Effective Field Theory for Physics Beyond the Standard Model

BNL Seminar, Oct 14, 2021

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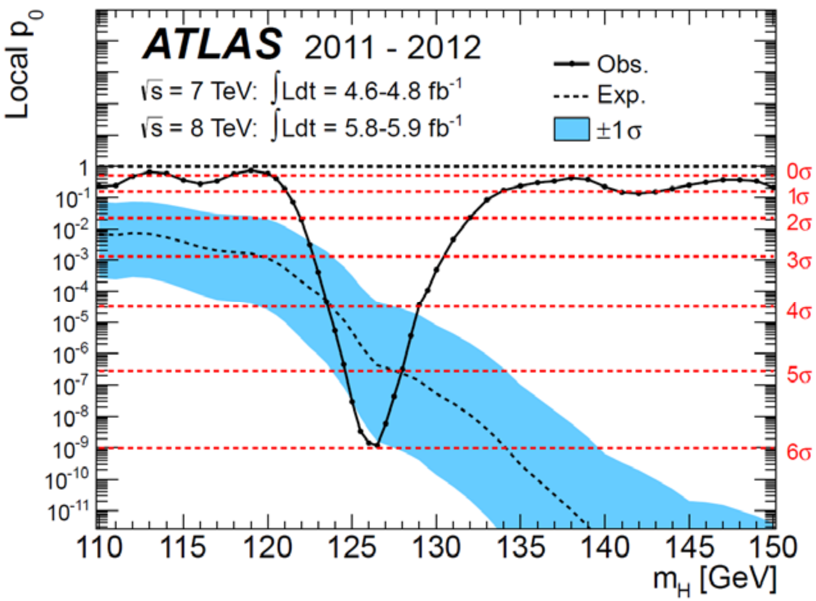
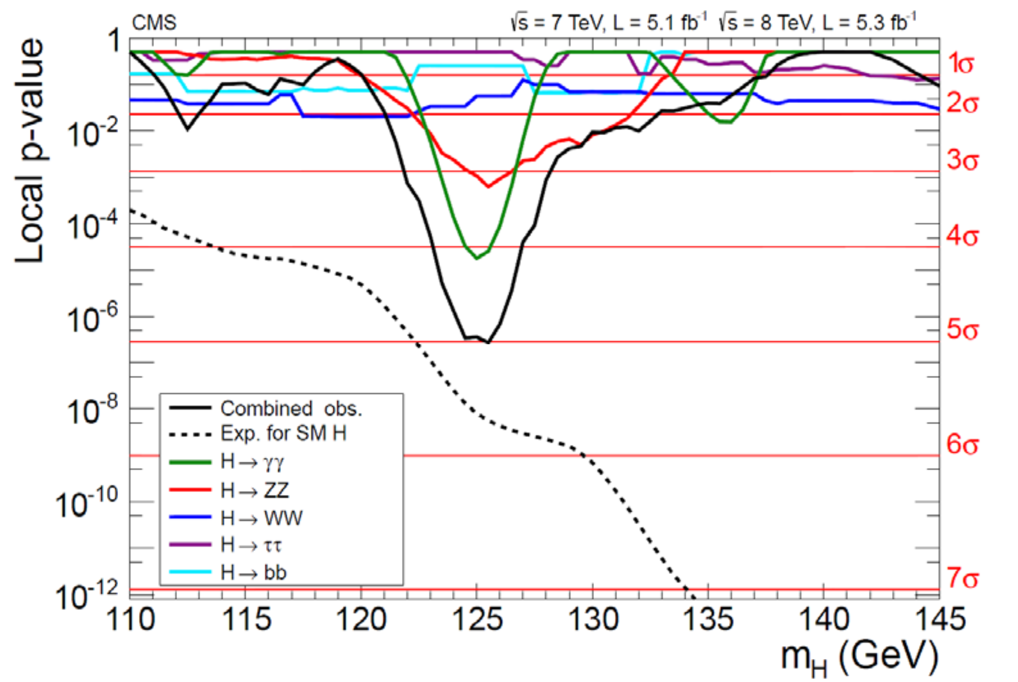
University of Oregon

We have found a Higgs boson!!

$$m_h \sim 125 \text{ GeV}$$



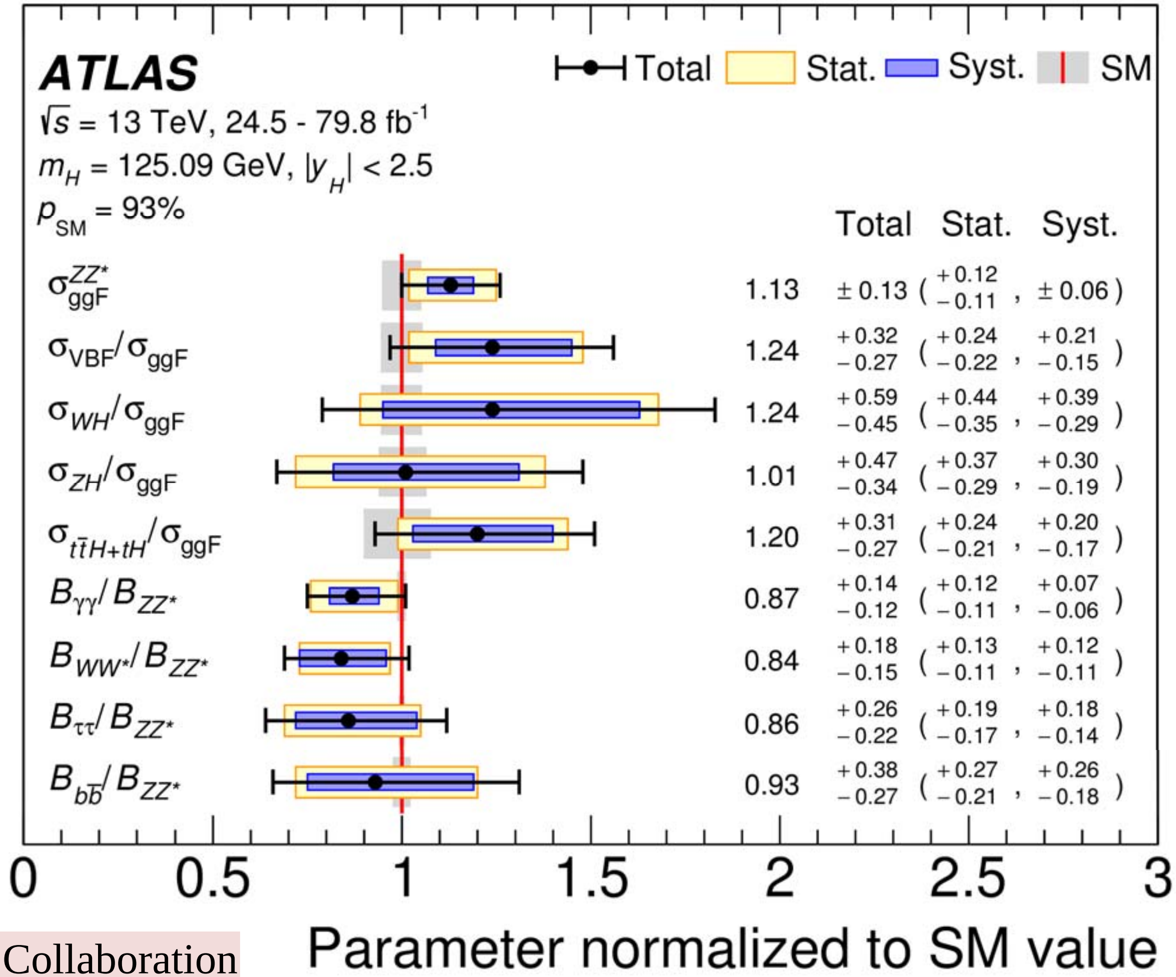
ATLAS Collaboration
arXiv: 1207.7214



CMS Collaboration
arXiv: 1207.7235

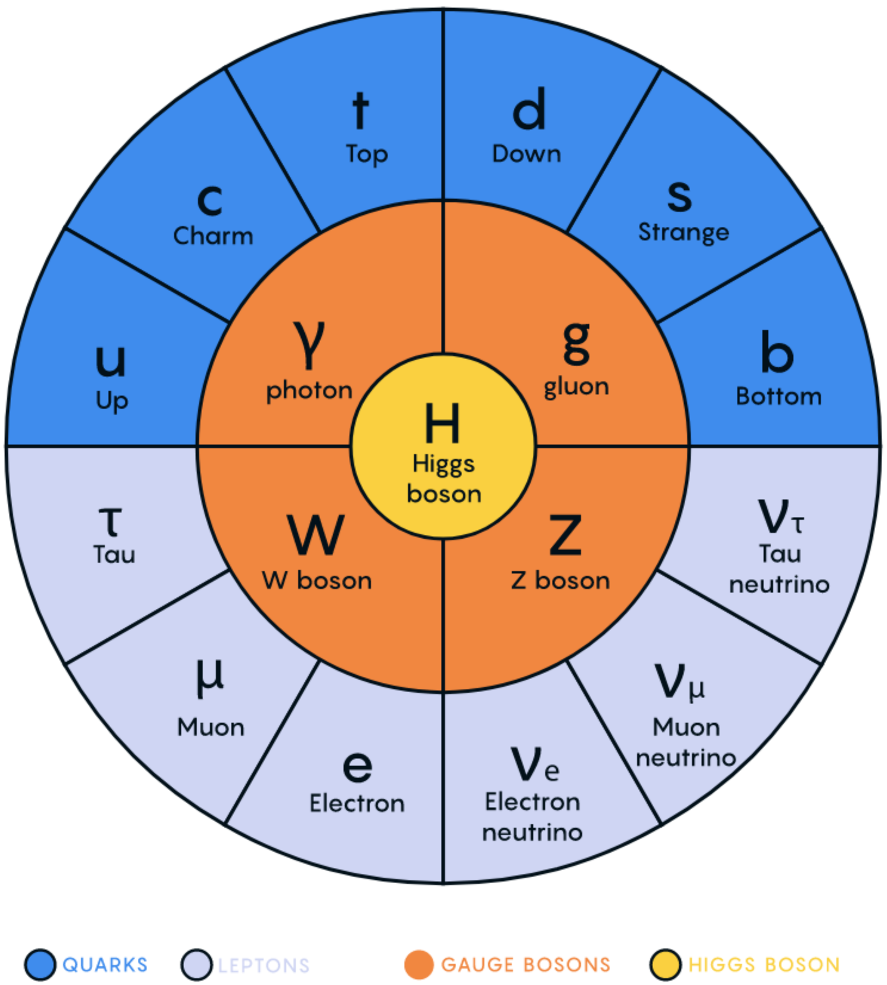
Motivation

h is consistent with Standard Model



ATLAS Collaboration
arXiv: 1909.02845

Standard Model of Elementary Particles



Motivation

Still many questions unanswered

Experimental observations:

- Dark matter
- Neutrino mass
- Baryogenesis
- ……

Theoretical concerns:

- Hierarchy problem
- Strong CP problem
- Cosmological Constant
- ……

Motivation

How to probe BSM physics?

UV models

- Supersymmetry?
- Composite Higgs?
- Neutral naturalness?
- Extra dimension?
- ……

A complementary parameterization?

Motivation

How to probe BSM physics?

UV models

- Supersymmetry?
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- ……

Standard Model
Effective Field Theory

$$\mathcal{L}_{\mathrm{SM}} + \sum_i c_i \mathcal{O}_i = \mathcal{L}_{\mathrm{SMEFT}}$$

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i c_i \mathcal{O}_i$$

Outline

- Establishing the Framework: What is SMEFT?

  --- non-renormalizable, defined with a truncation, operator basis

- Implementing the Framework: How to use SMEFT?

  --- interpreting experimental limits

  --- guide UV model building: Matching and Running

  --- additional restrictions to reduce degrees of freedom

- Re-examining the Framework: Is SMEFT enough?

  --- SMEFT / HEFT dichotomy

  --- geometric picture for non-analyticities and unitarity violation

  --- HEFT describes non-decoupling BSM physics

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i c_i \mathcal{O}_i$$

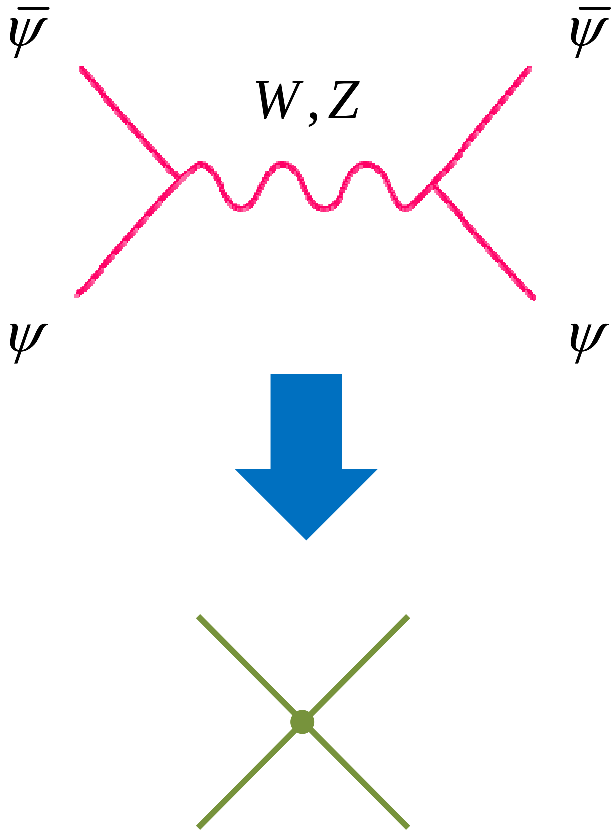
Outline

- Establishing the Framework: What is SMEFT?

  --- non-renormalizable, defined with a truncation, operator basis

What is SMEFT?

What are EFTs?

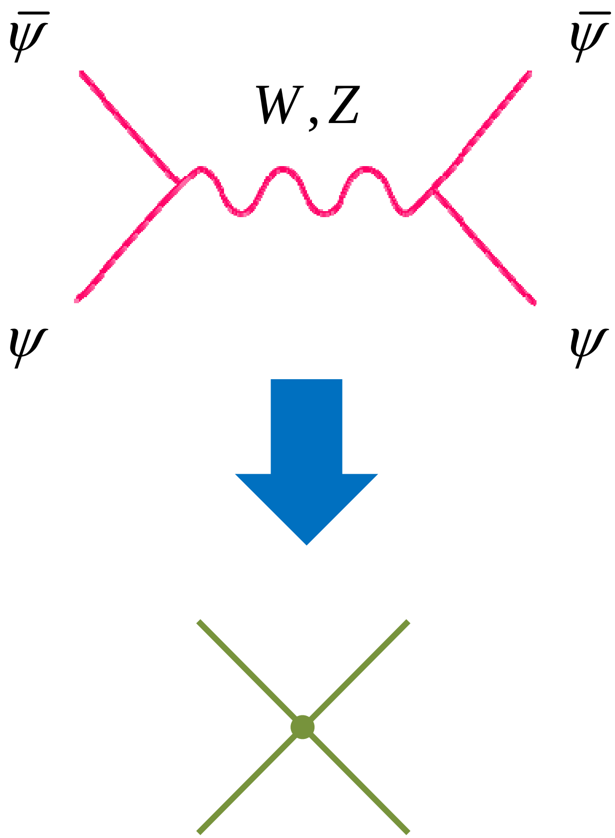


$$\mathcal{O}_6 \sim (\bar{\psi}\psi)(\bar{\psi}\psi)$$

- ➢ Fermi's Theory of Weak Interactions

What is SMEFT?

What are EFTs?



$$\mathcal{O}_6 \sim (\bar{\psi}\psi)(\bar{\psi}\psi)$$

- Fermi's Theory of Weak Interactions
- Mesonic QCD Chiral Lagrangian
- Heavy Quark Effective Theory (HQET)
- Soft Collinear Effective Theory (SCET)
- Low-energy Effective Field Theory (LEFT)
- SMEFT (or HEFT)

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \sum_i c_i \mathcal{O}_i$$

What is SMEFT?

Symmetries define the theory

field content + symmetries $\Rightarrow$ Lagrangian

Symmetries define the theory

field content +	symmetries	$\Rightarrow$ Lagrangian
real scalar ϕ	$Z_2\left(\phi \to -\phi\right)$	$\mathcal{L}\left(\phi\right) = \frac{1}{2}\left(\partial\phi\right)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{24}\phi^4$

Renormalizable
(up to dim-4)

What is SMEFT?

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field content $+$ symmetries $\Rightarrow$ Lagrangian

real scalar ϕ $\quad Z_2(\phi \to -\phi)$ $\quad \mathcal{L}(\phi) = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{24}\phi^4$

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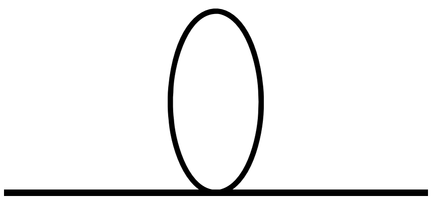
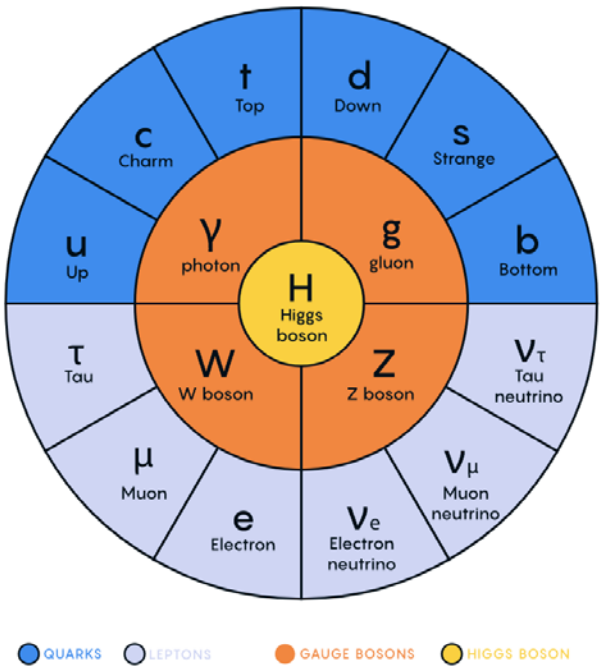


λ

What is SMEFT?



Standard Model		$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
	H	1	2	$+\frac{1}{2}$
ψ	q	3	2	$+\frac{1}{6}$
ψ	u	3	1	$+\frac{2}{3}$
ψ	d	3	1	$-\frac{1}{3}$
ψ	l	1	2	$-\frac{1}{2}$
ψ	e	1	1	-1
	$G^A_{\mu\nu}$	8	1	0
	$W^a_{\mu\nu}$	1	3	0
	$B_{\mu\nu}$	1	1	0

$$\mathcal{L}_{\mathrm{SM}} = |DH|^2 + \sum_{\psi=q,u,d,l,e} \bar{\psi} i \not{D} \psi - \frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$-\lambda \left(|H|^2 - \frac{1}{2} v^2 \right)^2 - \left(\bar{q} Y_u \tilde{H} u + \bar{q} Y_d H d + \bar{l} Y_e H e + \mathrm{h.c.} \right)$$

What is SMEFT?

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \underline{\sum_i c_i \mathcal{O}_i}$$

- Non-renormalizable interactions

--- truncate: finite number of counterterms (effective operators)

--- there are also EFTs that do not run (Conformal Field Theories)

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- Operator Redundancies:

--- Group identities

--- Integration by Part $\left(\text{IBP}\right)$ $\quad \mathcal{O}_1 = \mathcal{O}_2 + \partial_\mu \mathcal{O}^\mu$

--- Equations of Motion $\left(\text{EOM}\right)$ $\quad \mathcal{O}_1 = \mathcal{O}_2 + \mathcal{O}\dfrac{\delta \mathcal{L}^{(0)}}{\delta \Phi}$

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Operator Basis:

A minimal but complete set $\{\mathcal{O}_i\}$

SMEFT

- dim 6, $n_g = 1$ — 1986 — Buchmuller and Wyler
  Nucl. Phys. B 268 (1986) 621 — ~~80~~
  2010 — Grzadkowski, Iskrzynski, Misiak, and Rosiek
  arXiv: 1008.4884 — 59 + ~~5~~
- dim 6, general n_g — 2013 — Alonso, Jenkins, Manohar, and Trott
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- $\begin{cases} \text{dim 7, general } n_g \\ \text{dim 8, } n_g = 1 \end{cases}$ — 2014 - 15 — Lehman and Martin
  arXiv: 1410.4193, 1503.07537, 1510.00372 — ~~931~~

Henning, **XL**, Melia, and Murayama, arXiv: 1512.03433 — 993

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What is SMEFT?

dim 6, $n_g = 1$

Grzadkowski, Iskrzynski, Misiak, and Rosiek, arXiv: 1008.4884

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

Table 2: Dimension-six operators other than the four-fermion ones.

What is SMEFT?

dim 6, $n_g = 1$ Grzadkowski, Iskrzynski, Misiak, and Rosiek, arXiv: 1008.4884

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	Q	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi$		
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu}$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu}$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu}$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu}$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu}$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu}$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu}$
$Q_{\varphi\widetilde W B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu}$

Table 2: Dimension-six operators

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu\tau^I q_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu\tau^I l_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j\sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k\sigma^{\mu\nu} u_t)$		

Table 3: Four-fermion operators.

SMEFT

- dim 6, $n_g = 1$ — 1986 — Buchmuller and Wyler
  Nucl. Phys. B 268 (1986) 621 — $\cancel{80}$
  2010 — Grzadkowski, Iskrzynski, Misiak, and Rosiek
  arXiv: 1008.4884 — 59 + $\cancel{5}$
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$\boxed{\text{Henning, XL, Melia, and Murayama, arXiv: 1512.03433 \quad 993}}$

Hilbert Series

- Gauge invariants

  Benvenuti, Feng, Hanany, and He, arXiv: hep-th/0608050

  Feng, Hanany, and He, hep-th/0701063

  Gray, Hanany, He, Jejjala, and Mekareeya, arXiv: 0803.4257

- Flavor invariants

  Jenkins and Manohar, arXiv: 0907.4763

  Hanany, Jenkins, Manohar, and Torri, arXiv: 1010.3161

- SMEFT

  Lehman and Martin, arXiv: 1503.07537, 1510.00372

  Henning, **XL**, Melia, and Murayama, arXiv: 1507.07240, 1512.03433, 1706.08520

- Mesonic QCD Chiral Lagrangian

  Graf, Henning, **XL**, Melia, and Murayama, arXiv: 2009.01239

- NRQED and HQET

  Kobach and Pal, arXiv: 1704.00008

What is SMEFT?

- Operator Redundancies:
  - Group identities
  - Integration by Part
  - Equation of Motion

$$\phi \in \{H, q, u, d, l, e, G^A_{\mu\nu}, W^a_{\mu\nu}, B_{\mu\nu}\}$$

$$R_\phi = \begin{pmatrix} \phi \\ D_{\mu_1}\phi \\ D_{\{\mu_1}D_{\mu_2\}}\phi \\ D_{\{\mu_1}D_{\mu_2}D_{\mu_3\}}\phi \\ \vdots \end{pmatrix}$$

traceless symmetric
EOM removed

A Representation of
Gauge $\otimes$ Conformal group

$$\{\mathcal{O}_i\} = \bigotimes_\phi R_\phi$$

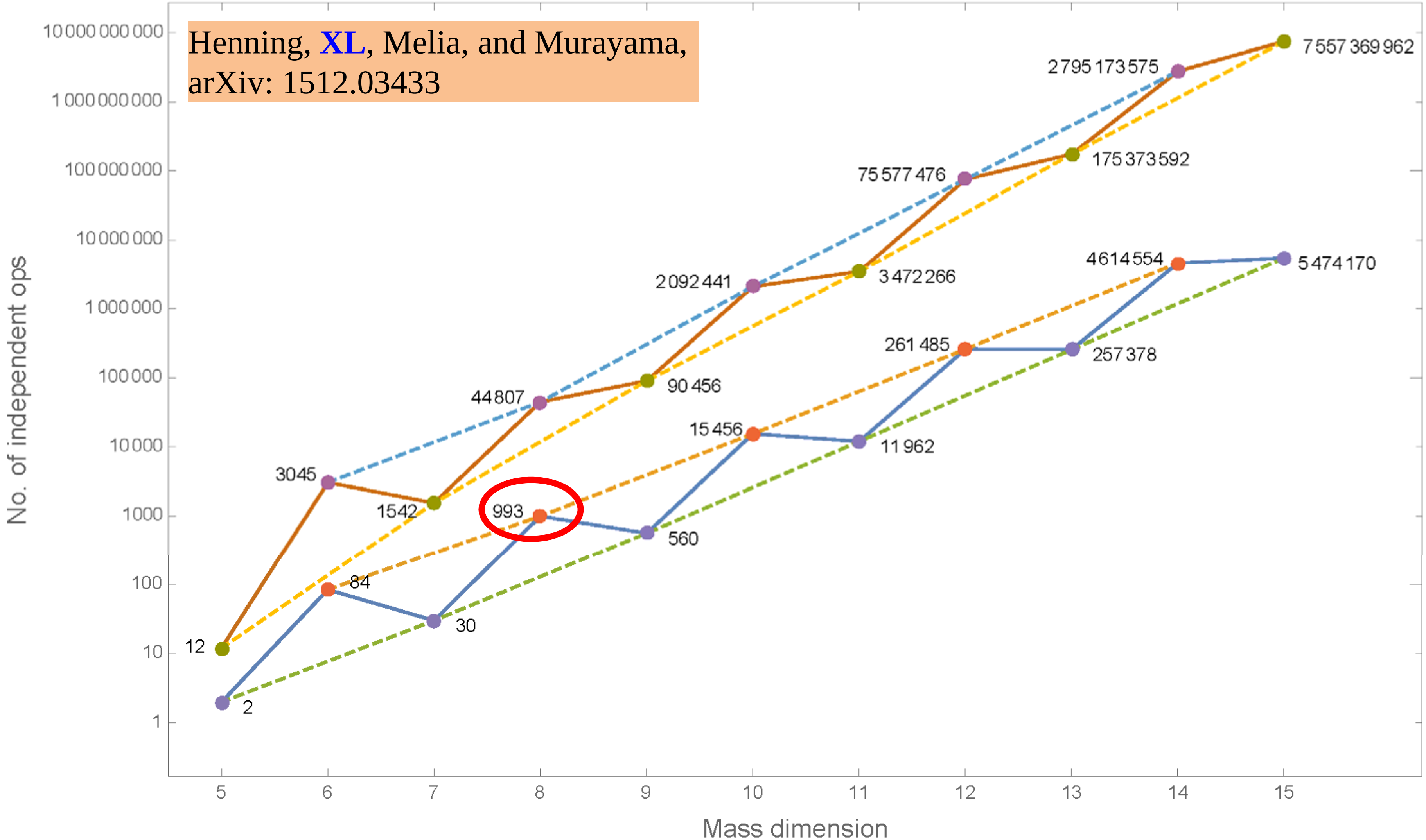
- Gauge singlets
- Conformal scalars and primaries

Operator Basis

What is SMEFT?

Number of SMEFT operators

Henning, **XL**, Melia, and Murayama,
arXiv: 1512.03433



993 dim-8 operators for $n_g = 1$

```
f =
  2*L^2*Ld^2*t^2 + 2*ee*ed*L*Ld*t^2 + ee^2*ed^2*t^2 + 2*d*dd*L*Ld*t^2 + 2*
  d*dd*ee*ed*t^2 + 2*d^2*dd^2*t^2 + ud^2*dd*ed*t^2 + 2*u*ud*L*Ld*t^2 + 2*u
  *ud*ee*ed*t^2 + 4*u*ud*d*dd*t^2 + u^2*d*ee*t^2 + 2*u^2*ud^2*t^2 + 2*Qd*
  dd*ee*L*t^2 + 3*Qd*ud*ed*Ld*t^2 + 2*Qd*u*d*Ld*t^2 + 3*Qd^2*ud*dd*t^2 +
  Qd^2*u*ee*t^2 + Qd^3*Ld*t^2 + 2*Q*d*ed*Ld*t^2 + 2*Q*ud*dd*L*t^2 + 3*Q*u*
  ee*L*t^2 + 4*Q*Qd*L*Ld*t^2 + 2*Q*Qd*ee*ed*t^2 + 4*Q*Qd*d*dd*t^2 + 4*Q*Qd
  *u*ud*t^2 + Q^2*ud*ed*t^2 + 3*Q^2*u*d*t^2 + 4*Q^2*Qd^2*t^2 + Q^3*L*t^2
   + Wr*L^2*Ld^2 + Wr*ee*ed*L*Ld + Wr*d*dd*L*Ld + Wr*u*ud*L*Ld + Wr*Qd*dd*
  ee*L + 3*Wr*Qd*ud*ed*Ld + Wr*Qd*u*d*Ld + 3*Wr*Qd^2*ud*dd + Wr*Qd^2*u*ee
   + 2*Wr*Qd^3*Ld + Wr*Q*d*ed*Ld + Wr*Q*ud*dd*L + 3*Wr*Q*Qd*L*Ld + Wr*Q*Qd
  *ee*ed + 2*Wr*Q*Qd*d*dd + 2*Wr*Q*Qd*u*ud + 2*Wr*Q^2*Qd^2 + Wr^2*L*Ld*t
   + Wr^2*Q*Qd*t + 2*Wr^4 + Wl*L^2*Ld^2 + Wl*ee*ed*L*Ld + Wl*d*dd*L*Ld +
  Wl*u*ud*L*Ld + Wl*Qd*dd*ee*L + Wl*Qd*u*d*Ld + Wl*Q*d*ed*Ld + Wl*Q*ud*dd*
  L + 3*Wl*Q*u*ee*L + 3*Wl*Q*Qd*L*Ld + Wl*Q*Qd*ee*ed + 2*Wl*Q*Qd*d*dd + 2*
  Wl*Q*Qd*u*ud + Wl*Q^2*ud*ed + 3*Wl*Q^2*u*d + 2*Wl*Q^2*Qd^2 + 2*Wl*Q^3*L
   + 2*Wl*Wr*L*Ld*t + Wl*Wr*ee*ed*t + Wl*Wr*d*dd*t + Wl*Wr*u*ud*t + 2*Wl*
  Wr*Q*Qd*t + Wl^2*L*Ld*t + Wl^2*Q*Qd*t + 2*Wl^2*Wr^2 + 2*Wl^4 + Gr*d*dd*L
  *Ld + Gr*d*dd*ee*ed + Gr*d^2*dd^2 + 3*Gr*ud^2*dd*ed + Gr*u*ud*L*Ld + Gr*
  u*ud*ee*ed + 4*Gr*u*ud*d*dd + Gr*u^2*ud^2 + Gr*Qd*dd*ee*L + 3*Gr*Qd*ud*
  ed*Ld + 2*Gr*Qd*u*d*Ld + 6*Gr*Qd^2*ud*dd + Gr*Qd^2*u*ee + 2*Gr*Qd^3*Ld
   + Gr*Q*d*ed*Ld + 2*Gr*Q*ud*dd*L + 2*Gr*Q*Qd*L*Ld + Gr*Q*Qd*ee*ed + 4*Gr
  *Q*Qd*d*dd + 4*Gr*Q*Qd*u*ud + Gr*Q^2*ud*ed + 2*Gr*Q^2*Qd^2 + Gr*Wr*Q*Qd*
  t + Gr*Wl*Q*Qd*t + Gr^2*d*dd*t + Gr^2*u*ud*t + Gr^2*Q*Qd*t + 2*Gr^2*Wr^2
   + Gr^2*Wl^2 + 3*Gr^4 + Gl*d*dd*L*Ld + Gl*d*dd*ee*ed + Gl*d^2*dd^2 + Gl*
  u*ud*L*Ld + Gl*u*ud*ee*ed + 4*Gl*u*ud*d*dd + 3*Gl*u^2*d*ee + Gl*u^2*ud^2
   + Gl*Qd*dd*ee*L + 2*Gl*Qd*u*d*Ld + Gl*Qd^2*u*ee + Gl*Q*d*ed*Ld + 2*Gl*Q
  *ud*dd*L + 3*Gl*Q*u*ee*L + 2*Gl*Q*Qd*L*Ld + Gl*Q*Qd*ee*ed + 4*Gl*Q*Qd*d*
  dd + 4*Gl*Q*Qd*u*ud + Gl*Q^2*ud*ed + 6*Gl*Q^2*u*d + 2*Gl*Q^2*Qd^2 + 2*Gl
  *Q^3*L + Gl*Wr*Q*Qd*t + Gl*Wl*Q*Qd*t + Gl*Gr*L*Ld*t + Gl*Gr*ee*ed*t + 3*
  Gl*Gr*d*dd*t + 3*Gl*Gr*u*ud*t + 3*Gl*Gr*Q*Qd*t + Gl*Gr*Wl*Wr + Gl^2*d*dd
  *t + Gl^2*u*ud*t + Gl^2*Q*Qd*t + Gl^2*Wr^2 + 2*Gl^2*Wl^2 + 3*Gl^2*Gr^2
   + 3*Gl^4 + Br*ee*ed*L*Ld + Br*d*dd*L*Ld + Br*d*dd*ee*ed + 2*Br*ud^2*dd*
  ed + Br*u*ud*L*Ld + Br*u*ud*ee*ed + 2*Br*u*ud*d*dd + Br*Qd*dd*ee*L + 3*
  Br*Qd*ud*ed*Ld + Br*Qd*u*d*Ld + 3*Br*Qd^2*ud*dd + Br*Qd^3*Ld + Br*Q*d*ed
  *Ld + Br*Q*ud*dd*L + 2*Br*Q*Qd*L*Ld + Br*Q*Qd*ee*ed + 2*Br*Q*Qd*d*dd + 2
  *Br*Q*Qd*u*ud + Br*Q^2*ud*ed + Br*Wr*L*Ld*t + Br*Wr*Q*Qd*t + Br*Wl*L*Ld*
  t + Br*Wl*Q*Qd*t + Br*Gr*d*dd*t + Br*Gr*u*ud*t + Br*Gr*Q*Qd*t + Br*Gr^3
   + Br*Gl*d*dd*t + Br*Gl*u*ud*t + Br*Gl*Q*Qd*t + Br*Gl^2*Gr + 2*Br^2*Wr^2
   + Br^2*Wl^2 + 2*Br^2*Gr^2 + Br^2*Gl^2 + Br^4 + Bl*ee*ed*L*Ld + Bl*d*dd*
  L*Ld + Bl*d*dd*ee*ed + Bl*u*ud*L*Ld + Bl*u*ud*ee*ed + 2*Bl*u*ud*d*dd + 2
  *Bl*u^2*d*ee + Bl*Qd*dd*ee*L + Bl*Qd*u*d*Ld + Bl*Qd^2*u*ee + Bl*Q*d*ed*
  Ld + Bl*Q*ud*dd*L + 3*Bl*Q*u*ee*L + 2*Bl*Q*Qd*L*Ld + Bl*Q*Qd*ee*ed + 2*
  Bl*Q*Qd*d*dd + 2*Bl*Q*Qd*u*ud + 3*Bl*Q^2*u*d + Bl*Q^3*L + Bl*Wr*L*Ld*t
   + Bl*Wr*Q*Qd*t + Bl*Wl*L*Ld*t + Bl*Wl*Q*Qd*t + Bl*Gr*d*dd*t + Bl*Gr*u*
  ud*t + Bl*Gr*Q*Qd*t + Bl*Gl*d*dd*t + Bl*Gl*u*ud*t + Bl*Gl*Q*Qd*t + Bl*Gl
  *Gr^2 + Bl*Gl^3 + Bl*Br*L*Ld*t + Bl*Br*ee*ed*t + Bl*Br*d*dd*t + Bl*Br*u*
  ud*t + Bl*Br*Q*Qd*t + Bl*Br*Wl*Wr + Bl*Br*Gl*Gr + Bl^2*Wr^2 + 2*Bl^2*
  Wl^2 + Bl^2*Gr^2 + 2*Bl^2*Gl^2 + Bl^2*Br^2 + Bl^4 + 3*Hd*ee*L^2*Ld*t +
  Hd*ee^2*ed*L*t + 3*Hd*d*dd*ee*L*t + 3*Hd*ud*d*ed*Ld*t + 2*Hd*ud^2*dd*L*t
   + 2*Hd*u*d^2*Ld*t + 3*Hd*u*ud*ee*L*t + 6*Hd*Qd*ud*L*Ld*t + 3*Hd*Qd*ud*
  ee*ed*t + 6*Hd*Qd*ud*d*dd*t + 3*Hd*Qd*u*d*ee*t + 3*Hd*Qd*u*ud^2*t + 3*Hd
  *Qd^2*d*Ld*t + Hd*Qd^3*ee*t + 6*Hd*Q*d*L*Ld*t + 3*Hd*Q*d*ee*ed*t + 3*Hd*
  Q*d^2*dd*t + 2*Hd*Q*ud^2*ed*t + 6*Hd*Q*u*ud*d*t + 6*Hd*Q*Qd*ee*L*t + 6*
  Hd*Q*Qd^2*ud*t + 3*Hd*Q^2*ud*L*t + 6*Hd*Q^2*Qd*d*t + Hd*Wr*ee*L*t^2 + 2*
  Hd*Wr*Qd*ud*t^2 + Hd*Wr*Q*d*t^2 + Hd*Wr^2*ee*L + 2*Hd*Wr^2*Qd*ud + Hd*
  Wr^2*Q*d + 2*Hd*Wl*ee*L*t^2 + Hd*Wl*Qd*ud*t^2 + 2*Hd*Wl*Q*d*t^2 + 2*Hd*
  Wl^2*ee*L + Hd*Wl^2*Qd*ud + 2*Hd*Wl^2*Q*d + 2*Hd*Gr*Qd*ud*t^2 + Hd*Gr*Q*
  d*t^2 + 2*Hd*Gr*Wr*Qd*ud + Hd*Gr*Wr*Q*d + Hd*Gr^2*ee*L + 3*Hd*Gr^2*Qd*ud
   + 2*Hd*Gr^2*Q*d + Hd*Gl*Qd*ud*t^2 + 2*Hd*Gl*Q*d*t^2 + Hd*Gl*Wl*Qd*ud +
  2*Hd*Gl*Wl*Q*d + Hd*Gl^2*ee*L + 2*Hd*Gl^2*Qd*ud + 3*Hd*Gl^2*Q*d + Hd*Br*
  ee*L*t^2 + 2*Hd*Br*Qd*ud*t^2 + Hd*Br*Q*d*t^2 + Hd*Br*Wr*ee*L + 2*Hd*Br*
  Wr*Qd*ud + Hd*Br*Wr*Q*d + 2*Hd*Br*Gr*Qd*ud + Hd*Br*Gr*Q*d + Hd*Br^2*ee*L
   + Hd*Br^2*Qd*ud + Hd*Br^2*Q*d + 2*Hd*Bl*ee*L*t^2 + Hd*Bl*Qd*ud*t^2 + 2*
  Hd*Bl*Q*d*t^2 + 2*Hd*Bl*Wl*ee*L + Hd*Bl*Wl*Qd*ud + 2*Hd*Bl*Wl*Q*d + Hd*
  Bl*Gl*Qd*ud + 2*Hd*Bl*Gl*Q*d + Hd*Bl^2*ee*L + Hd*Bl^2*Qd*ud + Hd*Bl^2*Q*
  d + Hd^2*ee^2*L^2 + Hd^2*ud*d*t^3 + Hd^2*ud*d*L*Ld + Hd^2*Qd*ud*ee*L + 2
  *Hd^2*Qd^2*ud^2 + 2*Hd^2*Q*d*ee*L + 2*Hd^2*Q*Qd*ud*d + 2*Hd^2*Q^2*d^2 +
  Hd^2*Wr*ud*d*t + Hd^2*Wl*ud*d*t + Hd^2*Gr*ud*d*t + Hd^2*Gl*ud*d*t + Hd^2
  *Br*ud*d*t + Hd^2*Bl*ud*d*t + 3*H*ed*L*Ld^2*t + H*ee*ed^2*Ld*t + 3*H*d*
  dd*ed*Ld*t + 2*H*ud*dd^2*L*t + 3*H*u*dd*ee*L*t + 3*H*u*ud*ed*Ld*t + 2*H*
  u^2*d*Ld*t + 6*H*Qd*dd*L*Ld*t + 3*H*Qd*dd*ee*ed*t + 3*H*Qd*d*dd^2*t + 6*
  H*Qd*u*ud*dd*t + 2*H*Qd*u^2*ee*t + 3*H*Qd^2*u*Ld*t + 3*H*Q*ud*dd*ed*t +
  6*H*Q*u*L*Ld*t + 3*H*Q*u*ee*ed*t + 6*H*Q*u*d*dd*t + 3*H*Q*u^2*ud*t + 6*H
  *Q*Qd*ed*Ld*t + 6*H*Q*Qd^2*dd*t + 3*H*Q^2*dd*L*t + 6*H*Q^2*Qd*u*t + H*
  Q^3*ed*t + 2*H*Wr*ed*Ld*t^2 + 2*H*Wr*Qd*dd*t^2 + H*Wr*Q*u*t^2 + 2*H*Wr^2
  *ed*Ld + 2*H*Wr^2*Qd*dd + H*Wr^2*Q*u + H*Wl*ed*Ld*t^2 + H*Wl*Qd*dd*t^2
   + 2*H*Wl*Q*u*t^2 + H*Wl^2*ed*Ld + H*Wl^2*Qd*dd + 2*H*Wl^2*Q*u + 2*H*Gr*
  Qd*dd*t^2 + H*Gr*Q*u*t^2 + 2*H*Gr*Wr*Qd*dd + H*Gr*Wr*Q*u + H*Gr^2*ed*Ld
   + 3*H*Gr^2*Qd*dd + 2*H*Gr^2*Q*u + H*Gl*Qd*dd*t^2 + 2*H*Gl*Q*u*t^2 + H*
  Gl*Wl*Qd*dd + 2*H*Gl*Wl*Q*u + H*Gl^2*ed*Ld + 2*H*Gl^2*Qd*dd + 3*H*Gl^2*Q
  *u + 2*H*Br*ed*Ld*t^2 + 2*H*Br*Qd*dd*t^2 + H*Br*Q*u*t^2 + 2*H*Br*Wr*ed*
  Ld + 2*H*Br*Wr*Qd*dd + H*Br*Wr*Q*u + 2*H*Br*Gr*Qd*dd + H*Br*Gr*Q*u + H*
  Br^2*ed*Ld + H*Br^2*Qd*dd + H*Br^2*Q*u + H*Bl*ed*Ld*t^2 + H*Bl*Qd*dd*t^2
   + 2*H*Bl*Q*u*t^2 + H*Bl*Wl*ed*Ld + H*Bl*Wl*Qd*dd + 2*H*Bl*Wl*Q*u + H*Bl
  *Gl*Qd*dd + 2*H*Bl*Gl*Q*u + H*Bl^2*ed*Ld + H*Bl^2*Qd*dd + H*Bl^2*Q*u + 4
  *H*Hd*L*Ld*t^3 + 2*H*Hd*L^2*Ld^2 + 2*H*Hd*ee*ed*t^3 + 2*H*Hd*ee*ed*L*Ld
   + H*Hd*ee^2*ed^2 + 2*H*Hd*d*dd*t^3 + 2*H*Hd*d*dd*L*Ld + H*Hd*d*dd*ee*ed
   + H*Hd*d^2*dd^2 + H*Hd*ud^2*dd*ed + 2*H*Hd*u*ud*t^3 + 2*H*Hd*u*ud*L*Ld
   + H*Hd*u*ud*ee*ed + 2*H*Hd*u*ud*d*dd + H*Hd*u^2*d*ee + H*Hd*u^2*ud^2 +
  2*H*Hd*Qd*dd*ee*L + 4*H*Hd*Qd*ud*ed*Ld + 2*H*Hd*Qd*u*d*Ld + 4*H*Hd*Qd^2*
  ud*dd + H*Hd*Qd^2*u*ee + 2*H*Hd*Qd^3*Ld + 2*H*Hd*Q*d*ed*Ld + 2*H*Hd*Q*ud
  *dd*L + 4*H*Hd*Q*u*ee*L + 4*H*Hd*Q*Qd*t^3 + 5*H*Hd*Q*Qd*L*Ld + 2*H*Hd*Q*
  Qd*ee*ed + 4*H*Hd*Q*Qd*d*dd + 4*H*Hd*Q*Qd*u*ud + H*Hd*Q^2*ud*ed + 4*H*Hd
  *Q^2*u*d + 3*H*Hd*Q^2*Qd^2 + 2*H*Hd*Q^3*L + 6*H*Hd*Wr*L*Ld*t + 2*H*Hd*Wr
  *ee*ed*t + 2*H*Hd*Wr*d*dd*t + 2*H*Hd*Wr*u*ud*t + 6*H*Hd*Wr*Q*Qd*t + 2*H*
  Hd*Wr^2*t^2 + H*Hd*Wr^3 + 6*H*Hd*Wl*L*Ld*t + 2*H*Hd*Wl*ee*ed*t + 2*H*Hd*
  Wl*d*dd*t + 2*H*Hd*Wl*u*ud*t + 6*H*Hd*Wl*Q*Qd*t + 2*H*Hd*Wl*Wr*t^2 + 2*H
  *Hd*Wl^2*t^2 + H*Hd*Wl^3 + 2*H*Hd*Gr*d*dd*t + 2*H*Hd*Gr*u*ud*t + 4*H*Hd*
  Gr*Q*Qd*t + H*Hd*Gr^2*t^2 + H*Hd*Gr^3 + 2*H*Hd*Gl*d*dd*t + 2*H*Hd*Gl*u*
  ud*t + 4*H*Hd*Gl*Q*Qd*t + H*Hd*Gl*Gr*t^2 + H*Hd*Gl^2*t^2 + H*Hd*Gl^3 + 4
  *H*Hd*Br*L*Ld*t + 2*H*Hd*Br*ee*ed*t + 2*H*Hd*Br*d*dd*t + 2*H*Hd*Br*u*ud*
  t + 4*H*Hd*Br*Q*Qd*t + 2*H*Hd*Br*Wr*t^2 + H*Hd*Br*Wr^2 + H*Hd*Br*Wl*t^2
   + H*Hd*Br^2*t^2 + 4*H*Hd*Bl*L*Ld*t + 2*H*Hd*Bl*ee*ed*t + 2*H*Hd*Bl*d*dd
  *t + 2*H*Hd*Bl*u*ud*t + 4*H*Hd*Bl*Q*Qd*t + H*Hd*Bl*Wr*t^2 + 2*H*Hd*Bl*Wl
  *t^2 + H*Hd*Bl*Wl^2 + H*Hd*Bl*Br*t^2 + H*Hd*Bl^2*t^2 + 6*H*Hd^2*ee*L*t^2
   + 6*H*Hd^2*Qd*ud*t^2 + 6*H*Hd^2*Q*d*t^2 + 2*H*Hd^2*Wr*Qd*ud + 2*H*Hd^2*
  Wl*ee*L + 2*H*Hd^2*Wl*Q*d + H*Hd^2*Gr*Qd*ud + H*Hd^2*Gl*Q*d + H*Hd^2*Br*
  Qd*ud + H*Hd^2*Bl*ee*L + H*Hd^2*Bl*Q*d + H*Hd^3*ud*d*t + H^2*ed^2*Ld^2
   + H^2*u*dd*t^3 + H^2*u*dd*L*Ld + 2*H^2*Qd*dd*ed*Ld + 2*H^2*Qd^2*dd^2 +
  H^2*Q*u*ed*Ld + 2*H^2*Q*Qd*u*dd + 2*H^2*Q^2*u^2 + H^2*Wr*u*dd*t + H^2*Wl
  *u*dd*t + H^2*Gr*u*dd*t + H^2*Gl*u*dd*t + H^2*Br*u*dd*t + H^2*Bl*u*dd*t
   + 6*H^2*Hd*ed*Ld*t^2 + 6*H^2*Hd*Qd*dd*t^2 + 6*H^2*Hd*Q*u*t^2 + 2*H^2*Hd
  *Wr*ed*Ld + 2*H^2*Hd*Wr*Qd*dd + 2*H^2*Hd*Wl*Q*u + H^2*Hd*Gr*Qd*dd + H^2*
  Hd*Gl*Q*u + H^2*Hd*Br*ed*Ld + H^2*Hd*Br*Qd*dd + H^2*Hd*Bl*Q*u + 3*H^2*
  Hd^2*t^4 + 4*H^2*Hd^2*L*Ld*t + H^2*Hd^2*ee*ed*t + H^2*Hd^2*d*dd*t + H^2*
  Hd^2*u*ud*t + 4*H^2*Hd^2*Q*Qd*t + 2*H^2*Hd^2*Wr*t^2 + 2*H^2*Hd^2*Wr^2 +
  2*H^2*Hd^2*Wl*t^2 + 2*H^2*Hd^2*Wl^2 + H^2*Hd^2*Gr^2 + H^2*Hd^2*Gl^2 +
  H^2*Hd^2*Br*t^2 + H^2*Hd^2*Br*Wr + H^2*Hd^2*Br^2 + H^2*Hd^2*Bl*t^2 + H^2
  *Hd^2*Bl*Wl + H^2*Hd^2*Bl^2 + H^2*Hd^3*ee*L + H^2*Hd^3*Qd*ud + H^2*Hd^3*
  Q*d + H^3*Hd*u*dd*t + H^3*Hd^2*ed*Ld + H^3*Hd^2*Qd*dd + H^3*Hd^2*Q*u + 2
  *H^3*Hd^3*t^2 + H^4*Hd^4;
```

Henning, **XL**, Melia, and Murayama, arXiv: 1512.03433

993 dim-8 operators for $n_g = 1$

```
f =
 2*L^2*Ld^2*t^2 + 2*ee*ed*L*Ld*t^2 + ee^2*ed^2*t^2 + 2*d*dd*L*Ld*t^2 + 2*
 d*dd*ee*ed*t^2 + 2*d^2*dd^2*t^2 + ud^2*dd*ed*t^2 + 2*u*ud*L*Ld*t^2 + 2*u
 *ud*ee*ed*t^2 + 4*u*ud*d*dd*t^2 + u^2*d*ee*t^2 + 2*u^2*ud^2*t^2 + 2*Qd*
 dd*ee*L*t^2 + 3*Qd*ud*ed*Ld*t^2 + 2*Qd*u*d*Ld*t^2 + 3*Qd^2*ud*dd*t^2 +
 Qd^2*u*ee*t^2 + Qd^3*Ld*t^2 + 2*Q*d*ed*Ld*t^2 + 2*Q*ud*dd*L*t^2 + 3*Q*u*
 ee*L*t^2 + 4*Q*Qd*L*Ld*t^2 + 2*Q*Qd*ee*ed*t^2 + 4*Q*Qd*d*dd*t^2 + 4*Q*Qd
 *u*ud*t^2 + Q^2*ud*ed*t^2 + 3*Q^2*u*d*t^2 + 4*Q^2*Qd^2*t^2 + Q^3*L*t^2
  + Wr*L^2*Ld^2 + Wr*ee*ed*L*Ld + Wr*d*dd*L*Ld + Wr*u*ud*L*Ld + Wr*Qd*dd*
 ee*L + 3*Wr*Qd*ud*ed*Ld + Wr*Qd*u*d*Ld + 3*Wr*Qd^2*ud*dd + Wr*Qd^2*u*ee
  + 2*Wr*Qd^3*Ld + Wr*Q*d*ed*Ld + Wr*Q*ud*dd*L + 3*Wr*Q*Qd*L*Ld + Wr*Q*Qd
 *ee*ed + 2*Wr*Q*Qd*d*dd + 2*Wr*Q*Qd*u*ud + 2*Wr*Q^2*Qd^2 + Wr^2*L*Ld*t
  + Wr^2*Q*Qd*t + 2*Wr^4 + Wl*L^2*Ld^2 + Wl*ee*ed*L*Ld + Wl*d*dd*L*Ld +
 Wl*u*ud*L*Ld + Wl*Qd*dd*ee*L + Wl*Q
 L + 3*Wl*Q*u*ee*L + 3*Wl*Q*Qd*L*Ld
 Wl*Q*Qd*u*ud + Wl*Q^2*ud*ed + 3*Wl*
  + 2*Wl*Wr*L*Ld*t + Wl*Wr*ee*ed*t +
 Wr*Q*Qd*t + Wl^2*L*Ld*t + Wl^2*Q*Qd
 *Ld + Gr*d*dd*ee*ed + Gr*d^2*dd^2 +
 u*ud*ee*ed + 4*Gr*u*ud*d*dd + Gr*u^
 ed*Ld + 2*Gr*Qd*u*d*Ld + 6*Gr*Qd^2*
  + Gr*Q*d*ed*Ld + 2*Gr*Q*ud*dd*L +
 *Q*Qd*d*dd + 4*Gr*Q*Qd*u*ud + Gr*Q^
 t + Gr*Wl*Q*Qd*t + Gr^2*d*dd*t + Gr
  + Gr^2*Wl^2 + 3*Gr^4 + Gl*d*dd*L*L
 u*ud*L*Ld + Gl*u*ud*ee*ed + 4*Gl*u*
  + Gl*Qd*dd*ee*L + 2*Gl*Qd*u*d*Ld +
 *ud*dd*L + 3*Gl*Q*u*ee*L + 2*Gl*Q*Q
 dd + 4*Gl*Q*Qd*u*ud + Gl*Q^2*ud*ed
 *Q^3*L + Gl*Wr*Q*Qd*t + Gl*Wl*Q*Qd*
 Gl*Gr*d*dd*t + 3*Gl*Gr*u*ud*t + 3*G
 *t + Gl^2*u*ud*t + Gl^2*Q*Qd*t + Gl
  + 3*Gl^4 + Br*ee*ed*L*Ld + Br*d*dd
 ed + Br*u*ud*L*Ld + Br*u*ud*ee*ed +
 Br*Qd*ud*ed*Ld + Br*Qd*u*d*Ld + 3*B
 *Ld + Br*Q*ud*dd*L + 2*Br*Q*Qd*L*Ld
 *Br*Q*Qd*u*ud + Br*Q^2*ud*ed + Br*Wr
 t + Br*Wl*Q*Qd*t + Br*Gr*d*dd*t + Br*Gr*u*ud*t + Br*Gr*Q*Qd*t + Br*Gr^3
  + Br*Gl*d*dd*t + Br*Gl*u*ud*t + Br*Gl*Q*Qd*t + Br*Gl^2*Gr + 2*Br^2*Wr^2
  + Br^2*Wl^2 + 2*Br^2*Gr^2 + Br^2*Gl^2 + Br^4 + Bl*ee*ed*L*Ld + Bl*d*dd*
 L*Ld + Bl*d*dd*ee*ed + Bl*u*ud*L*Ld + Bl*u*ud*ee*ed + 2*Bl*u*ud*d*dd + 2
 *Bl*u^2*d*ee + Bl*Qd*dd*ee*L + Bl*Qd*u*d*Ld + Bl*Qd^2*u*ee + Bl*Q*d*ed*
 Ld + Bl*Q*ud*dd*L + 3*Bl*Q*u*ee*L + 2*Bl*Q*Qd*L*Ld + Bl*Q*Qd*ee*ed + 2*
 Bl*Q*Qd*d*dd + 2*Bl*Q*Qd*u*ud + 3*Bl*Q^2*u*d + Bl*Q^3*L + Bl*Wr*L*Ld*t
  + Bl*Wr*Q*Qd*t + Bl*Wl*L*Ld*t + Bl*Wl*Q*Qd*t + Bl*Gr*d*dd*t + Bl*Gr*u*
 ud*t + Bl*Gr*Q*Qd*t + Bl*Gl*d*dd*t + Bl*Gl*u*ud*t + Bl*Gl*Q*Qd*t + Bl*Gl
 *Gr^2 + Bl*Gl^3 + Bl*Br*L*Ld*t + Bl*Br*ee*ed*t + Bl*Br*d*dd*t + Bl*Br*u*
 ud*t + Bl*Br*Q*Qd*t + Bl*Br*Wl*Wr + Bl*Br*Gl*Gr + Bl^2*Wr^2 + 2*Bl^2*
 Wl^2 + Bl^2*Gr^2 + 2*Bl^2*Gl^2 + Bl^2*Br^2 + Bl^4 + 3*Hd*ee*L^2*Ld*t +
 Hd*ee^2*ed*L*t + 3*Hd*d*dd*ee*L*t + 3*Hd*ud*d*ed*Ld*t + 2*Hd*ud^2*dd*L*t
  + 2*Hd*u*d^2*Ld*t + 3*Hd*u*ud*ee*L*t + 6*Hd*Qd*ud*L*Ld*t + 3*Hd*Qd*ud*
 ee*ed*t + 6*Hd*Qd*ud*d*dd*t + 3*Hd*Qd*u*d*ee*t + 3*Hd*Qd*u*ud^2*t + 3*Hd
 *Qd^2*d*Ld*t + Hd*Qd^3*ee*t + 6*Hd*Q*d*L*Ld*t + 3*Hd*Q*d*ee*ed*t + 3*Hd*
 Q*d^2*dd*t + 2*Hd*Q*ud^2*ed*t + 6*Hd*Q*u*ud*d*t + 6*Hd*Q*Qd*ee*L*t + 6*
 Hd*Q*Qd^2*ud*t + 3*Hd*Q^2*ud*L*t + 6*Hd*Q^2*Qd*d*t + Hd*Wr*ee*L*t^2 + 2*
 Hd*Wr*Qd*ud*t^2 + Hd*Wr*Q*d*t^2 + Hd*Wr^2*ee*L + 2*Hd*Wr^2*Qd*ud + Hd*
 Wr^2*Q*d + 2*Hd*Wl*ee*L*t^2 + Hd*Wl*Qd*ud*t^2 + 2*Hd*Wl*Q*d*t^2 + 2*Hd*
 Wl^2*ee*L + Hd*Wl^2*Qd*ud + 2*Hd*Wl^2*Q*d + 2*Hd*Gr*Qd*ud*t^2 + Hd*Gr*Q*
 d*t^2 + 2*Hd*Gr*Wr*Qd*ud + Hd*Gr*Wr*Q*d + Hd*Gr^2*ee*L + 3*Hd*Gr^2*Qd*ud
  + 2*Hd*Gr^2*Q*d + Hd*Gl*Qd*ud*t^2 + 2*Hd*Gl*Q*d*t^2 + Hd*Gl*Wl*Qd*ud +
 2*Hd*Gl*Wl*Q*d + Hd*Gl^2*ee*L + 2*Hd*Gl^2*Qd*ud + 3*Hd*Gl^2*Q*d + Hd*Br*
 ee*L*t^2 + 2*Hd*Br*Qd*ud*t^2 + Hd*Br*Q*d*t^2 + Hd*Br*Wr*ee*L + 2*Hd*Br*
Wr*Qd*ud + Hd*Br*Wr*Q*d + 2*Hd*Br*Gr*Qd*ud + Hd*Br*Gr*Q*d + Hd*Br^2*ee*L
 + Hd*Br^2*Qd*ud + Hd*Br^2*Q*d + 2*Hd*Bl*ee*L*t^2 + Hd*Bl*Qd*ud*t^2 + 2*
Hd*Bl*Q*d*t^2 + 2*Hd*Bl*Wl*ee*L + Hd*Bl*Wl*Qd*ud + 2*Hd*Bl*Wl*Q*d + Hd*
Bl*Gl*Qd*ud + 2*Hd*Bl*Gl*Q*d + Hd*Bl^2*ee*L + Hd*Bl^2*Qd*ud + Hd*Bl^2*Q*
d + Hd^2*ee^2*L^2 + Hd^2*ud*d*t^3 + Hd^2*ud*d*L*Ld + Hd^2*Qd*ud*ee*L + 2
*Hd^2*Qd^2*ud^2 + 2*Hd^2*Q*d*ee*L + 2*Hd^2*Q*Qd*ud*d + 2*Hd^2*Q^2*d^2 +
Hd^2*Wr*ud*d*t + Hd^2*Wl*ud*d*t + Hd^2*Gr*ud*d*t + Hd^2*Gl*ud*d*t + Hd^2
*Br*ud*d*t + Hd^2*Bl*ud*d*t + 3*H*ed*L*Ld^2*t + H*ee*ed^2*Ld*t + 3*H*d*
dd*ed*Ld*t + 2*H*ud*dd^2*L*t + 3*H*u*dd*ee*L*t + 3*H*u*ud*ed*Ld*t + 2*H*
u^2*d*Ld*t + 6*H*Qd*dd*L*Ld*t + 3*H*Qd*dd*ee*ed*t + 3*H*Qd*d*dd^2*t + 6*
H*Qd*u*ud*dd*t + 2*H*Qd*u^2*ee*t + 3*H*Qd^2*u*Ld*t + 3*H*Q*ud*dd*ed*t +
6*H*Q*u*L*Ld*t + 3*H*Q*u*ee*ed*t + 6*H*Q*u*d*dd*t + 3*H*Q*u^2*ud*t + 6*H
*Q*Qd*ed*Ld*t + 6*H*Q*Qd^2*dd*t + 3*H*Q^2*dd*L*t + 6*H*Q^2*Qd*u*t + H*
 + H*Wr*Q*u*t^2 + 2*H*Wr^2
Ld*t^2 + H*Wl*Qd*dd*t^2
 + 2*H*Wl^2*Q*u + 2*H*Gr*
Gr*Wr*Q*u + H*Gr^2*ed*Ld
2 + 2*H*Gl*Q*u*t^2 + H*
H*Gl^2*Qd*dd + 3*H*Gl^2*Q
Q*u*t^2 + 2*H*Br*Wr*ed*
Qd*dd + H*Br*Gr*Q*u + H*
*Ld*t^2 + H*Bl*Qd*dd*t^2
dd + 2*H*Bl*Wl*Q*u + H*Bl
2*Qd*dd + H*Bl^2*Q*u + 4
t^3 + 2*H*Hd*ee*ed*L*Ld
dd*L*Ld + H*Hd*d*dd*ee*ed
d*t^3 + 2*H*Hd*u*ud*L*Ld
2*d*ee + H*Hd*u^2*ud^2 +
Qd*u*d*Ld + 4*H*Hd*Qd^2*
*Q*d*ed*Ld + 2*H*Hd*Q*ud
Hd*Q*Qd*L*Ld + 2*H*Hd*Q*
 + H*Hd*Q^2*ud*ed + 4*H*Hd
Hd*Wr*L*Ld*t + 2*H*Hd*Wr
 + 6*H*Hd*Wr*Q*Qd*t + 2*H*
H*Hd*Wl*ee*ed*t + 2*H*Hd*
 + 2*H*Hd*Wl*Wr*t^2 + 2*H
*Hd*Wl^2*t^2 + H*Hd*Wl^3 + 2*H*Hd*Gr*d*dd*t + 2*H*Hd*Gr*u*ud*t + 4*H*Hd*
Gr*Q*Qd*t + H*Hd*Gr^2*t^2 + H*Hd*Gr^3 + 2*H*Hd*Gl*d*dd*t + 2*H*Hd*Gl*u*
ud*t + 4*H*Hd*Gl*Q*Qd*t + H*Hd*Gl*Gr*t^2 + H*Hd*Gl^2*t^2 + H*Hd*Gl^3 + 4
*H*Hd*Br*L*Ld*t + 2*H*Hd*Br*ee*ed*t + 2*H*Hd*Br*d*dd*t + 2*H*Hd*Br*u*ud*
t + 4*H*Hd*Br*Q*Qd*t + 2*H*Hd*Br*Wr*t^2 + H*Hd*Br*Wr^2 + H*Hd*Br*Wl*t^2
 + H*Hd*Br^2*t^2 + 4*H*Hd*Bl*L*Ld*t + 2*H*Hd*Bl*ee*ed*t + 2*H*Hd*Bl*d*dd
*t + 2*H*Hd*Bl*u*ud*t + 4*H*Hd*Bl*Q*Qd*t + H*Hd*Bl*Wr*t^2 + 2*H*Hd*Bl*Wl
*t^2 + H*Hd*Bl*Wl^2 + H*Hd*Bl*Br*t^2 + H*Hd*Bl^2*t^2 + 6*H*Hd^2*ee*L*t^2
 + 6*H*Hd^2*Qd*ud*t^2 + 6*H*Hd^2*Q*d*t^2 + 2*H*Hd^2*Wr*Qd*ud + 2*H*Hd^2*
Wl*ee*L + 2*H*Hd^2*Wl*Q*d + H*Hd^2*Gr*Qd*ud + H*Hd^2*Gl*Q*d + H*Hd^2*Br*
Qd*ud + H*Hd^2*Bl*ee*L + H*Hd^2*Bl*Q*d + H*Hd^3*ud*d*t + H^2*ed^2*Ld^2
 + H^2*u*dd*t^3 + H^2*u*dd*L*Ld + 2*H^2*Qd*dd*ed*Ld + 2*H^2*Qd^2*dd^2 +
H^2*u*ed*Ld + 2*H^2*Q*Qd*u*dd + 2*H^2*Q^2*u^2 + H^2*Wr*u*dd*t + H^2*Wl
*u*dd*t + H^2*Gr*u*dd*t + H^2*Gl*u*dd*t + H^2*Br*u*dd*t + H^2*Bl*u*dd*t
 + 6*H^2*Hd*ed*Ld*t^2 + 6*H^2*Hd*Qd*dd*t^2 + 6*H^2*Hd*Q*u*t^2 + 2*H^2*Hd
*Wr*ed*Ld + 2*H^2*Hd*Wr*Qd*dd + 2*H^2*Hd*Wl*Q*u + H^2*Hd*Gr*Qd*dd + H^2*
Hd*Gl*Q*u + H^2*Hd*Br*ed*Ld + H^2*Hd*Br*Qd*dd + H^2*Hd*Bl*Q*u + 3*H^2*
Hd^2*t^4 + 4*H^2*Hd^2*L*Ld*t + H^2*Hd^2*ee*ed*t + H^2*Hd^2*d*dd*t + H^2*
Hd^2*u*ud*t + 4*H^2*Hd^2*Q*Qd*t + 2*H^2*Hd^2*Wr*t^2 + 2*H^2*Hd^2*Wr^2 +
2*H^2*Hd^2*Wl*t^2 + 2*H^2*Hd^2*Wl^2 + H^2*Hd^2*Gr^2 + H^2*Hd^2*Gl^2 +
H^2*Hd^2*Br*t^2 + H^2*Hd^2*Br*Wr + H^2*Hd^2*Br^2 + H^2*Hd^2*Bl*t^2 + H^2
*Hd^2*Bl*Wl + H^2*Hd^2*Bl^2 + H^2*Hd^3*ee*L + H^2*Hd^3*Qd*ud + H^2*Hd^3*
Q*d + H^3*Hd*u*dd*t + H^3*Hd^2*ed*Ld + H^3*Hd^2*Qd*dd + H^3*Hd^2*Q*u + 2
*H^3*Hd^3*t^2 + H^4*Hd^4;
```


```
f =
 2*L^2*Ld^2*t^2 + 2*ee*ed*L*Ld*t^2 + ee^2*ed^2*t^2
 d*dd*ee*ed*t^2 + 2*d^2*dd^2*t^2 + ud^2*dd*ed*t^2 +
 *ud*ee*ed*t^2 + 4*u*ud*d*dd*t^2 + u^2*d*ee*t^2 + 2
 dd*ee*L*t^2 + 3*Qd*ud*ed*Ld*t^2 + 2*Qd*u*d*Ld*t^2
 Qd^2*u*ee*t^2 + Qd^3*Ld*t^2 + 2*Q*d*ed*Ld*t^2 + 2*
 ee*L*t^2 + 4*Q*Qd*L*Ld*t^2 + 2*Q*Qd*ee*ed*t^2 + 4*
 *u*ud*t^2 + Q^2*ud*ed*t^2 + 3*Q^2*u*d*t^2 + 4*Q^2*
  + Wr*L^2*Ld^2 + Wr*ee*ed*L*Ld + Wr*d*dd*L*Ld + Wr
 ee*L + 3*Wr*Qd*ud*ed*Ld + Wr*Qd*u*d*Ld + 3*Wr*Qd^2
```


Henning, **XL**, Melia, and Murayama, arXiv: 1512.03433

SMEFT dim-6 Operator Basis

➢ Warsaw basis

- 80 operators

  Buchmuller and Wyler,
  Nucl. Phys. B 268 (1986) 621.

- 59 independent operators

  arXiv: 1008.4884
  Grzadkowski, Iskrzynski,
  Misiak, and Rosiek,

- 2499 couplings RG running

  arXiv: 1308.2627
  arXiv: 1310.4838
  arXiv: 1312.2014
  Alonso, Jenkins, Manohar, and Trott

➢ SILH basis

Giudice, Grojean, Pomarol,
and Rattazzi, arXiv: hep-ph/0703164

Elias-Miro, Espinosa, Masso,
and Pomarol, arXiv: 1308.1879

Pomarol and Riva, arXiv: 1308.2803

➢ HISZ basis

Hagiwara, Ishihara, Szalapski, and
Zeppenfeld, Phys.Rev. **D 48**, 2182 (1993)

➢ EGGM basis

Elias-Miro, Grojean, Gupta,
and Marzocca, arXiv: 1312.2928

Higher dim Operator Basis

- SMEFT dim-7 — Lehman, arXiv: 1410.4193
  Liao and Ma, arXiv: 1607.07309
- SMEFT dim-8 — Li, Ren, Shu, Xiao, Yu and Zhen, arXiv: 2005.00008
  Murphy, arXiv: 2005.00059
- SMEFT dim-9 — Liao and Ma, arXiv: 2007.08125

Higher dim Operator Basis

- SMEFT dim-7 — Lehman, arXiv: 1410.4193; Liao and Ma, arXiv: 1607.07309
- SMEFT dim-8 — Li, Ren, Shu, Xiao, Yu and Zhen, arXiv: 2005.00008; Murphy, arXiv: 2005.00059
- SMEFT dim-9 — Liao and Ma, arXiv: 2007.08125

- νSMEFT (with right-handed neutrino)
  - dim-6 — del Aguila, Bar-Shalom, Soni, and Wudka, arXiv: 0806.0876; Aparici, Kim, Santamaria, and Wudka, arXiv: 0904.3244
  - dim-7 — Bhattacharya and Wudka, arXiv: 1505.05264; Liao and Ma, arXiv: 1612.04527
  - dim-8 and 9 — Li, Ren, Xiao, Yu and Zhen, arXiv: 2105.09329

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i c_i \mathcal{O}_i$$

Outline

- ✓ Establishing the Framework: What is SMEFT?

  --- non-renormalizable, defined with a truncation, operator basis

- Implementing the Framework: How to use SMEFT?

  --- interpreting experimental limits

  --- guide UV model building: Matching and Running

  --- additional restrictions to reduce degrees of freedom

- Re-examining the Framework: Is SMEFT enough?

  --- SMEFT / HEFT dichotomy

  --- geometric picture for non-analyticities and unitarity violation

  --- HEFT describes non-decoupling BSM physics

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i c_i \mathcal{O}_i$$

Outline

- Implementing the Framework: How to use SMEFT?

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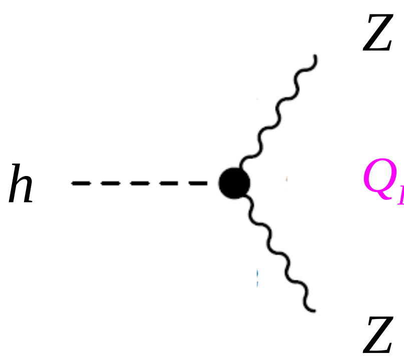
How to use SMEFT?

$$c_i(v) \quad \Rightarrow \quad \text{weak scale observables}$$

e.g. Henning, **XL**, and Murayama, arXiv: 1412.1837



$$Q_{HB} = |H|^2 B_{\mu\nu} B^{\mu\nu}$$

$$\mathcal{A}_{hZZ^*} = r_h^{1/2} r_Z \left(\mathcal{A}_{hZZ^*}^{\text{SM}} + \mathcal{A}_{hZZ^*}^{c} \right)$$

- Interference corrections

$$\Gamma_{hZZ^*}\left(g_1, g_2, v, c_i\right)$$

- Residue corrections

$$\Gamma_{hZZ^*}\left(\alpha, G_F, m_Z, c_i\right)$$

- Parametric corrections

How to use SMEFT?

Global Fitting Results

Ellis, Madigan, Mimasu, Sanz, and You, arXiv: 2012.02779

	Individual			Marginalised		
SMEFT Coeff.	Best fit $[\Lambda = 1\ \text{TeV}]$	95% CL range	Scale $\frac{\Lambda}{\sqrt{C}}$ [TeV]	Best fit $[\Lambda = 1\ \text{TeV}]$	95% CL range	Scale $\frac{\Lambda}{\sqrt{C}}$ [TeV]
C_{HWB}	0.00	[-0.0043, +0.0026]	17.0	0.18	[-0.36, +0.73]	1.4
C_{HD}	-0.01	[-0.023, +0.0027]	8.8	-0.39	[-1.6, +0.81]	0.91
C_{ll}	0.01	[-0.005, +0.019]	9.2	-0.03	[-0.084, +0.02]	4.4
$C_{Hl}^{(3)}$	0.00	[-0.01, +0.003]	12.0	-0.03	[-0.13, +0.055]	3.3
$C_{Hl}^{(1)}$	0.00	[-0.0044, +0.013]	11.0	0.11	[-0.19, +0.41]	1.8
C_{He}	0.00	[-0.015, +0.0071]	9.6	0.19	[-0.41, +0.79]	1.3
$C_{Hq}^{(3)}$	0.00	[-0.017, +0.012]	8.3	-0.05	[-0.11, +0.012]	4.1
$C_{Hq}^{(1)}$	0.02	[-0.1, +0.14]	2.9	-0.04	[-0.27, +0.18]	2.1
C_{Hd}	-0.03	[-0.13, +0.071]	3.1	-0.39	[-0.91, +0.13]	1.4
C_{Hu}	0.00	[-0.075, +0.073]	3.7	-0.19	[-0.63, +0.25]	1.5
$C_{H\Box}$	-0.27	[-1, +0.47]	1.2	-0.9	[-3, +1.2]	0.69
C_{HG}	0.00	[-0.0034, +0.0032]	17.0	0.00	[-0.014, +0.0086]	9.4
C_{HW}	0.00	[-0.012, +0.006]	11.0	0.12	[-0.38, +0.62]	1.4
C_{HB}	0.00	[-0.0034, +0.002]	19.0	0.07	[-0.09, +0.22]	2.5

M ~ a few TeV

How to use SMEFT?

Global Fitting Results

Ellis, Madigan, Mimasu, Sanz, and You, arXiv: 2012.02779

SMEFT Coeff.	Individual: Best fit [$\Lambda = 1$ TeV]	Individual: 95% CL range	Individual: Scale $\frac{\Lambda}{\sqrt{C}}$ [TeV]	Marginalised: Best fit [$\Lambda = 1$ TeV]	Marginalised: 95% CL range	Marginalised: Scale $\frac{\Lambda}{\sqrt{C}}$ [TeV]
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[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

M ~ a few TeV $< \sqrt{s} \sim 14$ TeV $\Rightarrow$ SMEFT validity at LHC?

SMEFT Validity Issue Emphasized

- Same-sign *WW* scattering

  Kalinowski, Kozow, Pokorski, Rosiek, Szleper, and Tkaczyk, arXiv: 1802.02366

  Kozow, Merlo, Pokorski, and Szleper, arXiv: 1905.03354

  Chaudhary, Kalinowski, Kaur, Kozow, Sandeep, Szleper, and Tkaczyk, arXiv: 1906.10769

- General vector boson scattering

  Lang, Liebler, Schafer-Siebert, and Zeppenfeld, arXiv: 2103.16517

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$$\sqrt{s} \sim 14 \text{ TeV} \xrightarrow{\text{PDF effects}} \sqrt{\hat{s}} \sim 2 \text{ TeV} < M \sim \text{ a few TeV}$$

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$$\boxed{\sqrt{s} \sim 14\ \text{TeV} \xrightarrow{\text{PDF effects}} \sqrt{\hat{s}} \sim 2\ \text{TeV} \;<\; M \sim \text{a few TeV}}$$

Cohen, Doss, and **XL**, arXiv: 2110.XXXXX

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i c_i \mathcal{O}_i$$

Outline

- Implementing the Framework: How to use SMEFT?

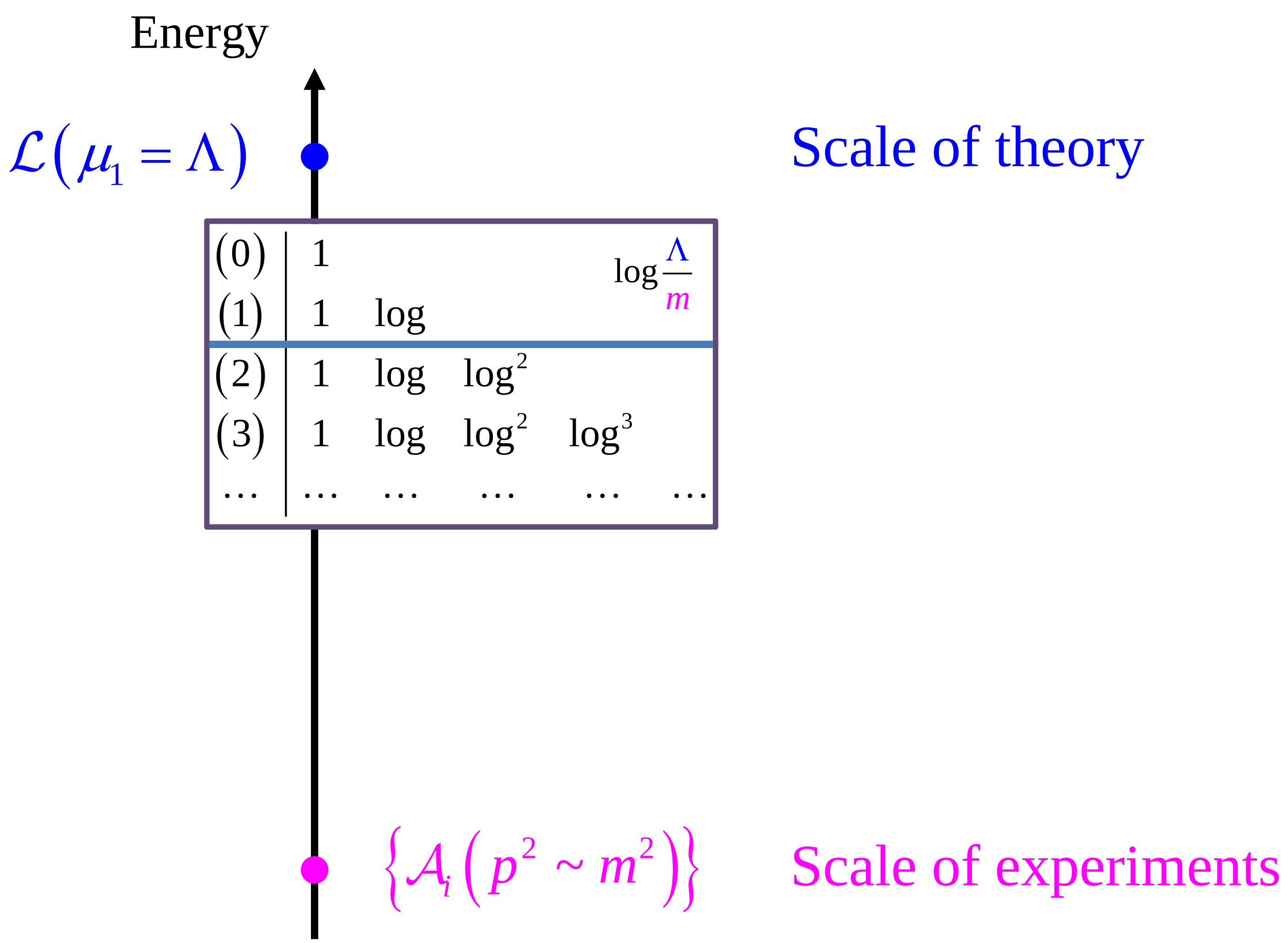
  ✓ --- interpreting experimental limits

  --- guide UV model building: Matching and Running

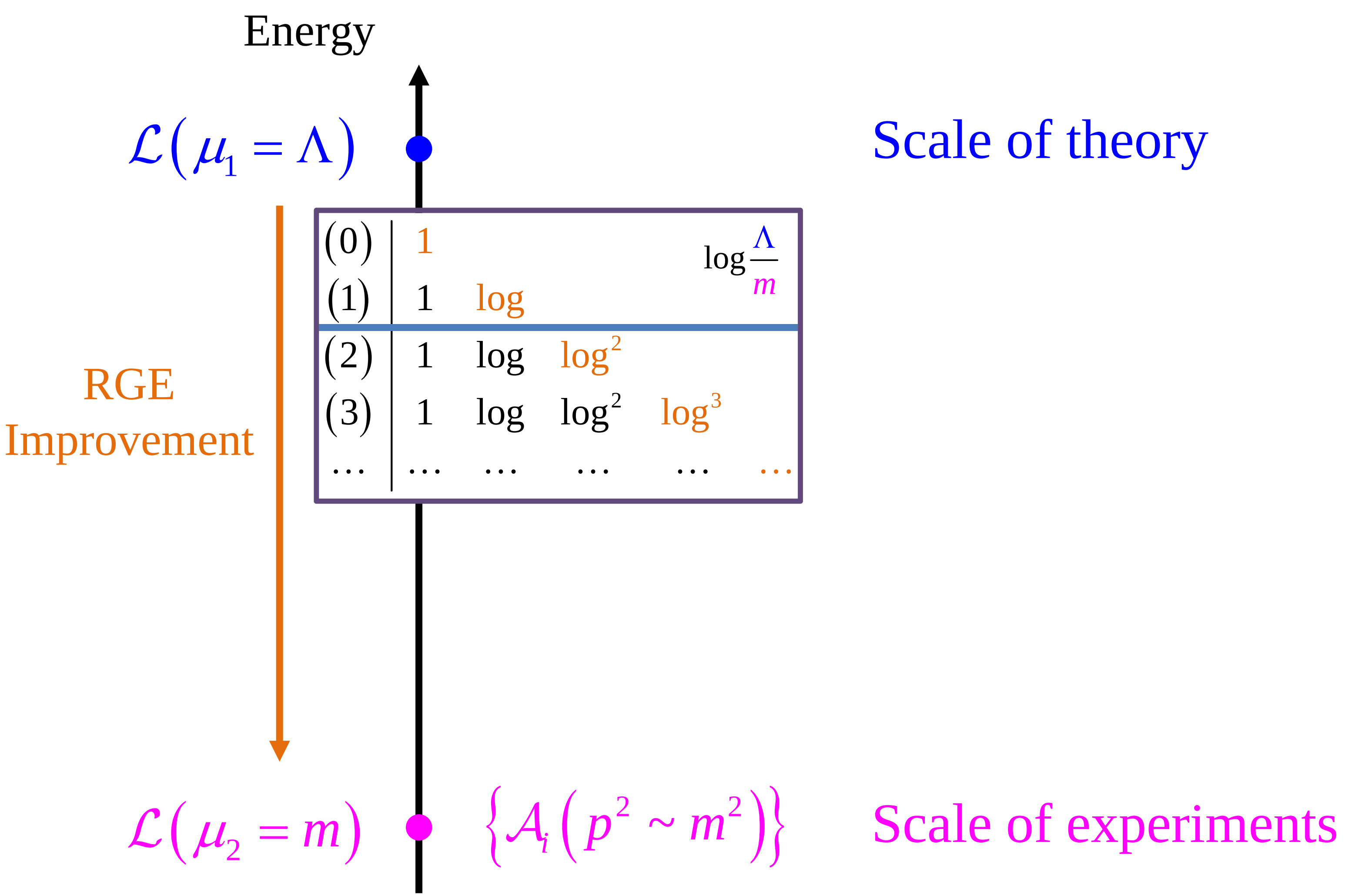
  --- additional restrictions to reduce degrees of freedom

How to use SMEFT?

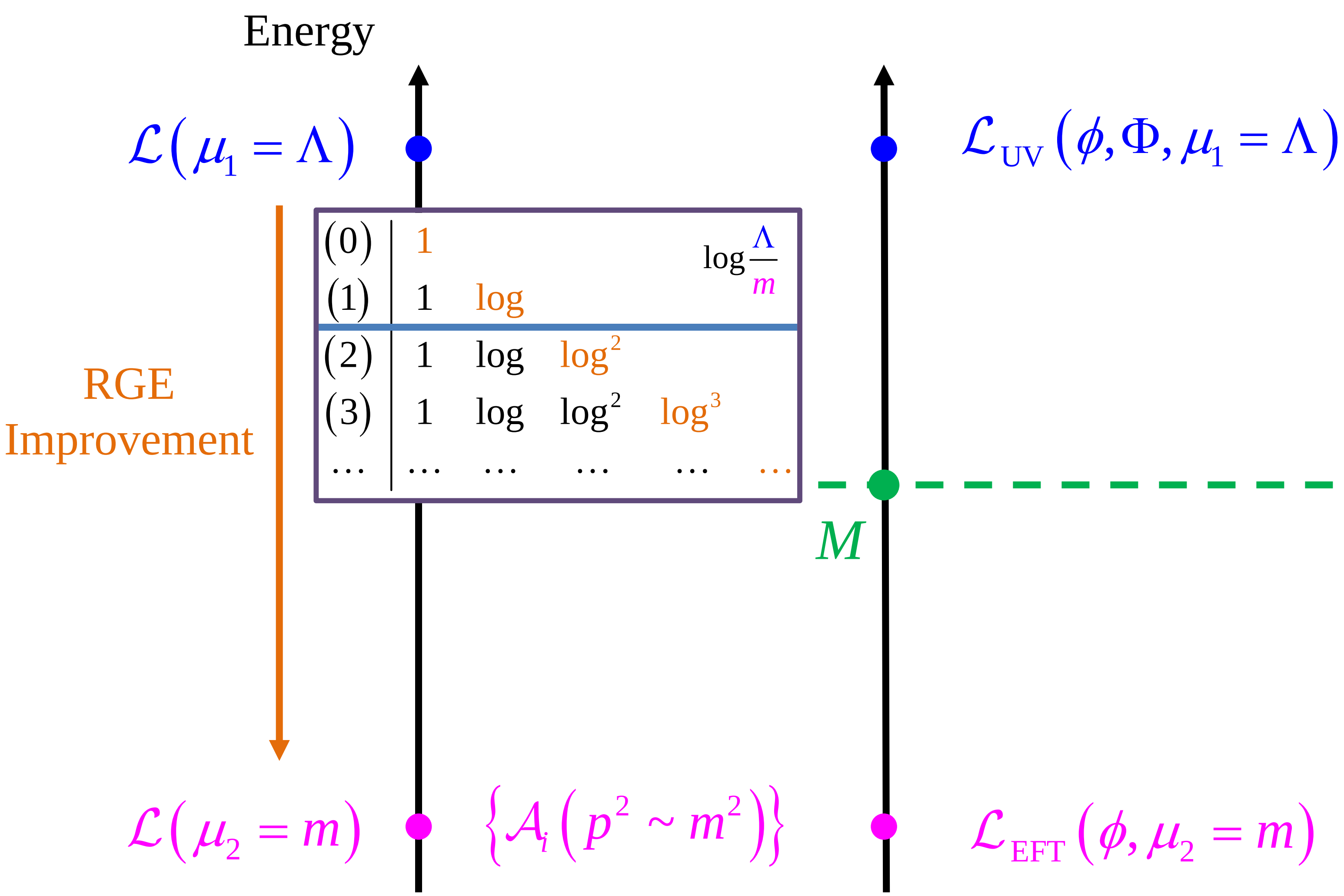
Matching and Running: systematically summing large logs



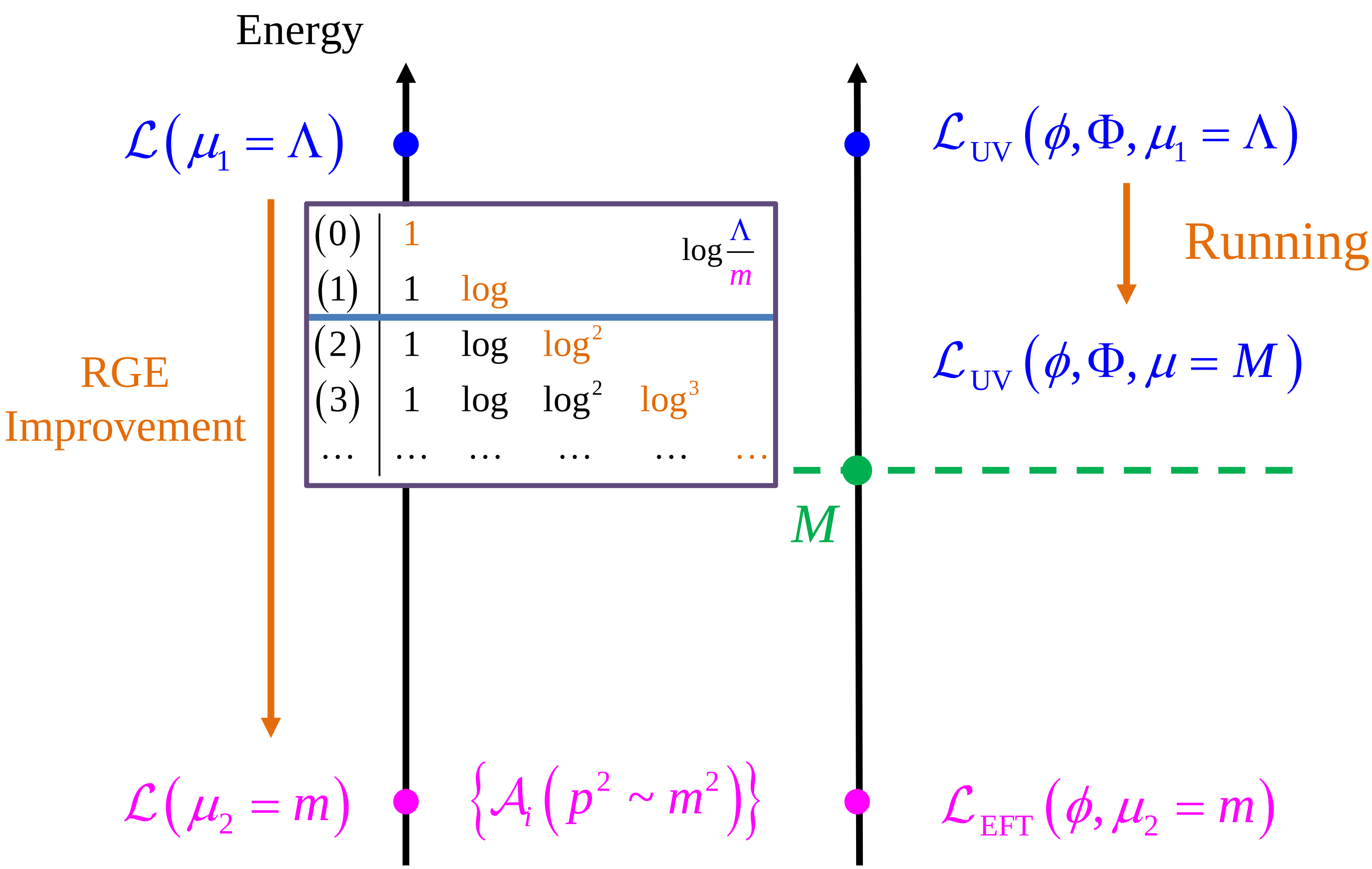
Matching and Running: systematically summing large logs



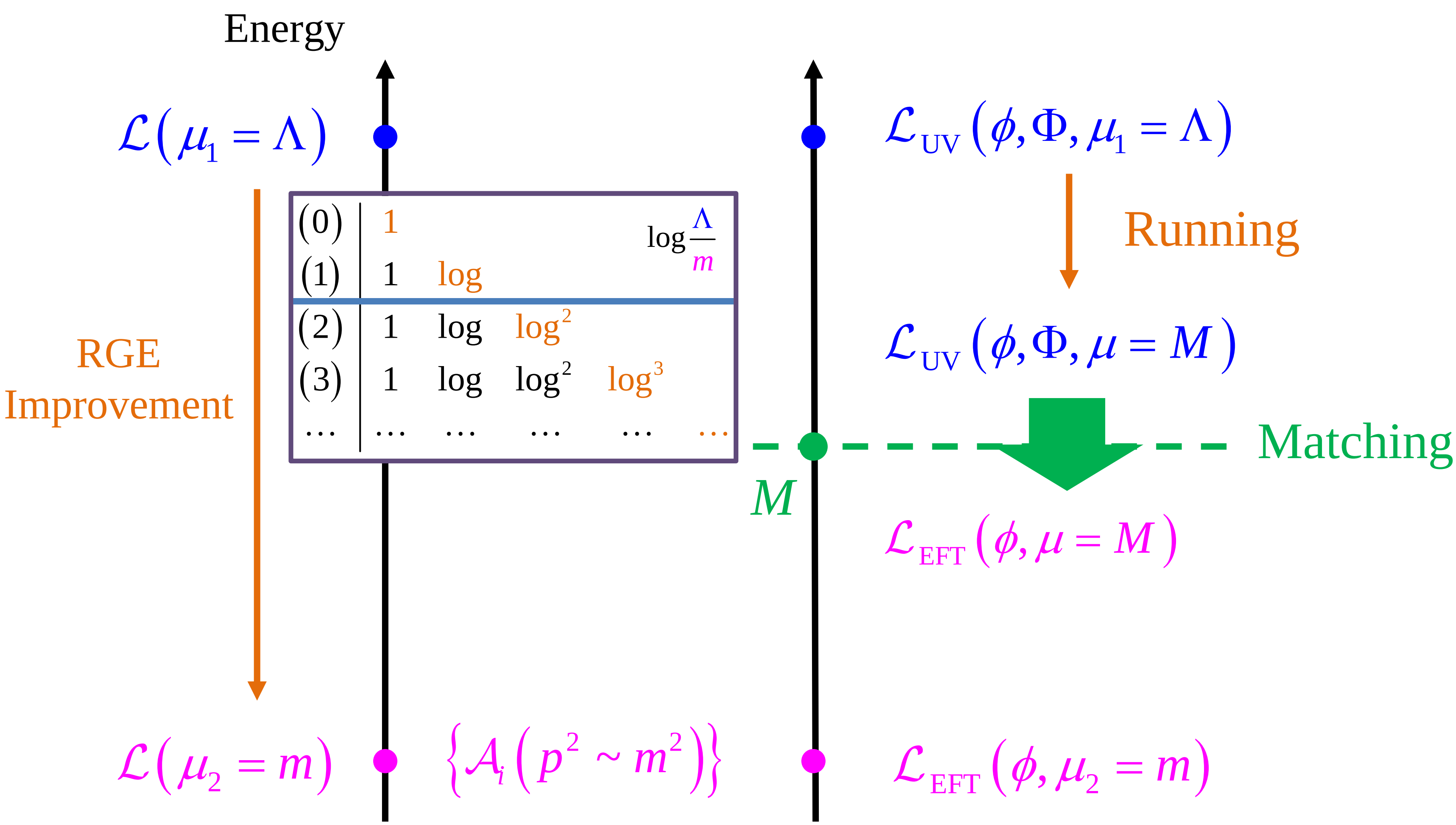
Matching and Running: systematically summing large logs



Matching and Running: systematically summing large logs

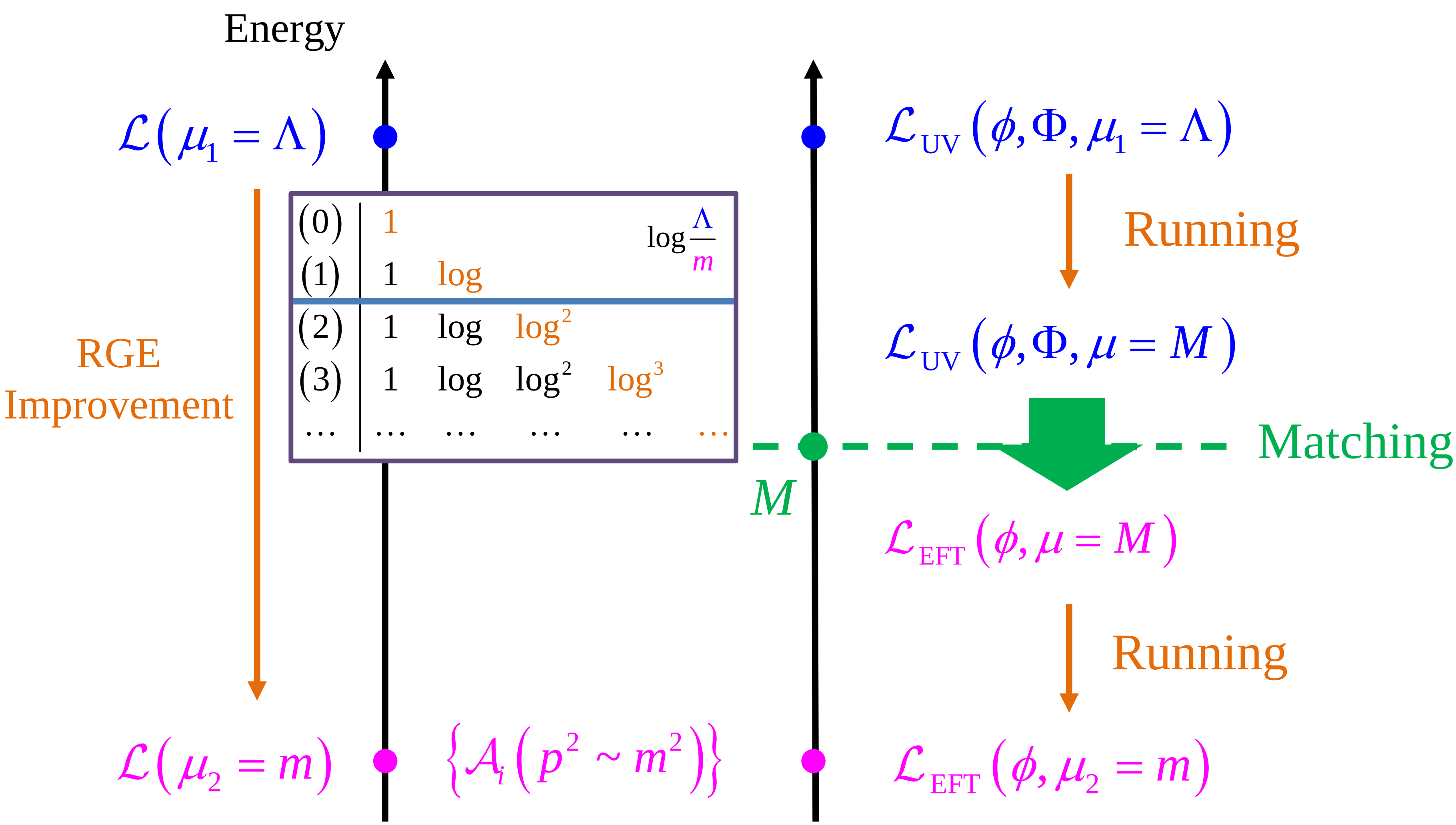


Matching and Running: systematically summing large logs



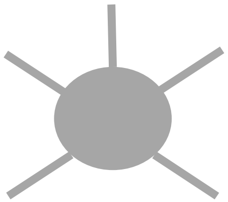
How to use SMEFT?

Matching and Running: systematically summing large logs



Match Amplitudes

$$\mathcal{L}_{\text{UV}} \longrightarrow \{\mathcal{A}_{\text{UV}}\}$$



$$\{\mathcal{A}_{\text{UV}}\} \longleftrightarrow \{\mathcal{A}_{\text{SMEFT}}(c_i)\} \longrightarrow c_i$$

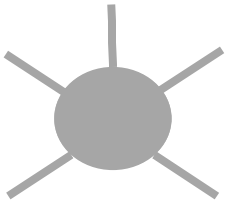
$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i c_i \mathcal{O}_i \longrightarrow \{\mathcal{A}_{\text{SMEFT}}(c_i)\}$$

Need $\mathcal{O}_i(\phi)$, $\{\mathcal{A}_i\}$

Match Amplitudes

$$\mathcal{L}_{\mathrm{UV}} \rightarrow \{\mathcal{A}_{\mathrm{UV}}\}$$

$$\mathcal{L}_{\mathrm{SMEFT}}(\mathcal{L}_{\mathrm{UV}}) \downarrow \qquad \updownarrow \; c_i$$



$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i c_i \mathcal{O}_i \rightarrow \{\mathcal{A}_{\mathrm{SMEFT}}(c_i)\}$$

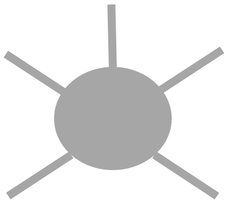
Need $\mathcal{O}_i(\phi)$, $\{\mathcal{A}_i\}$

How to use SMEFT?

Match Amplitudes

$\mathcal{L}_{\text{UV}} \rightarrow \{\mathcal{A}_{\text{UV}}\}$ $\quad \Gamma_{\text{UV, 1LPI}}[\phi]$

$\mathcal{L}_{\text{SMEFT}}(\mathcal{L}_{\text{UV}})$



c_i

Functional Matching

$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i c_i \mathcal{O}_i \rightarrow \{\mathcal{A}_{\text{SMEFT}}(c_i)\}$ $\quad \Gamma_{\text{EFT, 1PI}}[\phi]$

Need $\mathcal{O}_i(\phi)$, $\{\mathcal{A}_i\}$

$$\Gamma_{\text{UV, 1LPI}}[\phi] = \Gamma_{\text{EFT, 1PI}}[\phi] \quad \Rightarrow \quad \begin{cases} \mathcal{L}_{\text{EFT}}^{(\text{tree})} = \mathcal{L}_{\text{UV}}\left(\phi, \Phi = \Phi_{\text{c}}[\phi]\right) \quad , \quad \left.\dfrac{\delta S_{\text{UV}}}{\delta \Phi}\right|_{\Phi=\Phi_{\text{c}}[\phi]} = 0 \\ \displaystyle\int \mathrm{d}^4 x\, \mathcal{L}_{\text{EFT}}^{(\text{1-loop})} = \frac{i}{2}\,\mathrm{STr}\log\left[\left. -\frac{\delta^2 S_{\text{UV}}}{\delta(\phi,\Phi)^2}\right|_{\Phi=\Phi_{\text{c}}}\right]\Bigg|_{\text{hard}} \end{cases}$$

Henning, **XL**, and Murayama, arXiv: 1412.1837, 1604.01019

Ellis, Quevillon, You, and Zhang, arXiv: 1604.02445

Fuentes-Martin, Portoles, and Ruiz-Femenia, arXiv: 1607.02142

Zhang, arXiv: 1610.00710

How to use SMEFT?

$$\Gamma_{\text{UV, 1LPI}}[\phi] = \Gamma_{\text{EFT, 1PI}}[\phi] \quad \Rightarrow \quad \begin{cases} \mathcal{L}_{\text{EFT}}^{(\text{tree})} = \mathcal{L}_{\text{UV}}\left(\phi, \Phi = \Phi_{\text{c}}[\phi]\right) \quad , \quad \left.\dfrac{\delta S_{\text{UV}}}{\delta \Phi}\right|_{\Phi=\Phi_{\text{c}}[\phi]} = 0 \\ \displaystyle\int \mathrm{d}^4 x\, \mathcal{L}_{\text{EFT}}^{(\text{1-loop})} = \frac{i}{2} \operatorname{STr} \log \left[-\left.\frac{\delta^2 S_{\text{UV}}}{\delta(\phi,\Phi)^2}\right|_{\Phi=\Phi_{\text{c}}} \right]\Bigg|_{\text{hard}} \end{cases}$$

Method of regions

M. Beneke and V. A. Smirnov, "Asymptotic expansion of Feynman integrals near threshold," *Nucl. Phys.* **B522** (1998) 321–344, arXiv:hep-ph/9711391 [hep-ph].

V. A. Smirnov, "Applied asymptotic expansions in momenta and masses," *Springer Tracts Mod. Phys.* **177** (2002) 1–262.

$$-i \operatorname{STr}\left(\frac{1}{P^2 - M^2} U_1 \frac{1}{P^2 - m^2} U_2 \right) \supset -i \int \mathrm{d}^d x \int \frac{\mathrm{d}^d q}{(2\pi)^d} \operatorname{tr}\left[\frac{1}{q^2 - M^2} U_1 \underline{\frac{1}{q^2 - m^2}} U_2 \right]$$

$$|q| \sim M \gg m \quad \Rightarrow \quad \frac{1}{q^2 - m^2} = \frac{1}{q^2} + \frac{m^2}{q^4} + \frac{m^4}{q^6} + \cdots$$

How to use SMEFT?

$$\Gamma_{\text{UV, 1LPI}}[\phi] = \Gamma_{\text{EFT, 1PI}}[\phi] \quad \Rightarrow \quad \begin{cases} \mathcal{L}_{\text{EFT}}^{(\text{tree})} = \mathcal{L}_{\text{UV}}\left(\phi, \Phi = \Phi_{\text{c}}[\phi]\right) \quad , \quad \left.\dfrac{\delta S_{\text{UV}}}{\delta \Phi}\right|_{\Phi=\Phi_{\text{c}}[\phi]} = 0 \\ \displaystyle\int d^4x\, \mathcal{L}_{\text{EFT}}^{(\text{1-loop})} = \frac{i}{2} \operatorname{STr} \log \left[-\left.\frac{\delta^2 S_{\text{UV}}}{\delta(\phi,\Phi)^2}\right|_{\Phi=\Phi_{\text{c}}} \right]\Bigg|_{\text{hard}} \end{cases}$$

Applicability:

- Any spin: scalars, fermions, vector bosons
- Contributions from heavy-light loops
- Derivative interactions in UV
- Non-renormalizable interactions in UV
- Non-relativistic EFT matching, e.g. HQET

How to use SMEFT?

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- Any spin: scalars, fermions, vector bosons
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How to compute this functional SuperTrace?

How to use SMEFT?

$$\frac{i}{2}\,\mathrm{STr}\log\left[\left.-\frac{\delta^2 S_{\mathrm{UV}}}{\delta(\phi,\Phi)^2}\right|_{\Phi=\Phi_c}\right]\Bigg|_{\mathrm{hard}}$$

$$=\frac{i}{2}\,\mathrm{STr}\log\left(K-X\right)\Big|_{\mathrm{hard}}=\frac{i}{2}\,\mathrm{STr}\log K\Big|_{\mathrm{hard}}-\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\,\mathrm{STr}\left[\left(\frac{1}{K}X\right)^n\right]\Bigg|_{\mathrm{hard}}$$

How to use SMEFT?

$$\frac{i}{2}\,\mathrm{STr}\log\left[-\frac{\delta^2 S_{\mathrm{UV}}}{\delta(\phi,\Phi)^2}\bigg|_{\Phi=\Phi_c}\right]\Bigg|_{\mathrm{hard}}$$

Log-type

Power-type

$$=\frac{i}{2}\,\mathrm{STr}\log\left(K-X\right)\Big|_{\mathrm{hard}}=\frac{i}{2}\,\mathrm{STr}\log K\Big|_{\mathrm{hard}}-\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\,\mathrm{STr}\left[\left(\frac{1}{K}X\right)^n\right]\Bigg|_{\mathrm{hard}}$$

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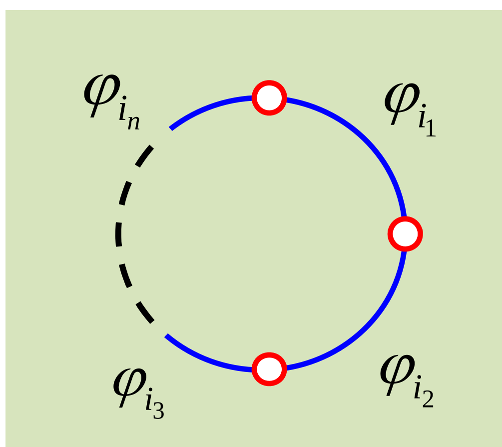
Covariant graphs
for enumeration



Cohen, **XL**, and Zhang, arXiv: 2011.02484

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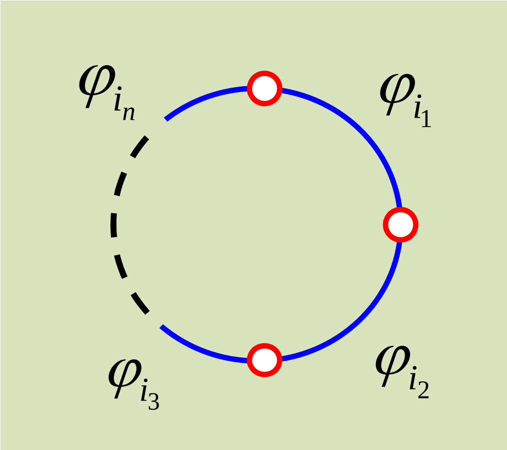
Covariant graphs for enumeration



"Covariant Derivative Expansion"

Gaillard, Nucl. Phys. B 268 (1986) 669

Chan, Phys. Rev. Lett. 57 (1986) 1199

Cheyette, Nucl. Phys. B 297 (1988) 183

Henning, **XL**, and Murayama, arXiv: 1404.1058, 1412.1837, 1604.01019

Cohen, Freytsis, and **XL**, arXiv: 1912.08814

Cohen, **XL**, and Zhang, arXiv: 2011.02484

How to use SMEFT?

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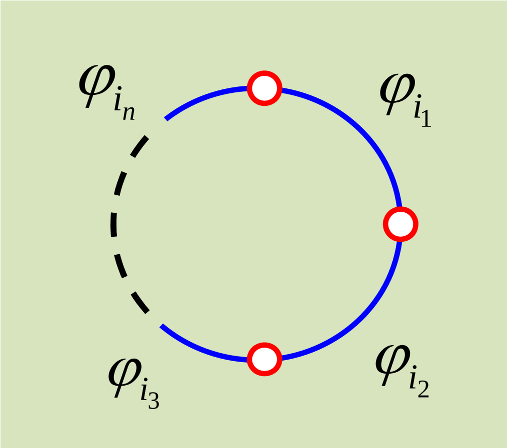
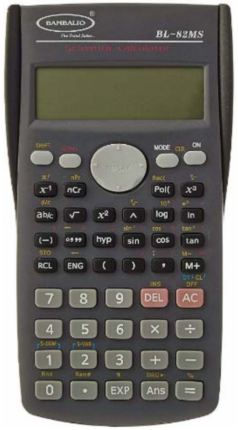
Log-type

Power-type

Covariant graphs for enumeration

STrEAM.m for evaluation





Cohen, **XL**, and Zhang, arXiv: 2011.02484, 2012.07851

How to use SMEFT?

Prescription up to one-loop order:

1. Derive heavy EOM(s) and $\mathcal{L}_{\text{EFT}}^{(\text{tree})}$

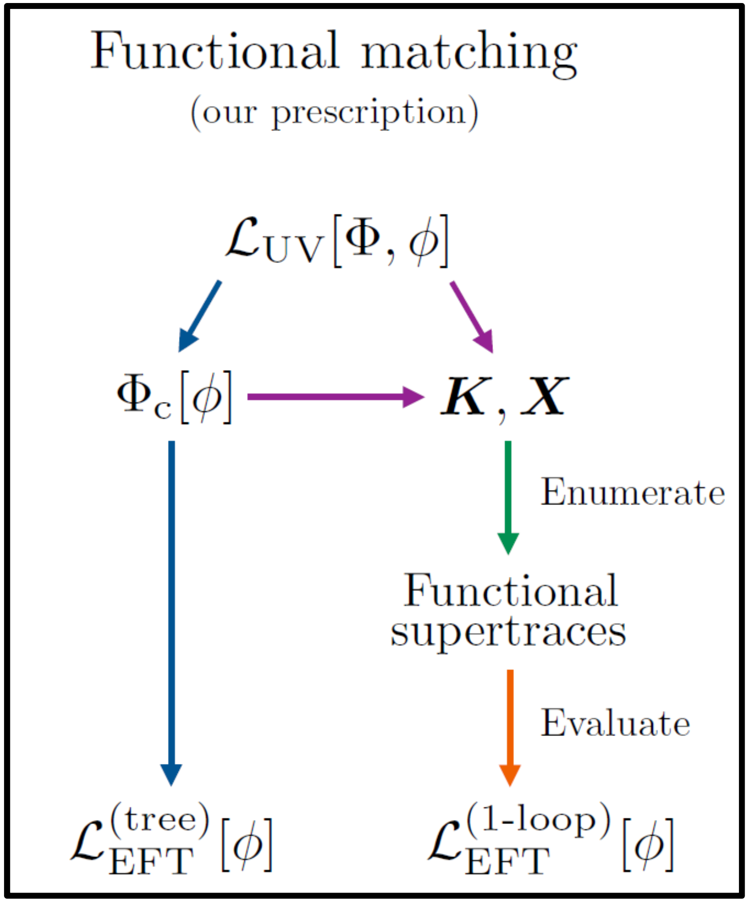
2. Derive K and X matrices

3. Enumerate supertraces

   Covariant graphs

4. Evaluate supertraces to obtain $\mathcal{L}_{\text{EFT}}^{(\text{1-loop})}$

   Mathematica package STrEAM.m



Cohen, **XL**, and Zhang, arXiv: 2011.02484, 2012.07851

How to use SMEFT?

1-loop functional matching examples

- Singlet Scalar — Cohen, **XL**, and Zhang, arXiv: 2011.02484
- Type-I Seesaw — Zhang and Zhou, arXiv: 2107.12133
- Scalar Leptoquark — Dedes and Mantzaropoulos, arXiv: 2108.10055

How about 2-loop and beyond?

Cohen, **XL**, and Zhang, in progress

How to use SMEFT?

- SMEFT dim-6 baryon preserving RGE

$$c_i(v) = c_i(M) + \frac{1}{16\pi^2}\gamma_{ij} c_j(M) \log\frac{v}{M}$$

Alonso, Jenkins, Manohar, and Trott, arXiv: 1308.2627, 1310.4838, 1312.2014

- Baryon violating RGE

Alonso, Chang, Jenkins, Manohar, and Shotwell, arXiv: 1405.0486

- SMEFT RGE Holomorphy

Alonso, Jenkins, and Manohar, arXiv: 1409.0868

Cheung and Shen, arXiv: 1505.01844

Functional methods with CDE for running

$$\mathcal{L}(\phi) \supset \mathcal{O}_K(\phi) + \lambda(\mu)\mathcal{O}_\lambda(\phi) \quad \Rightarrow \quad \beta_\lambda \equiv \mu \frac{d}{d\mu}\lambda(\mu) = ?$$

$$\Gamma[\phi] \supset \int d^4x \left[a_K(\mu)\mathcal{O}_K(\phi) + a_\lambda(\mu)\mathcal{O}_\lambda(\phi) \right]$$
$$\rightarrow \int d^4x \left[\mathcal{O}_K(\phi) + a'_\lambda(\mu)\mathcal{O}_\lambda(\phi) \right]$$

$$\text{RGE}: \qquad \mu \frac{d}{d\mu} a'_\lambda(\mu) = 0$$

Henning, **XL**, and Murayama, "One-loop Matching and Running with Covariant Derivative Expansion," arXiv: 1604.01019

Cohen, Freytsis, and **XL**, "Functional Methods for Heavy Quark Effective Theory," arXiv: 1912.08814

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i c_i \mathcal{O}_i$$

Outline

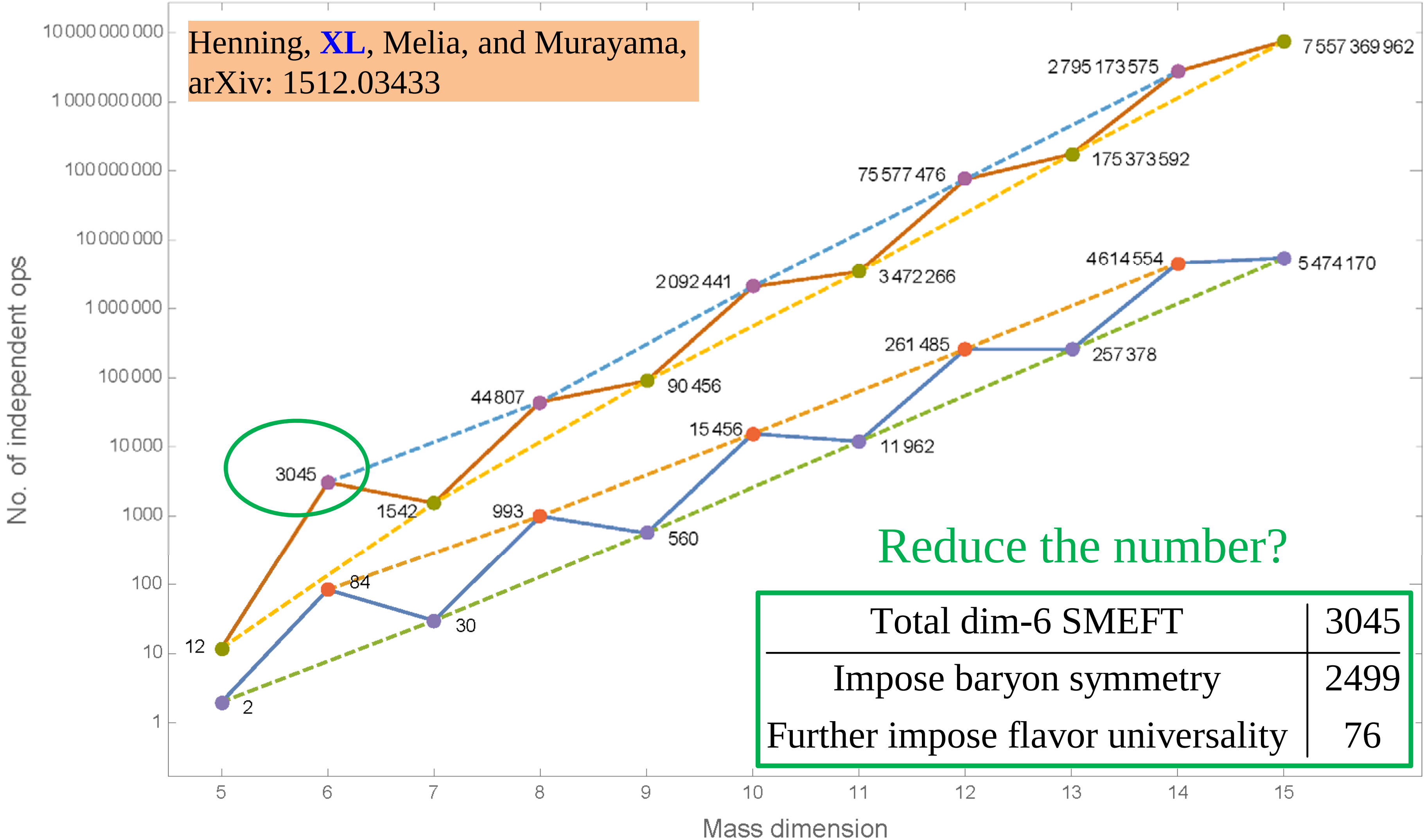
- Implementing the Framework: How to use SMEFT?

✓ --- interpreting experimental limits

✓ --- guide UV model building: Matching and Running

--- additional restrictions to reduce degrees of freedom

Number of SMEFT operators



Reduce the number?

Total dim-6 SMEFT	3045
Impose baryon symmetry	2499
Further impose flavor universality	76

Consider additional restrictions to SMEFT?

- Baryon symmetry
- One generation (flavor symmetry)
- Only bosonic operators (apt for “universal theories”)

Essentially an oblique framework

Consider additional restrictions to SMEFT?

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- One generation (flavor symmetry)
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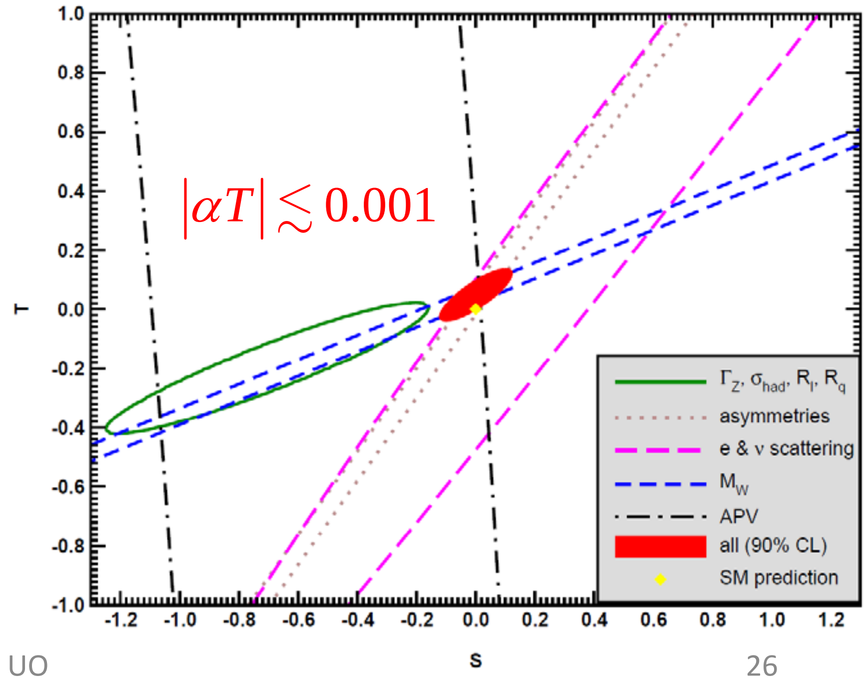
Essentially an oblique framework

- What about beyond oblique?

--- Custodial Symmetry?

Kribs, **XL**, Martin, and Tong, arXiv: 2009.10725

Particle Data Group Collaboration, P. Zyla et al., *PETP* 2020 (2020) no. 8, 083C01.



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Essentially an oblique framework

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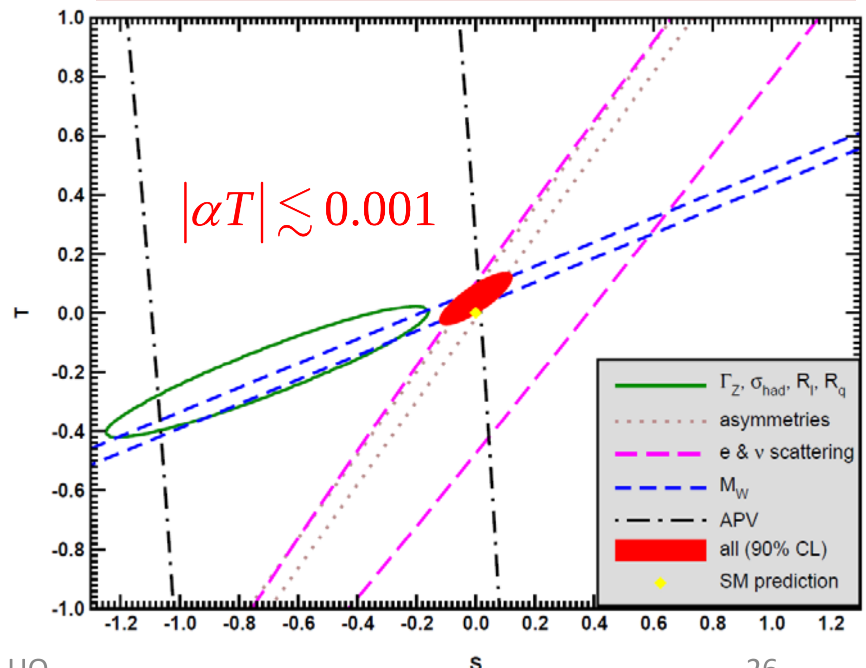
--- Custodial Symmetry?

Kribs, **XL**, Martin, and Tong, arXiv: 2009.10725

$$\alpha T = -\frac{1}{2} v^2 C_{HD}$$

$$\alpha \mathcal{T}_l \equiv \hat{\rho}_*(0) - 1 = -\frac{1}{2} v^2 \left[C_{HD} + 4 C_{Hl}^{(1)} \right]$$

Particle Data Group Collaboration, P. Zyla et al., *PETP* 2020 (2020) no. 8, 083C01.



$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i c_i \mathcal{O}_i$$

Outline

- ✓ Establishing the Framework: What is SMEFT?
  - --- non-renormalizable, defined with a truncation, operator basis
- ✓ Implementing the Framework: How to use SMEFT?
  - --- interpreting experimental limits
  - --- guide UV model building: Matching and Running
  - --- additional restrictions to reduce degrees of freedom
- Re-examining the Framework: Is SMEFT enough?
  - --- SMEFT / HEFT dichotomy
  - --- geometric picture for non-analyticities and unitarity violation
  - --- HEFT describes non-decoupling BSM physics

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i c_i \mathcal{O}_i$$

Outline

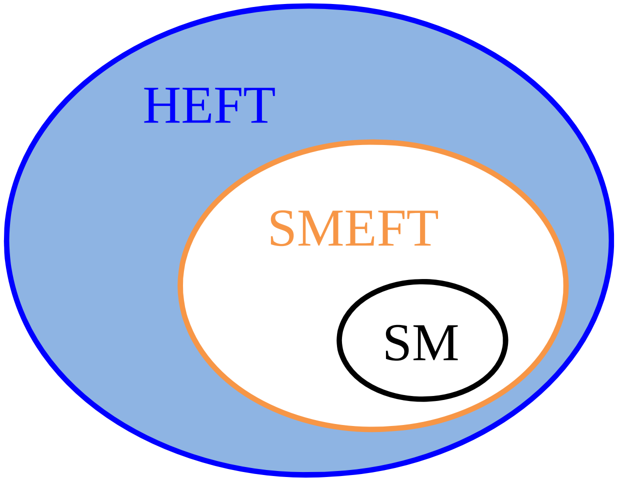
- Re-examining the Framework: Is SMEFT enough?

--- SMEFT / HEFT dichotomy

--- geometric picture for non-analyticities and unitarity violation

--- HEFT describes non-decoupling BSM physics

Is SMEFT enough?



HEFT / Electroweak Chiral Lagrangian

Feruglio, arXiv: hep-ph/9301281

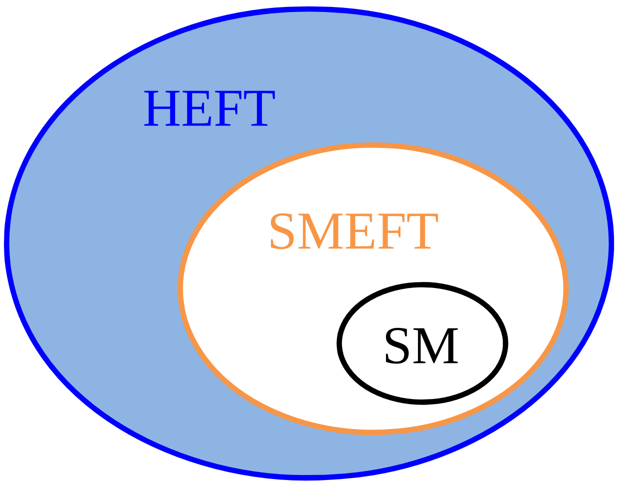
Bagger, Barger, Cheung, Gunion, Han, Ladinsk, Rosenfeld, and Yuan, arXiv: hep-ph/9306256

Koulovassilopoulos and Chivukula, arXiv: hep-ph/9312317

Burgess, Matias, and Pospelo, arXiv: hep-ph/9912459

Grinstein and Trott, arXiv: 0704.1505

Is SMEFT enough?

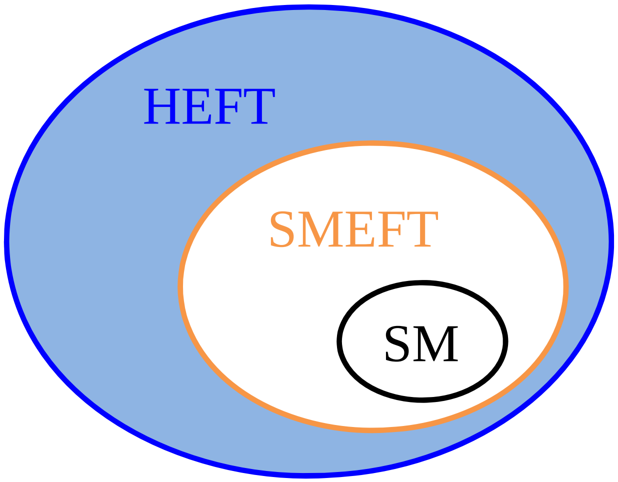


physical Higgs   Goldstones

$$\left\{h\,,\pi^{a}\right\}=H\quad\text{an } SU(2)_L \text{ doublet}$$

$$\mathcal{L}_{\text{SMEFT}}(H)=\mathcal{L}_{\text{SM}}+\frac{C_H}{\Lambda^2}|H|^6+\frac{C_{H\Box}}{\Lambda^2}|H|^2\partial^2|H|^2+\frac{C_R}{\Lambda^2}|H|^2|DH|^2+\cdots$$

Is SMEFT enough?



physical Higgs   Goldstones

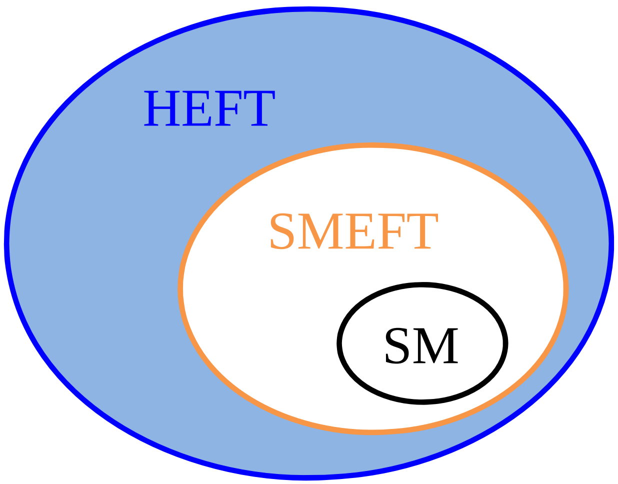
$$\{h, \pi^a\} = H \quad \text{an } SU(2)_L \text{ doublet}$$

$$h \;,\; U \equiv e^{i\pi^a t^a / v} \quad \text{separately}$$

$$\mathcal{L}_{\text{SMEFT}}(H) = \mathcal{L}_{\text{SM}} + \frac{C_H}{\Lambda^2}|H|^6 + \frac{C_{H\Box}}{\Lambda^2}|H|^2 \partial^2 |H|^2 + \frac{C_R}{\Lambda^2}|H|^2|DH|^2 + \cdots$$

$$\mathcal{L}_{\text{HEFT}}(h,U) = \frac{1}{2}\left[K(h)\right]^2 (\partial h)^2 - V(h) + \frac{1}{2}\left[vF(h)\right]^2 \frac{1}{2}\text{tr}\left[\left(D_\mu U\right)^\dagger \left(D^\mu U\right)\right] + \cdots$$

Is SMEFT enough?



physical Higgs   Goldstones

$$\{h\,,\pi^a\} = H \quad \text{an } SU(2)_L \text{ doublet}$$

$$h\ ,\ U \equiv e^{i\pi^a t^a/v} \quad \text{separately}$$

Linear

$$\mathcal{L}_{\text{SMEFT}}(H) = \mathcal{L}_{\text{SM}} + \frac{C_H}{\Lambda^2}|H|^6 + \frac{C_{H\Box}}{\Lambda^2}|H|^2\partial^2|H|^2 + \frac{C_R}{\Lambda^2}|H|^2|DH|^2 + \cdots$$

Nonlinear

$$\mathcal{L}_{\text{HEFT}}(h,U) = \frac{1}{2}[K(h)]^2(\partial h)^2 - V(h) + \frac{1}{2}[vF(h)]^2\frac{1}{2}\text{tr}\left[(D_\mu U)^\dagger(D^\mu U)\right] + \cdots$$

Is SMEFT enough?

$$\text{SMEFT} \quad \Rightarrow \quad \text{HEFT}$$

$$\begin{array}{cc} \tilde{H} & H \end{array}$$

$$\Sigma = \begin{pmatrix} H_2^* & H_1 \\ -H_1^* & H_2 \end{pmatrix} = \frac{1}{\sqrt{2}}(v+h)\,U$$

Is SMEFT enough?

$\text{SMEFT} \Rightarrow \text{HEFT}$ $\qquad$ $\text{HEFT} \Rightarrow \text{SMEFT}$

$$\begin{array}{ccc} & \tilde{H} & H \end{array}$$

$$\Sigma = \begin{pmatrix} H_2^* & H_1 \\ -H_1^* & H_2 \end{pmatrix} = \frac{1}{\sqrt{2}}(v+h)U$$

$$\begin{cases} v+h = \sqrt{2|H|^2} \\ U = \dfrac{1}{\sqrt{|H|^2}}\Sigma \end{cases}$$

Is SMEFT enough?

SMEFT $\Rightarrow$ HEFT

HEFT $\Rightarrow$ SMEFT

$$\begin{array}{cc} \tilde{H} & H \end{array}$$

$$\Sigma = \begin{pmatrix} H_2^* & H_1 \\ -H_1^* & H_2 \end{pmatrix} = \frac{1}{\sqrt{2}}(v+h)U$$

$$\begin{cases} v + h = \sqrt{2|H|^2} \\ U = \dfrac{1}{\sqrt{|H|^2}}\Sigma \end{cases}$$

Non-analyticities

Brivio and Trott, arXiv: 1706.08945

Falkowski and Rattazzi, arXiv: 1902.05936

$$\mathcal{L}_{\text{HEFT}} \supset (\partial h)^2 = \frac{1}{2|H|^2}\left(\partial |H|^2\right)^2$$

Is SMEFT enough?

$$V(H) \propto \frac{1}{4}(v+h)^2 + \frac{1}{4v}(v+h)^3 + \frac{1}{16v^2}(v+h)^4$$

$$= \frac{1}{4}\left(2H^\dagger H\right) + \frac{1}{4v}\left(\sqrt{2H^\dagger H}\right)^3 + \frac{1}{16v^2}\left(2H^\dagger H\right)^2$$

HEFT

$$= \left[\frac{1}{2}(v+h) + \frac{1}{4v}(v+h)^2\right]^2$$

$$= \left(v_1 + h_1\right)^2 = 2H_1^\dagger H_1$$

SMEFT

Field redefinition $h_1 \equiv h + \frac{1}{4v}h^2$

Is SMEFT enough?

$$V(H) \propto \frac{1}{4}(v+h)^2 + \frac{1}{4v}(v+h)^3 + \frac{1}{16v^2}(v+h)^4$$

$$= \frac{1}{4}(2H^\dagger H) + \frac{1}{4v}\left(\sqrt{2H^\dagger H}\right)^3 + \frac{1}{16v^2}(2H^\dagger H)^2$$

HEFT

$$= \left[\frac{1}{2}(v+h) + \frac{1}{4v}(v+h)^2\right]^2$$

$$= (v_1 + h_1)^2 = 2H_1^\dagger H_1$$

SMEFT

Field redefinition $h_1 \equiv h + \frac{1}{4v}h^2$

Alonso, Jenkins, and Manohar: (arXiv: 1605.03602)

One can convert the SMEFT Lagrangian to HEFT form using Eq. (2.11) to switch from Cartesian and polar coordinates. One can attempt to convert from HEFT to SMEFT form using

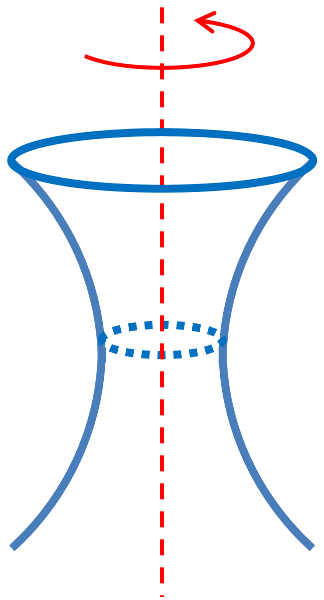
$$\frac{\phi}{(\phi\cdot\phi)^{1/2}} = \boldsymbol{n} \tag{2.30}$$

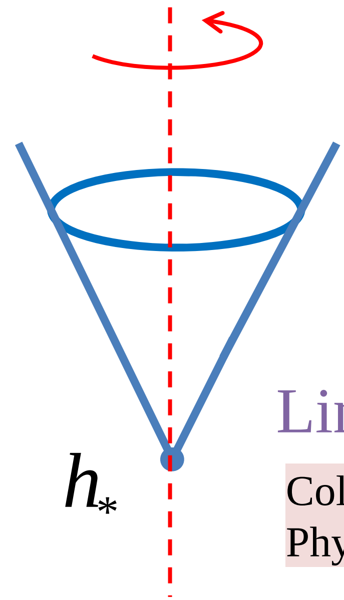
with $(\phi\cdot\phi)^{1/2}$ some function of h. This substitution gives a Lagrangian $L(\phi)$ that need not be analytic in ϕ. However, if there is an $O(4)$ fixed point, then there is a suitable change of variables such that the resulting Lagrangian is analytic in ϕ.

Is SMEFT enough?

$O(4)$ Fixed Point on the Scalar Manifold ?

$$\phi = \left(h\,,\,\pi^a\right)$$





Linearization lemma

Coleman, Wess, and Zumino, Phys. Rev. 177 (1969) 2239

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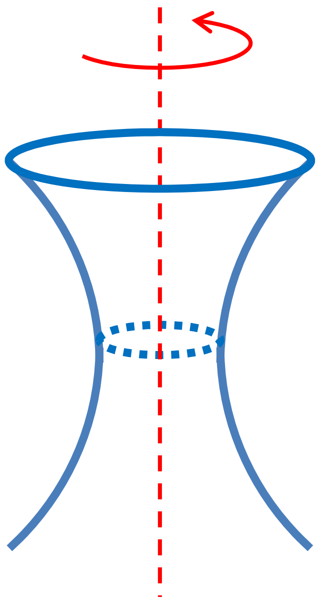

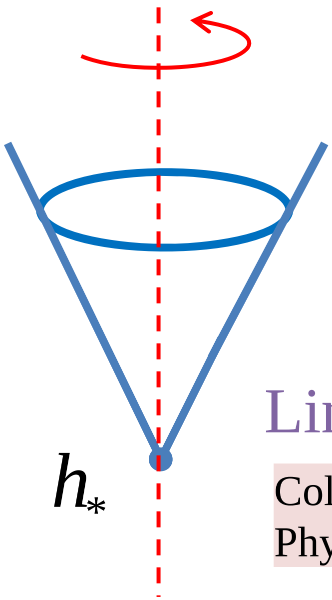

Linearization lemma

Coleman, Wess, and Zumino, Phys. Rev. 177 (1969) 2239

- metric of the Goldstones vanish $g_{\pi\pi}\left(h_*\right) = 0$

Alonso, Jenkins, and Manohar, arXiv: 1605.03602

$$\mathcal{L}_{\text{Kinetic}} = \frac{1}{2} g_{ab}\left(\phi\right)\left(\partial_\mu \phi_a\right)\left(\partial^\mu \phi_b\right)$$

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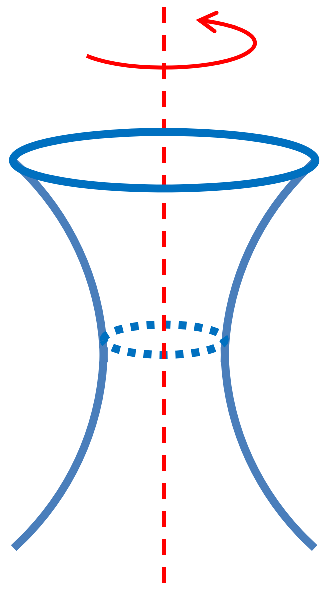

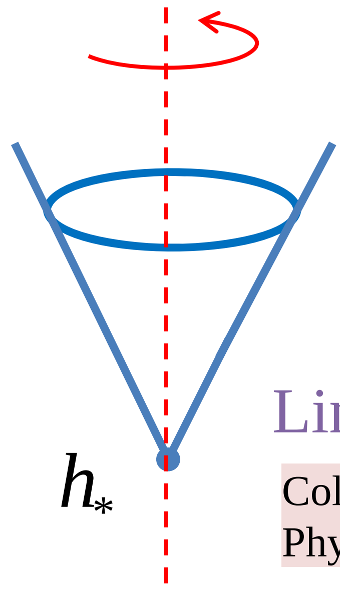

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  Alonso, Jenkins, and Manohar, arXiv: 1605.03602

- geometric invariants finite

  Cohen, Craig, **XL**, and Sutherland, arXiv: 2008.08597

SMEFT vs HEFT dichotomy

Non-analyticity at $H = 0$

Geometric Picture

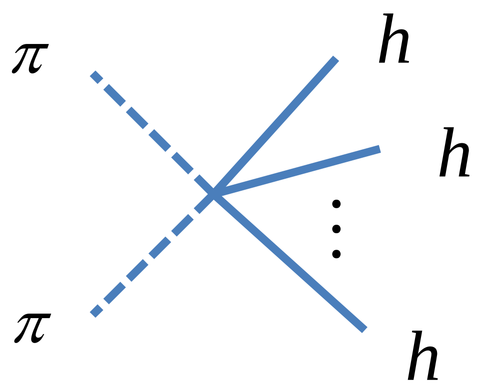
Unitarity Violation at $4\pi v$

Chang and Luty, arXiv: 1902.05556

Falkowski and Rattazzi, arXiv: 1902.05936

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} \pi_1 + i\pi_2 \\ v + h + i\pi_3 \end{pmatrix}$$

$$\sqrt{2|H|^2} = \sqrt{(v+h)^2 + \vec{\pi}^2} = (v+h) + \frac{1}{2(v+h)}\vec{\pi}^2 + \mathcal{O}\left(\vec{\pi}^4\right) \supset \pi\pi\frac{1}{(-v)^n}h^n$$



$$\mathcal{A}\left(\pi\pi \to h^n\right) \sim \frac{n!}{v^n}$$

SMEFT vs HEFT dichotomy

Non-analyticity at $H = 0$ $\longrightarrow$ Unitarity Violation at $4\pi v$

$\nwarrow$

Geometric Picture

SMEFT vs HEFT dichotomy

Non-analyticity at $H = 0$ $\longrightarrow$ Unitarity Violation at $4\pi v$

?

Cohen, Craig, **XL**, and Sutherland, arXiv: 2108.03240

Geometric Picture

Cohen, Craig, **XL**, and Sutherland, arXiv: 2108.03240

Geometric Picture for Unitarity Violation

$$\mathcal{A}\left(\pi\pi \to h^n\right) \supset E_{\text{cm}}^2 \left(\partial_h^{n-4} \mathcal{K}_h\right)\Big|_{h=0}$$

$$R_{\pi hh\pi} = -g_{hh} g_{\pi\pi} \mathcal{K}_h$$

Cohen, Craig, **XL**, and Sutherland, arXiv: 2108.03240

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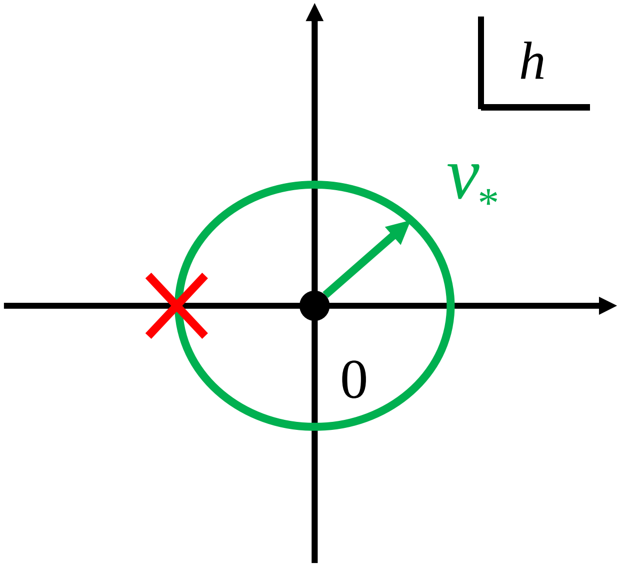

$$\left|\partial_h^n \mathcal{K}_h\right| \sim \frac{n!}{v_*^n}$$

Cauchy-Hadamard theorem

Unitarity Violation at $4\pi v_*$

Is SMEFT enough?

Cohen, Craig, **XL**, and Sutherland, arXiv: 2108.03240

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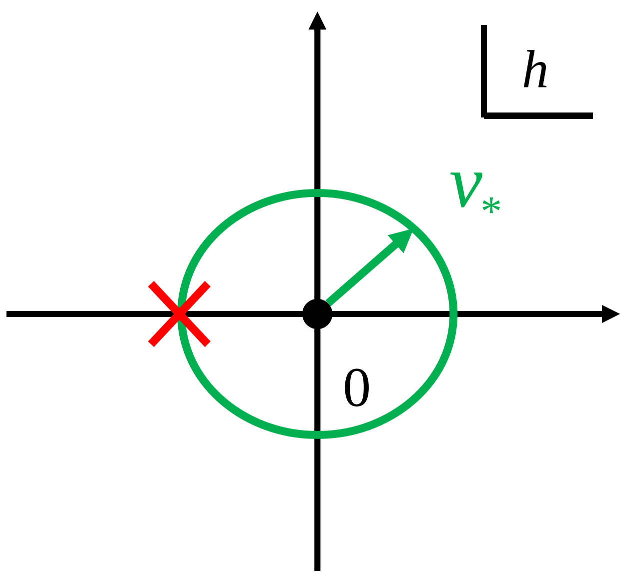
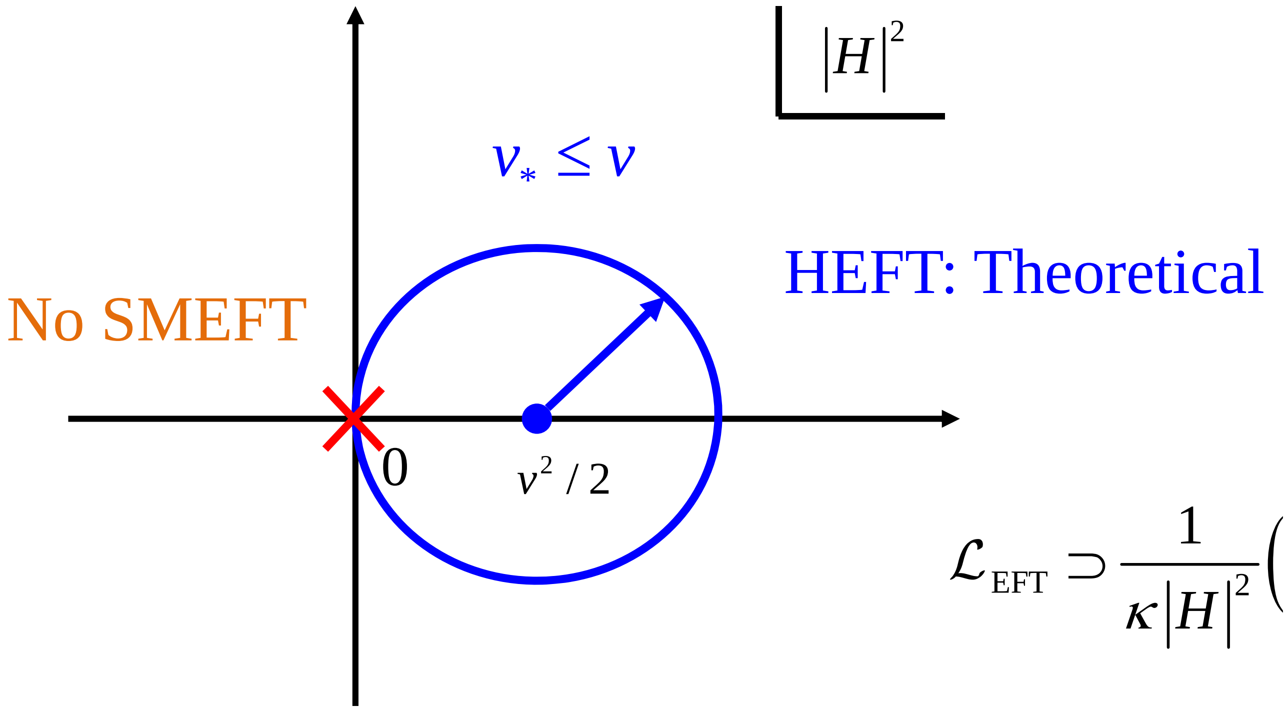


$$\left|\partial_h^n \mathcal{K}_h\right| \sim \frac{n!}{v_*^n}$$ Cauchy-Hadamard theorem

Unitarity Violation at $4\pi v_*$

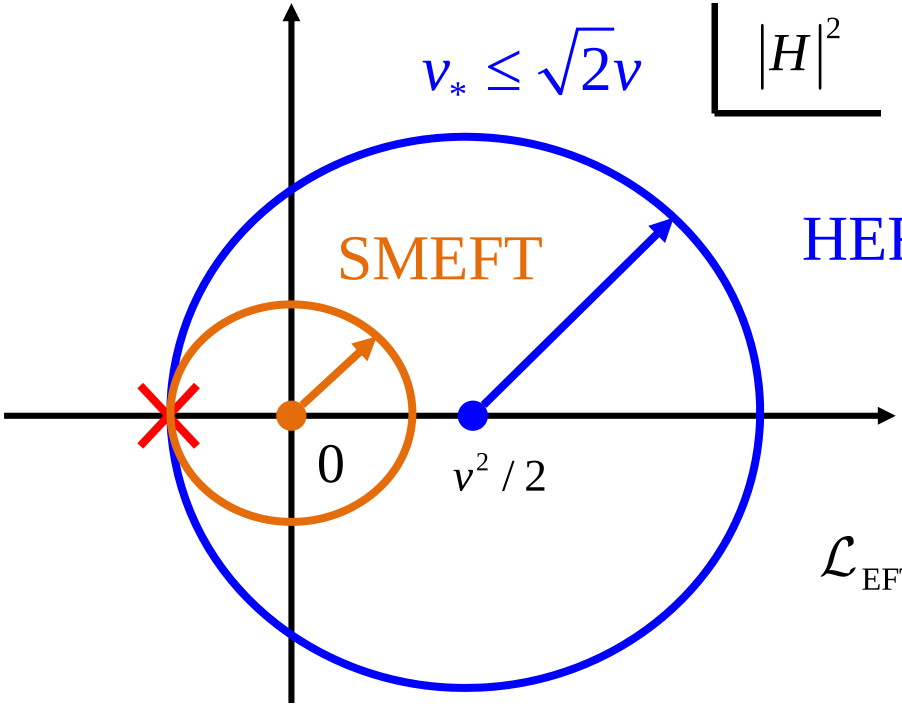
$$H = 0 \;\left(h = -v\right) \;\Rightarrow\; v_* = v$$

Is SMEFT enough?



$$\mathcal{L}_{\mathrm{EFT}} \supset \frac{1}{\kappa |H|^2} \left(\partial |H|^2 \right)^2$$

Is SMEFT enough?



HEFT: Practical

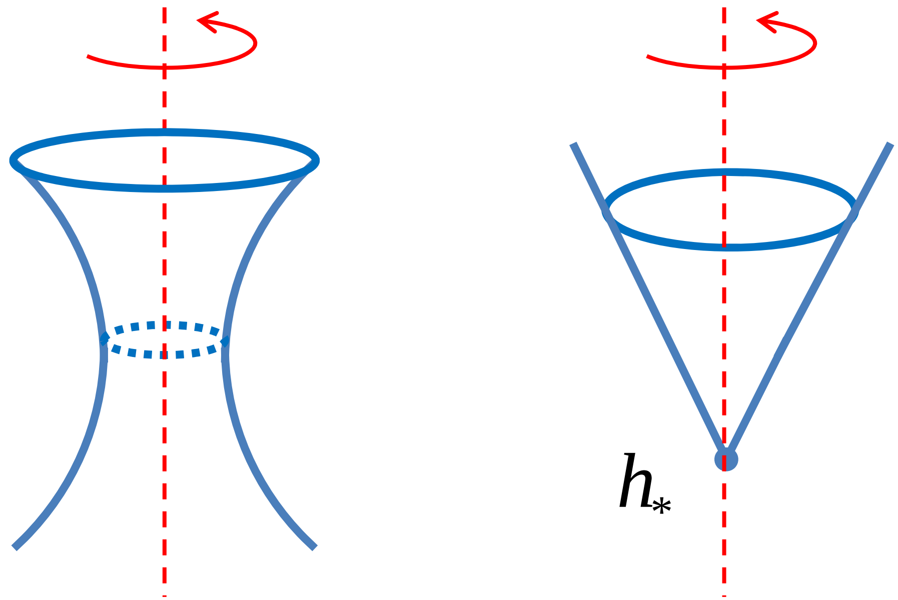
$$\mathcal{L}_{\text{EFT}} \supset \frac{1}{M^2 + \kappa |H|^2} \left(\partial |H|^2 \right)^2$$

HEFT describes non-decoupling BSM physics

Is SMEFT enough?

Cohen, Craig, **XL**, and Sutherland, arXiv: 2008.08597

UV theories that will generate HEFT?



$g_{\pi\pi}(h) \neq 0$

$g_{\pi\pi}(h_*) = 0 \quad , \quad R(h_*) = \infty$

Extra EWSB

BSM particle mass fully from EWSB

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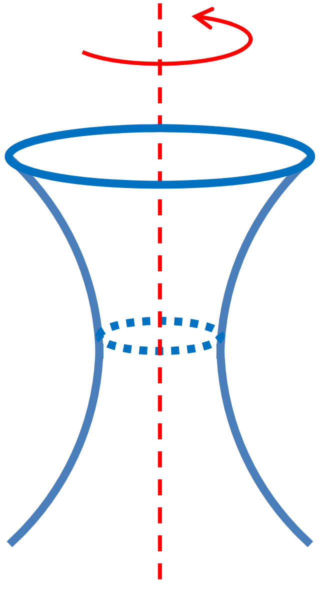

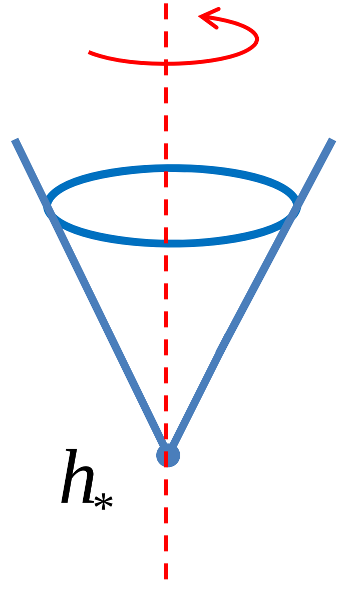

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more than ½ for practical HEFT

Cohen, Craig, **XL**, and Sutherland, arXiv: 2008.08597

Technique for Matching to all orders in fields

what we need

$$\mathcal{L}_{\text{EFT}} \supset \frac{1}{M^2+\kappa|H|^2}\left(\partial|H|^2\right)^2$$

usual truncated matching

$$= \frac{1}{M^2}\left(\partial|H|^2\right)^2 - \frac{\kappa|H|^2}{M^4}\left(\partial|H|^2\right)^2 + \frac{\kappa^2|H|^4}{M^6}\left(\partial|H|^2\right)^2 + \cdots$$

dim-6 dim-8 dim-10

Is SMEFT enough?

Cohen, Craig, **XL**, and Sutherland, arXiv: 2008.08597

Technique for Matching to all orders in fields

Generalizing Coleman-Weinberg Potential

$$\mathcal{L}_{\mathrm{UV}}\left[\phi,\Phi\right] = -\frac{1}{2}\Phi\left[\partial^2 + M^2 + U\left(\phi\right)\right]\Phi$$

$$\int \mathrm{d}^4x\, \mathcal{L}_{\mathrm{EFT}}^{(\text{1-loop})} = \frac{i}{2}\,\mathrm{STr}\log\left[\left.-\frac{\delta^2 S_{\mathrm{UV}}}{\delta\left(\phi,\Phi\right)^2}\right|_{\Phi=\Phi_c}\right]\Bigg|_{\mathrm{hard}} = \frac{i}{2}\,\mathrm{Tr}\log\left(\partial^2 + M^2 + U\right)$$

Cohen, Craig, **XL**, and Sutherland, arXiv: 2008.08597

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$$= \frac{i}{2}\int d^4x \int \frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\left[\ln\left(-p^2 + M^2 + U\right)\right]$$

$$= \frac{1}{2}\int d^4x \frac{1}{16\pi^2}\,\mathrm{tr}\left[\frac{1}{2}\left(M^2+U\right)^2\left(\ln\frac{\mu^2}{M^2+U} + \frac{3}{2}\right)\right]$$

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$$=\frac{i}{2}\int d^4x\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}\left[\ln\left(-p^2+M^2+U\right)+\frac{1}{6\left(p^2-M^2-U\right)^3}(\partial U)^2+\mathcal{O}\left(\partial^4\right)\right]$$

$$=\frac{1}{2}\int d^4x\frac{1}{16\pi^2}\mathrm{tr}\left[\frac{1}{2}\left(M^2+U\right)^2\left(\ln\frac{\mu^2}{M^2+U}+\frac{3}{2}\right)+\frac{1}{12}\frac{1}{M^2+U}(\partial U)^2+\mathcal{O}\left(\partial^4\right)\right]$$

Is SMEFT enough?

Cohen, Craig, **XL**, and Sutherland, arXiv: 2008.08597

A heavy singlet at one-loop level

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} - \frac{1}{2} S\left(\partial^2 + M^2 + \kappa |H|^2\right) S \quad , \quad m_S^2 = M^2 + \frac{1}{2}\kappa v^2$$

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$$R = \frac{1}{16\pi^2}\left[\frac{1}{2K^4}\frac{\kappa^2 M^2}{\left(M^2 + \kappa |H|^2\right)^2} + \frac{1}{2K^2}\frac{\kappa^2}{M^2 + \kappa |H|^2}\right] \quad , \quad K^2 = 1 + \frac{1}{96\pi^2}\frac{\kappa^2 |H|^2}{M^2 + \kappa |H|^2}$$

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$M^2 = 0$, HEFT : theoretical

$M^2 < \frac{1}{2}\kappa v^2$, HEFT : practical

Is SMEFT enough?

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$M^2 = 0$, HEFT : theoretical

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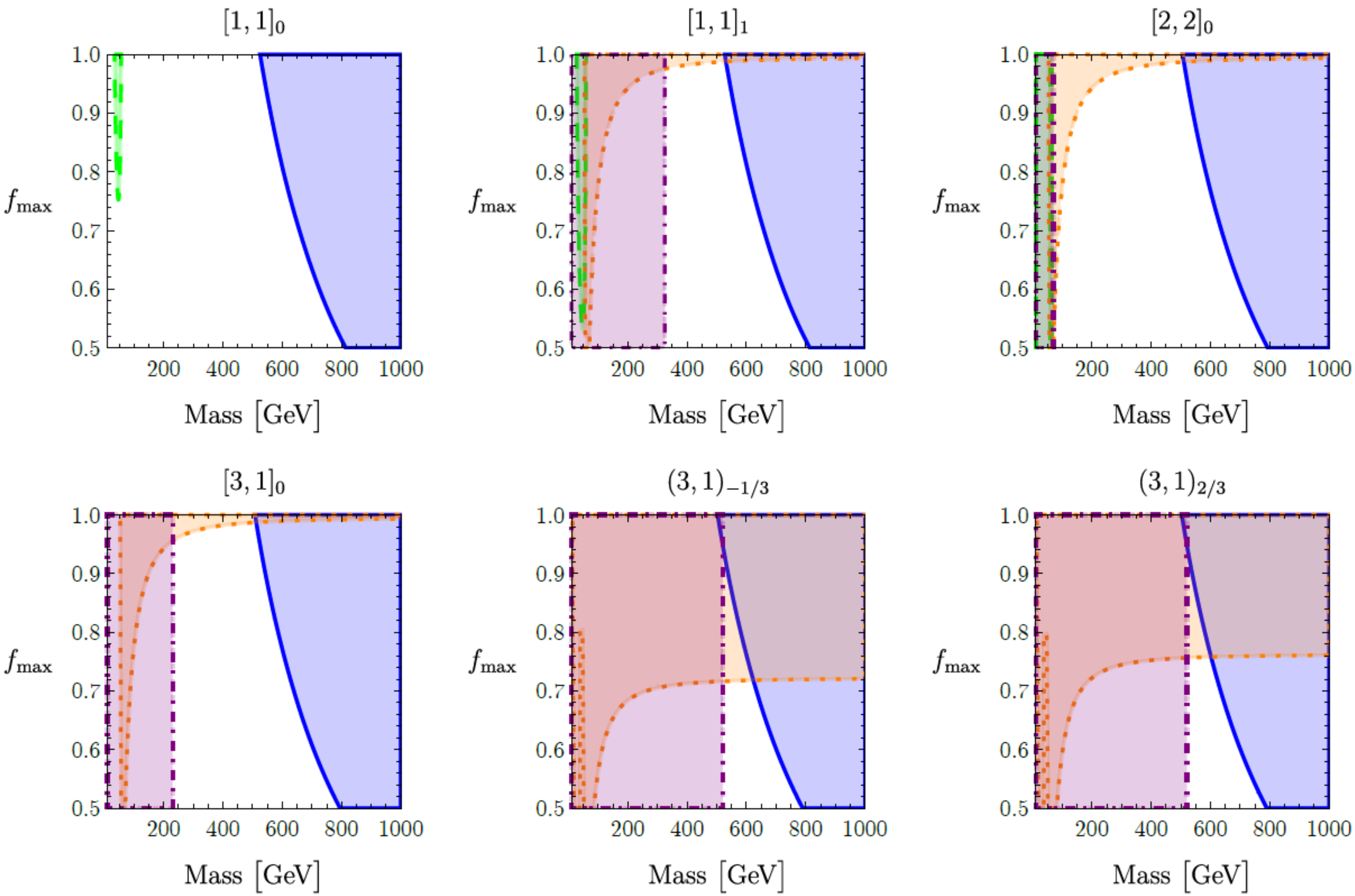
Banta, Cohen, Craig, **XL**, and Sutherland, arXiv: 2110.02967

$$f \equiv \frac{\kappa v^2 / 2}{m_S^2} > \frac{1}{2}$$ “Loryons”

Is SMEFT enough?

Banta, Cohen, Craig, **XL**, and Sutherland, arXiv: 2110.02967

Viable Scalar Loryons: $[L\,,R]_Y$, $(C\,,L)_Y$



unitarity

direct

$h\gamma\gamma$, hgg

h width

Summary

- SMEFT $\supset$ SM is a low-energy approximation of BSM physics

  --- robust parameterization --- non-renormalizable, need truncation

  --- operator basis Connection with amplitude approach, on-shell methods?

- To interpret experimental limits with SMEFT, we need to be cautious

  --- validity at LHC could be salvaged by PDFs More realistic cases?

- To guide UV model building with SMEFT, we need Matching and Running

  --- advantages of functional methods Beyond 1-loop? Anomaly matching?

- Additional restrictions to reduce SMEFT would be ideal for practical use

  Custodial symmetry? Other options? More systematically?

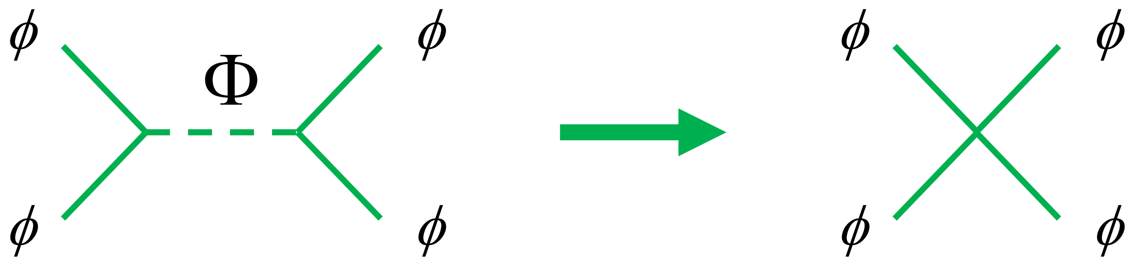
- SMEFT is not enough; HEFT is needed for certain non-decoupling physics

  --- geometry to address field redefinitions Field redefinitions with derivatives?

  --- non-decoupling BSM particles (“Loryons”) New searches?

What could go wrong when $\sqrt{\hat{s}} > M$: A naive toy case

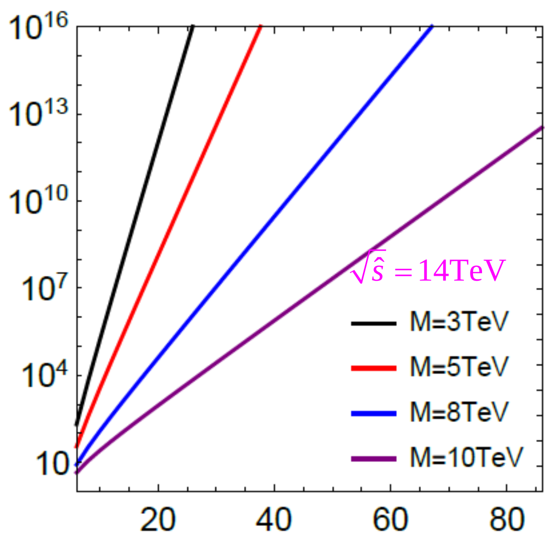
Cohen, Doss, and **XL**, arXiv: 2110.XXXXX



$$\mathcal{A}_{\rm UV} \sim \frac{1}{\hat{s} - M^2} \qquad \mathcal{A}_{\rm EFT} \sim -\frac{1}{M^2}\left[1 + \frac{\hat{s}}{M^2} + \left(\frac{\hat{s}}{M^2}\right)^2 + \left(\frac{\hat{s}}{M^2}\right)^3 + \cdots\right]$$

Not converging if $\sqrt{\hat{s}} \geq M$

$$\left|\frac{\sigma_{\rm EFT}^{[k]}}{\sigma_{\rm UV}} - 1\right|$$



$\dim = 6 + 2k$

$$\sigma_{\rm EFT}^{[k]} \equiv \sum_{r=0}^{k} \sigma_{\rm EFT}^{(r)} \overset{?}{\simeq} \sigma_{\rm UV}$$

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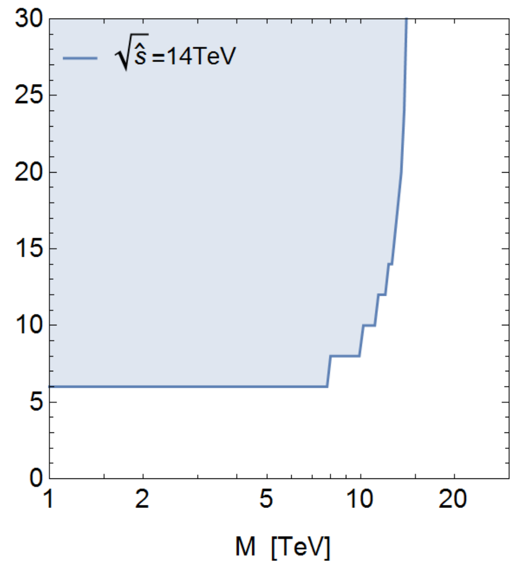
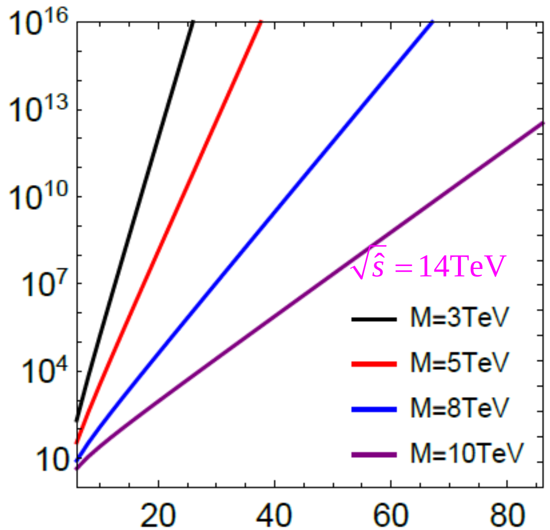


Unitarity Bound

$$\left| \frac{\sigma_{\text{EFT}}^{[k]}}{\sigma_{\text{UV}}} - 1 \right|$$



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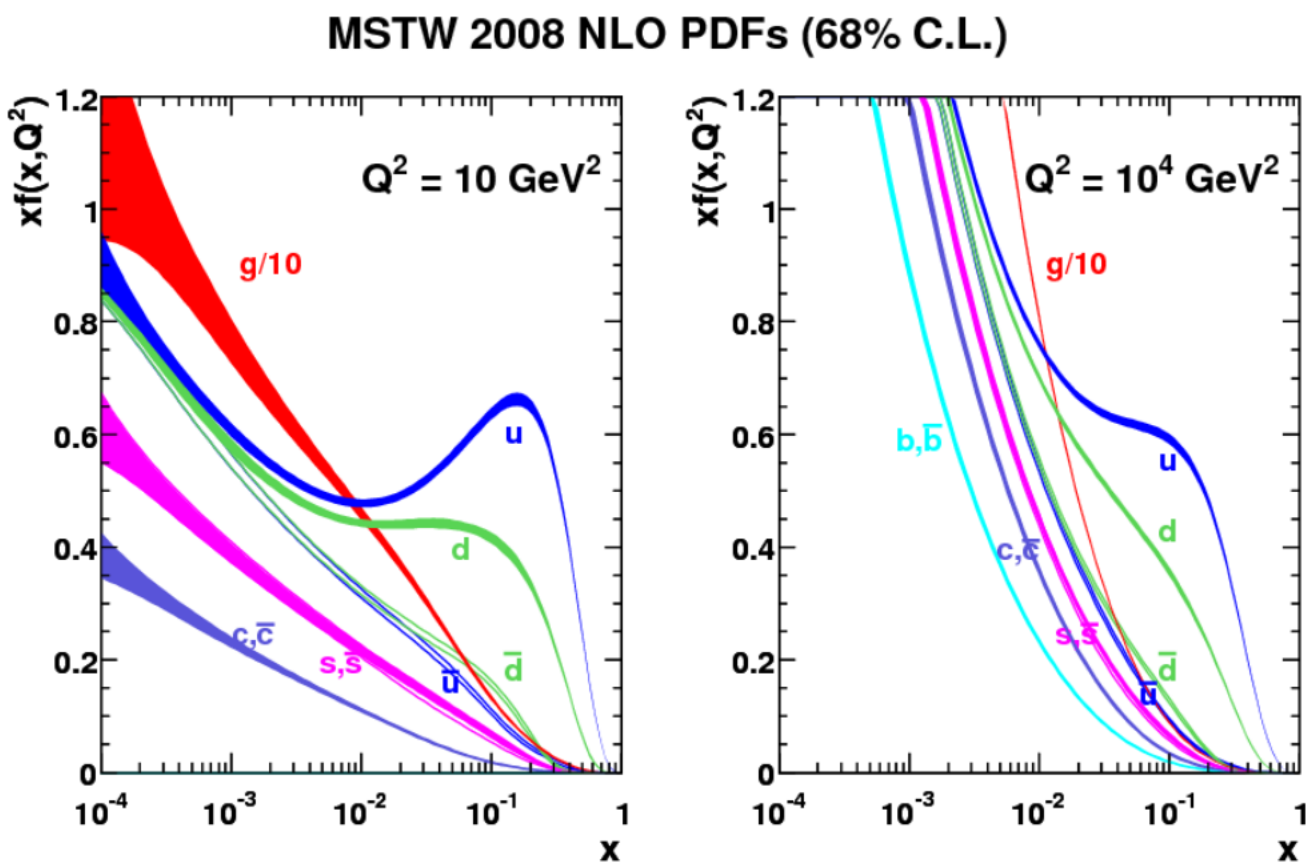
$\dim = 6 + 2k$

Backup: How to use SMEFT?

For an inclusive enough search, we only know $\sqrt{s} > M$: $\sqrt{\hat{s}} \ll \sqrt{s}$



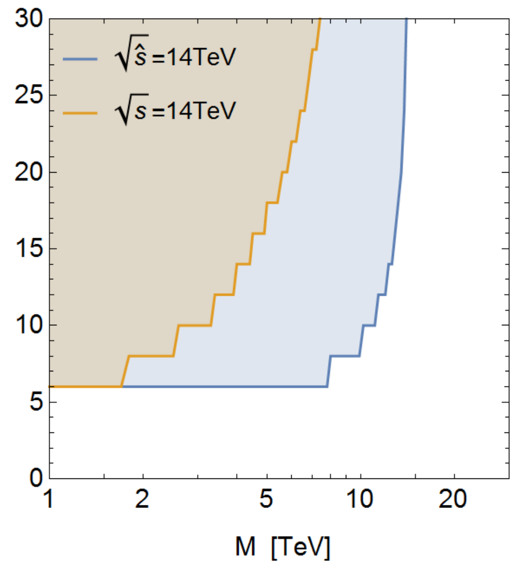
$$\mathcal{A}_{\text{EFT}}^{(r)} \sim \left(\frac{\hat{s}}{M^2}\right)^r$$

Backup: How to use SMEFT?

Cohen, Doss, and **XL**, arXiv: 2110.XXXXX

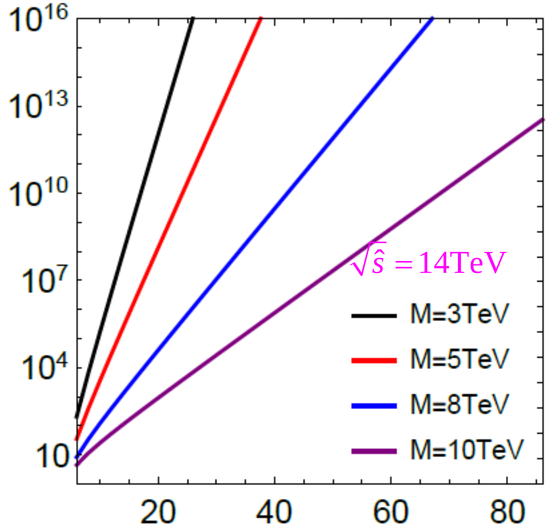
PDF effects on the naïve toy case:

$\dim = 6 + 2k$



Unitarity Bound

$$\left|\frac{\sigma_{\mathrm{EFT}}^{[k]}}{\sigma_{\mathrm{UV}}} - 1\right|$$



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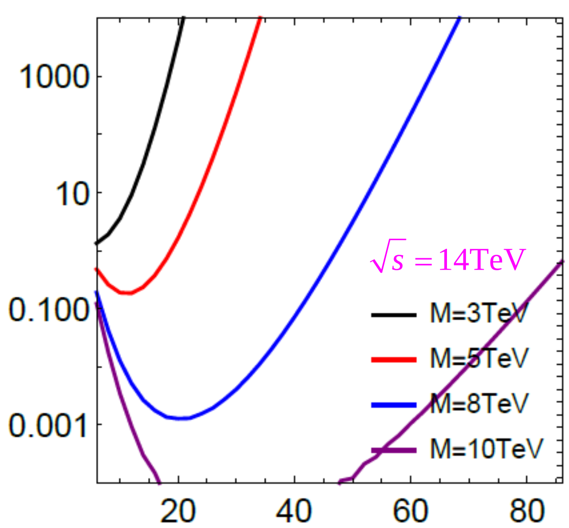

$\dim = 6 + 2k$

A closely related work:

Fukaya, arXiv: 2109.11147

Anomaly Matching

Cohen, **XL**, and Zhang, in progress

$$\frac{i}{2}\,\mathrm{STr}\log K\Big|_{\text{hard}} = \frac{i}{2}\log \mathrm{S}\det\underline{\left(i\not{D}-M\right)}$$

has zero modes

$$\left\langle \partial_\mu\left(\bar{\psi}\gamma^\mu\gamma^5\psi\right) - 2iM\bar{\psi}\gamma^5\psi\right\rangle_A = \left.\frac{\delta J[\alpha]}{\delta\alpha}\right|_{\alpha=0} = \frac{1}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\left(\left[iD_\mu, iD_\nu\right]\left[iD_\rho, iD_\sigma\right]\right)$$

$$J[\alpha] = i\,\mathrm{Tr}\log\left[e^{-i\alpha\gamma^5}\left(i\not{D}-M\right)e^{-i\alpha\gamma^5}\right]$$

$$\supset \mathrm{Tr}\left\{\frac{1}{i\not{D}-M}\left[\gamma^5\alpha\left(i\not{D}-M\right)+\left(i\not{D}-M\right)\gamma^5\alpha\right]\right\} \quad \rightarrow \quad \mathrm{Tr}\left(2\gamma^5\alpha\right)$$

$$= \mathrm{Tr}\left\{\frac{1}{i\not{D}-M}\left[-2M\gamma^5\alpha + \gamma^5\left(-i\not{D}\alpha\right)_x\right]\right\}$$

$$= \int d^dx\left\{\alpha(x)\frac{1}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{tr}\left(\left[iD_\mu, iD_\nu\right]\left[iD_\rho, iD_\sigma\right]\right)\right\}$$

Backup: How to use SMEFT?

Kribs, **XL**, Martin, and Tong,
arXiv: 2009.10725

$$\hat{\rho}_{\text{Veltman}} = 1 + \frac{\alpha}{c_{2\theta}}\left(-\frac{1}{2}S + c_\theta^2 T + \frac{c_{2\theta}}{4s_\theta^2}U\right)$$

$$\hat{\rho}_*(0) \sim \frac{\mathcal{M}_{\text{NC}}(0)}{\mathcal{M}_{\text{CC}}(0)}$$

wrong

No longer observable!

$$\alpha T \equiv \frac{\Pi_{WW}(0) - \Pi_{33}(0)}{m_W^2} \sim 0 \quad \Leftrightarrow \quad \rho = \boxed{\frac{m_W^2}{m_Z^2\cos^2\theta} = 1 + \alpha\, T} \sim 1$$

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$$\boxed{\frac{m_W^2}{m_Z^2} - c_0^2 = \frac{\alpha c^2}{c^2 - s^2}\left[-\frac{1}{2}S + c^2 T + \frac{c^2 - s^2}{4s^2}U\right],}$$

$$s_*^2(q^2) - s_0^2 = \frac{\alpha}{c^2 - s^2}\left(\tfrac{1}{4}S - s^2c^2T\right),$$

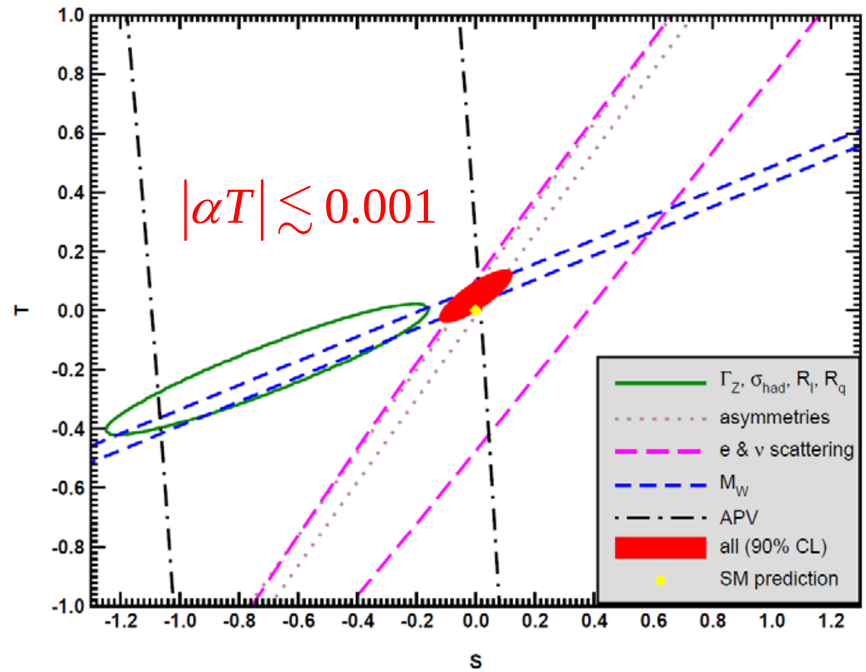
$$\boxed{\rho_*(0) - 1 = \alpha T\ ,} \qquad (3.13)$$

$$Z_{Z*}(q^2) - 1 = \frac{\alpha}{4s^2c^2}S\ .$$

$$Z_{W*}(q^2) - 1 = \frac{\alpha}{4s^2}(S + U)\ .$$

Peskin and Takeuchi, Phys. Rev. D 46 (1992) 381

$|\alpha T| \lesssim 0.001$



Global Fitting Results

Ellis, Madigan, Mimasu, Sanz, and You, arXiv: 2012.02779

	Individual			Marginalised		
SMEFT Coeff.	Best fit $[\Lambda = 1$ TeV$]$	95% CL range	Scale $\frac{\Lambda}{\sqrt{C}}$ [TeV]	Best fit $[\Lambda = 1$ TeV$]$	95% CL range	Scale $\frac{\Lambda}{\sqrt{C}}$ [TeV]
C_{HWB}	0.00	[-0.0043, +0.0026]	17.0	0.18	[-0.36, +0.73]	1.4
C_{HD}	-0.01	[-0.023, +0.0027]	8.8	-0.39	[-1.6, +0.81]	0.91
C_{ll}	0.01	[-0.005, +0.019]	9.2	-0.03	[-0.084, +0.02]	4.4
$C_{Hl}^{(3)}$	0.00	[-0.01, +0.003]	12.0	-0.03	[-0.13, +0.055]	3.3
$C_{Hl}^{(1)}$	0.00	[-0.0044, +0.013]	11.0	0.11	[-0.19, +0.41]	1.8
$C_{H[illegible]}$	0.00	[-0.015, +0.0071]	9.6	0.19	[-0.41, +0.79]	1.3

$$\alpha T = -\frac{1}{2} v^2 C_{HD}$$

$$\alpha \mathscr{T}_l \equiv \hat{\rho}_*(0) - 1 = -\frac{1}{2} v^2 \left[C_{HD} + 4 C_{Hl}^{(1)} \right] = -2\hat{c}_{HL}$$

Falkowski and Riva, arXiv: 1411.0669

Kribs, **XL**, Martin, and Tong, arXiv: 2009.10725

$$\frac{\Lambda}{\sqrt{C_{HD} + 4C_{Hl}^{(1)}}} \gtrsim \quad 4.7 \quad , \quad 2.4 \quad \text{TeV}$$