Quarkonia and TMD Factorization

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Physics Opportunities with Heavy Quarkonia at the EIC
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Review of Quarkonium Production Theory

vNRQCD, vNRQCD w/ Soft Wilson Lines

RPI and P-wave operators

p_T Quarkonium Shape Functions

Summary/Outlook

Color-Singlet Model (pre-1995)

$$\sigma(pp \to J/\psi + X) = f_{g/p} \otimes f_{g/p} \otimes \sigma[gg \to c\bar{c}(^3S_1^{(1)}) + X]\, |\psi_{c\bar{c}}(0)|^2$$

$\bar{c}c$ pair produced with same quantum numbers as J/ψ

Predictive Formalism

$\sigma[gg \to c\bar{c}(^3S_1^{(1)}) + X]$ calculable in QCD perturbation theory

$|\psi_{c\bar{c}}(0)|^2$ fixed by $\Gamma[J/\psi \to \ell^+\ell^-]$

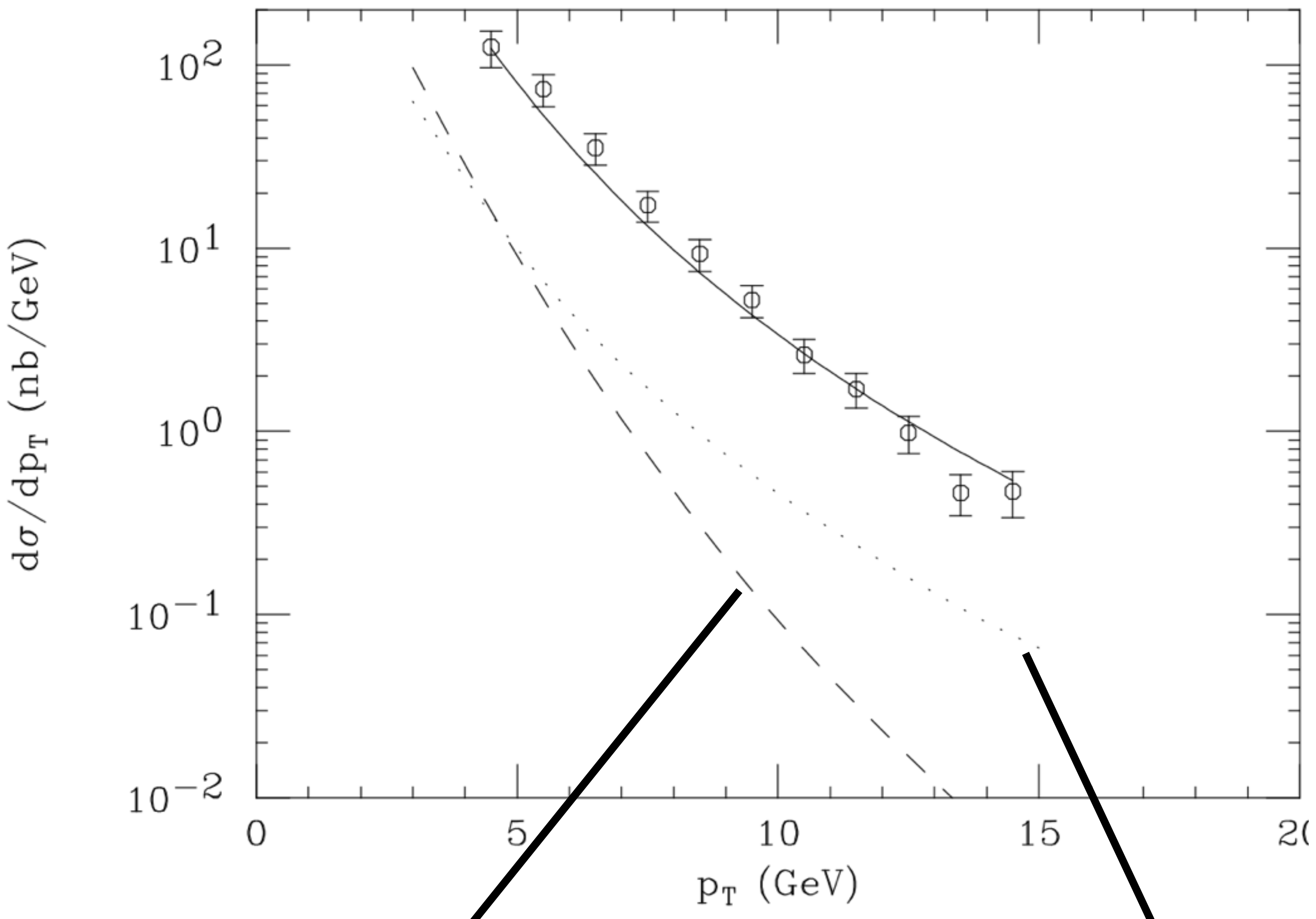
Suffers from theoretical inconsistencies when applied to χ_{cJ}

$\Gamma[\chi_{cJ} \to \text{hadrons}] = |\psi'_{c\bar{c}}(0)|^2 \sigma(c\bar{c}(^3P_J^{(1)}) \to gg)$ ⟵ **Not IR Safe**

J/ψ production at Tevatron (1996)

CSM badly underpredicts J/ψ and ψ’ production at large p_T

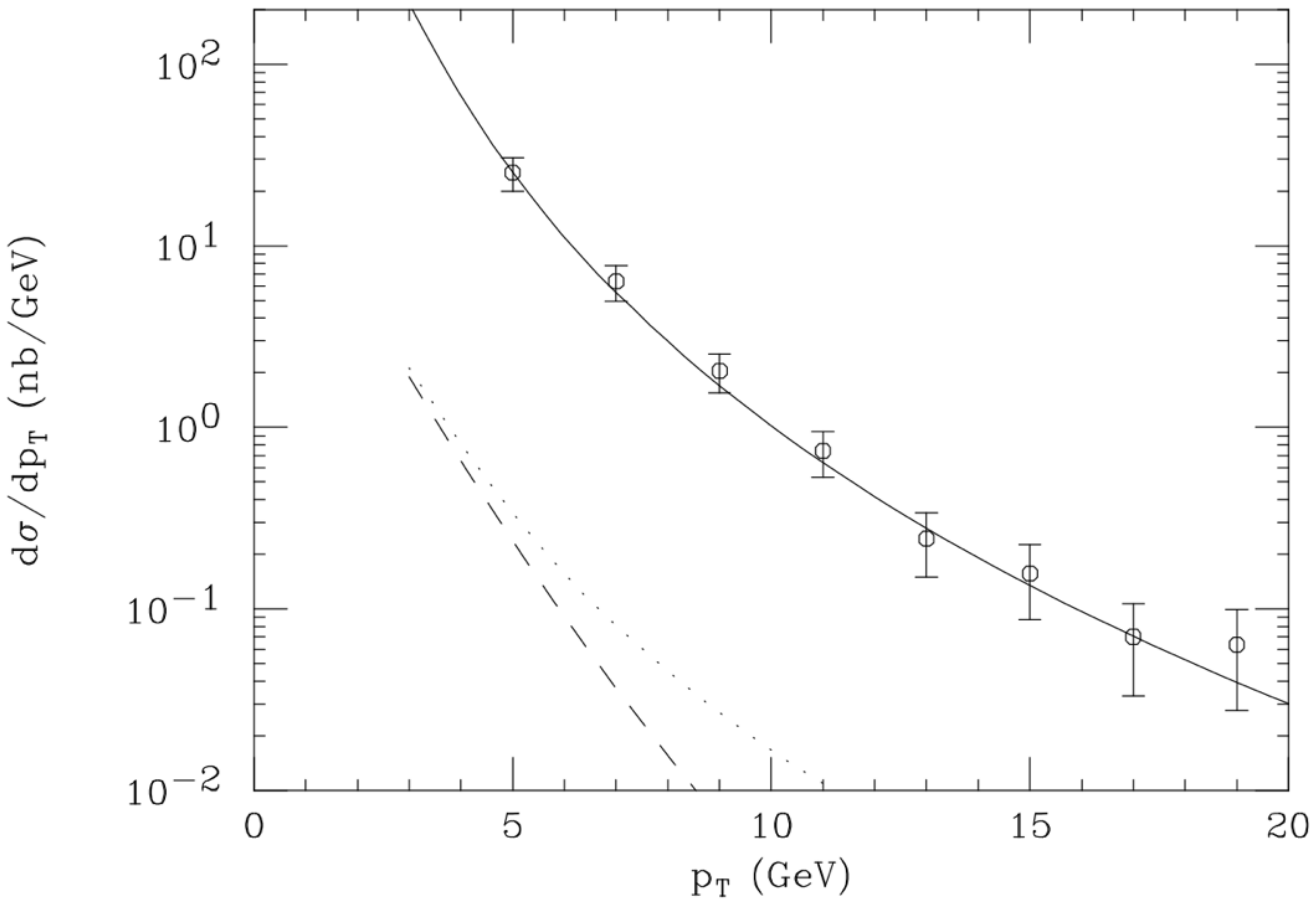
J/ψ



CSM (LO)

CSM
(Fragmentation)

ψ’



Non-Relativistic QCD (NRQCD) Factorization Formalism

(Bodwin, Braaten, Lepage)

$$\sigma(gg \to J/\psi + X) = \sum_n \sigma(gg \to c\bar{c}(n) + X)\langle \mathcal{O}^{J/\psi}(n)\rangle$$

$$n - {}^{2S+1}L_J^{(1,8)}$$

double expansion in α_s, v

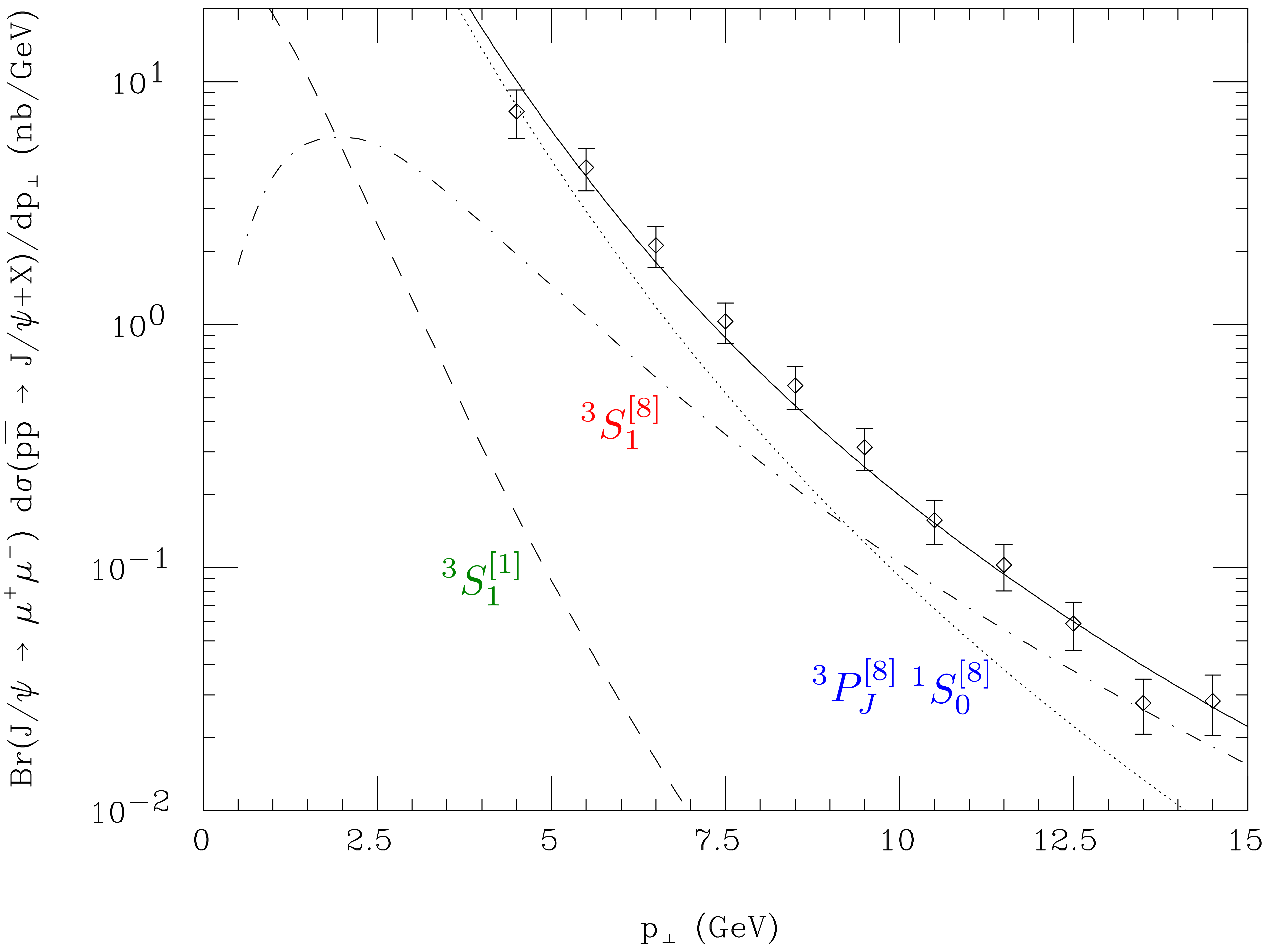
NRQCD long-distance matrix element (LDME)

$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]})\rangle \sim v^3$$

CSM - lowest order in v

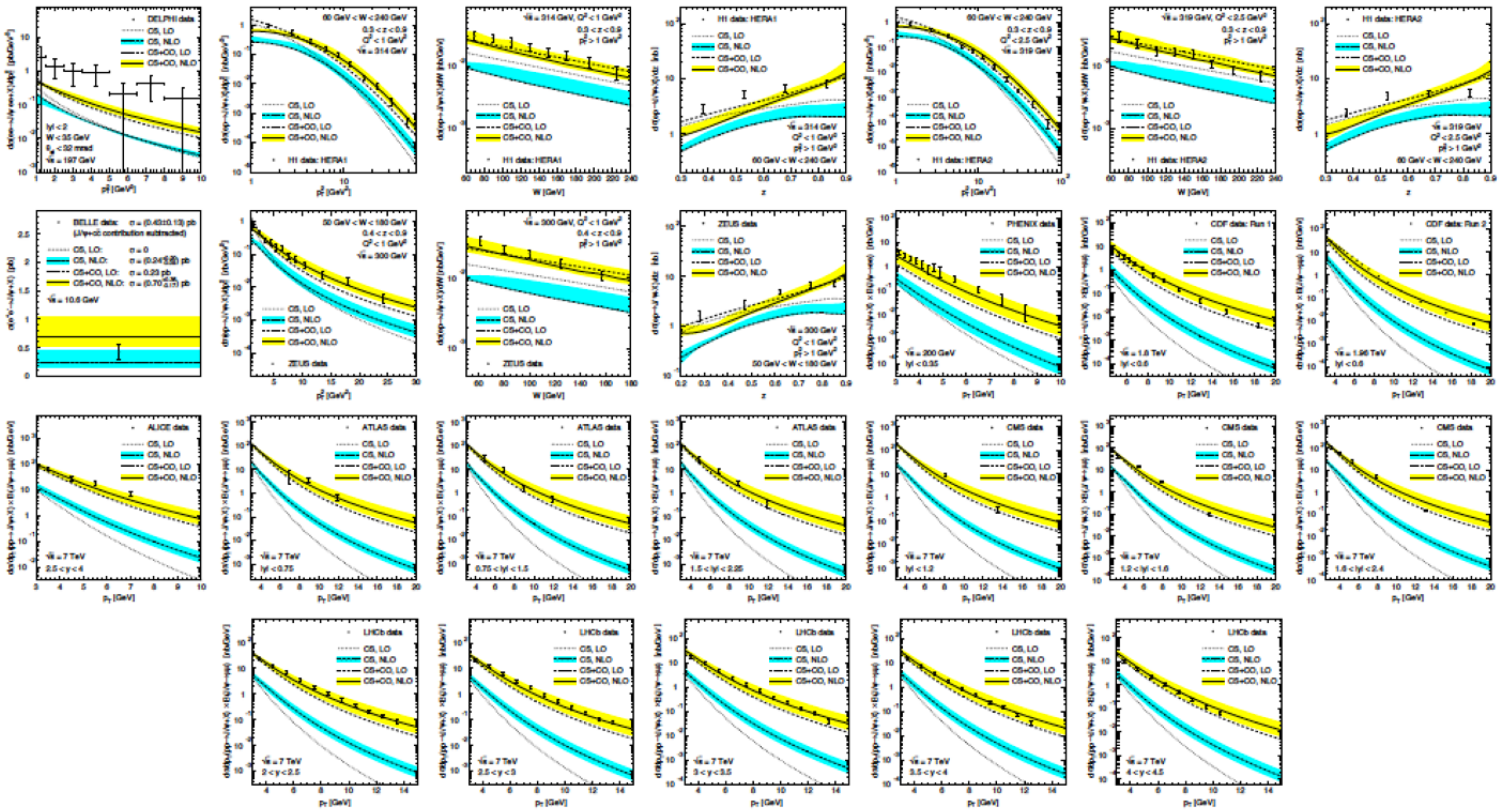
$$\langle \mathcal{O}^{J/\psi}({}^3S_1^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle, \langle \mathcal{O}^{J/\psi}({}^3P_J^{[8]})\rangle \sim v^7$$

color-octet mechanisms



10^1
10^0
10^{-1}
10^{-2}
0
2.5
5
7.5
10
12.5
15
$^3S_1^{[8]}$
$^3S_1^{[1]}$
$^3P_J^{[8]}$ $^1S_0^{[8]}$
$p_\perp$ (GeV)
$\mathrm{Br}(J/\psi \to \mu^+\mu^-)\ d\sigma(p\bar{p} \to J/\psi + X)/dp_\perp$ (nb/GeV)

Global Fits with NLO CSM + COM

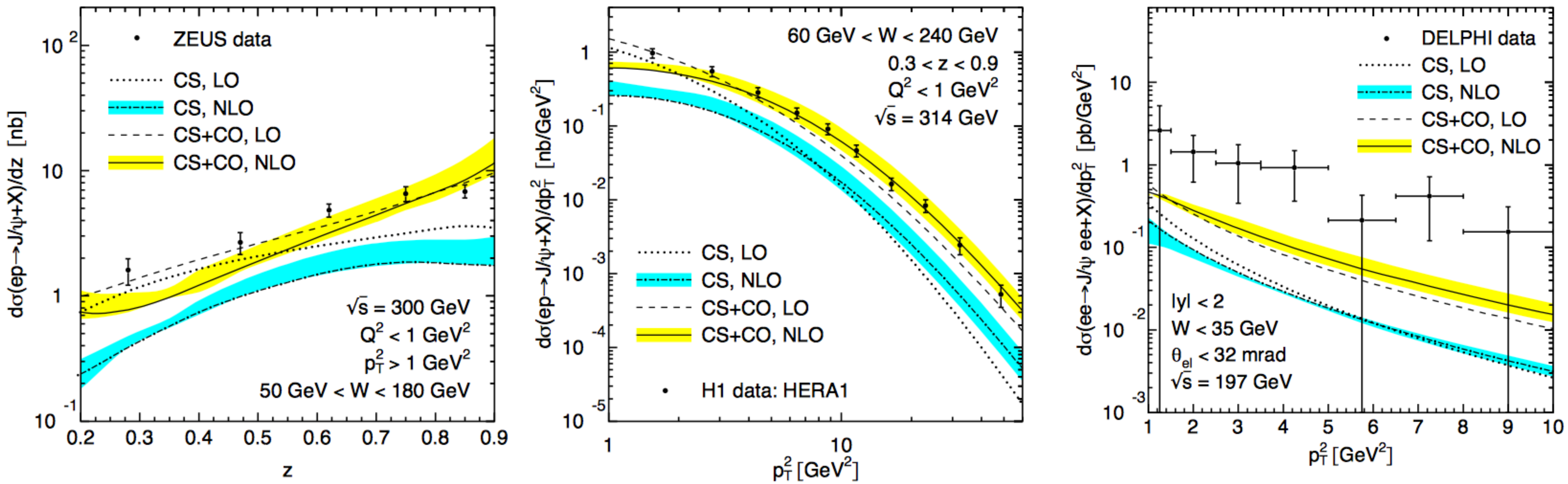


$$e^+e^-, \gamma\gamma, \gamma p, p\bar{p}, pp \to J/\psi + X$$

fit to 194 data points, 26 data sets,

Butenschoen and Kniehl, PRD 84 (2011) 051501

NLO: CSM + COM Required to Fit Data



$$ep \to J/\psi + X$$

$$\gamma^*\gamma^* \to J/\psi + X$$

Status of NRQCD approach to J/ψ Production

NLO: COM + CSM required for most processes

extracted LDME satisfy NRQCD v-scaling

$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]})\rangle \;=\; 1.32\ \text{GeV}^3$$

$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$	$(4.97 \pm 0.44) \times 10^{-2}\ \text{GeV}^3$
$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]})\rangle$	$(2.24 \pm 0.59) \times 10^{-3}\ \text{GeV}^3$
$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]})\rangle$	$(-1.61 \pm 0.20) \times 10^{-2}\ \text{GeV}^5$

$$\chi^2_{\text{d.o.f.}} = 857/194 = 4.42$$

NRQCD

Lagrangian

$$\mathcal{L}_{\text{NRQCD}} = \mathcal{L}_{\text{light}} + \mathcal{L}_{\text{heavy}} + \delta\mathcal{L}.$$

$$\mathcal{L}_{\text{heavy}} = \psi^\dagger \left(iD_t + \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(iD_t - \frac{\mathbf{D}^2}{2M} \right) \chi$$

LDME Operators

$$\begin{aligned} \mathcal{O}_n^H &= \chi^\dagger \mathcal{K}_n \psi \left(\sum_X \sum_{m_J} |H + X\rangle\langle H + X| \right) \psi^\dagger \mathcal{K}'_n \chi \\ &= \chi^\dagger \mathcal{K}_n \psi \left(a_H^\dagger a_H \right) \psi^\dagger \mathcal{K}'_n \chi, \end{aligned}$$

$$\mathcal{O}_8^H({}^3S_1) = \chi^\dagger \sigma^i T^a \psi \left(a_H^\dagger a_H \right) \psi^\dagger \sigma^i T^a \chi.$$

$$\mathcal{O}_1^H({}^3P_0) = \frac{1}{3}\, \chi^\dagger (-\tfrac{i}{2} \overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma}) \psi \left(a_H^\dagger a_H \right) \psi^\dagger (-\tfrac{i}{2} \overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma}) \chi$$

$$\mathcal{O}_1^H({}^3S_1) = \chi^\dagger \sigma^i \psi \left(a_H^\dagger a_H \right) \psi^\dagger \sigma^i \chi,$$

$$\mathcal{O}_8^H({}^1S_0) = \chi^\dagger T^a \psi \left(a_H^\dagger a_H \right) \psi^\dagger T^a \chi,$$

vNRQCD

Luke, Manohar, Rothstein, PRD 61(2000) 074025

heavy quarks, heavy antiquarks
potential gluons: $(p^0, \mathbf{p}) \sim (mv^2, mv)$

soft gluons: $p^\mu \sim mv$

ultrasoft gluons: $p^\mu \sim mv^2$

potential gluons are off-shell are integrated out and matched onto potentials for the heavy (anti-)quarks

$O(mv)$ $O(mv^2)$

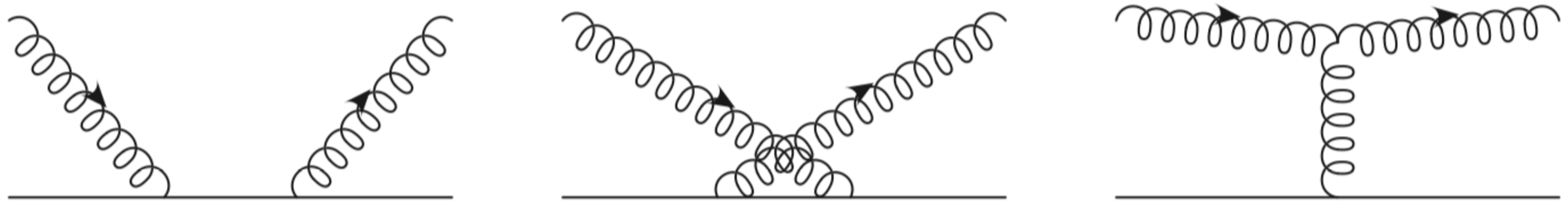
Label formalism: $\psi(x) = \sum e^{-i\mathbf{p}\cdot\mathbf{x}} \psi_{\mathbf{p}}(x)$ $i\partial_\mu \to \mathbf{P}_\mu + i\partial_\mu$

$$\mathcal{L} = \mathcal{L}_u + \mathcal{L}_p + \mathcal{L}_s$$

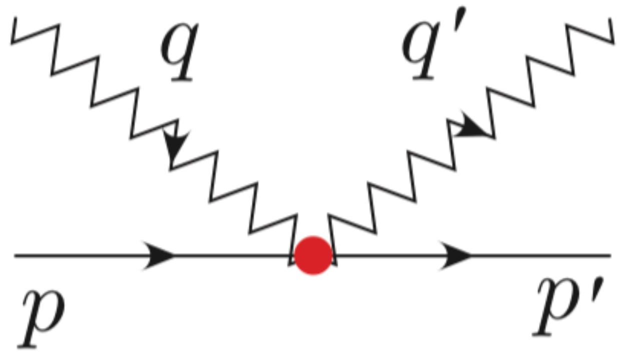
$$\mathcal{L}_u = \bar{\varphi}_{us}\, i\not{D}\, \varphi_{us} - \frac{1}{4} G_u^{\mu\nu} G_{u,\mu\nu} + \ldots$$

$$\mathcal{L}_p = \sum_{\mathbf{p}} \Big\{ \psi_{\mathbf{p}}^\dagger \Big[iD^0 - \frac{(\mathbf{p} - i\mathbf{D})^2}{2m} + \frac{\mathbf{p}^4}{8m^3} + \frac{c_F g_u}{2m} \sigma \cdot \mathbf{B}_u \Big] \psi_{\mathbf{p}} + (\psi \to \chi) \Big\}$$
$$- \sum_{\mathbf{p},\mathbf{p}'} \mu_s^{2\epsilon} \iota^\epsilon V(\mathbf{p}, \mathbf{p}')\, \psi_{\mathbf{p}'}^\dagger \psi_{\mathbf{p}} \chi_{-\mathbf{p}'}^\dagger \chi_{-\mathbf{p}} + \mathcal{L}_{pu} + \ldots\,,$$

Soft Lagrangian



(a) (b) (c)



(d)

$$\mathcal{L}_s^{\rm int} = -g_s^2 \mu_S^{2\epsilon} \iota^{\epsilon} \sum_{\mathbf{p},\mathbf{p}',q,q',\sigma} \Big\{ \frac{1}{2}\, \psi^\dagger_{\mathbf{p}'} [A^\mu_{q'}, A^\nu_q] U^{(\sigma)}_{\mu\nu} \psi_{\mathbf{p}} + \frac{1}{2}\, \psi_{\mathbf{p}'} \{A^\mu_{q'}, A^\nu_q\} W^{(\sigma)}_{\mu\nu} \psi_{\mathbf{p}}$$

$$+\ \psi^\dagger_{\mathbf{p}'} [\bar{c}_{q'}, c_q] Y^{(\sigma)}\, \psi_{\mathbf{p}} + (\, \psi^\dagger_{\mathbf{p}'} T^B Z^{(\sigma)}_\mu\, \psi_{\mathbf{p}})\, (\bar{\varphi}_{q'} \gamma^\mu T^B \varphi_q) \Big\} + (\psi \to \chi,\, T \to \bar{T})$$

$$U^{(0)}_{00} = \frac{1}{q^0}\,, \qquad U^{(0)}_{0i} = -\frac{(2p' - 2p - q)^i}{(\mathbf{p}' - \mathbf{p})^2}\,, \qquad U^{(0)}_{i0} = -\frac{(p - p' - q)^i}{(\mathbf{p}' - \mathbf{p})^2}\,, \qquad U^{(0)}_{ij} = \frac{(-\delta^{ij}) 2q^0}{(\mathbf{p}' - \mathbf{p})^2}$$

Soft Lagrangian with Soft Wilson Lines

Rothstein, Shrivastava, Stewart, ,Nucl.Phys. B939 (2019) 405

Soft Wilson line: $$S_{\rm v}(x,-\infty) = P\exp\left(-ig_s\int_{-\infty}^{0} d\lambda\, {\rm v}\cdot A(\lambda {\rm v}+x)\right)$$

Soft gauge invariant fields:
$$B^\mu(x) = -\frac{1}{g_s} S_{\rm v}^\dagger(x,-\infty)\, iD_s^\mu(x)\, S_{\rm v}(x,-\infty)\,,$$
$$\Xi(x) = S_{\rm v}^\dagger(x,-\infty)\,\varphi(x)\,,$$

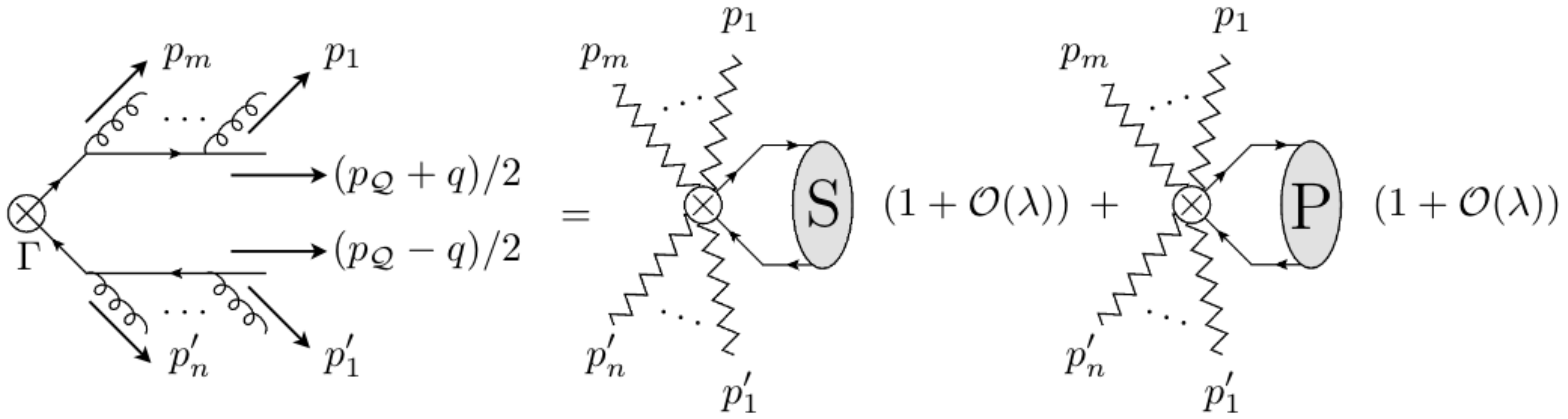
Simplified soft Lagrangian:

$$\begin{aligned}\mathcal{L}_s^{\rm int} = -g_s^2\mu_S^{2\epsilon}\iota^\epsilon \sum_{\mathbf{p},\mathbf{p}',q,q',\sigma} \Big\{ &\frac{i}{2} f^{abc}\, U_{ij}^{(\sigma)}(\psi^\dagger_{\mathbf{p}'}T^c\psi_{\mathbf{p}})(B_{q'}^{i,a}B_q^{j,b}) + \frac{1}{2}d^{abc}\, W_{ij}^{(\sigma)}(\psi_{\mathbf{p}'}T^c\psi_{\mathbf{p}})(B_{q'}^{i,a}B_q^{j,b}) \\ &+ \frac{1}{2}R_{ij}^{(\sigma)}\delta^{ab}(\psi^\dagger_{\mathbf{p}'}\psi_{\mathbf{p}})(B_{q'}^{i,a}B_q^{j,b}) + (\psi^\dagger_{\mathbf{p}'}T^B Z_\mu^{(\sigma)}\psi_{\mathbf{p}})\,(\bar\Xi_{q'}\gamma^\mu T^B\Xi_q) + (\psi^\dagger_{\mathbf{p}'}Z_\mu'^{(\sigma)}\psi_{\mathbf{p}})\,(\bar\Xi_{q'}\gamma^\mu\Xi_q)\Big\} \\ &+ (\psi\to\chi,\, T\to\bar T)\,. \end{aligned} \tag{3.6}$$

$$U_{ij}^{(0)}(q,q',\mathbf{p},\mathbf{p}') = -\frac{2q^0\delta_{ij}}{(\mathbf{p}'-\mathbf{p})^2}$$

reduced soft operator basis, simplifies matching, anomalous dimension calculations

NRQCD Matching Calculation w/ Soft Wilson Line



S-wave: q=0, match onto nonrelativistic spinors

P-wave: expand to linear order in q

$$\Gamma(\mathbf{q}) = \Gamma^{(0)} + \mathbf{q} \cdot \mathbf{\Gamma}^{(1)} + \ldots$$

S-waves

$$d_\Gamma^{(0)}(m,n) = \left(u^{(0)}\right)^\dagger \Big[(-g)^m \prod_{s=1}^{m} \frac{A_s^0}{p_t^0(s)}\Big] \Gamma^{(0)}(p_t(m), p_t'(n)) \Big[g^n \prod_{s=1}^{n} \frac{A^0_{(n+1-s)'}}{p_t'^0(s)}\Big] v^{(0)}$$

$$d_\Gamma^{(0)} = \left(u^{(0)}\right)^\dagger S_v^\dagger\, \Gamma^{(0)}\; S_v v^{(0)}$$

P-waves

$$\begin{aligned} d_\Gamma^{(1)} = d_V^{(1)} + d_D^{(1)} + d_\gamma^{(1)} &= \frac{g}{2m}\left(u^{(0)}\right)^\dagger \Big\{ S_v^\dagger \Gamma^{(0)} S_v, \Big[\frac{1}{v\cdot\mathcal{P}} \mathbf{q}\cdot\mathbf{B}_s\Big]\Big\} v^{(0)} \\ &+ \left(u^{(0)}\right)^\dagger S_v^\dagger\, \mathbf{q}\cdot\Big(\mathbf{\Gamma}^{(1)} - \frac{1}{4m}\Big\{\Gamma^{(0)}, \boldsymbol{\gamma}\Big\}\Big) S_v v^{(0)} \end{aligned}$$

Reparametrization Invariance and P-wave Operators

Instead of matching onto Wilson lines, heavy quark fields with label

$$v^\mu = (1, \mathbf{0})$$

Match onto Wilson lines with labels

$$v^\mu_\pm = \left(\sqrt{1+\frac{\mathbf{q}^2}{4m^2}}, \pm\frac{\mathbf{q}}{2m}\right) = v^\mu + \frac{q^\mu}{2m} + O\left(\frac{q^2}{m^2}\right) \qquad v_\pm^2 = 1$$

LO Matching $\qquad \psi^\dagger_{\mathbf{p},+} S^\dagger_{v_+}\, \Gamma^{(0)}\, S_{v_-} \chi_{\mathbf{p},-}$

$$\psi_{\mathbf{p},\pm} = \psi_{\mathbf{p}} \pm \frac{\not{q}}{4m}\psi_{\mathbf{p}} \qquad\qquad S_{v,+} = S_v + \delta S_v$$

$$\chi_{\mathbf{p},\pm} = \chi_{\mathbf{p}} \mp \frac{\not{q}}{4m}\chi_{\mathbf{p}} \qquad\qquad = S_v - \frac{g}{2m} S_v \frac{1}{v\cdot\mathcal{P}} q\cdot B$$

RPI transformations

M. Luke and A. Manohar, PLB286 (1992) 348

TMD Shape Functions for Quarkonia

$pp \to \eta_c + X$ small p_T

M. G. Echevarria, arXiv: 1907.06494

$$\frac{d\sigma}{dy\, d^2q_\perp} = \frac{4M^4\, H(M^2,\mu^2)}{2sM^2(N_c^2-1)} \Gamma^*_{\rho\sigma}\Gamma_{\mu\nu}(2\pi) \int d^2\boldsymbol{k}_{n\perp} d^2\boldsymbol{k}_{\bar{n}\perp} d^2\boldsymbol{k}_{s\perp}\, \delta^{(2)}\left(\boldsymbol{q}_\perp - \boldsymbol{k}_{n\perp} - \boldsymbol{k}_{\bar{n}\perp} - \boldsymbol{k}_{s\perp}\right)$$
$$\times\, G^{\sigma\nu}_{g/A}(x_A, \boldsymbol{k}_{n\perp}, S_A; \zeta_A, \mu)\, G^{\rho\mu}_{g/B}(x_B, \boldsymbol{k}_{\bar{n}\perp}, S_B; \zeta_B, \mu)\, S_{\eta_Q}\left[{}^1S_0^{[1]}\right](\boldsymbol{k}_{s\perp}; \mu)\,, \qquad (16)$$

TMD Quarkonium Shape Function

$$S^{(0)}_{\eta_Q}\left[{}^1S_0^{[1]}\right] = \frac{1}{N_c^2-1} \int \frac{d^2\boldsymbol{\xi}_\perp}{(2\pi)^2}\, e^{i\boldsymbol{\xi}_\perp \boldsymbol{k}_{s\perp}}\, \langle 0|\left[\mathcal{Y}_n^{\dagger ab}\mathcal{Y}_{\bar{n}}^{bc}\chi^\dagger\psi\right](\boldsymbol{\xi}_\perp)\, a^\dagger_{\eta_Q} a_{\eta_Q}\left[\mathcal{Y}_{\bar{n}}^{\dagger cd}\mathcal{Y}_n^{da}\psi^\dagger\chi\right](0)\,|0\rangle\ .$$

IR safe H at one-loop, non-trivial check of factorization

$$\chi_J \to q\bar{q} \to H_1 + H_2 + X$$

measure relative p_T of hadrons

$$\frac{d\Gamma}{dz_1 dz_2 d^2p_\perp} = \Gamma_0 \sum_{n={}^3S_1^{[8]},{}^3P_J^{[1]}} H_{[n]}(M_\chi,\mu)\int d^2k_\perp \int d^2q_\perp \int d^2r_\perp \delta^{(2)}(k_\perp + q_\perp + r_\perp)$$
$$\times S_{[n]}^{\perp}(\vec{r}_\perp) D_{q/H_1}(z_1,\vec{k}_\perp) D_{\bar{q}/H_2}(z_2,\vec{q}_\perp,\vec{p}_\perp)\,,$$

TMD Quarkonium Shape Functions

S. Fleming, Y. Makris, T.M., arXiv:1910.03586

$$S^{\perp}_{\mathcal{Q}\to {}^3S_1^{[8]}} = \frac{1}{t_F}\,\frac{d-2}{d-1}\mathrm{Tr}\Big\langle \chi_J \Big| \mathcal{O}_2^{a,i}({}^3S_1^{[8]})\mathcal{S}_v^{ba}(S_{\bar{n}}^\dagger T^b S_n)\delta^{(2)}(\mathbf{q}_\perp - \boldsymbol{\mathcal{P}}_\perp) \times (S_n^\dagger T^c S_{\bar{n}})\mathcal{S}_v^{dc}[\mathcal{O}_2^{d,is}({}^3S_1^{[8]})]^\dagger \Big| \chi_J \Big\rangle$$

$$S^{\perp}_{\mathcal{Q}\to {}^3P_J^{[1]}} = \frac{2g^2}{N_c t_F}\,\mathcal{A}_J^{ij}\,\mathrm{Tr}\Big\langle \chi_J \Big| \mathcal{O}_2^{\{mn\ldots\}}({}^3P_J^{[1]})\Big[\frac{B_s^{a,i}}{m\,v\cdot\mathcal{P}}\Big]\mathcal{S}_v^{ba}(S_{\bar{n}}^\dagger T^b S_n)$$
$$\times\ \delta^{(2)}(\mathbf{q}_\perp - \boldsymbol{\mathcal{P}}_\perp)(S_n^\dagger T^c S_{\bar{n}})\mathcal{S}_v^{dc}\Big[\frac{B_s^{d,j}}{m\,v\cdot\mathcal{P}}\Big][\mathcal{O}_2^{\{mn\ldots\}}({}^3P_J^{[1]})]^\dagger \Big| \chi_J \Big\rangle$$

$$\mathcal{O}_2^{a,i}({}^3S_1^{(8)}) = \psi^\dagger \sigma^i T^a \chi$$

$$\mathcal{O}_2({}^3P_0^{[1]}) = \frac{1}{6\sqrt{2N_c}}[\psi^\dagger \boldsymbol{\sigma}\cdot \overleftrightarrow{\boldsymbol{\mathcal{P}}}\,\chi]$$

Some Important Points

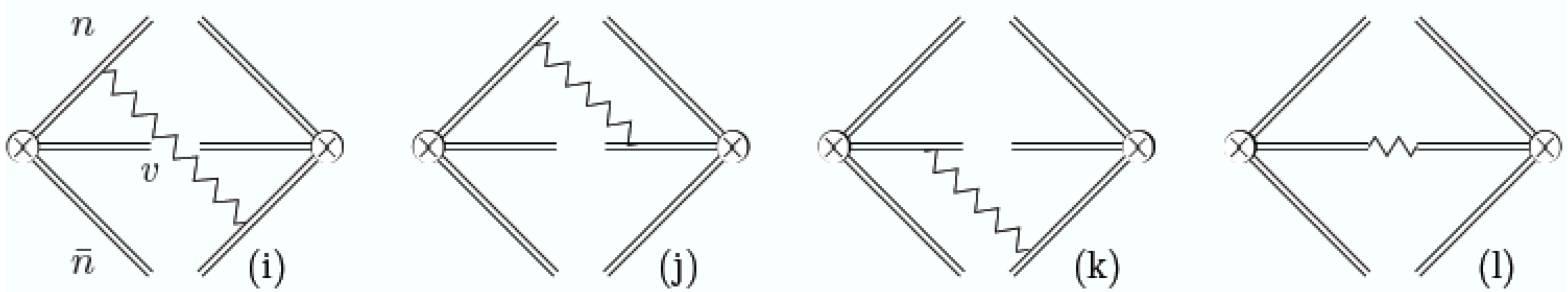
IR Safety - checked to NLO

$^3S_1^{(8)}, ^3P_J^{(1)}$ both required for IR safety (old NRQCD story)

p_T shape functions linked by RPI (new story)

octet mechanisms - final state radiation from soft Wilson line introduce additional logs

Evolution is slightly different than TMD resummation for, e.g., Drell-Yan

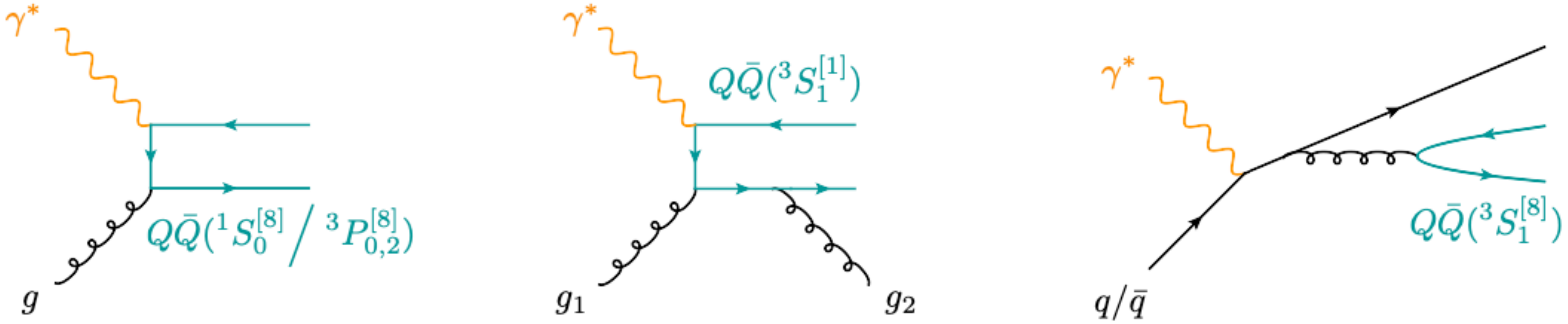


$$d_{(\mathrm{i}+\bar{\mathrm{i}})+(\mathrm{j}+\bar{\mathrm{j}})+(\mathrm{k}+\bar{\mathrm{k}})+\mathrm{l}} = \frac{2}{3}\langle {}^3S_1^{[8]}\rangle_{\mathrm{LO}}\Big(S^{\perp}_{\mathrm{DY}} + \frac{\alpha_s C_A}{2\pi}\Big\{\frac{1}{\epsilon}\delta^{(2)}(\mathbf{p}_T) - 2\mathcal{L}_0(\mathbf{p}_T^2,\mu^2)\Big\}\Big)$$

$$S^{\perp}_{\mathrm{DY}} = \frac{\alpha_s C_F}{2\pi}\Big\{\frac{4}{\eta}\Big[2\mathcal{L}_0(\mathbf{p}_T^2,\mu^2) - \frac{1}{\epsilon}\delta^{(2)}(\mathbf{p}_T)\Big] + \frac{2}{\epsilon}\Big[\frac{1}{\epsilon} - \ln\Big(\frac{\nu^2}{\mu^2}\Big)\Big]\delta^{(2)}(\mathbf{p}_T) - \frac{\pi^2}{6}\delta(\mathbf{p}_T)$$
$$- 4\mathcal{L}_1(\mathbf{p}_T^2,\mu^2) + 4\mathcal{L}_0(\mathbf{p}_T^2,\mu^2)\ln\Big(\frac{\nu^2}{\mu^2}\Big)\Big\}$$

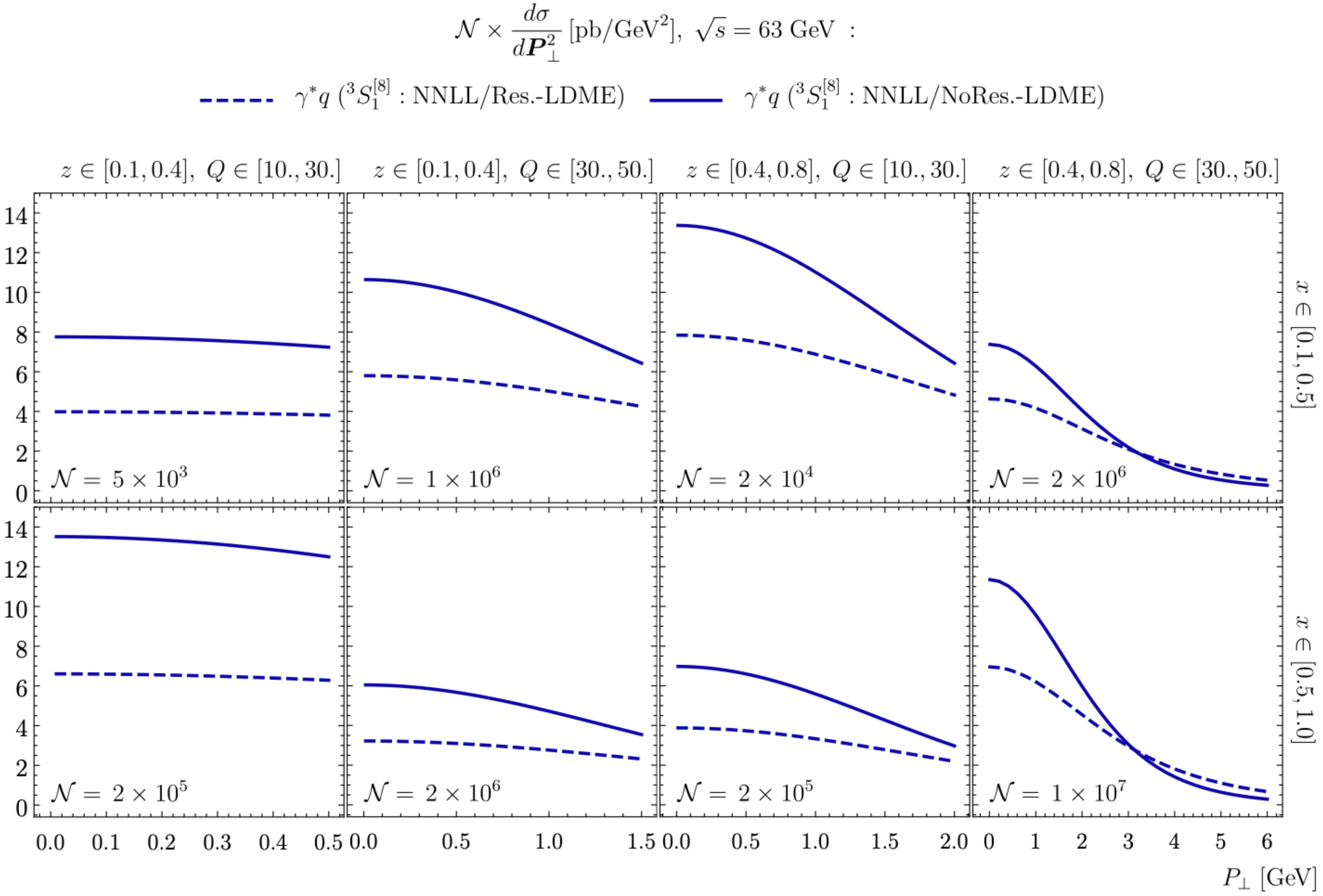
Quarkonia Production in SIDIS at EIC

M Echevarria, Y. Makris, I. Scimemi, arXiv:2007.05547

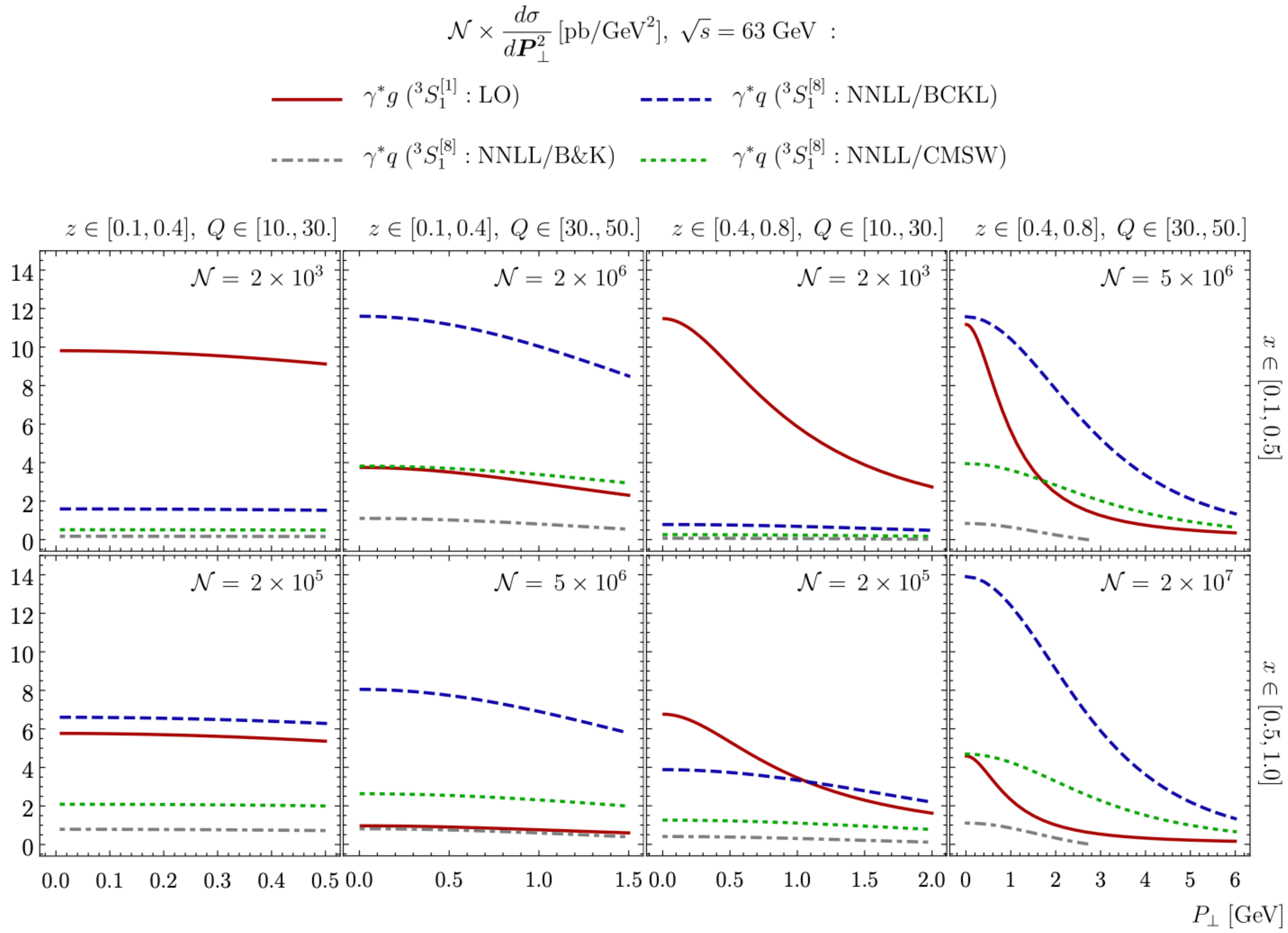


perform NLL resummation

compared light quark fragmentation TMDFF with photon-gluon fusion

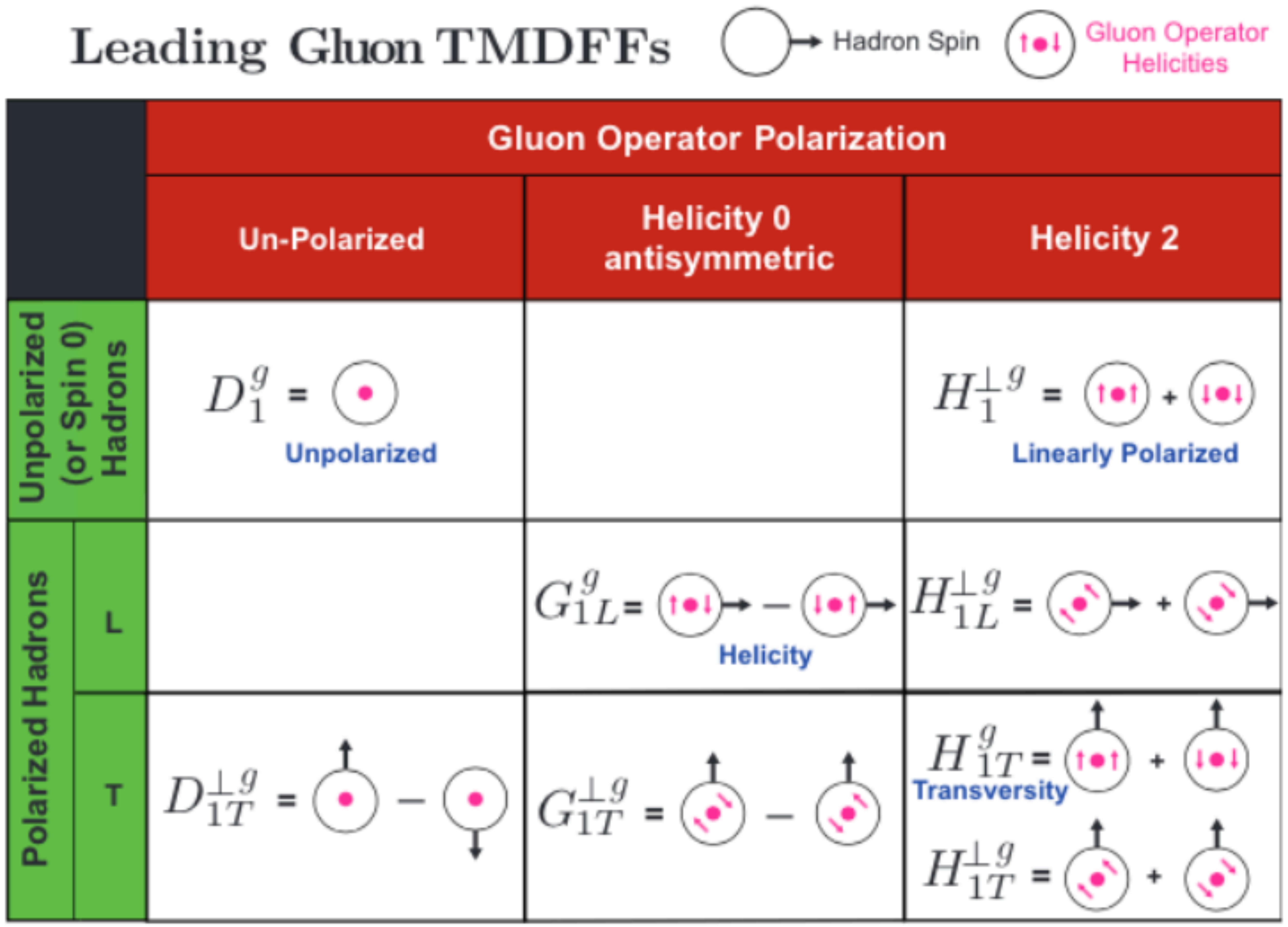


light quark fragmentation



photon-gluon fusion

More TMD FFs to be explored



from TMD handbook

polarized gluon TMD FFs to spin-1 hadrons yet to be classified

Summary/Outlook

TMD Factorization for quarkonium production requires p_T shape functions

M. G. Echevarria, arXiv: 1907.06494

S. Fleming, Y. Makris, T.M., arXiv:1910.03586

S-wave and P-wave shape functions related by RPI

Modified TMD evolution

Important for future quarkonium studies at, e.g., EIC

M Echevarria, Y. Makris, I. Scimemi, arXiv:2007.05547

$$\gamma^{(*)} g \to J/\psi + X$$

$$e + p \to e + J/\psi + p$$