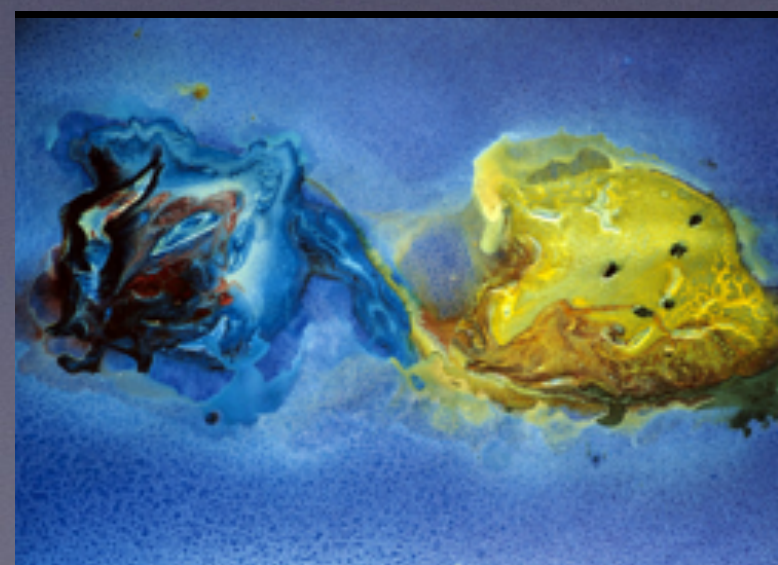


Quarkonium as a probe of hot/dense matter with potential NonrelativisticQCD and Open Quantum Systems



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- To do involves extending the description to charmonium, B_c

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it can be a hot medium or a dense medium, weakly or strongly coupled
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(Born-Oppenheimer EFT that we developed) and open quantum systems

Material for discussion/references

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, Phys. Rev. D **96** (2017) no.3, 034021 [arXiv:1612.07248 [hep-ph]].

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, Phys. Rev. D **97** (2018) no.7, 074009 [arXiv:1711.04515 [hep-ph]].

N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend, Phys. Rev. D **100** (2019) no.5, 054025 [arXiv:1903.08063 [hep-ph]].

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[M. Escobedo](#) *Phys.Rev.D* 103 (2021) 3, 034010 • e-Print: 2010.10424 [

Non-equilibrium
evolution in QGP

#4

[N. Brambilla](#), [J. Ghiglieri](#), [A. Vairo](#), [Peter Petreczky](#)
Phys.Rev.D 78 (2008) 014017 • e-Print: [0804.0993](#) [hep-ph]

[N. Brambilla](#), [M. Escobedo](#), [J. Ghiglieri](#), [A. Vairo](#)
JHEP 05 (2013) 130 • e-Print: [1303.6097](#) [hep-ph]

[N. Brambilla](#), [M. Escobedo](#), [J. Ghiglieri](#), [A. Vairo](#)
JHEP 12 (2011) 116 • e-Print: [1109.5826](#) [hep-ph]

[N. Brambilla](#), [M. Escobedo](#), [J. Ghiglieri](#), [J. Soto](#), [A. Vairo](#)
JHEP 09 (2010) 038 • e-Print: [1007.4156](#) [hep-ph]

Potential and energies
in medium

N. Brambilla, V. Leino, Peter Petreczky, A. Vairo
Phys.Rev.D 102 (2020) 074503 • e-Print 2007.10078 [hep-ph]

Lattice calculation of the
heavy quark transport coefficient

Heavy quarks are QGP probes

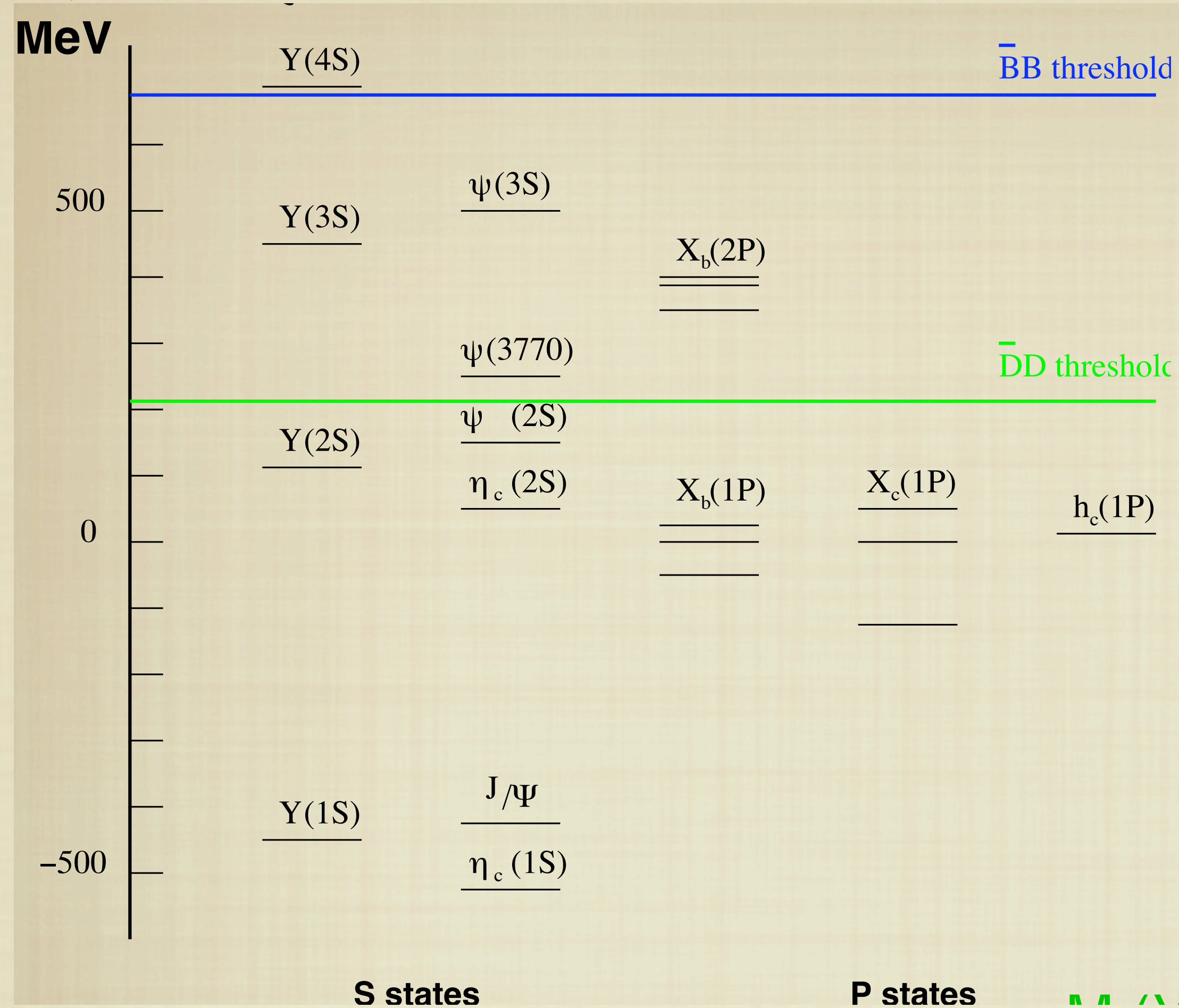
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Low-energy contributions may be accessible via lattice calculations.

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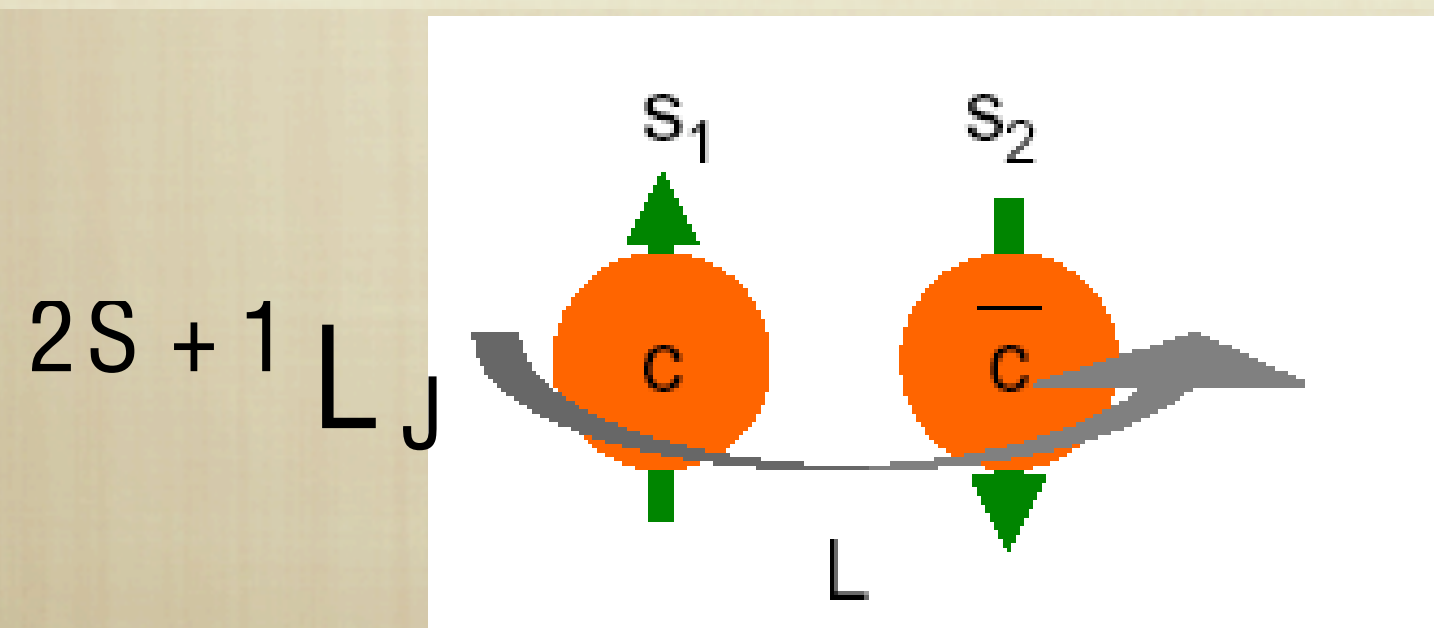
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Quarkonia are better hard probes
because they are multi scale systems

Quarkonium scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$



$$M(Y(1S)) = 9460 \text{ GeV}$$

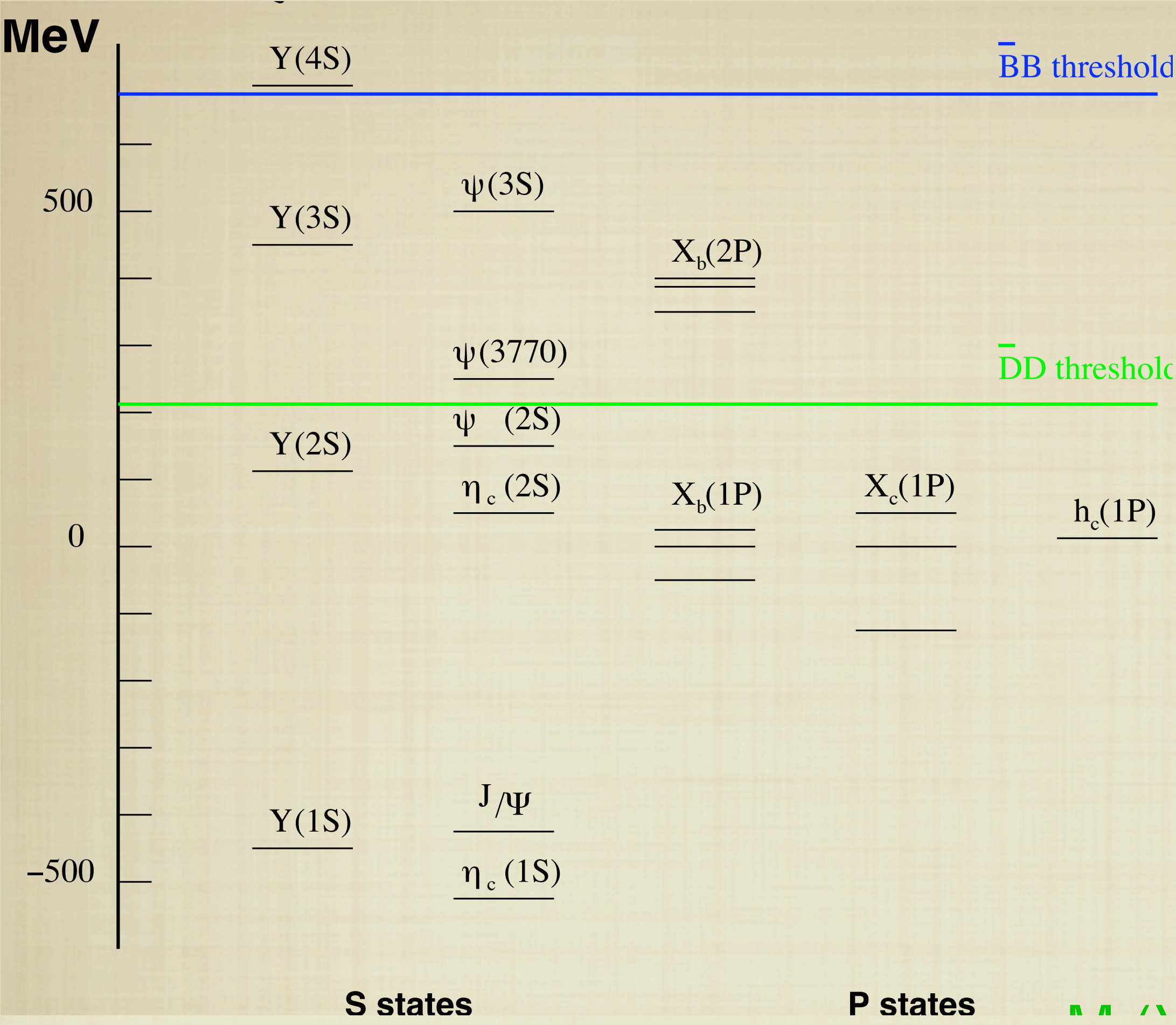
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THE MASS SCALE IS PERTURBATIVE

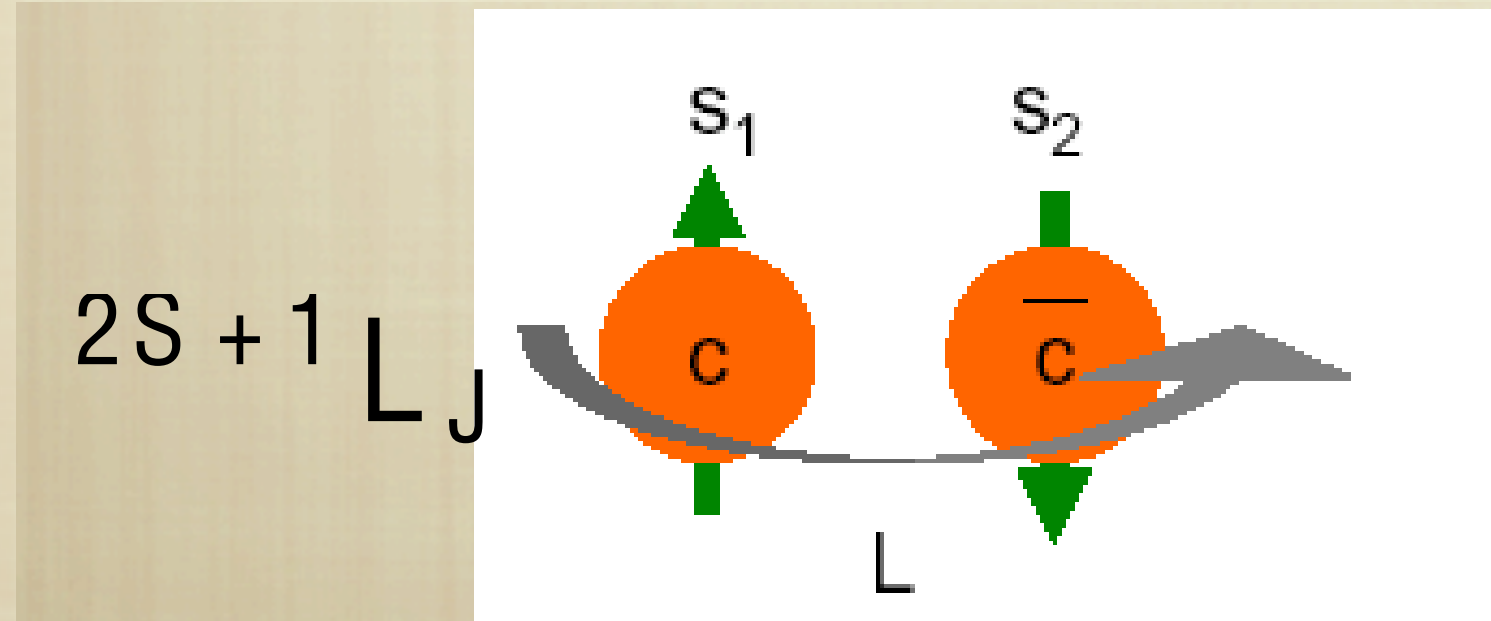
$$m_Q \gg \Lambda_{\text{QCD}}$$

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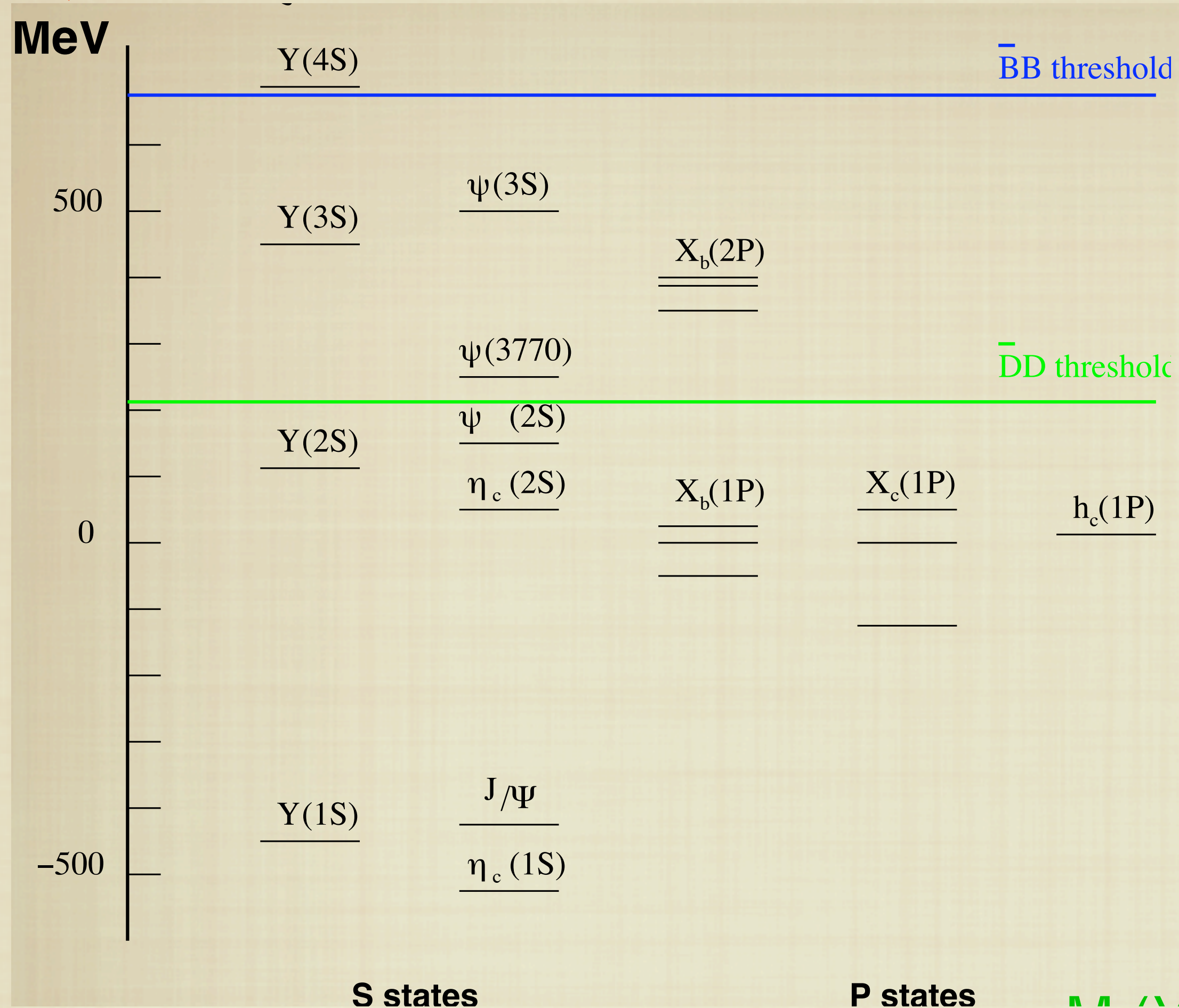
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$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$
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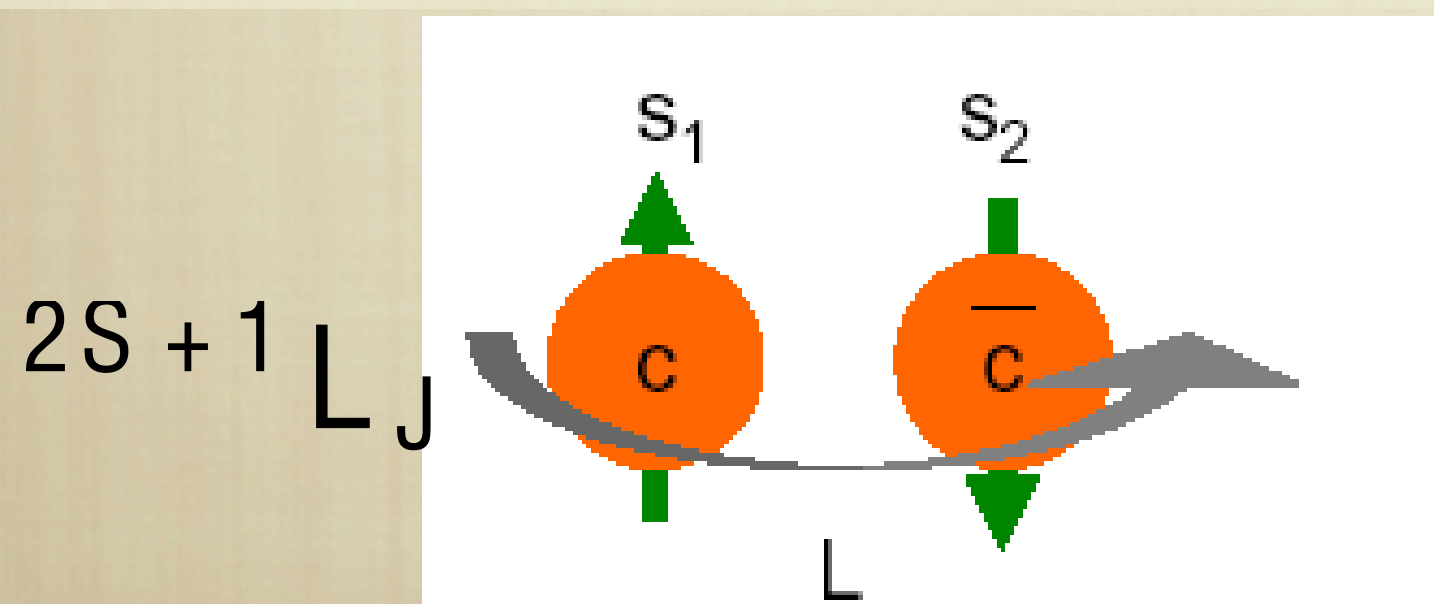
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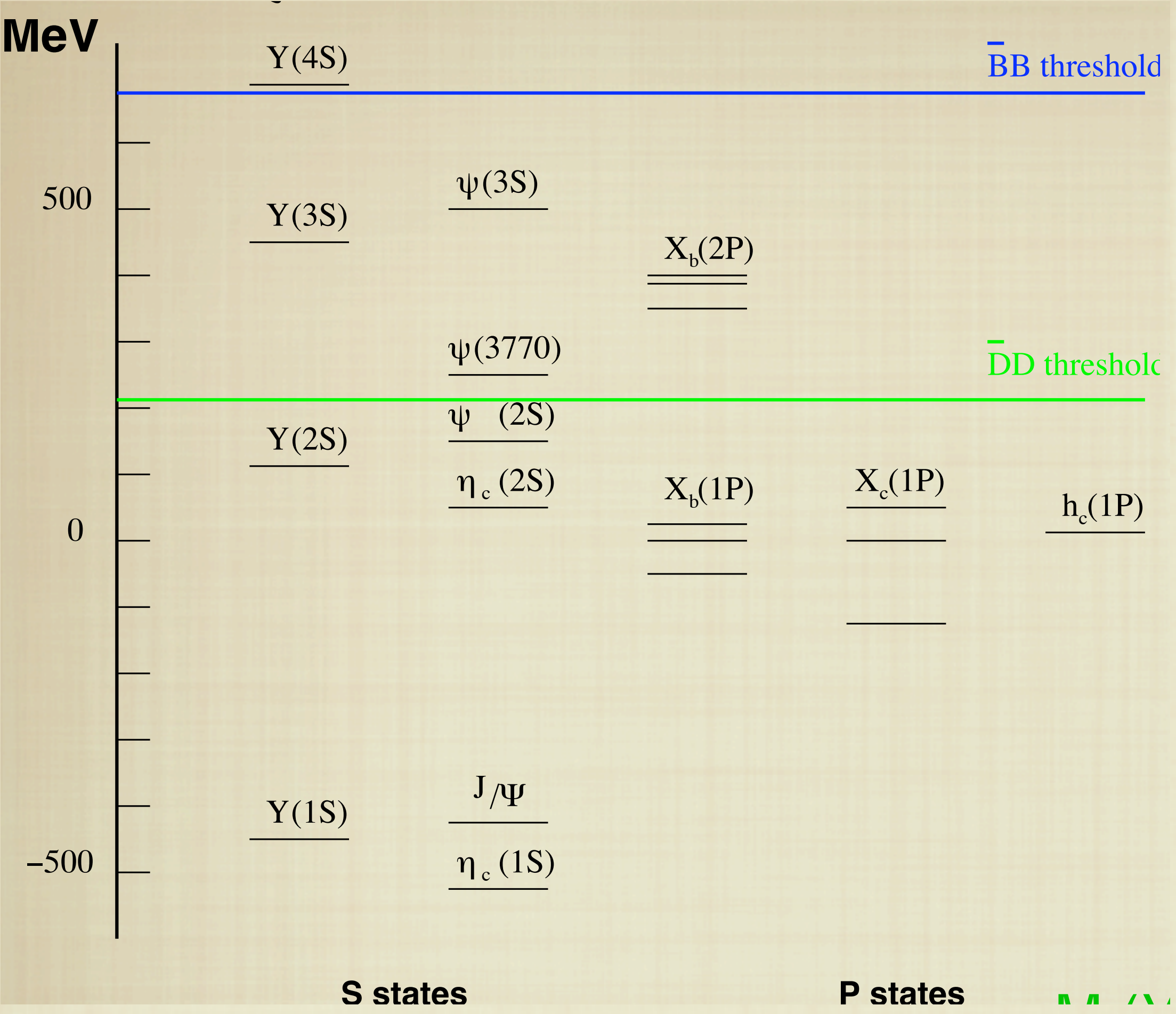
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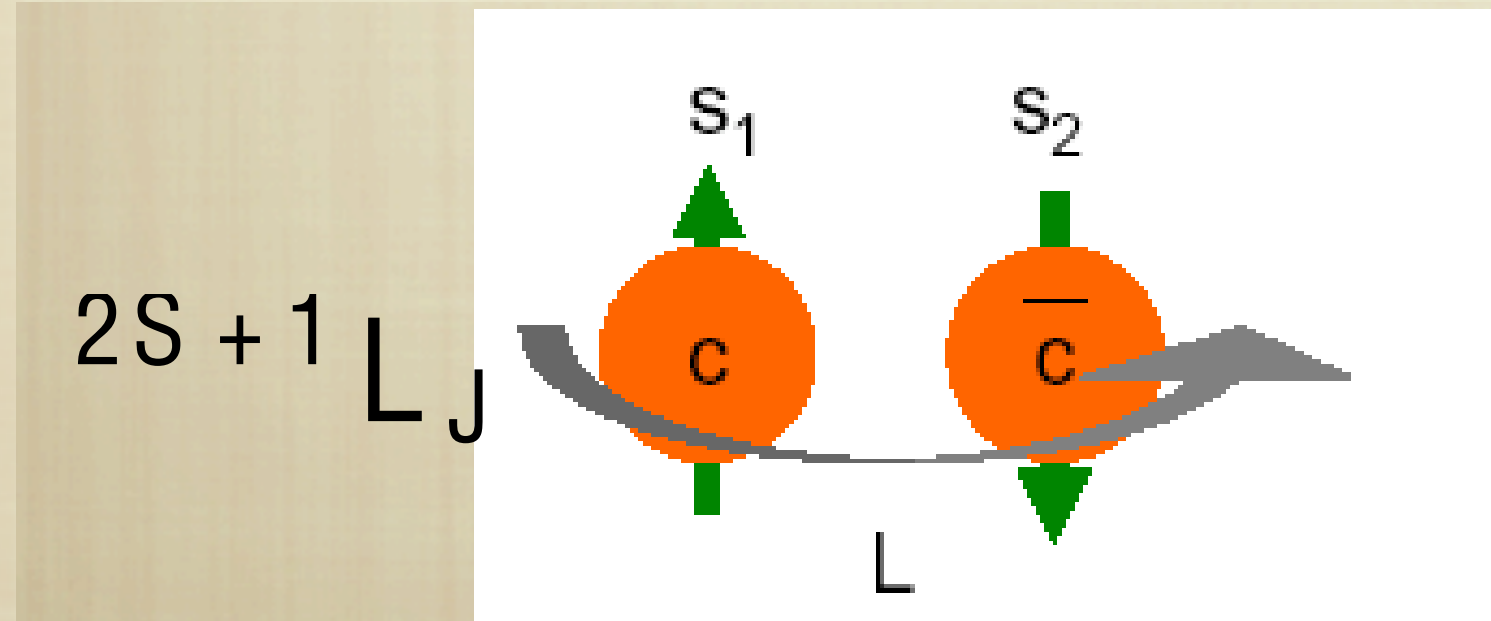
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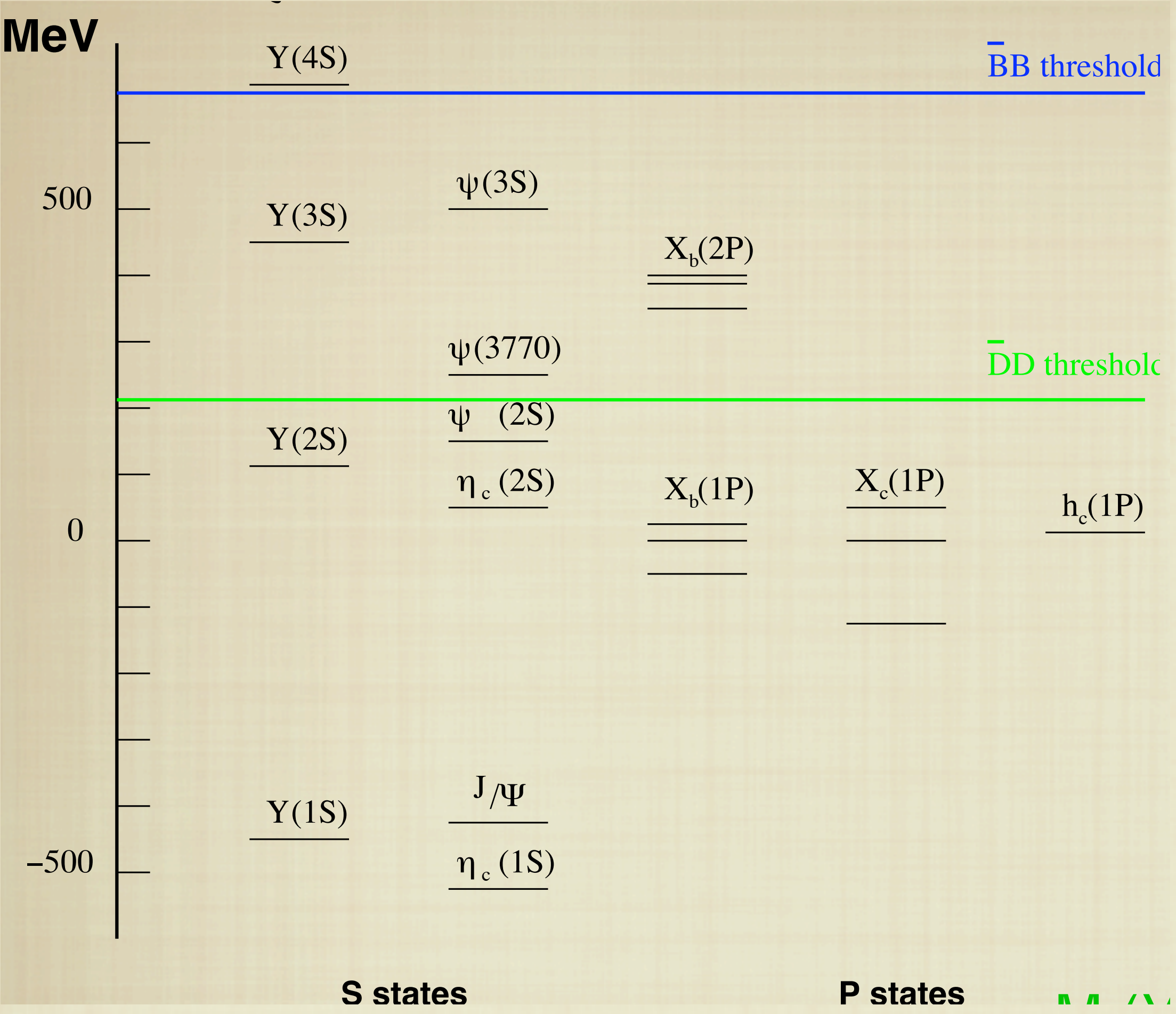
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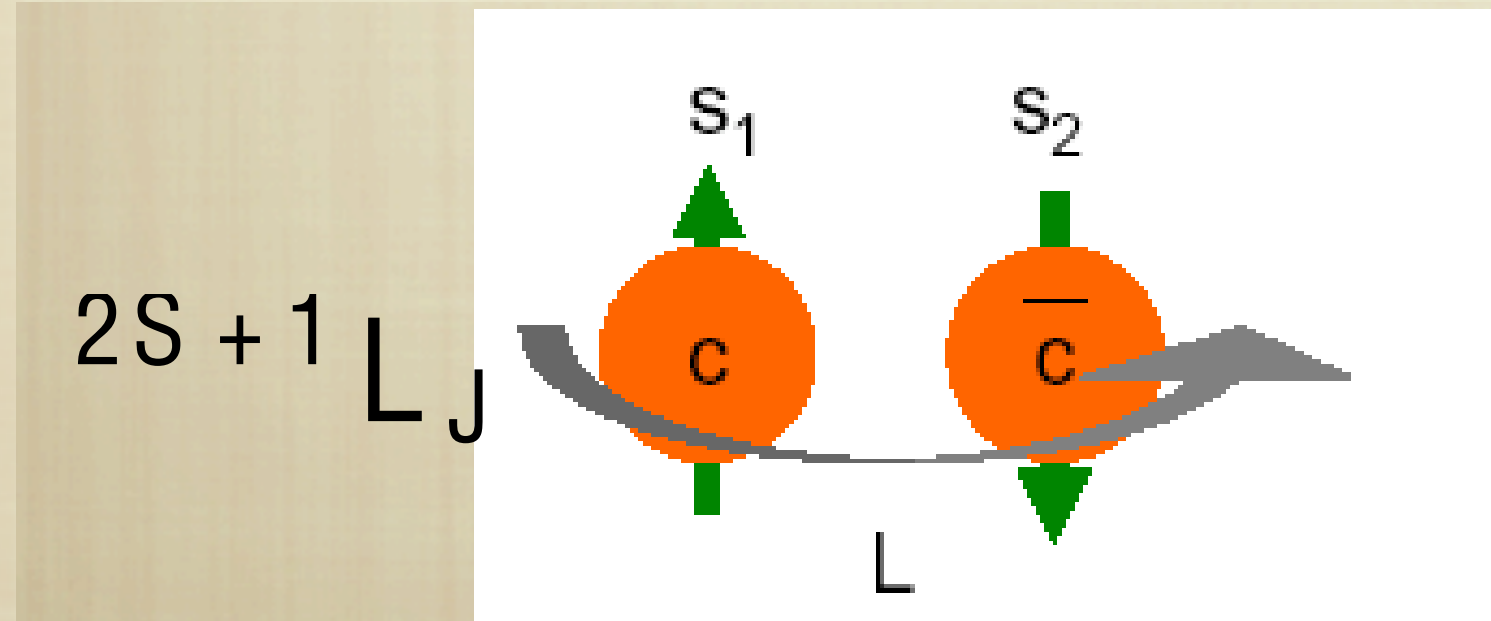
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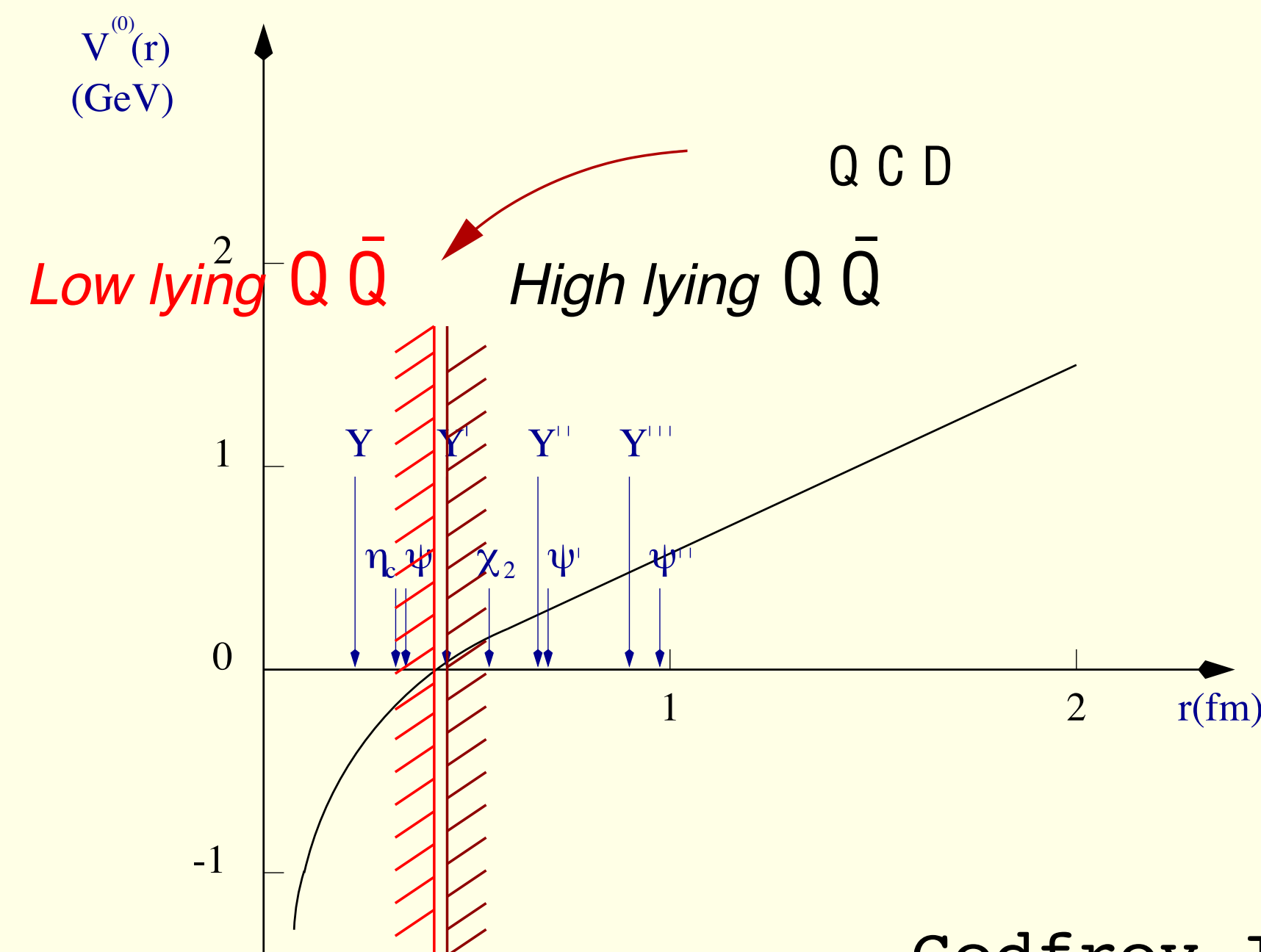
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Quarkonium as a confinement probe

The rich structure of separated energy scales makes $Q\bar{Q}$ an ideal probe

At zero temperature

- The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.



Godfrey Isgur PRD 32(85)189

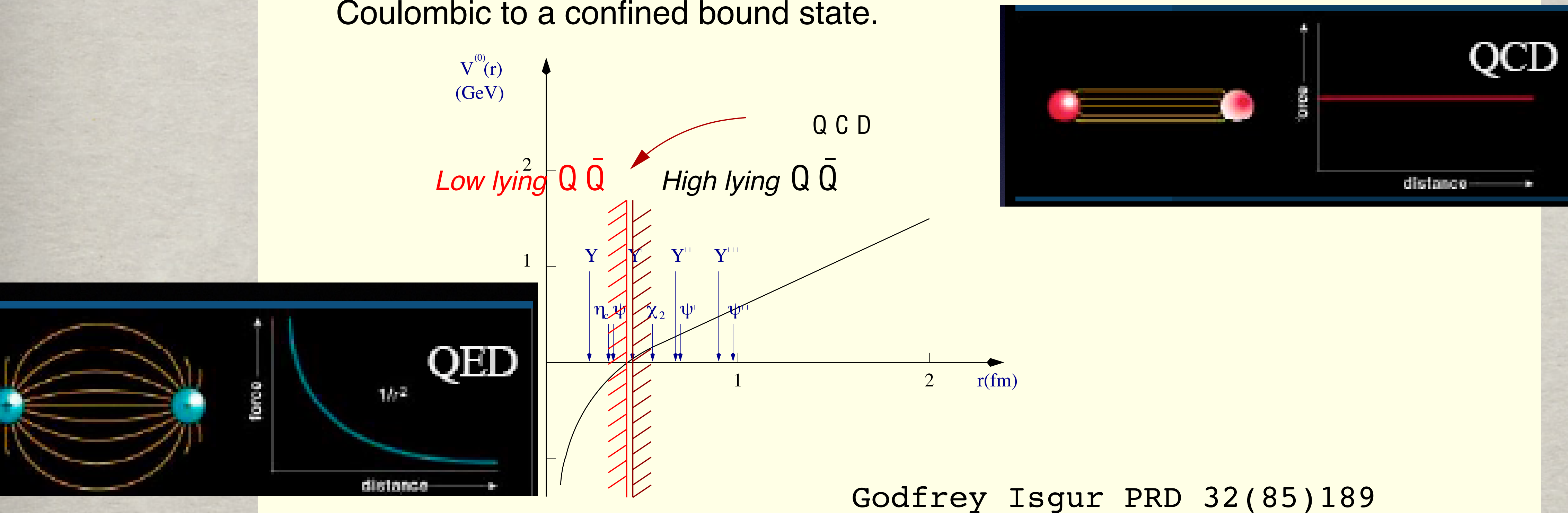
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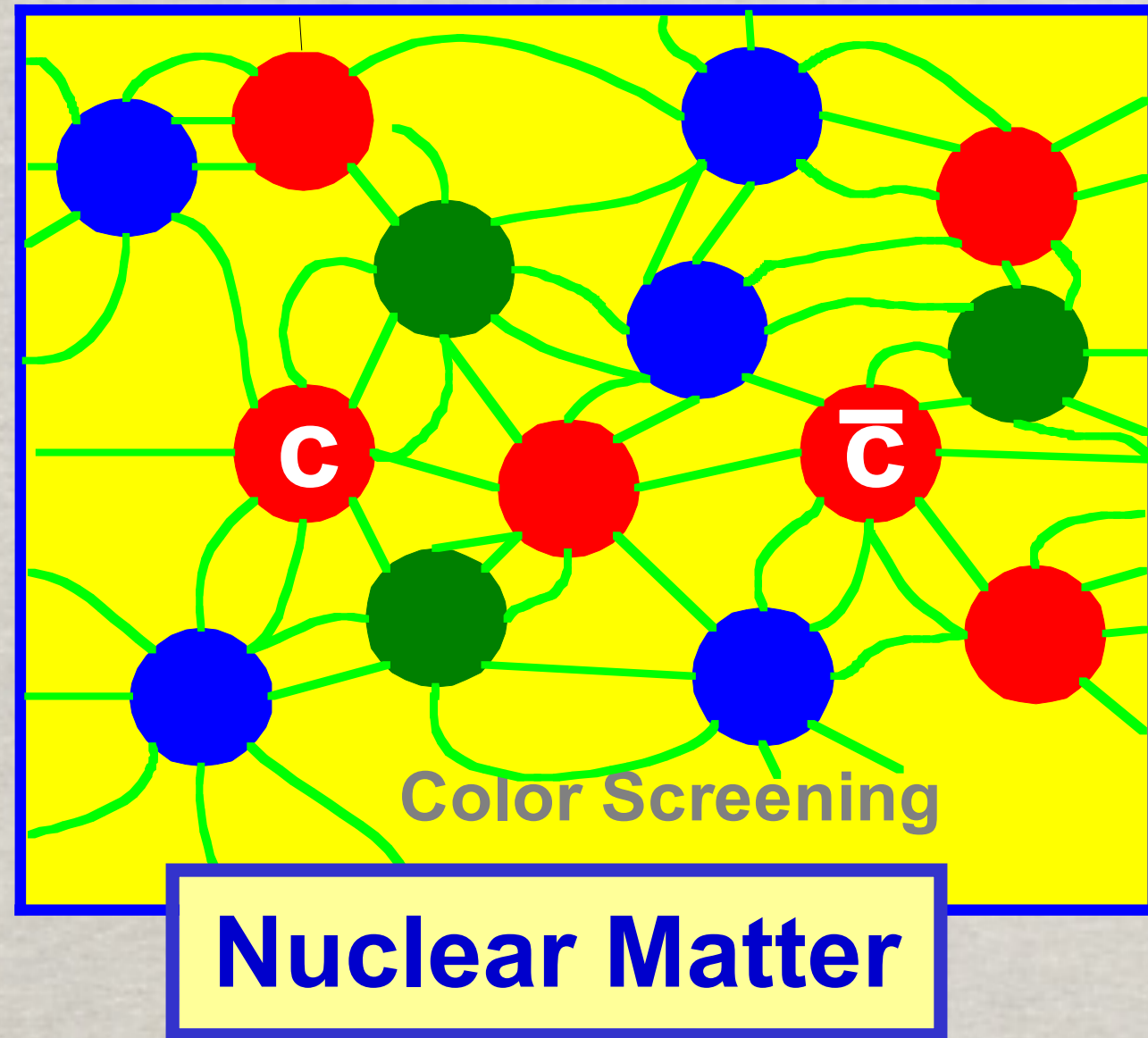
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Quarkonium as a confinement and deconfinement probe

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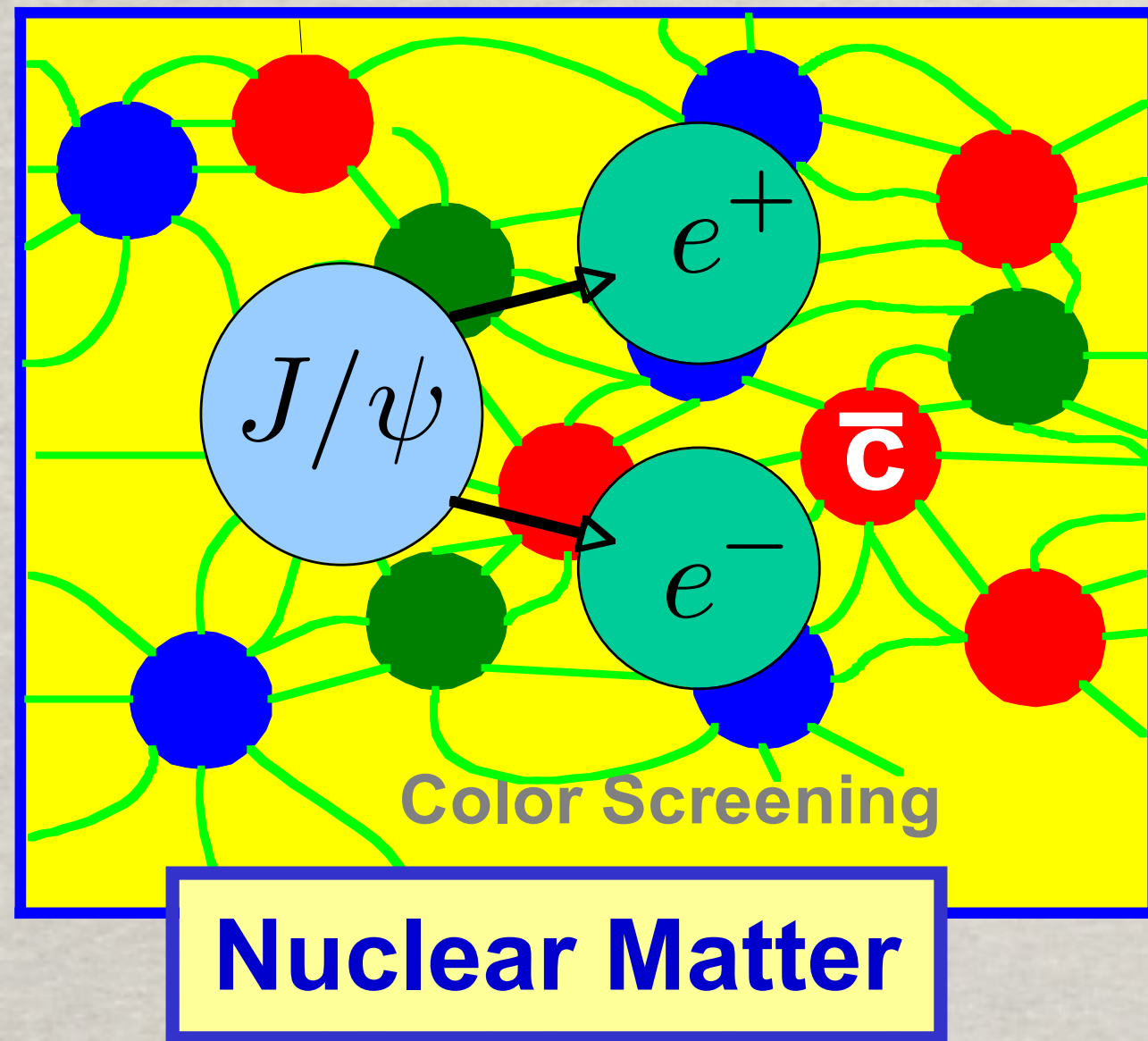
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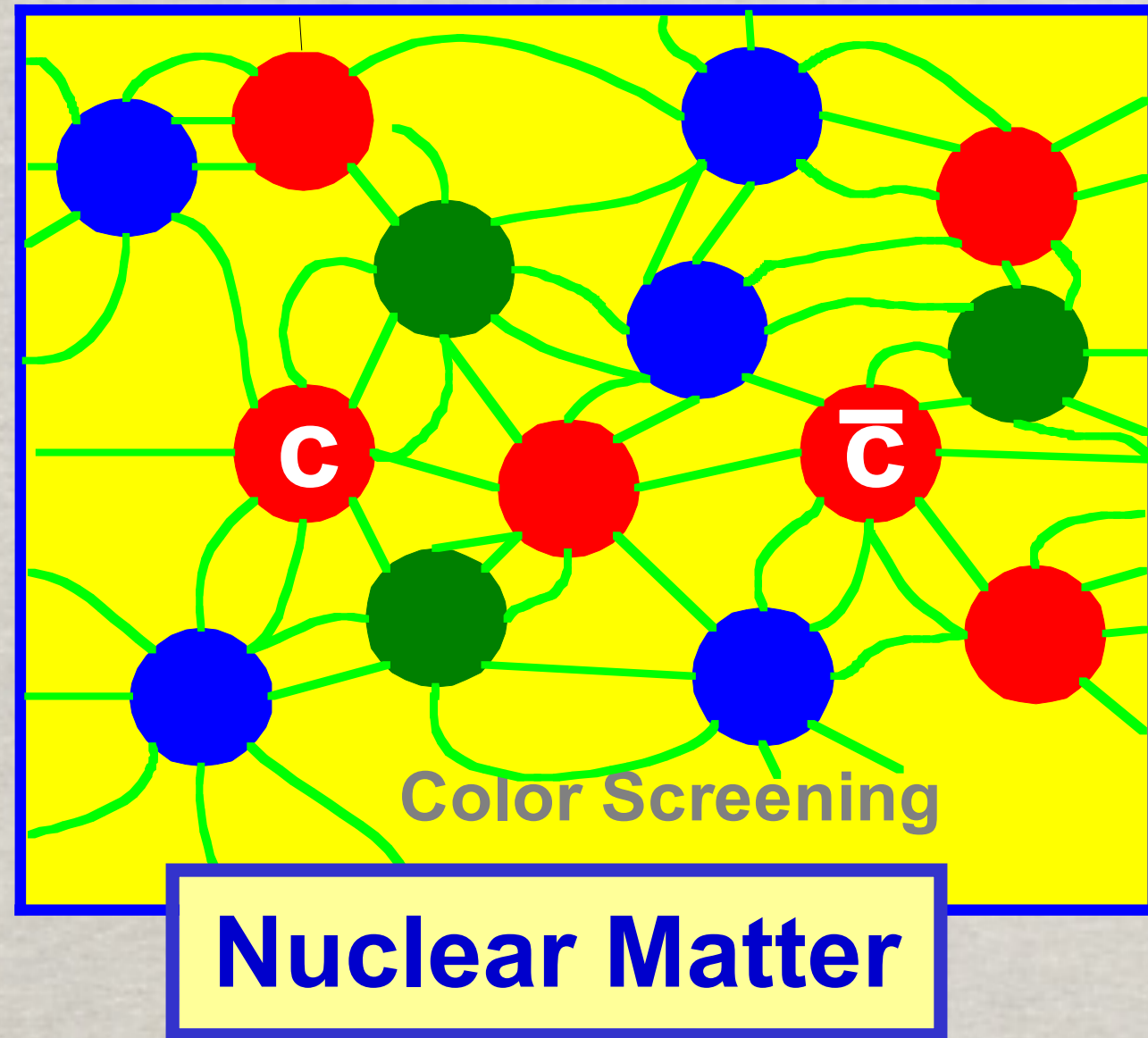
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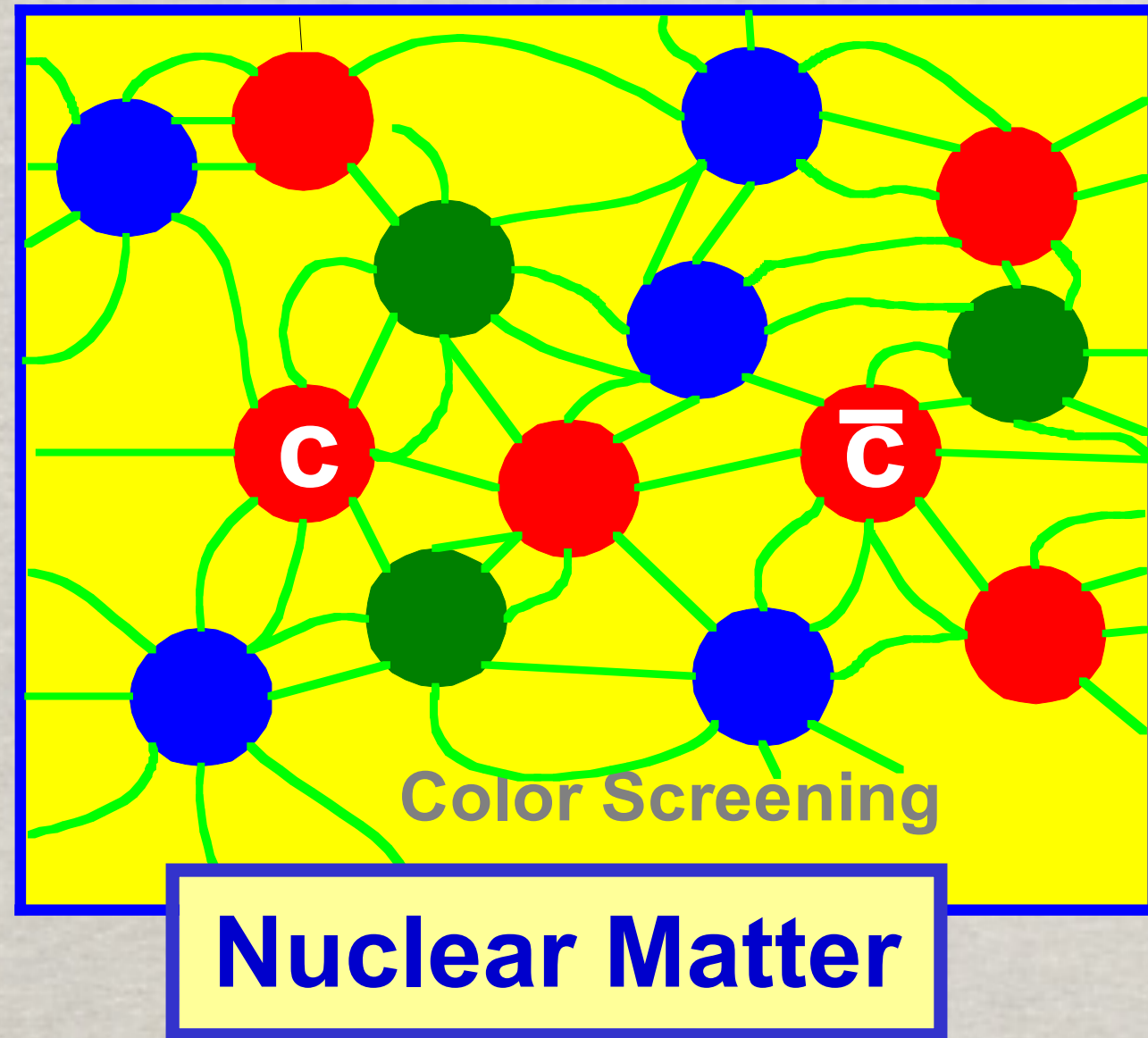
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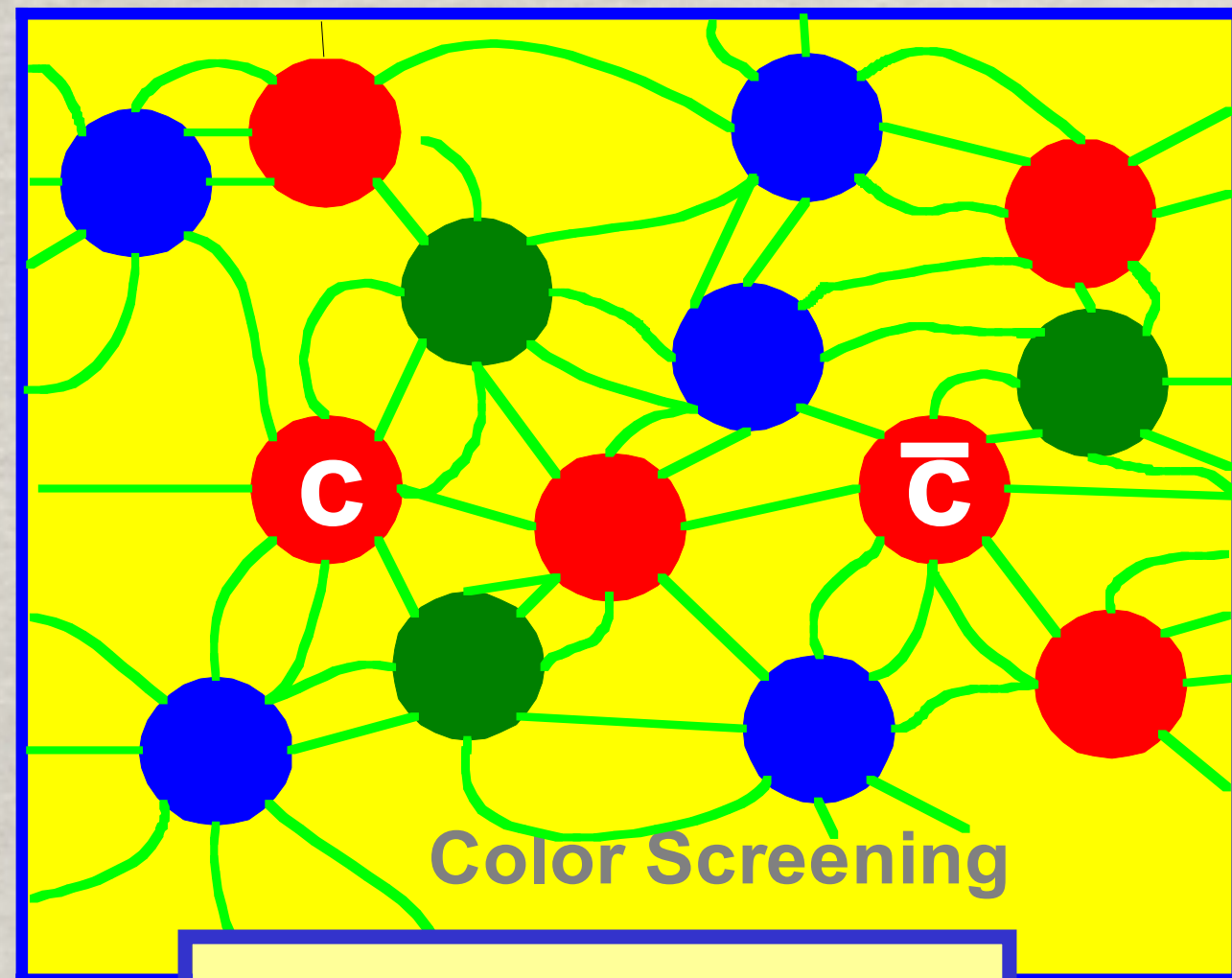
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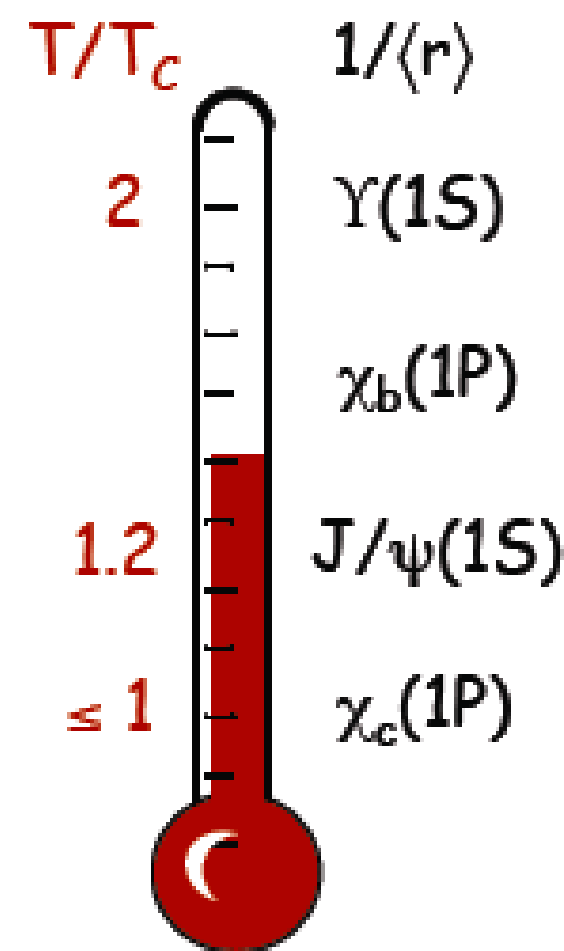
Nuclear Matter

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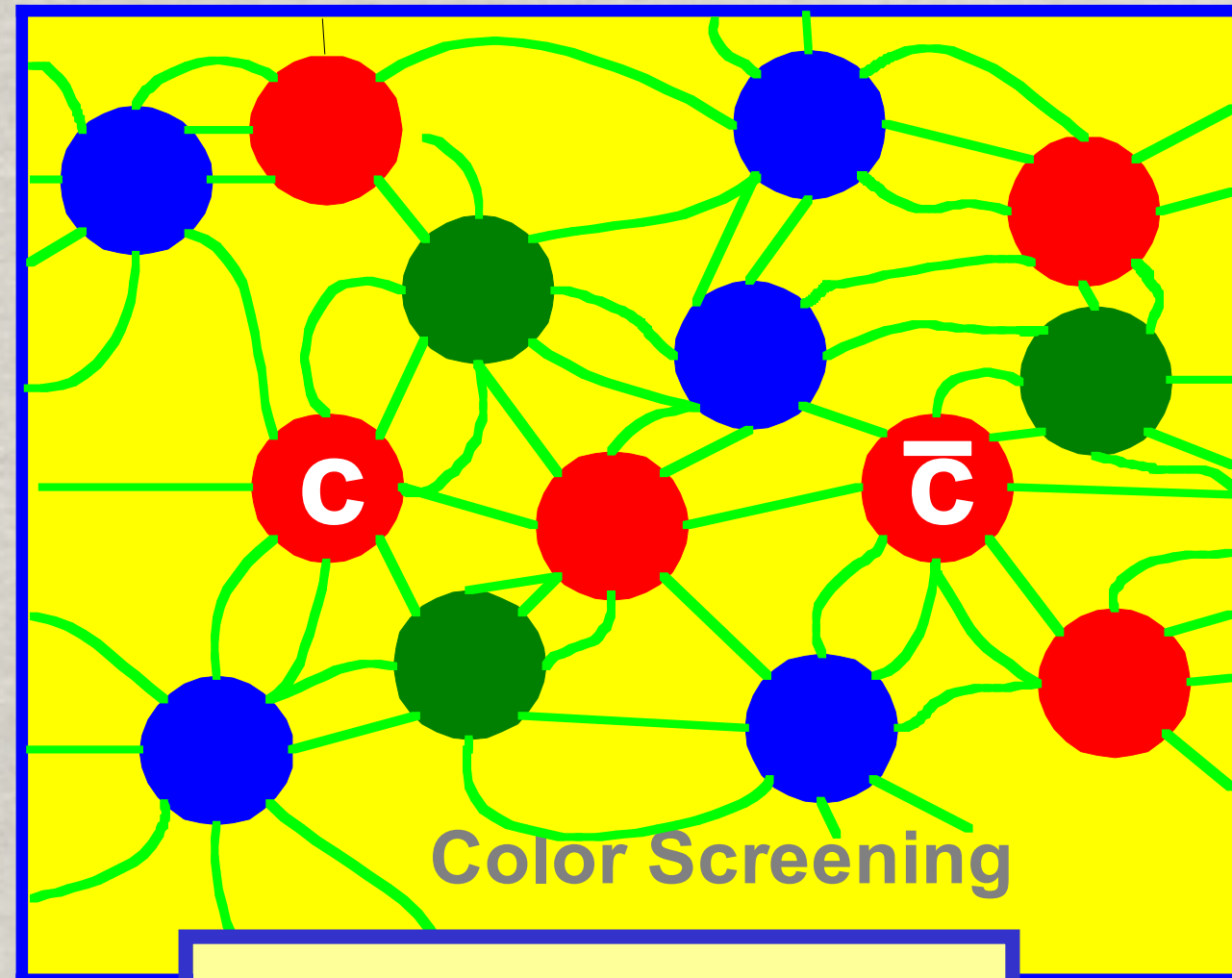
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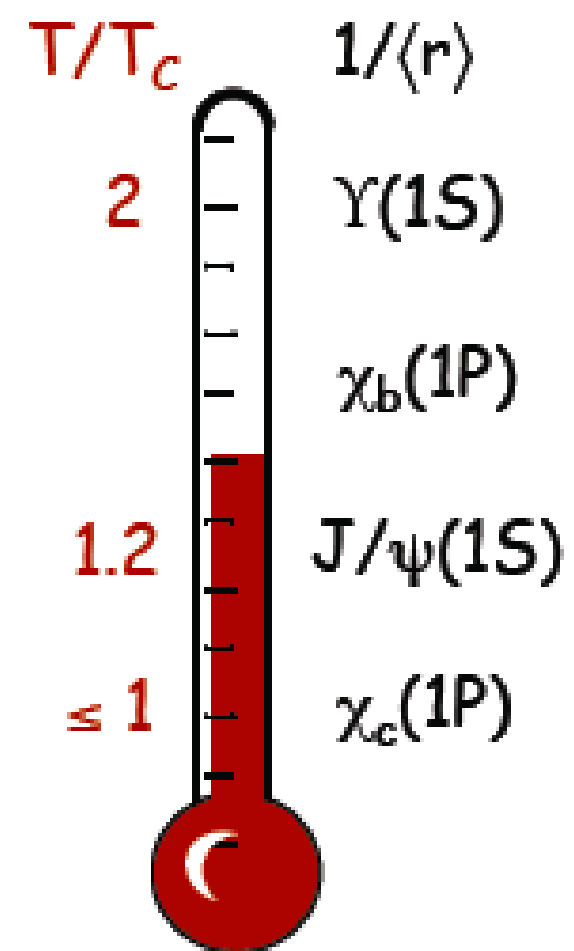
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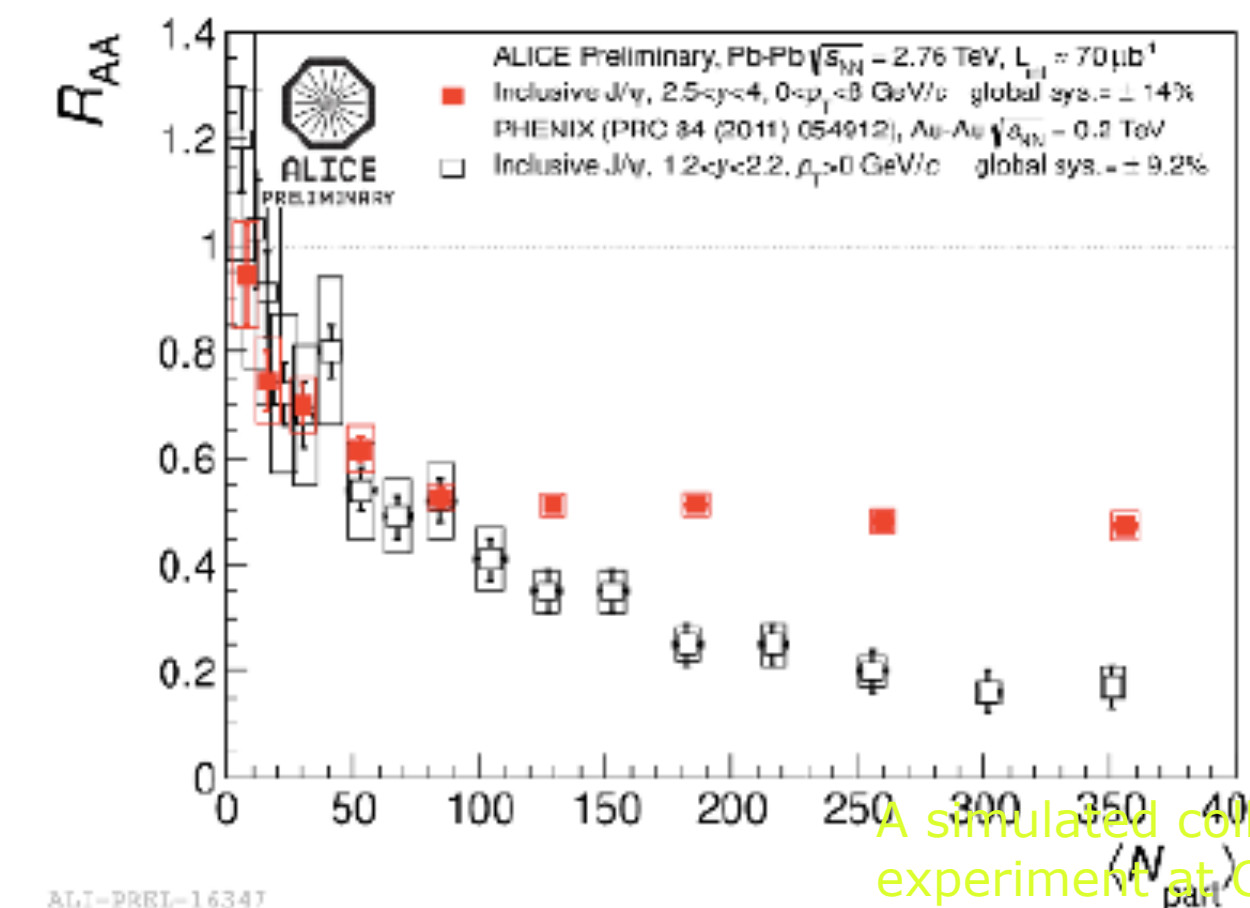
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nuclear modification

$$R_{AA} = \frac{Yield_{AA}^{q\bar{q}}}{\langle N_{coll} \rangle \times Yield_{pp}^{q\bar{q}}}$$



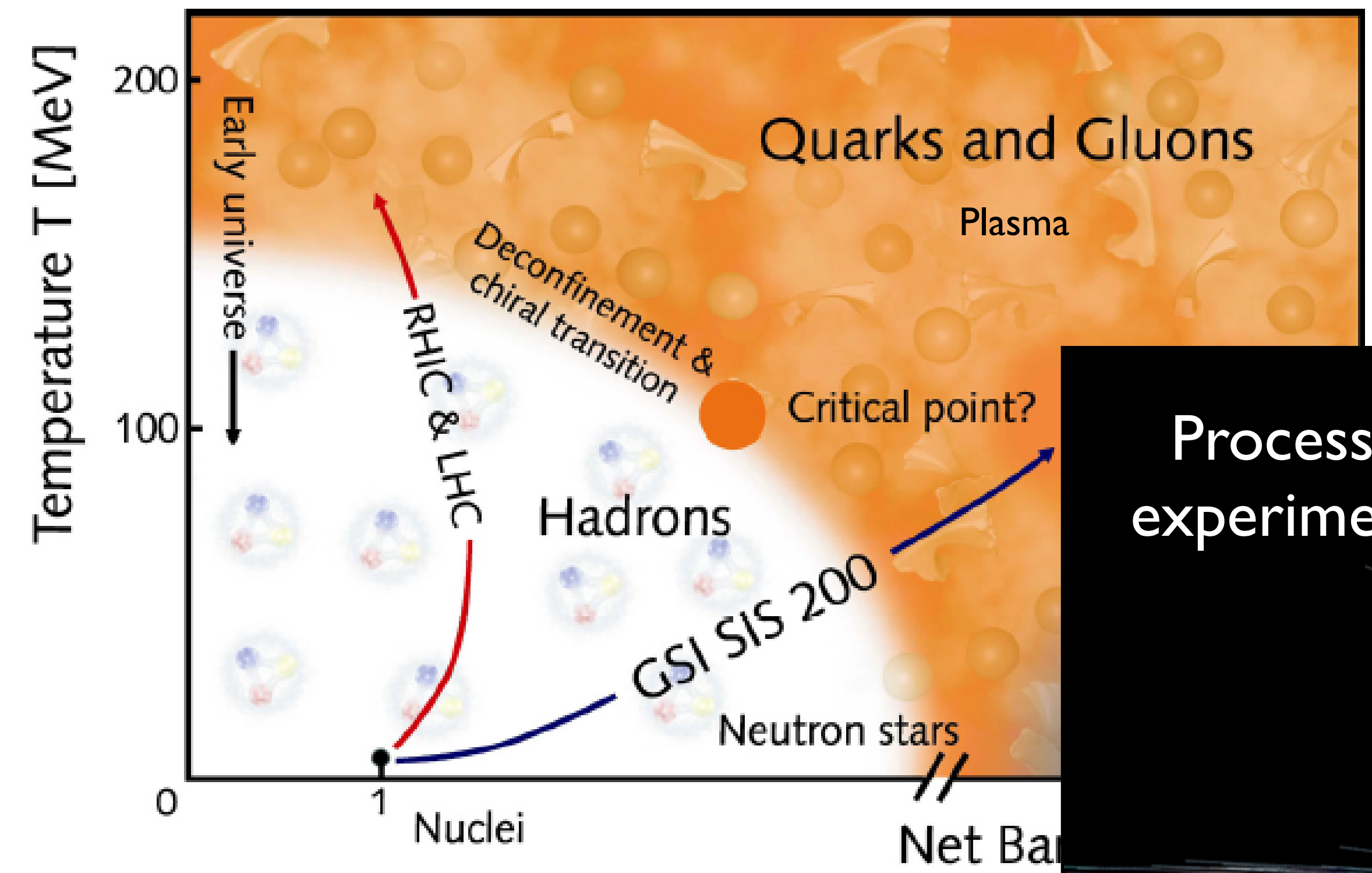
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A simulated collision of lead ions, courtesy the ALICE experiment at CERN

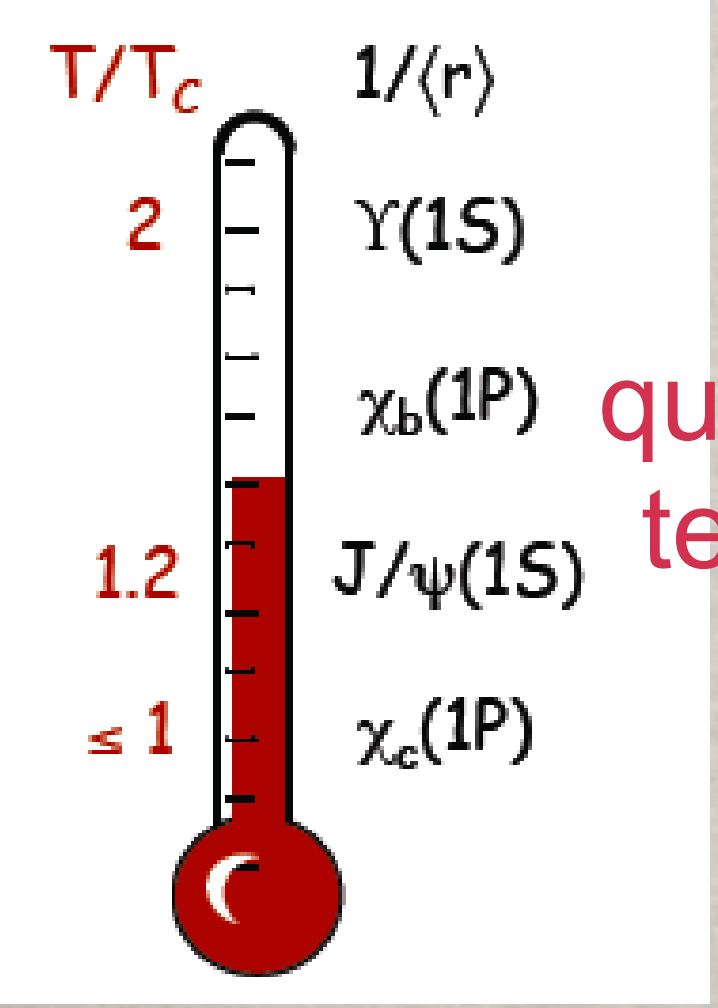
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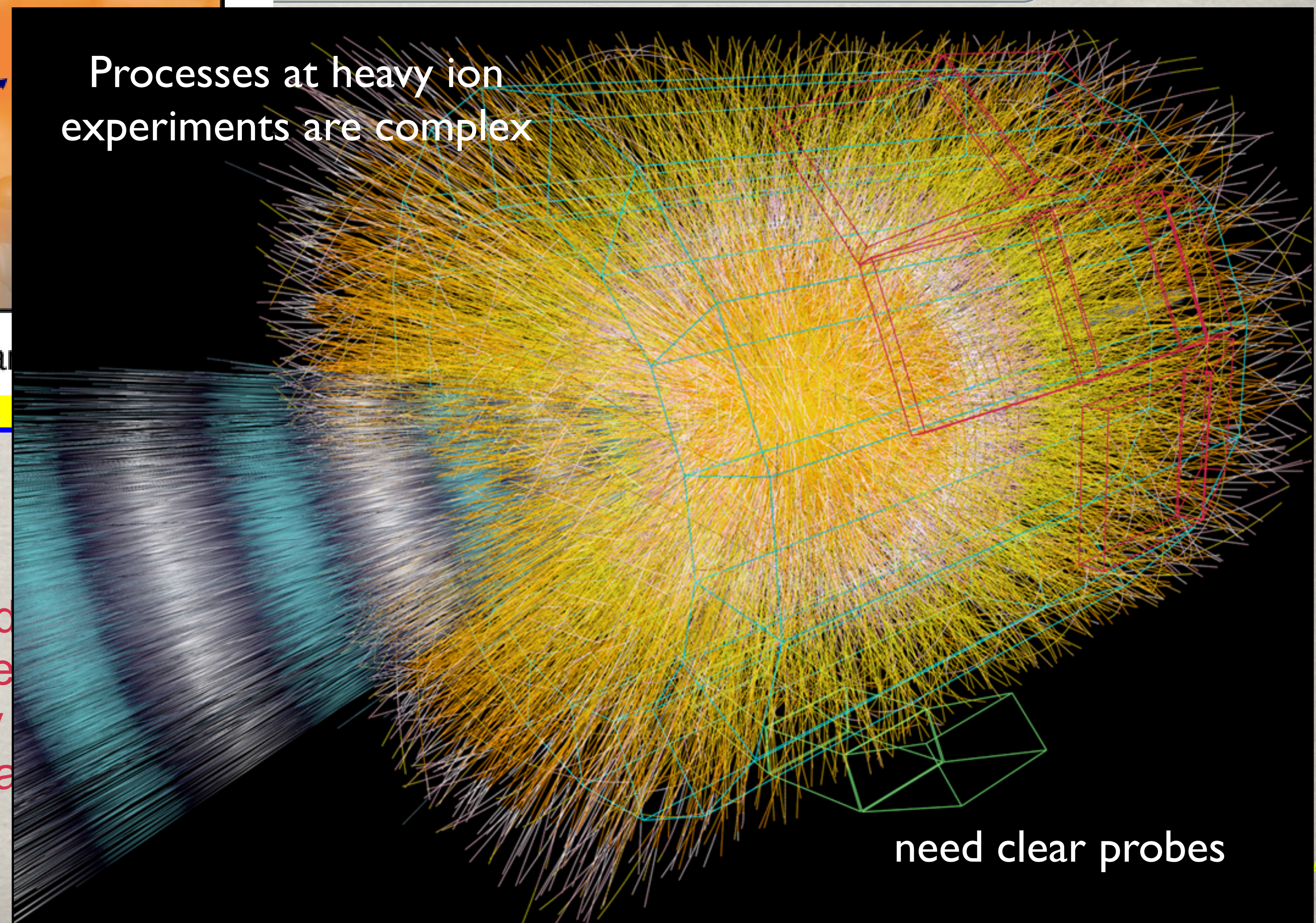


Processes at heavy ion experiments are complex

Nuclear Matter



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need clear probes

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Nonrelativistic Effective Field Theories (NREFTs) can give an answer to this
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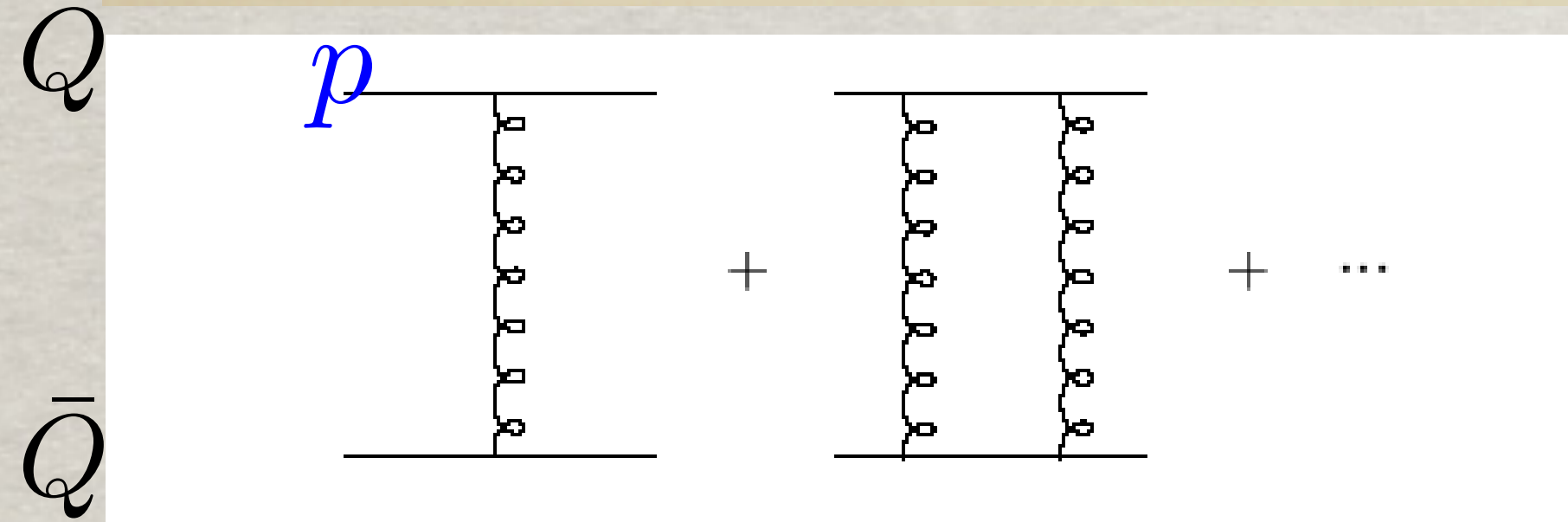
QCD theory of Quarkonium: a very hard problem even at $T=0$

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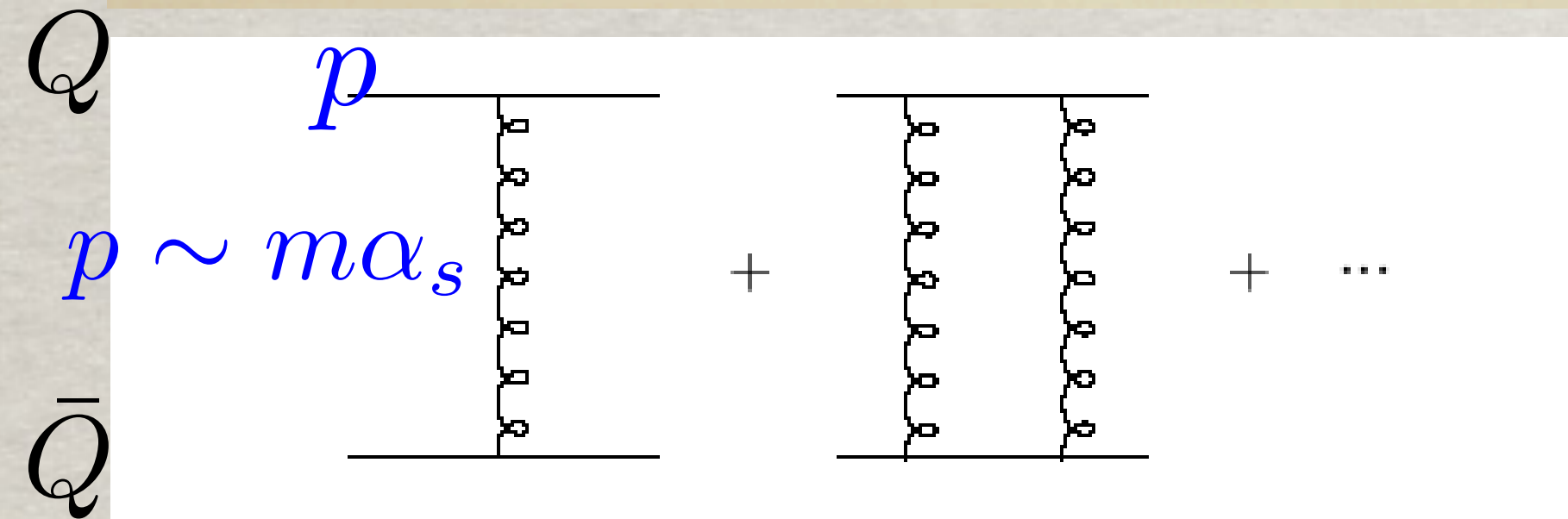
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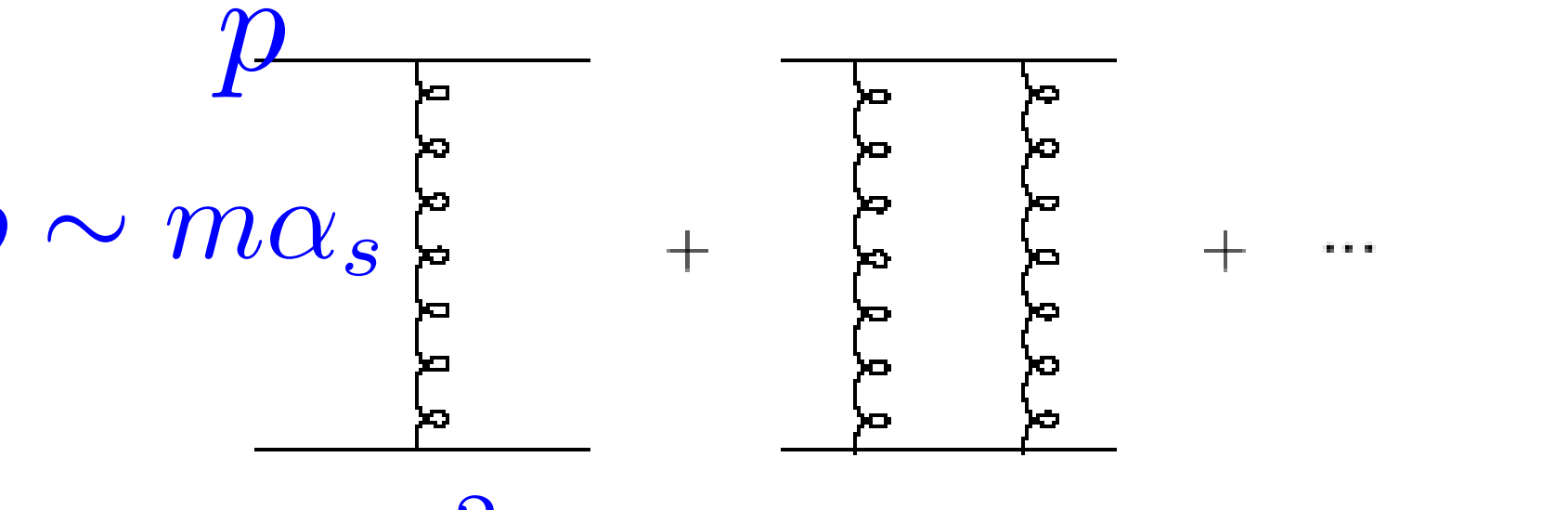
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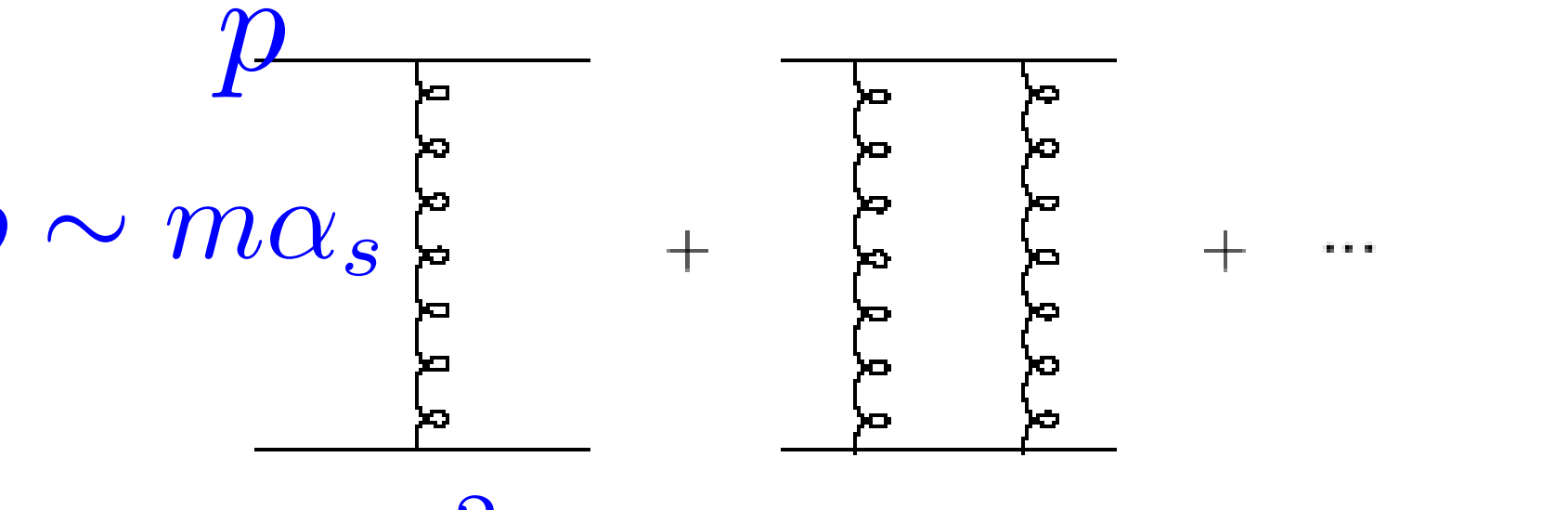


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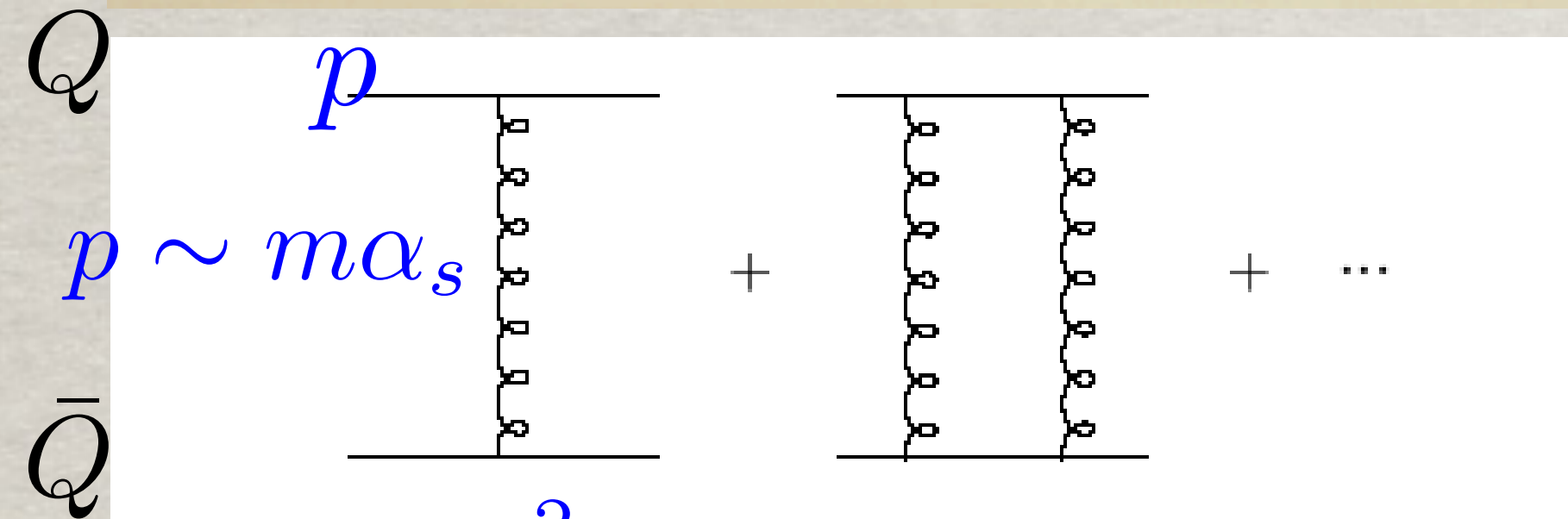


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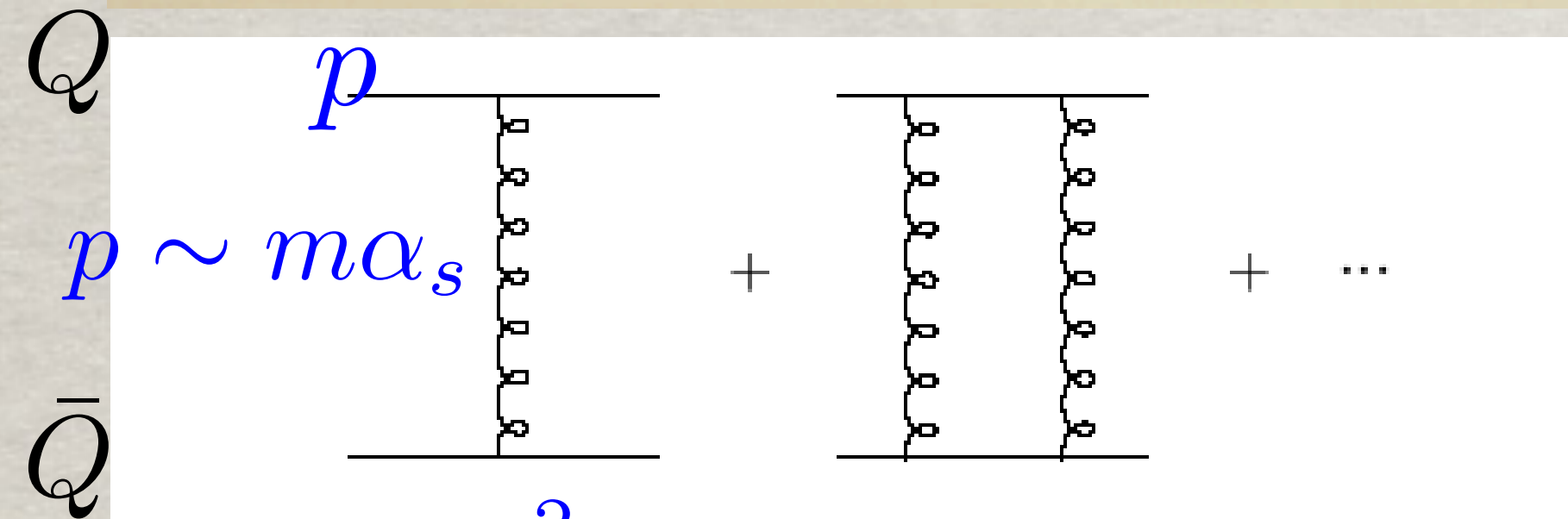
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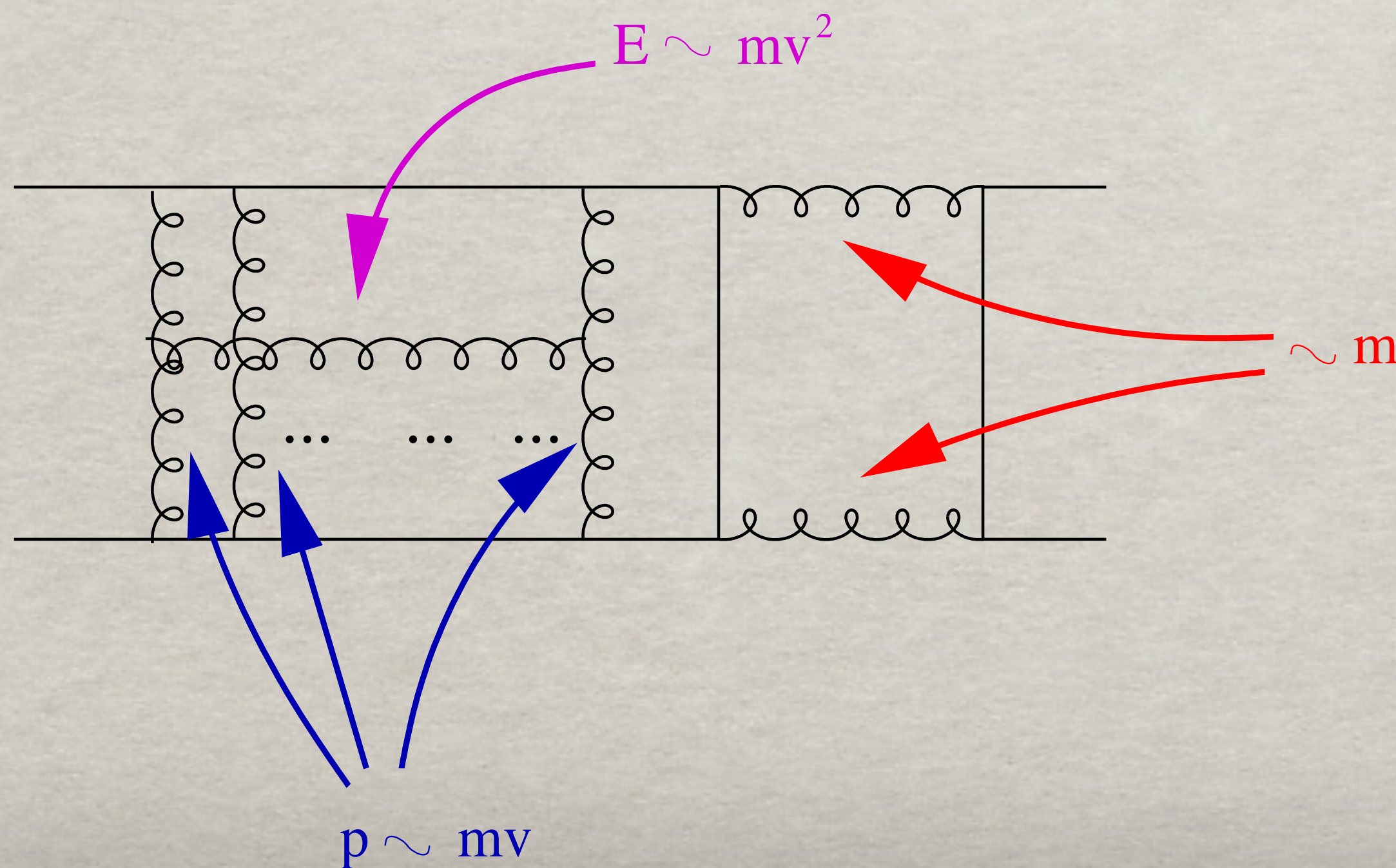
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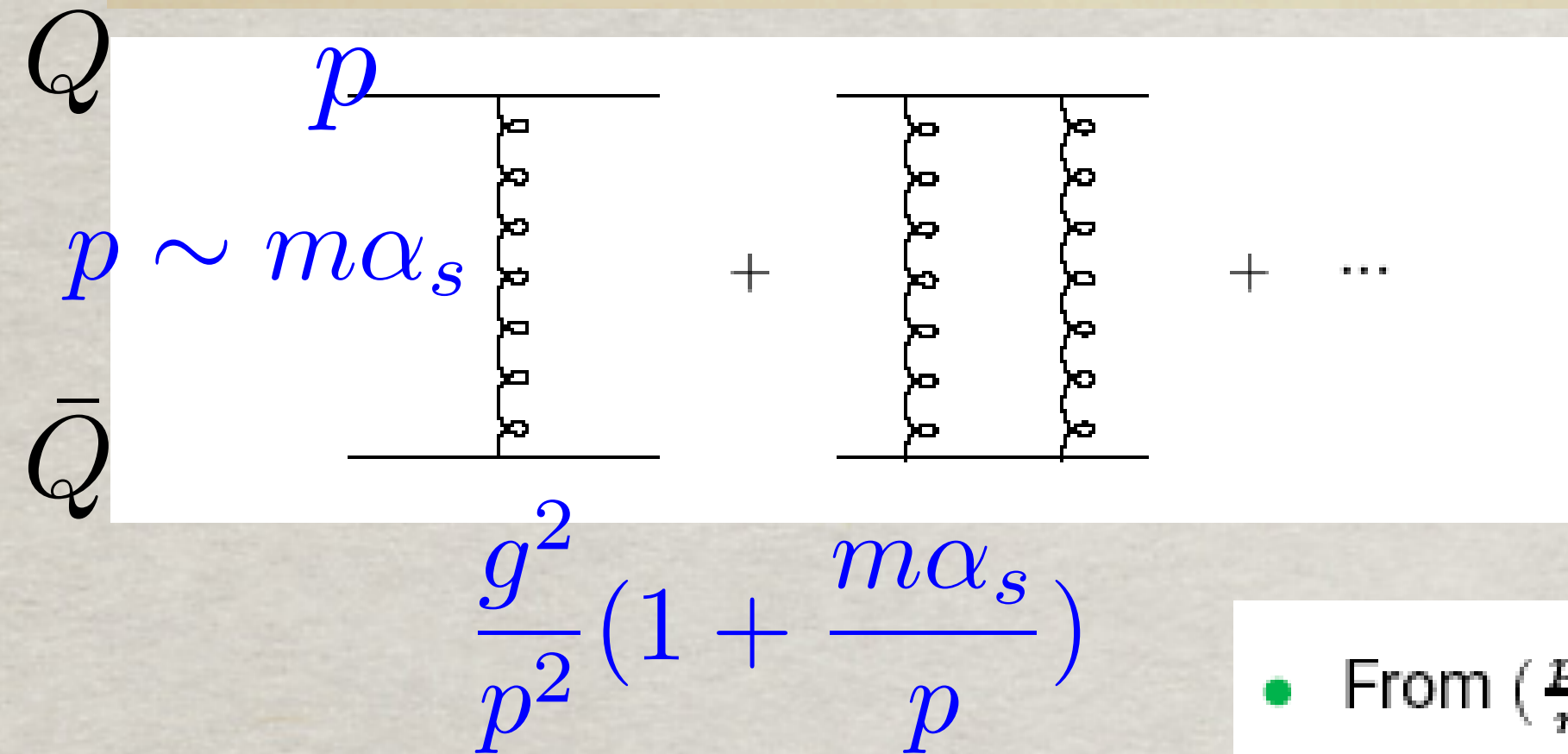
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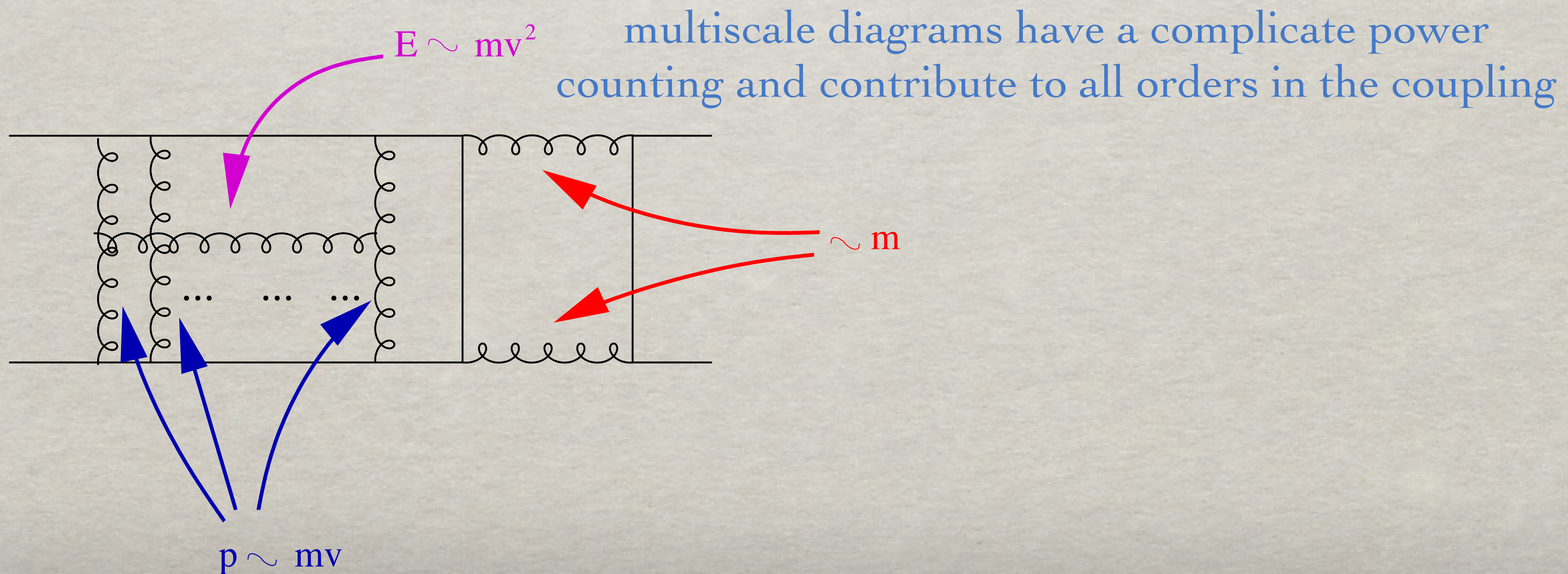
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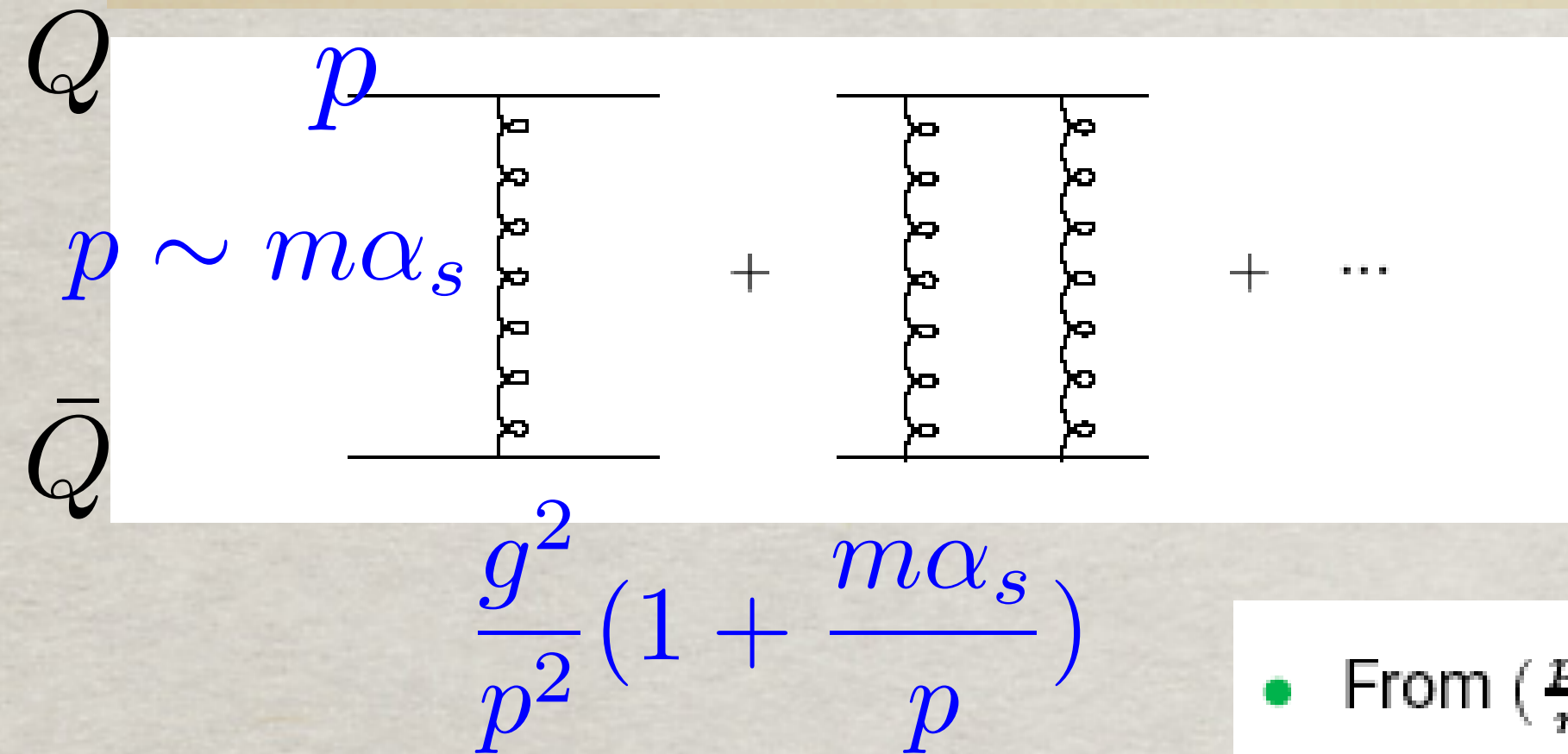
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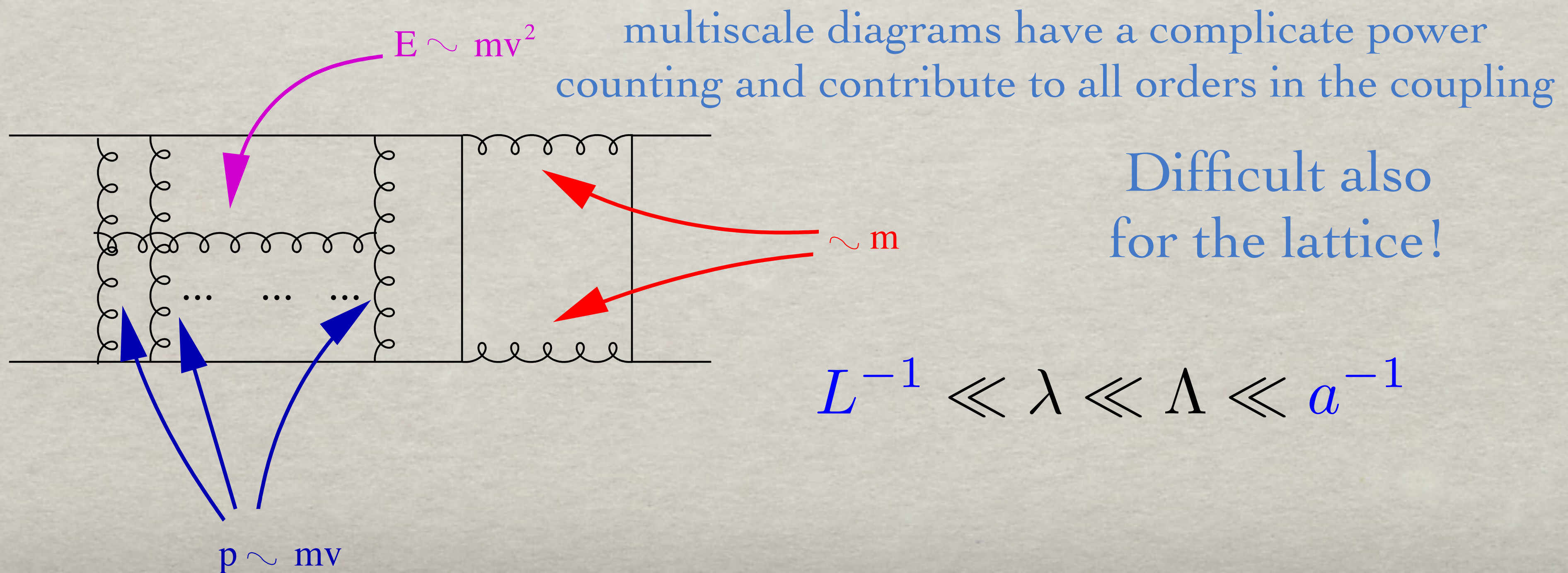
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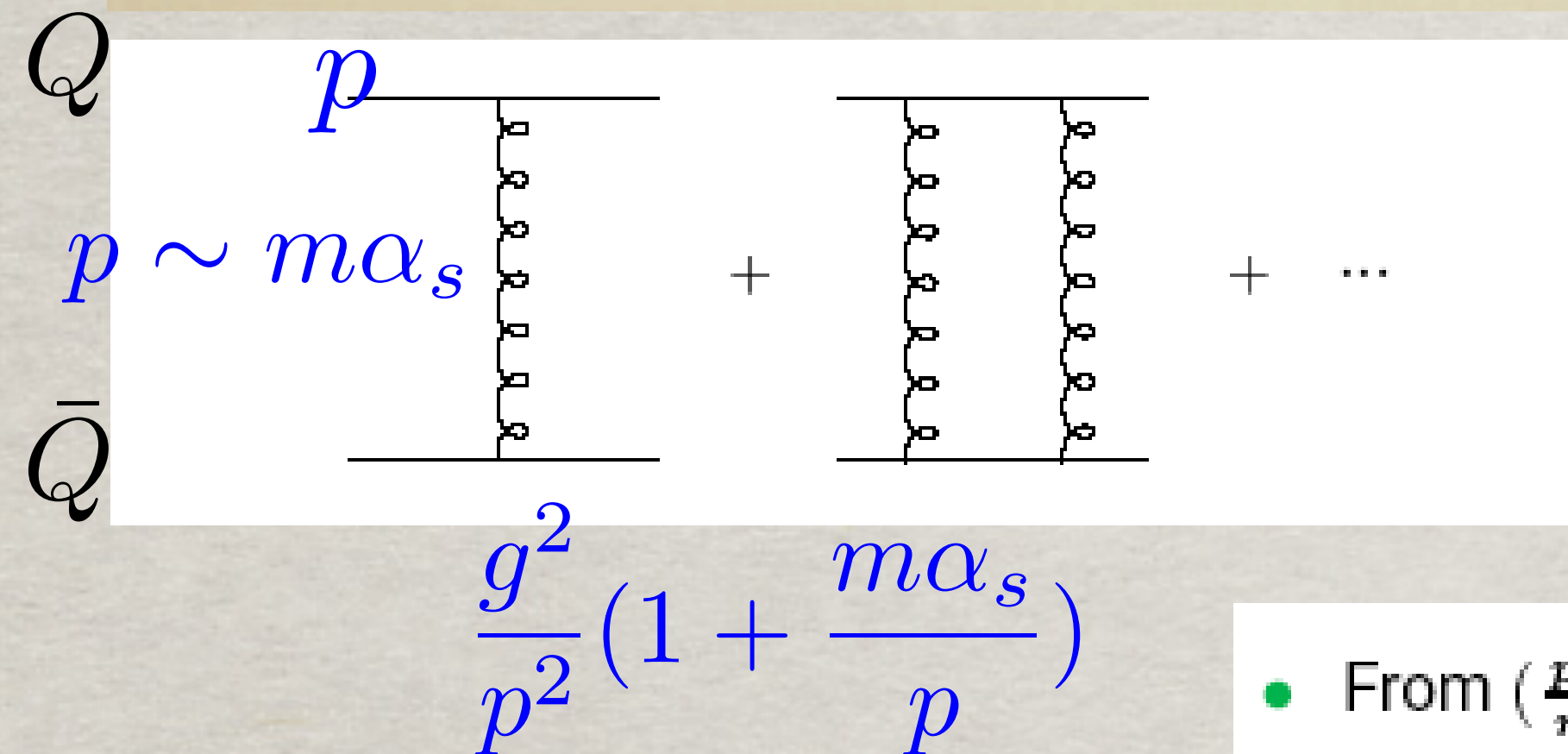
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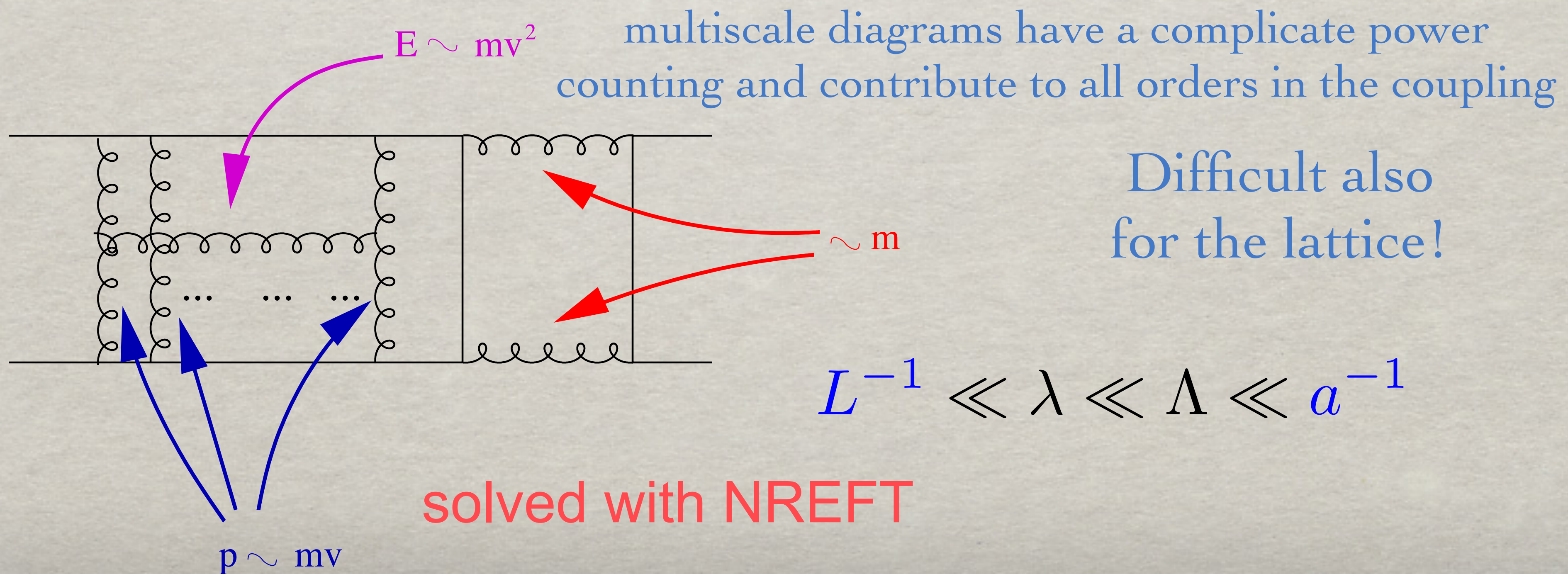
QCD theory of Quarkonium: a very hard problem even at T=0

Close to the bound state $\alpha_s \sim v$



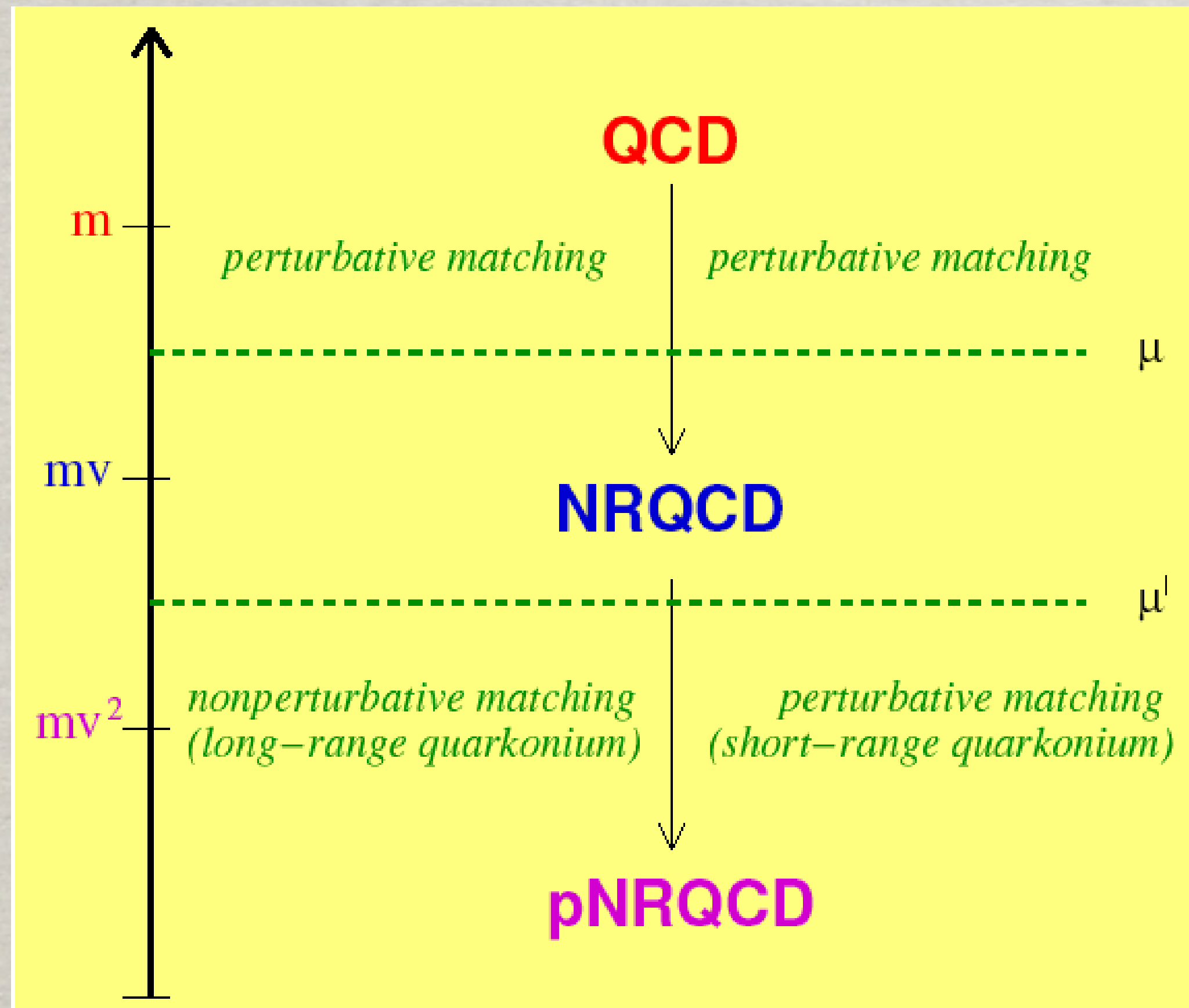
$$\sim \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$

- From $\left(\frac{p^2}{m} + V\right)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$.



Quarkonium with NR EFT

Color degrees of freedom
 $3 \times 3 = 1 + 8$
singlet and octet $Q\bar{Q}$



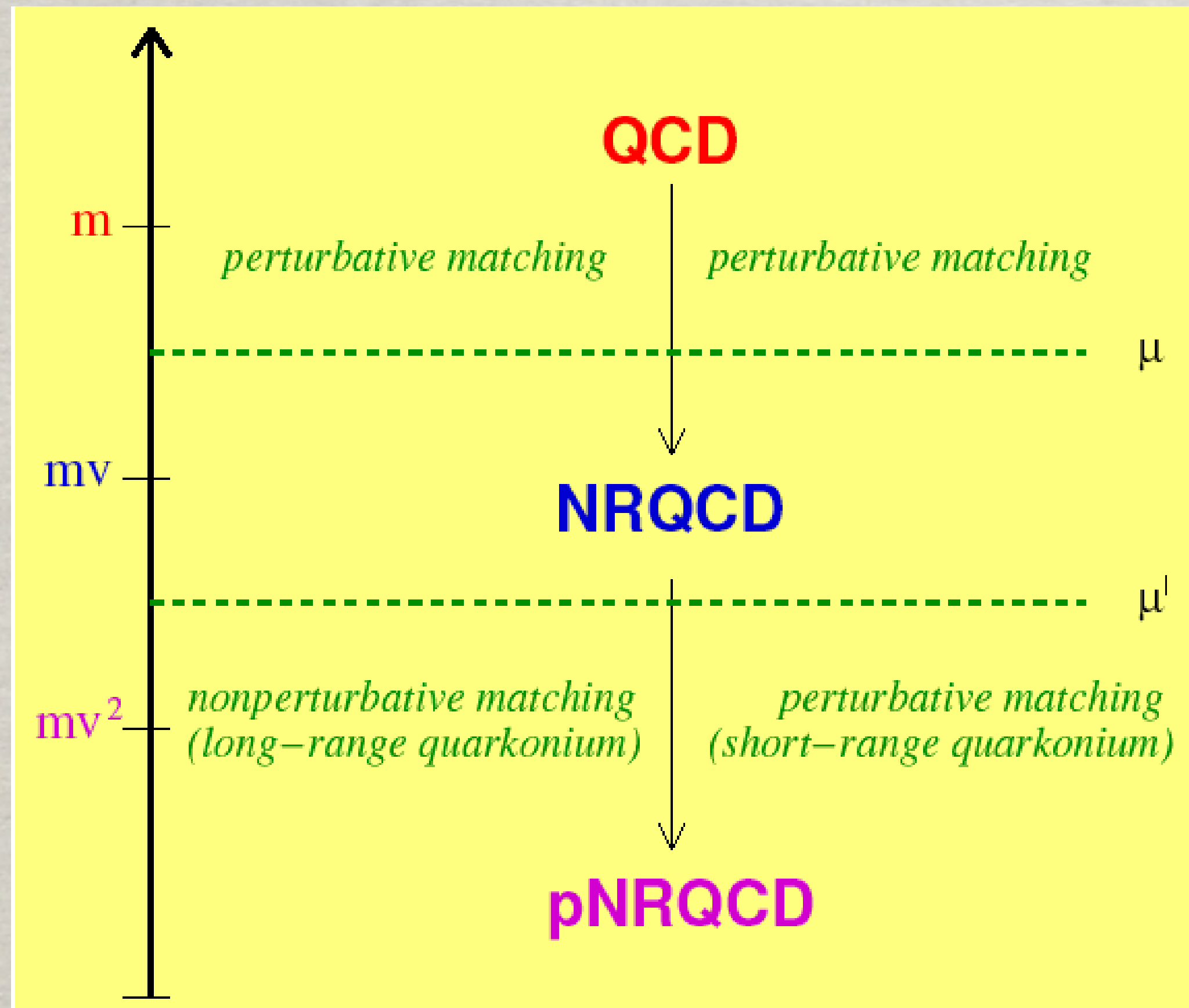
Hard

Soft
(relative
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Ultrasoft
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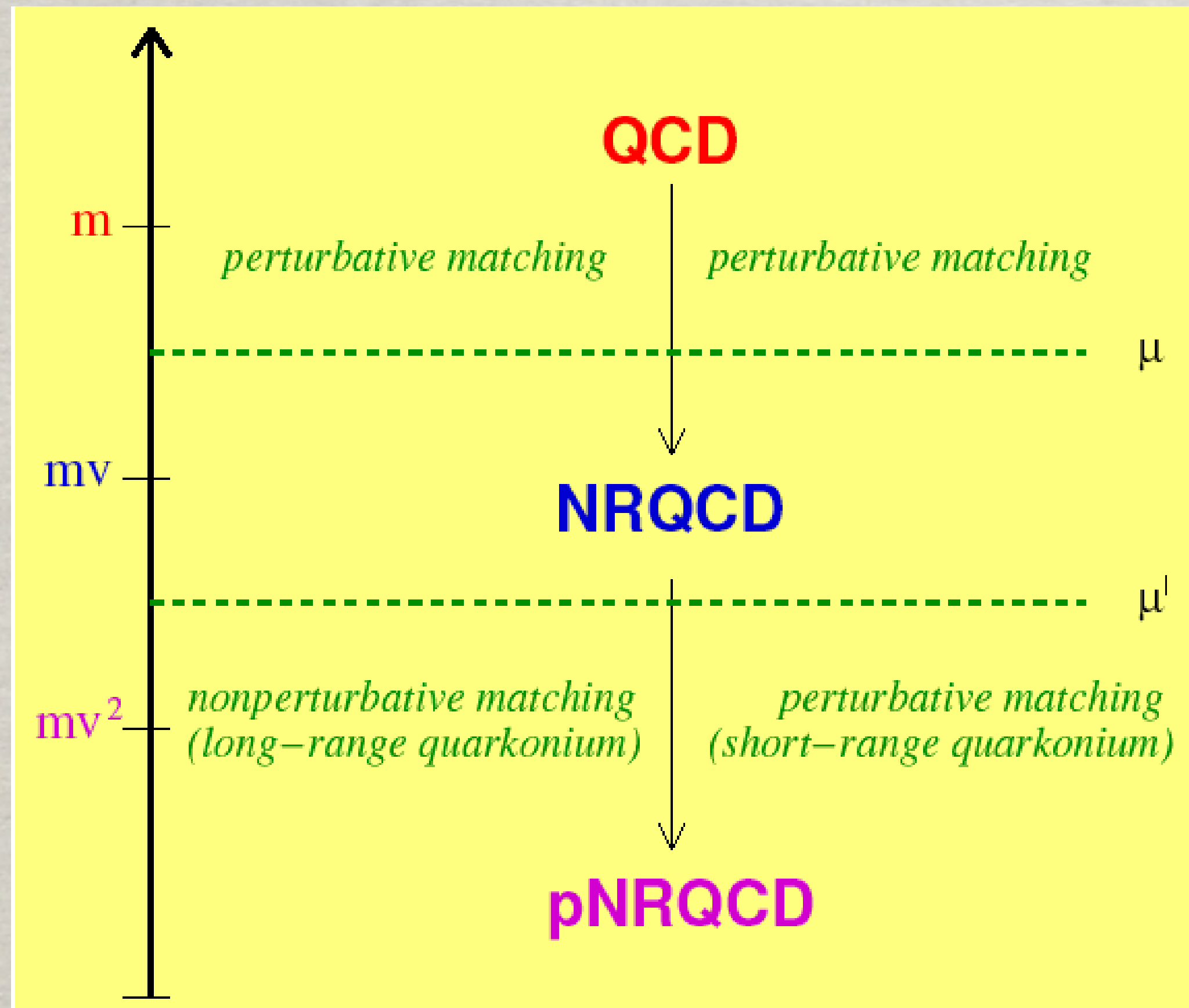
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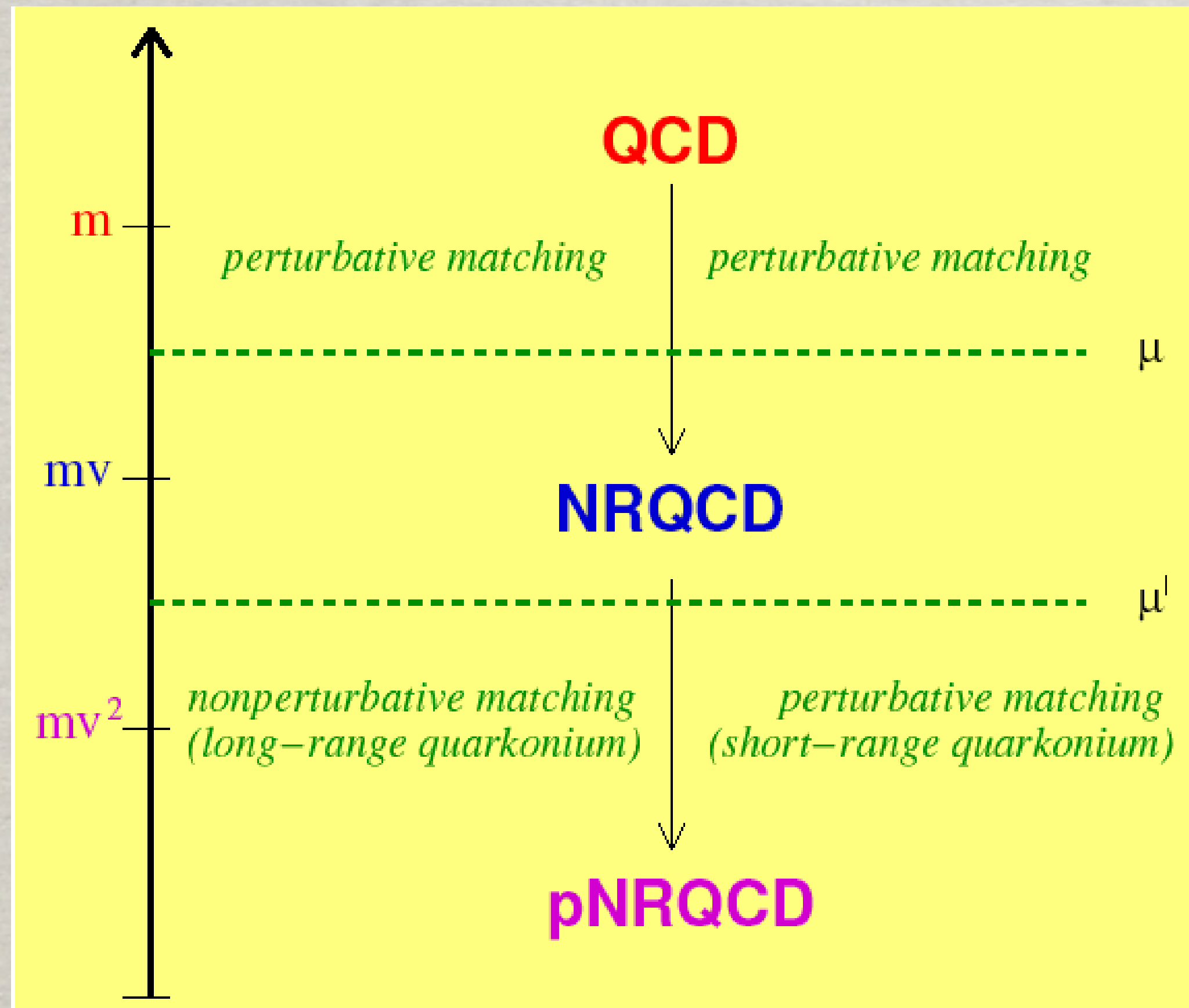
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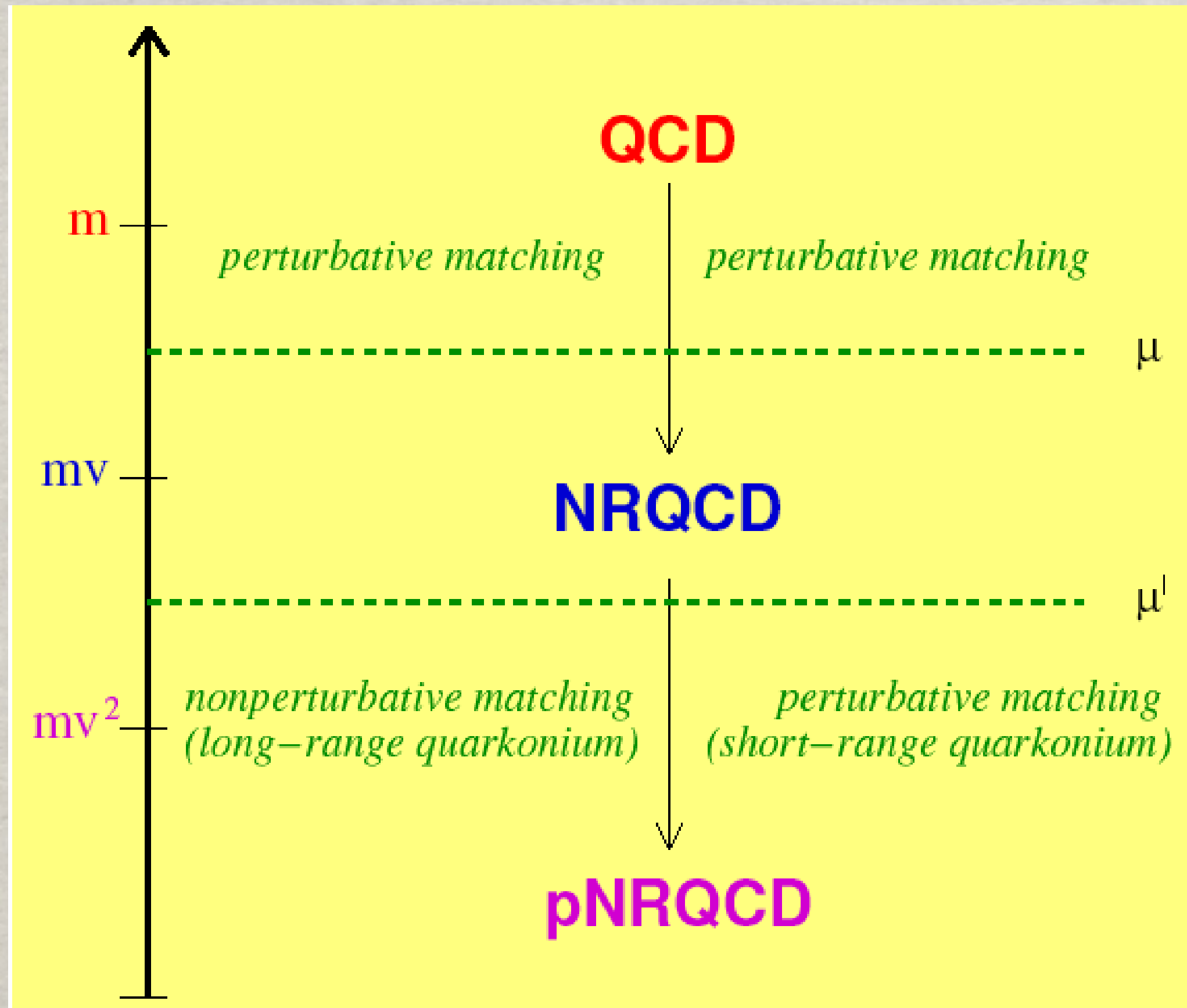
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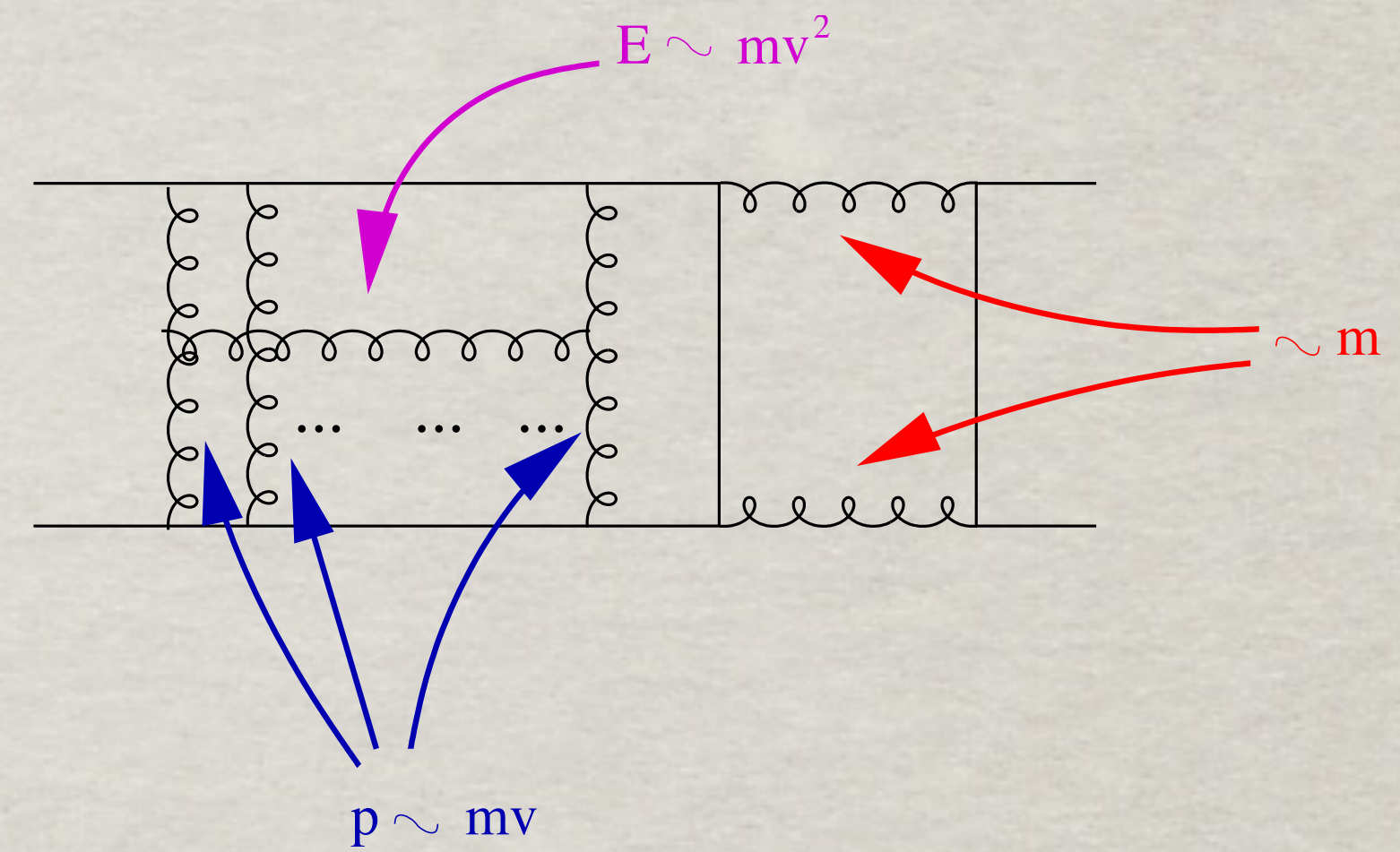
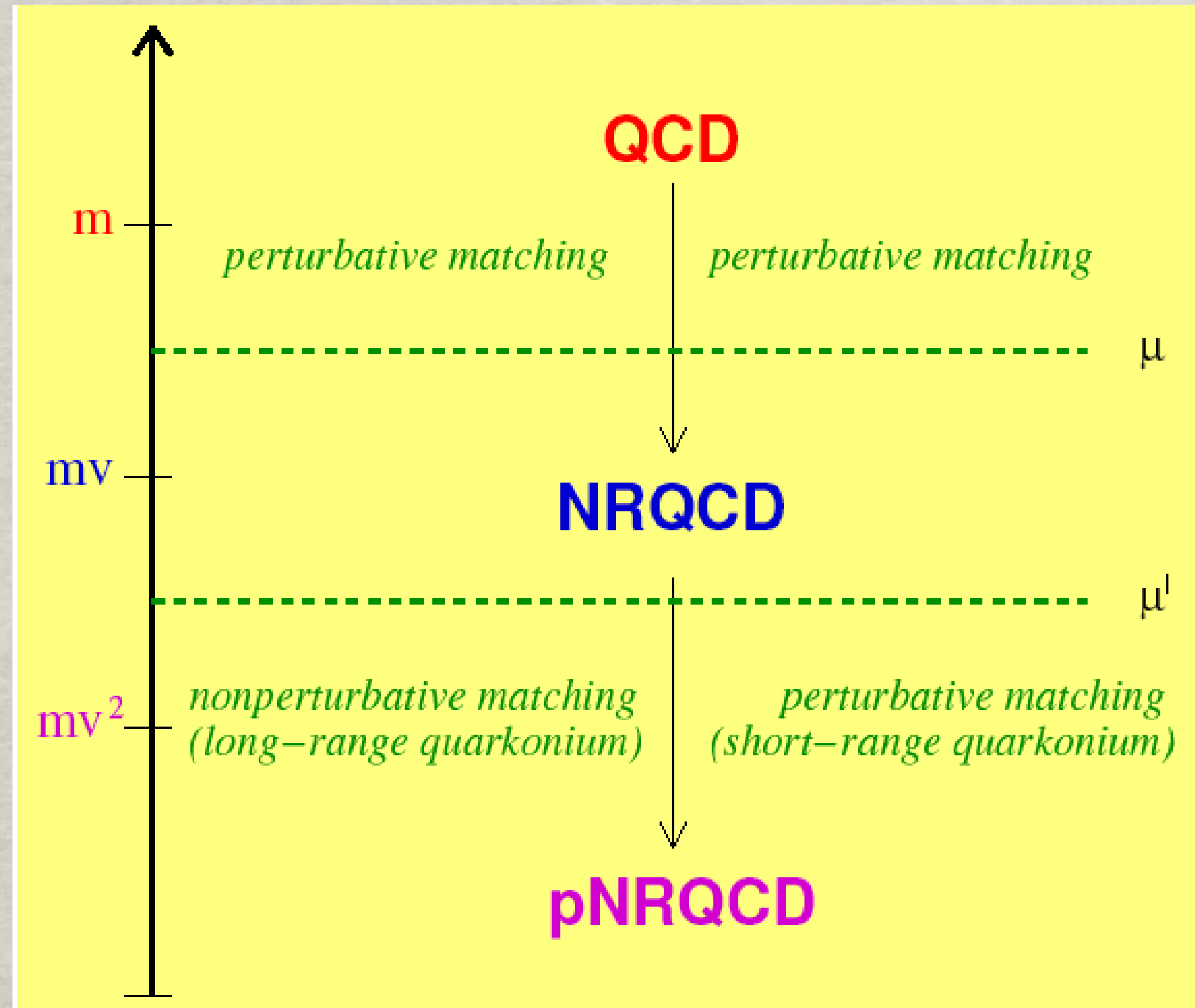
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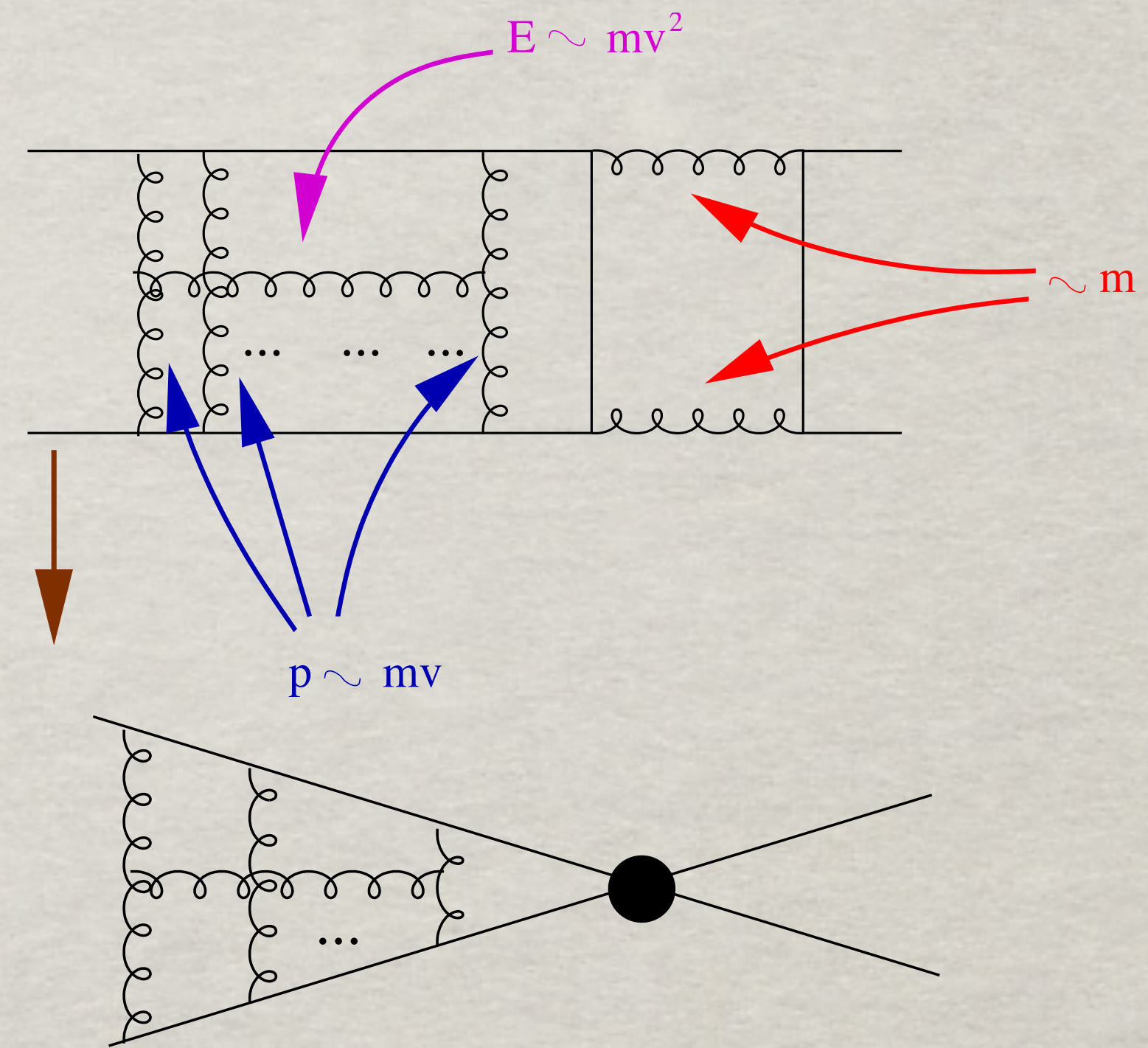
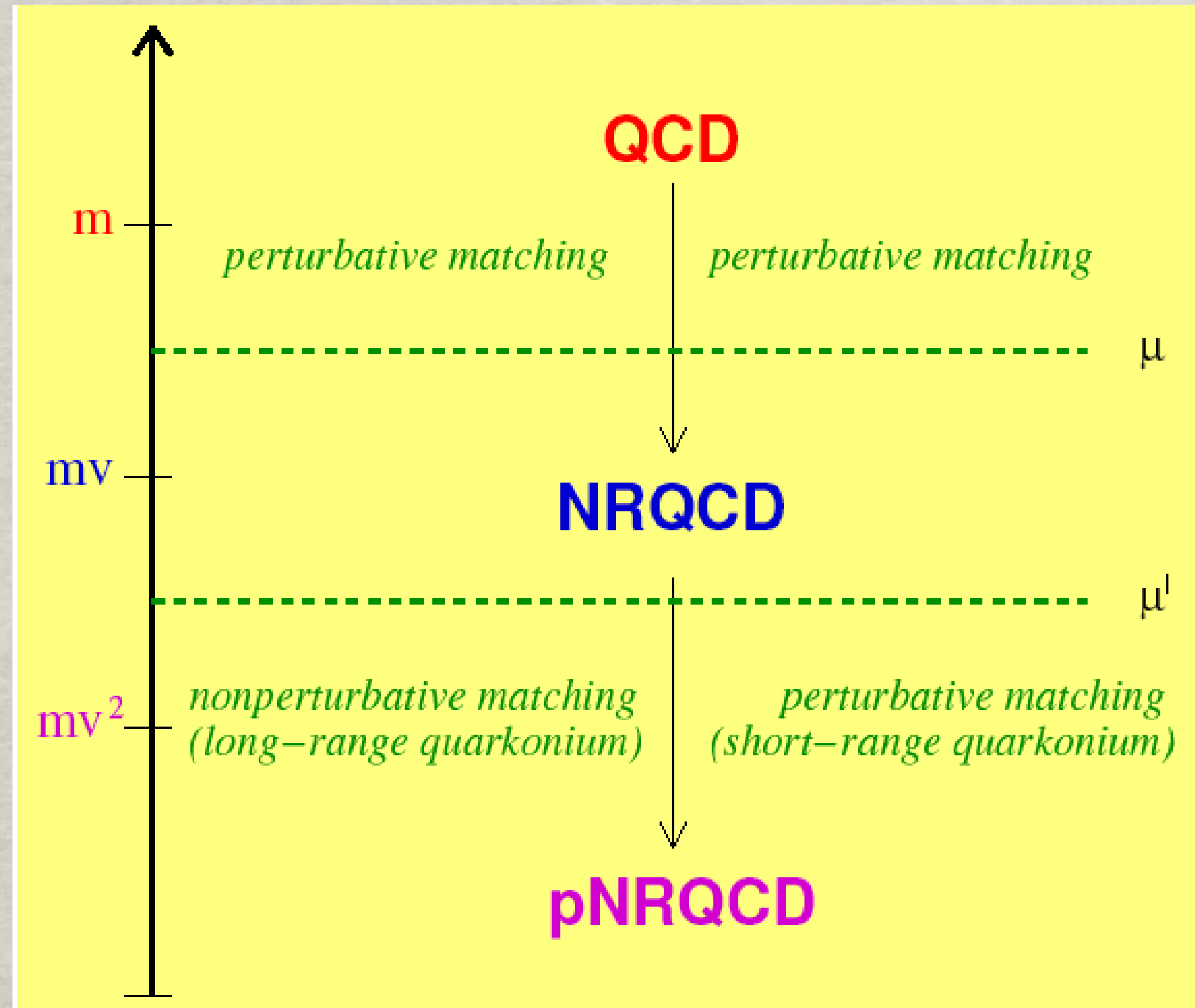
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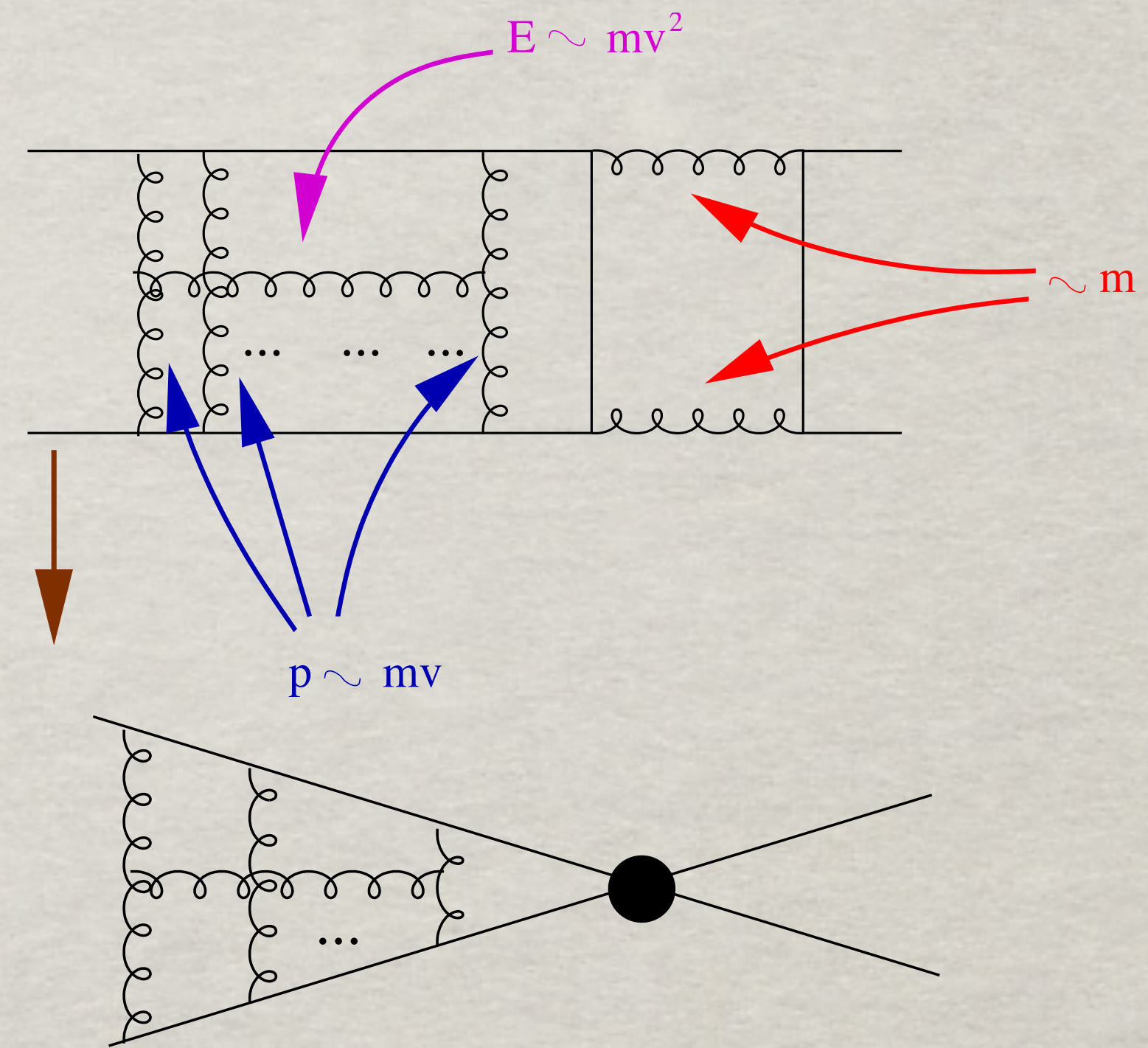
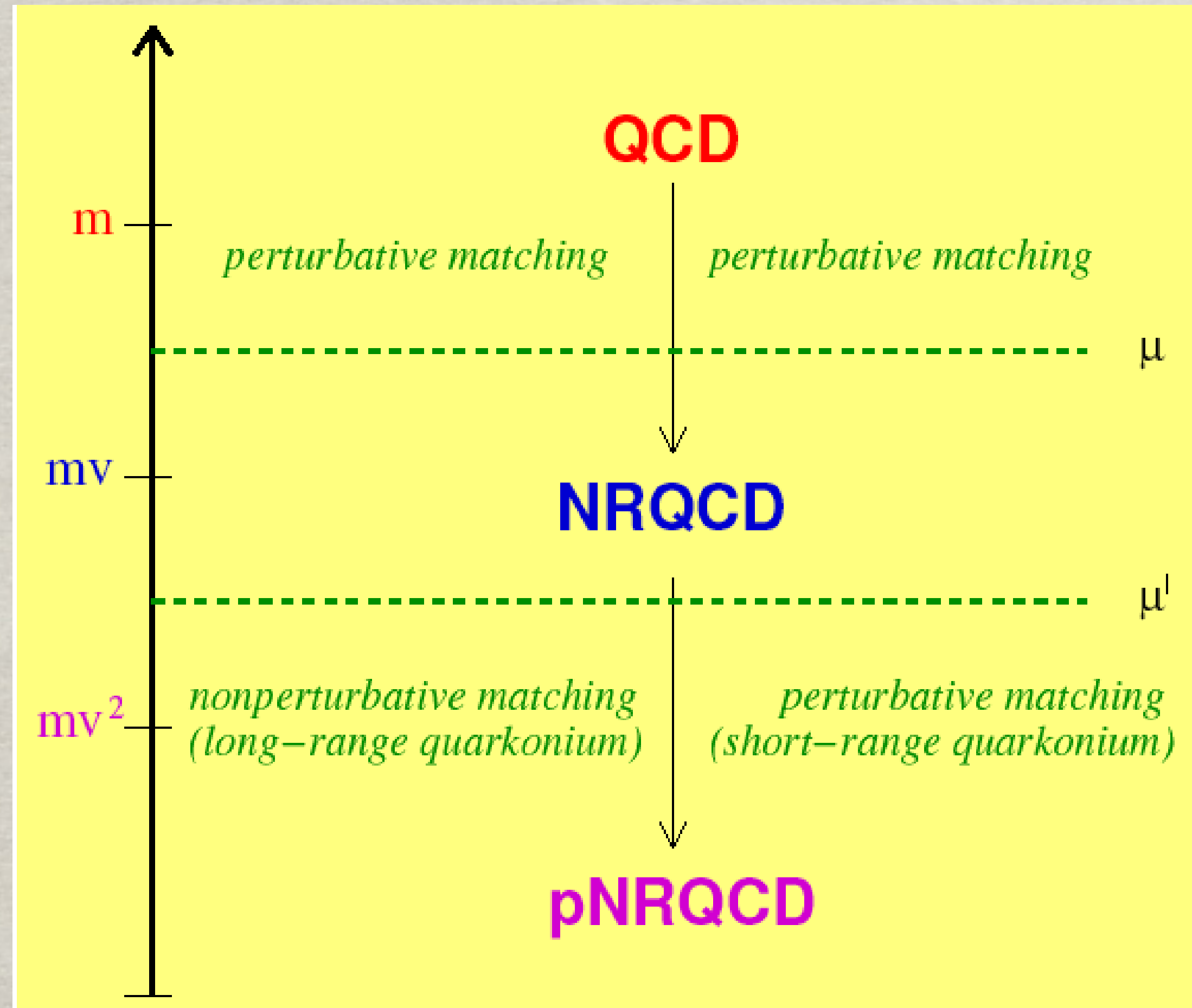
Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)



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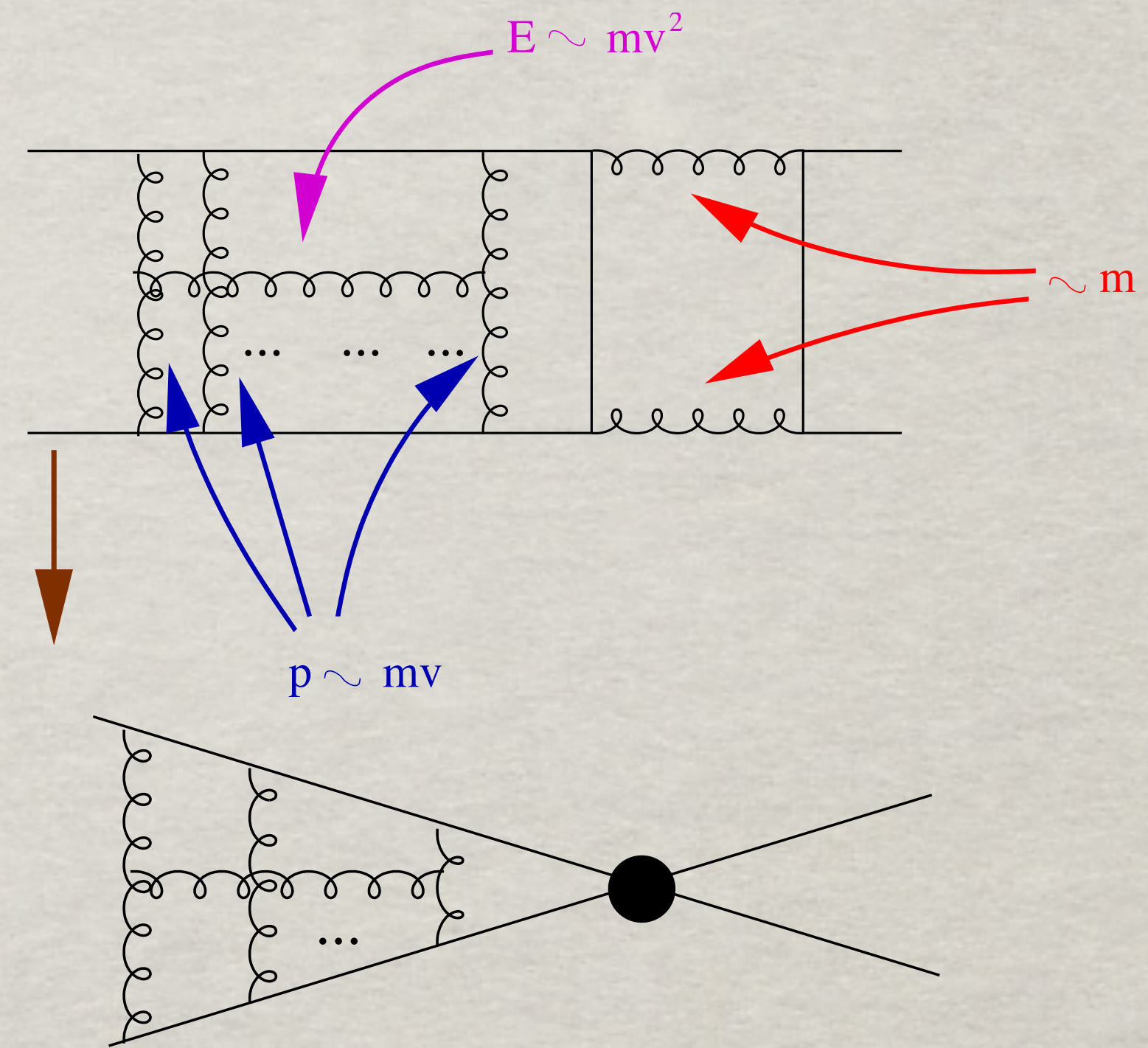
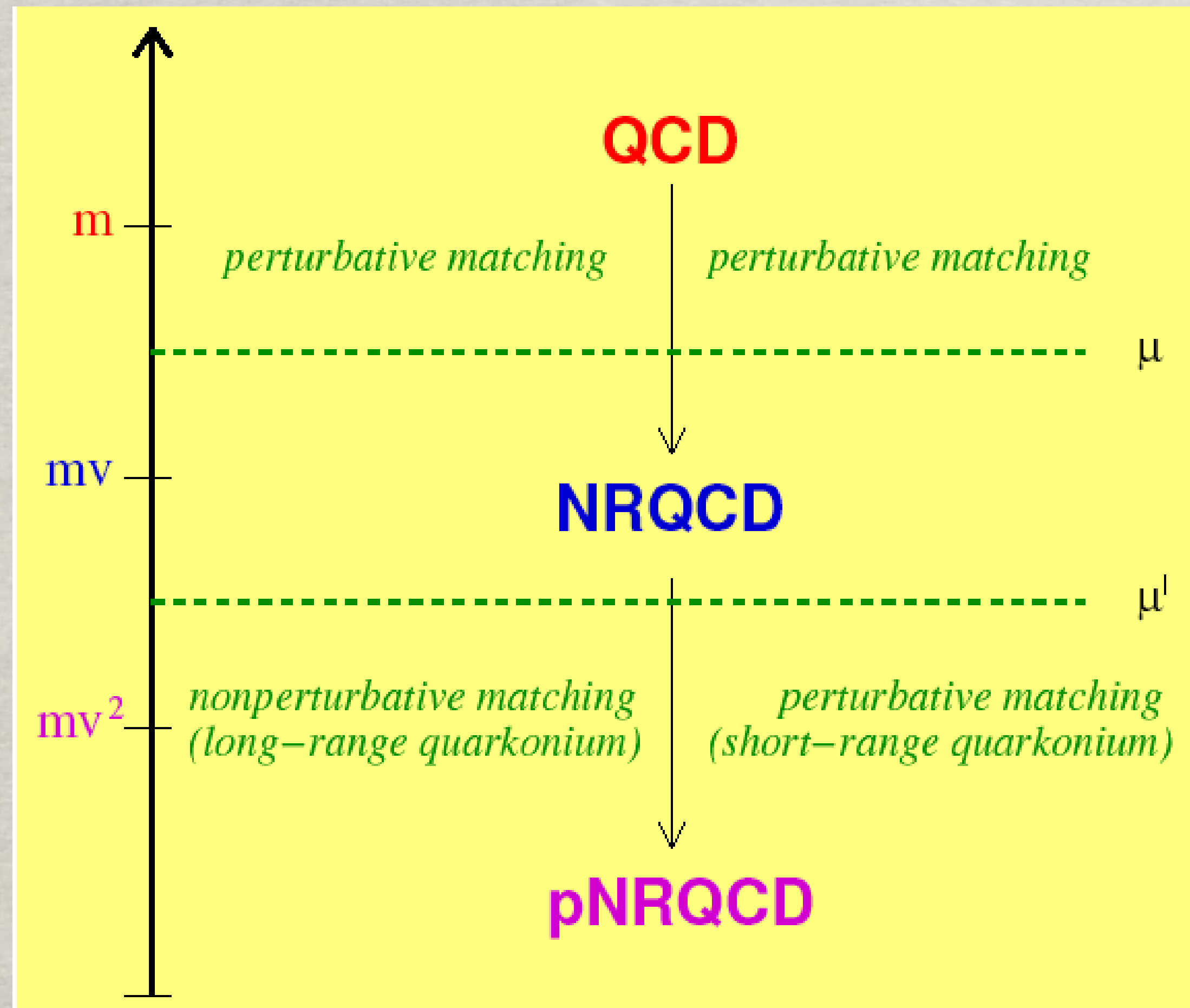


Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)

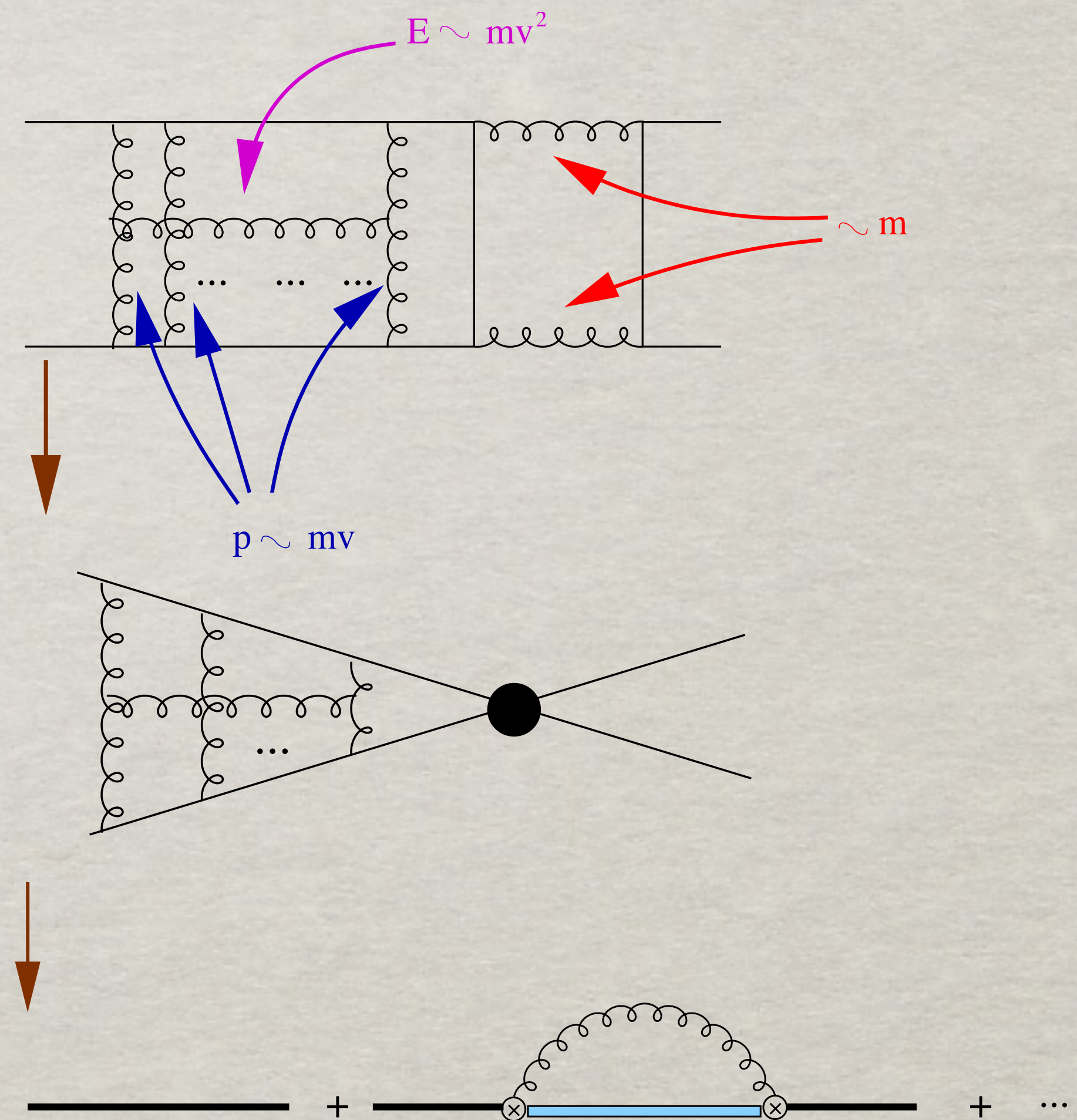
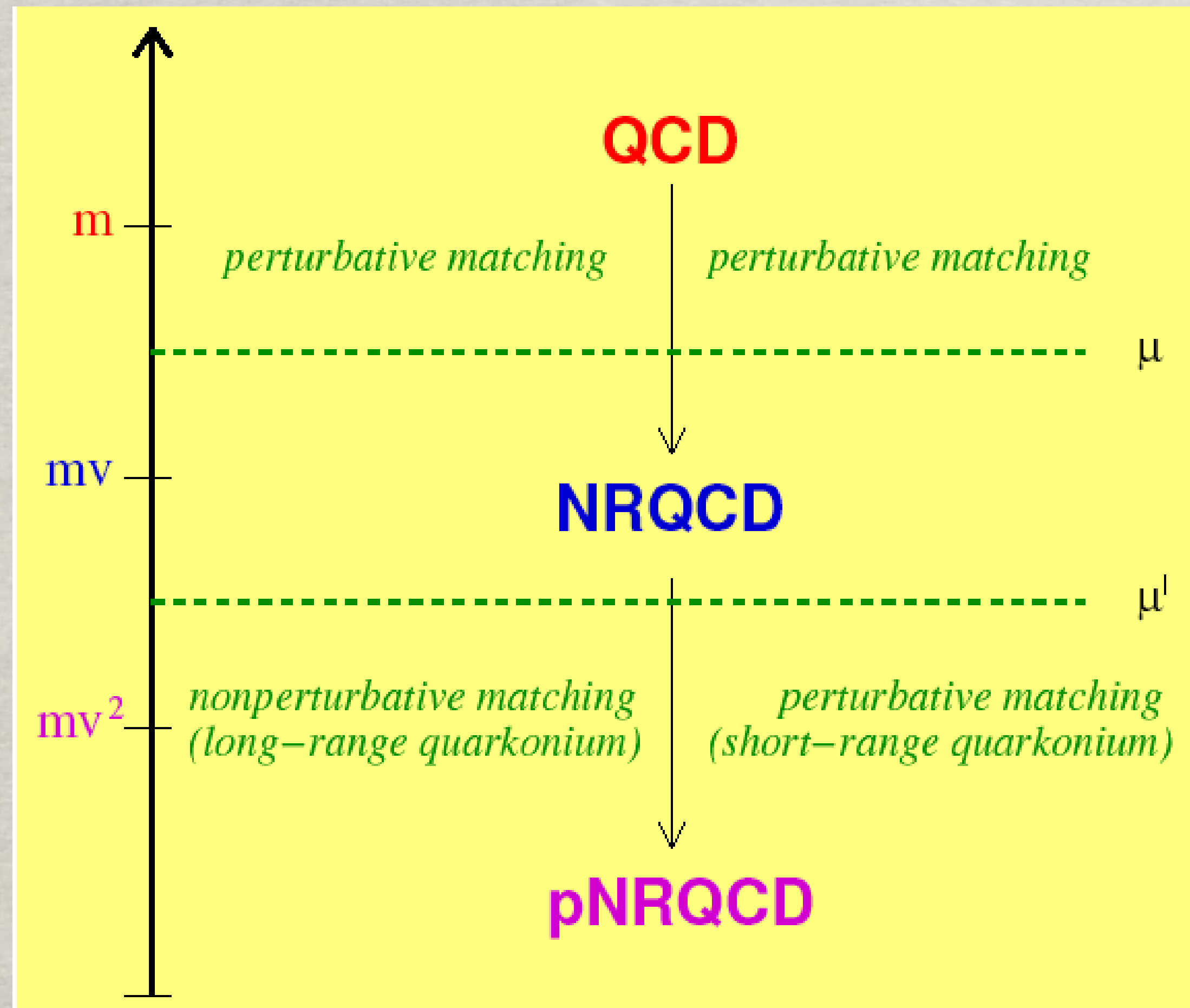


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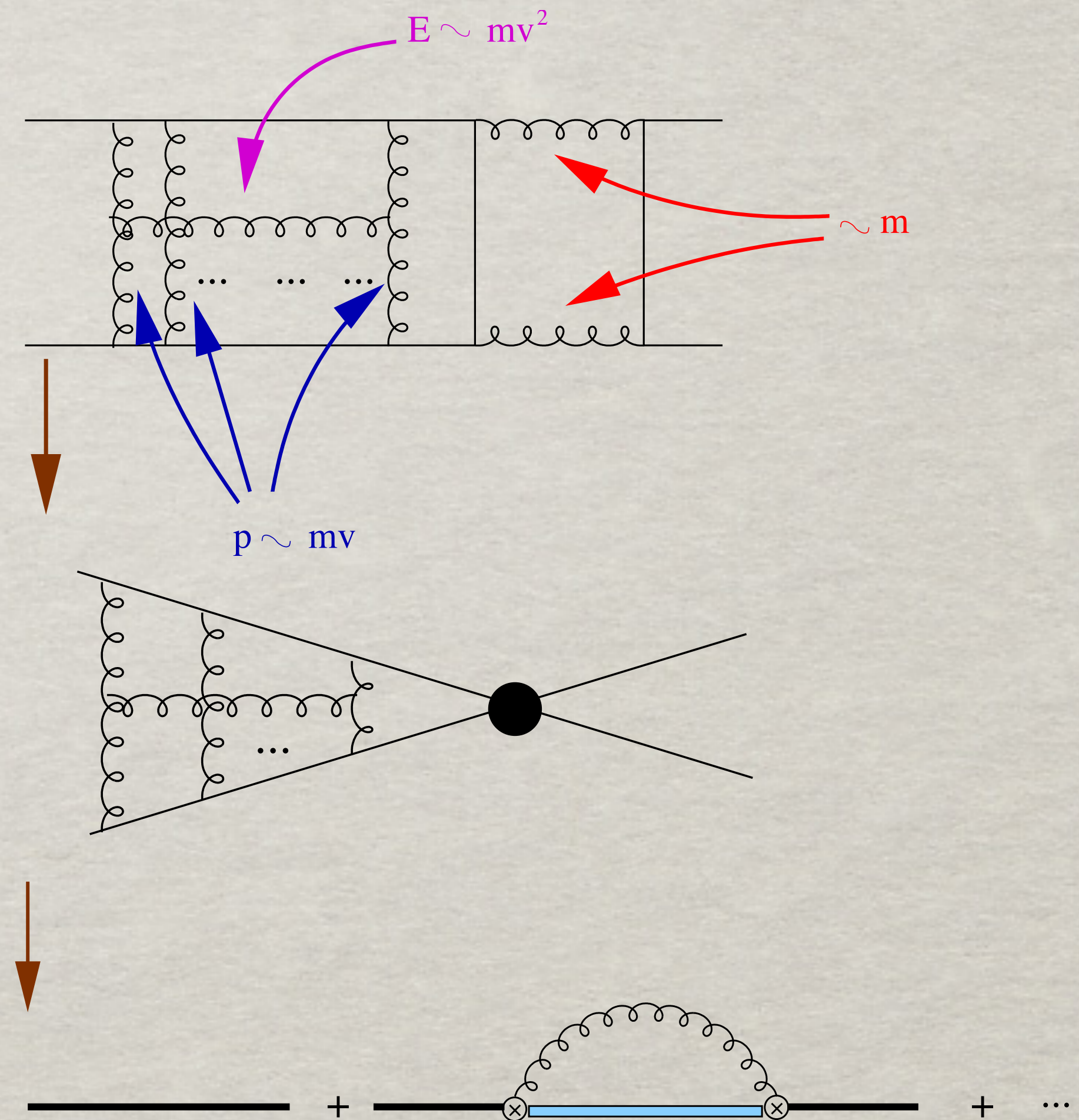
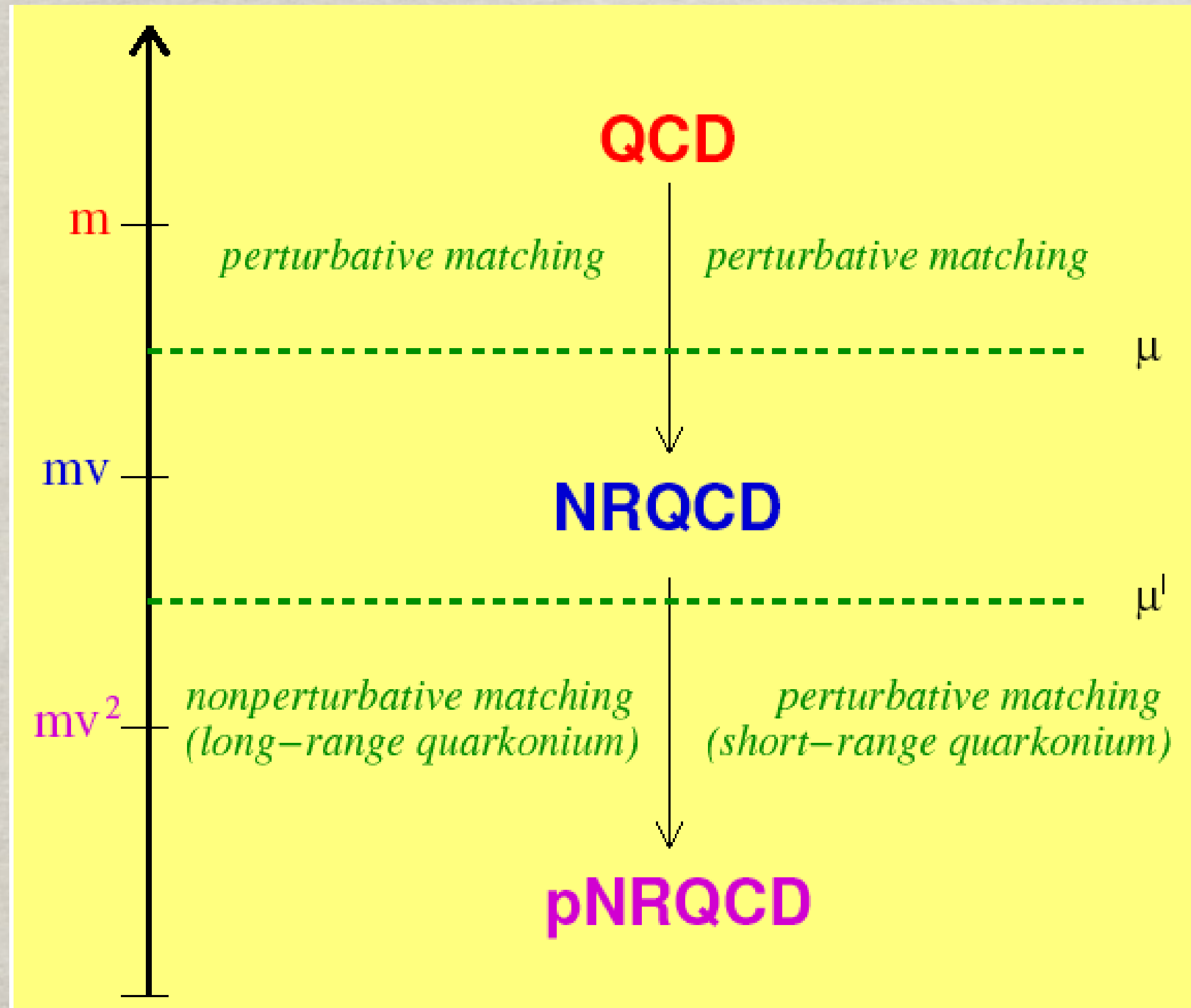
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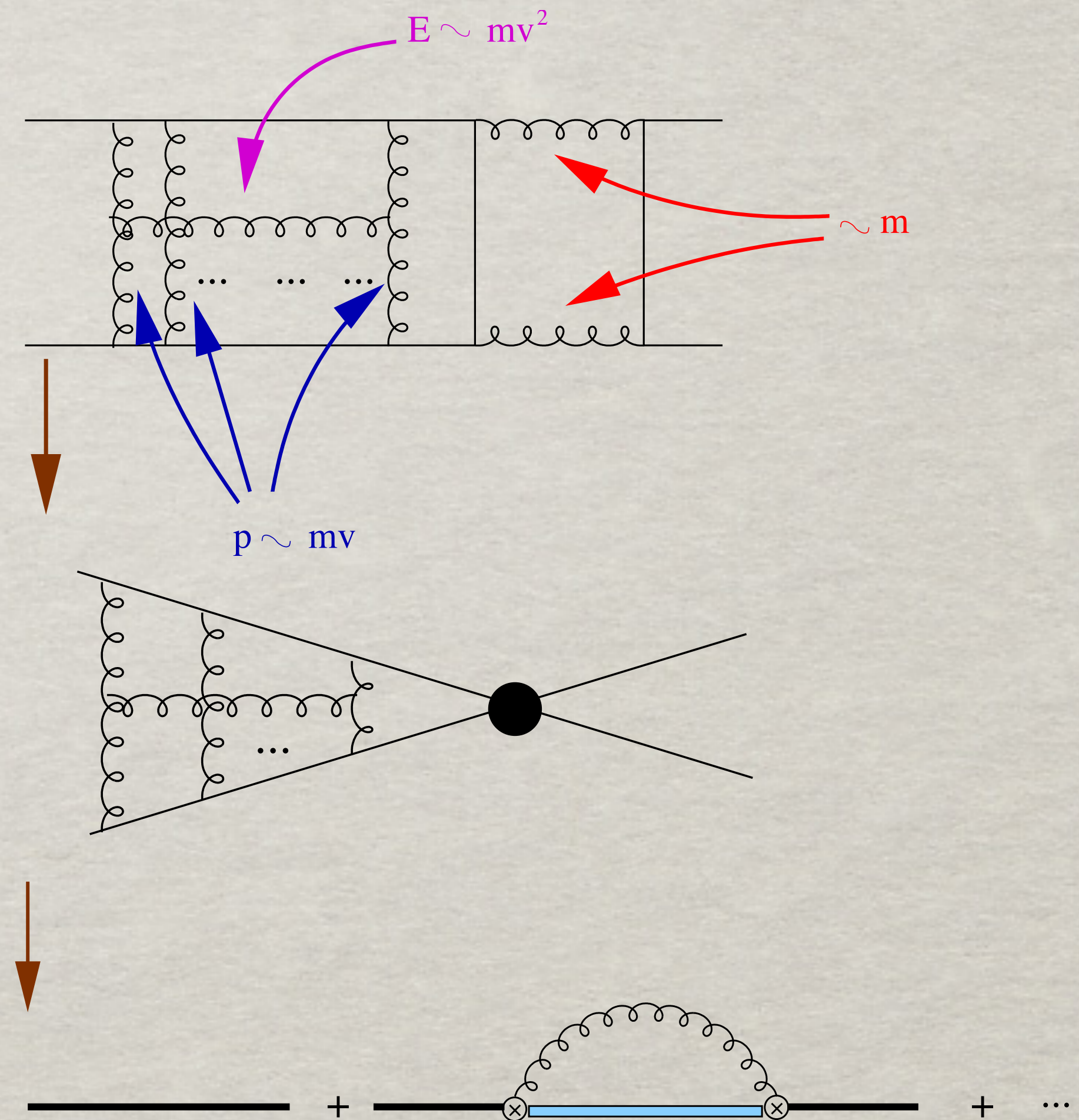
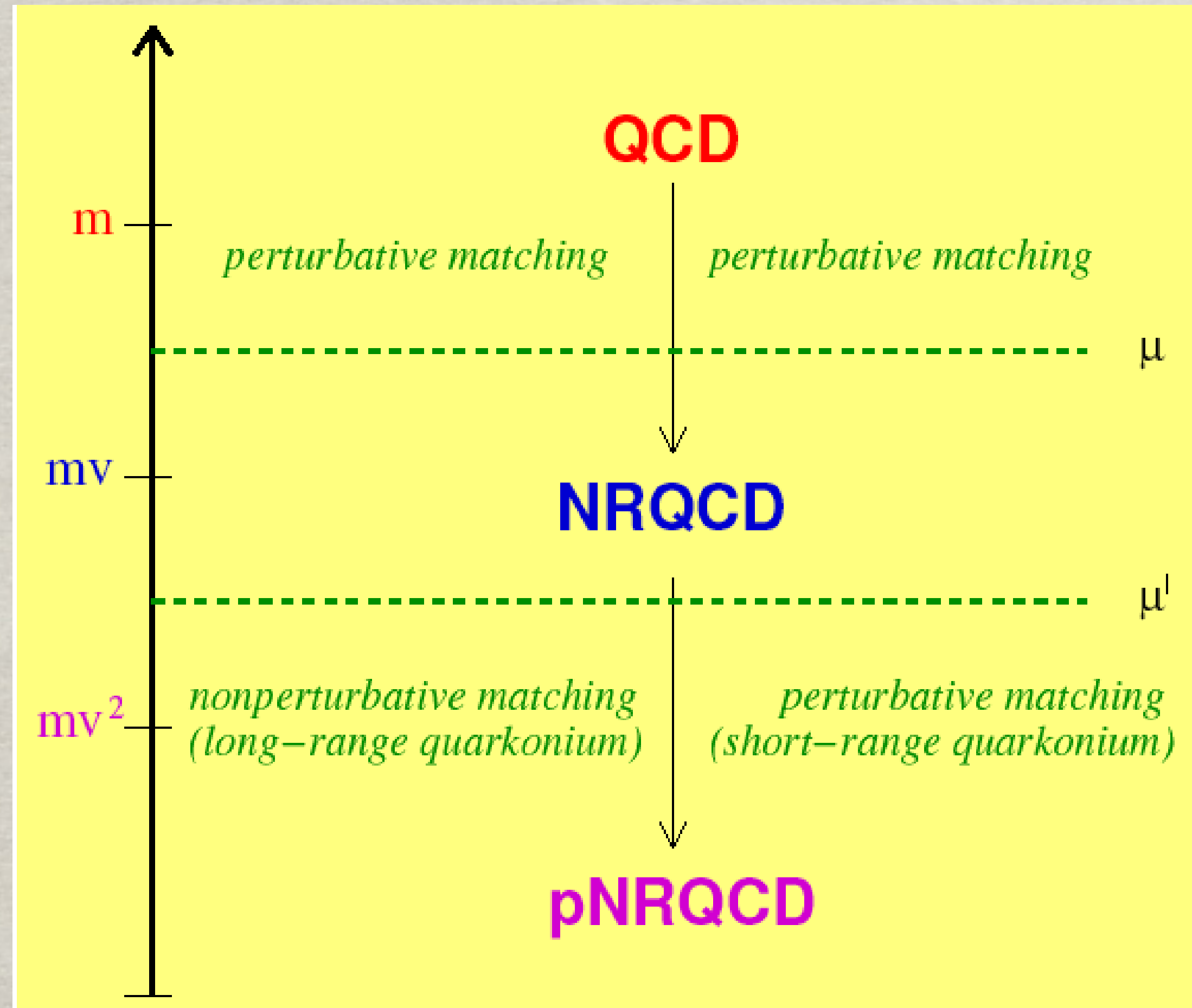


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$$\mathcal{L}_{\text{pNRQCD}} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda) r^n$$

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pNRQCD (potential NonRelativistic QCD) EFT for quarkonium for $r \ll \Lambda_{\text{QCD}}^{-1}$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \sum_{i=1}^{n_f} \bar{q}_i i \not{D} q_i + \int d^3r \text{Tr} \left\{ S^\dagger (i\partial_0 - h_s) S + O^\dagger (iD_0 - h_o) O \right\}$$

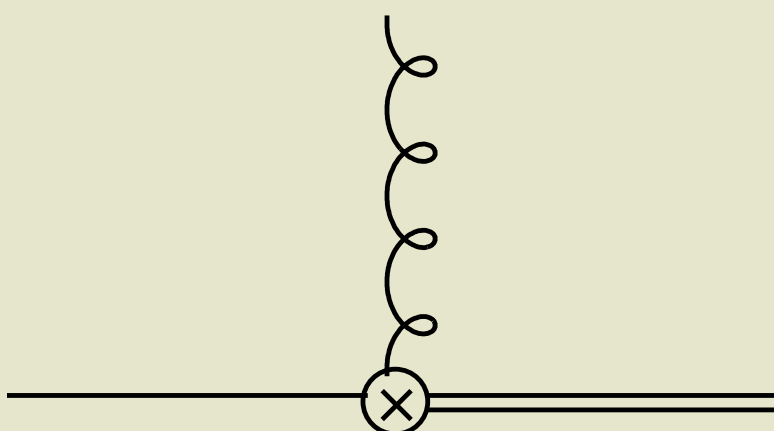
• LO in r

$$\overline{\hspace{1.5cm}} \quad \quad \quad \underline{\hspace{1.5cm}}$$

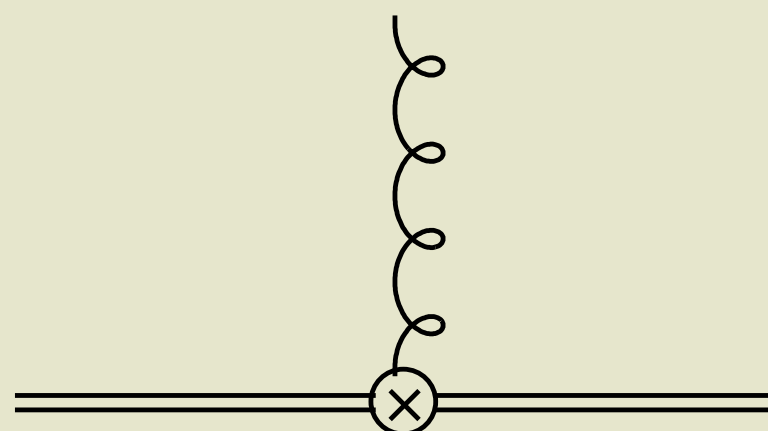
$$\theta(T) e^{-iT h_s} \quad \quad \theta(T) e^{-iT h_o} \left(e^{-i \int dt A^{\text{adj}}} \right)$$

$$+ V_A \text{Tr} \left\{ O^\dagger \mathbf{r} \cdot g \mathbf{E} S + S^\dagger \mathbf{r} \cdot g \mathbf{E} O \right\} + \frac{V_B}{2} \text{Tr} \left\{ O^\dagger \mathbf{r} \cdot g \mathbf{E} O + O^\dagger O \mathbf{r} \cdot g \mathbf{E} \right\}$$

• NLO in r



$$O^\dagger \mathbf{r} \cdot g \mathbf{E} S$$



$$O^\dagger \{ \mathbf{r} \cdot g \mathbf{E}, O \}$$

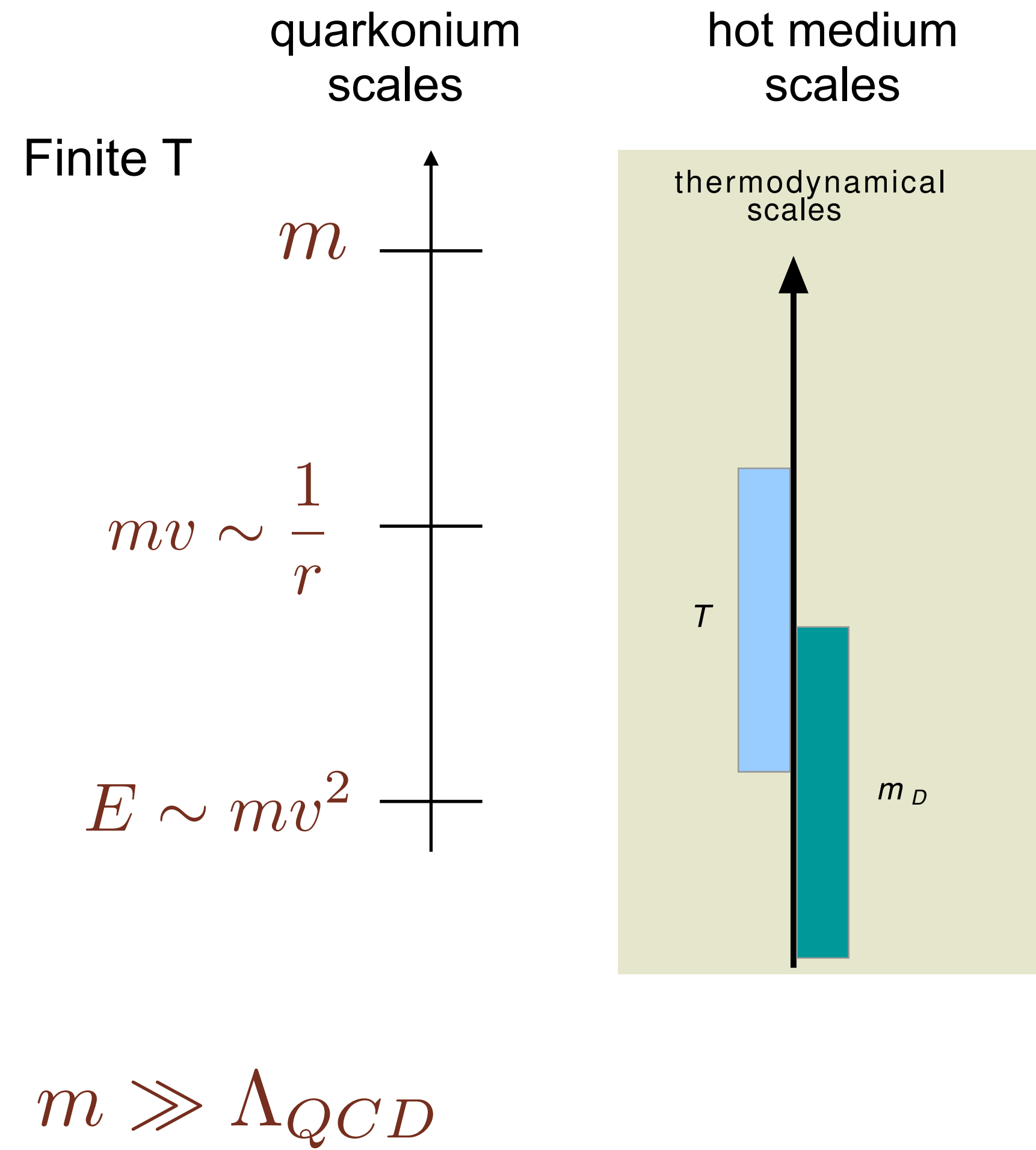
Degrees of freedom: colour singlet S and colour octet O and low energy gluons (multipole expanded)

The potentials are the matching coefficients of pNRQCD : they are calculated via a well defined matching procedure

the finite T potential in equilibrium

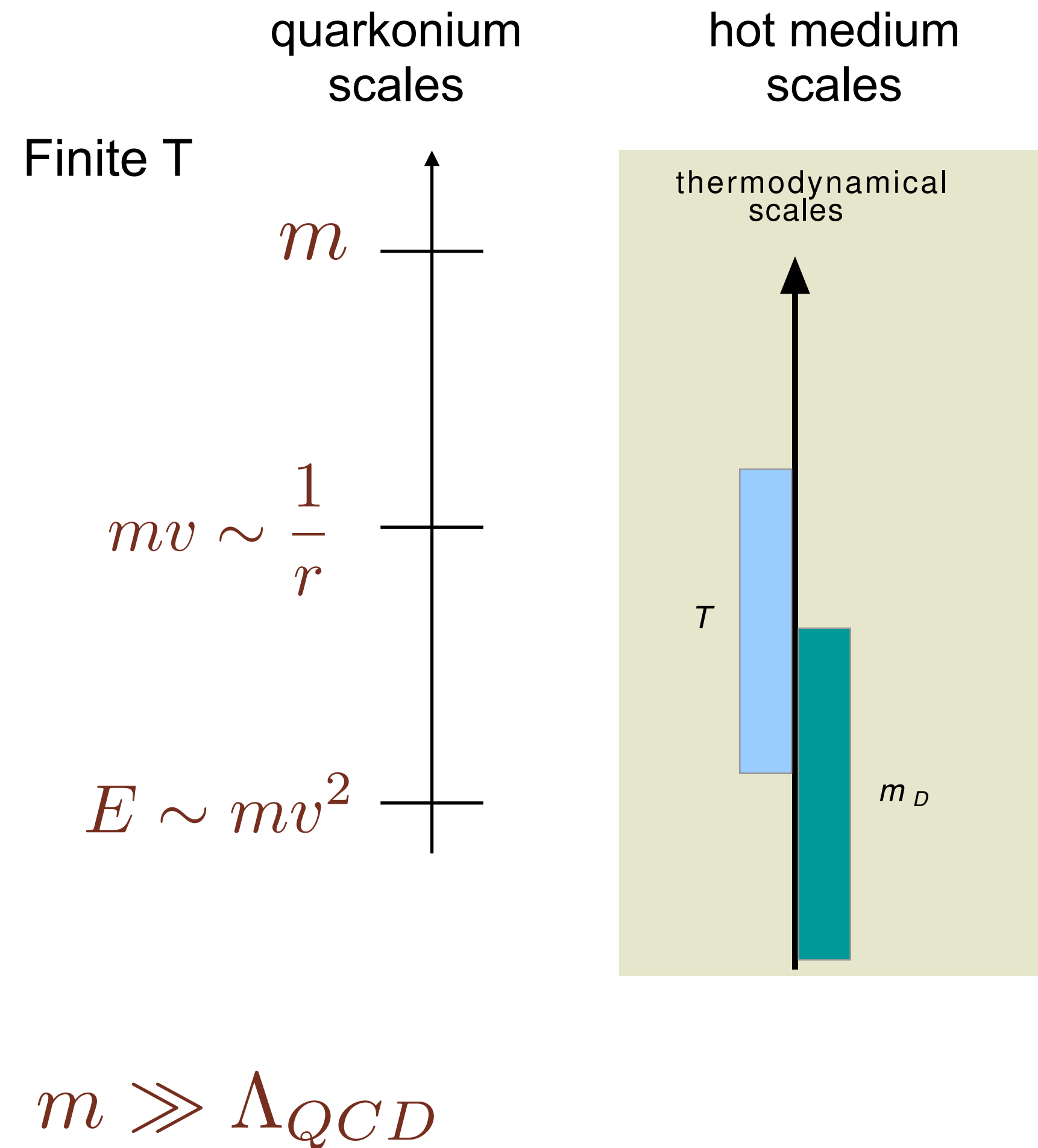
the change of paradigm from the screening to the imaginary part of the potential

pNRQCD at finite T: the static potential



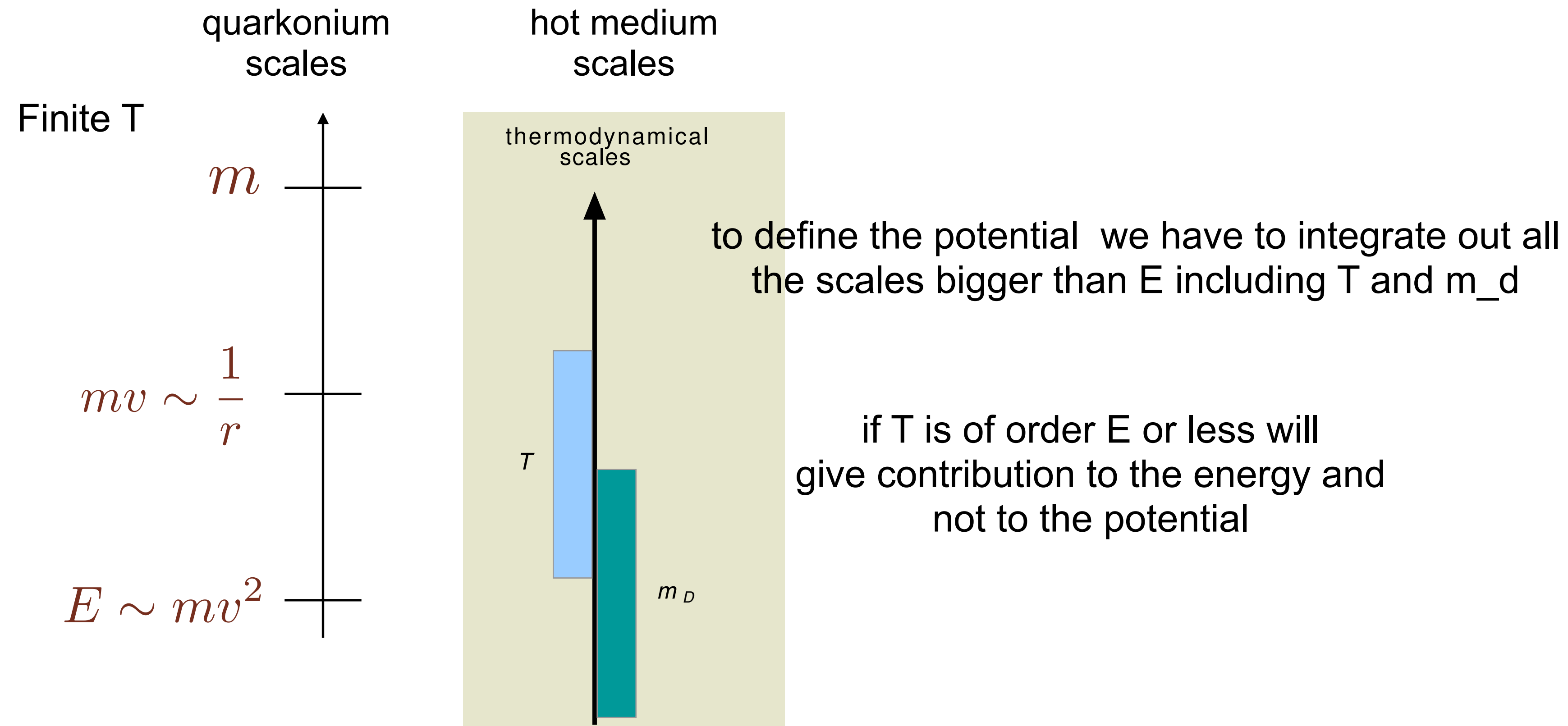
pNRQCD at finite T: the static potential

in pNRQCD the **potential** has a clear definition: it a matching coefficient and comes from the integration of all scales from mv up to (and not included) the energy mv^2



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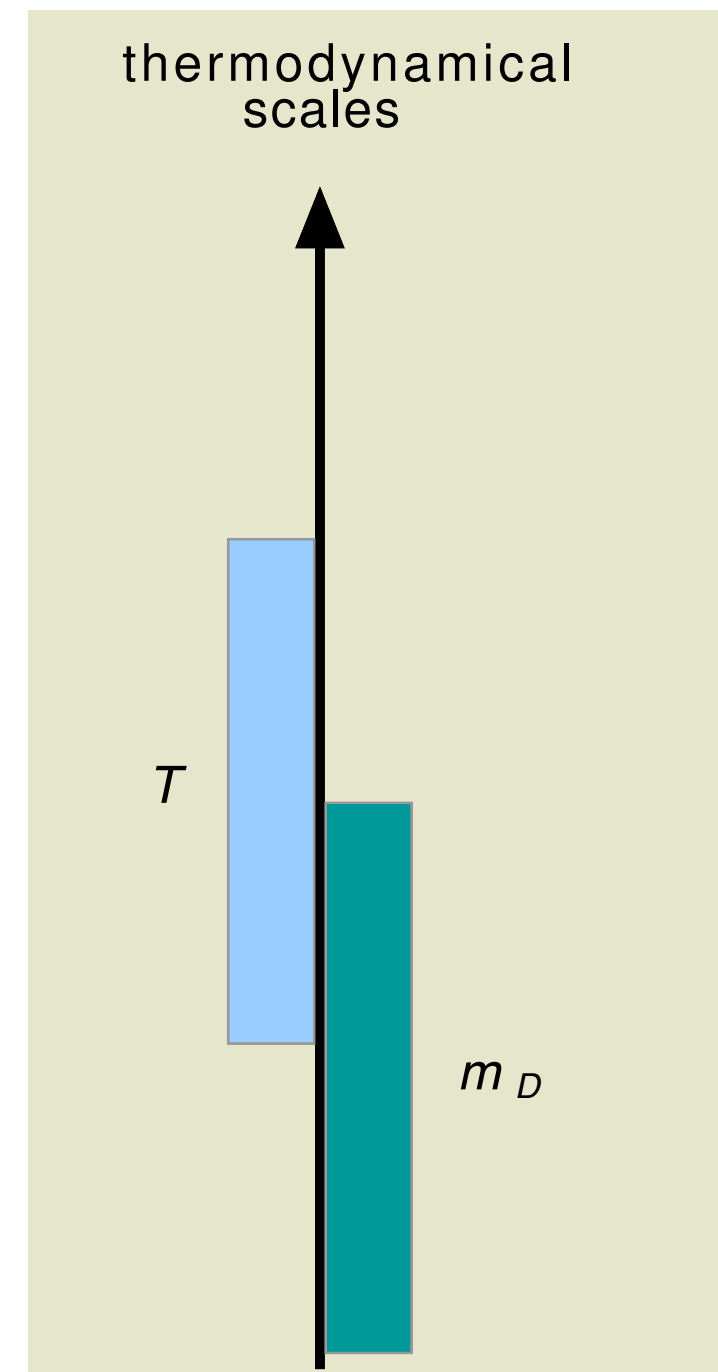
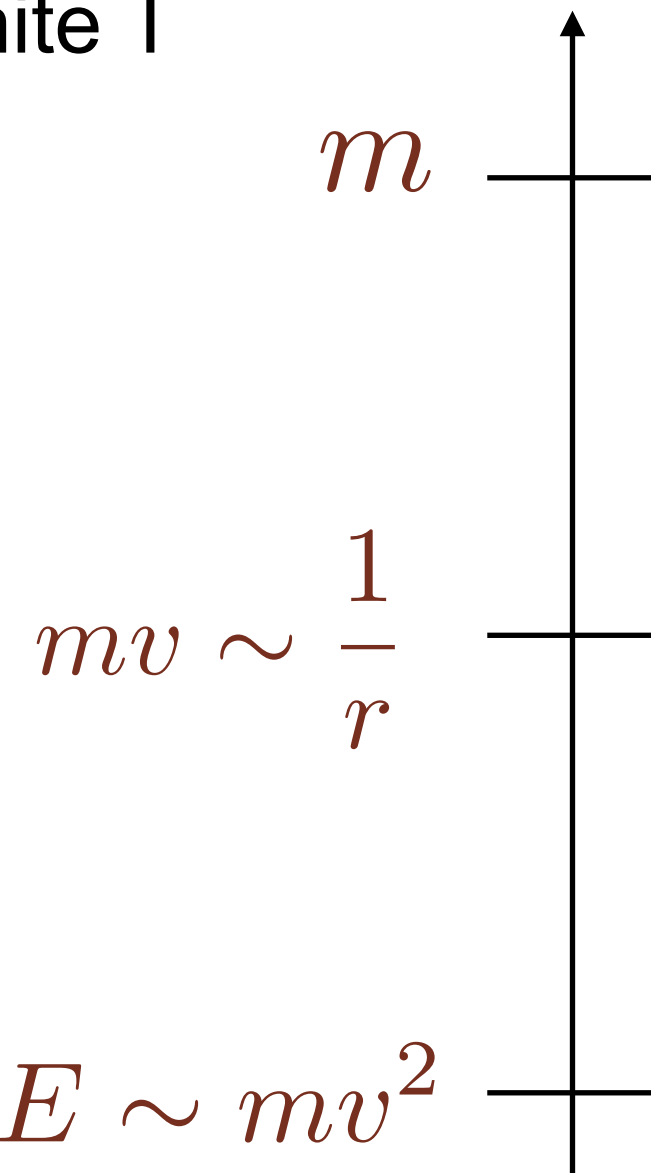
Notice:

The potential $V(r, T)$ dictates through the Schroedinger equation the real time evolution of the $Q\bar{Q}$ in the medium

The finite T potential: how to obtain it

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Finite T



We assume that bound states exist for

- $T \ll m$
- $1/r \sim mv \gtrsim m_D$

We neglect smaller thermodynamical scales.

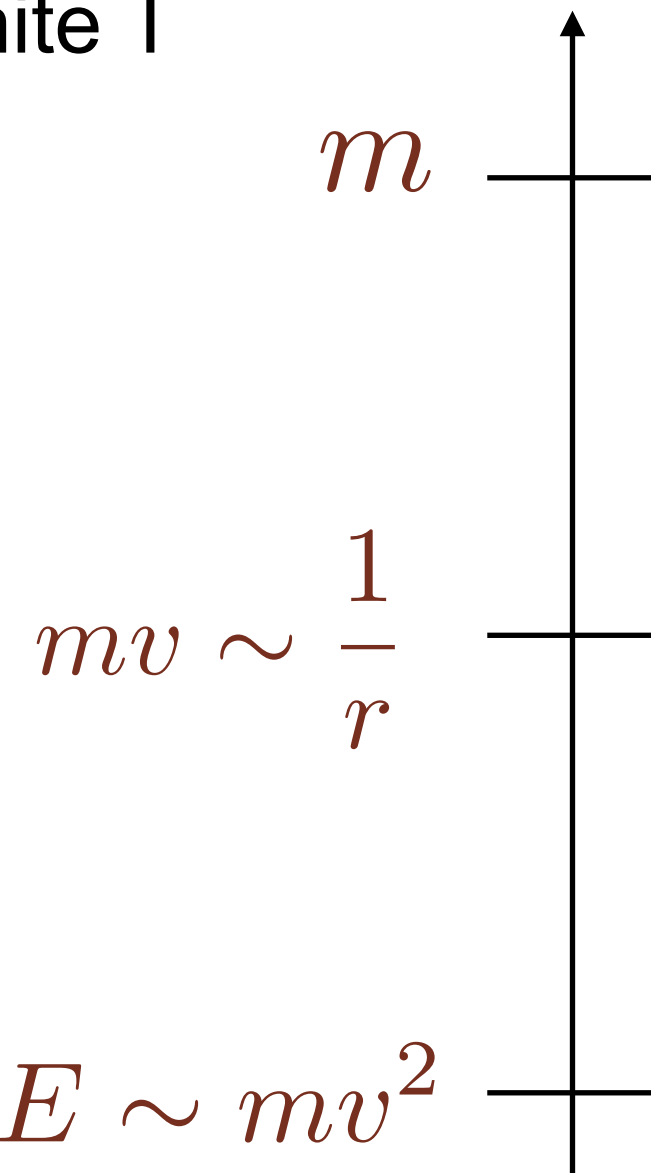
Inside these constraints
we consider all the possible
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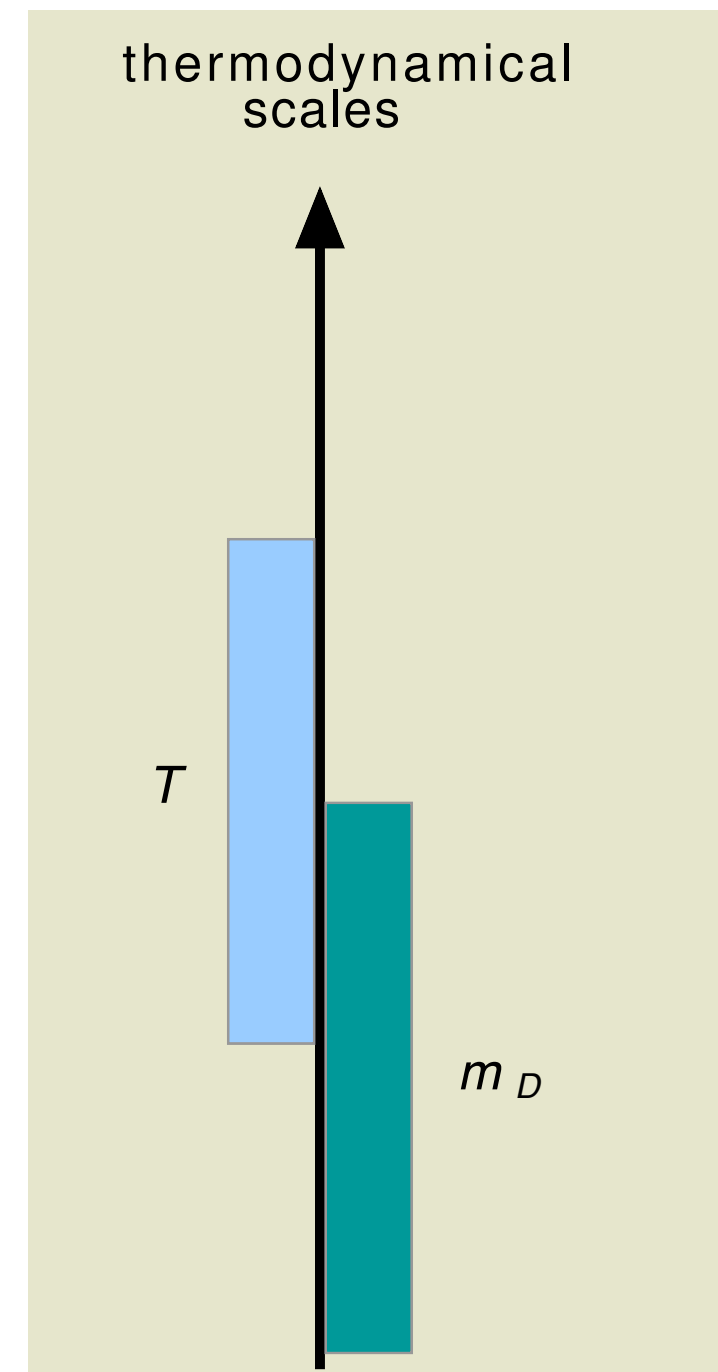
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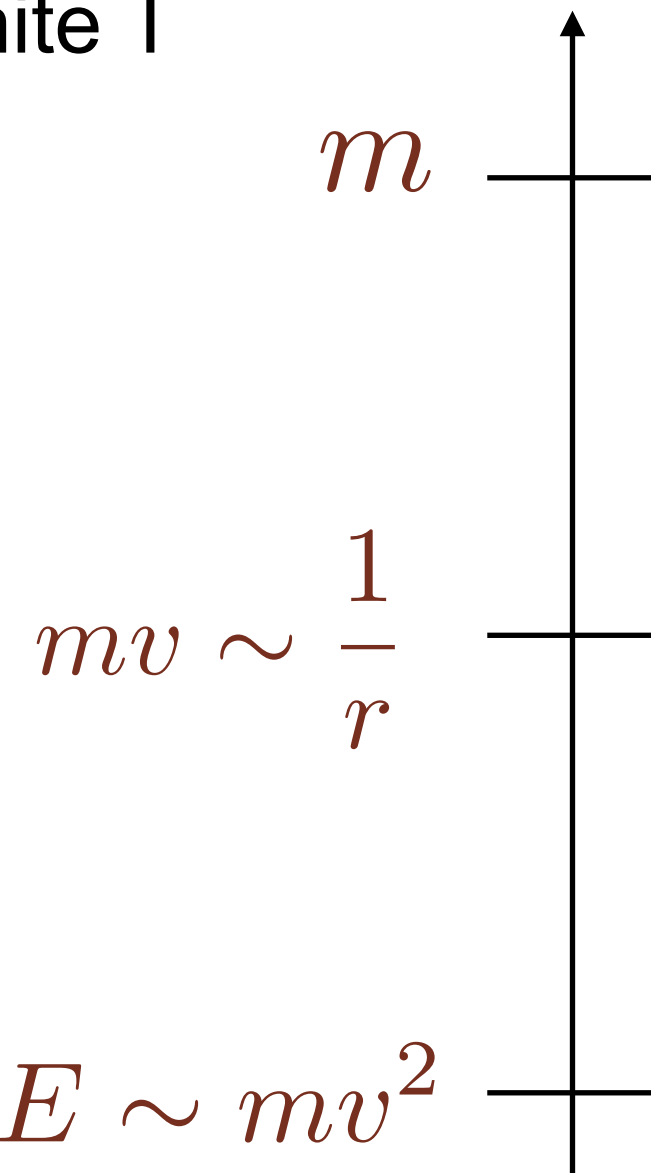
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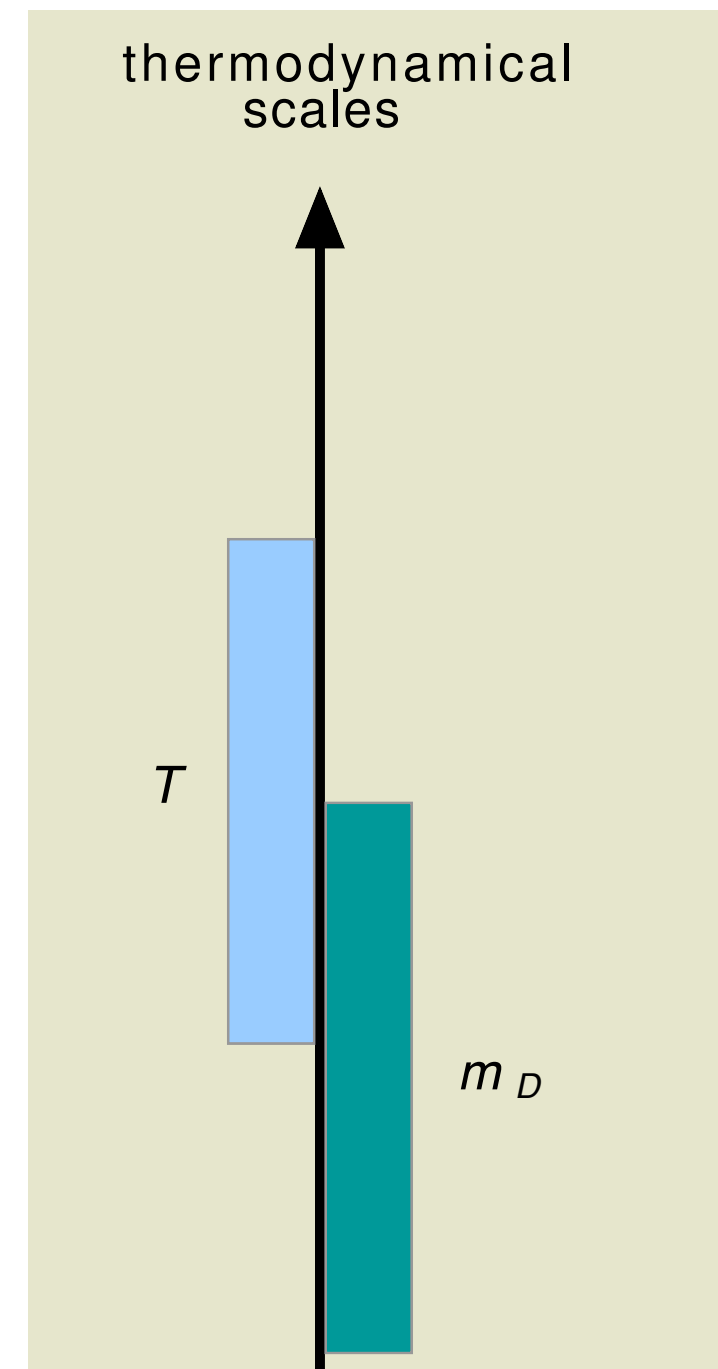
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for the nonperturbative
regime \rightarrow lattice calculation of the Wilson loop

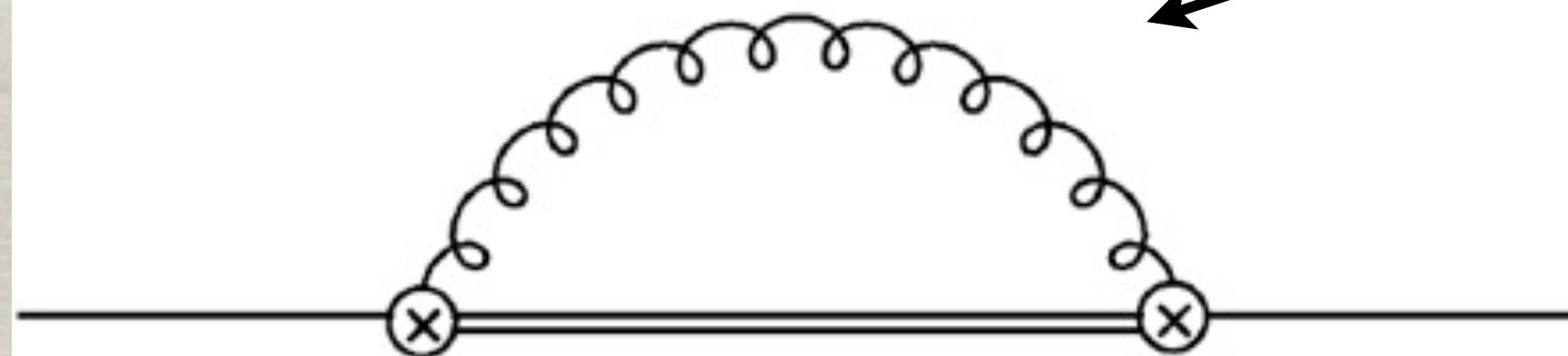
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$\text{Re}V_s(r,T)$

$\text{Im}V_s(r,T)$

thermal width of $Q\bar{Q}$

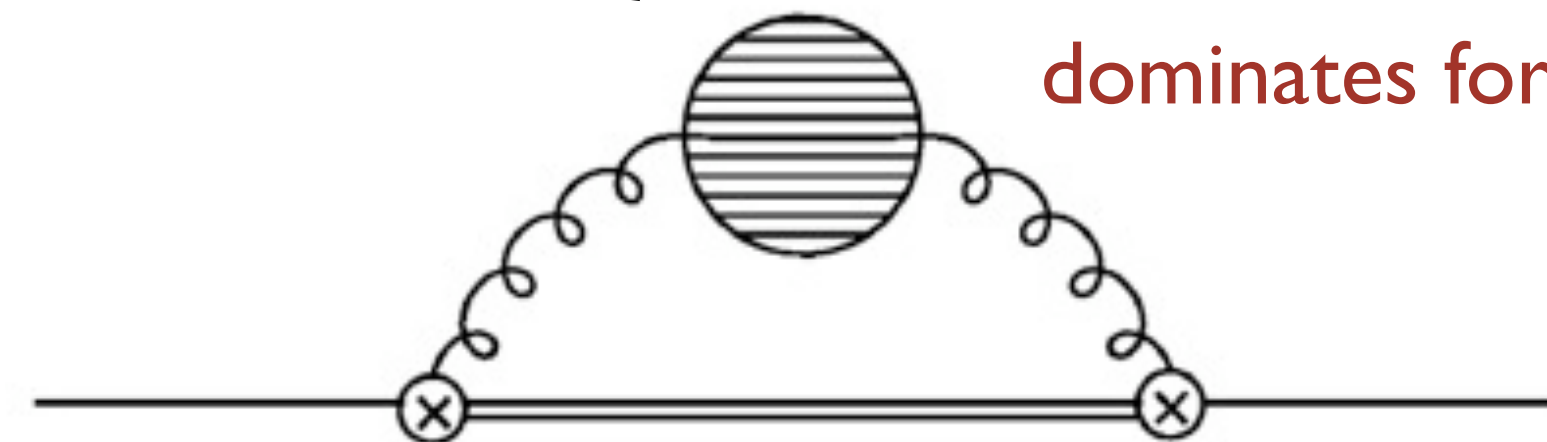
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Singlet-to-octet

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Landau damping

Laine et al 07, Escobedo Soto 07

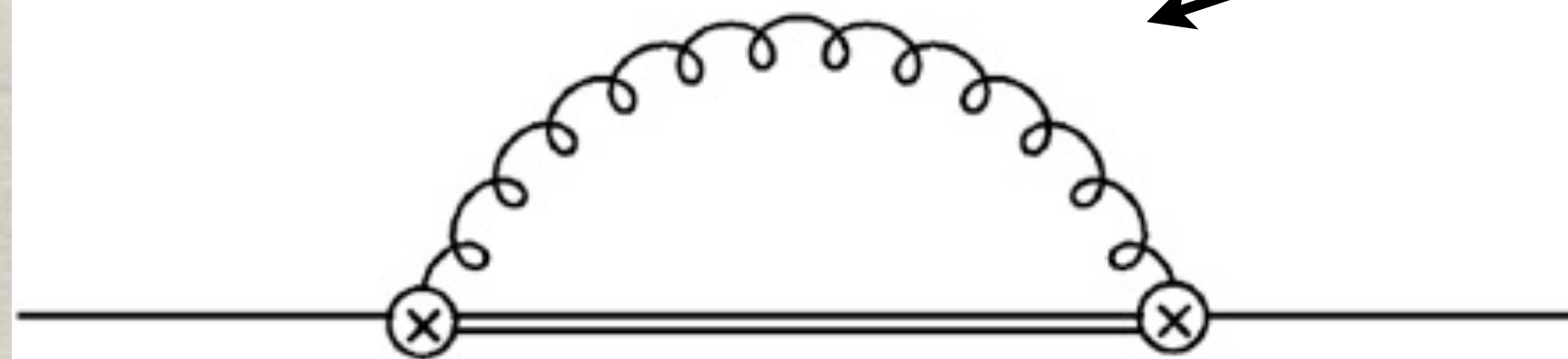
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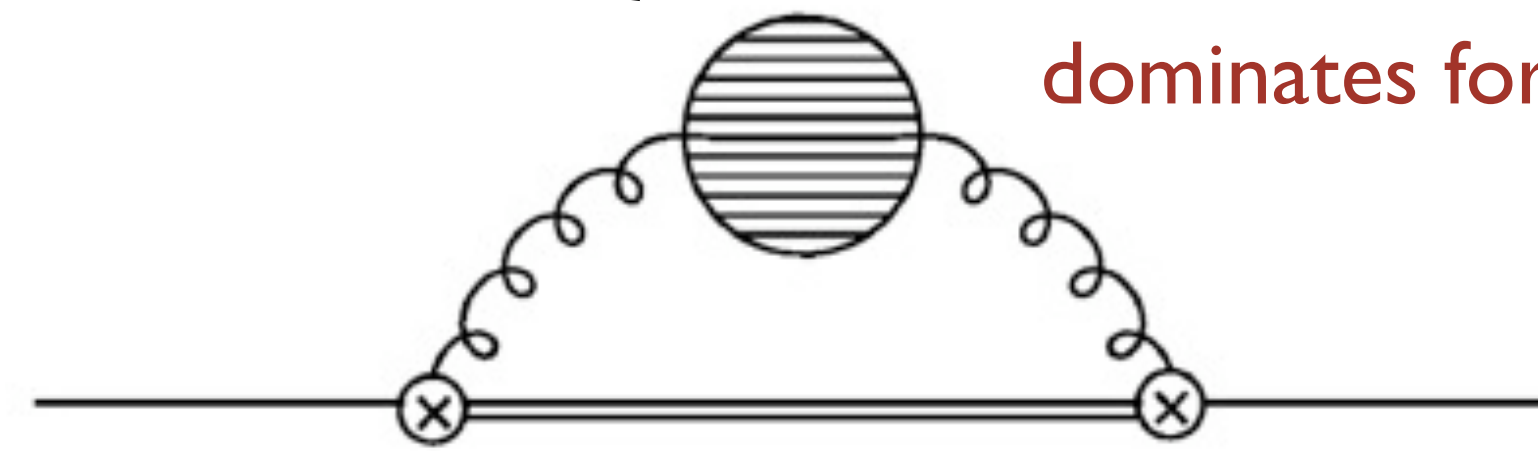


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N. B. Escobedo, Ghiglieri , Vairo 2011

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N. B. Escobedo, Ghiglieri , Vairo 2013

The singlet static potential and the static energy

you always have a real and an imaginary part

- Temperature effects can be other than screening

$$T > 1/r \text{ and } 1/r \sim m_D \sim gT$$

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no exponential screening but power-like T corrections

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imaginary parts in the potential have subsequently
been found also for a strongly coupled plasma on the lattice
(A. Rothkopf et al, Petreczky, Weber..) and in strings calculations

Change in the paradigm of dissociation

- The imaginary part is bigger than the real part before the screening $\exp\{-m_D r\}$ sets in

->the imaginary part is responsible for QQbar dissociation

$$T \gg 1/r \gg m_D \gg V$$

- Quarkonium dissociates at a temperature such that $\text{Im } V_s(r) \sim \text{Re } V_s(r) \sim \alpha_s/r$:

E_{binding}

$$\pi T_{\text{dissociation}} \sim mg^{4/3}$$

Escobedo Soto arXiv:0804.0691

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The **bottomonium ground state**, which is a weakly coupled non-relativistic bound state: $mv \sim m\alpha_s$, $mv^2 \sim m\alpha_s^2 \gtrsim \Lambda_{\text{QCD}}$, produced in the QCD medium of heavy-ion collisions at the LHC may possibly realize the hierarchy

$$m \approx 5 \text{ GeV} > m\alpha_s \approx 1.5 \text{ GeV} > \pi T \approx 1 \text{ GeV} > m\alpha_s^2 \approx 0.5 \text{ GeV} \gtrsim m_D, \Lambda_{\text{QCD}}$$

$T_{\text{dissociation}}$ in the $\Upsilon(1S)$ case is about 450 MeV.

bottomonium 1S below the melting temperature T_d

The complete mass and width up to $\mathcal{O}(m\alpha_s^5)$

$$\begin{aligned}\delta E_{1S}^{(\text{thermal})} &= \frac{34\pi}{27} \alpha_s^2 T^2 a_0 + \frac{7225}{324} \frac{E_1 \alpha_s^3}{\pi} \left[\ln \left(\frac{2\pi T}{E_1} \right)^2 - 2\gamma_E \right] \\ &\quad + \frac{128 E_1 \alpha_s^3}{81\pi} L_{1,0} - 3a_0^2 \left\{ \left[\frac{6}{\pi} \zeta(3) + \frac{4\pi}{3} \right] \alpha_s T m_D^2 - \frac{8}{3} \zeta(3) \alpha_s^2 T^3 \right\} \\ \Gamma_{1S}^{(\text{thermal})} &= \frac{1156}{81} \alpha_s^3 T + \frac{7225}{162} E_1 \alpha_s^3 + \frac{32}{9} \alpha_s T m_D^2 a_0^2 I_{1,0} \\ &\quad - \left[\frac{4}{3} \alpha_s T m_D^2 \left(\ln \frac{E_1^2}{T^2} + 2\gamma_E - 3 - \ln 4 - 2 \frac{\zeta'(2)}{\zeta(2)} \right) + \frac{32\pi}{3} \ln 2 \alpha_s^2 T^3 \right] a_0^2\end{aligned}$$

where $E_1 = -\frac{4m\alpha_s^2}{9}$, $a_0 = \frac{3}{2m\alpha_s}$ and $L_{1,0}$ (similar $I_{1,0}$) is the Bethe logarithm.

◦ Brambilla Escobedo Ghiglieri Soto Vairo JHEP 1009 (2010) 038

first systematic
calculation
of the thermal
contributions to
quarkonium
mass and width

Consistent with lattice calculations of spectral functions

Aarts Allton Kim Lombardo Oktay Ryan Sinclair Skullerud
JHEP 1111 (2011) 103

To describe quarkonium in QGP we should account for: screening,
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—>The non equilibrium evolution of quarkonium in QGP

Using pNRQCD and Open Quantum Systems (OQS)
we could use bottomonium as a probe of
a strongly coupled QGP and obtain
master equations for the singlet and octet matrix
density evolution

The equations are
quantum, nonabelian and conserve
the number of heavy quarks

Quarkonium in the fireball

- After the heavy-ion collisions, heavy quark-antiquarks propagate freely up to 0.6 fm.
- From 0.6 fm to the freeze-out time t_F they propagate in the medium.
- We assume the medium infinite, homogeneous and isotropic.
- We assume the heavy quarks comoving with the medium.
- We assume the medium to be locally in thermal equilibrium,
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Quarkonium as a small radius probe: bottomonium

Hierarchy of scales:

$$m \gg 1/r \sim m\alpha_s \gg T \sim gT \gg E$$

Coulombic bound state:

quark-antiquark color singlet Hamiltonian	$h_s = \frac{\mathbf{p}^2}{m} - \frac{4}{3} \frac{\alpha_s}{r}$
quark-antiquark color octet Hamiltonian	$h_o = \frac{\mathbf{p}^2}{m} + \frac{1}{6} \frac{\alpha_s}{r}$

The octet potential describes an unbound quark-antiquark pair.

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Open Quantum system

- **Subsystem:** heavy quarks/quarkonium
- **Environment:** quark gluon plasma

N.B., J. Soto, M. Escobedo, A. Vairo 2016, 2018 (1612.07248, 1711.04515)

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We may define a density matrix in pNRQCD for the heavy quark-antiquark pair in a singlet and octet configuration:

$$\begin{aligned} \langle \mathbf{r}', \mathbf{R}' | \rho_s(t'; t) | \mathbf{r}, \mathbf{R} \rangle &\equiv \text{Tr} \{ \rho_{\text{full}}(t_0) S^\dagger(t, \mathbf{r}, \mathbf{R}) S(t', \mathbf{r}', \mathbf{R}') \} \\ \langle \mathbf{r}', \mathbf{R}' | \rho_o(t'; t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{8} &\equiv \text{Tr} \{ \rho_{\text{full}}(t_0) O^{a\dagger}(t, \mathbf{r}, \mathbf{R}) O^b(t', \mathbf{r}', \mathbf{R}') \} \end{aligned}$$

Quarkonium in the fireball

- After the heavy-ion collisions, heavy quark-antiquarks propagate freely up to 0.6 fm.
- From 0.6 fm to the freeze-out time t_F they propagate in the medium.
- We assume the medium infinite, homogeneous and isotropic.
- We assume the heavy quarks comoving with the medium.
- We assume the medium to be locally in thermal equilibrium, i.e., the temperature T of the medium changes (slowly) with time:

Open Quantum system

- **Subsystem:** heavy quarks/quarkonium
- **Environment:** quark gluon plasma

N.B., J. Soto, M. Escobedo, A. Vairo 2016,
2018 (1612.07248, 1711.04515)

The system is in **non-equilibrium** because through interaction with the environment (quark gluon plasma) singlet and octet quark-antiquark states continuously transform in each other although **the number of heavy quarks is conserved**: $\text{Tr}\{\rho_s\} + \text{Tr}\{\rho_o\} = 1$.

Quarkonium as a small radius probe: bottomonium

Hierarchy of scales:

$$m \gg 1/r \sim m\alpha_s \gg T \sim gT \gg E$$

Coulombic bound state:

quark-antiquark **color singlet** Hamiltonian $h_s = \frac{\mathbf{p}^2}{m} - \frac{4}{3} \frac{\alpha_s}{r}$

quark-antiquark **color octet** Hamiltonian $h_o = \frac{\mathbf{p}^2}{m} + \frac{1}{6} \frac{\alpha_s}{r}$

The octet potential describes an unbound quark-antiquark pair.

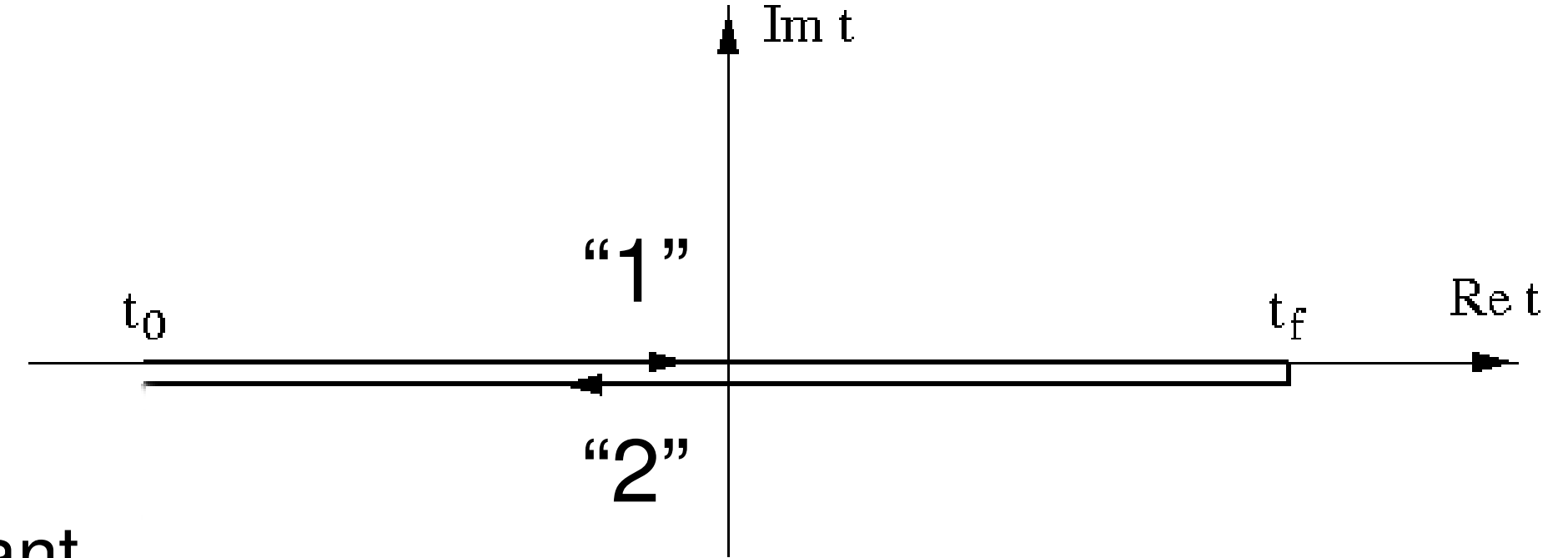
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Closed time path formalism

In the **closed-time path formalism** we can represent the density matrices as 12 propagators on a closed time path:

$$\begin{aligned}\langle \mathbf{r}', \mathbf{R}' | \rho_s(t'; t) | \mathbf{r}, \mathbf{R} \rangle &= \langle S_1(t', \mathbf{r}', \mathbf{R}') S_2^\dagger(t, \mathbf{r}, \mathbf{R}) \rangle \\ \langle \mathbf{r}', \mathbf{R}' | \rho_o(t'; t) | \mathbf{r}, \mathbf{R} \rangle \frac{\delta^{ab}}{8} &= \langle O_1^b(t', \mathbf{r}', \mathbf{R}') O_2^{a\dagger}(t, \mathbf{r}, \mathbf{R}) \rangle\end{aligned}$$



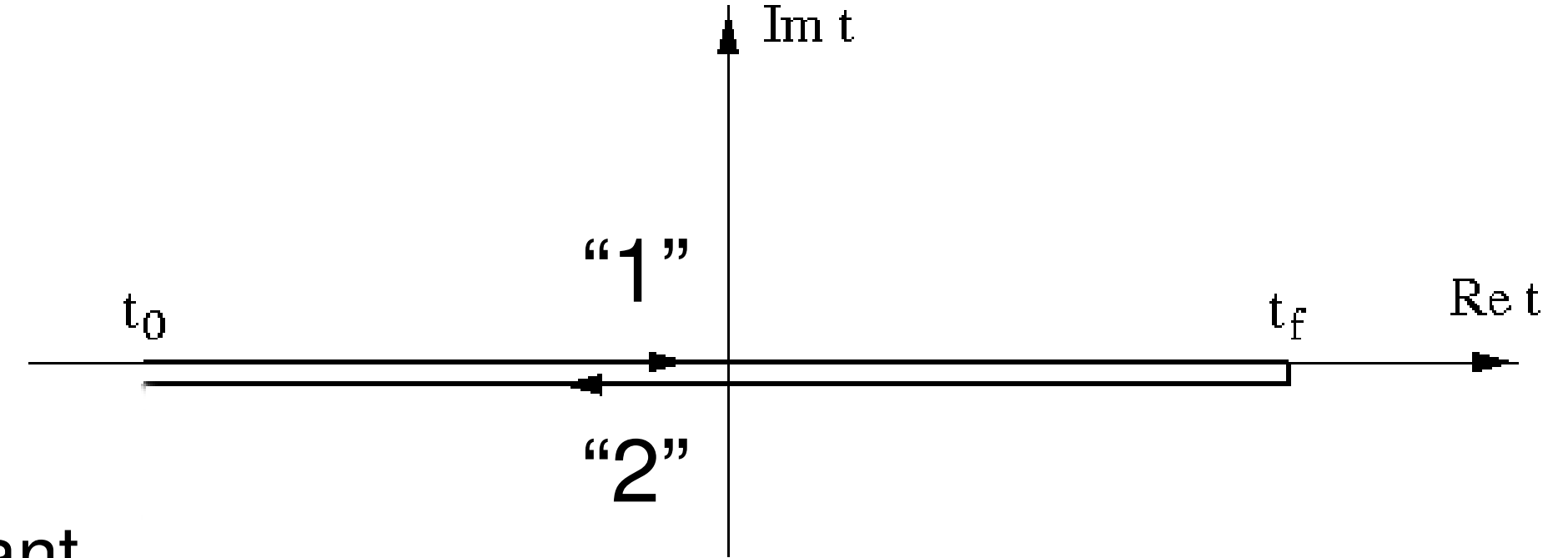
Differently from the thermal equilibrium case 12 propagators are relevant (in thermal equilibrium they are exponentially suppressed).

12 propagators are not time ordered, while 11 and 22 operators select the forward time direction $\propto \theta(t - t'), \theta(t' - t)$.

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Expansions

- The density of heavy quarks is much smaller than the one of light quarks: we expand at **first order in the heavy quark-antiquark density**.
- We consider **T much smaller than the Bohr radius** of the quarkonium: we expand up to **order r^2 in the multipole expansion**.

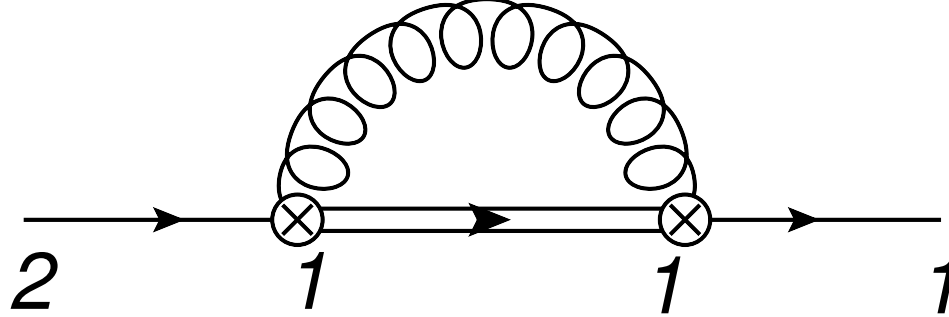
LO and NLO evolution

For $t > t_0$, the **LO singlet density matrix** is

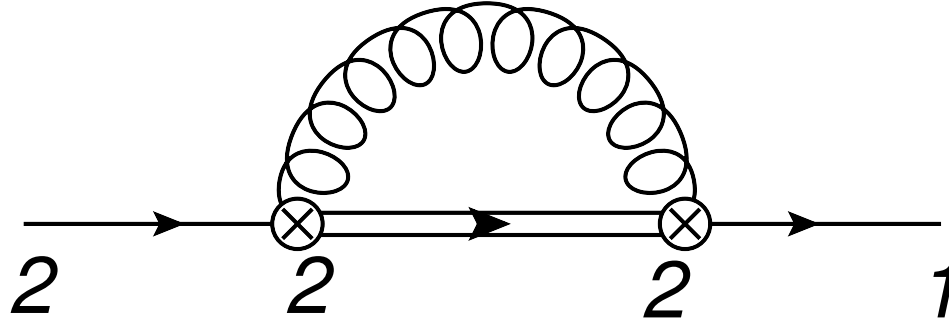
$$\overrightarrow{2} \xrightarrow{\quad} 1 = e^{-ih_s(t-t_0)} \rho_s(t_0; t_0) e^{ih_s(t-t_0)}$$

($h_{s,o}$ = singlet/octet pNRQCD Hamiltonian = $V_{s,o}$ in the static limit)

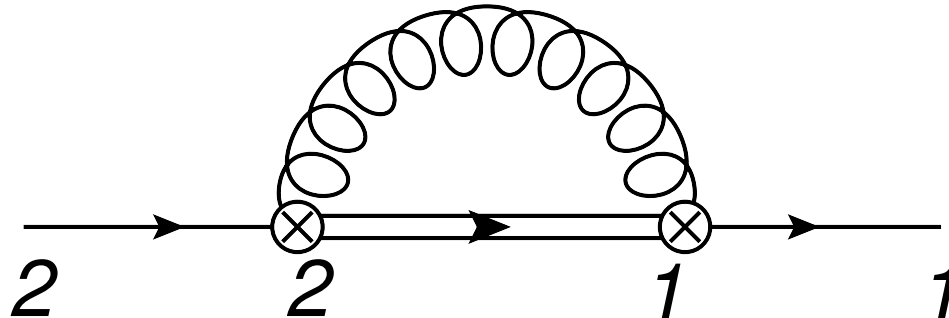
and the **NLO (in the multipole expansion)** corrections are at first order in the density



$$= - \int_{t_0}^t dt_1 e^{-ih_s(t-t_1)} \Sigma_s(t_1) e^{-ih_s(t_1-t_0)} \rho_s(t_0; t_0) e^{ih_s(t-t_0)}$$



$$= - \int_{t_0}^t dt_1 e^{-ih_s(t-t_0)} \rho_s(t_0; t_0) e^{ih_s(t_1-t_0)} \Sigma_s^\dagger(t_1) e^{ih_s(t-t_1)}$$



$$= \int_{t_0}^t dt_1 e^{-ih_s(t-t_1)} \Xi_{so}(\rho_o(t_0; t_0), t_1) e^{ih_s(t-t_1)}$$

and similar for the octet

$$\Sigma_s(t) \quad = \quad \frac{g^2}{2N_c} \int_{t_0}^t dt_2 \, r^i \, e^{-i h_o(t-t_2)} \, r^j \, e^{i h_s(t-t_2)} \, \langle E^{a,i}(t, \mathbf{0}) E^{a,j}(t_2, \mathbf{0}) \rangle$$

$$\begin{aligned} \Xi_{so}(\rho_o(t_0; t_0), t) \quad = \quad & \frac{g^2}{2N_c(N_c^2 - 1)} \int_{t_0}^t dt_2 \, \left[r^i \, e^{-i h_o(t-t_0)} \, \rho_o(t_0; t_0) \, e^{i h_o(t_2-t_0)} \right. \\ & \left. \times r^j \, e^{i h_s(t-t_2)} \, \langle E^{a,j}(t_2, \mathbf{0}) E^{a,i}(t, \mathbf{0}) \rangle + \text{H.c.} \right] \end{aligned}$$

and similar for the octet

A Wilson line in the adjoint representation is understood in the chromoelectric correlators.

$$\Sigma_s(t) = \frac{g^2}{2N_c} \int_{t_0}^t dt_2 r^i e^{-ih_o(t-t_2)} r^j e^{ih_s(t-t_2)} \langle E^{a,i}(t, \mathbf{0}) E^{a,j}(t_2, \mathbf{0}) \rangle$$

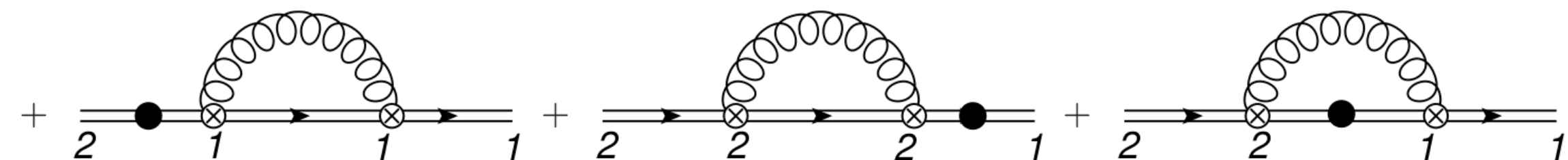
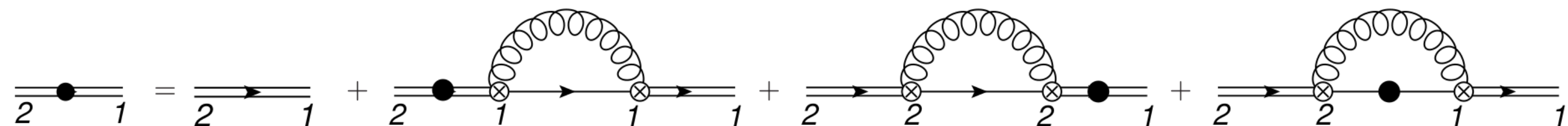
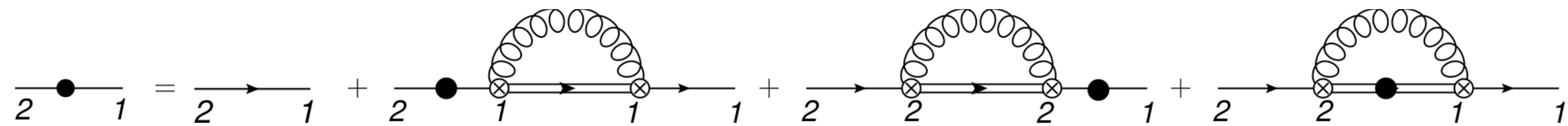
$$\Xi_{so}(\rho_o(t_0; t_0), t) = \frac{g^2}{2N_c(N_c^2 - 1)} \int_{t_0}^t dt_2 \left[r^i e^{-ih_o(t-t_0)} \rho_o(t_0; t_0) e^{ih_o(t_2-t_0)} \right. \\ \left. \times r^j e^{ih_s(t-t_2)} \langle E^{a,j}(t_2, \mathbf{0}) E^{a,i}(t, \mathbf{0}) \rangle + \text{H.c.} \right]$$

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Diagrams and resummation

Resumming $(t - t_0) \times$ self-energy contributions à la Schwinger–Dyson ...



Singlet and octet density matrix evolution equations

... and differentiating over time we obtain the coupled evolution equations:

$$\frac{d\rho_s(t;t)}{dt} = -i[h_s, \rho_s(t;t)] - \Sigma_s(t)\rho_s(t;t) - \rho_s(t;t)\Sigma_s^\dagger(t) + \Xi_{so}(\rho_o(t;t), t)$$

$$\begin{aligned} \frac{d\rho_o(t;t)}{dt} = & -i[h_o, \rho_o(t;t)] - \Sigma_o(t)\rho_o(t;t) - \rho_o(t;t)\Sigma_o^\dagger(t) + \Xi_{os}(\rho_s(t;t), t) \\ & + \Xi_{oo}(\rho_o(t;t), t) \end{aligned}$$

The evolution equations are Markovian.

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Interpretation

- The self energies Σ_s and Σ_o provide the in-medium induced mass shifts, $\delta m_{s,o}$, and widths, $\Gamma_{s,o}$, for the color-singlet and color-octet heavy quark-antiquark systems respectively:

$$-i\Sigma_{s,o}(t) + i\Sigma_{s,o}^\dagger(t) = 2 \operatorname{Re}(-i\Sigma_{s,o}(t)) = 2\delta m_{s,o}(t)$$

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- Ξ_{so} accounts for the production of singlets through the decay of octets, and Ξ_{os} and Ξ_{oo} account for the production of octets through the decays of singlets and octets respectively. There are two octet production mechanisms/octet chromoelectric dipole vertices in the pNRQCD Lagrangian.

Singlet and octet density matrix evolution equations

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The conservation of the trace of the sum of the densities, i.e., the conservation of the number of heavy quarks, follows from

$$\begin{aligned}\mathrm{Tr} \left\{ \rho_s(t; t) \left(\Sigma_s(t) + \Sigma_s^\dagger(t) \right) \right\} &= \mathrm{Tr} \{ \Xi_{os}(\rho_s(t; t), t) \} \\ \mathrm{Tr} \left\{ \rho_o(t; t) \left(\Sigma_o(t) + \Sigma_o^\dagger(t) \right) \right\} &= \mathrm{Tr} \{ \Xi_{so}(\rho_o(t; t), t) + \Xi_{oo}(\rho_o(t; t), t) \}\end{aligned}$$

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Time scales

Environment correlation time: $\tau_E \sim \frac{1}{T}$

System intrinsic time scale: $\tau_S \sim \frac{1}{E}$

System relaxation time: $\tau_R \sim \frac{1}{\text{self-energy}} \sim \frac{1}{\alpha_s a_0^2 \Lambda^3}$

$a_0 = r$

$a_0 = \text{Bohr radius}, \Lambda = T, E$

- Because we have assumed $1/a_0 \gg \Lambda$, it follows $\tau_R \gg \tau_S, \tau_E$ which, after resummation, qualifies the system as **Markovian**.
- If $T \gg E$ then $\tau_S \gg \tau_E$ which qualifies the motion of the system as **quantum Brownian**.

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◦ Akamatsu PRD 91 (2015) 056002

From the evolution equations to the Linblad equations

Under the Markovian

$$\tau_R \gg \tau_S, \tau_E \quad \text{or} \quad \frac{1}{a_0} \gg E, T$$

and quantum Brownian motion condition

$$\tau_S \gg \tau_E \quad \text{or} \quad T \gg E$$

at least at LO in E/T the evolution equations can be written in the **Lindblad form**.

nonequilibrium evolution of quarkonium: Linblad equations

If $E \ll T \sim m_D$ the Lindblad equation for a strongly coupled plasma reads

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_i (C_i \rho C_i^\dagger - \frac{1}{2} \{C_i^\dagger C_i, \rho\})$$

C collapse
operators

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2} \gamma(t) \begin{pmatrix} 1 & 0 \\ 0 & \frac{7}{16} \end{pmatrix},$$

$$C_i^0 = \sqrt{\frac{\kappa(t)}{8}} r^i \begin{pmatrix} 0 & 1 \\ \sqrt{8} & 0 \end{pmatrix}, \quad C_i^1 = \sqrt{\frac{5\kappa(t)}{16}} r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

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the sQGP is characterised by two nonperturbative parameters (transport coefficients) kappa and gamma that must be calculated on the lattice

κ is the heavy-quark momentum diffusion coefficient:

$$\kappa = \frac{g^2}{18} \text{Re} \int_{-\infty}^{+\infty} ds \langle \text{T} E^{a,i}(s, \mathbf{0}) \phi^{ab}(s, 0) E^{b,i}(0, \mathbf{0}) \rangle$$

$$\gamma = \frac{g^2}{18} \text{Im} \int_{-\infty}^{+\infty} ds \langle \text{T} E^{a,i}(s, \mathbf{0}) \phi^{ab}(s, 0) E^{b,i}(0, \mathbf{0}) \rangle$$

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the EFT allows to use lattice QCD equilibrium calculation to study the non equilibrium evolution! EFT is intermediate layer to non equilibrium

Our evolution equations depend
on two transport coefficients κ and γ that inside
pNRQCD acquire a field theoretical definition
as gauge invariant correlators of chromoelectric fields

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How to calculate these nonperturbative
transport coefficients?

use lattice QCD

The heavy quark diffusion coefficient

D in real space, related to κ in momentum space

Langevin dynamics of the heavy quark in the medium

$$\frac{dp_i}{dt} = -\eta_D p_i + \xi_i(t)$$

$$\langle \xi(t) \xi(t') \rangle = \kappa \delta(t - t')$$

$$\langle x^2(t) \rangle = 6Dt$$

$$\eta_D = \frac{\kappa}{2MT}$$

$$D = \frac{2T^2}{\kappa}$$

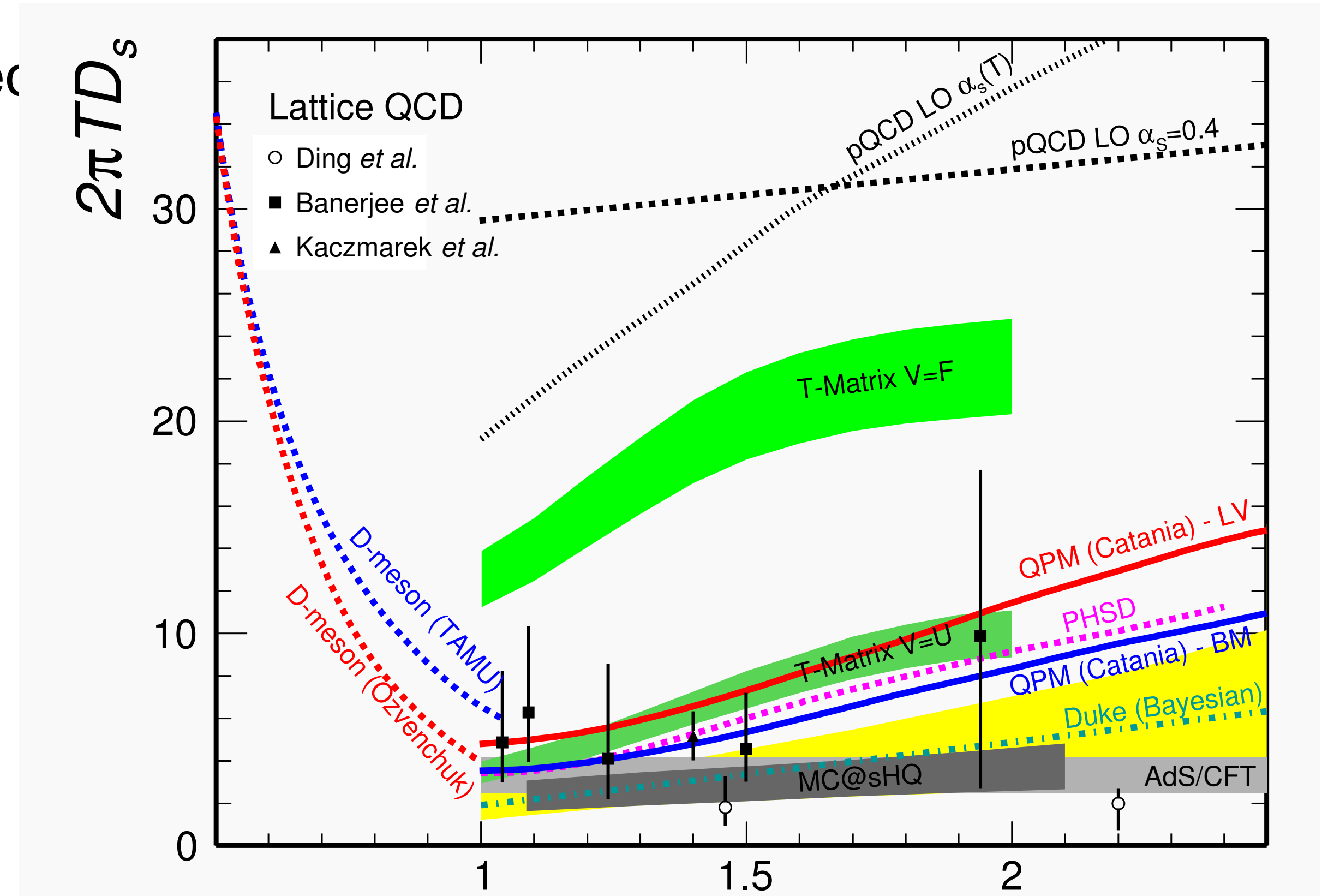


Figure from: X. Dong CIPANP (2018)

$\frac{T}{T_c}$

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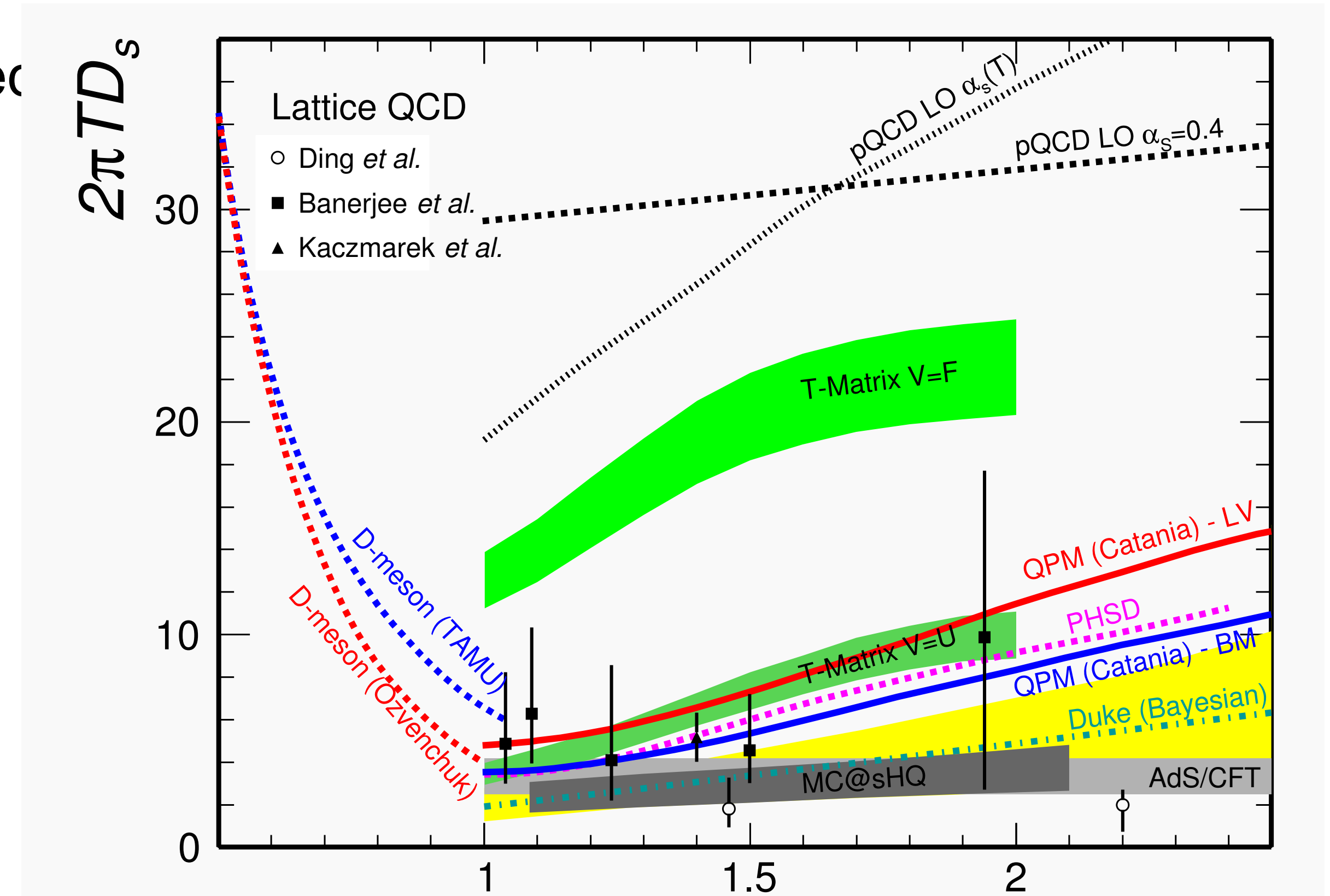
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in the limit in which the mass M of the heavy quark is the biggest scale one can integrate it out non relativistic effective field theory and from the current current correlator obtain



X. Dong CIPANP (2018)

$\frac{T}{T_c}$

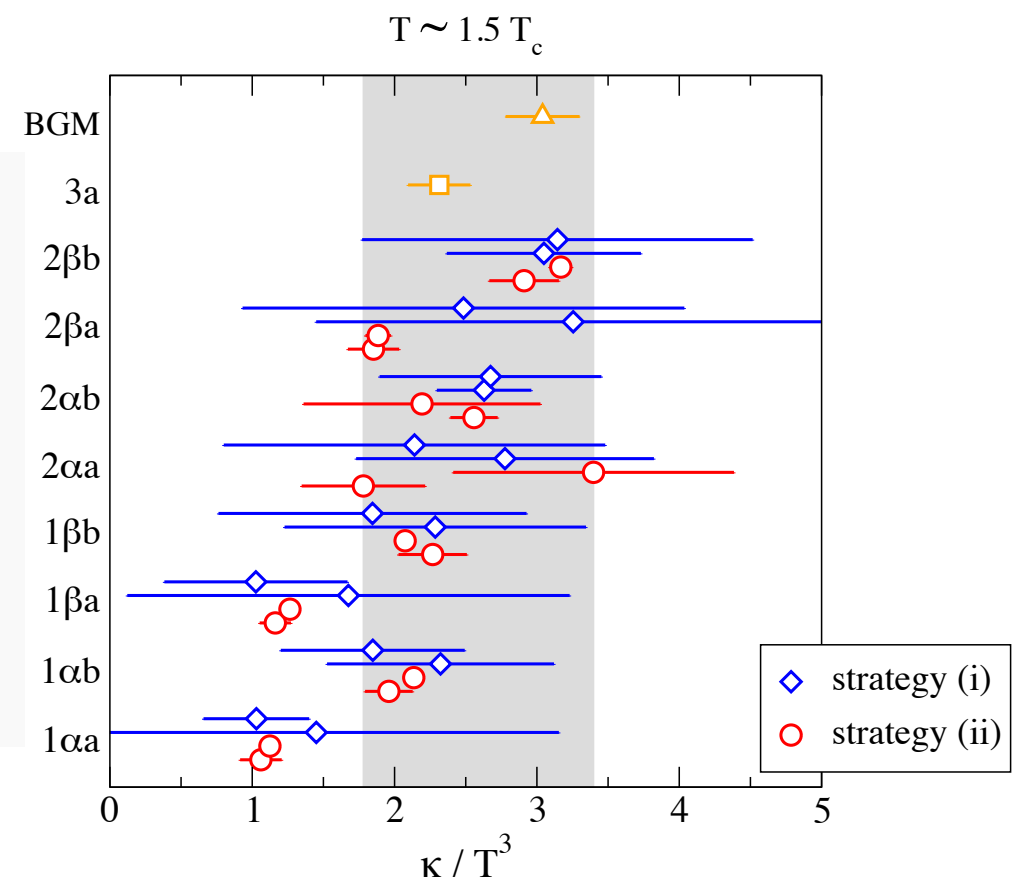
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which is the same transport coefficient kappa that we found studying the non equilibrium evolution of quarkonium in the QGP!

1903.08063

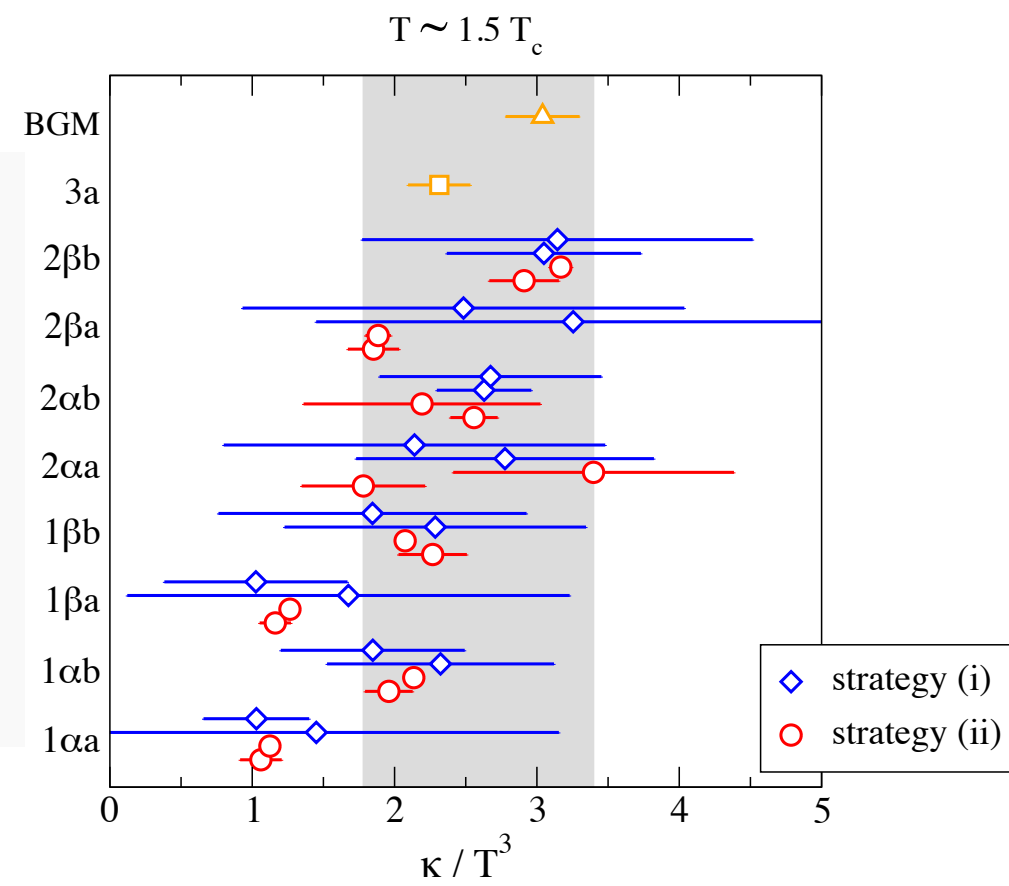
this object was already studied on quenched lattice

Meyer NJP13 (2011),
Ding *et.al.*JPG38 (2011),
Banarjee *et.al.* PRD85 (2012),
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$$1.91 < \frac{\kappa}{T^3} < 5.4 \text{ for } T = 1.1 T_c ,$$

$$1.31 < \frac{\kappa}{T^3} < 3.64 \text{ for } T = 1.5 T_c ,$$

$$0.63 < \frac{\kappa}{T^3} < 2.20 \text{ for } T = 3 T_c ,$$

$$0.43 < \frac{\kappa}{T^3} < 1.05 \text{ for } T = 6 T_c ,$$

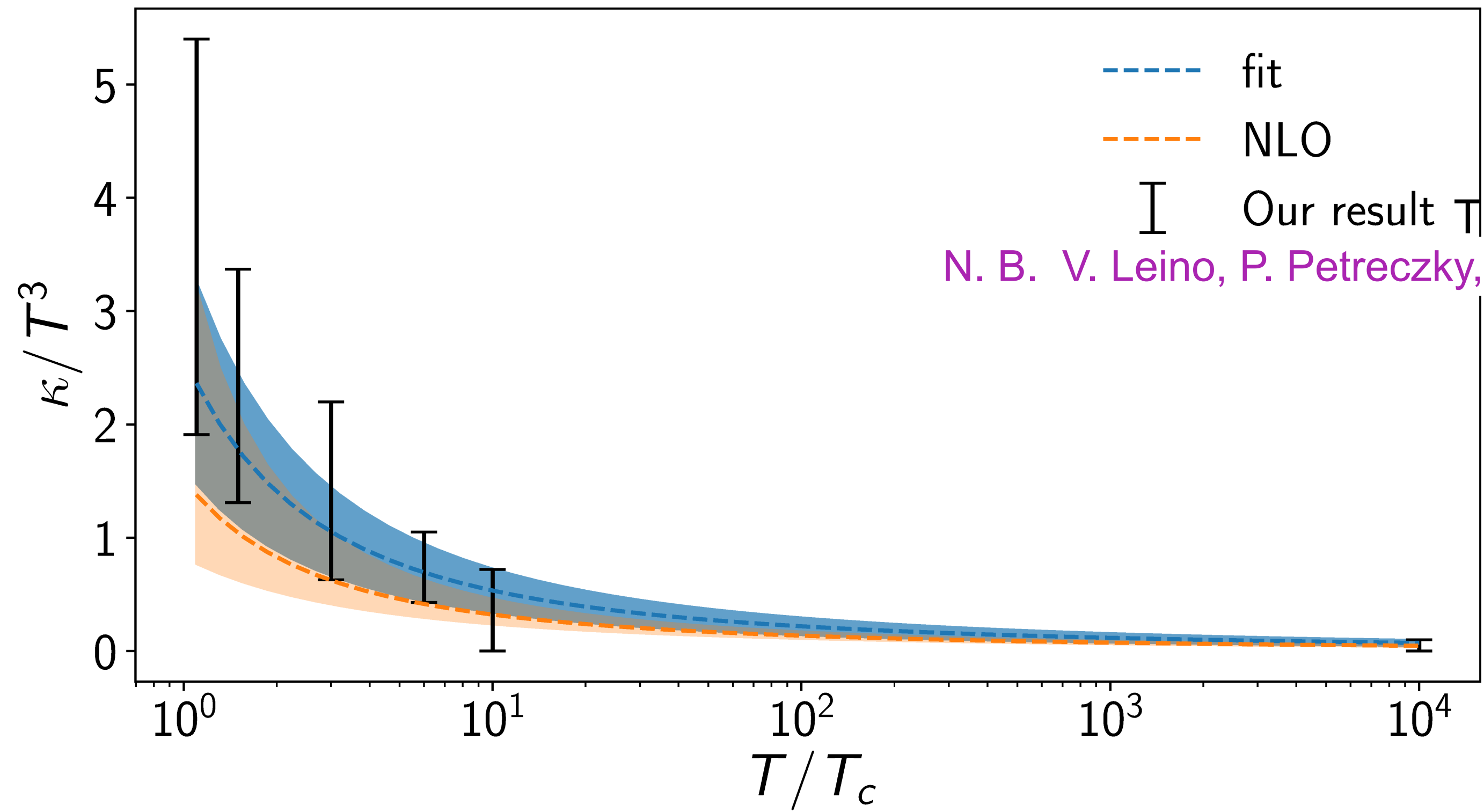
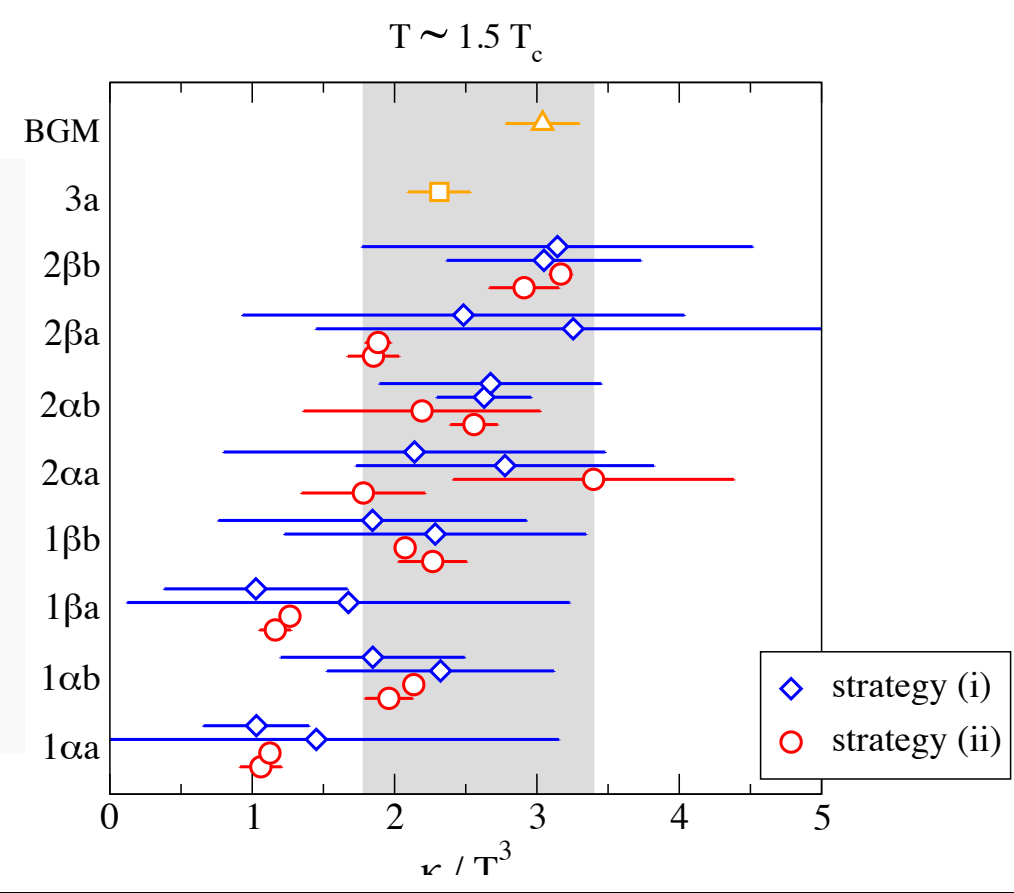
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in TUMQCD we studied kappa on
quenched latticed with the multilevel algorithm
in **a window of T** never attempted before
-> we get the T dependence of kappa

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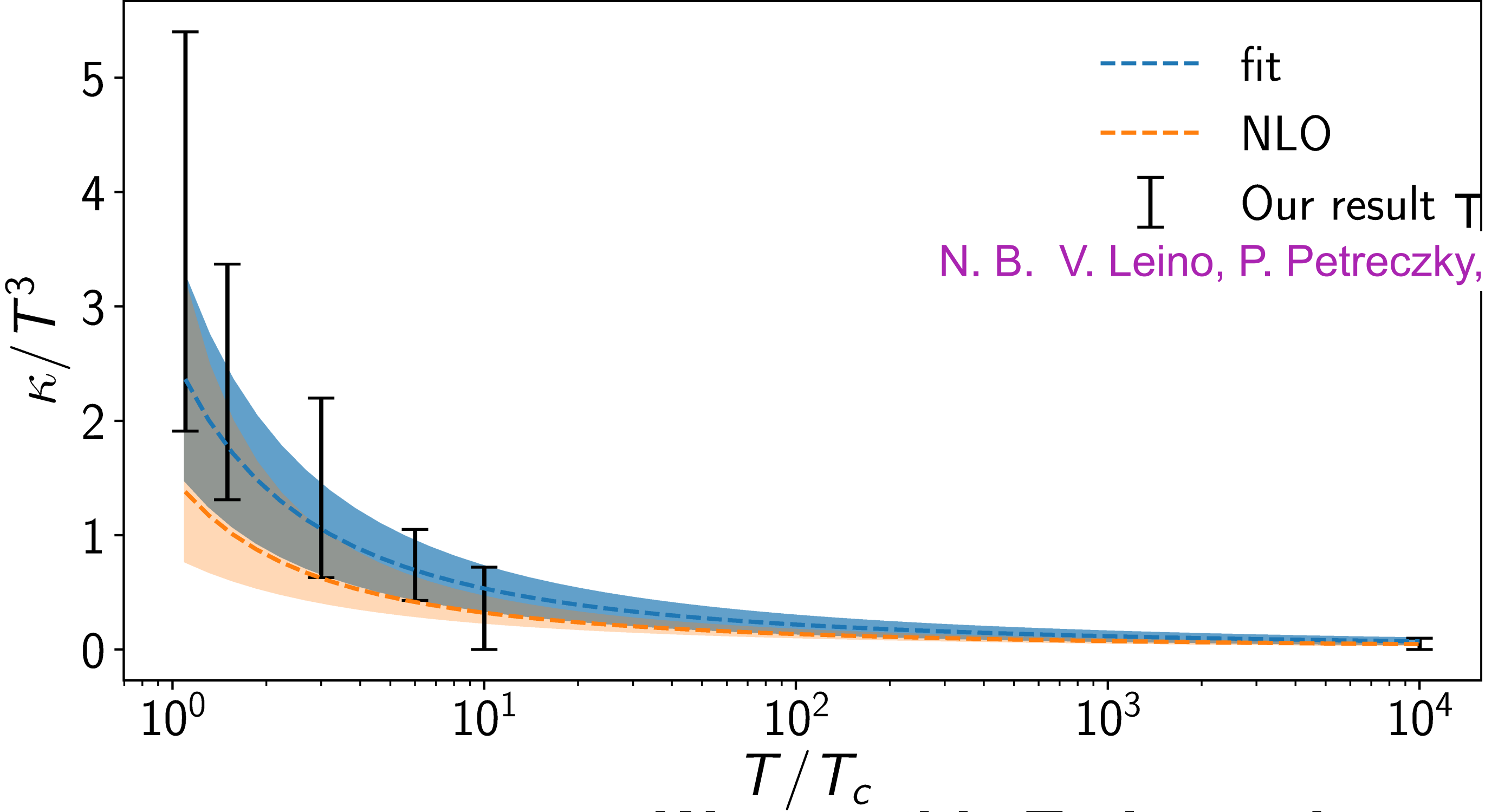
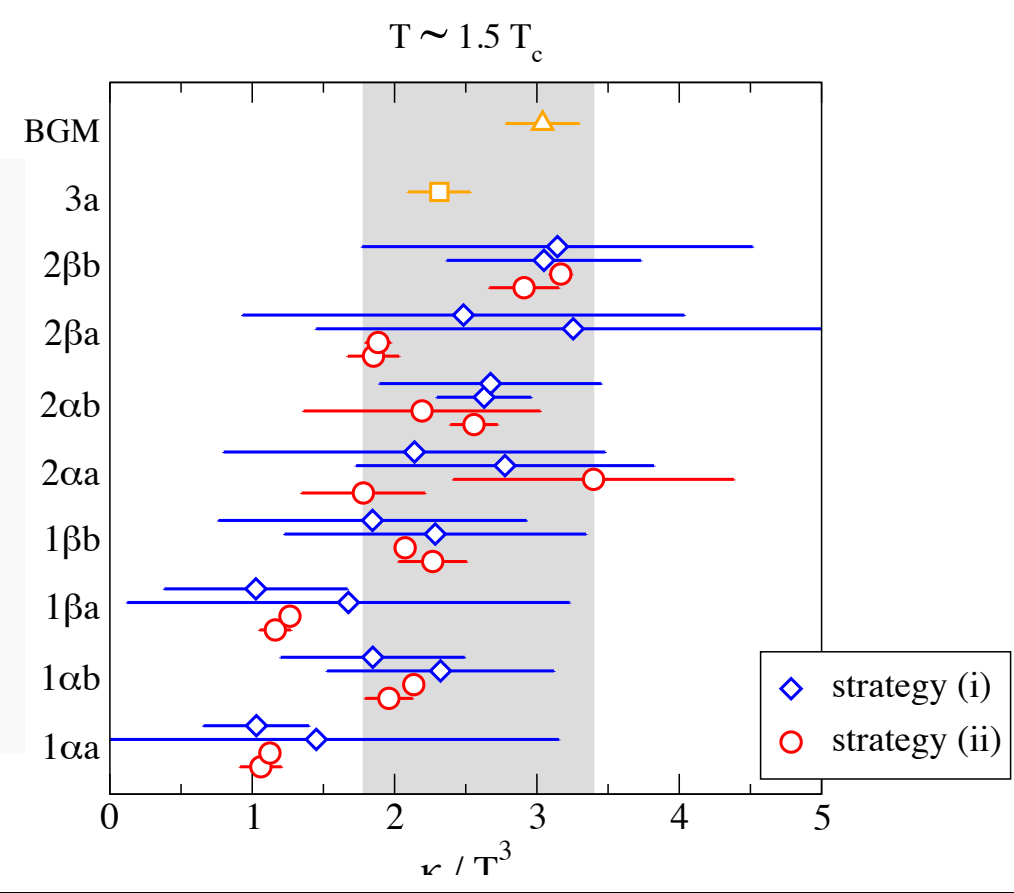
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within the errors the lattice results are compatible with the next-to-leading order perturbative results

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Our result TUMQCD
N. B. V. Leino, P. Petreczky, A. Vairo 2020

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$$0.63 < \frac{\kappa}{T^3} < 2.20 \text{ for } T = 3 T_c ,$$
$$0.43 < \frac{\kappa}{T^3} < 1.05 \text{ for } T = 6 T_c ,$$
$$0 < \frac{\kappa}{T^3} < 0.72 \text{ for } T = 10 T_c ,$$
$$0 < \frac{\kappa}{T^3} < 0.10 \text{ for } T = 10^4 T_c ,$$

We use this T dependence of kappa in our Linblad equation

within the errors the lattice results are compatible with the next-to-leading order perturbative results

Use the EFT to relate kappa (and gamma) to observables: quarkonium thermal mass shift and thermal widths

N.B., M. Escobedo, A. Vairo, P. vander Griend Phys.Rev. D100 (2019) no.5, 054025

kappa is related to the thermal decay width of quarkonium

in the hierarchy $\frac{1}{r} \gg T \gg E$ pNRQCD predicts for 1S states

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Therefore we can use **unquenched lattice data** on **quarkonium thermal mass shift and widths** to get **unquenched determination of these transport coefficients**

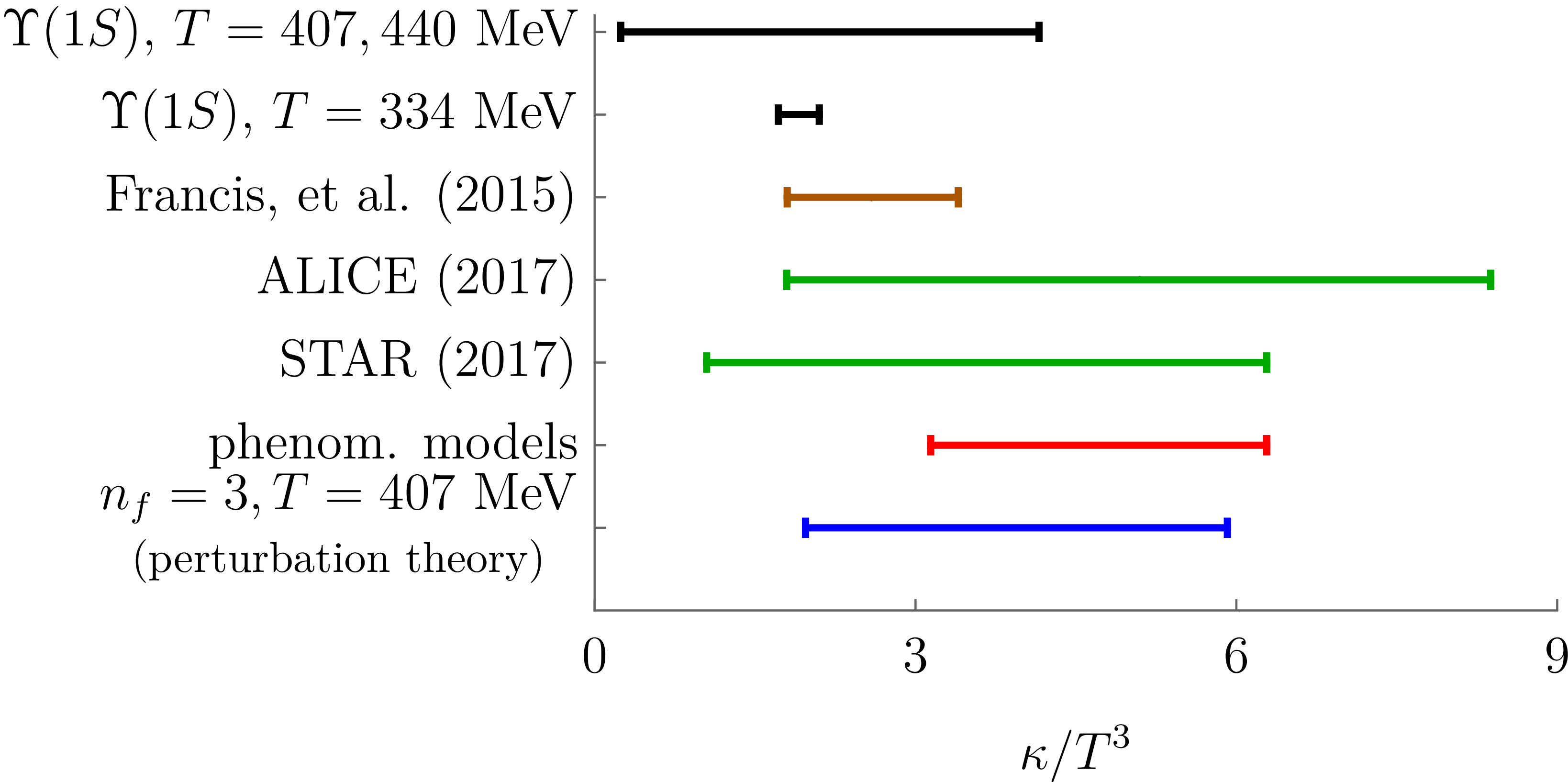
Use lattice data from Aarts,
Allton, Kim, Lombardo, Oktay,
Ryan, Sinclair and Skullerud
(2011) and Kim, Petreczky and
Rothkopf (2018). Unquenched.

and the new data from R. Larsen,
S. Meinel, S. Mukherjee, P. Petreczky
2019, 2020

Unquenched determinations of kappa and gamma

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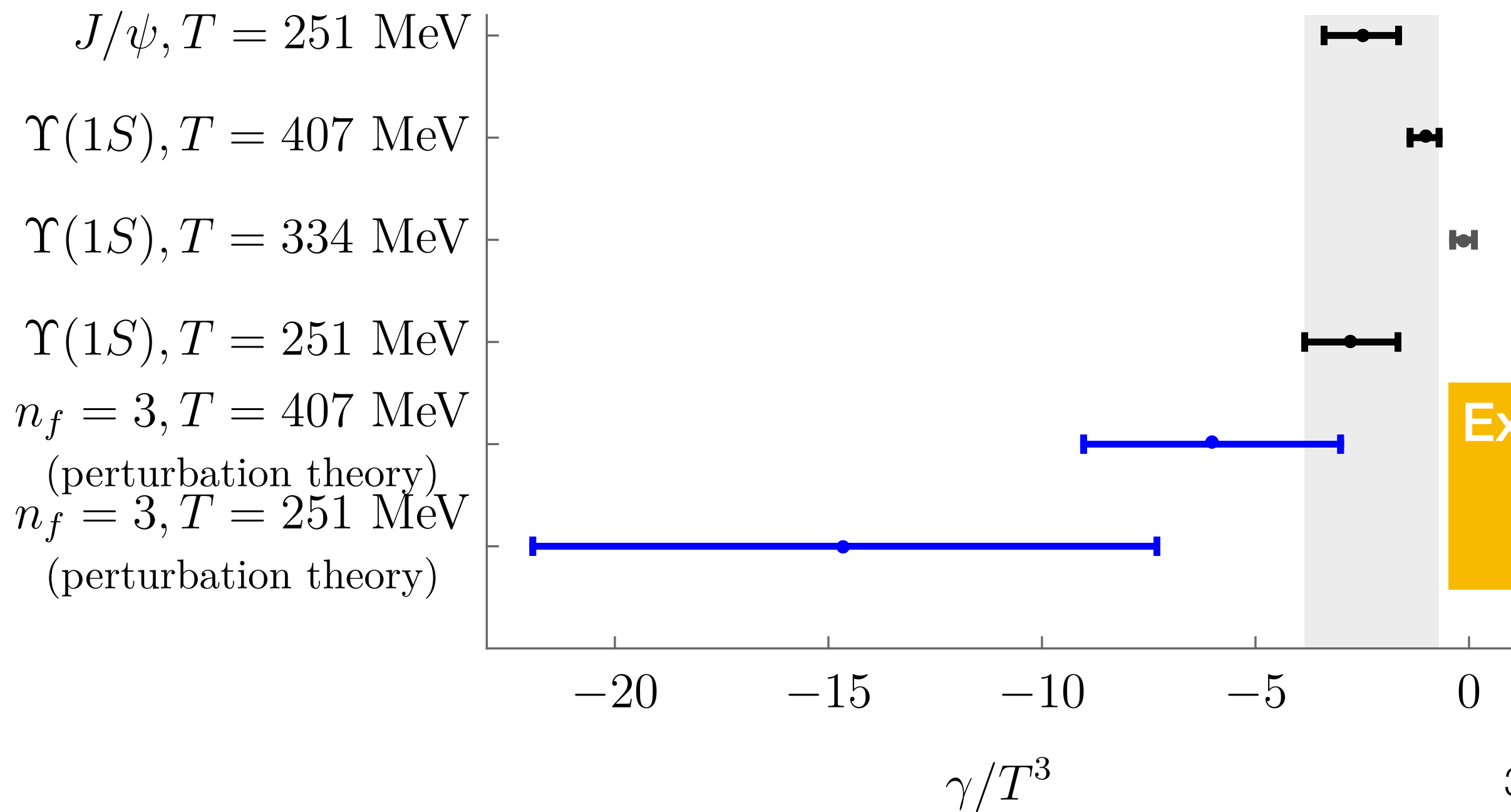
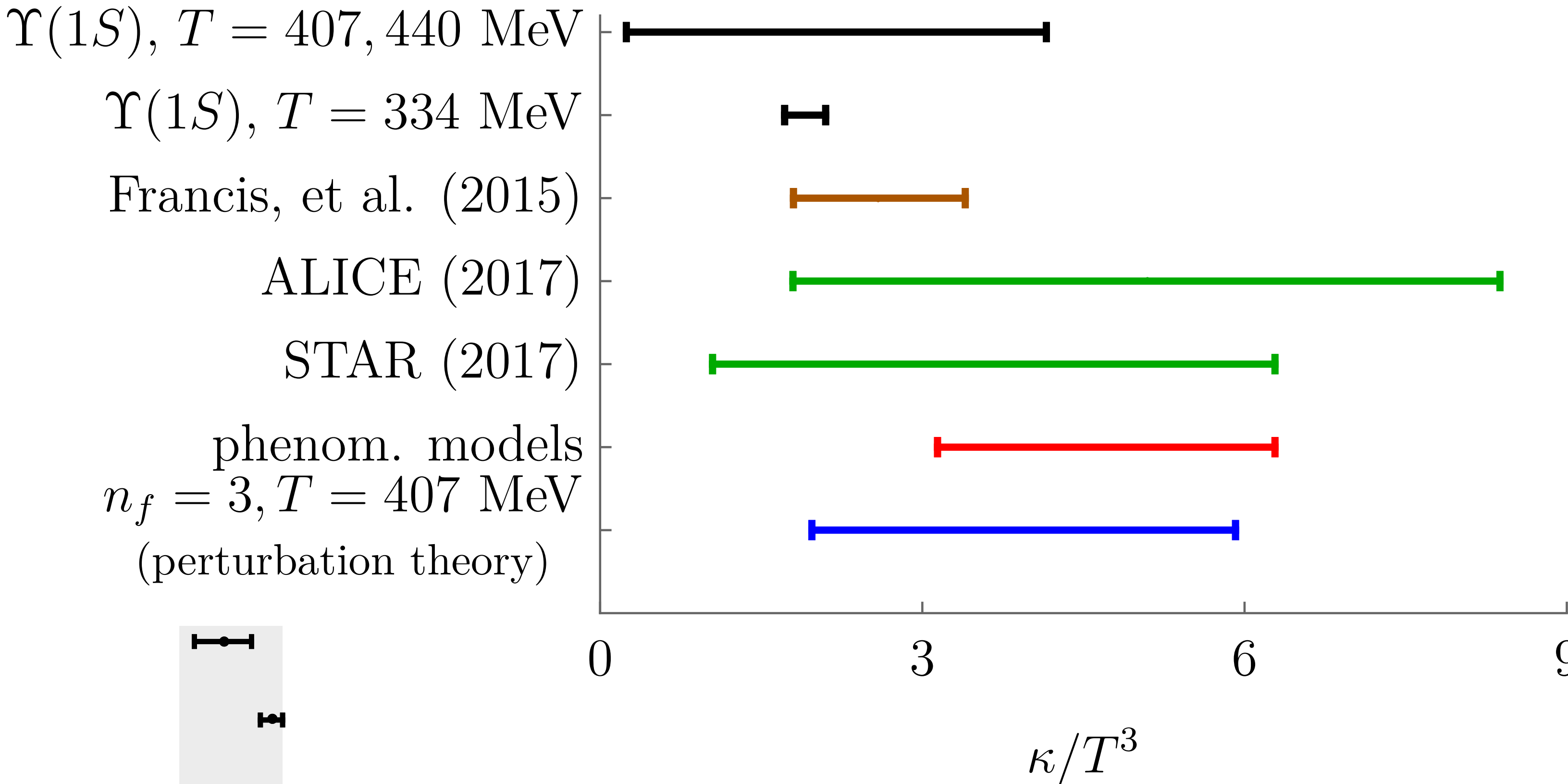
Extraction of kappa from unquenched lattice data for the thermal width of the $\Upsilon(1S)$ (black lines) in comparison to a quenched lattice determination (brown), determinations from the D meson v2 from Alice and Star data (green), model compilation from 1903.07709) (red) and the perturbative calculation (truncated g^5) (blue)



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Extraction of gamma from unquenched lattice data for the thermal width of the $\Upsilon(1S)$ and J/ψ (black lines) in comparison to NLO order perturbation theory at two different T

Solving the Linblad equation

Initial conditions

- The production of singlets is α_s suppressed compared to that of octets.

◦ Cho Leibovich PRD 53 (1996) 6203

$$\rho_s = N|\mathbf{0}\rangle\langle\mathbf{0}|, \quad \rho_o = \frac{1}{\alpha_s}\rho_s$$

N is fixed by $\text{Tr}\{\rho_s\} + \text{Tr}\{\rho_o\} = 1$

evolve in QGP from $t_0 = 0.6$ fm up $T = 250$ MeV

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$$T = T_0 \left(\frac{t_0}{t} \right)^{v_s^2}, \quad t_0 = 0.6 \text{ fm}, \quad v_s^2 = \frac{1}{3} \text{ (sound velocity)}$$

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◦ Brambilla Escobedo Soto Vairo PRD 96 (2017) 034021

Recently thanks to the collaboration with Mike Strickland we developed a much more efficient (embarrassingly parallel) program based on the quantum trajectory algorithm (Qtraj) and we coupled this to the hydrodynamical evolution of the QGP using a 3+1D dissipative hydrodynamics code (aHydro3p1)

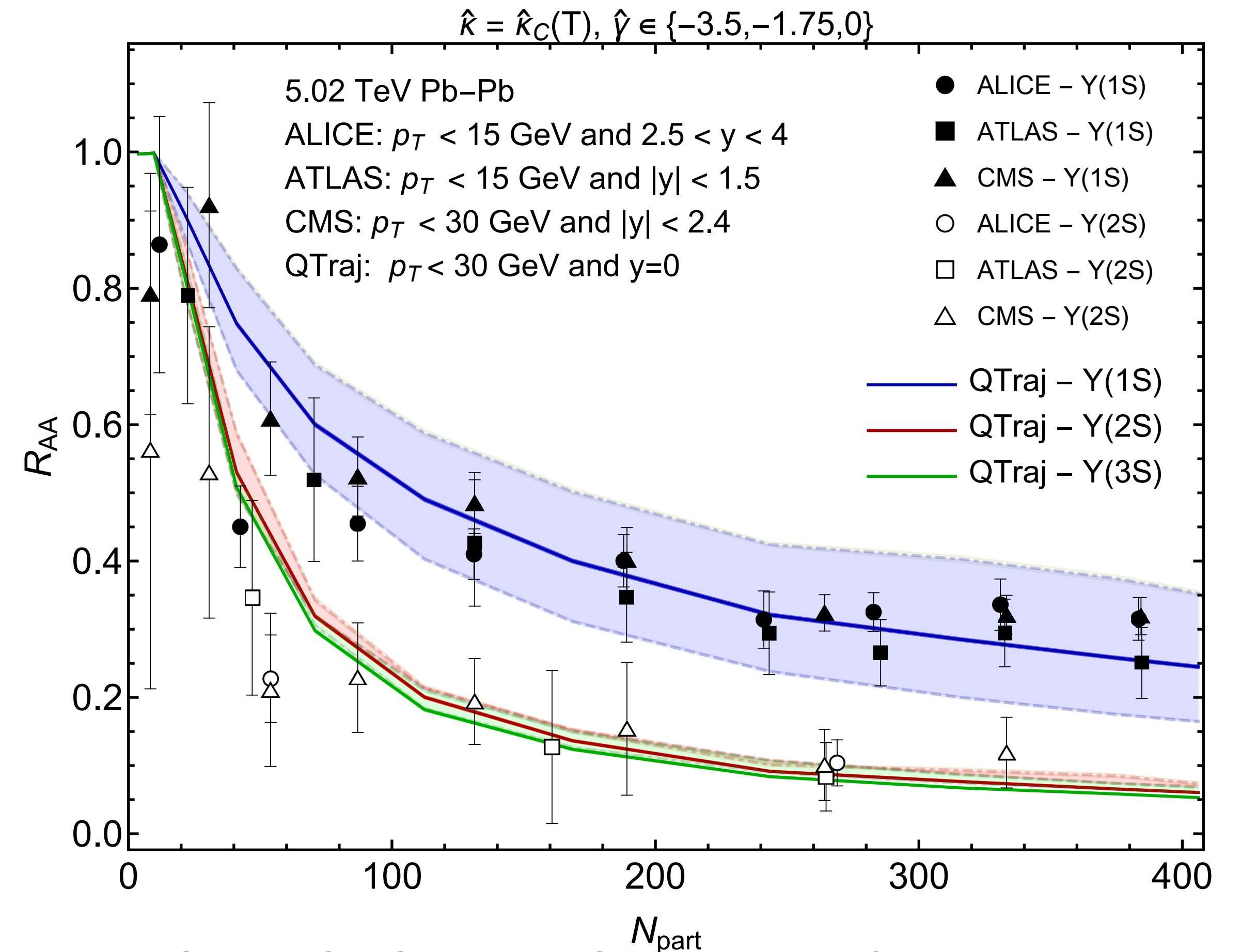
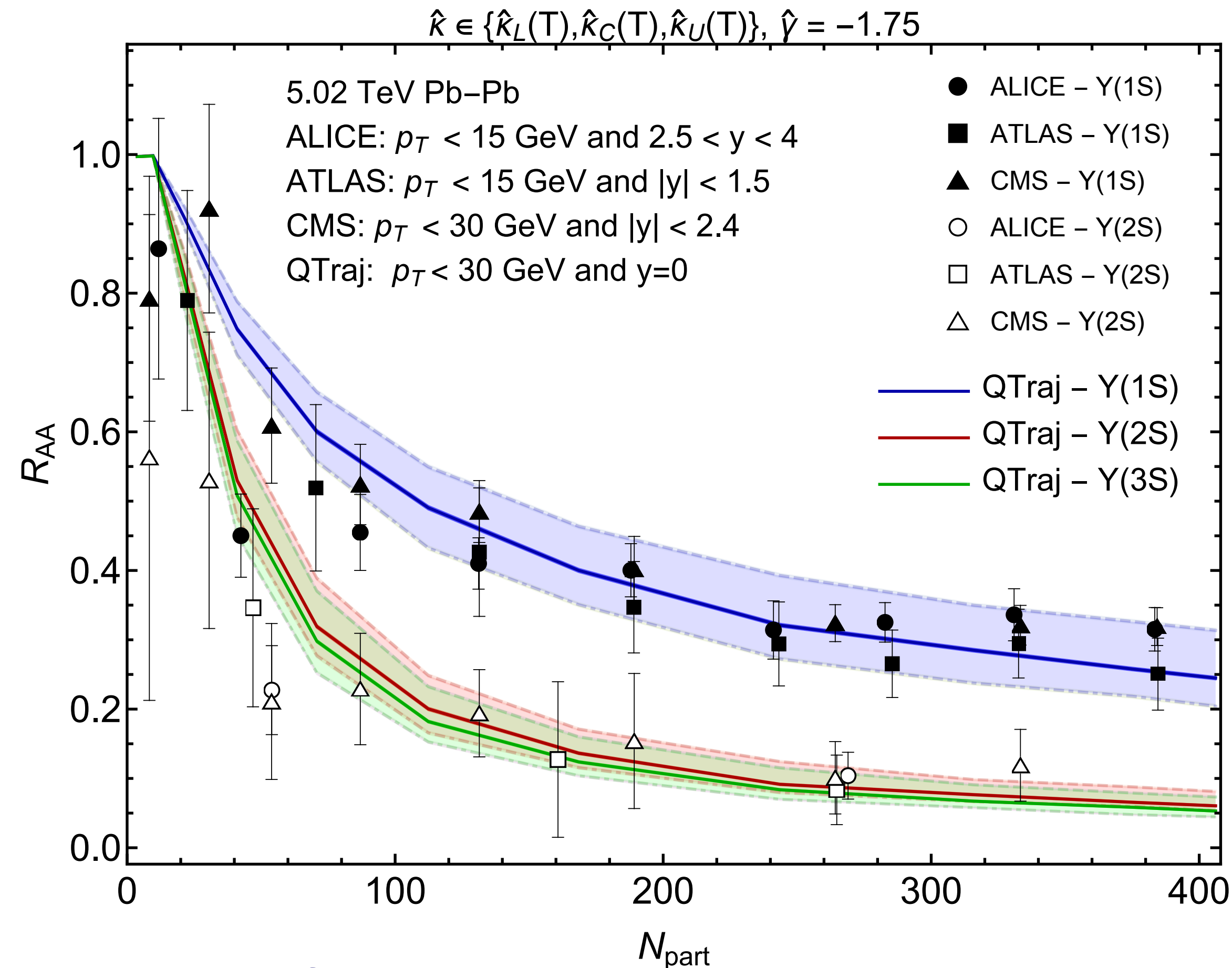
N.B. Escobedo , Strickland, Vairo, Vander Griend, Weber, 2012.01240

nonequilibrium evolution of quarkonium in medium: nuclear modification factor R_{AA}

We compute the **nuclear modification factor** R_{AA} :

$$R_{AA}(nS) = \frac{\langle n, \mathbf{q} | \rho_s(t_F; t_F) | n, \mathbf{q} \rangle}{\langle n, \mathbf{q} | \rho_s(0; 0) | n, \mathbf{q} \rangle}$$

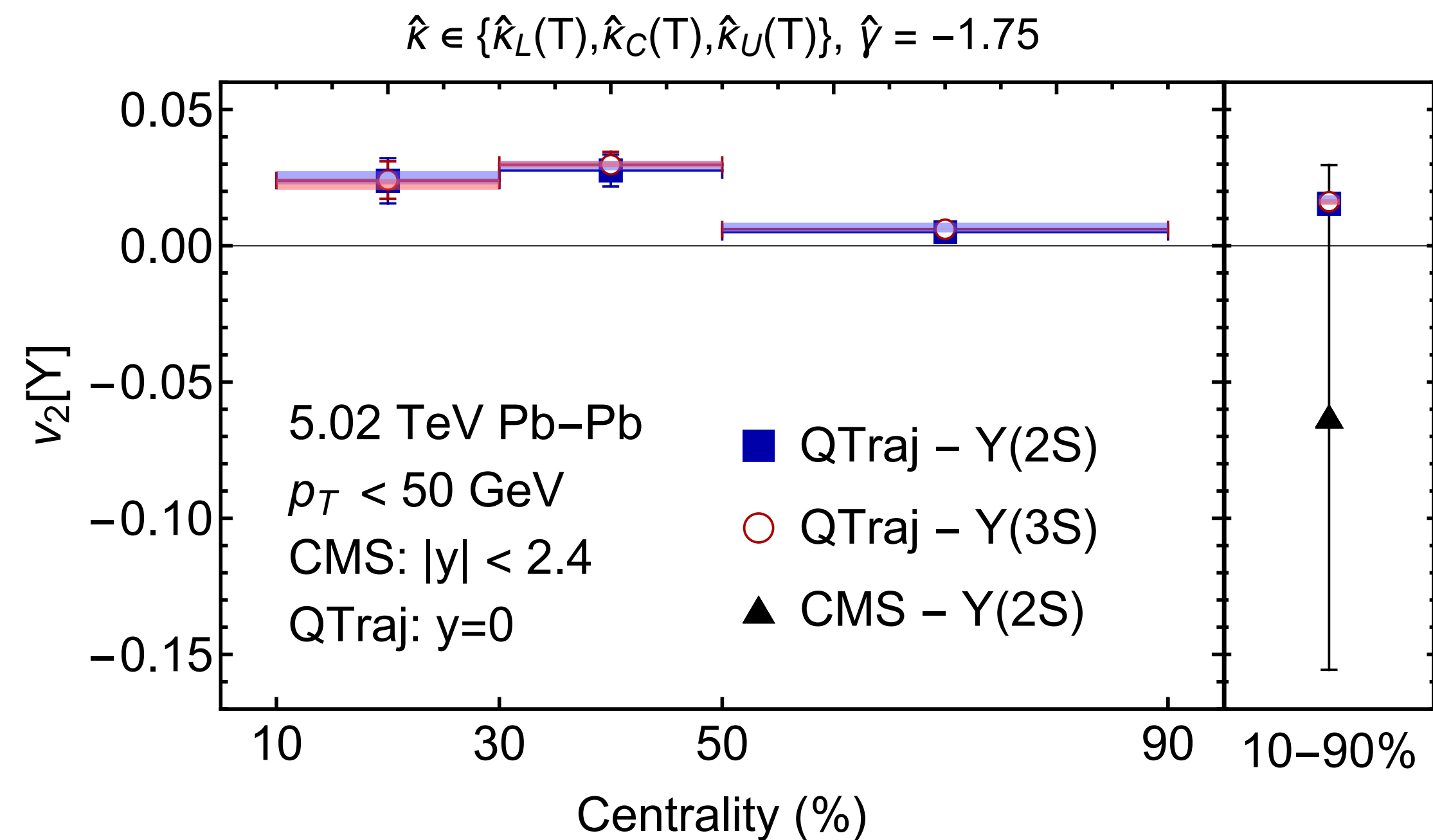
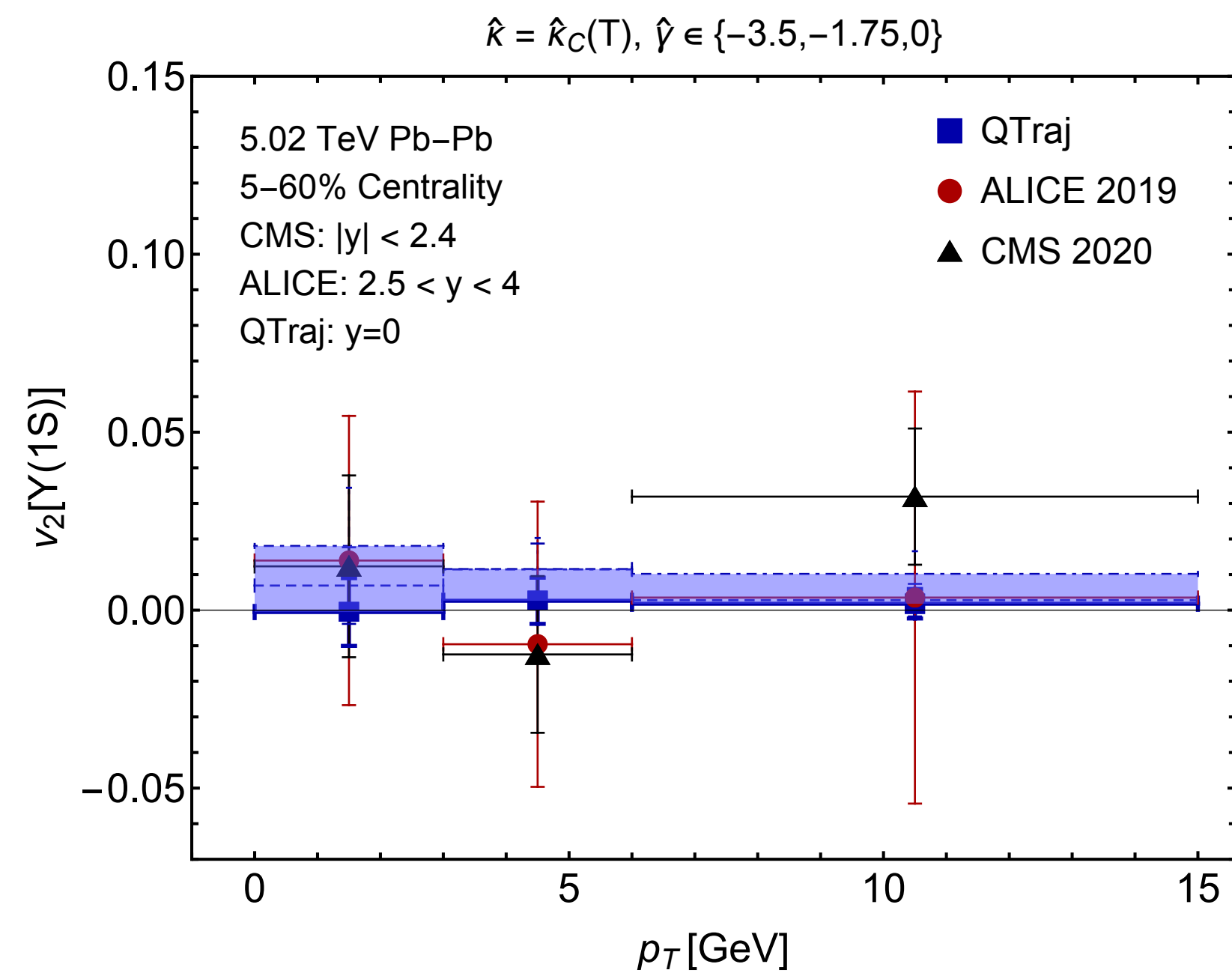
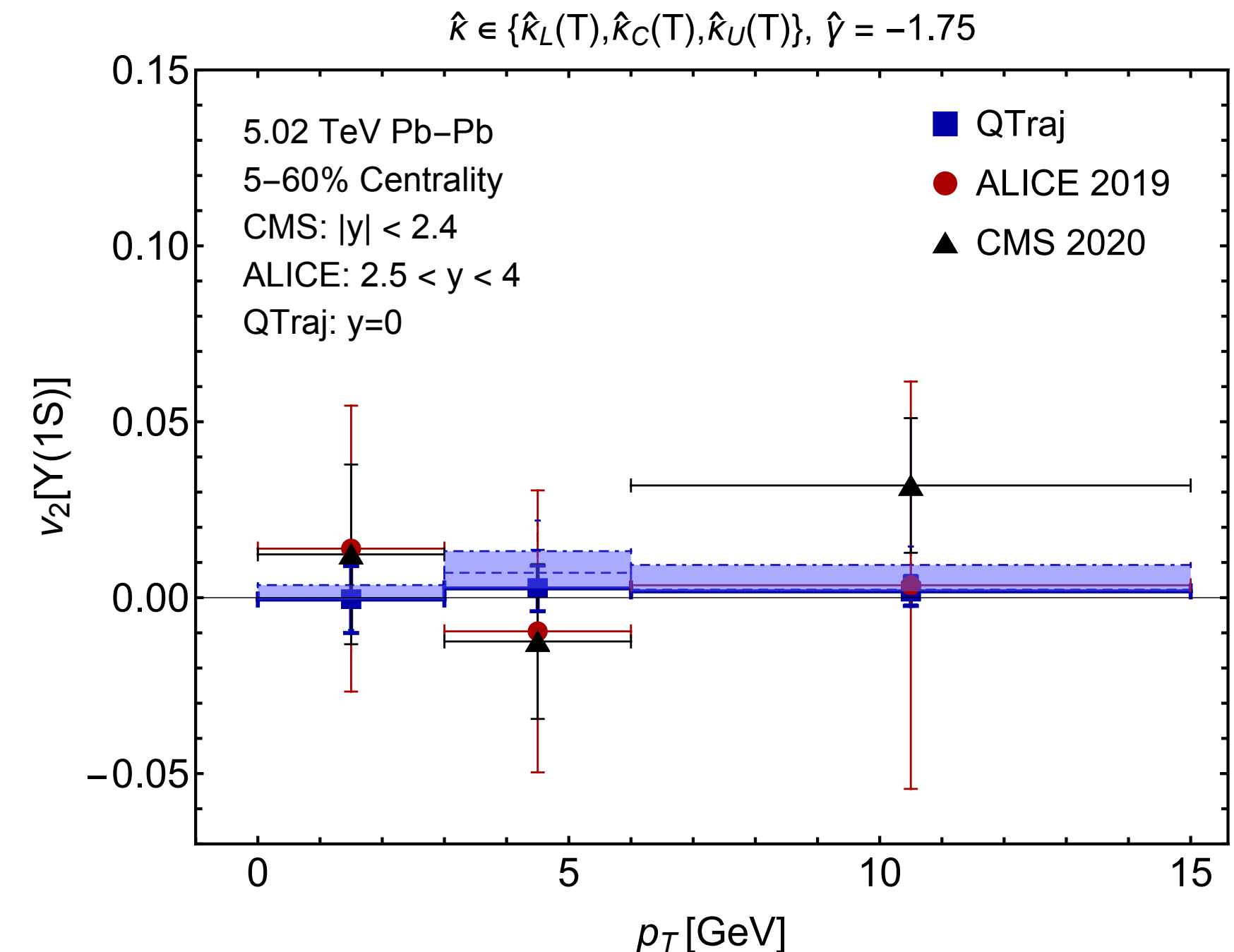
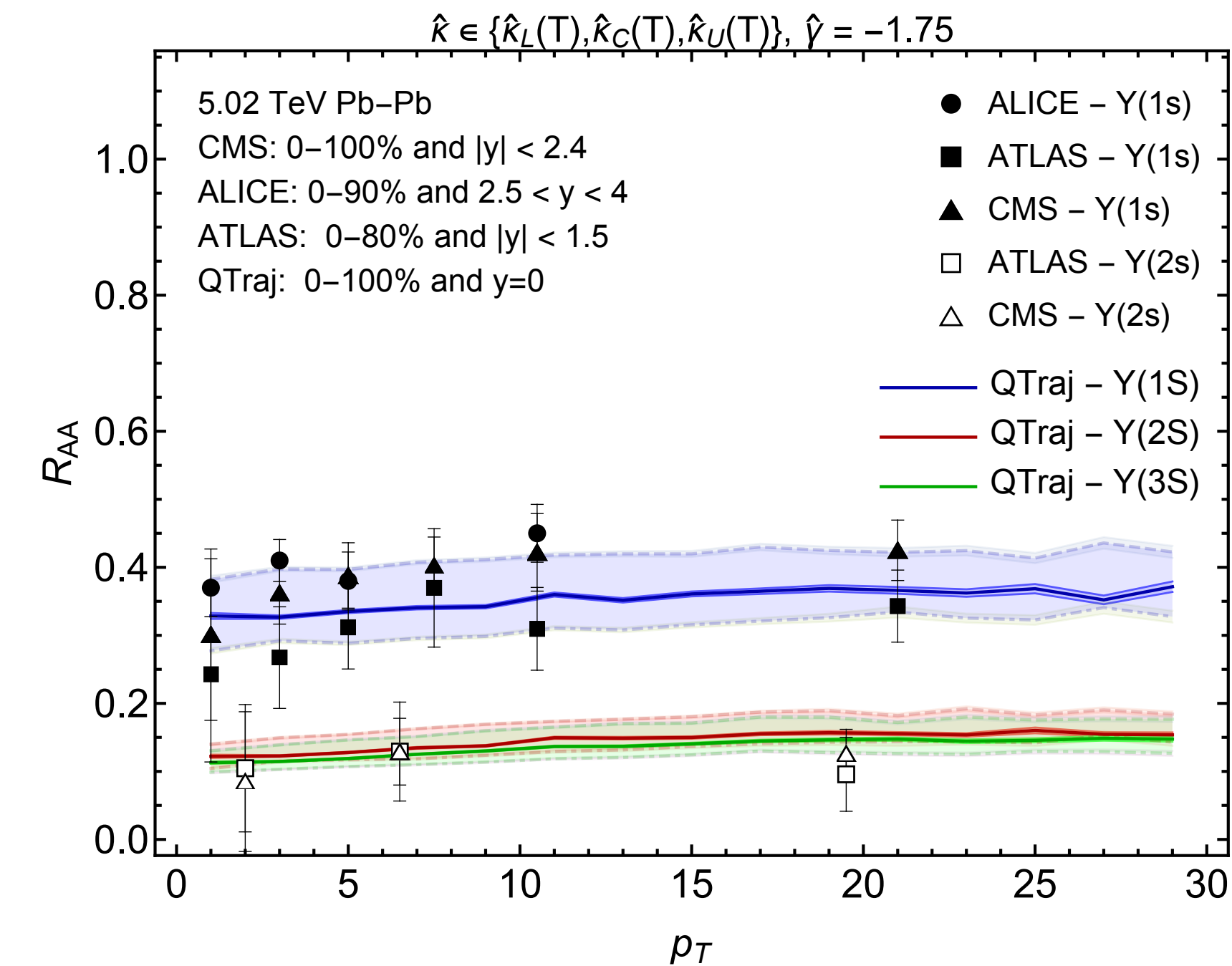
calculation with no
free parameters, results depends
on kappa function
of T (calculated on the lattice)
and gamma (extracted from the lattice)



R_{AA} of singlet Bottomonium in comparison to ALICE, ATLAS and CMS data, **left plot** bands from variation in kappa, **right plot** variation in gamma —> we can use R_{AA} to learn about the QGP!

Differential quantities

**Full diagnostic of the medium
in terms of objects
with a proper field
theoretical
definition, evaluated
on the lattice**



■ This calculation with no free parameter can reproduce *inside errors* all the experimental data on bottomonia 1S 2S 3S:

■ The band in our prediction depends on the indetermination on the transport coefficients

■ Recombination is there but it is small for $\Upsilon(1S)$ bottomonium

■ The evolution equations we obtained do not make any special assumption on the medium

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- R_{AA} as a function of N_{part} and as a function of p_T
- Ratios of R_{AA} , v_2

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They could be used far from equilibrium or for a medium with a scale different from $T \rightarrow$ use different methods to evaluate kappa and gamma (kinetic theory, classical simulations)

Conclusions

We have shown a realistic particle physics example where a complex full system made out of a **multiscale subsystem** (quarkonium) interacting with a rich and inherently **non-perturbative environment** (the quark-gluon plasma) could be studied in its **out-of-equilibrium evolution** with the methods of

- **effective field theories**, to **factorize** contributions coming from different energy scales. Contributions coming from high-energy scales (mass, ...) can be computed in perturbation theory.
- **lattice QCD**, to compute numerically on a space-time lattice low-energy non-perturbative contributions.
- **open quantum systems**, to compute the out-of-equilibrium evolution of the subsystem and its non-trivial interaction with the environment (production, dissociation and recombination of quarkonium).

As a result the study describes for the first time quarkonium dissociation taking into account **the conservation of the total number of heavy quarks, the non-Abelian nature of QCD, without any classical approximation.**

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- Quarkonium suppression may be systematically studied with the use of **effective field theories** and **lattice QCD**
- **In equilibrium** properties like **dissociation width, cross section, mass shift...** have been computed as expansions in the small parameters of the system.
- **Out of equilibrium** properties, like octet recombination, can be studied by treating quarkonium as an open quantum system. Lattice input is crucial.

The evolution equations follow from assuming the inverse size of the quark-antiquark system to be larger than any other scale of the medium and from being accurate at first non-trivial order in the multipole expansion and at first order in the heavy-quark density.

Under some conditions (large time, quasistatic evolution, temperature much larger than the inverse evolution time of the quarkonium) the evolution equations are of the Lindblad form. Their numerical solution provides $R_{AA}(nS)$ close to experimental data.

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Outlook

- We need precise and unquenched determinations of the kappa and gamma transport coefficients
- Recombination effects are small for bottomonium for not for charmonium:
we should go beyond the linear density approximation in that case
- We should investigate the effect of quarkonium moving with respect to the QGP and the anisotropy
- We should investigate the full master equations farther out of equilibrium: all the calculations holds
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Inside EFT and OQS and with the help of the lattice
quarkonium holds the promise to be a golden probe of QGP!