Generation of Angular Momentum in Fission

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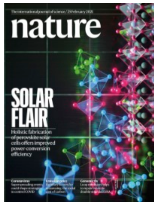
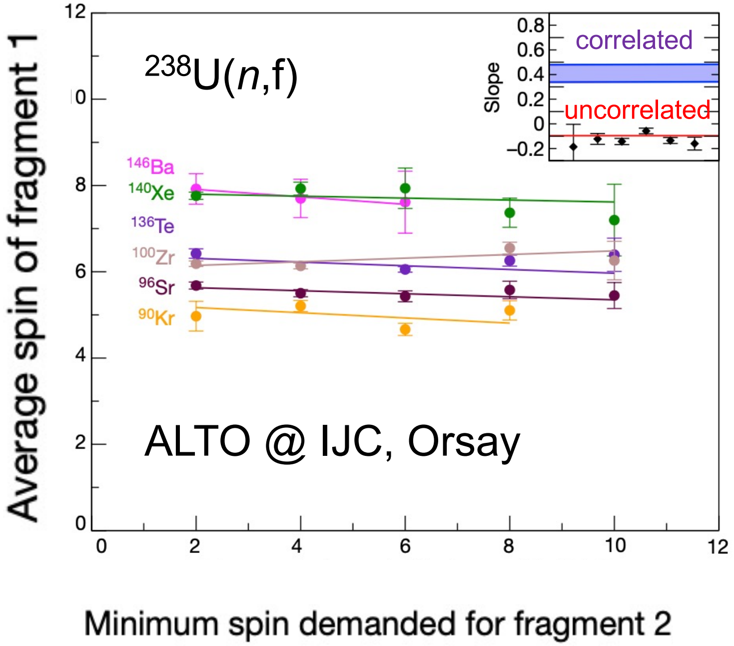
Recent experimental information on spin correlations

Angular momentum generation in nuclear fission,
J. N. Wilson *et al.,* Nature **590** (2021) 566





OBSERVATION:

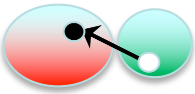
``There is no significant correlation between the spins of the fragments"

INTERPRETATION:

Therefore ``the fragment spins are generated after the nucleus splits", i.e. ``after the fragments have become two separate, independent systems"

Mechanism of fragment spin generation: Nucleon exchange in the dinucleus

Relevant theory of nucleon exchange



Damped heavy-ion collisions, W.U. Schröder and J.R. Huizenga, Ann. Rev. Nucl. Sci. (1977) 465

Intimate relationship between nucleon exchange and energy dissipation

Theory of transfer-induced transport in nuclear collisions, J. Randrup, Nucl. Phys. **A327** (1979) 490:

Each transfer changes the nucleon numbers and the excitation energies of the fragments, as well as their linear & angular momenta

Transport of angular momentum in damped nuclear reactions, J. Randrup, Nucl. Phys. **A383** (1982) 468:

Mobility (friction) tensor: anisotropic

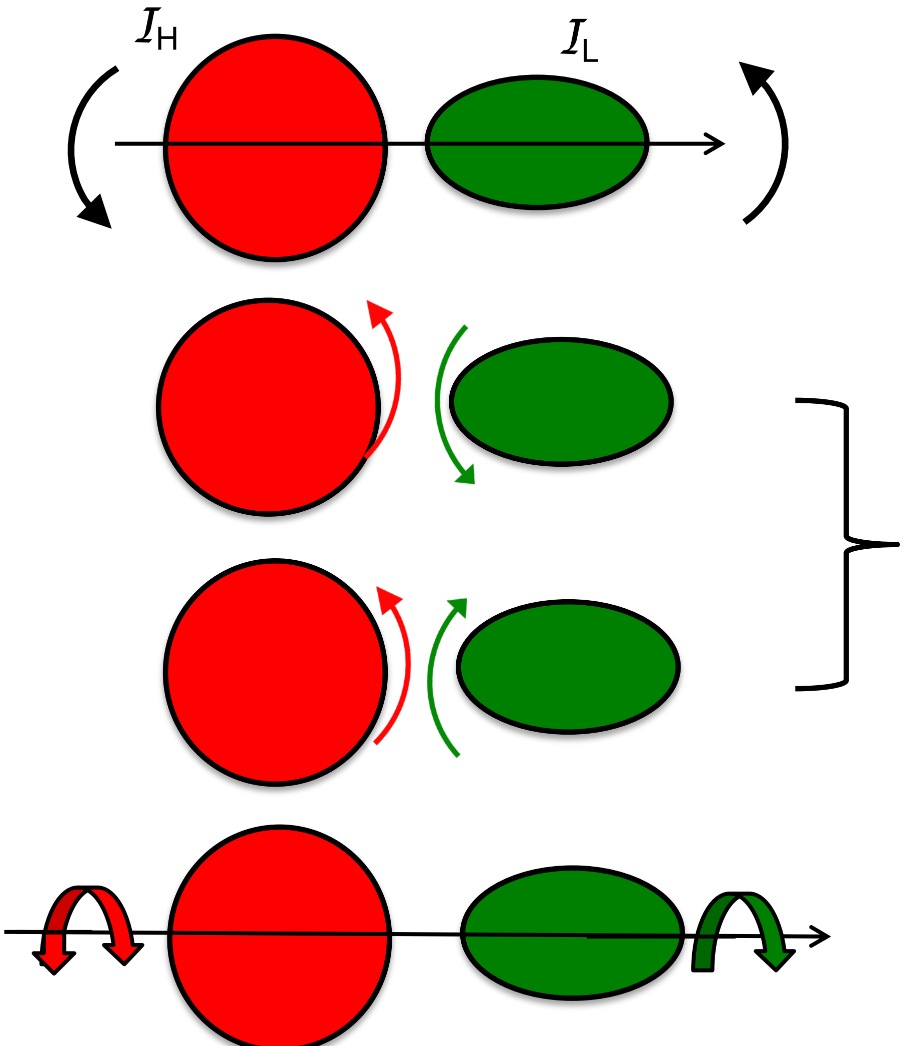
*Dynamical evolution of angular momentum in damped nuclear reac*tions, T. Døssing and J. Randrup, Nucl. Phys. **A433** (1985) 215:

Relaxation times $t_{\text{wriggling}} \ll t_{\text{bending}}$ & $t_{\text{twisting}} \lll t_{\text{tilting}}$

fast slow

Normal rotational modes of a dinucleus at scission

Studies in the liquid-drop theory of nuclear fission, J.R. Nix and W.J. Swiatecki, NP **71** (1965) 1



Rigid rotation:

Wriggling (2):

Bending (2):

perpendicular
to the fission axis
(doubly degenerate)

Tilting & twisting:

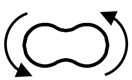
parallel to the fission axis

Diagonalization of the rotational energy in fission

All rotational modes of the dinucleus are assumed to be in thermal equilibrium

Angular momentum conservation leaves six independent rotational modes

$$E_{\rm rot} = \boldsymbol{S}_A^2/2\mathcal{I}_{\rm A} + \boldsymbol{S}_B^2/2\mathcal{I}_{\rm B} + \boldsymbol{L}^2/2\mathcal{I}_{\rm R}$$

$E_{\rm rot} = \boldsymbol{S}_0^2/2\mathcal{I}_0$  rigid rotation around

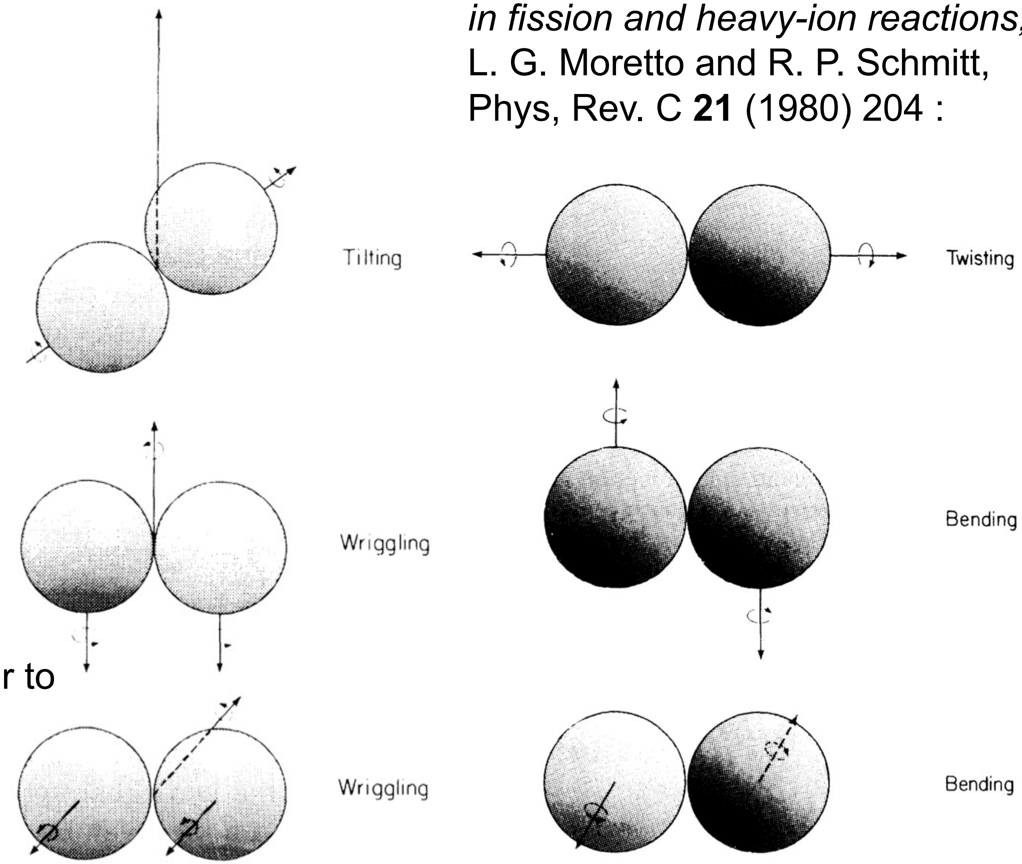
$+\ \boldsymbol{s}_{\rm wrig}^2/2\mathcal{I}_{\rm wrig}$

$+\ \boldsymbol{s}_{\rm bend}^2/2\mathcal{I}_{\rm bend}$ } perpendicular to

$+\ s_{\rm twist}^2/2\mathcal{I}_{\rm twist}$

$+\ s_{\rm tilt}^2/2\mathcal{I}_{\rm tilt}$ } parallel to fission axis

Equilibrium statistical treatment of angular momenta associated with collective modes in fission and heavy-ion reactions,
L. G. Moretto and R. P. Schmitt,
Phys, Rev. C **21** (1980) 204 :

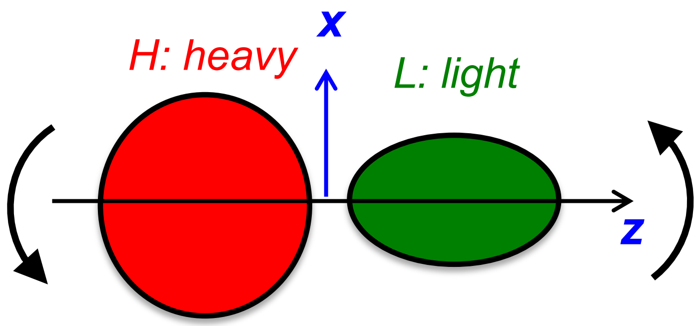


Positive modes:
Spins are *parallel*
$\boldsymbol{S}_{\rm A} \parallel \boldsymbol{S}_B$

Negative modes:
Spins are *opposite*
$\boldsymbol{S}_{\rm A} + \boldsymbol{S}_B = \boldsymbol{0}$

The fragment spins are determined at scission

Average:
Rigid rotation:



$\mathbf{z} = \mathbf{R}/R$

$\mathbf{y} = \mathbf{S}_0/S_0$

$\mathbf{x} = \mathbf{y} \times \mathbf{z}$

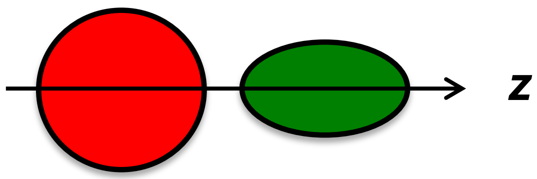
Individual fragments: $\underline{\mathbf{S}}_\mathrm{L} = (\mathcal{I}_\mathrm{L}/\mathcal{I}_\mathrm{tot})\,\mathbf{S}_0$ $\quad \underline{\mathbf{S}}_\mathrm{H} = (\mathcal{I}_\mathrm{H}/\mathcal{I}_\mathrm{tot})\,\mathbf{S}_0$

Relative motion: $\underline{\mathbf{L}} = (\mathcal{I}_\mathrm{rel}/\mathcal{I}_\mathrm{tot})\,\mathbf{S}_0 = \mathbf{R} \times \mathbf{P}$

Conservation:

$\underline{\mathbf{S}}_\mathrm{L} + \underline{\mathbf{S}}_\mathrm{H} + \underline{\mathbf{L}} = \mathbf{S}_0$

Fluctuations:
wriggling (s_+) &
bending (s_-)



Sample fluctuations of rotational modes

$\mathbf{s}_\pm = (s_\pm^x, s_\pm^y, 0)$:
$P(\mathbf{s}_\pm) \sim \exp(-\mathbf{s}_\pm^2/2\mathcal{I}_\pm T_\mathrm{S})$

Spin temperature: $T_\mathrm{S} = c_\mathrm{S} T_\mathrm{sciss}$

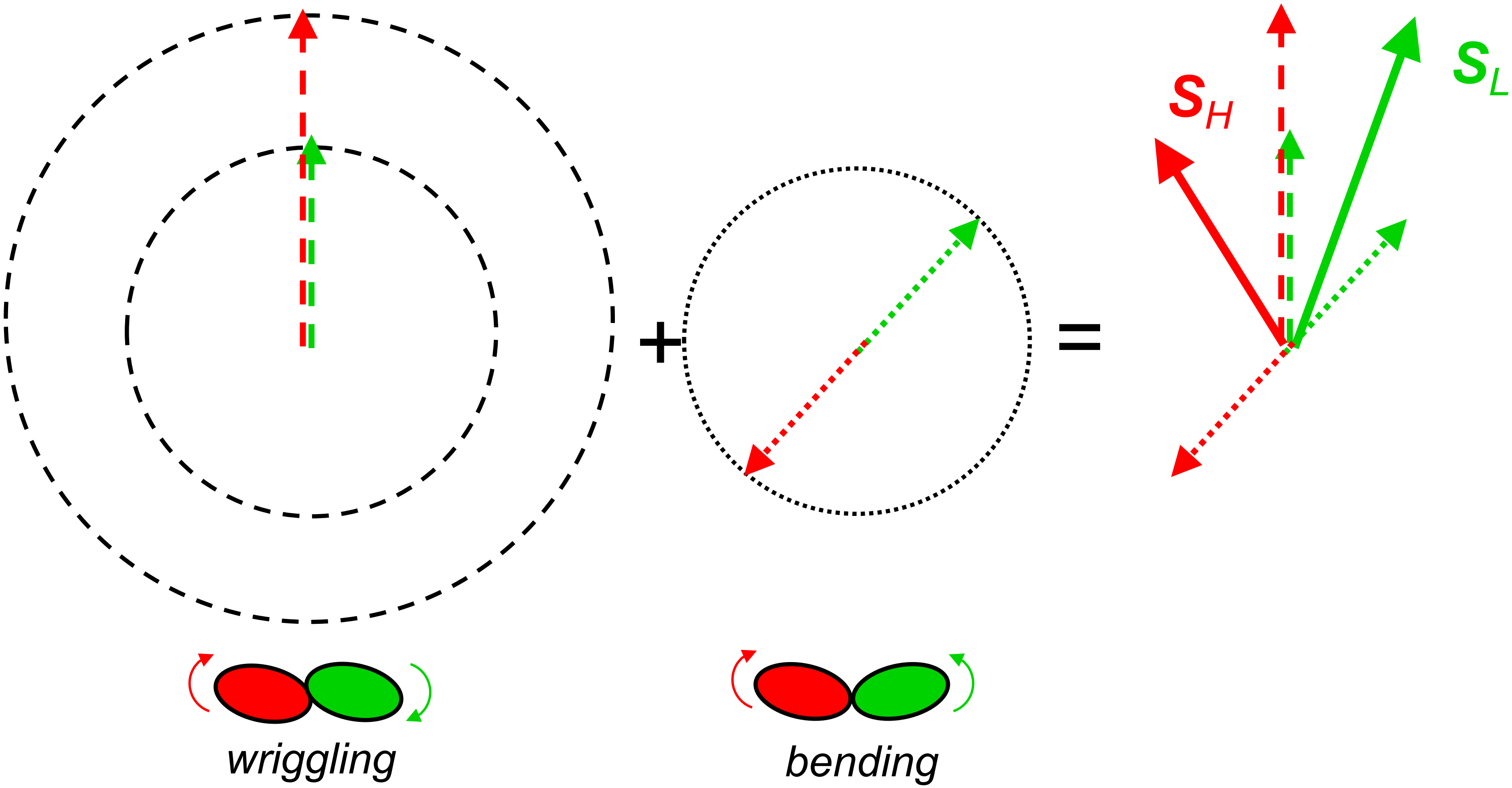
Fluctuations in fragment spin from rotational modes: $\delta S_\mathrm{L}^k = (\mathcal{I}_\mathrm{L}/\mathcal{I}_+)\, s_+^k + s_-^k$ $\quad \delta S_\mathrm{H}^k = (\mathcal{I}_\mathrm{H}/\mathcal{I}_+)\, s_+^k - s_-^k$

Total spin includes fluctuations: $\mathbf{S}_\mathrm{L} = \underline{\mathbf{S}}_\mathrm{L} + \delta\mathbf{S}_\mathrm{L}$ $\quad \mathbf{S}_\mathrm{H} = \underline{\mathbf{S}}_\mathrm{H} + \delta\mathbf{S}_\mathrm{H}$ $\quad \mathbf{L} = \underline{\mathbf{L}} - \delta\mathbf{S}_\mathrm{L} - \delta\mathbf{S}_\mathrm{H}$

Total angular momentum still conserved!

Fragment angular momenta are correlated due to normal modes but are nearly independent nonetheless

Why?



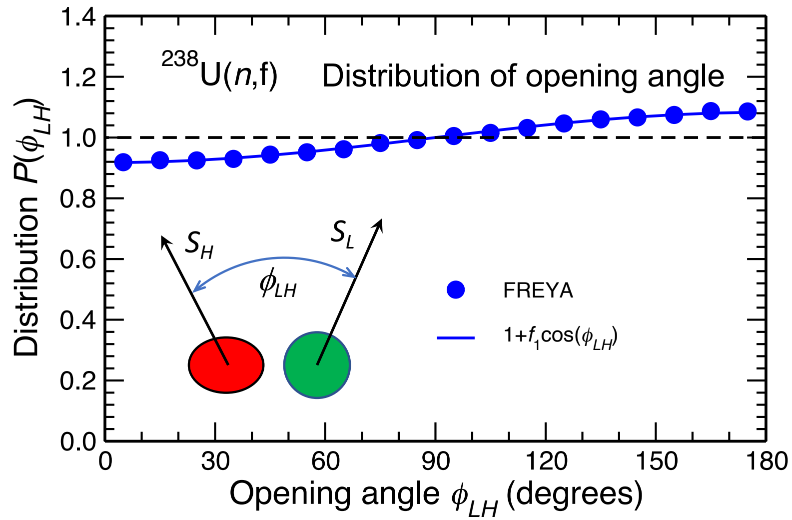
Dominance of fluctuations results in very weak fragment spin correlation

We can calculate the direction and magnitude of spin correlations

spin-spin correlation coefficient:

$$c(\boldsymbol{S}_L, \boldsymbol{S}_H) \equiv \frac{\langle \delta\boldsymbol{S}_L \cdot \delta\boldsymbol{S}_H \rangle}{[\langle \delta S_L^2 \rangle \langle \delta S_H^2 \rangle]^{1/2}} = -\left[\frac{\mathcal{I}_L \qquad \mathcal{I}_H}{(\mathcal{I}_R + \mathcal{I}_L)(\mathcal{I}_R + \mathcal{I}_H)}\right]^{\frac{1}{2}} \ll 1$$

Correlation between the spin *directions*:



Correlation between the spin *magnitudes*:

Case:	^{235}U(n,f)	^{238}U(n,f)	^{239}Pu(n,f)	^{252}Cf(sf)
$\bar{S}_L = \langle S_L \rangle$	4.27	4.43	4,58	5.08
$\bar{S}_H = \langle S_H \rangle$	5.66	5.80	5.93	6.33
$c(S_L, S_H)$ (%)	0.2	0.2	0.1	0.1
f_1 (%)	−8.2	−8.3	−8.3	−8.4

magnitude correlation coefficient:

$$c(S_L, S_H) \equiv \frac{\langle \delta S_L \delta S_H \rangle}{[\langle \delta S_L^2 \rangle \langle \delta S_H^2 \rangle]^{1/2}}$$

JR & RV, PRL **127** (2021) 062502, RV & JR, PRC **103** (2021) 014610

Even highly correlated contributions can result in uncorrelated results

Many rolls of the dice:

The red & blue dice are cast for many rounds: Alice's score is the sum of the top faces; Bob's score is the sum of the top of the red dice and the bottom of the blue dice

Alice: 6 + 5 = 11

Bob: 6 + 2 = 8

same as *Alice* — opposite of *Alice*

Alice	*Bob*
6 + 5 = 11	6 + 2 = 8
2 + 1 = 3	2 + 6 = 8
1 + 5 = 6	1 + 2 = 3
3 + 3 = 6	3 + 4 = 7
5 + 4 = 9	5 + 3 = 8
2 + 4 = 6	2 + 3 = 5
3 + 5 = 8	3 + 2 = 5
5 + 3 = 8	5 + 4 = 7
1 + 4 = 5	1 + 3 = 4
2 + 1 = 3	2 + 6 = 8
…..	…..

The two score sequences are *not* correlated

In each round, the score contributions for Bob are fully correlated with the corresponding score contributions for Alice

Nevertheless, Bob's scores are uncorrelated with Alice's scores

Wilson et al. also measured fragment spins:

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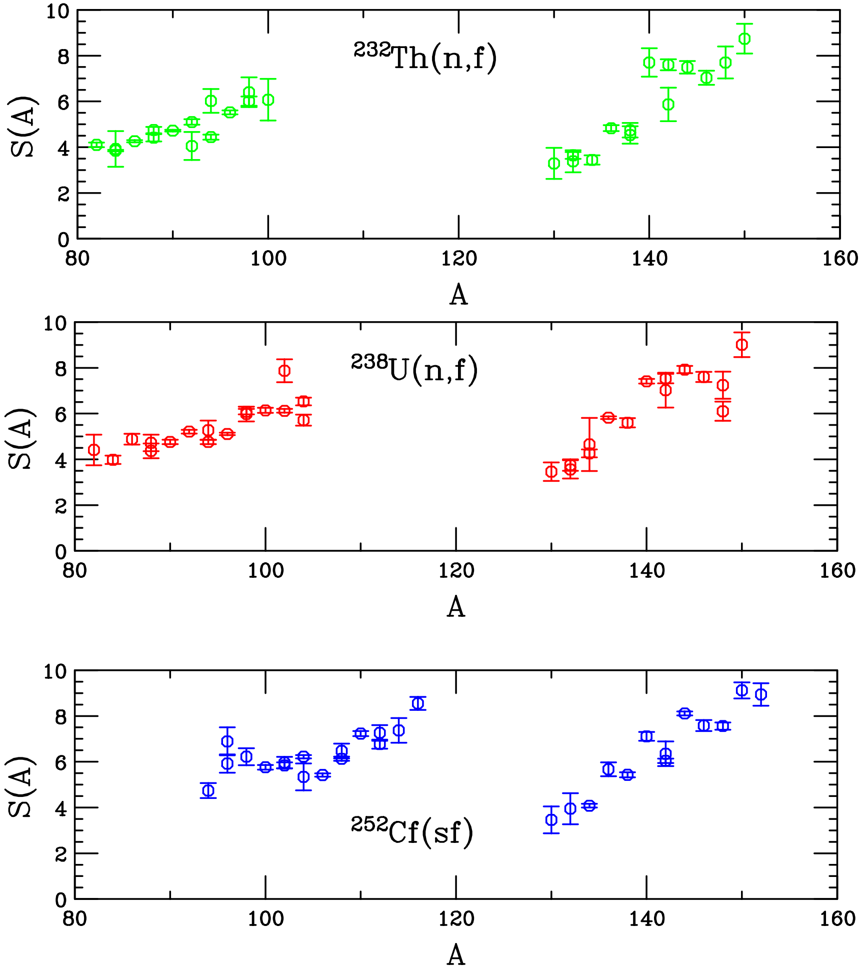


Measured S(A) is sawtooth-like, similar to ν(A), and, possibly, ν_γ(A) although new measurements should be made to confirm this behavior

We can model S(A) of the fragments & compare to data

Default moments of Inertia in FREYA have a simple dependence on mass:

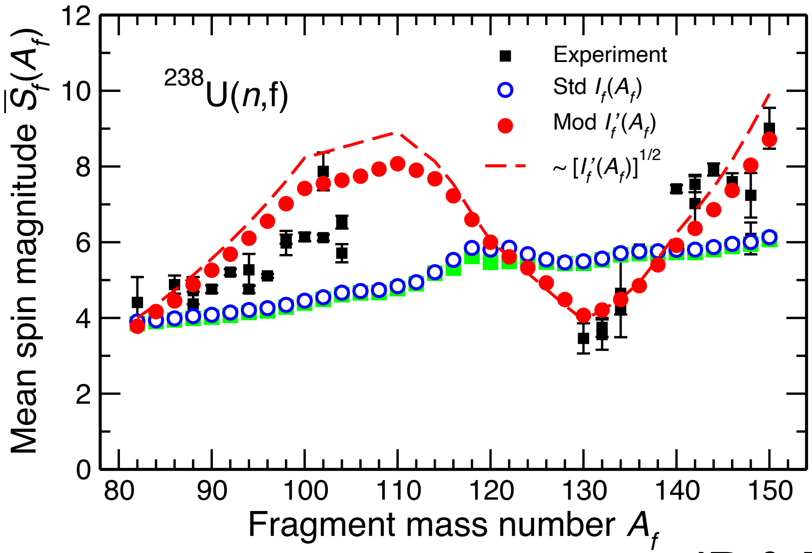
$$\mathcal{I}_L \propto (1/2) M_L R_L^2 \qquad \mathcal{I}_H \propto (1/2) M_H R_H^2$$

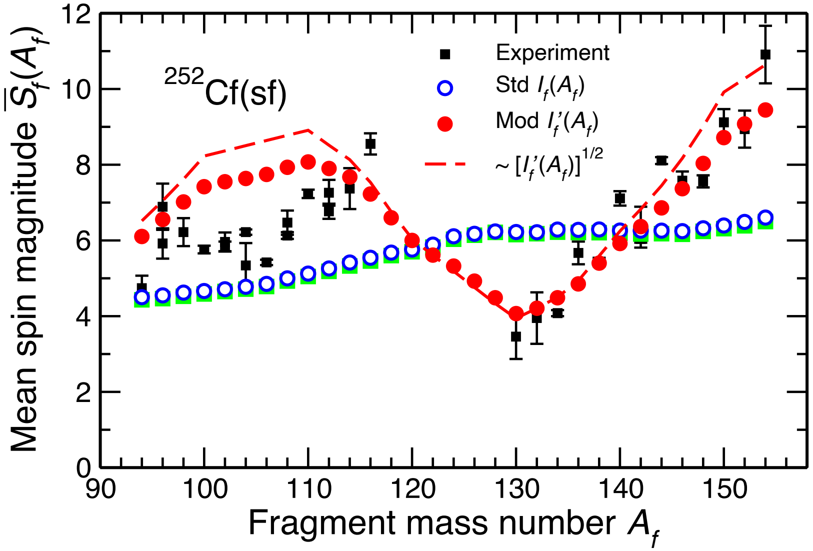
This simple dependence means that S(A) has a weak dependence on A

If the default moments of inertia are replaced by moments of inertia that schematically depend on the ground state deformation of the fragments,

$$I'_f(A_f) = 0.2[I_{rig}(A_f;0) + 10(I_{rig}(A_f;\varepsilon(A_f)) - I_{rig}(A_f;0))],$$

where ε is obtained from a fit to the ground state deformations





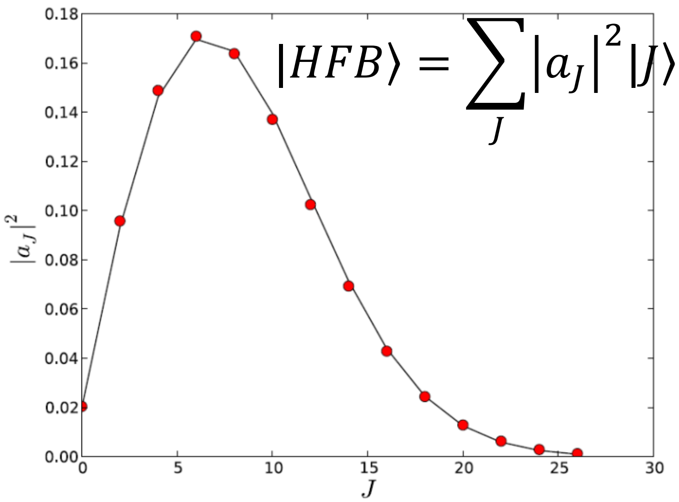
JR & RV, PRL **127** (2021) 062502

Modeling of moments of inertia reinforced by microscopic models

Spin distributions calculated recently in microscopic models

P. Marevic, N. Schunck, J. Randrup and R. Vogt, PRCL 021601 (2021) – Editor’s Suggestion

$$|HFB\rangle = \sum_J |a_J|^2 |J\rangle$$



Spin distribution fitted by:

$$|a_J|^2 \propto (2J+1)e^{-J(J+1)/2\sigma^2}$$

Where $\sigma^2 = = \mathcal{J}$, the spin cut-off parameter $\propto$ the moment of inertia

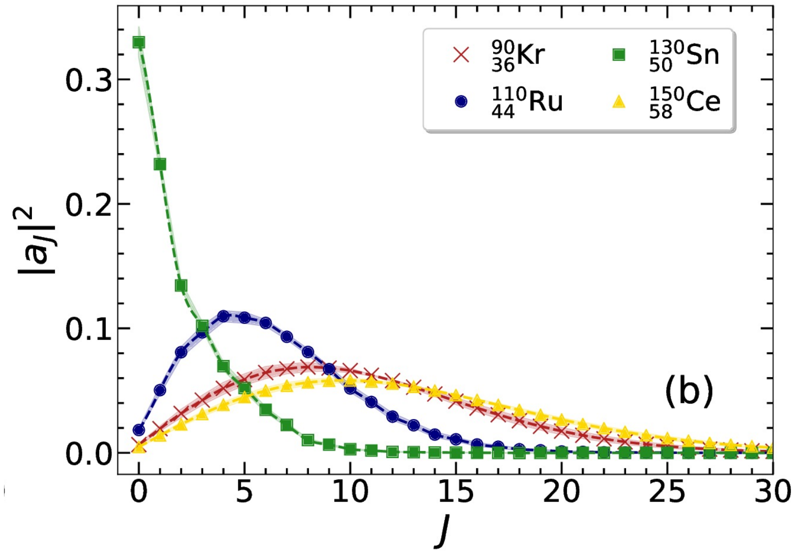
However, $\mathcal{J}$ is the moment of inertia at scission

- Analytical formulas are rough approximations
- Deformation at scission could be very different than in the ground state

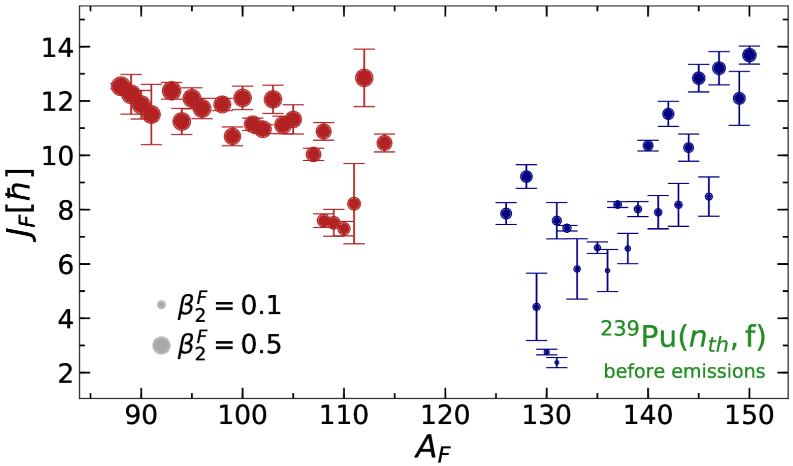
Angular Momentum Projection Allows Us to Calculate S(A) in Microscopic Models

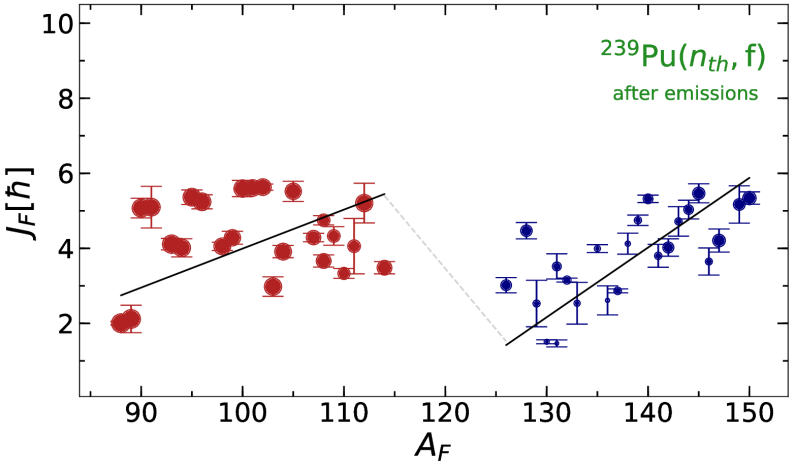
First-ever microscopic prediction of spin distributions across a broad range of Fragmentations, used to simulate γ emission in FREYA: good agreement with Wilson et al (albeit for ^{239}Pu(n,f))

- Extract spin distribution from HFB solutions by angular momentum projection



P. Marevic, N. Schunck, J. Randrup and R. Vogt, PRCL 021601 (2021)





FREYA references

- **FREYA** developed in collaboration with J. Randrup (LBNL); neutron-transport code integration by J. Verbeke (LLNL); available in MCNP6.2
- **FREYA** journal publications: Phys. Rev. C **80** (2009) 024601, 044611; **84** (2011) 044621; **85** (2012) 024608; **87** (2013) 044602; **89** (2014) 044601; **90** (2014) 064623; **96** (2017) 064620; **99** (2019) 054619; **103** (2021) 014610; Phys. Rev. Lett. **127** (2021) 062502;
- Parameter optimization for spontaneous fission: NIM A **922** (2019) 36
- **FREYA** published in Comp. Phys. Comm. **191** (2015) 178; **222** (2018) 263.
- “Nuclear Fission”, Chapter 5 of ‘100 Years of Subatomic Physics’, World Scientific, 2013
- Review in Eur. Phys. J. A **54** (2018) 9
- Papers with experimentalists: neutron polarization in photofission: Mueller *et al,* Phys. Rev. C **89** (2014) 034615; photon production: Gjerstvang *et al.,* Phys. Rev. C **103** (2021) 034609; neutron-gamma correlations: Wang *et al*, Phys. Rev. C **93** (2016) 014606, Marcath *et al*, Phys. Rev. C **97** (2018) 044622, Marin et al, NIM A **968** (2020) 163907 , PRC **104** (2021) 024602; neutron-neutron correlations, Schuster et al, Phys. Rev. C **100** (2019) 014605; Verbeke *et al*, Phys. Rev. C **97** (2018) 044601; Pozzi *et al,* Nucl. Sci. Eng. **178** (2014) 250.
- Fission in Astrophysics: Vassh *et al.,* J. Phys. G **46** (2019) 065202; Wang *et al.,* Ap. J. Lett. **903** (2020) L3
- Isotopes currently included: spontaneous fission of ^{252}Cf, ^{244}Cm, ^{238,240,242}Pu, ^{238}U and neutron-induced fission of ^{233,235,238}U(n,f), ^{239,241}Pu(n,f) for $E_n \leq 20$ MeV