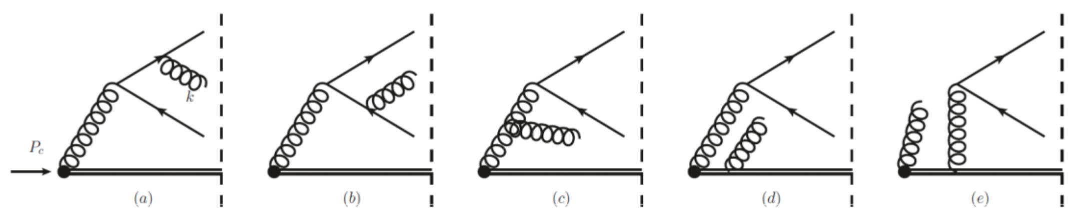
FF: $g \to Q\overline{Q}\,({}^3S_1^{[8]}) + X$ up to NLO

- **Feynman diagrams**

Chen, Jin, YQM, Meng, 2103.15121







Hard part for $g \to Q\overline{Q}\,({}^3S_1^{[8]}) + X$

➢ NRQCD

$$\hat{d}^{(2)}_{g\to {}^3S_1^{[8]}} = \frac{1}{12C_F}\Big[A(\mu_0)\delta(1-z) + \frac{1}{N_c}P_{gg}(z)\Big(\ln(\frac{\mu_0^2}{4m_Q^2}) - 1\Big) + \frac{2(1-z)}{z} - \frac{4(1-z+z^2)^2}{z}\Big(\frac{\ln(1-z)}{1-z}\Big)_+\Big],$$

Braaten, Lee, 0004228
YQM, Qiu, Zhang, 1311.7078

- **Double logs as $z \to 1$ (threshold logs)**

➢ SGF

Chen, Jin, YQM, Meng, 2103.15121

$$\hat{D}^{LO,(0)}_{[SS]}(\hat{z}, M_H/x, \mu, \mu_f) = \frac{\pi\alpha_s}{(N_c^2-1)}\frac{8x^3}{M_H^3}\delta(1-\hat{z}), \tag{5.28a}$$

$$\begin{aligned}
&\hat{D}^{NLO,(0)}_{[SS]}(\hat{z}, M_H/x, \mu, \mu_f) \\
&= \frac{4\alpha_s^2 N_c x^3}{(N_c^2-1)M_H^3}\Bigg[\frac{1}{2}\delta(1-\hat{z})\Bigg(2A(\mu, M_H/x) + \frac{2\beta_0}{N_c}\ln\Big(\frac{x^2\mu_f^2 e^{-1}}{M_H^2}\Big) + \ln^2\Big(\frac{x^2\mu_f^2 e^{-1}}{M_H^2}\Big) \\
&\quad + \frac{\pi^2}{6} - 1\Bigg) + \frac{1}{N_c}P^{(0)}_{gg}(\hat{z})\ln\Big(\frac{\mu^2}{\mu_f^2}\Big) + \Big(\frac{2(1-\hat{z})}{\hat{z}} + \hat{z}(4+2\hat{z}^2) + \frac{2\hat{z}^4}{9}(5+\hat{z})\Big) \\
&\quad \times\Big(\ln\Big(\frac{x^2\mu_f^2 e^{-1}}{M_H^2}\Big) - 2\ln(1-\hat{z})\Big) + \frac{2(1-\hat{z})}{\hat{z}} - \Big(\frac{4\hat{z}^4}{1-\hat{z}} - \frac{4\hat{z}^4}{9}(5+\hat{z})\Big)\ln\hat{z}\Bigg].
\end{aligned} \tag{5.28b}$$

- **No threshold logs in hard part**
- **Logs are factorized to SGDs and then resummed by using REGs**

Nonperturbative models

➢ The first class of models

$$F^{\rm mod}(\omega') = M_H N_H \frac{b^b}{\Gamma(b)} \frac{\omega'^{b-1}}{\bar\Lambda^b} e^{-b\omega'/\bar\Lambda}, \quad \omega' = M_H(1/x-1),$$ Fleming, Leibovich, Mehen, 0306139

Model-1: $F^{\rm mod}(\omega')|_{\bar\Lambda=0.6{\rm GeV},b=2}$, Model-2: $F^{\rm mod}(\omega')|_{\bar\Lambda=0.6{\rm GeV},b=1}$,

Model-3: $F^{\rm mod}(\omega')|_{\bar\Lambda=0.6{\rm GeV},b=3}$, Model-4: $F^{\rm mod}(\omega')|_{\bar\Lambda=0.5{\rm GeV},b=2}$,

Model-5: $F^{\rm mod}(\omega')|_{\bar\Lambda=0.7{\rm GeV},b=2}$,

the zeroth, first and second moments are $M_H N_H$, $M_H N_H \bar\Lambda$ and $M_H N_H \bar\Lambda^2(\frac{1}{b}+1)$

➢ The other models

Model-6: $4M_H N_H[\theta(w' \geq \frac{19}{40}) - \theta(w' > \frac{29}{40})]$,

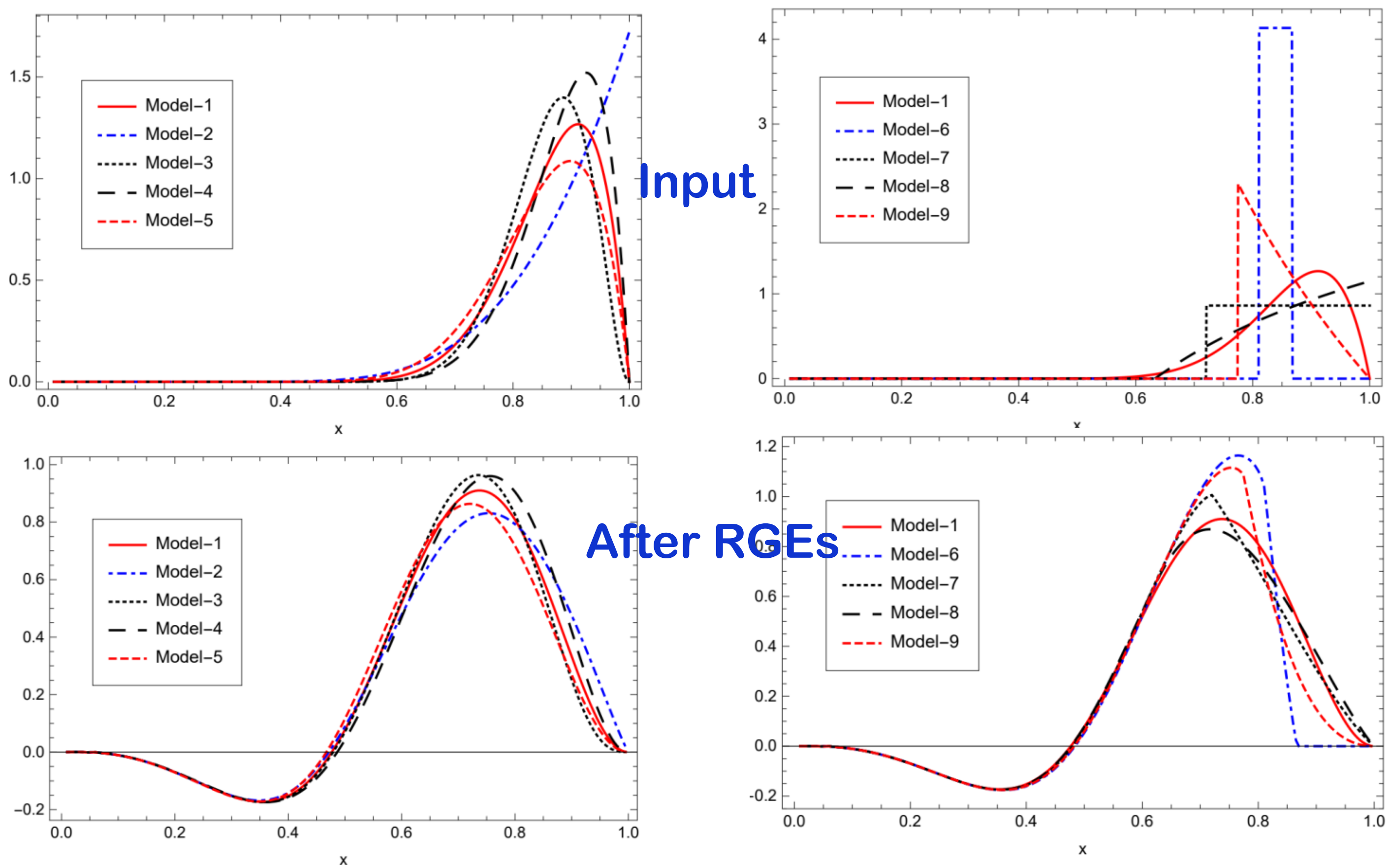
Model-7: $\frac{5}{6} M_H N_H[\theta(w' \geq 0) - \theta(w' > \frac{6}{5})]$,

Model-8: $\begin{cases} M_H N_H(-\frac{50}{81}w' + \frac{10}{9}), & 0 \leq w' \leq \frac{9}{5}, \\ 0, & w' > \frac{9}{5}, \end{cases}$

Model-9: $\begin{cases} \frac{200}{81} M_H N_H w', & 0 \leq w' \leq \frac{9}{10}, \\ 0, & w' > \frac{9}{10}. \end{cases}$

RGEs effects

- **Model dependence is significantly reduced after using RGEs**



Numerical: $g \to Q\overline{Q}\,({}^3S_1^{[8]}) + X$

- **Gluon FFs in ${}^3S_1^{[8]}$ channel**

Chen, Jin, YQM, Meng, 2103.15121

$\bar{\Lambda}$: average momentum emitted

$$D_{g\to H}(z, M_H, m_Q, M_H) = \int_z^1 \frac{dx}{x} \hat{D}_{[SS]}(\hat{z}, M_H/x, m_Q, M_H, M_H) \times F_{[SS]\to H}(x, M_H, m_Q, M_H),$$

$$D^{(0)}_{g\to H}(z, M_H, m_Q, M_H) = \int_z^1 \frac{dx}{x} \hat{D}^{(0)}_{[SS]}(\hat{z}, M_H/x, M_H, M_H) \times F_{[SS]\to H}(x, M_H, m_Q, M_H).$$

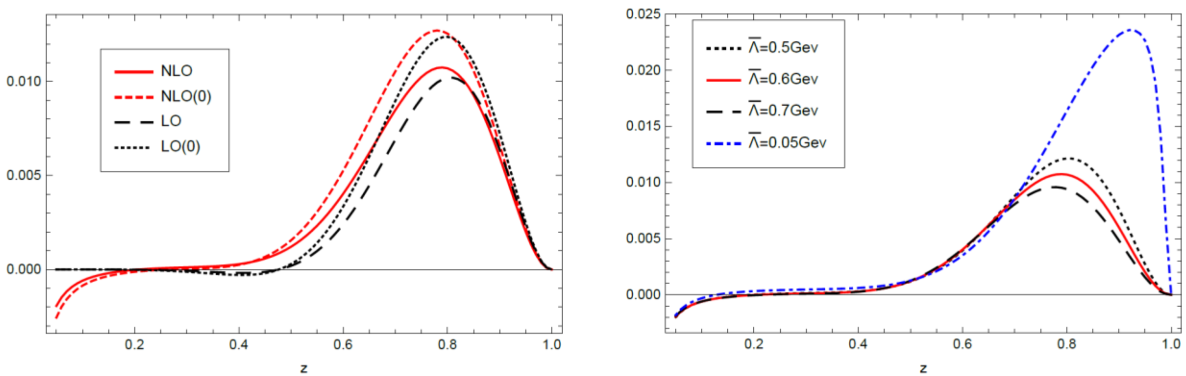
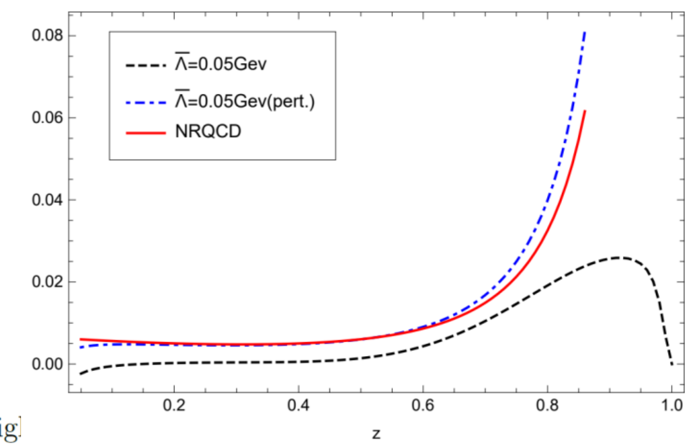


Figure 7. Left figure: Comparison of the gluon FF obtained in different approximations. Rig[ht] figure: $\bar{\Lambda}$ dependence of gluon FF at NLO.



$$R^X(n) \equiv \frac{\int_0^1 dz\, z^n D^X_{g\to H}(z, M_H, m_Q, \mu)}{\int_0^1 dz\, z^n D_{g\to H}(z, M_H, m_Q, \mu)},$$

$R^{NRQCD} \approx 6$

Summary

➢ NRQCD factorization: universality problem

- Different groups got different LDMEs; Inconsistent with data.
- Possible reason: high order in v^2 expansion needed

➢ Resummation of powers in v^2 expansion

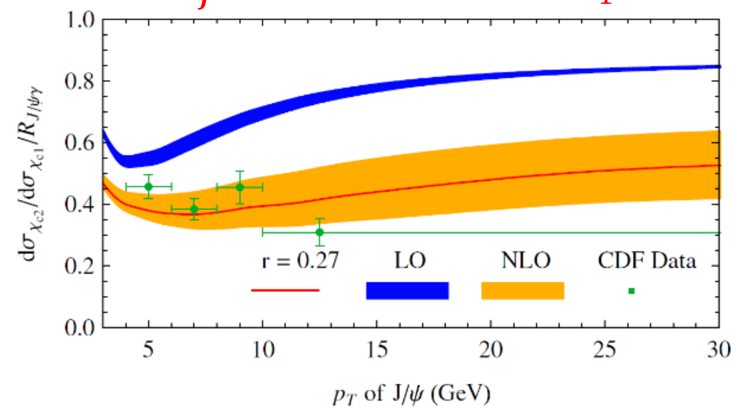
- Soft gluon factorization: equivalent to NRQCD, but with relativistic corrections (due to kinematic effects) resummed, better convergence in v expansion
- Phenomenological difficulties encountered in NRQCD should be restudied in the new framework
- Still lots of theoretical works to do. Stay tuned!

Thank you!

Achievement: χ_{cJ} production

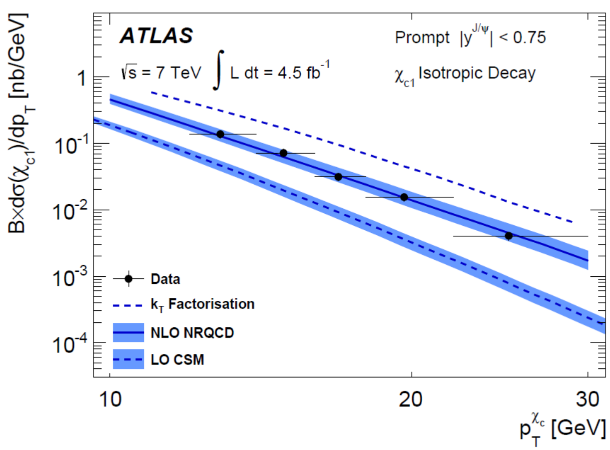
- χ_{cJ} **production:** $d\sigma_{\chi_{cJ}}/(2J+1) \approx d\hat{\sigma}_{^3P_J^{[1]}} \langle O\left(^3P_0^{[1]}\right)\rangle + d\hat{\sigma}_{^3S_1^{[8]}} \langle O\left(^3S_1^{[8]}\right)\rangle$

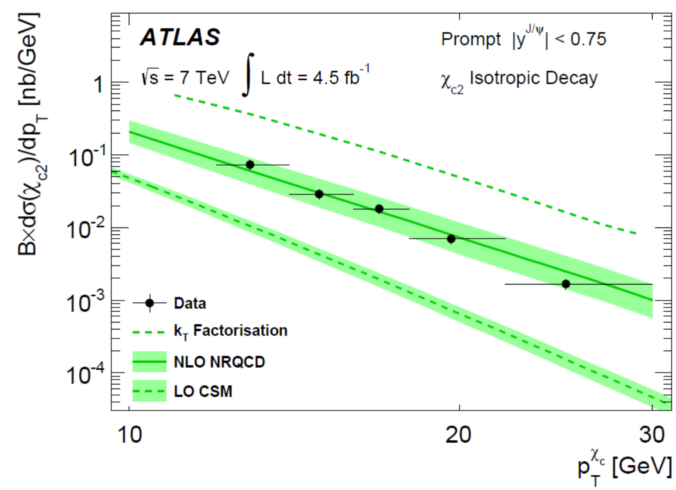
YQM, Wang, Chao, 1002.3987



- **Predictions agree with new data**

ATLAS, 1404.7035

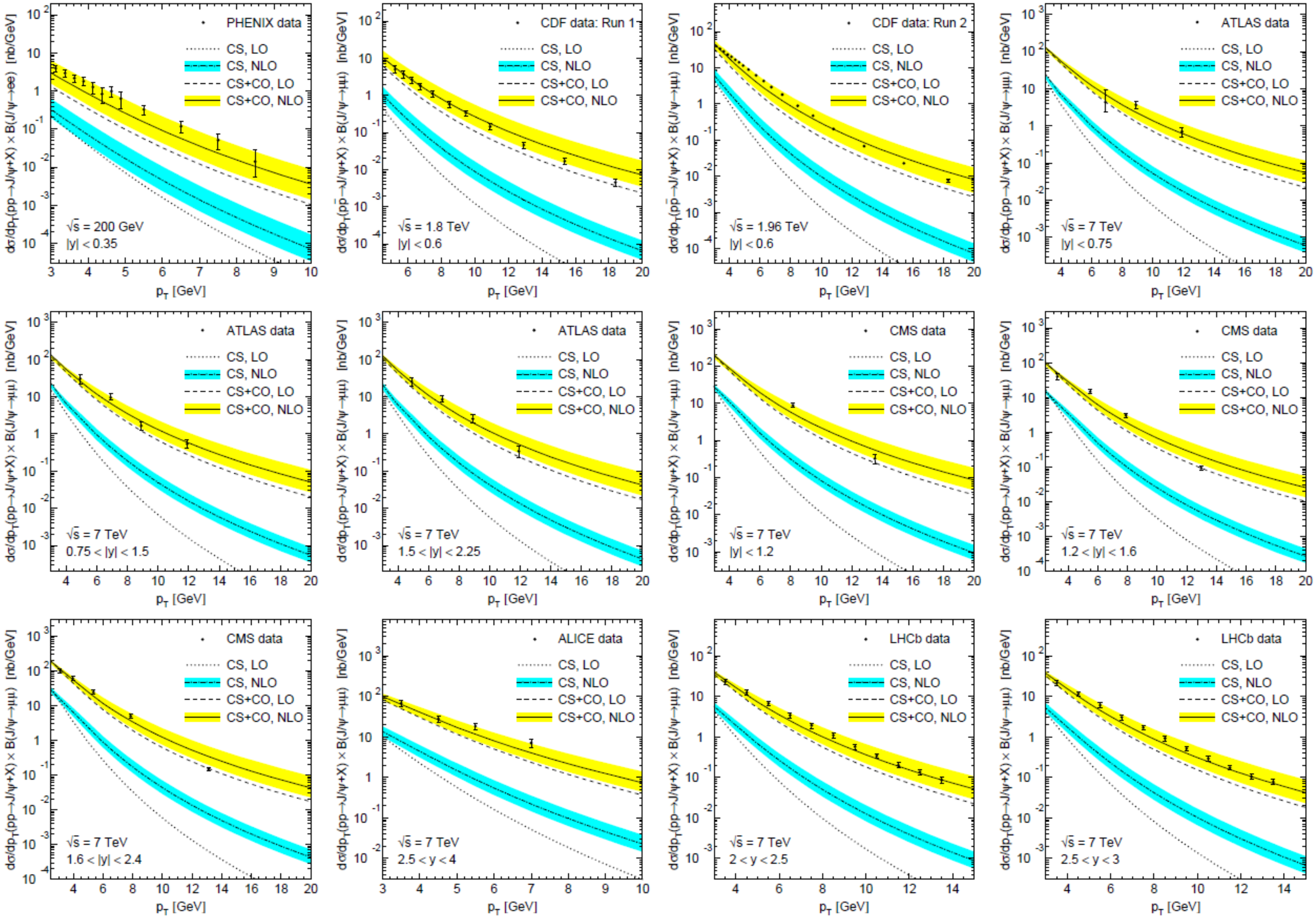




Global fit by Butenschoen and Kniehl

➢ **NLO NRQCD V.S. RHIC, Tevatro, LHC data**

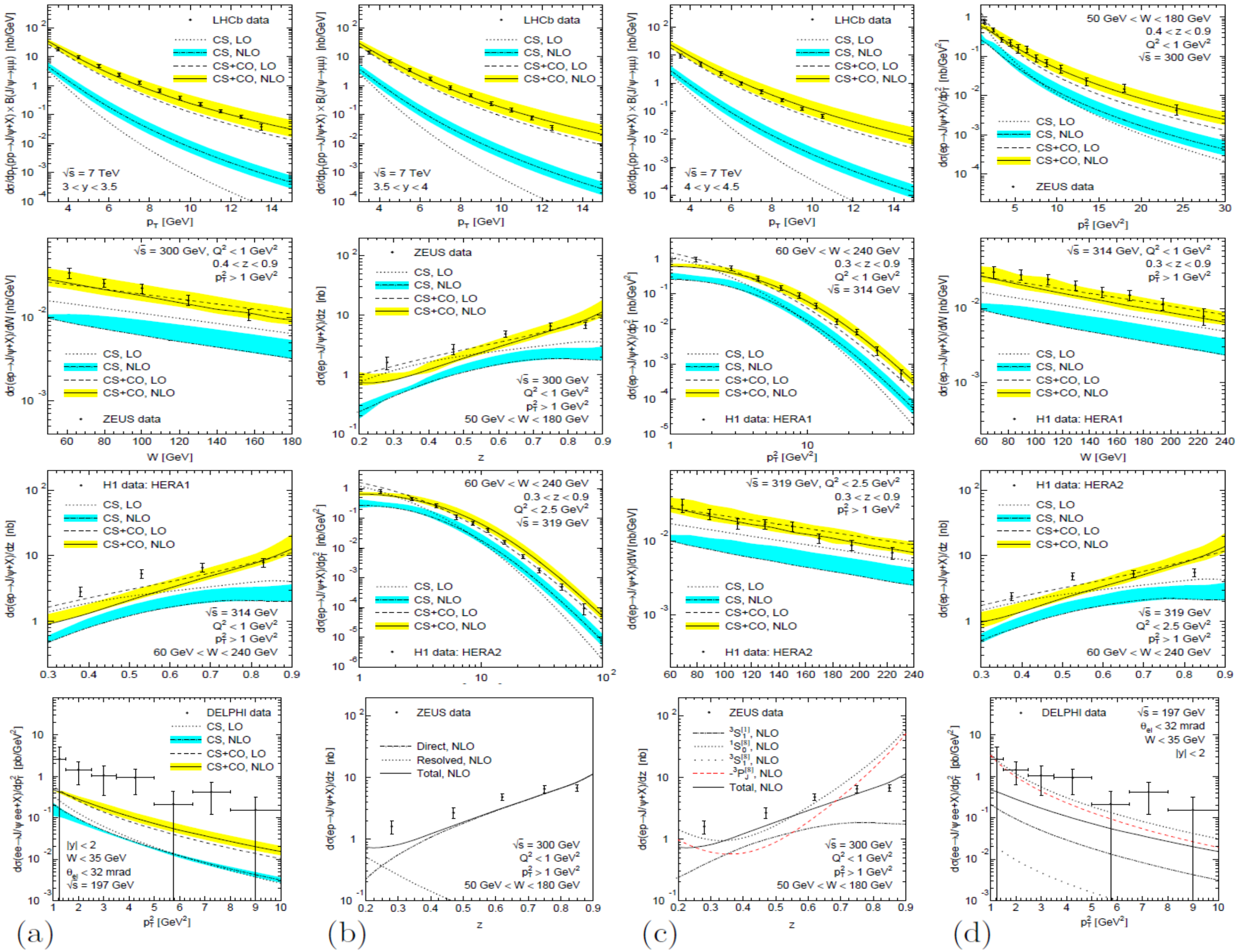
Butenschoen, Kniehl, 1105.0820



Global fit by Butenschoen and Kniehl

- **NLO NRQCD V.S. LHC, HERA, LEP data**

Butenschoen, Kniehl, 1105.0820



Pheno difficulty: polarization puzzle

➢ LO NRQCD

- **Dominated by $^3S_1^{[8]}$, LO NRQCD predicts transversely polarized $\psi(\mathrm{nS})$ at high p_T, contradicts with Tevatron and LHC data**

CDF, 0704.0638

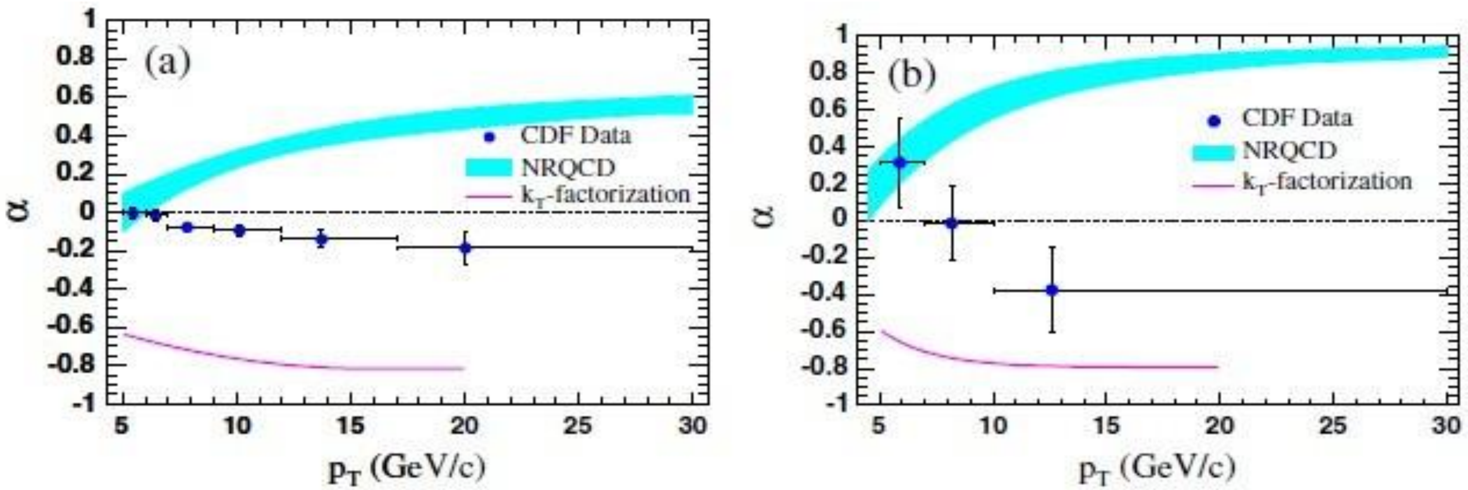
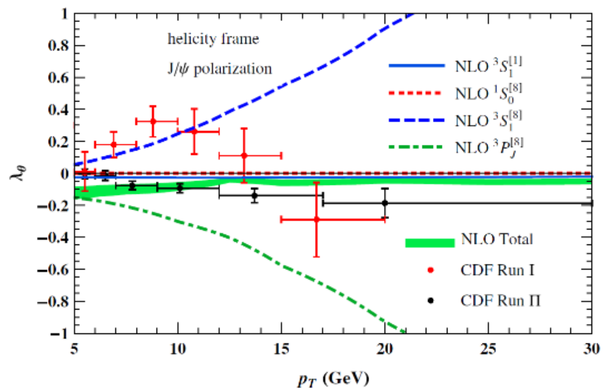


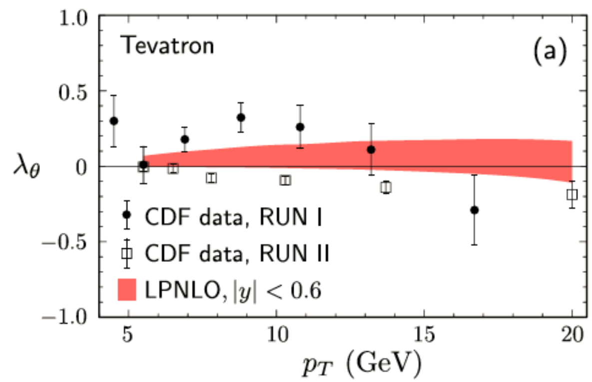
FIG. 4 (color online). Prompt polarizations as functions of p_T: (a) J/ψ and (b) $\psi(2S)$. The band (line) is the prediction from NRQCD [4] (the k_T-factorization model [9]).

Pheno difficulty: polarization puzzle

- J/ψ at NLO: transverse polarization largely canceled (***natural?***) between ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$

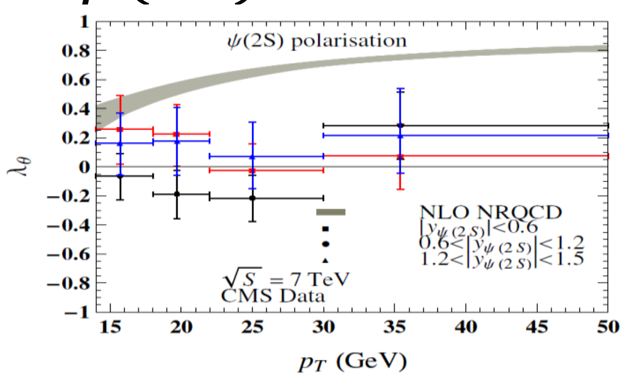


Chao,YQM,Shao,Wang, Zhang,1201.2675

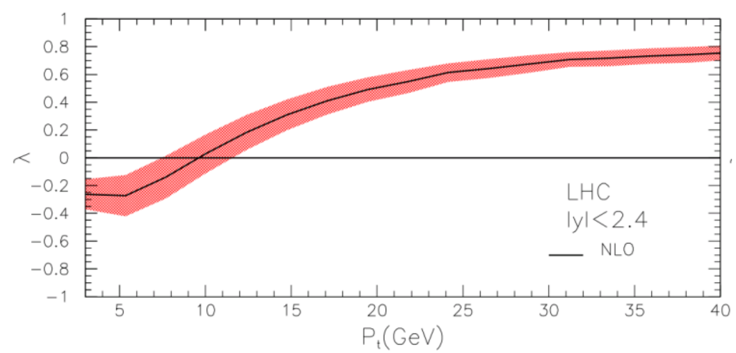


Bodwin, Chung, Kim, Lee, 1403.3612

- $\psi(2S)$: cancelation weak, hard to understand



Shao, Han, YQM, Meng, Zhang, Chao, 1411.3300



Gong, Wan, Wang, Zhang, 1205.6682

Summary of NRQCD factorization

- **Rigorousness**
  - Based on EFT of QCD: NRQCD
  - Factorization has been tested to NNLO

Nayak, Qiu, Sterman, 0509021
Bodwin, Chung, Ee, Kim, Lee, 1910.05497
Zhang, Meng, YQM, Chao, 2011.04905

- **Color-octet mechanism: great success in solving theoretical issues and explaining data**
- **Color-octet mechanism: final-state radiation of soft gluons results in large power and log corr.**
  - Should be responsible for phenomenological failures
- **Soft gluon factorization: resum a dominant series of power corrections (kinematic effects) and log corrections**