Ivan Vitev

Effective theories of quarkonia in matter

Based on: ArXiv: 1906.04186, 1709.02372 , in preparation
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Physics Opportunities with Heavy Quarkonia at the EIC workshop, Oct. 25 – 27, 2021

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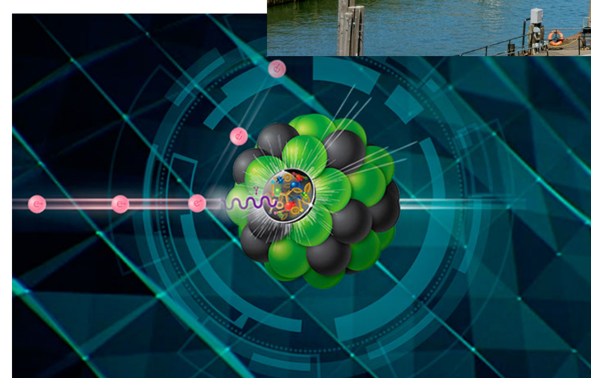
Outline of the talk

- A brief introduction to effective field theories (EFTs). A motivation to develop an EFT of quarkonia in matter.
- The high p_T limit of NRQCD and tension with energy loss phenomenology
- An effective theory of qaurkonia in matter – $NRQCD_G$. Derivation of the leading order and next to leading order $NRQCD_G$ Lagrangian using different methods
- Connection to quarkonium dissociation in matter and existing phenomenology
- Conclusions

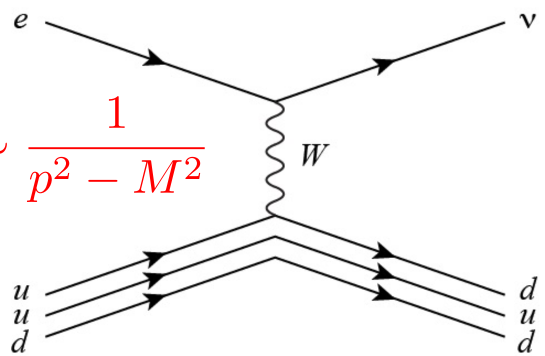


Talk intended to be given by Y. Makris who could not attend

Introduction

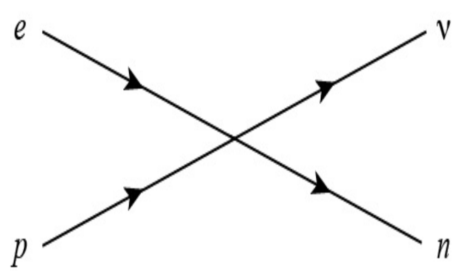
The Fermi interaction

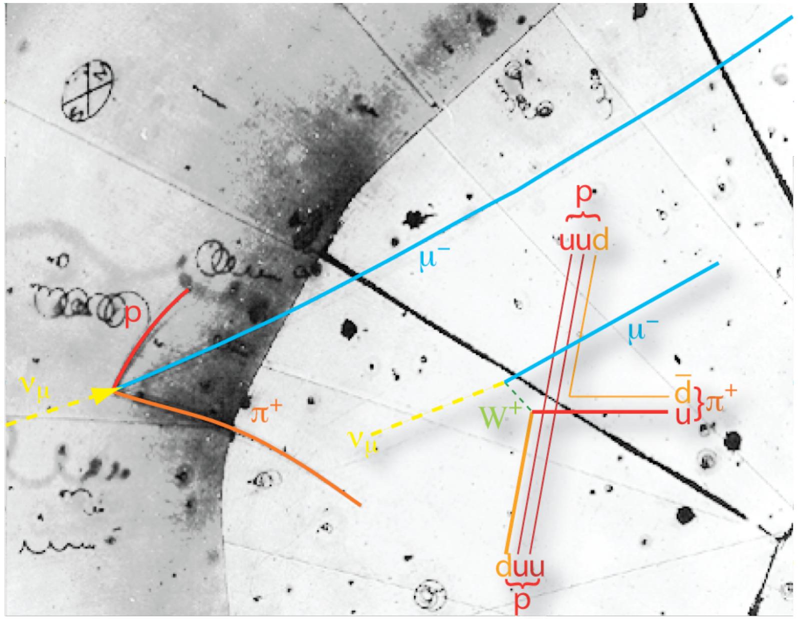
- The first, probably best known, effective theory is the Fermi interaction

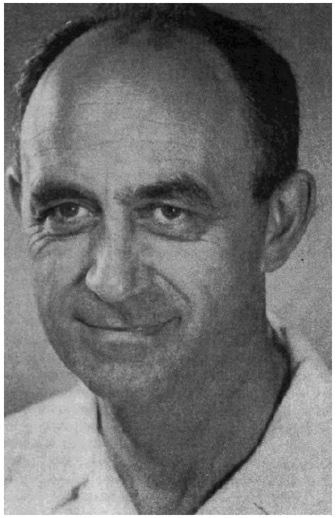


$$\sim \frac{1}{p^2 - M^2}$$

$$p \ll M$$



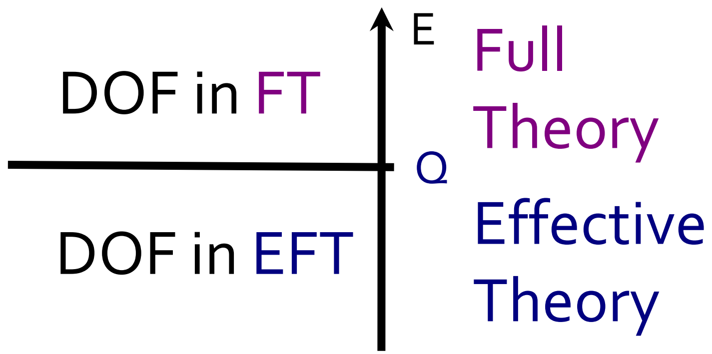




E. Fermi
(Nobel Prize)

- First direct observation of the neutrino, Nov. 1970

Examples of effective field theories [EFTs]

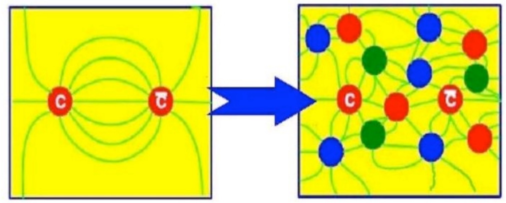


- Focus on the significant degrees of freedom [DOF]. Manifest power counting

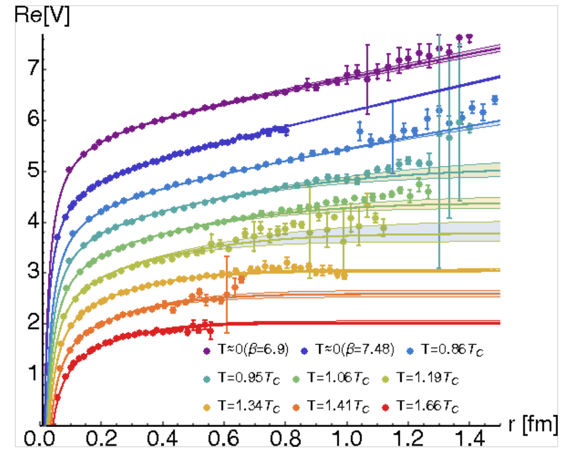
	Q	power counting	DOF in FT	DOF in EFT
Chiral Perturbation Theory (ChPT)	Λ_{QCD}	p/Λ_{QCD}	q, g	K,π
Heavy Quark Effective Theory (HQET)	m_b	Λ_{QCD}/m_b	ψ,A	h_v, A_s
Soft Collinear Effective Theory (SCET)	Q	$p_\perp/Q$	ψ,A	ξ_n, A_n, A_s
Non-Relativistic QCD (NRQCD)	m_Q	p/m_Q	ψ,A	ψ_Q, A_s, A_{us}

Quarkonia in the QGP

- Quarkonia (e.g. J/ψ,ϒ), bound states of the heaviest elementary particles, long considered standard candle to characterize QGP properties
- Most sensitive to the space-time temperature profile



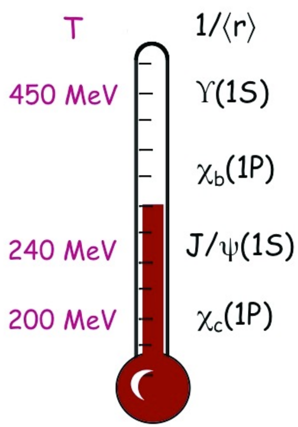
Matsui *et al.* (1986)



Rothkopf *et al.* (2016)

$$\left[-\frac{1}{2\mu_{\mathrm{red}}}\frac{\partial^2}{\partial r^2}+\frac{l(l+1)}{2\mu_{\mathrm{red}}r^2}+V(r)\right]rR_{nl}(r)=(E_{nl}-2m_Q)rR_{nl}(r)$$

$$\psi(\mathbf{r}) = Y_l^m(\hat{r})R_{nl}(r)$$



l	n	E_{nl} (GeV)	$\sqrt{\langle r^2\rangle}$ (GeV^{-1})	k^2 (GeV^2)	Meson
0	1	0.700	2.24	0.30	J/ψ
0	2	0.086	5.39	0.05	$\psi(2S)$
1	1	0.268	3.50	0.20	χ_c
0	1	1.122	1.23	0.99	$\Upsilon(1S)$
0	2	0.578	2.60	0.22	$\Upsilon(2S)$
0	3	0.214	3.89	0.10	$\Upsilon(3S)$
1	1	0.710	2.07	0.58	$\chi_b(1P)$
1	2	0.325	3.31	0.23	$\chi_b(2P)$
1	3	0.051	5.57	0.08	$\chi_b(3P)$

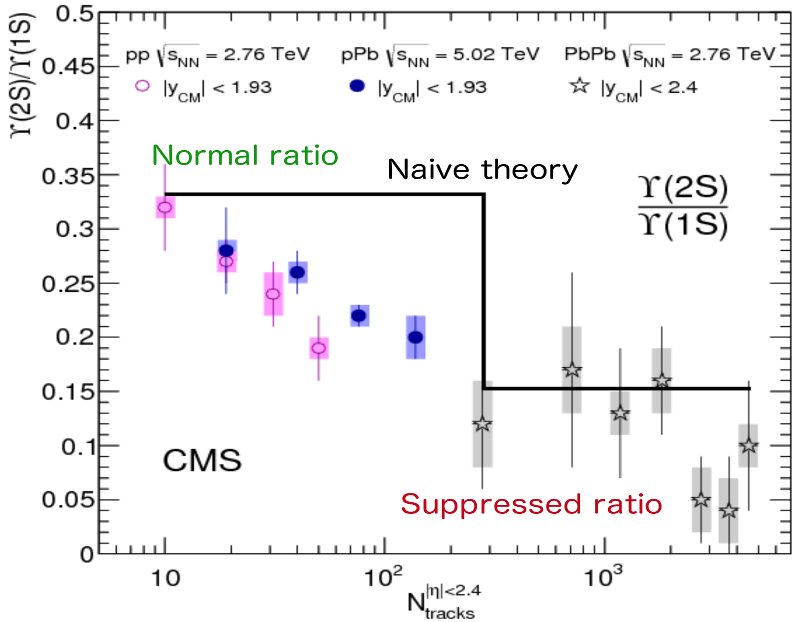
Mocsy *et al.* (2007)

Bazavov *et al.* (2013)

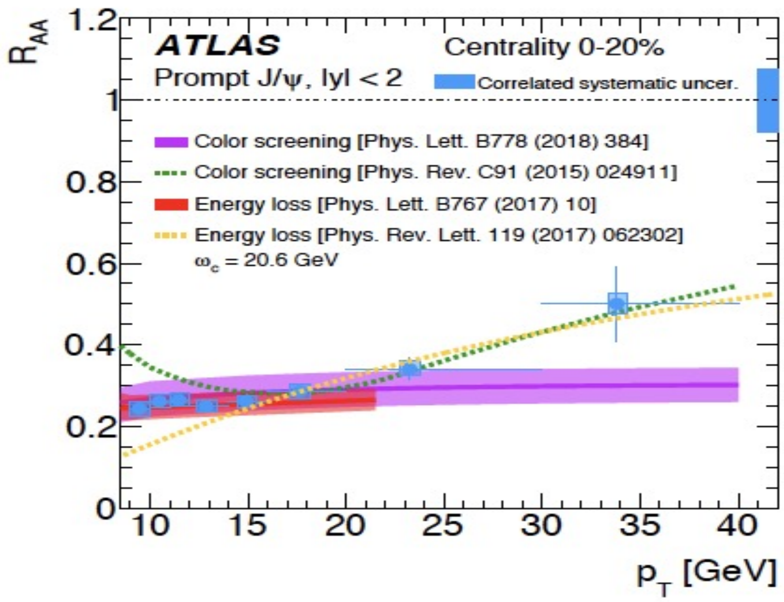
Challenges and hypothesae

- Suppression puzzle - similar dissociation behavior observed in small system, p+A and even in p+p (where QGP is not expected)
- Co-mover dissociation model, energy loss model – need cross check and microscopic explanation

E. Ferreiro (2014)

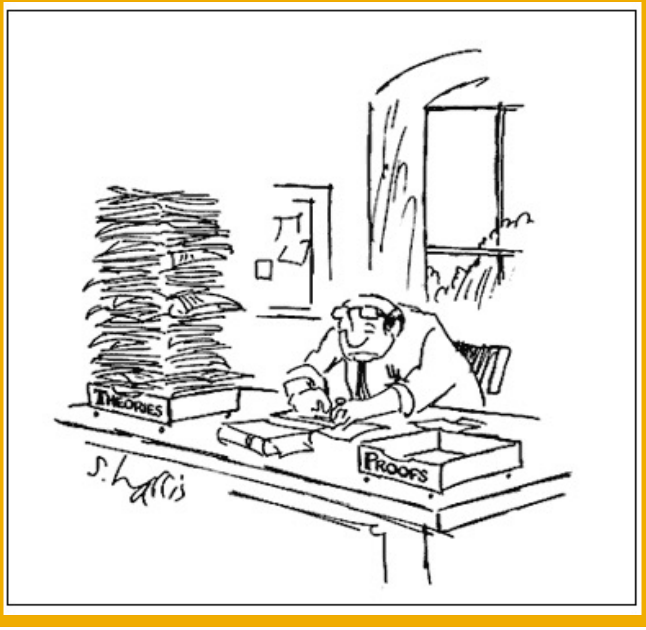


Chatrachyan *et al.* (2014)



- EFT - capture the interactions without explicitly specifying their nature

NQCD, Leading power factorization & E-loss

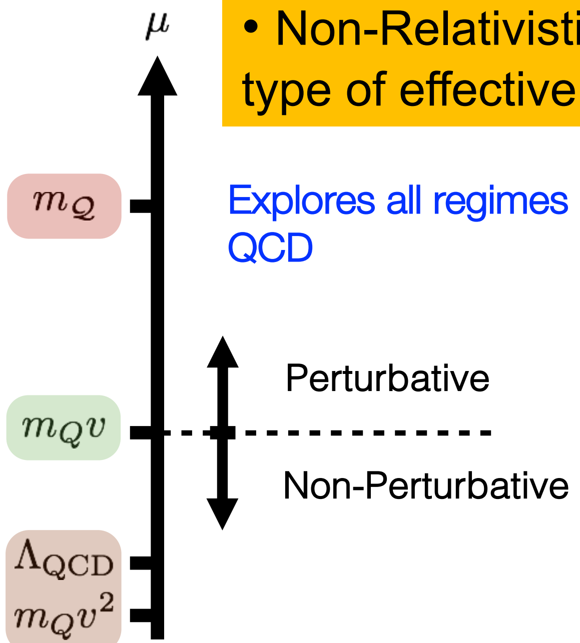


Production of quarkonia at intermediate and high p_T

- Non-Relativistic QCD (NRQCD) -a particular type of effective theory (EFT)

Bodwin *et al.* (1995)

Cho *et al.* (1996)



Explores all regimes of QCD

$b\bar{b}: \quad v^2 \sim 0.1$

$c\bar{c}: \quad v^2 \sim 0.3$

$p_s^\mu \sim m_Q v(1,1,1,1)$

$p_{us}^\mu \sim m_Q v^2(1,1,1,1)$

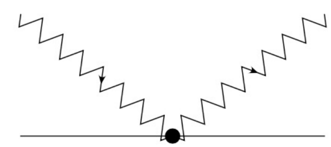
Ultra-soft

$$\mathcal{L}_{\rm NRQCD} = \mathcal{L}_{\rm light} + \psi^\dagger\left(iD_0 + \frac{\mathbf{D}^2}{2M}\right)\psi + \chi^\dagger\left(iD_0 - \frac{\mathbf{D}^2}{2M}\right)\chi$$

QCD without the heavy flavor

ultra-soft

+ heavy - soft interactions at NLO



typical momentum if heavy quark: $|\mathbf{p}_Q| \sim m_Q v$

typical kinetic energy if heavy quark: $K_Q \sim m_Q v^2$

- NRQCD factorization formula. Short distance cross sections (perturbatively calculable) and long distance matrix elements (fit to data, scaling relations)

$$d\sigma(a+b\to \mathcal{Q}+X) = \sum_n d\sigma(a+b\to Q\overline{Q}(n)+X)\langle \mathcal{O}_n^{\mathcal{Q}}\rangle$$

NRQCD example

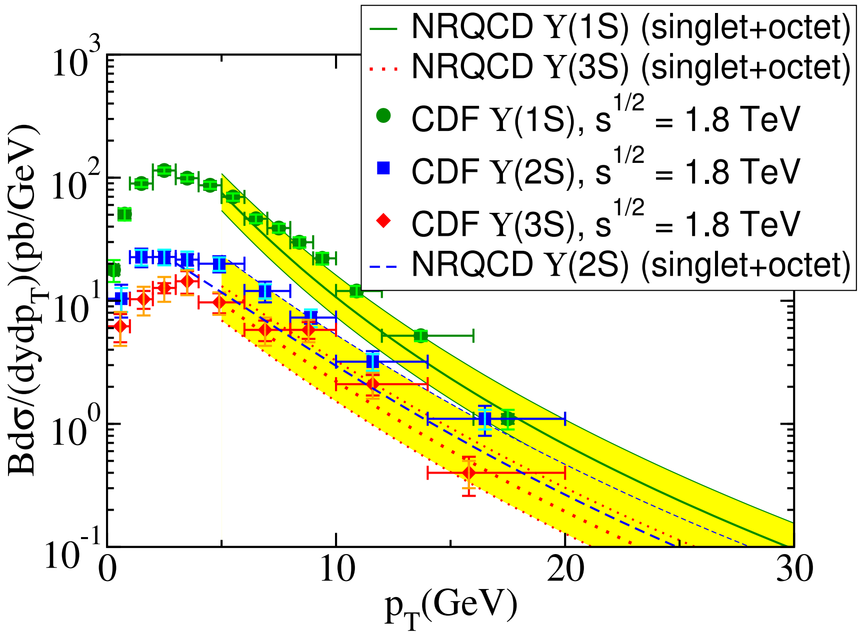
• One has to be careful, the simple power counting approximately manifest in the LDMEs can be affected by the partonic cross section – a large number of singlet and octet; S wave and P wave terms enter

$$d\sigma(J/\psi) = d\sigma(Q\bar{Q}([^3S_1]_1))\langle\mathcal{O}(Q\bar{Q}([^3S_1]_1) \to J/\psi)\rangle + d\sigma(Q\bar{Q}([^1S_0]_8))\langle\mathcal{O}(Q\bar{Q}([^1S_0]_8) \to J/\psi)\rangle$$
$$+ d\sigma(Q\bar{Q}([^3S_1]_8))\langle\mathcal{O}(Q\bar{Q}([^3S_1]_8) \to J/\psi)\rangle + d\sigma(Q\bar{Q}([^3P_0]_8))\langle\mathcal{O}(Q\bar{Q}([^3P_0]_8) \to J/\psi)\rangle$$
$$+ d\sigma(Q\bar{Q}([^3P_1]_8))\langle\mathcal{O}(Q\bar{Q}([^3P_1]_8) \to J/\psi)\rangle + d\sigma(Q\bar{Q}([^3P_2]_8))\langle\mathcal{O}(Q\bar{Q}([^3P_2]_8) \to J/\psi)\rangle + \cdots$$

• The situation is similar for bottomonia. Excited states have their own expansion

The question is – is there a simplification at high p_T where the p_T dependence of the short distance cross section dominates? Also large logarithms arise, spoil fixed order expansion and require resummation

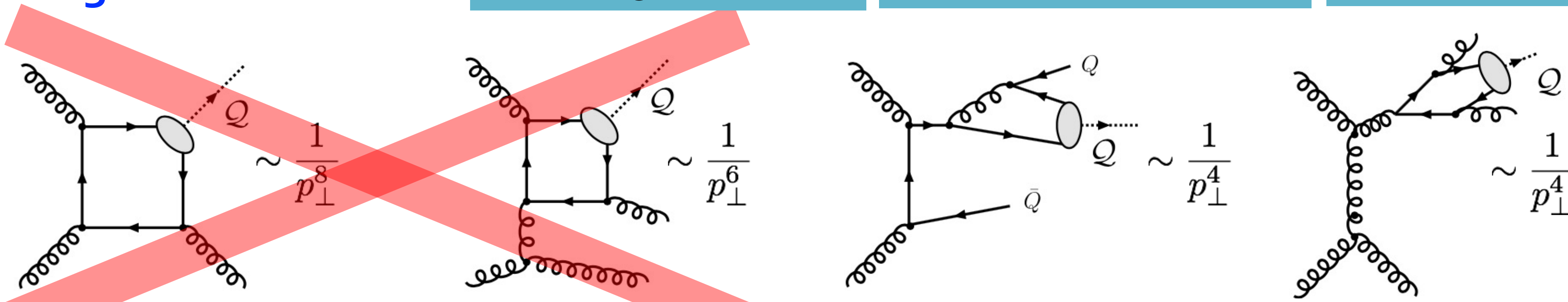
$$\alpha_s^m \ln^n(p_T/2m_Q)$$



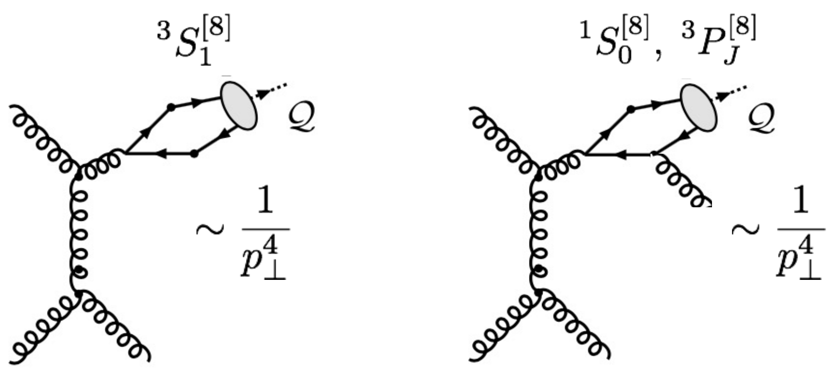
Leading power factorization

Singlet contribution

S. Fleming *et al.* (2012) | M. Baumgart *et al.* (2014) | Y. Ma *et al.* (2014)

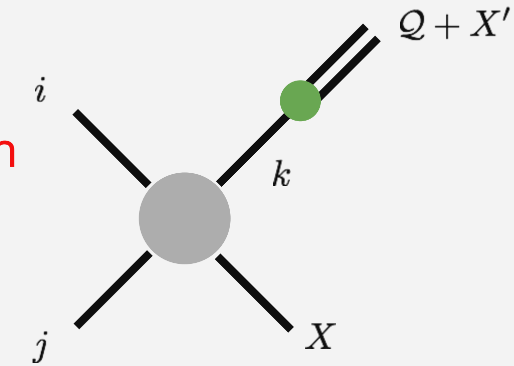


LO NLO NLO NNLO



LO NLO

(single) Parton fragmentation process



Octet contribution

Only a subset of contributions survive, now interpretable as parton fragmentation in quarkonia

LP example and applicability

$$\frac{d\sigma_h}{dp_\perp}(p_\perp) = \sum_i \int_z^1 \frac{dx}{x}\, \frac{d\sigma_i}{dp_\perp}\left(\frac{p_\perp}{x}, \mu\right) D_{i/h}(x,\mu) + \mathcal{O}\left(\frac{m_h^2}{p_\perp^2}\right)$$

$p_T \gg m_Q$

$$\ln\left(\frac{\mu}{p_T}\right) - \ln\left(\frac{\mu}{2m_Q}\right) \quad d_{i/n}(x,\mu)\langle \mathcal{O}_n^h \rangle$$

DGLAP Evolution

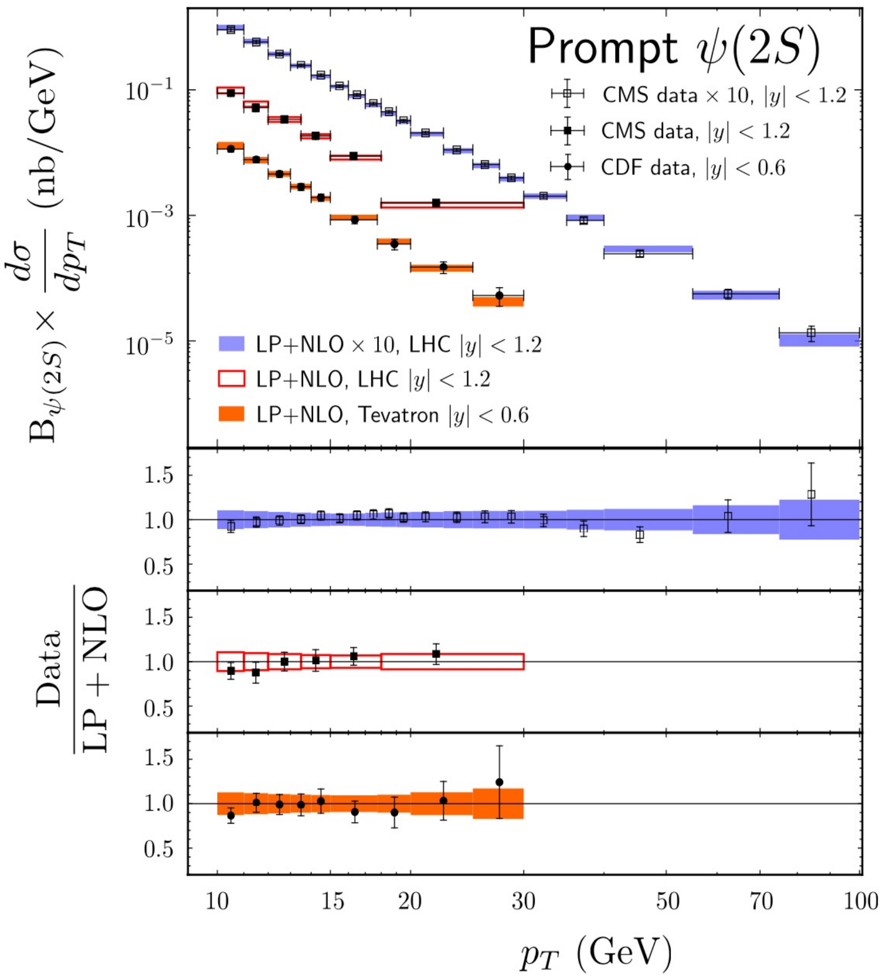
$$\mu \frac{d}{d\mu} D_{i/h}(z,\mu) = \sum_i \int_z^1 \frac{dx}{x}\, P_{ij}(x) D_{i/h}\left(\frac{z}{x}, \mu\right)$$

Resummation of $\ln(p_T/m_h)$

Contributions we take

χ_{cJ} ; $J/\psi(1S)/\psi(2S)$

Mechanism	Initiating parton	$^3P_J^{[1]}$	$^3S_1^{[8]}$	$^3P_J^{[8]}/^1S_0^{[8]}$	$^3S_1^{[1]}$
	g	α_s^2	α_s	α_s^2	α_s^3
	Q	α_s^2	α_s^2	α_s^3	α_s^2
	q	α_s^3	α_s^2	α_s^3	α_s^4



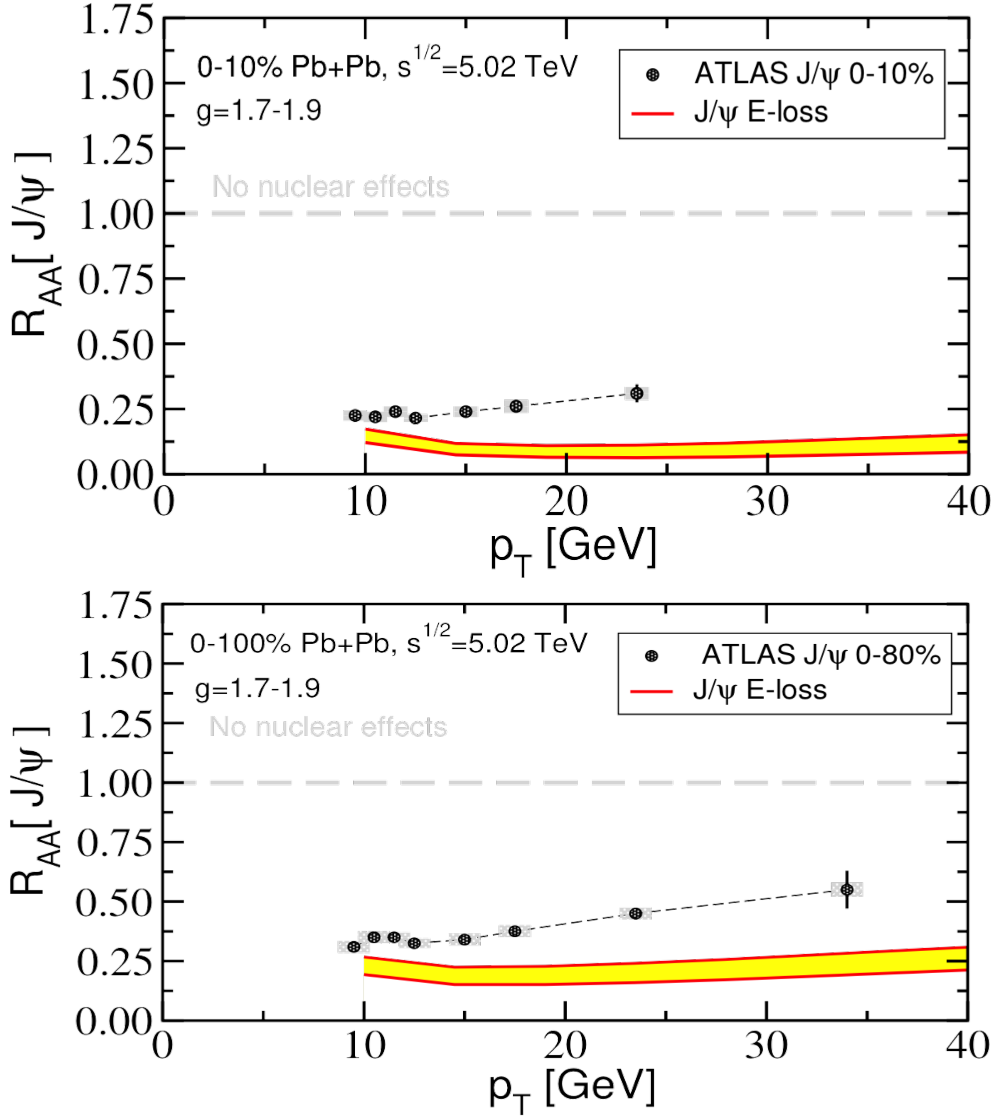
Resummation: G. Bodwin *et al. (2016)*

Comparison of energy loss phenomenology to data

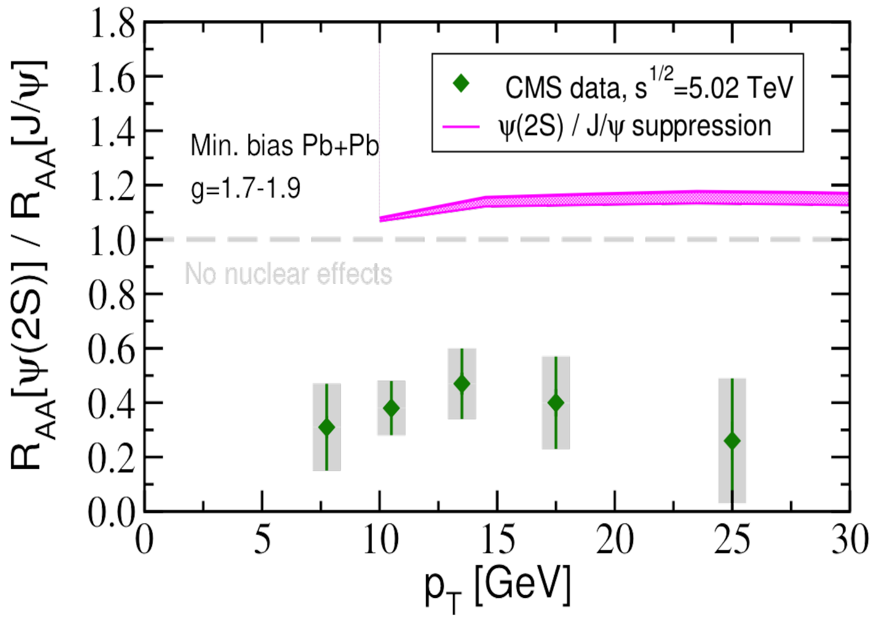
- Suppression of J/ψ overestimated by factor of 2 to 3. Included χ_c and ψ(2S) feeddown.
- Persists over centralities. Somewhat different p_T dependence
- Differences are significan

$$R_{AA}^{\text{min. bias}}(p_T) = \frac{\sum_i R_{AA}(\langle b_i \rangle) W_i}{\sum_i W_i}$$

$$W_i = \int_{b_i\,\text{min}}^{b_i\,\text{max}} N_{\text{coll.}}(b)\, \pi\, b\, db$$

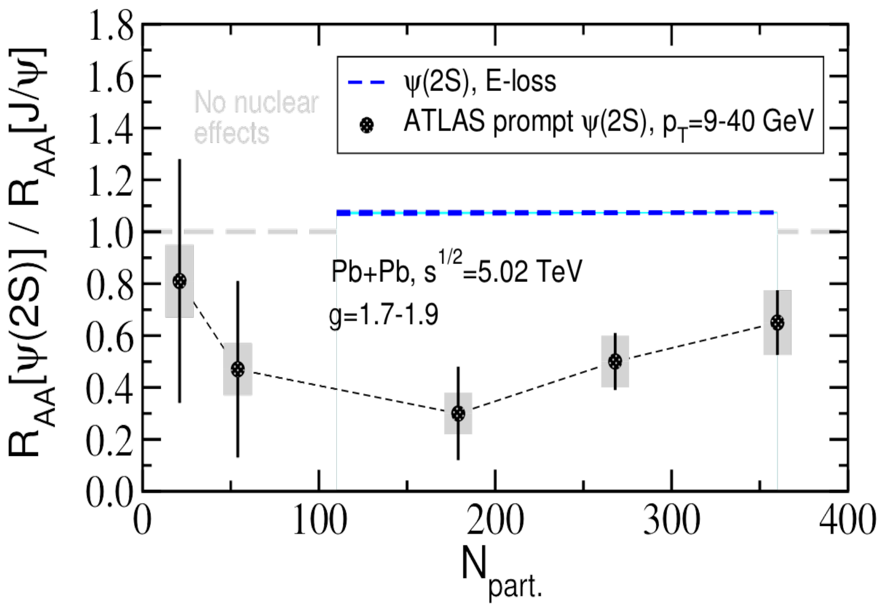


Double suppression ratio ψ(2S) / J/ψ



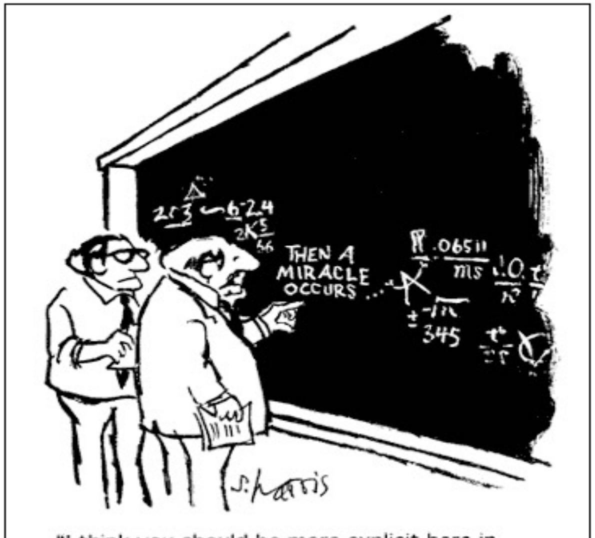
Y. Makris et al. (2019)

- In the double suppression ratio $R_{AA}(\psi(2S))/R_{AA}(J/\psi)$ the discrepancy is not simply in magnitude. There is a discrepancy in the sign of prediction



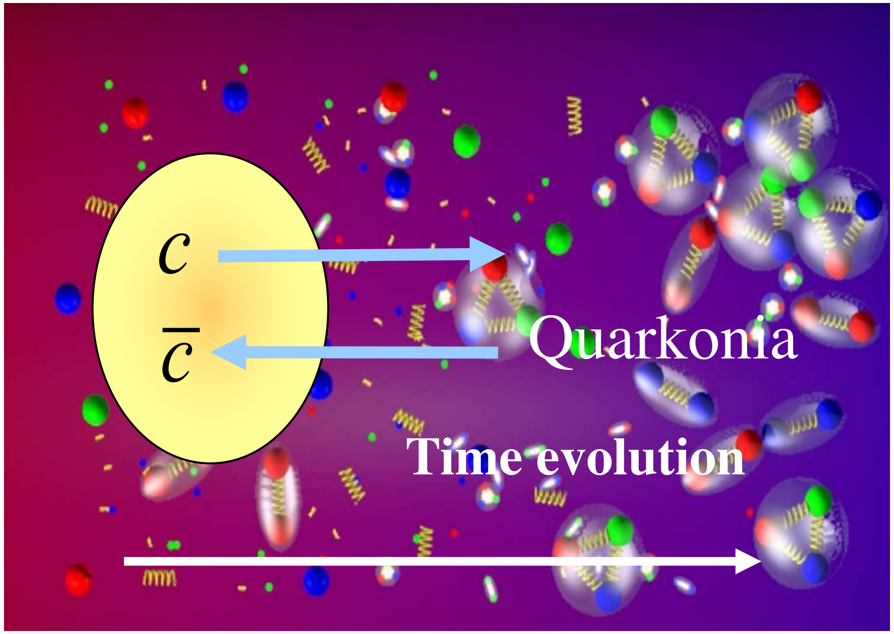
The energy loss picture of quarkonium suppression in the p_T range measured by ATLAS and CMS (up to 30 GeV) is strongly challenged as along as we see a hierarchy of suppression

NRQCD with Glauber Gluons & phenomenology



"I think you should be more explicit here in step two."

NRQCD in a background medium



- Take a closer look at the NRQCD Lagrangian below

M. Luke *et al.* (2000)

Scales in the problem

$p_s^\mu \sim m_Q v(1,1,1,1)$ soft ~ λ

$p_{us}^\mu \sim m_Q v^2(1,1,1,1)$ ultrasoft ~ λ²

- Ultrasoft gluons included in covariant derivatives

- Soft gluons are included explicitly

  - Double soft gluon emission
  - Heavy quark-antiquark potential
  - (can also be interaction with soft particles)

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_p \left|p^\mu A_p^\nu - p^\nu A_p^\mu\right|^2 + \sum_{\mathbf{p}} \psi_{\mathbf{p}}^\dagger \left\{ iD^0 - \frac{(\mathbf{p} - i\mathbf{D})^2}{2m} \right\} \psi_{\mathbf{p}}$$

$$-4\pi\alpha_s \sum_{q,q'\mathbf{p},\mathbf{p}'} \left\{ \frac{1}{q^0} \psi_{\mathbf{p}'}^\dagger \left[A_{q'}^0, A_q^0\right] \psi_{\mathbf{p}} \right.$$

$$\left. + \frac{g^{\nu 0}(q'-p+p')^\mu - g^{\mu 0}(q-p+p')^\nu + g^{\mu\nu}(q-q')^0}{(\mathbf{p}'-\mathbf{p})^2} \psi_{\mathbf{p}'}^\dagger \left[A_{q'}^\nu, A_q^\mu\right] \psi_{\mathbf{p}} \right\}$$

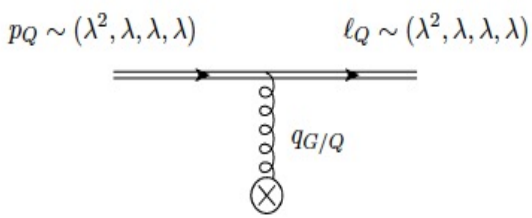
$$+\psi \leftrightarrow \chi,\ T \leftrightarrow \bar{T}$$

$$+\sum_{\mathbf{p},\mathbf{q}} \frac{4\pi\alpha_s}{(\mathbf{p}-\mathbf{q})^2} \psi_{\mathbf{q}}^\dagger T^A \psi_{\mathbf{p}} \chi_{-\mathbf{q}}^\dagger \bar{T}^A \chi_{-\mathbf{p}} + \dots$$

Allowed interactions in the medium

- At the level of the Lagrangian

$$\mathcal{L}_{\mathrm{NRQCD}_G} = \mathcal{L}_{\mathrm{NRQCD}} + \mathcal{L}_{Q-G/C}(\psi, A^{\mu,a}_{G/C}) + \mathcal{L}_{g-G/C}(A^{\mu,b}_{s}, A^{\mu,a}_{G/C}) + \psi \longleftrightarrow \chi$$



Possible scaling for the virtual gluons interacting with the heavy quarks

	0 1 2 3	+ − ⊥
(1)	$q_G \sim (\lambda^2, \lambda^1, \lambda^1, \lambda^2)$	$\sim (\lambda^2, \lambda^2, \boldsymbol{\lambda}_\perp)_n$
(2)	$q_C \sim (\lambda^2, \lambda^1, \lambda^1, \lambda^1)$	$\sim (\lambda^1, \lambda^1, \boldsymbol{\lambda}_\perp)_n$

- Energy component must always be suppressed
- Glauber gluons - transverse to the direction of propagation contribution
- Coulomb gluons - isotropic momentum distribution

- Calculated the leading power and next to leading power contributions 3 different ways

Background field method	**Perform a shift in the gluon field in the NRQCD Lagrangian then perform the power-counting**
Hybrid method	**From the full QCD diagrams for single effective Glauber/Coulomb gluon perform the corresponding power-counting, read the Feynman rules**
Matching method	**Full QCD diagrams describing the forward scattering of incoming heavy quark and a light quark or a gluon. We also derive the tree level expressions of the effective fields in terms of the QCD ingredients**

Example of the background field method

- Perform the label momentum representation and field substitution (u.s. -> u.s. + Glauber)

$$\psi(x) \to \sum_{\mathbf{p}} \psi_{\mathbf{p}}(x) ,$$

$$iD_\mu \to \mathcal{P}_\mu + i\partial_\mu - g(A^\mu_U + A^\mu_{G/C})$$

$$iD_t = \underbrace{i\partial_t - gA^0_U - gA^0_G}_{\sim \lambda^2} ,$$

$$i\mathbf{D} = \underbrace{\boldsymbol{\mathcal{P}}}_{\sim\lambda} - (\underbrace{i\boldsymbol{\partial} + g\mathbf{A}_U + g\mathbf{n}A^{\mathbf{n}}_G}_{\sim\lambda^2}) + \mathcal{O}(\lambda^3) ,$$

$$\mathbf{E} = \partial_t(\mathbf{A}_U + \mathbf{A}_G) + (\boldsymbol{\partial} + i\boldsymbol{\mathcal{P}})(A^0_U + A^0_G) + gT^c f^{cba}(A^0_U + A^0_G)^b(\mathbf{A}_U + \mathbf{A}_G)^a$$
$$= \underbrace{i\boldsymbol{\mathcal{P}}_\perp A^0_G}_{\sim\lambda^3} + \mathcal{O}(\lambda^4) ,$$

$$\mathbf{B} = -(\boldsymbol{\partial} + i\boldsymbol{\mathcal{P}}) \times (\mathbf{A}_U + \mathbf{A}_G) + \frac{g}{2}T^c f^{cba}(\mathbf{A}_U + \mathbf{A}_G)^b(\mathbf{A}_U + \mathbf{A}_G)^a$$
$$= -\underbrace{(i\boldsymbol{\mathcal{P}}_\perp \times \mathbf{n})\, A^{\mathbf{n}}_G}_{\sim\lambda^3} + \mathcal{O}(\lambda^4) .$$

Example for a collinear source (note results depend on the type of source)

Substitute, expand and collect terms up to order λ^3

- Results: depend on the type of the source of scattering in the medium

Leading medium corrections

$$\mathcal{L}^{(0)}_{Q-G/C}(\psi, A^{\mu,a}_{G/C}) = \sum_{\mathbf{p},q_T} \psi^\dagger_{\mathbf{p}+\mathbf{q}_T}\Big(-gA^0_{G/C}\Big)\psi_{\mathbf{p}} \quad (collinear/static/soft).$$

Sub-leading medium corrections

$$\mathcal{L}^{(1)}_{Q-G}(\psi, A^{\mu,a}_G) = g\sum_{\mathbf{p},q_T} \psi^\dagger_{\mathbf{p}+\mathbf{q}_T}\Big(\frac{2A^{\mathbf{n}}_G(\mathbf{n}\cdot\boldsymbol{\mathcal{P}}) - i\Big[(\boldsymbol{\mathcal{P}}_\perp \times \mathbf{n})A^{\mathbf{n}}_G\Big]\cdot\boldsymbol{\sigma}}{2m}\Big)\psi_{\mathbf{p}} \quad (collinear)$$

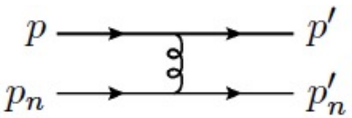
$$\mathcal{L}^{(1)}_{Q-C}(\psi, A^{\mu,a}_C) = 0 \quad (static)$$

$$\mathcal{L}^{(1)}_{Q-C}(\psi, A^{\mu,a}_C) = g\sum_{\mathbf{p},q_T} \psi^\dagger_{\mathbf{p}+\mathbf{q}_T}\Big(\frac{2\mathbf{A}_C\cdot\boldsymbol{\mathcal{P}} + [\boldsymbol{\mathcal{P}}\cdot\mathbf{A}_C] - i\Big[\boldsymbol{\mathcal{P}}\times\mathbf{A}_C\Big]\cdot\boldsymbol{\sigma}}{2m}\Big)\psi_{\mathbf{p}} \quad (soft)$$

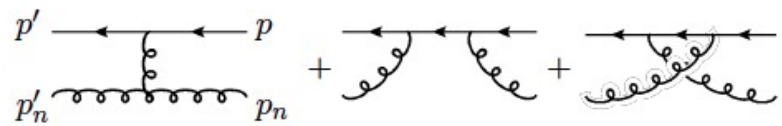
The QCD forward scattering diagram expansion

- Looking at t-channel scattering we can also extract the form of the Glauber/Coulomb fields in terms of QCD ingredients (and recover Lagrangian)

$t_{coll.} =$



$t_{g-coll.} =$



$$= t^{(0)}_{g-coll.} + t^{(1)}_{g-coll.} + \mathcal{O}(\lambda^2)\,.$$

Glauber field for collinear source

$$A_G^{\mu,a} = \frac{n^\mu}{\mathbf{q}_T^2} \sum_\ell \bar{\xi}_{n,\ell-\mathbf{q}_T} \frac{\bar{\not n}}{2} (gT^a) \xi_{n,\ell}$$

Coulomb field for soft source

$$A_C^{\mu,a} \equiv \frac{1}{\mathbf{q}^2} \sum_\ell \bar{\phi}_{\ell-\mathbf{q}} \gamma^\mu (gT^A) \phi_\ell$$

Glauber field for collinear source

$$A_G^{\mu,a} = \frac{i}{2} g f^{abc} \frac{n^\mu}{\mathbf{q}_T^2} \sum_\ell \left[\bar{n} \cdot \mathcal{P} \left(\mathbf{B}^{b(0)}_{n\perp,\ell-\mathbf{q}_T} \cdot \mathbf{B}^{c(0)}_{n\perp,\ell} \right) \right]$$

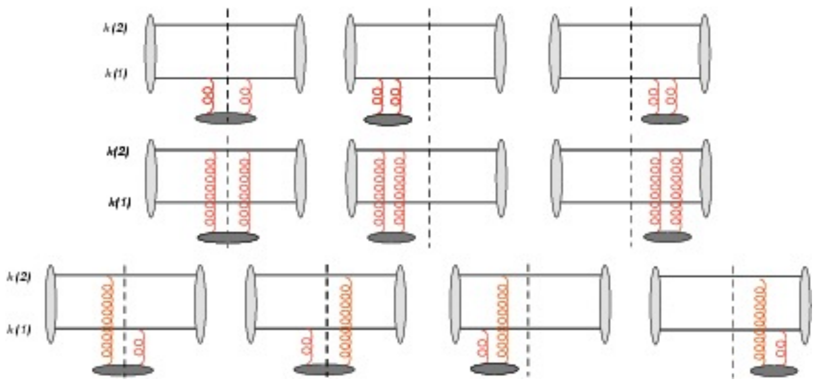
Coulomb field for soft source

Y. Makris et al. (2019)

$$A_C^{\mu,a} = f^{abc} \frac{ig}{2\,\mathbf{q}^2} \sum_\ell \left\{ \left[\mathcal{P}^\mu \left(\mathbf{B}^{b(0)}_{s,\ell-\mathbf{q}} \cdot \mathbf{B}^{c(0)}_{s,\ell} \right) \right] - 2 (\mathbf{B}^{c(0)}_{s,\ell} \cdot \left[\boldsymbol{\mathcal{P}} \right) B^{\mu,b(0)}_{s,\ell-\mathbf{q}} \right] - 2 (\mathbf{B}^{b(0)}_{s,\ell-\mathbf{q}} \cdot \left[\boldsymbol{\mathcal{P}} \right) B^{\mu,c(0)}_{s,\ell} \right] \right\}$$

- Note that for the gluon the last 2 diagrams are necessary for gauge invariance but the first diagram the leading forward scattering contribution
- In the medium the momentum exchange can get dressed ~ Debye screening

Collisional interactions of heavy meson states in matter



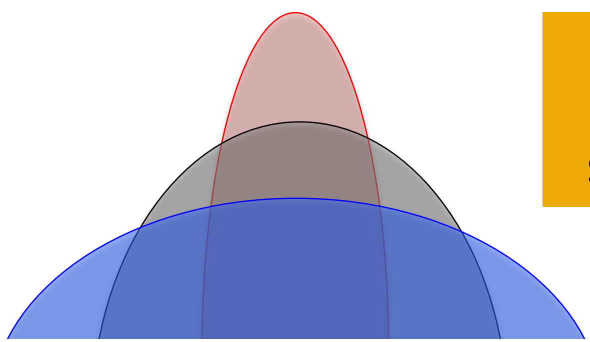
Adil *et al.* (2006) Sharma *et al.* (2012)

Heavy meson acoplanarity & distortion of the light cone wave function (meson decay)

- Resum in impact parameter space, make Gaussian approximation

$$|\psi_f(\mathbf{K},\Delta\mathbf{k})|^2 = \left[\frac{e^{-\frac{\mathbf{K}^2}{4\chi\mu^2\xi}}}{4\pi\chi\mu^2\xi}\right]\left[\mathrm{Norm}^2\,\frac{x(1-x)\Lambda^2}{\chi\mu^2\xi+x(1-x)\Lambda^2} \times e^{-\frac{\Delta\mathbf{k}^2}{4(\chi\mu^2\xi+x(1-x)\Lambda^2)}}\, e^{-\frac{m_1^2(1-x)+m_2^2 x}{x(1-x)\Lambda^2}}\right] .$$

$$P_{\mathrm{surv.}}\left(\frac{\mu^2}{\lambda}L\xi\right) = \left|\int dx d^2\Delta k_\perp\, \psi^*_f(x,\Delta k_\perp)\psi_i(x,\Delta k_\perp)\right|^2$$



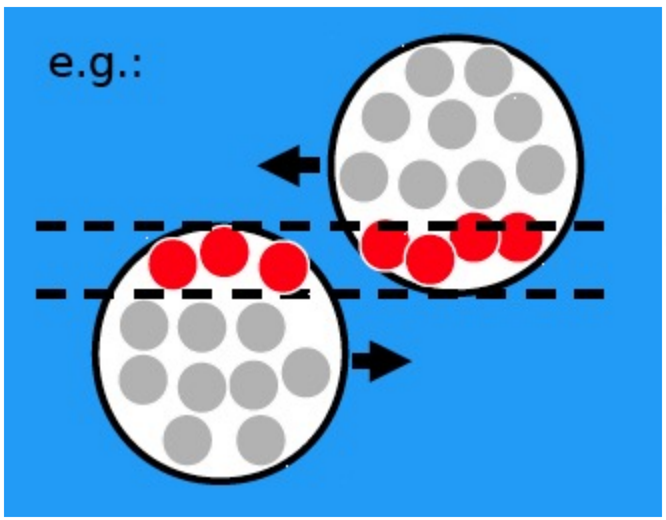
S. Aronson *et al.* (2017)

Momentum space picture – may be counter intuitive (note that broadening in configuration space is narrowing in momentum space)

- **Initial wavefunction ~ vacuum**
- **Collisional broadening**
- **Thermal narrowing**

Effects of the medium and centrality

- Reminder about the geometry in heavy ion collisions

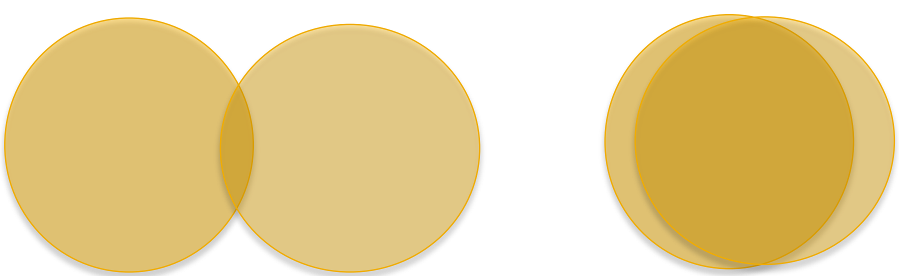


Number of binary collisions

$N_1 \times N_2$

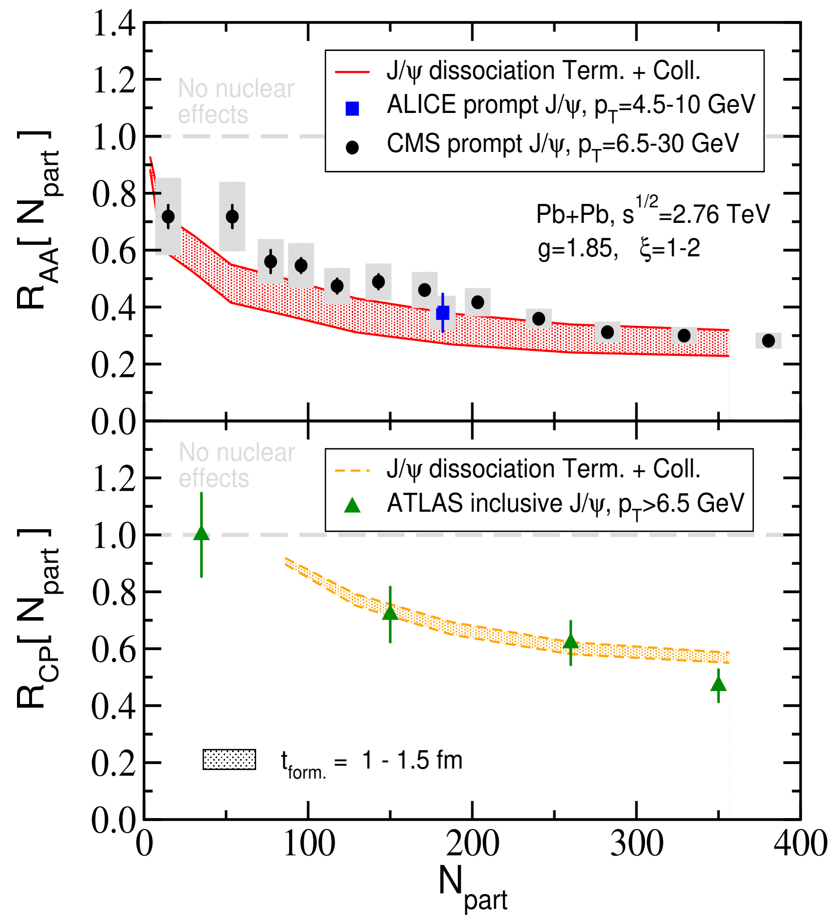
Number of participants

$N_1 + N_2$



Peripheral **Central**

Perform full feed down

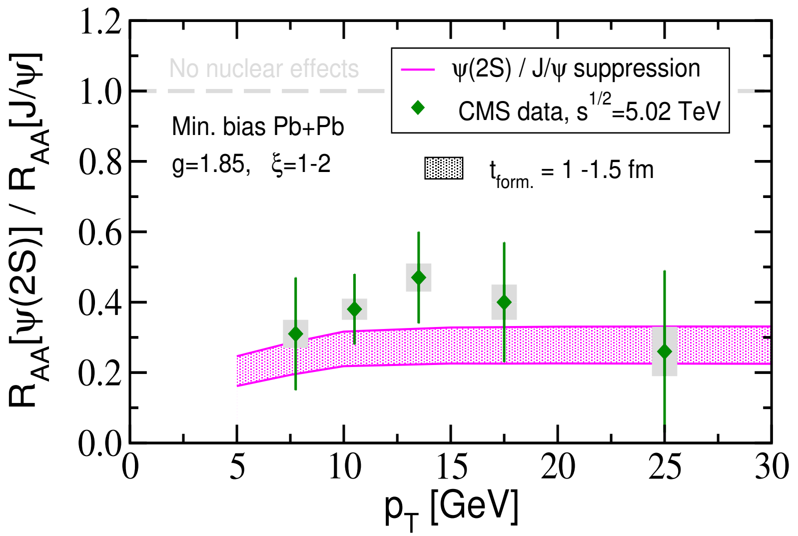


S. Aronson *et al.* (2017)

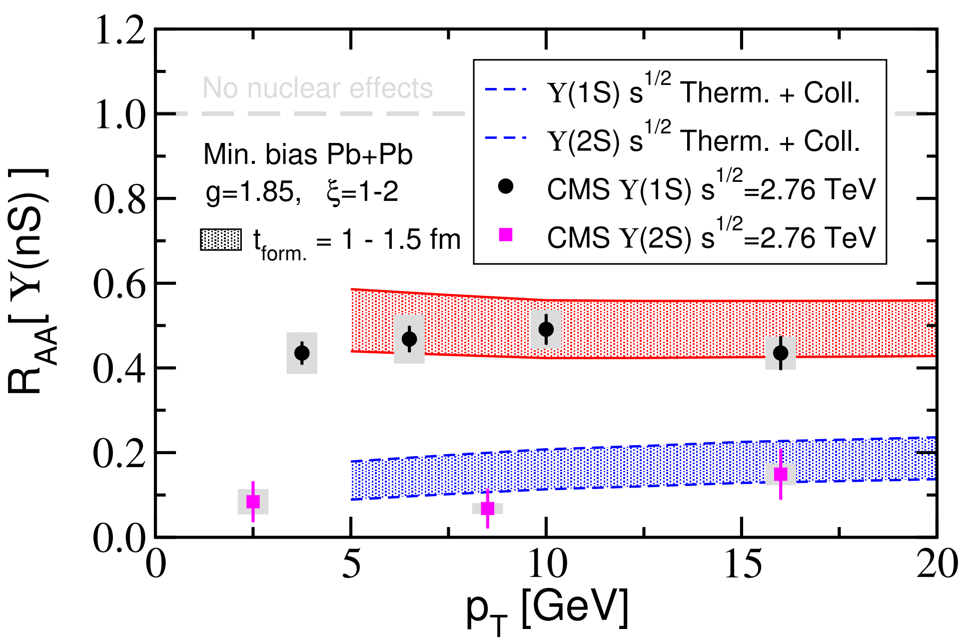
Phenomenological results

- We have centrality and p_T dependence at 2.76 TeV and 5.02 TeV around midrapidity
- Both ground and excited quarkonium states with consistent feed down

S. Aronson *et al.* (2017)



Approximately flat p_T dependence



- Good separation the suppression of the ground and excited

The theory is general and is suitable for implementation to cold nuclear matter (with suitable mods). See student talk:

I. Olivant *et al.* (in progress)

Conclusions

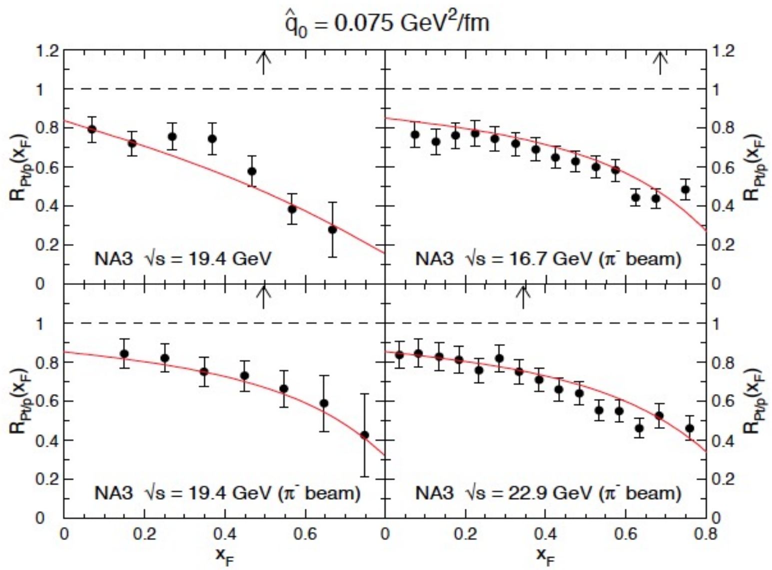
- Effective theories of QCD have enabled important conceptual and technical breakthroughs in our understanding of strong interactions and very significant improvement in the accuracy of the theoretical predictions
- In the the leading power factorization (high p_T) limit of NRQCD we investigated energy loss phenomenology and showed that it severely overpredicts the J/ψ modification and gives the wrong hierarchy of ground/excited suppression
- Motivated by this we constructed an effective theory of quarkonia in matter - $NRQCD_G$. Derived the Feynman rules (3 different ways) to leading and subleading power for different sources of interactions in the medium. We showed the connection to existing quarkonium dissociation phenomenology
- The theory is general and applicable to both hot (QGP) and cold (large nucleus) nuclear matter. Some interesting very preliminary results on dissociation in cold nuclear matter available

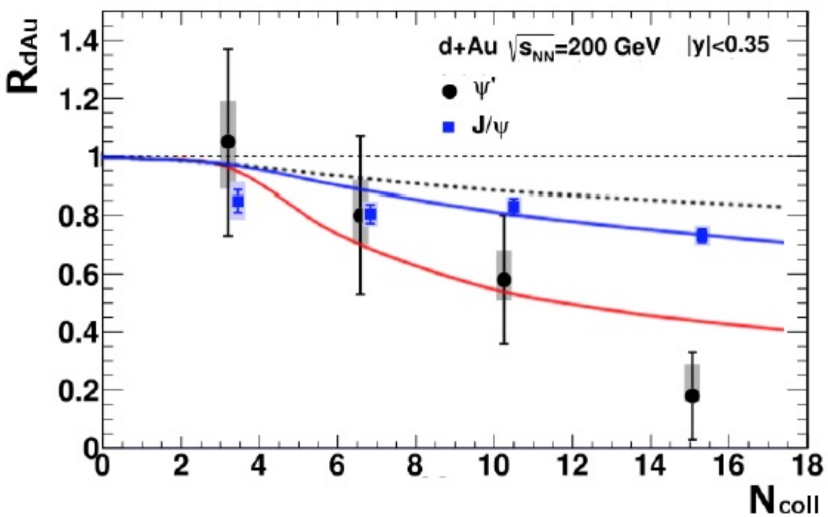
Energy loss for quarkonia in nuclei and co-mover dissociation

- Another radiative energy loss approach – Radiation off of a heavy quark. The Bertsch-Gunion spectrum is integrated from M to the cumulative broadening scale. It is suppressed by M_T at high p_T.

F. Arleo *et al.* (2012)

$$\Delta E \equiv \int_0^E \mathrm{d}\omega\, \omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \underset{M \gg \ell_\perp}{\simeq} N_c\, \alpha_s \frac{\ell_\perp - \Lambda}{M} E$$





- Co-mover dissociation model– phenomenological cross section to break up quarkonia in a co-moving hadron gas.

E. Ferreiro (2014)

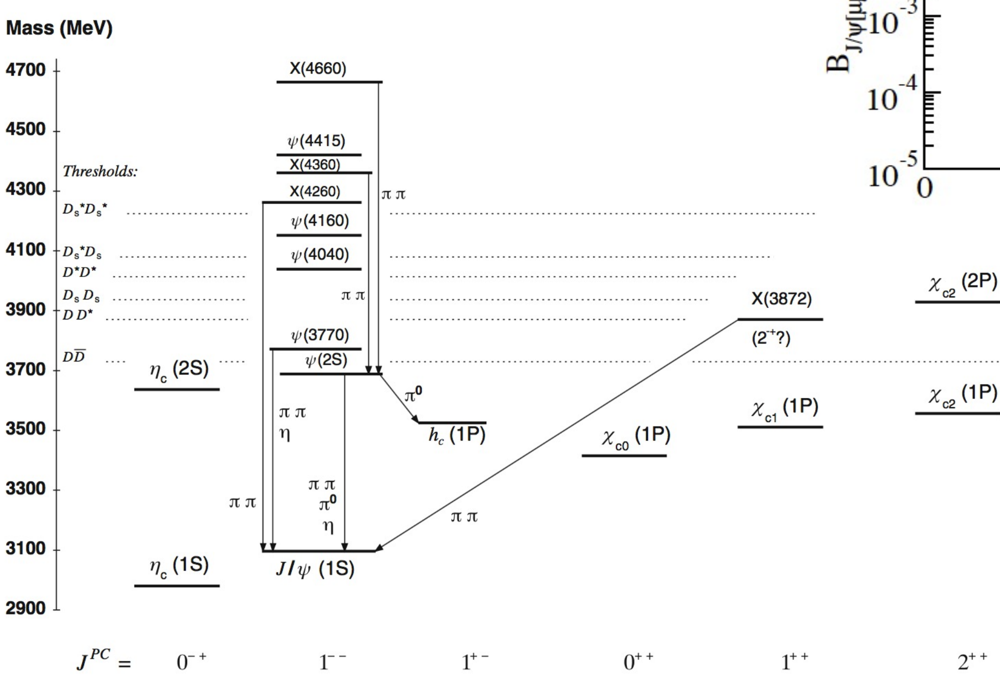
$$\tau \frac{\mathrm{d}\rho^{\psi}}{\mathrm{d}\tau}(b,s,y) = -\sigma^{co-\psi}\, \rho^{co}(b,s,y)\, \rho^{\psi}(b,s,y)$$

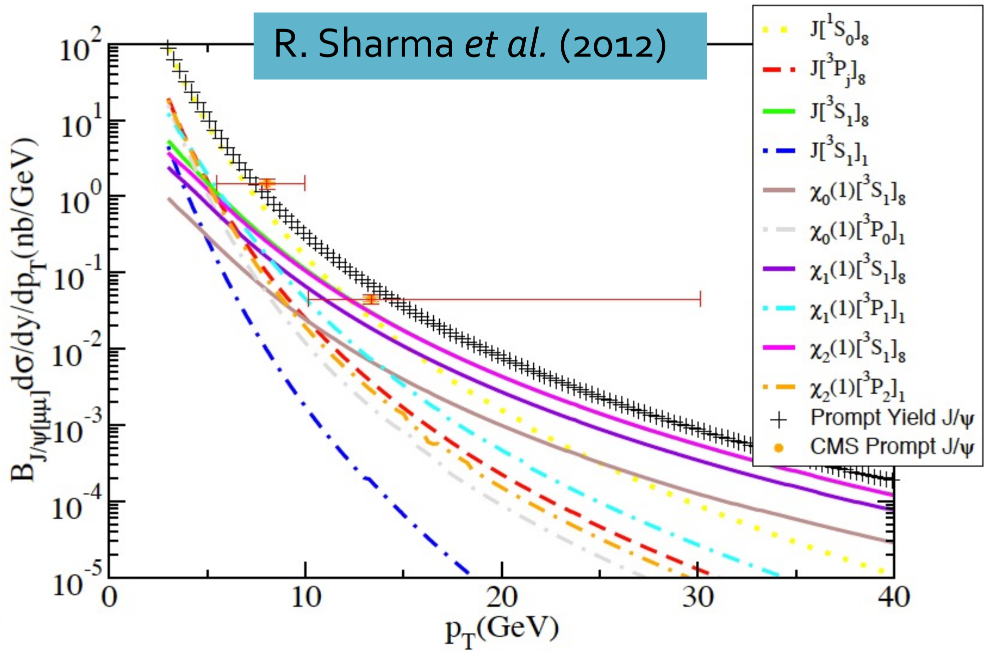
$$S_{\psi}^{co}(b,s,y) = \exp\left\{-\sigma^{co-\psi}\, \rho^{co}(b,s,y)\, \ln\left[\frac{\rho^{co}(b,s,y)}{\rho_{pp}(y)}\right]\right\}$$

Feeddown is important

- Example of NRQCD calculation. You see both different high p_T behavior and feeddown

Charmonium states





Following feeddown contributions taken, others small

$$\psi(2S):\quad \mathrm{Br}\left[\psi(2S)\to J/\psi+X\right]=61.4\pm0.6\%$$

$$\chi_{c1}:\quad \mathrm{Br}\left[\chi_{c1}\to J/\psi+\gamma\right]=34.3\pm1.0\%$$

$$\chi_{c2}:\quad \mathrm{Br}\left[\chi_{c2}\to J/\psi+\gamma\right]=19.0\pm0.5\%$$