Black hole Inspirals and Quarkonia: Field Theoretic Cousins

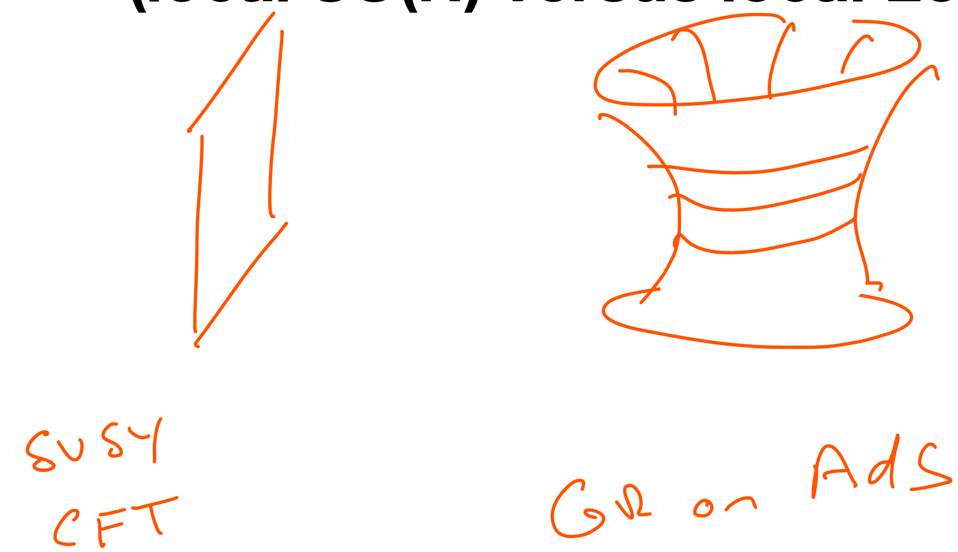
Ira Rothstein (CMU) in Collaboration with Walter Goldberger

Physics Opportunities with Quarkonia at the EIC 10/25/21

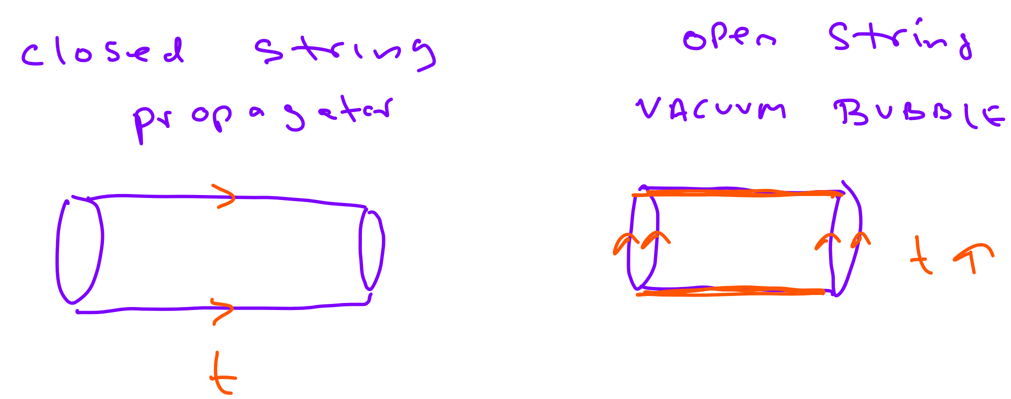
Connections between QCD (YM) and GR

Qualitatively: Both are describable as gauge theories (local SU(N) versus local Lorentz).

Ads-CFT



quantitatively : **KLT-Relations**



BCJ relations and color-kinematic duality: $c \to n^2$ **"double copy"**

Classical double copy: $$h_{\mu\nu} = \phi(t,x)k_\mu k_\nu \leftrightarrow A^a_\mu = c^a\phi(t,x)k_\mu$$

(static) **(Monteiro, O'connell and White)**

Crucial Distinctions:

-Gravitational corrections are controlled by a dimensional parameter:

$$G_N \equiv \frac{1}{M_{pl}^2} \qquad \textbf{vs.} \qquad \alpha_s$$

-The gravitational charge is a continuous parameters of the theory:

$$T_{\mu\nu} \qquad \textbf{vs.} \qquad \alpha_s C_{F,A}$$

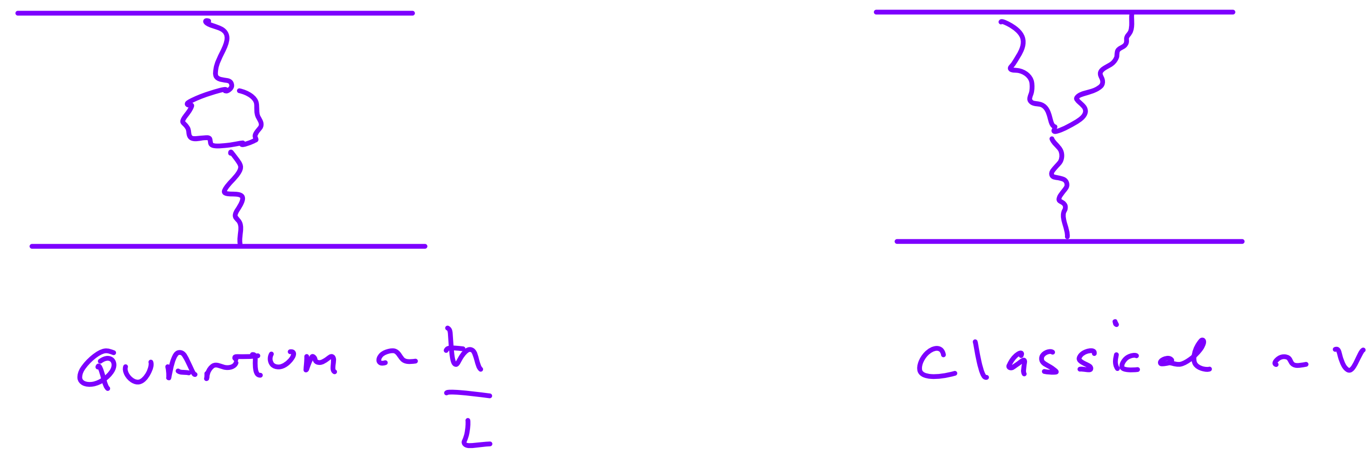
Ramifications: (for potentials)

For GR there are two classes of non-linearities: quantum and classical and they're scaling could not be more distinct.

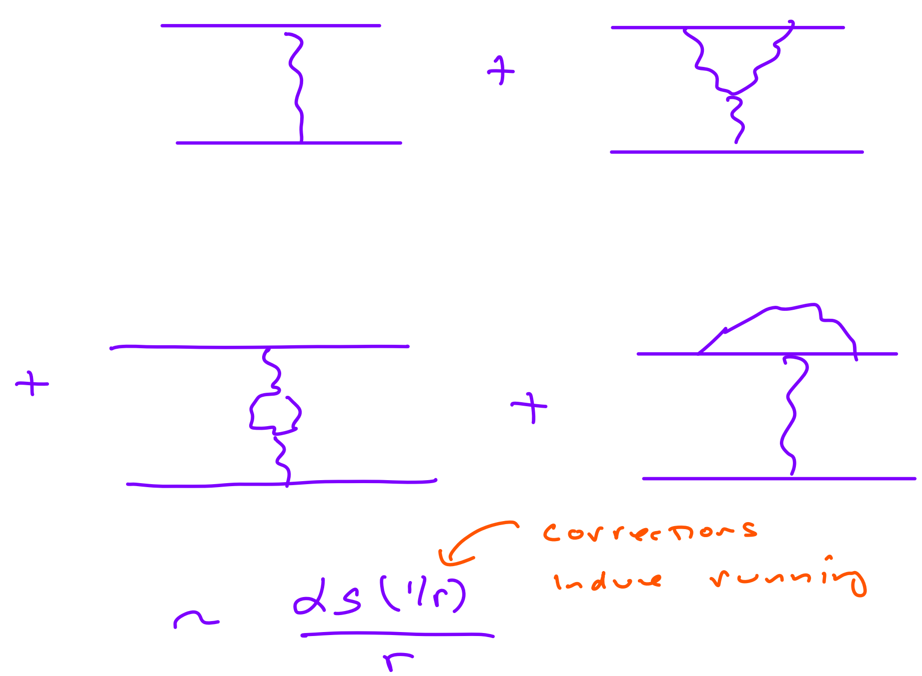
Quantum non-linearities are down by $\hbar/L^{-1} \sim \hbar/rmv$

Classical non-linearities are down by v

Topologically easy to distinguish



For QCD the mass plays no roll for classical sources, its conformally invariant all corrections are quantum mechanical



	Onia	Binary
Short distances	Weak Coupling Coulomb Phase	Strong coupling
Long Distances	Confinement	Minkowski Space
Non-Linearities	$\alpha_s \sim v$	Controlled by v^2
Quantum Effects	Controlled by $\alpha_s \sim v$	Controlled by $(M_{pl} r)^2 \sim \hbar / L$

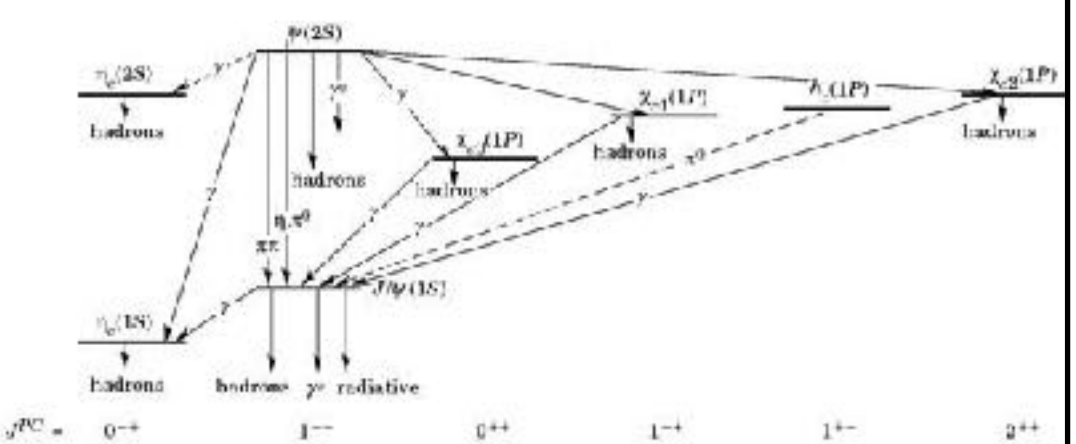
Allows for Strong Classical gravity

Physical Observables:

Onia

Decay Products

Extract: Spectrum



$m \qquad \alpha_s(mv)$

$\langle O \rangle$ Condensates

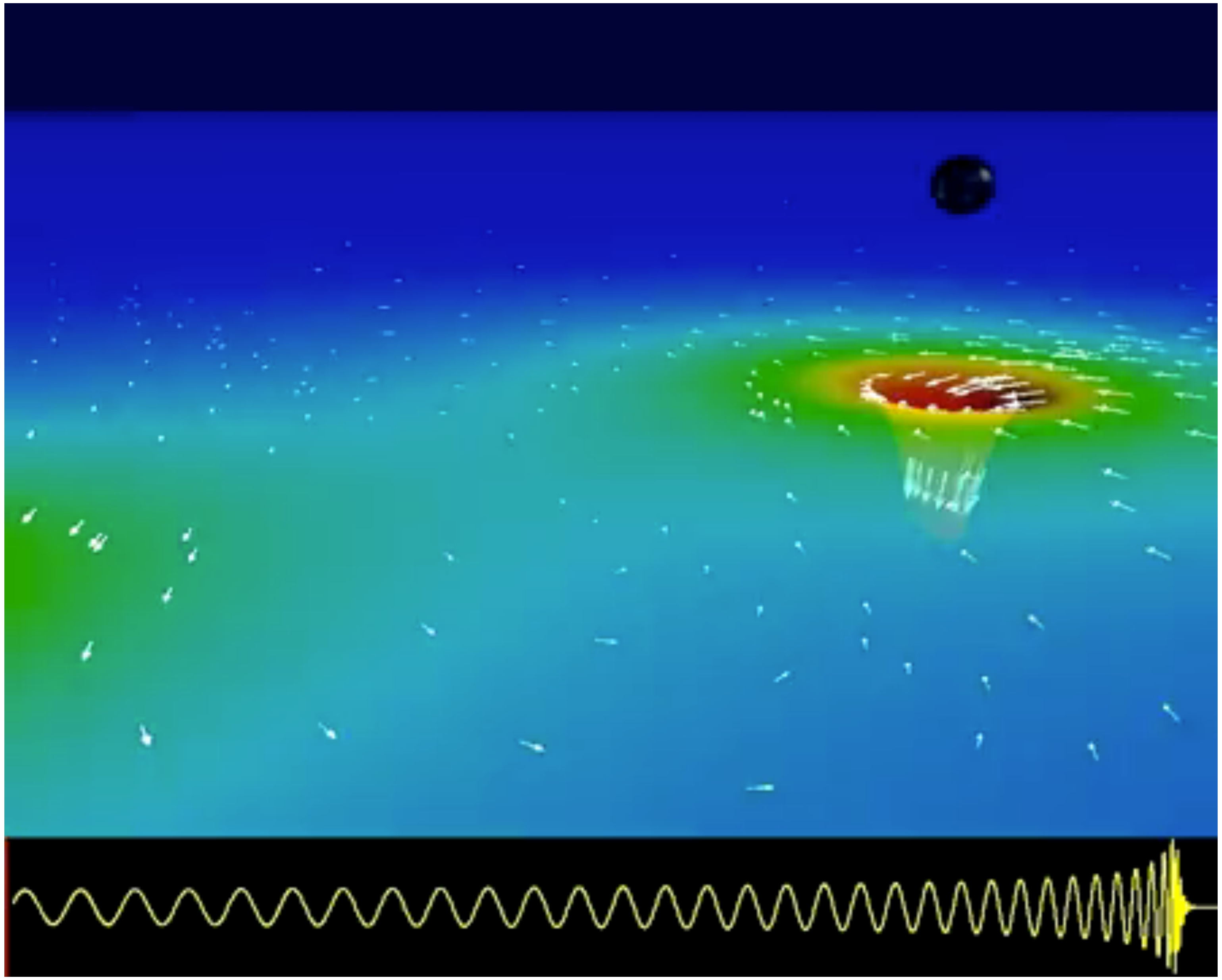
Binaries

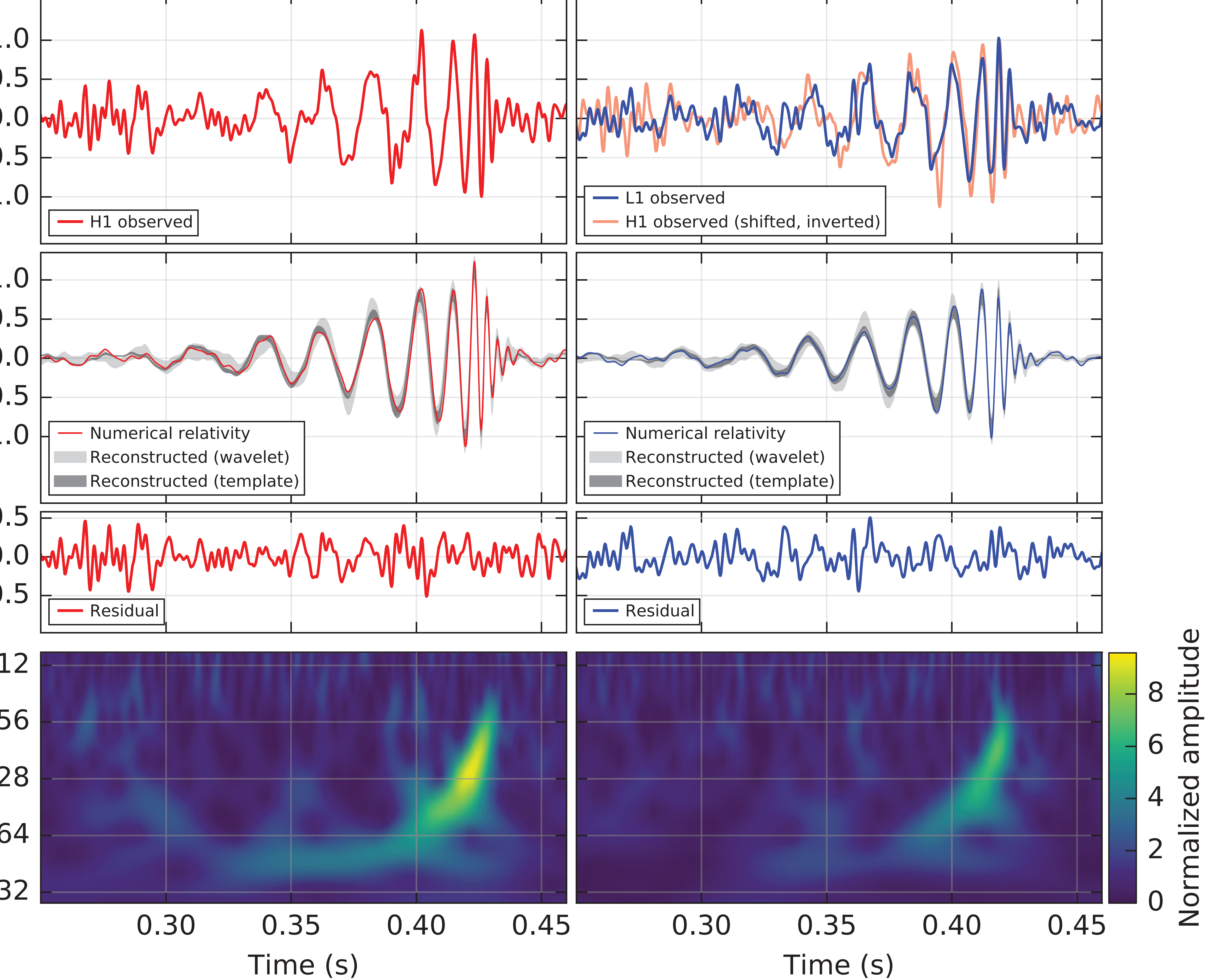
LIGO strain

$$h_{\mu\nu}(r \to \infty)$$

Phase and Amplitude contains all information

$$m_i, S_i, L_i^A$$




H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Residual
0.30
0.35
0.40
0.45
Time (s)
0.30
0.35
0.40
0.45
Time (s)
Normalized amplitude
0
2
4
6
8

For smaller mass binaries we can study the signal over hundreds of cycles (hours) where the velocity is small and study the details of the signal.

Early stages of the inspiral as well approximated by the ``Post-Newtonian'' expansion, which is an expansion in relative velocity.

$$g_{\nu\nu} = \eta_{\mu\nu} + \delta^0_\mu \delta^0_\nu [(GM/r^2) \sim v^2] + O(v^4) + \ldots.$$

For small velocities non-linearities are under systematic control analytically, but this is just like quarkonia (in some important ways).

Build an analog to NRQCD
NRGR (Non-Relativistic General Relativity)

(W. Goldberger/IZR)

Key Distinctions:

- Classical Sources. No recoil from individual potential graviton exchange.
- No Soft Fields (in NRQCD responsible for quantum effects)
- Theory has UV cut-off (when constituents overlap)
- Source are not fundamental (i.e. have internal structure)
- Necessitates extra stage of matching relative to NRQCD

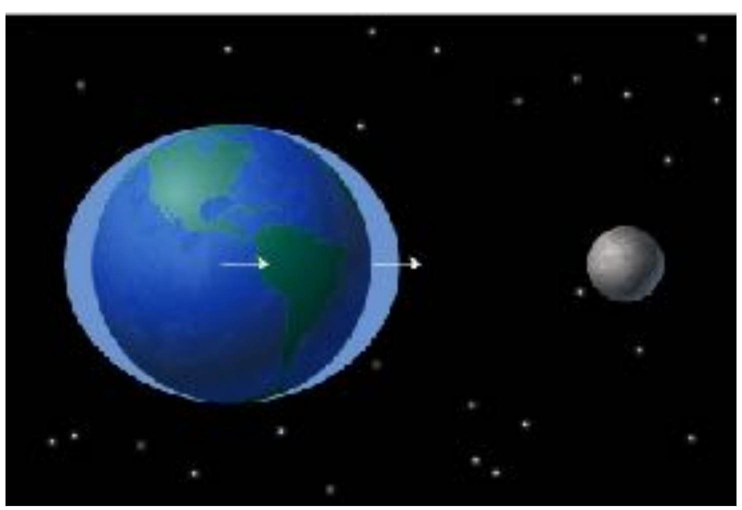
- So the leading order non-geodesic flow is due to

$$L_{size} = \int (C_e E^2 + C_b B^2) d\tau$$

$$C_{e,b} \propto r^2$$

Wilson coefficients calculable via calculation of

C_e, C_b Tidal ``Love numbers", static susceptability



Determined by matching calculations scales as (v^10) (5PN) ``Effacement Theorem" (Damour)

All static susceptabilities vanish for BH's!!

(Generalization of the no hair theorem)

This does not mean that the system does not deform

This seems to be a severe fine-tuning (classical)

$$ds^2_{\text{near-zone}} = -\frac{\Delta}{r_s^2}\mathrm{d}t^2 + \frac{r_s^2}{\Delta}\mathrm{d}r^2 + r_s^2\mathrm{d}\Omega^2$$

(Hui, Joyce, Penco, Santoni, Solomon)

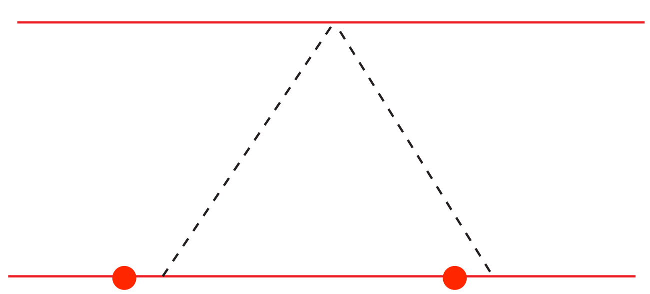
Extra killing vectors, (only shown for scalar perturbations at this point) explains absence of Wilson coefficient.

Next stage of matching

$$k_{pot} \sim (v/r, 1/r) \qquad k_{rad} \sim (v/r, v/r)$$

No soft mode and no analog of quark potential mode. In addition we have another power counting parameter relative to NRQCD, 1/L.

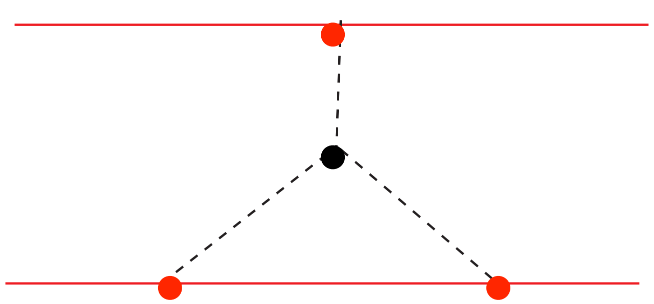
(a)



$$\sim \left[dt_1 \left(\frac{m_1}{m_{Pl}^2}\right) H_{00}^2(1)\right] \left[dt_2 \left(\frac{m_2}{m_{Pl}}\right) H_{00}(2)\right]^2$$

$$\sim \frac{m^3}{m_{Pl}^4}(dt)^3 H_{00}^4 \sim m(v^2 r)^2 \left(\frac{r}{v}\right)^3 \frac{v^2}{r^4} \sim Lv^2$$

(b)



$$\sim \left[dt \left(\frac{m}{m_{Pl}}\right) H_{00}\right]^3 \left[dx^0 d^3\mathbf{x}\, \partial_i{}^2 \frac{H^3}{m_{Pl}}\right]$$

$$\sim \frac{m^3}{m_{Pl}^4}(dt)^4 d^3\mathbf{x}\, \partial_i{}^2 H^6 \sim m(v^2 r)^2 \left(\frac{r}{v}\right)^4 r \frac{v^3}{r^6} \sim Lv^2$$

$$(a) = \frac{i}{2}\int dt \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{|\mathbf{x}_1 - \mathbf{x}_2|^2} \qquad (b) = -2(a)$$

$$
\begin{aligned}
\mathcal{L}^{(4PN)}_{G^3} = \frac{G^3 m_1^3 m_2}{r^3} \Big[& v_2.a_1 \left(\frac{3763}{240} v_2^r - \frac{18719}{720} v_1^r\right) r + a_1^r \left(-\frac{18719}{1440} v_1^2 - \frac{95119}{7200} v^2 + \frac{1309}{48} v_2^{r2} - \frac{75}{4} v_1^r v_2^r\right) \\
& + \frac{3763}{480} r a_2^r v_1^2 - \frac{231}{160} v_1^4 + \frac{1397}{480} v_1^2 v_2^2 - \frac{433}{60} v_1^2 v_1.v_2 + \frac{43}{2} v_1.v_2 v.v_2 + \frac{91}{16} v_2^4 \\
& + v_1^2 \left(\frac{15349}{480} v_1^{r2} - \frac{4381}{60} v_1^r v_2^r + \frac{16729}{480} v_2^{r2}\right) + v_1.v_2 \left(7 v_1^r v_2^r + \frac{43}{16} v_1^{r2} - 2 v_2^{r2}\right) \\
& + v_2^2 \left(\frac{7}{4} v_2^{r2} - \frac{1}{8} v_1^{r2} - \frac{7}{2} v_1^r v_2^r\right) + \left(\frac{43}{6} v_1^{r2} - \frac{119}{48} v_1^r v_2^r - \frac{15}{4} v_2^{r2}\right) v_1^{r2} \Big] \\
& + \frac{G^3 m_1^2 m_2^2}{r^3} \Big\{ a_1^r \left[\left(\frac{349207}{7200} - \frac{43}{128}\pi^2\right) v^2 + \left(\frac{123\pi^2}{128} - \frac{2005}{96}\right) v_2^{r2} \right] r \\
& + r (2 v_1^r v_2.a_1 + a_1^r v_1^2) \left(\frac{1099}{288} - \frac{41\pi^2}{128}\right) + \frac{383}{192} v_1^4 + \left(\frac{21427}{480} + \frac{133\pi^2}{1024}\right) (v_1^2 v^2 - 2 v_1.v_2 v.v_1) \\
& - \frac{55}{24} v_1^2 v_1.v_2 + v_1.v_2 \left(\frac{31687}{150} - \frac{447\pi^2}{256}\right) v^r v_1^r - 4 v_1.v_2 v_1^r v_2^r \\
& + v_1^2 \left[\frac{260921}{1200} v_1^r v_2^r - \frac{265721}{2400} v_1^{r2} - \frac{62399}{600} v_2^{r2} + \frac{447\pi^2}{512} v^{r2}\right] \\
& + \left[\frac{155563}{2880} v_1^{r2} - \frac{155593}{720} v_1^r v_2^r + \frac{78719}{480} v_2^{r2} - \frac{2155\pi^2}{1024} \left(v_1^{r2} - 4 v_1^r v_2^r + 3 v_2^{r2}\right)\right] v_1^{r2} \Big\} \\
& + \log \bar{r} \Big\{ \frac{G^3 m_1^3 m_2}{r^3} \Big[v_2.a_1 \left(28 v_1^r - \frac{62}{3} v_2^r\right) r + r a_1^r \left(14 v_1^2 + \frac{338}{15} v^2 - 25 v_2^{r2} + 28 v_1^r v_2^r\right) \\
& - \frac{31}{3} v_1^2 \left(r a_2^r - v^2\right) + v_1^2 \left(14 v_1^{r2} - 31 v^{r2}\right) - 14 v_1.v_2 v_1^{r2} - \frac{70}{3} v_1^{r3} v^r \Big] \\
& + \frac{G^3 m_1^2 m_2^2}{r^3} \left[\frac{34}{15} r a_1^r v^2 + 4 (v_1^2 v^2 - 2 v_1.v_2 v.v_1) - \frac{62}{5} (v_1^2 v^r - 2 v_1.v_2 v_1^r) v^r + \frac{2}{3} v_1^{r2} \left(v_1^{r2} - 4 v_1^r v_2^r + 3 v_2^{r2}\right) \right] \Big\} .
\end{aligned}
$$

(2

$$
\begin{aligned}
\mathcal{L}^{(4PN)}_{G^4} = \frac{G^4 m_1^4 m_2}{r^4} & \left[-\frac{177749}{3600} v_1^2 + \frac{175049}{3600} v_1.v_2 + \frac{15}{16} v_2^2 + \frac{242309}{900} v_1^{r2} - \frac{243659}{900} v_1^r v_2^r + \frac{9}{4} v_2^{r2} \right] \\
+ \frac{G^4 m_1^3 m_2^2}{r^4} & \Big[-\left(\frac{91369}{7200} + \frac{15}{32}\pi^2\right) v_1^2 + \left(\frac{103}{16}\pi^2 - \frac{18083}{240}\right) v_1.v_2 + \left(\frac{714259}{7200} - \frac{191}{32}\pi^2\right) v_2^2
\end{aligned}
$$

Potentials have been calculated to v^10 (Foffa and Sturani)

(Blanchet at al)

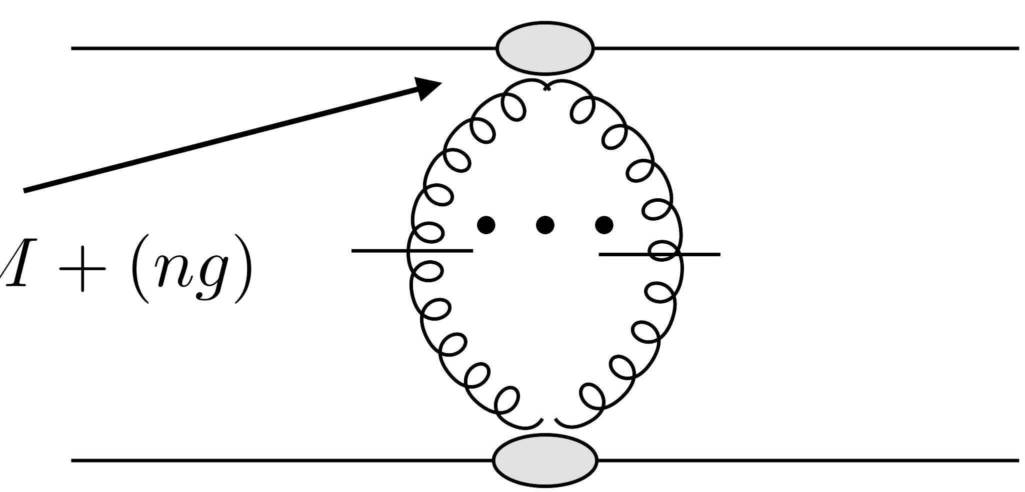
Number of diagrams grows faster than QCD, 500 at v^10

Instead use scattering amplitudes to extract potentials (Duff Neill/IZR)

on-shell

$$M + (ng) \to M + (ng)$$

(BCFW)



More importantly use double copy formalism to calculate in QCD

Including Spin Effects

Allow local frame basis to rotate (R Porto)

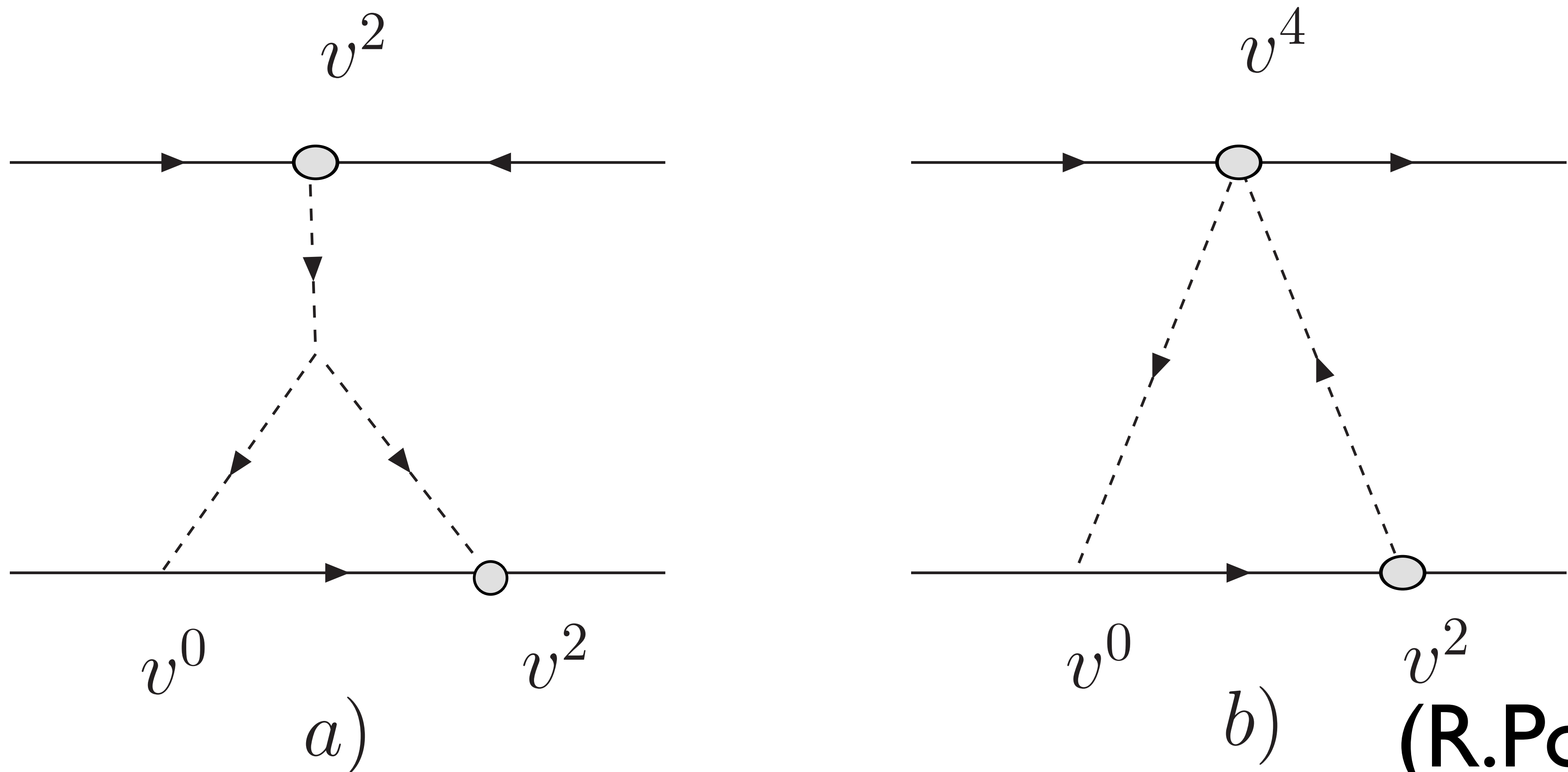
Introduce degrees of freedom which relates observer frame to co-rotating frame.

$e_I^\mu(\tau)$ Spin is the conjugate momentum.

Hamiltonian coupling spin to gravity is uniques fixed by symmetries

$$H \equiv H(S,h)$$

$$\mathrm{H} \quad = \frac{1}{4M_{pl}^2} S^{ij}\left(h^{ik}(-h^{k0,j} + h^{j0,k}) + h^{i0}h^{00,j}\right)$$



(R.Porto/IZR)

$$V_{3PN}^{spin} = \frac{-G_N}{2r^3}\left[\vec{S}_1\cdot\vec{S}_2\left(\frac{3}{2}\vec{v}_1\cdot\vec{v}_2 - 3\vec{v}_1\cdot n\vec{v}_2\cdot n - \left(\vec{v}_1^2+\vec{v}_2^2\right)\right) - \vec{S}_1\cdot\vec{v}_1\vec{S}_2\cdot\vec{v}_2 - \frac{3}{2}\vec{S}_1\cdot\vec{v}_2\vec{S}_2\cdot\vec{v}_1 + \vec{S}_1\cdot\vec{v}_2\vec{S}_2\cdot\vec{v}_2\right.$$
$$+\vec{S}_2\cdot\vec{v}_1\vec{S}_1\cdot\vec{v}_1 + 3\vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}\left(\vec{v}_1\cdot\vec{v}_2 + 5\vec{v}_1\cdot\vec{n}\vec{v}_2\cdot\vec{n}\right) + 3\vec{S}_1\cdot\vec{v}_1\vec{S}_2\cdot\vec{n}\vec{v}_2\cdot\vec{n} + 3\vec{S}_2\cdot\vec{v}_2\vec{S}_1\cdot\vec{n}\vec{v}_1\cdot\vec{n}$$
$$+3(\vec{v}_2\times\vec{S}_1)\cdot\vec{n}(\vec{v}_2\times\vec{S}_2)\cdot\vec{n} + 3(\vec{v}_1\times\vec{S}_1)\cdot\vec{n}(\vec{v}_1\times\vec{S}_2)\cdot\vec{n} - \frac{3}{2}(\vec{v}_1\times\vec{S}_1)\cdot\vec{n}(\vec{v}_2\times\vec{S}_2)\cdot\vec{n}$$
$$\left. - 6(\vec{v}_1\times\vec{S}_2)\cdot\vec{n}(\vec{v}_2\times\vec{S}_1)\cdot\vec{n}\right] + \frac{3G_N^2(m_1+m_2)}{r^4}\left(\vec{S}_1\cdot\vec{S}_2 - 3\vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}\right).$$

Final Stage of matching: Match onto Composite Object with multipole moments

E+M

$$S = \int dt\ \vec{p}(t) \cdot \vec{E}(t)$$

$$p_i(t) = \sum_A q_A x_i^A(t)$$

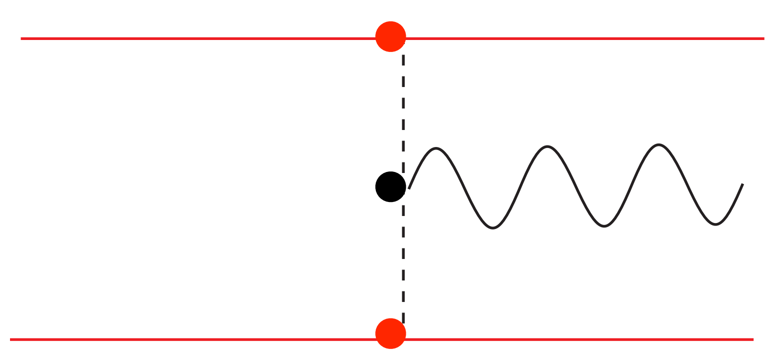
GR

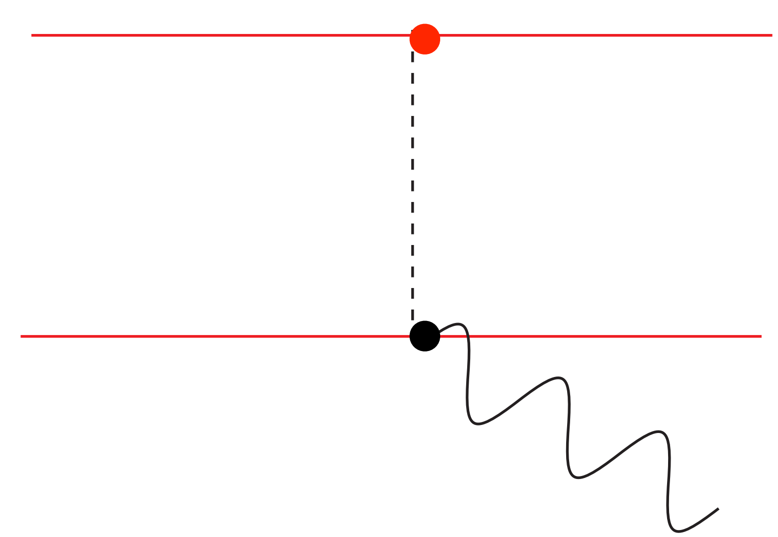
$$S = \int dt\ \vec{Q}_{ij}(t) \cdot \vec{E}_{ij}(t)$$

$$Q_{ij}(t) = \sum_A M_A (x_i^A(t) x_j^A(t) - \frac{\delta_{ij}}{3} \vec{x}^{A2})$$

Matching is achieved by multipole expand the full solution to extract moment

graviton acts as source in GR





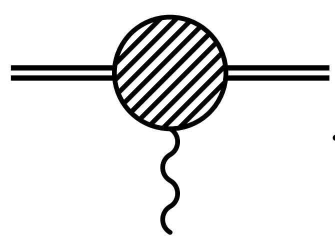
Radiation

One we have integrated out the potentials we match onto another point particle theory, endowed with moments of binary.

$$S = -\int M d\tau - \frac{1}{2}\int dx^\mu \omega_\mu^{ab} L_{ab} + \int d\tau \left(\frac{1}{2} Q_{ab} E^{ab} - \frac{4}{3} J^{ab} B_{ab} + \frac{1}{3} O^{abc} \nabla_c E_{ab} + \ldots ..\right)$$

source moments (worked out to all orders (Ross))

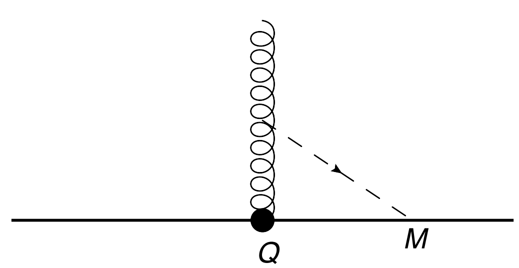
Power Loss can be calculated via in-out S matrix elements $A_h(k) =_{out} \langle \epsilon(k) \mid 0 \rangle_{in}$

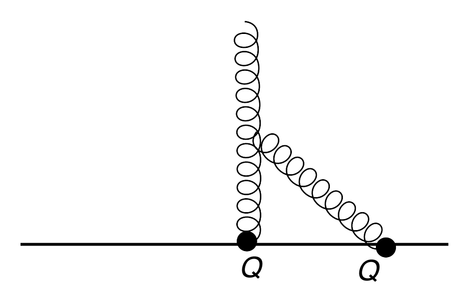
$i\mathcal{A}_h(\mathbf{k}) =$  .

$$d\Gamma_h(\mathbf{k}) = \frac{1}{T} \frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|} |\mathcal{A}_h(\mathbf{k})|^2$$

Free to use Feynman prescription for propagator poles

note that higher order effects involving calculation within this final theory: e.g. tail and memory effects





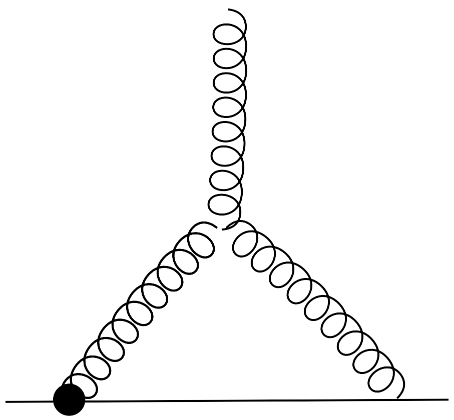
``radiative moments``

Renormalization of the Radiation Theory and Log Resummation

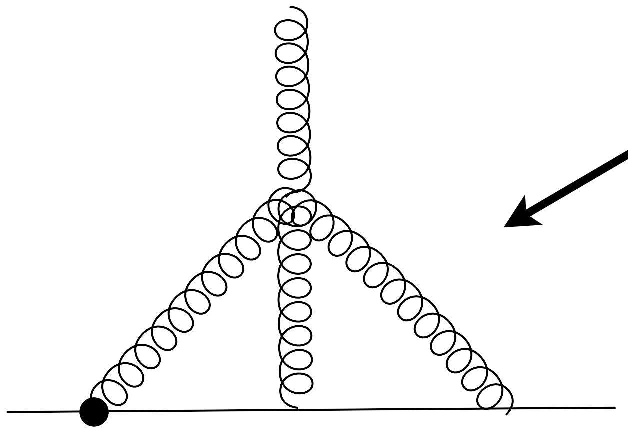
Quadrapole renormalization (Goldberger and Ross)

Quadrapole moments are scale dependent via

IR div. Coulomb phase (cancels in any physical observable)



UV div. physical log



$$\frac{1}{\epsilon_{UV}} + ALog(\omega^2/\mu^2)$$

+....

Divergence gets absorbed into renormalized quadrapole $Q^R_{ij} = Z^{-1}(\omega,\mu) Q^B_{ij}$

$$Z^{\bar{M}S} = 1 + \frac{107}{105}(Gm\omega)^2\left(\frac{1}{\epsilon_{UV}} + \gamma_E + Log(4\pi)\right)$$

$$\mu\frac{d}{d\mu}Q^B = 0$$

$$\mu\frac{d}{d\mu}Q^R = -\frac{214}{105}(Gm\omega)^2 Q^R$$

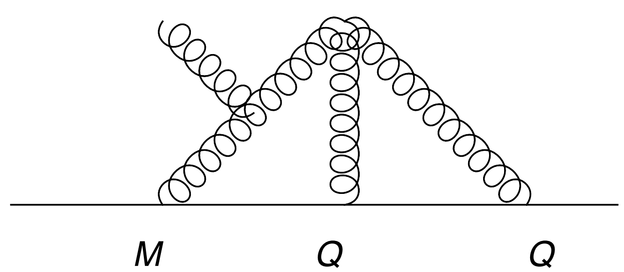
By Choosing $\mu = \omega$ we eliminate the logs in the amplitude.

$$Q^R(\omega,\mu) = (\mu/\mu_0)^{(-214/105(Gm\omega)^2)} Q(\omega,\mu_0)$$

Infinite sum of log enhanced terms $\sum_n C_n (Gm\omega)^{2n} Log^n(r\omega)$

$$-\frac{39201376}{3472875}(Gm\omega)^6 Log^3(\omega r) \sim v^{18}$$ checked in test mass limit (Fujita)

Mass Renormalization (Goldberger, Ross, IZR)



$\beta_Q = -214/105$

+...

Lagrangian mass parameter is asymptotically free

$$\mu \frac{d}{d\mu}\bar{m} = -2G^2 \langle Q_{ij}^{(3)} Q_{ij}^{(3)} \rangle$$ (Avg. of period)

$$\frac{\bar{m}(\mu)}{\bar{m}(\mu_0)} = exp\left[\frac{\langle Q_{ij}^{(2)} Q_{ij}^{(2)}\rangle_{\mu_0} - \langle Q_{ij}^{(2)} Q_{ij}^{(2)}\rangle_{\mu}}{\beta_Q \bar{m}_0^2}\right] = 1 - \frac{1}{2}\frac{\langle Q_{ij}^{(3)} Q_{ij}^{(3)}\rangle}{\bar{m}_0^2} r_s^2 Ln(v) + \frac{107}{420}\frac{\langle Q_{ij}^{(4)} Q_{ij}^{(4)}\rangle}{\bar{m}_0^2} r_s^4 Ln^2(v) - \frac{11449}{132300}\frac{\langle Q_{ij}^{(5)} Q_{ij}^{(5)}\rangle}{\bar{m}_0^2} r_s^6 Ln^3(v) + \ldots$$

$$E(\Omega) = -\frac{\mu}{2}\frac{448}{15}\nu x^5 \ln x + \ldots$$ Agrees with (Blanchet,Detweiler, Le Tiec and Whitting)

$$x = (G\bar{m}_0\Omega)^{2/3}$$

Additional Effects (necessitate IN-IN)

-Dissipation (tidal Heating)

-Radiation Reaction forces

-Hawking Radiation

The theory of binary inspirals has been informed by the physics quarkonia

EFT construction share much in common despite the gulf in the nature of the observables and the relevant of quantum mechanics. Perhaps this connection will bear more fruit, perhaps in the opposite direction?