Near threshold heavy quarkonium photoproduction

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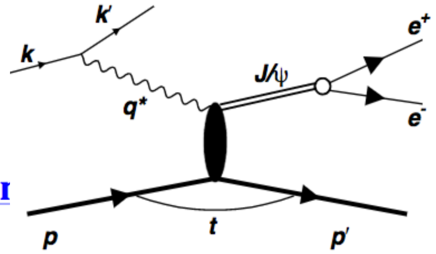
Refs: Sun, Tong, Yuan, Phys.Lett.B 822 (2021) 136655;
arXiv: 2110.xxxx;

Heavy quarkonium production near threshold: a very hot topic



Near threshold heavy vector meson photoproduction at LHC and EicC,
Ya-Ping Xie, V.P. Gonçalves, e-Print: 2103.12568 [hep-ph]
QCD Analysis of Near-Threshold Photon-Proton Production of Heavy Quarkonium.
Yuxun Guo, Xiangdong Ji, Yizhuang Liu, e-Print: 2103.11506 [hep-ph]
Trace Anomaly of Proton Mass with Vector Meson Near-Thresholds Photoproductio
Wei Kou, Rong Wang, Xurong Chen e-Print: 2103.10017 [hep-ph]
Nucleon mass radii and distribution: Holographic QCD, Lattice QCD and GlueX data,
Kiminad A. Mamo, Ismail Zahed, e-Print: 2103.03186 [hep-ph]
ϕ-meson lepto-production near threshold and the strangeness D-term,
Yoshitaka Hatta, Mark Strikman e-Print: 2102.12631 [hep-ph]
Proton mass decomposition: naturalness and interpretations,
Xiangdong Ji, e-Print: 2102.07830 [hep-ph]
Extraction of the proton mass radius from the vector meson photoproductions near thresholds,
Rong Wang, Wei Kou, Ya-Ping Xie, Xurong Chen e-Print: 2102.01610 [hep-ph]
The mass radius of the proton,
Dmitri E. Kharzeev, e-Print: 2102.00110 [hep-ph]
Quantum Anomalous Energy Effects on the Nucleon Mass,
Xiangdong Ji, Yizhuang Liu e-Print: 2101.04483 [hep-ph]

The proton mass distribution/mass radius is one of focuses

The mass radius of the proton

Dmitri E. Kharzeev (Stony Brook U. and RIKEN BNL) (Ja

e-Print: 2102.00110 [hep-ph]

Nucleon mass radii and distribution: Holographic QCD, Lattice QCD and GlueX data

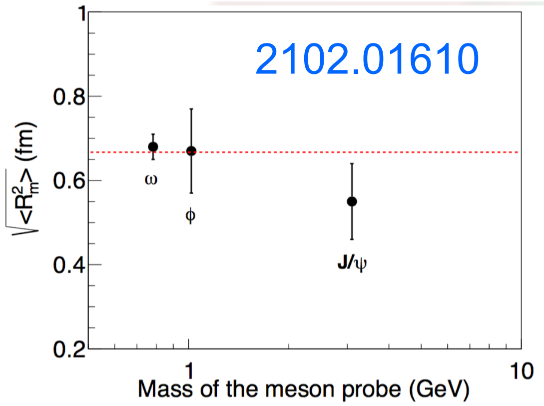
Kiminad A. Mamo (Argonne), Ismail Zahed (SUNY, Stony Brook) (Mar 4, 2021)

Published in: *Phys.Rev.D* 103 (2021) 9, 094010 • e-Print: 2103.03186 [hep-ph]

Extraction of the proton mass radius from the vector meson photoproductions near thresholds

Rong Wang (Lanzhou, Inst. Modern Phys. and Beijing, GUCAS), Wei Kou (Lanzhou, Inst. Modern Phys. and Beijing, GUCAS), Ya-Ping Xie (Lanzhou, Inst. Modern Phys. and Beijing, GUCAS), Xurong Chen (Lanzhou, Inst. Modern Phys. and Beijing, GUCAS and South China Normal U.) (Feb 2, 2021)

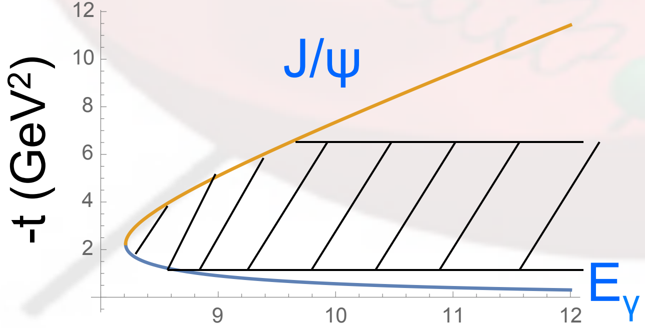
Published in: *Phys.Rev.D* 103 (2021) 9, L091501 • e-Print: 2102.01610 [hep-ph]

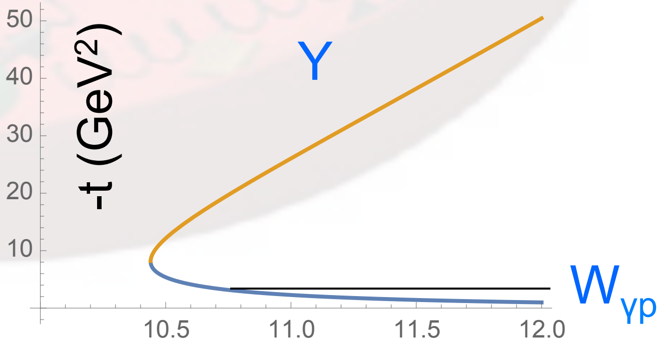


- All these are models
  - Vector meson dominance model
  - Holographic model
- How rigorous (in QCD) that we can measure the proton mass distribution?

This talk focuses on the special kinematics: large momentum transfer

- We can compute both the cross section and the form factors separately in perturbative QCD, then we can check that if there is/not direct connection between the near threshold production and the gluonic gravitational form factors





Outline

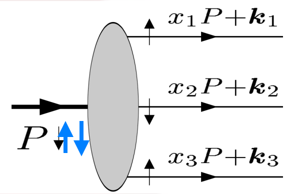
- Light-cone wave functions/distribution amplitudes
- Form factor calculations
- Threshold heavy quarkonium production
- Discussions

Light-cone Wave Functions

Brodsky-Lepage 1981

- They are building blocks for the hadron structure

$$|P\rangle = \sum_{n,\lambda_i} \int \prod_i \frac{dx_i d^2 k_{\perp i}}{\sqrt{x_i} 16\pi^3} \phi_n(x_i, k_{\perp i}, \lambda_i) |n : x_i, k_{\perp i}, \lambda_i\rangle$$



- Fock state of n-partons: momentum fractions, transverse momenta, helicities
- Can be used to calculate the form factors, GPDs, and hard exclusive scattering amplitudes, including near threshold heavy quarkonium production

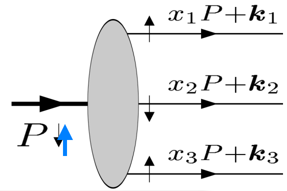
Nucleon's 3-quarks WF

Ji, Ma, Yuan, 2002

- According to the general structure, six independent light-cone wave functions for three quarks component:

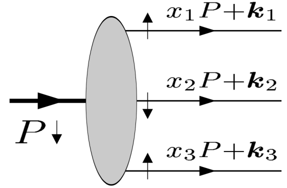
$$|P\uparrow\rangle_{1/2} = \int d[1]d[2]d[3] \left(\tilde{\psi}^{(1)}(1,2,3)\right) \times \frac{\epsilon^{abc}}{\sqrt{6}} u^\dagger_{a\uparrow}(1)\left(u^\dagger_{b\downarrow}(2)d^\dagger_{c\uparrow}(3) - d^\dagger_{b\downarrow}(2)u^\dagger_{c\uparrow}(3)\right)|0\rangle$$

$L_z=0$



$$|P\downarrow\rangle_{1/2} = \int d[1]d[2]d[3] \left((k_1^x - ik_1^y)\tilde{\psi}^{(3)}(1,2,3) + (k_2^x - ik_2^y)\tilde{\psi}^{(4)}(1,2,3)\right) \times \frac{\epsilon^{abc}}{\sqrt{6}} \left(u^\dagger_{a\downarrow}(1)u^\dagger_{b\uparrow}(2)d^\dagger_{c\uparrow}(3) - d^\dagger_{a\downarrow}(1)u^\dagger_{b\uparrow}(2)u^\dagger_{c\uparrow}(3)\right)|0\rangle\ ,$$

$|L_z|=1$



Distribution amplitudes

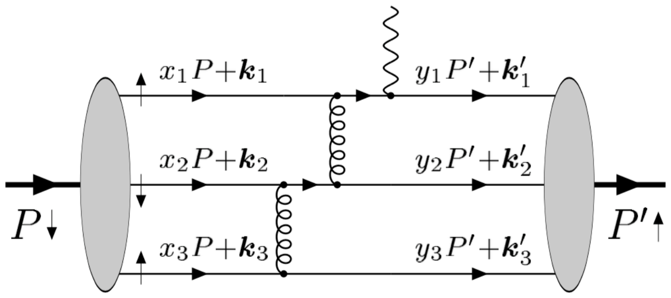
- Integrate out the transverse momentum
  - Twist-three (leading-twist)

$$\Phi_3(y_i) = -2\sqrt{6}\int \frac{d^2\vec{k}'_{1\perp} d^2\vec{k}'_{2\perp} d^2\vec{k}'_{3\perp}}{(2\pi)^6}\delta^{(2)}(\vec{k}'_{1\perp}+\vec{k}'_{2\perp}+\vec{k}'_{3\perp})\tilde{\psi}^{(1)}(1,2,3)$$

  - Twist-four (Braun-Fries-Mahnke-Stein 2000)

$$\begin{aligned}\Psi_4(x_1,x_2,x_3) &= -\frac{2\sqrt{6}}{x_2 M}\int \frac{d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp} d^2\vec{k}_{3\perp}}{(2\pi)^6}\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp}) \\ &\quad \times \vec{k}_{2\perp}\cdot\left[\vec{k}_{1\perp}\tilde{\psi}^{(3)}(1,2,3)+\vec{k}_{2\perp}\tilde{\psi}^{(4)}(1,2,3)\right] . \\ \Phi_4(x_2,x_1,x_3) &= -\frac{2\sqrt{6}}{x_3 M}\int \frac{d^2\vec{k}_{1\perp} d^2\vec{k}_{2\perp} d^2\vec{k}_{3\perp}}{(2\pi)^6}\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp}) \\ &\quad \times \vec{k}_{3\perp}\cdot\left[\vec{k}_{1\perp}\tilde{\psi}^{(3)}(1,2,3)+\vec{k}_{2\perp}\tilde{\psi}^{(4)}(1,2,3)\right] .\end{aligned}$$

Form factor calculations



Compute the partonic scattering amplitudes, convert to hadron's Leading-twist: direct integration of k_t, higher-twist: need k_t-expansion

- Two gluon exchanges are needed to generate large momentum transfer
- Helicity-non-flip has power behavior, F_1~1/t^2
- Helicity-flip amplitude has power behavior, F_2~1/t^3

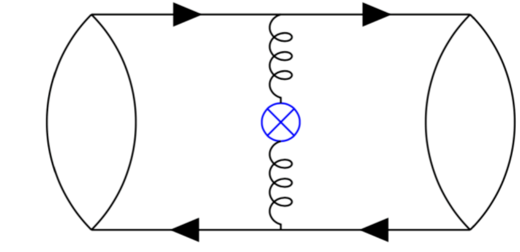
Brodsky-Lepage 1981 for F_1
Belitsky-Ji-Yuan 2002 for F_2

Gravitational form factors: No much difference, only some surprises

- Pion case

$$\langle P'|T_g^{\mu\nu}|P\rangle = 2\bar{P}^\mu \bar{P}^\nu A_g^\pi(t) + \frac{1}{2}(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)C_g^\pi(t) + 2m^2 g^{\mu\nu}\overline{C}_g^\pi(t)$$

$$A_g^\pi(t) = C_g^\pi(t) = \frac{4m^2}{t}\overline{C}_g^\pi(t) = \frac{4\pi\alpha_s C_F}{-t}\int dx_1 dy_1 \phi^*(y_1)\phi(x_1)\left(\frac{1}{x_1\bar{x}_1} + \frac{1}{y_1\bar{y}_1}\right)$$



Tong-Ma-Yuan, 2103.12047;
Different from Tanaka, PRD 2018

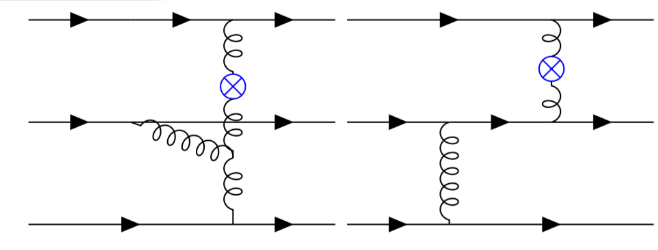
➢ May introduce difficulty in the interpretation, since integral over t is not convergent

Polyakov-Schweitzer 2018
Freese-Miller 2021

- $A_g=C_g$!!
- Cbar cancels between quarks and gluons
  - Quark part from GPD quark at large-t (Hoodbhoy-Ji-Yuan 2003)

Nucleon case

$$\langle P', s'|T_g^{\mu\nu}|P, s\rangle = \bar{U}_{s'}(P')\left[A_g(t)\gamma^{\{\mu}\bar{P}^{\nu\}} + C_g(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} + \cdots\right]U_s(P)$$

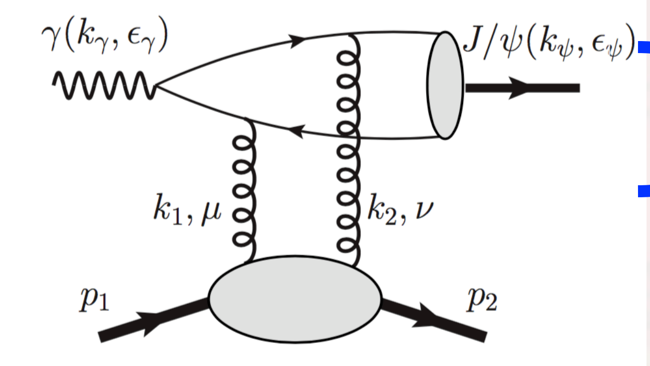


- No contribution from three-gluon vertex diagram
- A_g~1/t^2
- B_g, C_g scale as 1/t^3,Cbar$_g$ scales as 1/t^2

$$\mathcal{A} = \frac{4\pi^2\alpha_s^2 C_B^2}{3t^2}\left(I_{13} + I_{12} + I_{31} + I_{32}\right), \quad I_{ij} = \frac{x_i + y_i}{\bar{x}_i\bar{y}_i x_i x_j y_i y_j}$$

Tong-Ma-Yuan, 2103.12047

Near threshold production: kinematics



Photon-J/ψ Part: $\mathcal{M}_\psi^{\mu\nu}$

- Two limits
  - Threshold: $\chi = \frac{M_V^2 + 2M_pM_V}{W_{\gamma p}^2 - M_p^2}$ → 1, (1-χ) a small parameter, Brodsky et al 2001
  - Heavy quark limits

$$W_{\gamma p}^2 \sim M_V^2 \gg (-t) \gg \Lambda_{QCD}^2 \ ,$$

$$p_1 \cdot k_\gamma \sim p_1 \cdot k_\psi \sim M_V^2$$
$$p_2 \cdot k_\gamma \sim p_2 \cdot k_\psi \ll M_V^2$$

$\mathcal{M}_{\psi}^{\mu\nu}$

- NRQCD for heavy quarkonium production   Bodwin-Braaten-Lepage 1995
- Propagators are of order heavy quark mass, $\sim 1/M_V$
- Take transverse polarization for the incoming photon

$$\mathcal{M}_{\psi}^{\mu\nu} = \frac{M_c|R(0)|}{(k_1^2 - k_1\cdot k_\psi)(k_2^2 - k_2\cdot k_\psi)(k^2 - k\cdot k_\psi)} \times \left[\epsilon_\psi^* \cdot \epsilon_\gamma \mathcal{W}_T^{\mu\nu} + \epsilon_\psi^* \cdot k \mathcal{W}_L^{\mu\nu} + \mathcal{W}_s^{\mu\nu}\right] ,$$

$$\mathcal{W}_T^{\mu\nu} = -k_1\cdot k_\gamma k_2\cdot k_\gamma g^{\mu\nu} - k_1\cdot k_2 k_\gamma^\mu k_\gamma^\nu + k_1\cdot k_\gamma k_2^\mu k_\gamma^\nu + k_2\cdot k_\gamma k_1^\nu k_\gamma^\mu$$

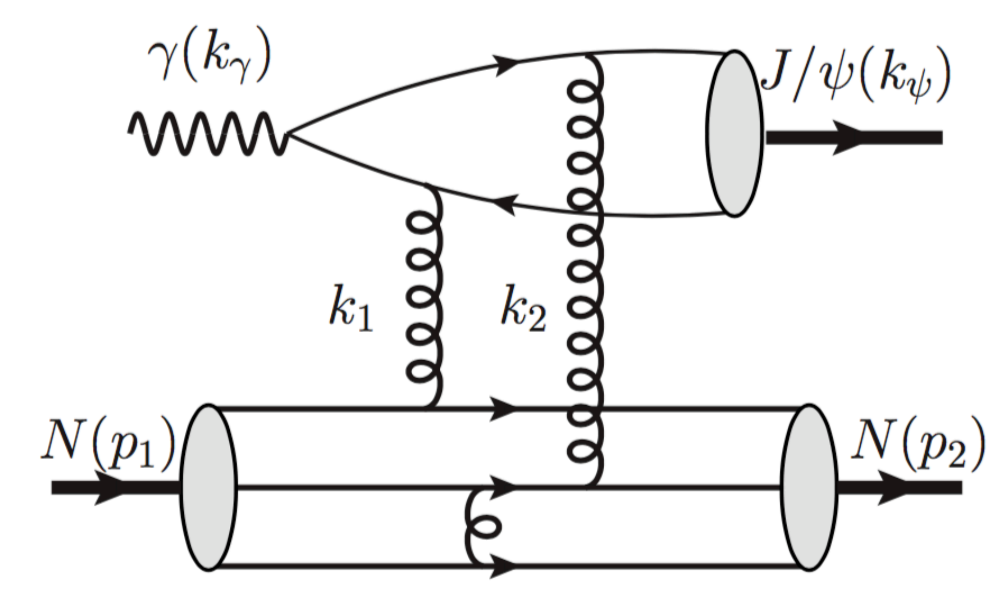
Leading terms

$$\mathcal{W}_L^{\mu\nu} = k_1\cdot k_\gamma \epsilon_\gamma^\nu k_2^\mu + k_2\cdot k_\gamma \epsilon_\gamma^\mu k_1^\nu$$

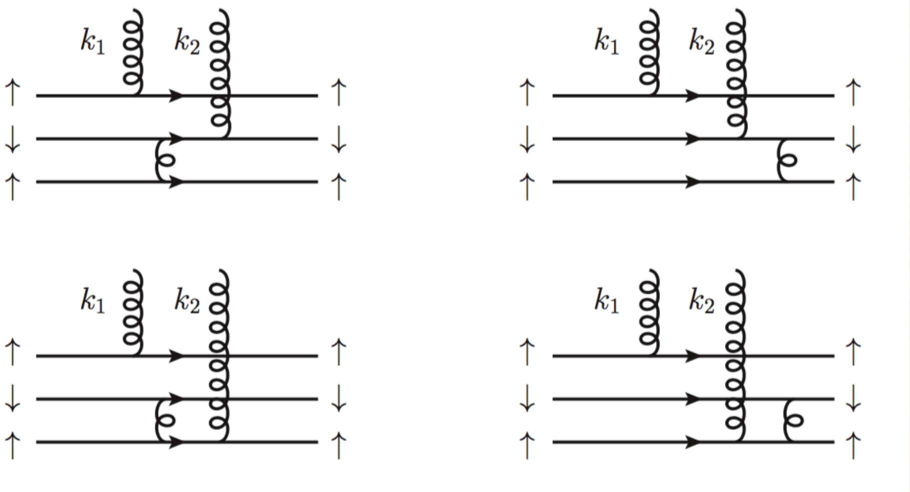
$$\mathcal{W}_S^{\mu\nu} = -k_1\cdot k_2 \left(k_1\cdot k_\gamma \epsilon_\psi^{*\mu} \epsilon_\gamma^\nu + k_2\cdot k_\gamma \epsilon_\psi^{*\nu}\epsilon_\gamma^\mu + k_1\cdot \epsilon_\psi^* k_\gamma^\nu \epsilon_\gamma^\mu + k_2\cdot \epsilon_\psi^* k_\gamma^\mu \epsilon_\gamma^\nu\right) .$$

Couple to the Nucleon

- Additional gluon exchange to generate large-t
- Nucleon spin configurations
  - Helicity conserved
  - Helicity-flip



Partonic scattering: I

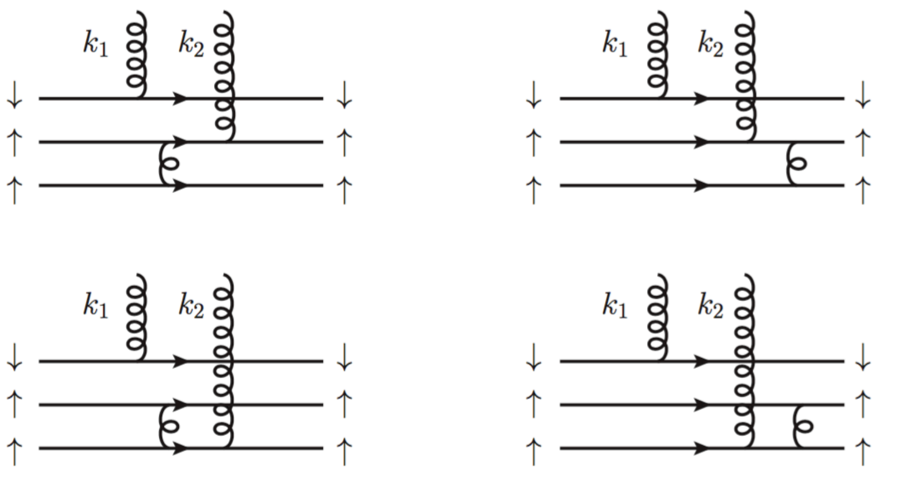


$$\bar{U}(p_2)\gamma^\mu U(p_1)\mathrm{Tr}\left[\frac{1+\gamma_5}{2}\not{p}_2\cdots\gamma^\nu\cdots\frac{1+\gamma_5}{2}\not{p}_1\cdots\right]$$

$$\bar{U}(p_2)\gamma^\mu U(p_1)\bar{P}^\nu$$

- k_1 attaches the helicity-up quark line

Partonic scattering: II



$$\bar{U}(p_2)\gamma^\rho U(p_1)\mathrm{Tr}\left[\frac{1+\gamma_5}{2}\not{p}_2\cdots\gamma^\nu\cdots\frac{1+\gamma_5}{2}\not{p}_1\cdots\gamma^\rho\cdots\right]$$

$$\bar{U}(p_2)\gamma^\mu U(p_1)\bar{P}^\nu$$

- k_1 attaches the helicity-down quark line

Final amplitude

$$\mathcal{A}_3 = \langle J/\psi(\epsilon_\psi), N'_\uparrow|\gamma(\epsilon_\gamma), N_\uparrow\rangle$$
$$= \int [dx][dy]\Phi(x_1,x_2,x_3)\Phi^*(y_1,y_2,y_3)\frac{1}{(-t)^2}$$
$$\times \bar{U}_\uparrow(p_2)\not{k}_\gamma U_\uparrow(p_1)\mathcal{M}_\psi^{(3)}(\epsilon_\gamma,\epsilon_\psi,\{x_i\},\{y_i\}) \ ,$$

$$\mathcal{M}_\psi^{(3)} = R_\psi\left(2\mathcal{H}_3 + \mathcal{H}'_3\right),$$

$$\mathcal{H}_3 = I_{13} + I_{31} + I_{12} + I_{32},$$

$$I_{ij} = \frac{1}{x_i x_j y_i y_j \bar{x}_i^2 \bar{y}_i}$$

$$R_\psi \equiv \frac{8 e_c e g_s^6}{27\sqrt{3M_\psi^{10}}}\psi_J(0)$$

- Similar structure as A_g form factor or GPD H_g contribution

Amplitude squared

$$|\overline{\mathcal{A}_3}|^2 = (1-\chi) G_\psi G_{p3}(t) G^*_{p3}(t)$$

$$G_\psi = C_N^2 \frac{64\pi^2 \alpha e_c^2 (4\pi\alpha_s)^6}{3M_\psi^3} \langle \mathcal{O}_1^\psi(^3S_1)\rangle ,$$

$$G_{p3}(t) = \frac{1}{t^2} \int [dx][dy] \Phi_3(\{x\}) \Phi_3^*(\{y\}) \left[2\mathcal{H}_3 + \mathcal{H}'_3\right]$$

- Suppressed at the threshold, χ→1
- This behavior is similar to H_g contribution to J/ψ production in the GPD formalism with 1-ξ suppression factor
  - Hoodbhoy 1996, see also, Koempel-Kroll-Metz-Zhou 2012, Guo-Ji-Liu 2021
- Power behavior of 1/t^4

Twist-four contribution

$$\mathcal{A}_4 = \langle J/\psi(\epsilon_\psi), N'_\uparrow | \gamma(\epsilon_\gamma), N_\downarrow \rangle$$
$$= \int [dx][dy]\Psi_4(\{x\})\Phi_3^*(\{y\})\mathcal{M}_\psi^{(4)}(\{x\},\{y\})$$
$$\times \bar{U}_\uparrow(p_2)U_\downarrow(p_1)\frac{M_p}{(-t)^3} ,$$

$$|\overline{\mathcal{A}_4}|^2 = \widetilde{m}_t^2 G_\psi G_{p4}(t) G_{p4}^*(t) \quad \widetilde{m}_t^2 = M_p^2/(-t)$$

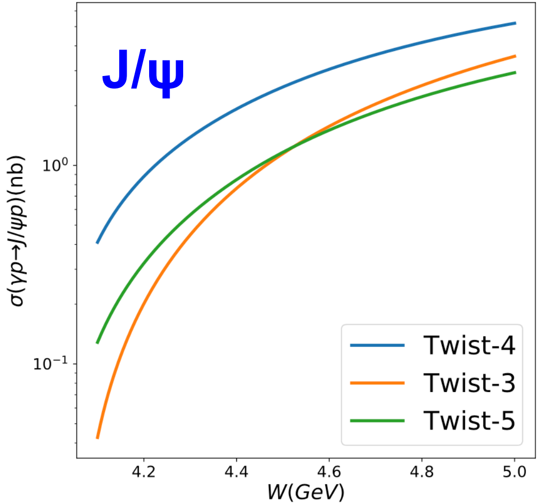
$$G_{p4}(t) = \frac{1}{t^2}\int [dx][dy]\Psi_4(\{x\})\Phi_3^*(\{y\})\mathcal{H}_{\Psi 4}$$

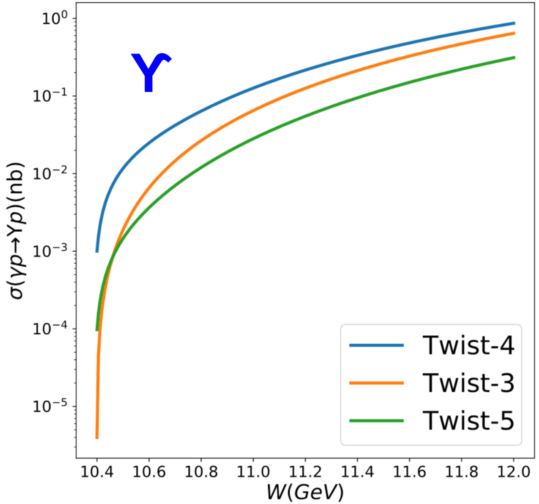
- Helicity-flip amplitude
- kt-expansion, similar to F_2 form factor
- There is no interference between twist-3 and twist-4
- Power behavior~1/t^5

Differential cross section

$$\frac{d\sigma}{dt}\Big|_{(-t)\gg\Lambda^2_{QCD}} = \frac{1}{16\pi(W^2_{\gamma p}-M^2_p)^2}\left(|\overline{\mathcal{A}_3}|^2+|\overline{\mathcal{A}_4}|^2\right)$$

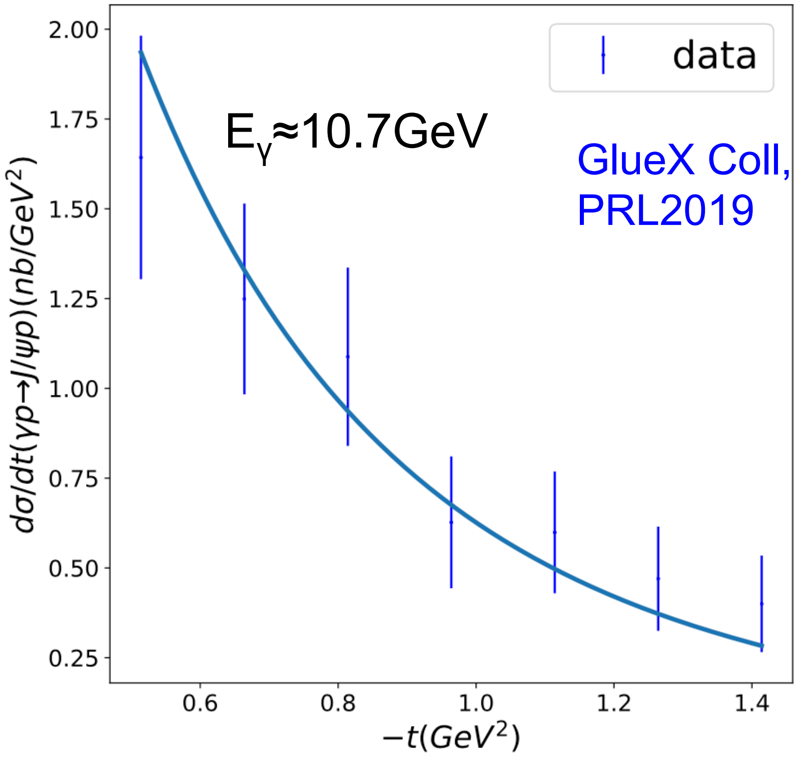
$$\approx \frac{1}{(-t)^4}\left[(1-\chi)\mathcal{N}_3+\tilde{m}_t^2\mathcal{N}_4\right] ,$$

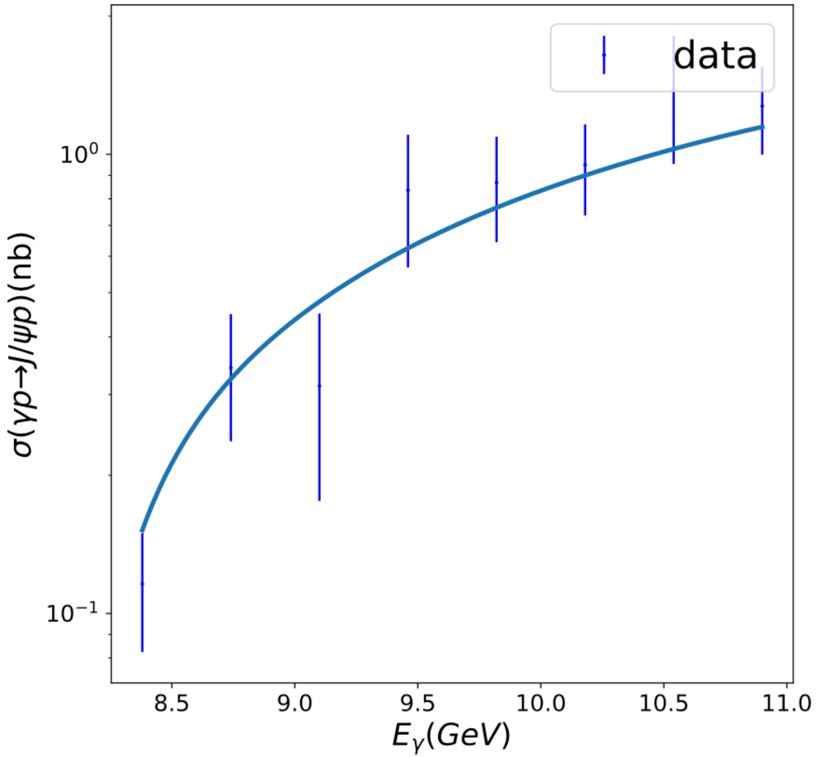




- Total cross section contributions
  - Smooth to low-t by replace $-t \rightarrow \Lambda^2$-t
  - Parametric comparison, only power counting
- Clearly, twist-4 dominates near threshold

Twist-four fit to GlueX data



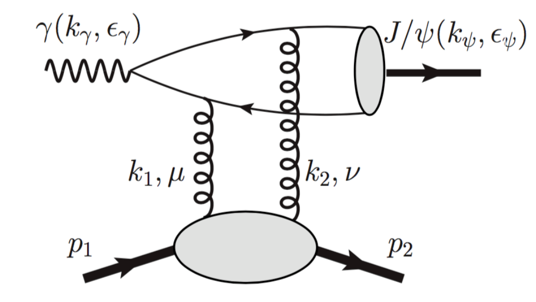


Comments

- Power behavior for near threshold heavy quarkonium production at large-t is derived, and the leading contribution comes from non-zero OAM three-quark state, scales as 1/t^5
  - Agree with the GlueX data
  - Precision data in the future will be able to test different power
- There is no direct connection to the gravitational form factors

Discussion: construct the gluon operators

- Take the leading contribution of heavy quark mass limit



} Photon-J/ψ Part: $\mathcal{M}_\psi^{\mu\nu}$

$$\mathcal{M}_\psi^{\mu\nu} = \frac{M_c|R(0)|}{(k_1^2 - k_1 \cdot k_\psi)(k_2^2 - k_2 \cdot k_\psi)(k^2 - k \cdot k_\psi)} \times \left[\epsilon_\psi^* \cdot \epsilon_\gamma \mathcal{W}_T^{\mu\nu} + \epsilon_\psi^* \cdot k \mathcal{W}_L^{\mu\nu} + \mathcal{W}_s^{\mu\nu}\right] ,$$

$$\mathcal{W}_T^{\mu\nu} = -k_1 \cdot k_\gamma k_2 \cdot k_\gamma g^{\mu\nu} - k_1 \cdot k_2 k_\gamma^\mu k_\gamma^\nu + k_1 \cdot k_\gamma k_2^\mu k_\gamma^\nu + k_2 \cdot k_\gamma k_1^\nu k_\gamma^\mu$$

$$\frac{k_\gamma^\alpha k_\gamma^\beta}{k_1 \cdot k_\gamma k_2 \cdot k_\gamma} \mathcal{W}_T^{\alpha\beta\mu\nu}$$

$$\mathcal{W}_T^{\alpha\beta\mu\nu} = -k_1^\alpha k_2^\beta g^{\mu\nu} - k_1 \cdot k_2 g^{\alpha\mu} g^{\beta\nu} + k_1^\nu k_2^\beta g^{\alpha\mu} + k_2^\mu k_1^\alpha g^{\beta\nu} ,$$

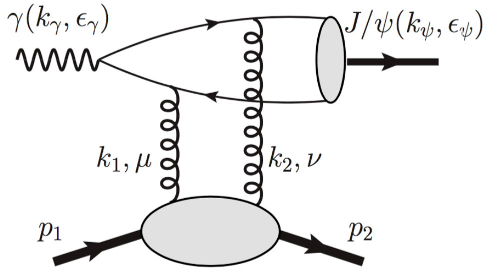
Connect to gravitational form factors?

$$\mathcal{A} \propto \int d^4\eta_1 d^4\eta_2 d^4k_1 d^4k_2 e^{ik_1\cdot\eta_1+ik_2\cdot\eta_2} \frac{k_\gamma^\alpha k_\gamma^\beta}{k_1\cdot k_\gamma k_2\cdot k_\gamma}$$

$$\times\langle N'|F^\alpha{}_\rho(\eta_1)F^{\beta\rho}(\eta_2)|N\rangle \ . \qquad ($$

- We have to make approximations: the two gluons in the t-channel carry the same momentum

$$\mathcal{A} \propto \frac{k_\gamma^\alpha k_\gamma^\beta}{\langle k_1\cdot k_\gamma k_2\cdot k_\gamma\rangle}\langle T_g^{\alpha\beta}\rangle$$



Discussion: compare to the GPD formalism

$$\mathcal{M}(\varepsilon_V,\varepsilon)=\frac{8\sqrt{2}\pi\alpha_S(M_V)}{M_V^2}\phi^*(0)G(t,\xi)(\varepsilon_V^*\cdot\varepsilon)$$

Guo-Ji-Liu 2021;
See also
Hatta-Strikman 2021

GPDs

$$G(t,\xi)=\frac{1}{2\xi}\int_{-1}^{1}\mathrm{d}x\mathcal{A}(x,\xi)F_g(x,\xi,t)$$

$$\mathcal{A}(x,\xi)\equiv\frac{1}{x+\xi-i0}-\frac{1}{x-\xi+i0}$$

- Guo-Ji-Liu 2021 argue that this can also apply near the threshold
  - Very strong argument
- Large-t gluon GPD can be calculated in perturbative QCD
  - Ji-Hoodbhoy-Yuan 2004, for quark GPDs
  - Consistency is checked

Earlier references in GPD formalism:
Hoodbhoy 1996; Koempel-Kroll-Metz-Zhou 2012 and references therein

Taylor expansion of the hard coefficient

$$G(t,\xi) = \sum_{n=0}^{\infty} \frac{1}{\xi^{2n+2}} \int_{-1}^{1} dx x^{2n} F_g(x,\xi,t)$$

- Only the leading term corresponds to the gluonic gravitational form factors

$$G(t,\xi) = \frac{1}{\xi^2(\bar{P}+)^2} \langle P'| \frac{1}{2} \sum_{a,i} F^{a,+i}(0)\, F^{a,+}{}_{i}(0)\, |P\rangle$$

$$= \frac{1}{2\xi^2(\bar{P}+)^2} \langle P' | T_g^{++} | P \rangle \ ,$$

Boussarie-Hatta 2020;
Hatta-Strikman 2021;
Guo-Ji-Liu 2021;

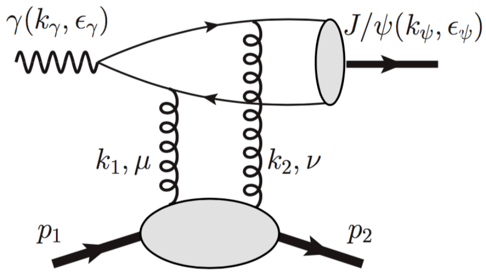
What does this approximation mean?

- The leading term is equivalent to: no x-dependence in the hard part

$$\mathcal{A}(x,\xi) \equiv \frac{1}{\not{x} + \xi - i0} - \frac{1}{\not{x} - \xi + i0}$$

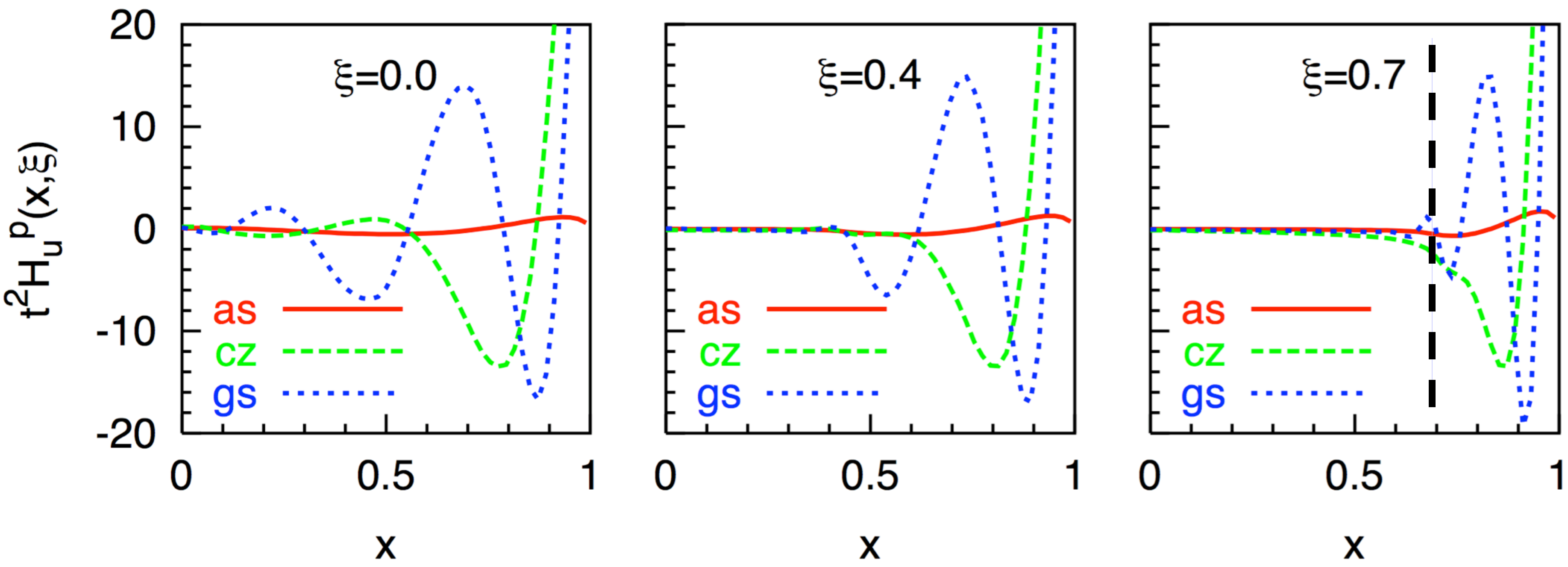
- Which means that same momentum for the two gluons

$$\mathcal{A} \propto \frac{k_\gamma^\alpha k_\gamma^\beta}{\langle k_1 \cdot k_\gamma k_2 \cdot k_\gamma \rangle} \langle T_g^{\alpha\beta} \rangle$$



- If the GPD gluon distributions have power behaviors $(\xi^2-x^2)^2$, this approximation is not that bad
  - Guo-Ji-Liu 2021, Hatta-Strikman 2021, ~20% corrections
  - Power behavior is supported by evolution at asymptotic scale (see, review by Markus Diehl and references therein)
- However
  - There may not be a simple Taylor expansion at high order perturbative QCD
  - At low/moderate scale, a power behavior may not be manifest for the GPD gluon distributions
    - Cross section contribution has divergence around $\xi=x$, the Taylor expansion will be completely wrong

An example



Ji-Hoodbhoy-Yuan PRL2004

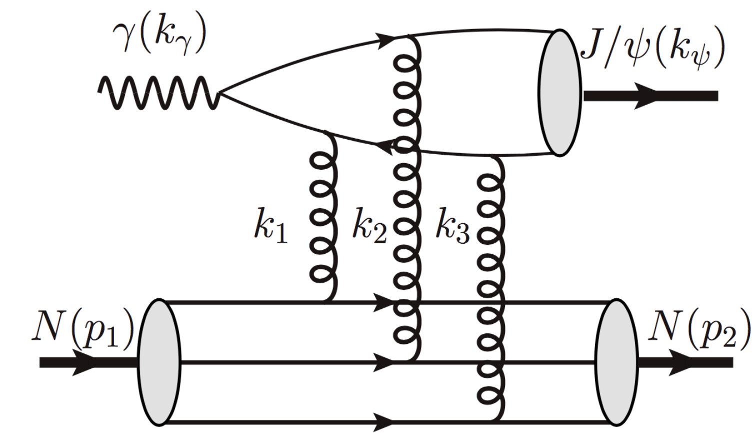
Summary

- There is no direct connection between the near threshold photoproduction of heavy quarkonium and the gluonic gravitational form factors
  - All previous results/claims should be re-evaluated
- Looking forward: phenomenological study in terms of the gluon GPDs is greatly needed

Back-ups

Vanishing of three-gluon exchange

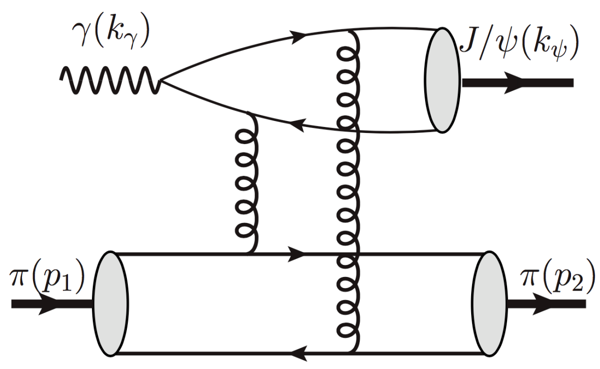
- Suggested by Brodsky et al, 2001, and widely accepted by exp. and claimed that
  - Two-gluon exchange suppressed by $(1-x)^2$, where three-gluon dominates at threshold
- Due to C-parity conservation, there is no contribution from the three-gluon exchange



$$\epsilon^{ijk}\epsilon^{lmn}T^a_{il}T^b_{jm}T^c_{kn} \propto d^{abc}$$

Pion: illustrate all major points

$$\mathcal{A}^\pi = \int dx_1 dy_1 \phi^*(y_1)\phi(x_1)\mathcal{M}_\psi^{\mu\nu}(\epsilon_\gamma, \epsilon_\psi, x_1, y_1) \times \frac{1}{k_1^2}\frac{1}{k_2^2}\mathrm{Tr}\left[\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\right] ,$$



- Take the threshold and heavy quark mass limits

$$|\overline{\mathcal{A}^\pi}|^2 = G_\psi G_\pi(t) G_\pi^*(t)$$

$$G_\psi = C_N^2 \frac{32\pi^2\alpha e_c^2(4\pi\alpha_s)^6}{3M_\psi^3}\langle 0|\mathcal{O}(^3S_1^{(1)})|0\rangle$$

$$G_\pi(t) = \frac{1}{(-t)}\int dx_1 dy_1 \phi^*(y_1)\phi(x_1)\frac{1}{x_1\bar{x}_1 y_1\bar{y}_1}$$