Electroproduction of Heavy Quarkonia: Probing the Spin-2 (Gravitational) and Resummed Spin-j (Pomeron) Form Factors of Proton

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Argonne NATIONAL LABORATORY

References

This talk is based on the references **2106.00722** (PRD), **2103.03186** (PRD), and **1910.04707** (PRD) in collaboration with **Ismail Zahed**

Spin-2 (Gravitational) Form Factor of Proton

- the gravitational form factors of proton are defined as

$$\langle p_2|T^{\mu\nu}(0)|p_1\rangle = \overline{u}(p_2)\left(A(k)\gamma^{(\mu}p^{\nu)} + B(k)\frac{ip^{(\mu}\sigma^{\nu)\alpha}k_\alpha}{2m_N} + C(k)\frac{k^\mu k^\nu - \eta^{\mu\nu}k^2}{m_N}\right)u(p_1)$$

with $a^{(\mu}b^{\nu)} = \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$, $k^2 = (p_2 - p_1)^2 = t$, $p = (p_1 + p_2)/2$, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] = i(\gamma^\mu\gamma^\nu - \eta^{\mu\nu})$, and the normalizations $\overline{u}u = 2m_N$ and $m_N \times \bar{u}(p_1)\gamma^\mu u(p_2) = \bar{u}(p_1)p^\mu u(p_2)$

- the energy-momentum tensor in the proton state is conserved and tracefull
- throughout, $D(k) = 4C(k)$ will be used interchangeably
- the spin-2 (gravitational) form factor of proton corresponds to

$$\langle p_2|T^{xy}(0)|p_1\rangle = \overline{u}(p_2)\left(A(k)\gamma^{(x}p^{y)}\right)u(p_1)$$

after setting $k_z \neq 0$, $k_x = k_y = 0$, and $p_z = 0$ with $p_x \neq 0$, and $p_y \neq 0$

Spin-2 (Gravitational) Form Factor of Proton

- note that the spin-0 (scalar) form factor of proton corresponds to

$$\left\langle p_2 \right| T^{\mu}_{\mu}(0) \left| p_1 \right\rangle = \overline{u}(p_2)\left(A(k)\gamma^{\mu}p_{\mu} + B(k)\frac{ip^{\mu}\sigma_{\mu}^{\ \alpha}k_{\alpha}}{2m_N} - 3C(k)\frac{k^2}{m_N}\right) u(p_1)$$

- therefore we have $\langle p|T^{xy}(0)|p\rangle = \langle p|T^{\mu}_{\mu}(0)|p\rangle = \overline{u}(p)\left(A(0)\gamma^{\mu}p_{\ \mu}\right)u(p) = \overline{u}(p)\left(A(0)\gamma^{(x}p^{y)}\right)u(p)$ for $k = 0$, and arbitrary values of $B(0)$ and $C(0)$
- in general $\langle p_2|T^{xy}(0)|p_1\rangle \neq \langle p_2|T^{\mu}_{\mu}(0)|p_1\rangle$ unless $B(k) = C(k) = 0$
- indeed in holographic QCD with degenerate mass spectrum for 2^{++} and 0^{++} glueballs, we have $B(k) = C(k) = 0$
- but in lattice QCD with non-degenerate mass spectrum for 2^{++} and 0^{++} glueballs, we have $B(k) = 0$, and $C(k) \neq 0$

Spin-2 (Gravitational) Form Factor of Proton

- in holographic QCD, the gravitational form factors, can be computed using Witten diagrams in AdS

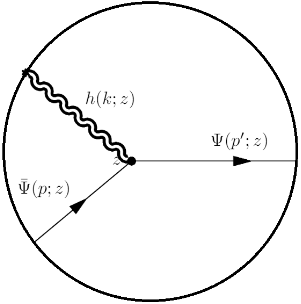

Figure: Witten diagram for the spin-2 gravitational form factor $\langle p_2|T^{xy}(0)|p_1\rangle$ due to the exchange of spin-2 glueball resonances.

Spin-2 (Gravitational) Form Factor of Proton

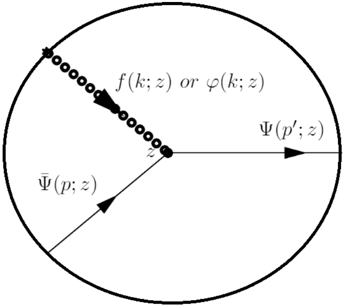

Figure: Witten diagram for the spin-0 gravitational form factor $\langle p_2 | T^{\mu}_{\ \mu}(0) | p_1 \rangle$ due to the exchange of spin-0 glueball resonances.

Spin-2 (Gravitational) Form Factor of Proton

- the gravitational form factors of proton follow from the coupling of the irreducible representations of the metric fluctuations $h_{\mu\nu}$ to a bulk Dirac fermion
- and the bulk metric fluctuations can be decomposed in terms of the $2 \oplus 1 \oplus 0$ invariant tensors [Kanitscheider:2008]

$$h_{\mu\nu}(k,z) = \left[\epsilon^{TT}_{\mu\nu} h(k,z) + k_\mu k_\nu H(k,z)\right] + \left[k_\mu A^\perp_\nu(k,z) + k_\nu A^\perp_\mu(k,z)\right] + \left[\frac{1}{3}\eta_{\mu\nu} f(k,z)\right]$$

which is the spin-2 made of the transverse-traceless part h plus the longitudinal-tracefull part H, the spin-1 made of the transverse vector $A^\perp_\mu$, and the spin-0 tracefull part f

we have found the spin-2 (gravitational) form factor of proton to be

$$\tilde{A}(K) = \frac{A(K)}{A(0)} \;\; = \;\; 6 \times \frac{\Gamma\left(2 + a_K\right)}{\Gamma\left(4 + a_K\right)} \times \, {}_2F_1\left(3, a_K; a_K + 4; -1\right) \; ,$$

with $a_K = K^2/8\tilde{\kappa}_N^2$ at $\tilde{\kappa}_N = 0.350 \; GeV$ fixed by the mass of rho meson and proton

from the spin-2 (gravitational) form factor of proton, we find the mass radius of proton to be

$$\left\langle r^2_{Mass} \right\rangle_p = -6 \left(\frac{d \ln A(K)}{dK^2} \right)_0 = \frac{1.04}{\kappa_N^2} \, \hbar^2 c^2 = (0.57 \, \mathrm{fm})^2$$

Spin-2 (Gravitational) Form Factor of Proton

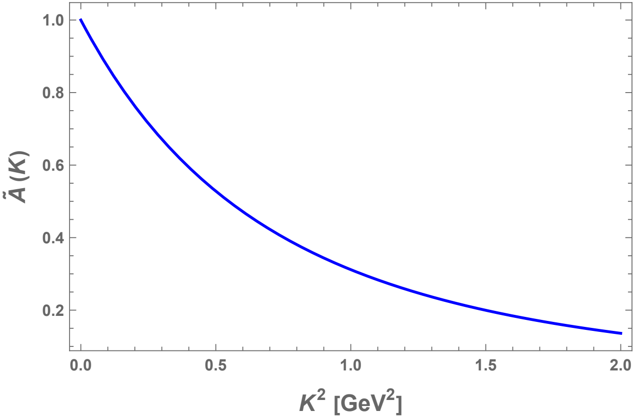

Figure: The spin-2 nucleon gravitational form factor $\tilde{A}(K)$.

Resummed Spin-j (Pomeron) Form Factor of Proton

- the spin-j form factors of proton can be defined as

$$\langle p_2|T^{\mu_1\mu_2\cdots\mu_j}(0)|p_1\rangle = \overline{u}(p_2)\Bigg(\mathcal{A}(k,j)\gamma^{\mu_1}p^{\mu_2}...p^{\mu_j} + \mathcal{B}(k,j)\frac{ip^{\mu_1}p^{\mu_3}...p^{\mu_j}\sigma^{\mu_2\alpha}k_\alpha}{2m_N}$$
$$+\,\mathcal{C}(k,j)\frac{k^{\mu_1}k^{\mu_2}...k^{\mu_j} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}...\eta^{\mu_{j-1}\mu_j}k^2}{m_N}\Bigg)u(p_1)$$

- setting $\mu_1 = x$, $\mu_2 = y$, $k_z \neq 0$, $k_x = k_y = 0$, and $p_z = 0$ with $p_x \neq 0$, and $p_y \neq 0$, we have

$$\langle p_2|T^{xy\mu_3\cdots\mu_j}(0)|p_1\rangle = \overline{u}(p_2)\Big(\mathcal{A}(k,j)\gamma^x p^y p^{\mu_3}...p^{\mu_j}\Big)u(p_1)$$

Resummed Spin-j (Pomeron) Form Factor of Proton

- and we define the resummed spin-j (Pomeron) form factor of proton as

$$\sum_j \langle p_2 | T^{xy\mu_3\ldots\mu_j}(0) | p_1 \rangle = \sum_j \overline{u}(p_2) \left(\mathcal{A}(k,j) \gamma^x p^y p^{\mu_3} \ldots p^{\mu_j} \right) u(p_1)$$

Resumed Spin-j (Pomeron) Form Factor of Proton

- in holographic QCD, the resummed spin-j (Pomeron) form factor, can be computed using Witten diagrams in AdS

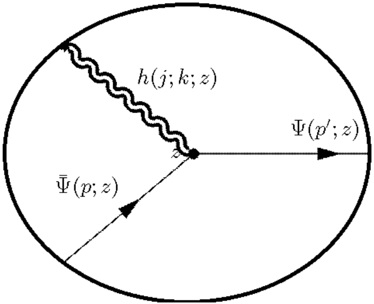

Figure: Witten diagram for the spin-j gravitational form factor $\langle p_2|T^{xy\mu_3\cdots\mu_j}(0)|p_1\rangle$ due to the exchange of spin-j glueball resonances.

Resummed Spin-j (Pomeron) Form Factor of Proton

- we have found the resummed spin-j (Pomeron) form factor of proton to be

$$
\begin{aligned}
\mathcal{A}(j=j_0,\tau,\Delta,K) &= \Gamma\left(a_K+\frac{\Delta(j)}{2}\right) \\
\times\quad & \frac{2^{1-\Delta}}{\Gamma(\tau)}\Bigg((\tau-1)\Gamma\left(\frac{j}{2}+\tau-\frac{\Delta}{2}\right)\Gamma\left(\frac{1}{2}(j+\Delta+2\tau-4)\right) \\
\times\quad & {}_2F_1\left(\frac{1}{2}(j-\Delta+2\tau),\frac{1}{2}(-\Delta+2a_k+4);\frac{1}{2}(j+2\tau+2a_k);-1\right) \\
+\quad & \Gamma\left(\frac{j}{2}+\tau-\frac{\Delta}{2}+1\right)\Gamma\left(\frac{1}{2}(j+\Delta+2\tau-2)\right) \\
\times\quad & {}_2F_1\left(\frac{1}{2}(j-\Delta+2\tau+2),\frac{1}{2}(-\Delta+2a_k+4);\frac{1}{2}(j+2\tau+2a_k+2);-1\right)\Bigg),
\end{aligned}
$$

with $j_0=2-\frac{2}{\sqrt{\lambda}}$, $\tau=3$ (fixed by the electromagnetic form factor of proton), and $a_k=K^2/8\tilde{\kappa}_N^2$ at $\tilde{\kappa}_N=0.350\ GeV$ (fixed by the mass of rho meson and proton)

Resumed Spin-j (Pomeron) Form Factor of Proton

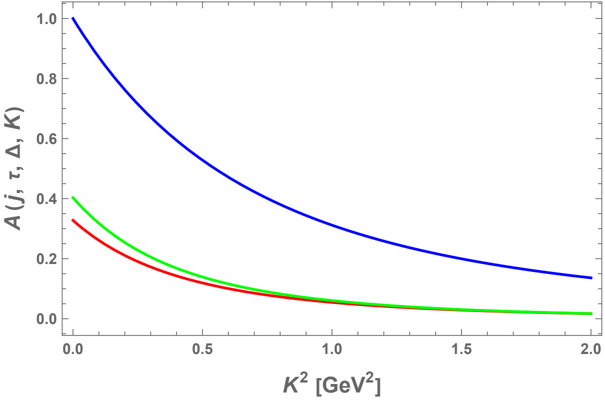

Figure: The spin-j nucleon form factor $\mathcal{A}(j, \tau, \Delta(j), K)$: Upper-blue-curve is $j = j_0 = 2$ and $\Delta(j = 2) = 4$; Middle-green curve is $j = j_0 = 2 - 2/\sqrt{\lambda}$ and $\Delta(j = j_0) = 2$ with $\sqrt{\lambda} = \infty$; Lower-red-curve is $j = j_0 = 2 - 2/\sqrt{\lambda}$ and $\Delta(j = j_0) = 2$ with $\sqrt{\lambda} = \sqrt{11.243}$.

Resummed Spin-j (Pomeron) Form Factor of Proton

- we find the Pomeron mass radius of proton to be

$$\left\langle r^2_{Pomeron} \right\rangle_p = -6 \left(\frac{d \ln \mathcal{A}(j_0, \tau, \Delta(j_o), K)}{dK^2} \right)_0 = (0.425\,\mathrm{fm})^2$$

for $\sqrt{\lambda} = \sqrt{11.243}$, and $\left\langle r^2_{Pomeron} \right\rangle_p = (0.482\,\mathrm{fm})^2$ for $\sqrt{\lambda} = \infty$

Electroproduction of heavy mesons near threshold

- the spin-2 (gravitational) form factor can be measured from photo or electroproduction of heavy vector mesons (J/ψ or Υ) near threshold

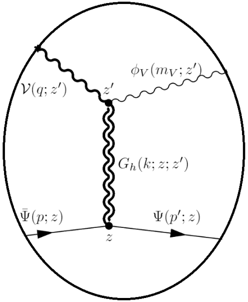

Figure: Witten diagram for the diffractive photoproduction of vector mesons with a bulk wave function ϕ_V. The thick lines or thick wiggles represent the propagators of summed over vector meson or glueball resonances. The thin lines or thin wiggles correspond to a single vector meson and proton.

Electroproduction of heavy mesons near threshold

- the differential cross section for electroproduction of heavy vector mesons (J/ψ or ϕ), near threshold, is given by

$$
\begin{aligned}
\frac{d\sigma(s,t,Q,M_{J/\Psi},\epsilon_T,\epsilon'_T)}{dt} &\propto \mathcal{I}^2(Q,M_{J/\Psi}) \times \left(\frac{s}{\tilde{\kappa}_N^2}\right)^2 \\
&\times \left(-\frac{t}{4m_N^2}+1\right) \times \tilde{A}^2(t)\,, \\
\frac{d\sigma(s,t,Q,M_{J/\Psi},\epsilon_L,\epsilon'_L)}{dt} &\propto \frac{1}{9} \times \frac{Q^2}{M_{J/\Psi}^2} \times \mathcal{I}^2(Q,M_{J/\Psi}) \times \left(\frac{s}{\tilde{\kappa}_N^2}\right)^2 \\
&\times \left(-\frac{t}{4m_N^2}+1\right) \times \tilde{A}^2(t)\,,
\end{aligned}
$$

Electroproduction of heavy mesons near threshold

- where we defined the transition form factor that controls the Q dependence as

$$\mathcal{I}(Q, M_{J/\Psi}) = \frac{\mathcal{I}(0, M_{J/\Psi})}{\frac{1}{6} \times \left(\frac{Q^2}{4\tilde{\kappa}^2_{J/\Psi}} + 3\right)\left(\frac{Q^2}{4\tilde{\kappa}^2_{J/\Psi}} + 2\right)\left(\frac{Q^2}{4\tilde{\kappa}^2_{J/\Psi}} + 1\right)},$$

with $\mathcal{I}(0, M_{J/\Psi}) = \frac{g_5 f_{J/\Psi}}{4M_{J/\Psi}}$, and $\tilde{\kappa}_{J/\Psi} = 2^{-3/4}\sqrt{g_5 f_{J/\Psi} M_{J/\Psi}}$

- we have also defined the spin-2 (gravitational) form factor of proton that controls the t dependence as

$$\tilde{A}(t) = \frac{A(t)}{A(0)} = 6 \times \frac{\Gamma(2 + a_K)}{\Gamma(4 + a_K)} \times {}_2F_1(3, a_K; a_K + 4; -1),$$

with $a_K = -t/8\tilde{\kappa}^2_N$ at $\tilde{\kappa}_N = 0.350\ GeV$ fixed by the mass of rho meson and proton

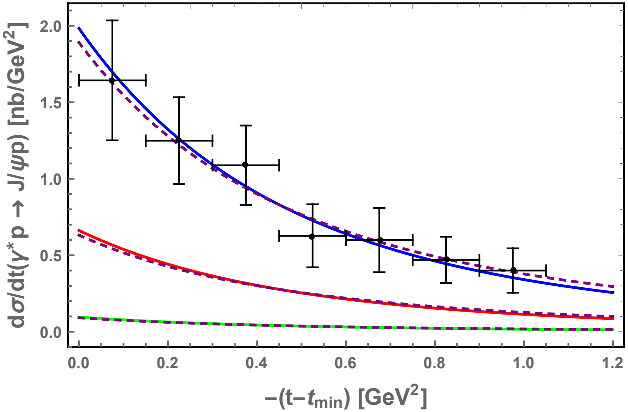

Figure: The variation of the total differential cross section with t and Q^2 for $s = 21\ GeV^2$, $\tilde{\kappa}_{J/\Psi} = 1.03784$ GeV, and $A^2(0) \times \tilde{n}' = 30.7944\ [nb/GeV^2]$.The blue curve is for $Q = 0$. The red curve is for $Q = 1.2\ GeV$. The green curve is for $Q = 2.2\ GeV$. The gapped purple lines are the total differential cross sections using our kinematic factors, and the lattice dipole gravitational form factor with $m_A = 1.13$ GeV. The data is from GlueX collaboration at JLab in 2019.

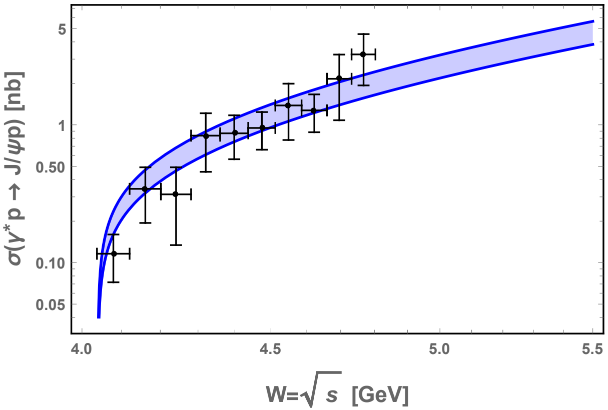

Figure: The total cross section near threshold, for $e = 0.3$, $m_N = 0.94$ GeV, $\tilde{\kappa}_N = 0.350$ GeV, $f_{J/\Psi} = 0.405$ GeV, $M_{J/\Psi} = 3.1$ GeV, $A^2(0) \times \tilde{n} = 240 \pm 47$, $\tilde{\kappa}_{J/\Psi} = 1.03784$ GeV, and $Q^2 = 0$. The data is from GlueX collaboration at JLab in 2019.

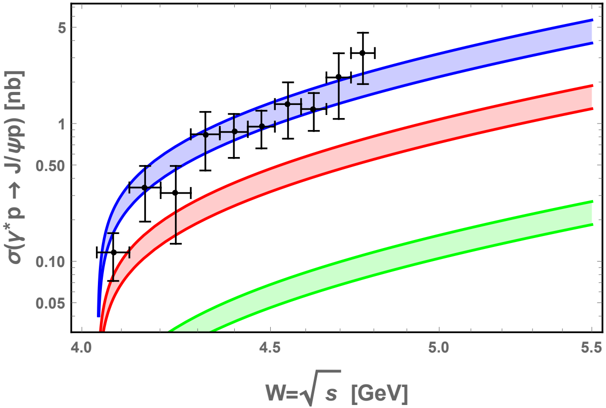

Figure: The variation of the total cross section (near threshold) with Q^2 and $\sqrt{s}$. The blue band is for $Q^2 = 0$ (the data is from GlueX in 2019), the red band is for $Q^2 = 1.2^2$ GeV^2, the green band is for $Q^2 = 2.2^2$ GeV^2.

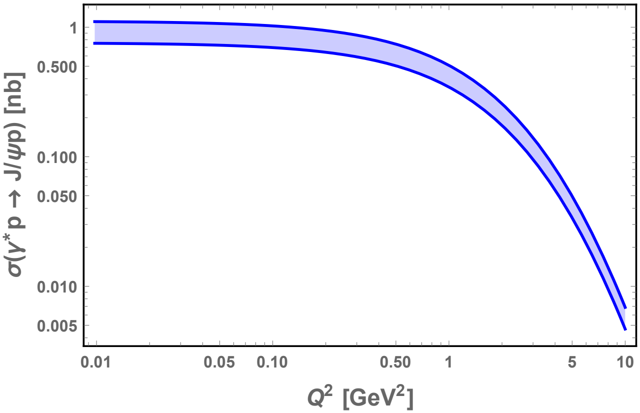

Figure: The variation of the total cross section with Q^2 (near threshold), for $\tilde{\kappa}_N = 0.350$ GeV, $s = W^2 = 4.4^2$ GeV^2, $e = 0.3$, $f_{J/\Psi} = 0.405$ GeV, $M_{J/\Psi} = 3.1$ GeV, $A^2(0) \times \tilde{n} = 240 \pm 47$, $\tilde{\kappa}_{J/\Psi} = 1.03784$ GeV.

- the resummed spin-j (Pomeron) form factor can be measured from photo or electroproduction of heavy vector mesons (J/ψ or Υ) far from threshold

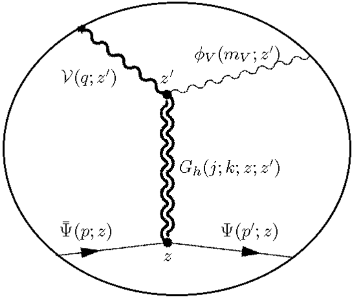

Figure: Witten diagram for the diffractive photoproduction of vector mesons with a bulk wave function ϕ_V. The thick lines or thick wiggles represent the propagators of summed over vector meson or spin-j glueball resonances. The thin lines or thin wiggles correspond to a single proton or vector meson .

Electroproduction of heavy mesons far from threshold

- the differential cross section for electroproduction of heavy vector mesons (J/ψ or ϕ), far from threshold, is given by

$$
\begin{aligned}
\frac{d\sigma(s,t,Q,M_{J/\Psi},\epsilon_T,\epsilon'_T)}{dt} &\propto \mathcal{I}^2(j_0,Q,M_{J/\Psi})\times\left(\frac{s}{\tilde{\kappa}_N^2}\right)^{2\left(1-\frac{2}{\sqrt{\lambda}}\right)} \\
&\times \left(-\frac{t}{4m_N^2}+1\right)\times\mathcal{A}^2(j_0,\tau,\Delta,t)\,, \\
\frac{d\sigma(s,t,Q,M_{J/\Psi},\epsilon_L,\epsilon'_L)}{dt} &\propto \left(\frac{1}{2-\frac{1}{\sqrt{\lambda}}}\right)^2\times\frac{Q^2}{M_{J/\Psi}^2}\times\mathcal{I}^2(j_0,Q,M_{J/\Psi})\times\left(\frac{s}{\tilde{\kappa}_N^2}\right)^{2\left(1-\frac{2}{\sqrt{\lambda}}\right)} \\
&\times \left(-\frac{t}{4m_N^2}+1\right)\times\mathcal{A}^2(j_0,\tau,\Delta,t)\,,
\end{aligned}
$$

Electroproduction of heavy mesons far from threshold

- where we defined the transition form factor that controls the Q dependence as

$$\begin{aligned}
\mathcal{I}(j_0, Q, M_{J/\Psi}) &= \frac{1}{2}\frac{g_5 f_{J/\Psi}}{M_{J/\Psi}} \times \left(2 - \frac{1}{\sqrt{\lambda}}\right) \times \frac{1}{4}\Gamma^2\left(2 - \frac{1}{\sqrt{\lambda}}\right) \\
&\times \frac{\Gamma\left(\frac{Q^2}{4\tilde{\kappa}_{J/\Psi}^2} + 1\right)}{\Gamma\left(\frac{Q^2}{4\tilde{\kappa}_{J/\Psi}^2} + 1 - \frac{1}{\sqrt{\lambda}}\right)} \\
&\times \frac{1}{\left(\frac{Q^2}{4\tilde{\kappa}_{J/\Psi}^2} + 2 - \frac{1}{\sqrt{\lambda}}\right)\left(\frac{Q^2}{4\tilde{\kappa}_{J/\Psi}^2} + 1 - \frac{1}{\sqrt{\lambda}}\right)},
\end{aligned}$$

with $\mathcal{I}(0, M_{J/\Psi}) = \frac{g_5 f_{J/\Psi}}{4M_{J/\Psi}}$, and $\tilde{\kappa}_{J/\Psi} = 2^{-3/4}\sqrt{g_5 f_{J/\Psi} M_{J/\Psi}}$

Electroproduction of heavy mesons far from threshold

- we have also defined the resummed spin-j (Pomeron) form factor of proton that controls the t dependence as

$$\begin{aligned}
\mathcal{A}(j=j_0,\tau,\Delta,t) &= \Gamma\left(a_K+\frac{\Delta(j)}{2}\right)\\
\times\quad & \frac{2^{1-\Delta}}{\Gamma(\tau)}\Bigg((\tau-1)\Gamma\left(\frac{j}{2}+\tau-\frac{\Delta}{2}\right)\Gamma\left(\frac{1}{2}(j+\Delta+2\tau-4)\right)\\
\times\quad & {}_2F_1\left(\frac{1}{2}(j-\Delta+2\tau),\frac{1}{2}(-\Delta+2a_k+4);\frac{1}{2}(j+2\tau+2a_k);-1\right)\\
+\quad & \Gamma\left(\frac{j}{2}+\tau-\frac{\Delta}{2}+1\right)\Gamma\left(\frac{1}{2}(j+\Delta+2\tau-2)\right)\\
\times\quad & {}_2F_1\left(\frac{1}{2}(j-\Delta+2\tau+2),\frac{1}{2}(-\Delta+2a_k+4);\frac{1}{2}(j+2\tau+2a_k+2);-1\right)\Bigg),
\end{aligned}$$

with $j_0 = 2-\frac{2}{\sqrt{\lambda}}$, $\tau = 3$ (fixed by the electromagnetic form factor of proton), and $a_k = -t/8\tilde{\kappa}_N^2$ at $\tilde{\kappa}_N = 0.350\ GeV$ (fixed by the mass of rho meson and proton)

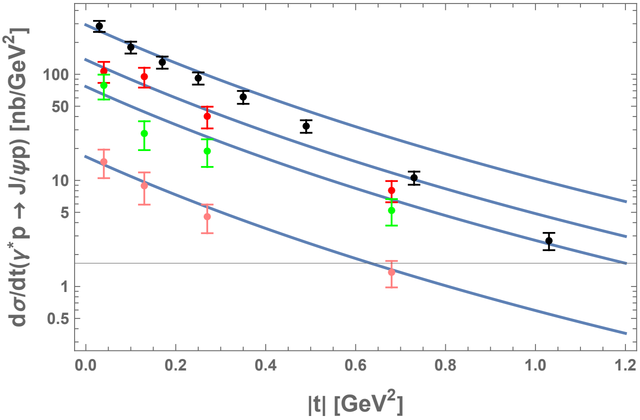

Figure: The total differential cross section (far from threshold) for $40^2\ GeV^2 < s < 160^2\ GeV^2$. The black data points are for $Q = \sqrt{0.05}\ GeV$, and $A^2(0) \times n' = 9.33 \times 10^{13}\ [nb/GeV^2]$. The red data points are for $Q = \sqrt{3.2}\ GeV$, and $A^2(0) \times n' = 1.98 \times 10^{14}\ [nb/GeV^2]$. The green data points are for $Q = \sqrt{7}\ GeV$, and $A^2(0) \times n' = 3.62 \times 10^{14}\ [nb/GeV^2]$. The pink data points are for $Q = \sqrt{22.4}\ GeV$, and $A^2(0) \times n' = 1.01 \times 10^{15}\ [nb/GeV^2]$. The data is from H1 collaboration at HERA in 2005.

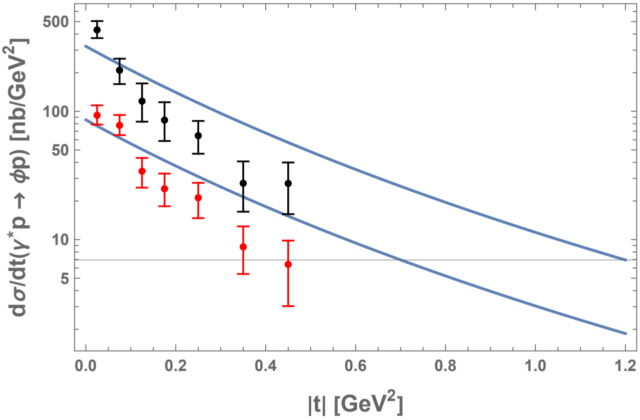

Figure: The total differential cross section (far from threshold)for $s = 75^2\ GeV^2$. The black data points are for $Q = \sqrt{3.3}\ GeV$, and $A^2(0) \times n' = 1.98 \times 10^{10}\ [nb/GeV^2]$. The red data points are for $Q = \sqrt{6.6}\ GeV$, and $A^2(0) \times n' = 2.89 \times 10^{10}\ [nb/GeV^2]$. The data is from H1 collaboration at HERA in 2010.

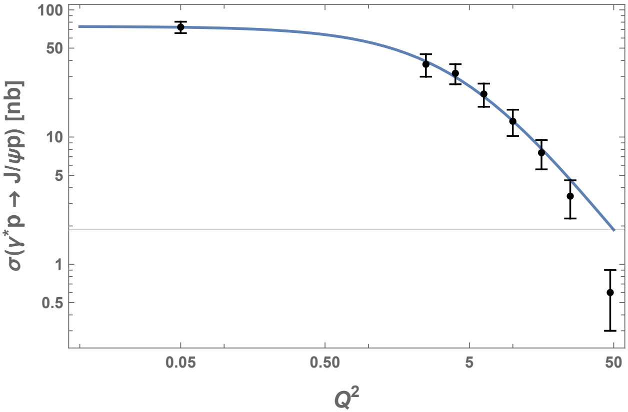

Figure: The variation of the total cross section (far from threshold) with Q^2, for $s = 90^2$ GeV^2, $\lambda = 11.243$, $f_{J/\Psi} = 0.405$ GeV, $M_{J/\Psi} = 3.1$ GeV, $A^2(0) \times n = 206,556$ $[nb]$, $\mathcal{N}_R = 0.6$, and $\tilde{\kappa}_{J/\Psi} = 1.03784$ GeV. The data is from H1 collaboration at HERA in 2005.

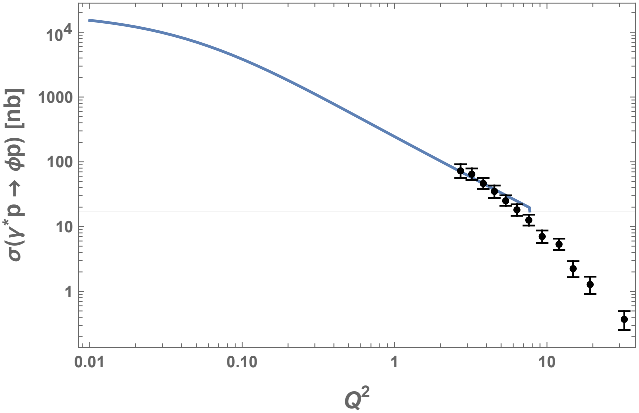

Figure: The variation of the total cross section (far from threshold) with Q^2 for ϕ meson, $s = 75^2$ GeV^2, $\lambda = 11.243$, $f_\phi = 0.233$ GeV, $M_\phi = 1.019$ GeV, $A^2(0) \times n = 3,624.56$ $[nb]$, $\mathcal{N}_R = 2.6$, and $\tilde{\kappa}_\phi = 0.1042$ GeV. The data is from H1 collaboration at HERA in 2010.

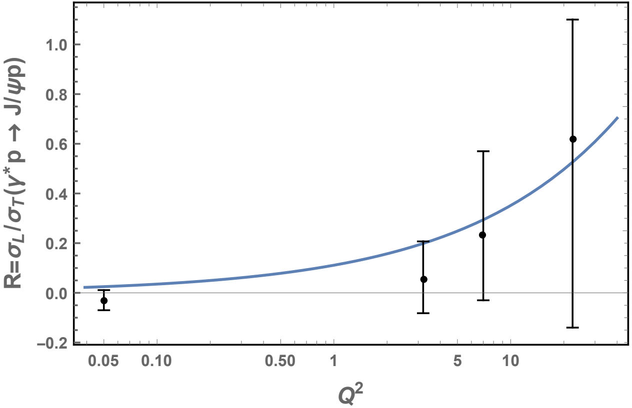

Figure: The ratio of the longitudinal and transverse total cross sections (far from threshold) which is simply given by $R = \frac{\sigma_L}{\sigma_R} = \mathcal{N}_R \times \left(\frac{1}{2 - \frac{1}{\sqrt{11.243}}}\right) \times \frac{Q}{3.1\mathrm{GeV}}$ where the arbitrary normalization coefficient $\mathcal{N}_R = 0.6$ for the blue curve. The data is from H1 collaboration at HERA in 2005.

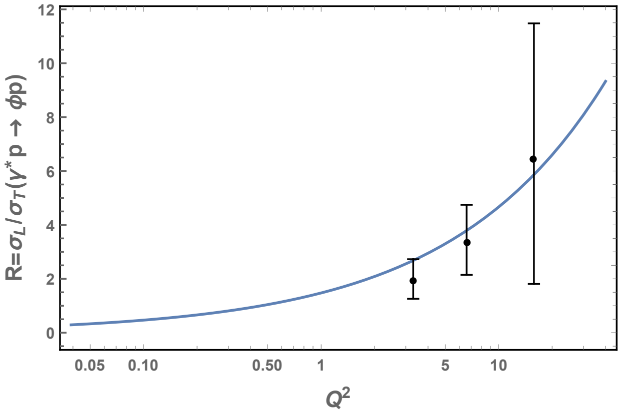

Figure: The ratio of the longitudinal and transverse total cross sections (far from threshold), after replacing $M_{J/\Psi}$ by M_ϕ, which is simply given by $R = \frac{\sigma_L}{\sigma_R} = \mathcal{N}_R \times \left(\frac{1}{2-\frac{1}{\sqrt{11.243}}}\right) \times \frac{Q}{1.019\text{GeV}}$ where the arbitrary normalization coefficient $\mathcal{N}_R = 2.6$ for the blue curve. The data is from H1 collaboration at HERA in 2010.

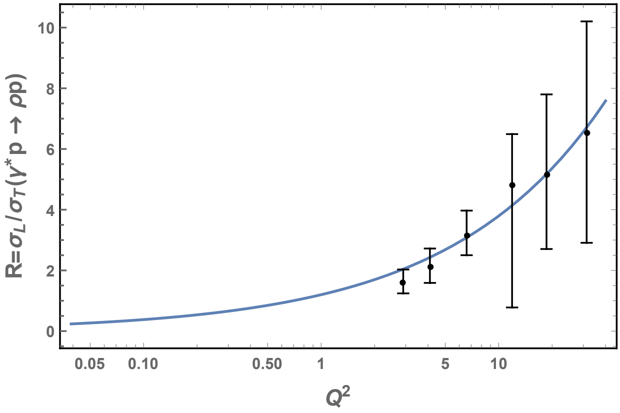

Figure: The ratio of the longitudinal and transverse total cross sections (far from threshold), after replacing $M_{J/\Psi}$ by M_ρ, which is simply given by $R = \frac{\sigma_L}{\sigma_R} = \mathcal{N}_R \times \left(\frac{1}{2-\frac{1}{\sqrt{11.243}}}\right) \times \frac{Q}{0.775\mathrm{GeV}}$ where the arbitrary normalization coefficient $\mathcal{N}_R = 1.6$ for the blue curve. The data is from H1 collaboration at HERA in 2010.

Thank You!