Factorization and k_T dependent pdfs

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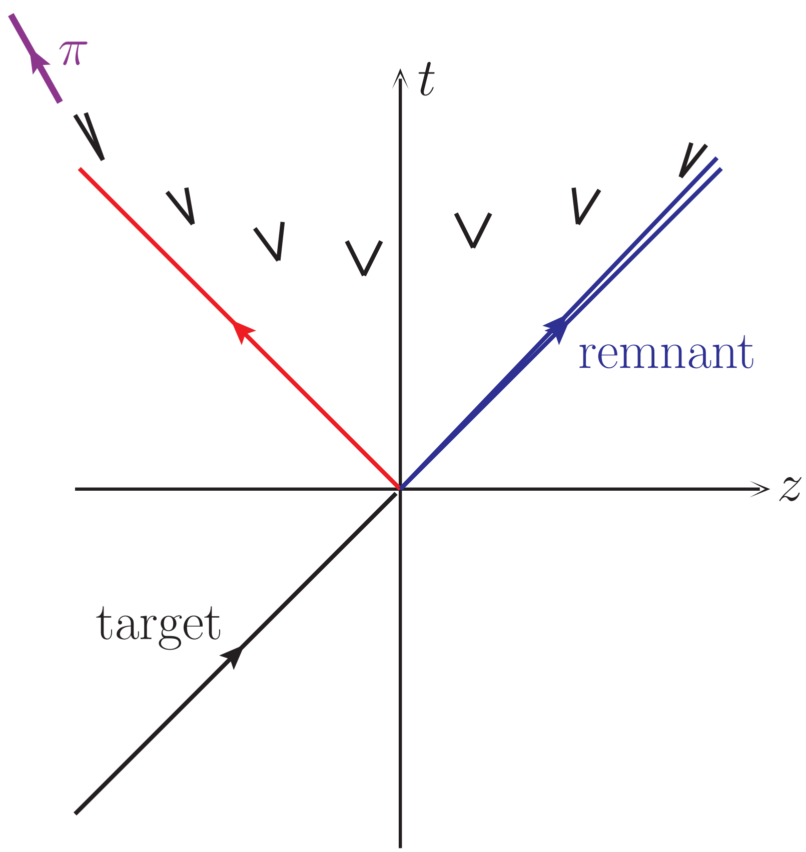
Overall issue: How to get accurate relationship between TMD functions and MCEG.

- Basic orientation.
- Kinematic approximations used to get factorization, especially TMD.
- Evolution.
- What's different in a MCEG?

Hard scattering and SIDIS (and context of full final state)

SIDIS: $e + N \to e(x, Q) + \pi(z, \boldsymbol{p}_{\pi\,\mathrm{T}}) + X$.

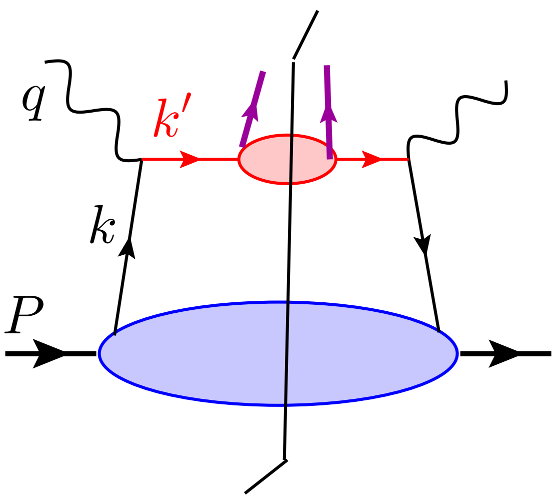
Basic motivation for factorization:

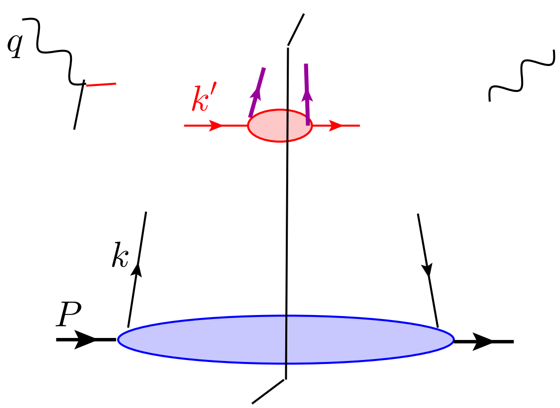


- Short distance, $1/Q$, hard scattering.
- Ejected parton + target remnant.
- $\sim$ Lorentz contracted, time dilated fast objects.
- String fragmentation.
- Remember 2 transverse directions.
- Remember *QCD complications*
  - High k_T partons
  - All intermediate scales contribute
  - Gluon radiation to all rapidities

QFT/momentum-space approximation for TMD factorization

Regress to parton model level to start.





- Use light-front coordinates:

$$P^\mu = (P^+, M^2/2p^+, \mathbf{0}_T)$$
$$q^\mu = (-xP^+, Q^2/2xP^+, \mathbf{0}_T)$$

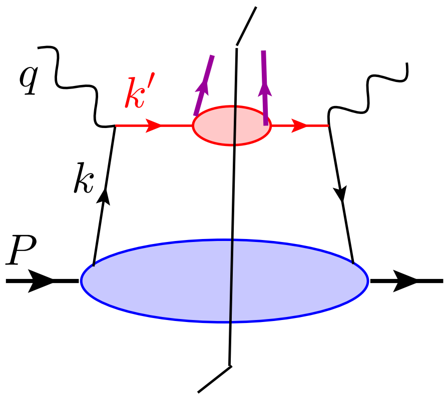
- Replace quark momenta at hard scattering by on-shell values, no transverse momentum.
  Neglect k^- in out-going quark subgraph.
  Neglect k'^+ in target subgraph.
  But preserve $\boldsymbol{k}_T$ flow.

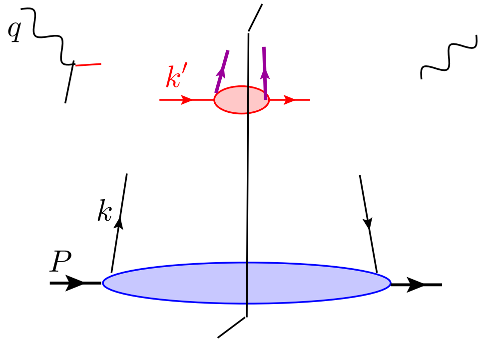
- Hence:

$k' \mapsto (k'^+, q^-, \boldsymbol{k}_T)$ in out-going quark subgraph

$k \mapsto (xP^+, k^-, \boldsymbol{k}_T)$ in target subgraph

Basic TMD factorization





- Hence momentum conservation at photon vertex entails

$$k' \mapsto (k'^{+}, q^{-}, \boldsymbol{k}_T) \text{ in out-going quark subgraph}$$

$$k \mapsto (xP^{+}, k^{-}, \boldsymbol{k}_T) \text{ in target subgraph}$$

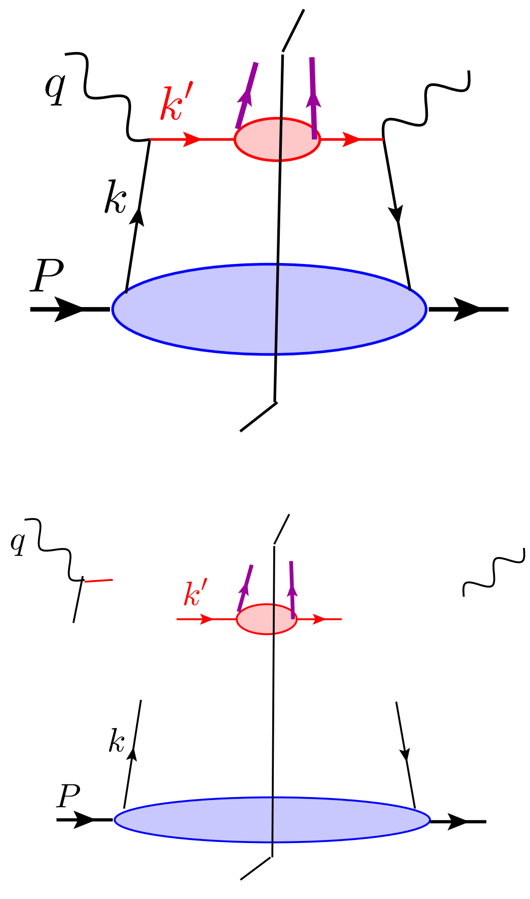
- k'^{+} integral stays inside out-going quark subgraph, k^{-} integral stays inside target subgraph.

$\Longrightarrow$ Gives *basic* operator definitions of

- TMD fragmentation function: function of z, and of $\boldsymbol{p}_{\pi\,\mathrm{T}} - z\boldsymbol{k}_T$ (pion transverse momentum relative to struck quark)
- TMD pdf: function of x and $\boldsymbol{k}_T$

On-shell LO hard scattering.
Convolution of TMD functions in transverse momentum.

Cross section integrated over $p_{\pi\,\mathsf{T}}$



- Approximations: as before, but also ignore $\boldsymbol{k}_T$ in out-going quark subgraph.
- Hence momentum conservation at photon vertex entails

  $$k' \mapsto (k'^+, q^-, \mathbf{0}_T) \text{ in out-going quark subgraph}$$

  $$k \mapsto (xP^+, k^-, \boldsymbol{k}_T) \text{ in target subgraph}$$

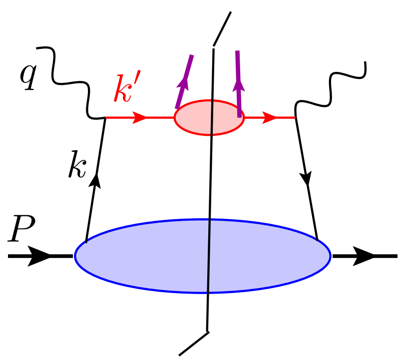
- k'^+ and $\boldsymbol{p}_T$ integrals stay inside fragmentation part;
  k^- and $\boldsymbol{k}_T$ integrals stay inside pdf part.

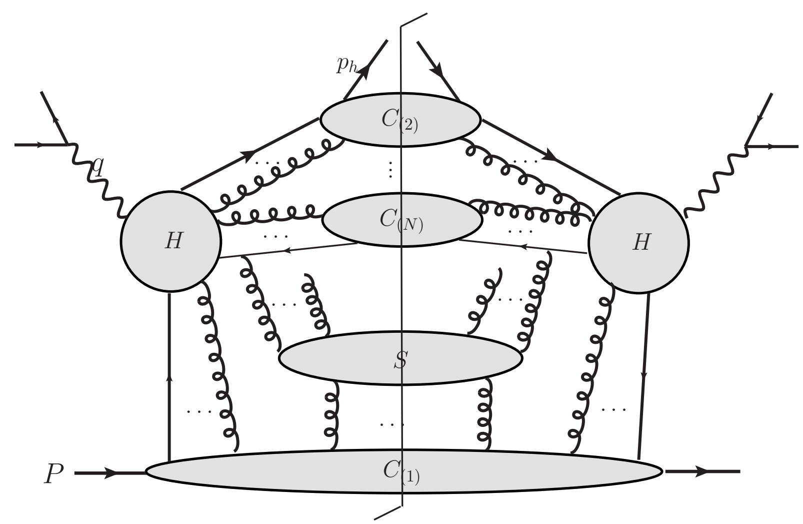
$\Longrightarrow$ Gives *basic* operator definitions of

- Collinear fragmentation function: function of z of pion relative to struck quark
- Collinear pdf: function of x

Complications v. basic picture

- If $\boldsymbol{p}_{\pi\,\mathsf{T}}$ unrestricted, k_{T} can range up to Q. ("k_{T} not limited to hadronic scale").
  Hence degradation in parton-model approximation.
  And multi-jet final states.
- Gluon emission fills rapidity gap.
- Hence much more complicated dominant momentum configurations:

Not: 

but: 

- Sort out with extra approximations, Ward identities, and subtractions in deriving factorization.
- Final state string not allowed for.

TMD factorization and evolution

- Detailed definition of TMD pdfs, fragmentation functions:
  - Arrange effective split between the TMD functions by rapidity.
  - Arrange effective cut off on transverse momentum by renormalization at scale μ.
- Define ζ such that $\frac{1}{2}\ln(\zeta/m^2)$ is essentially rapidity difference target v. q.
- TMD factorization and evolution (CSS), *for* $p_{\pi\,T} \ll Q$,

$$\mathrm{d}\sigma(x,Q,z,\boldsymbol{p}_{\pi\,\mathsf{T}}) = H(Q,\mu)\; f(x,\boldsymbol{k}_T,\mu,\zeta) \otimes_{\boldsymbol{k}_T} d(z,\boldsymbol{p}_{\pi\,\mathsf{T}} - z\boldsymbol{k}_T,\mu,\zeta),$$

$$\frac{\partial \dots}{\ln\mu} = \text{Simple RGE}$$

$$\frac{\partial f}{\partial \ln\zeta} = K(\boldsymbol{k}'_T,\mu) \otimes_{\boldsymbol{k}'_T} f(x,\boldsymbol{k}_T - \boldsymbol{k}'_T,\mu,\zeta)$$

$$\frac{\partial d}{\partial \ln\zeta} = \text{similar}$$

  K = gluon emission per unit rapidity, *non-perturbative at low* $\boldsymbol{k}'_T$.
  Separation of scales.
- Factorization for TMD functions in terms of collinear functions at large k_T. (Small transverse distance b_T actually.)
  DGLAP enters here.

Collinear factorization and evolution

Collinear factorization and evolution (DGLAP)

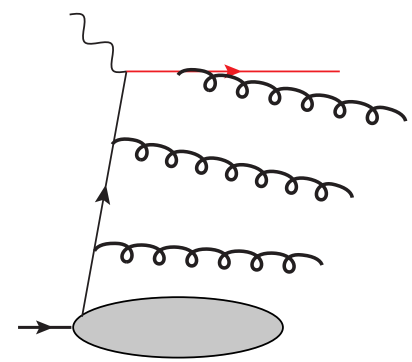
$$\mathrm{d}\sigma(x,Q,z) = C(x/x',z',Q,\mu) \otimes_{x',z'} f(x',\mu)\, d(z/z',\mu),$$

$$\frac{\partial f}{\partial \ln \mu} = P(x/x',\alpha_s(\mu),Q/\mu) \otimes_{x'} f(x',\mu),$$ and similarly for d

In hard scattering C, subtraction of collinear parts to prevent double counting.

Orientation about gluon emission and DLLA (old story)

- Consider full DIS cross section, and aim to decompose by final state.
- Simplify to



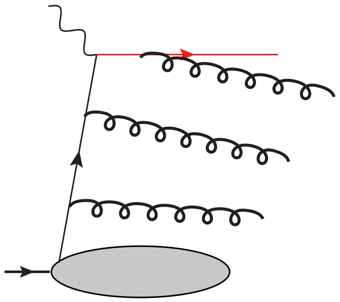
- One-gluon emission uniform in rapidity y (and $\ln k_\mathsf{T}$)

$$\int\int_{\text{accessible}} \mathrm{d}\ln k_\mathsf{T}\,\mathrm{d}y \ \text{ constant}$$

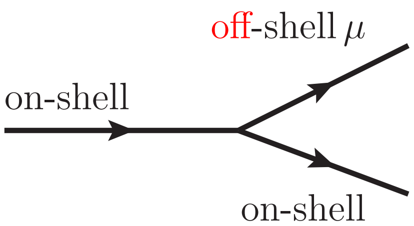
  with corrections at edges.

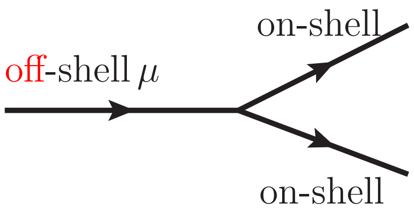
- Low $\boldsymbol{p}_{\pi\,\mathsf{T}} \implies k_\mathsf{T}$ restricted.
- Similarly for multiple strongly ordered gluons.
- No clear separation between right and left movers.
- (Also different partonic transitions, loop corrections, ...)

Use in MCEG



- Basic calculation is from same graphs as to obtain DGLAP

Initial state: 

. Final state: 

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- Splitting: function of ζ and parton virtuality or transverse momentum
- Generate configurations (MC); use with string
- Reinterpret same variables as applying to off-shell partons.
- Reconstruct actual momenta using momentum conservation for each splitting.
- Fix up to get exact momentum conservation for whole even.

Matches and mismatches between MCEG and TMD factorization

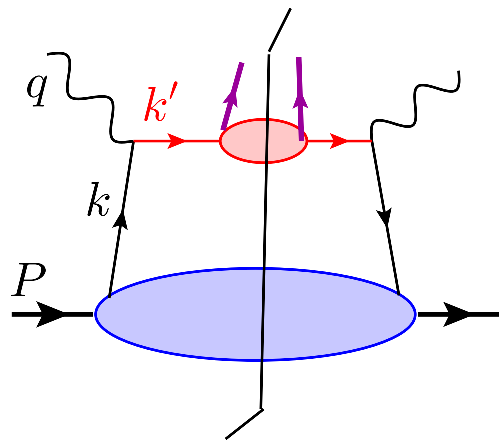
- TMD factorization: "Intrinsic transverse momentum" etc allowed for in TMD functions (measured!).
  MCEG needs/has something like this.

- TMD factorization does better job on parton kinematics than collinear factorization, but is restricted to $p_{\pi\,\mathrm{T}} \ll Q$.
  Collinear factorization does worse, but for inclusive cross sections that are insensitive to "intrinsic k_{T}" etc.
  MCEG needs even better parton kinematics.

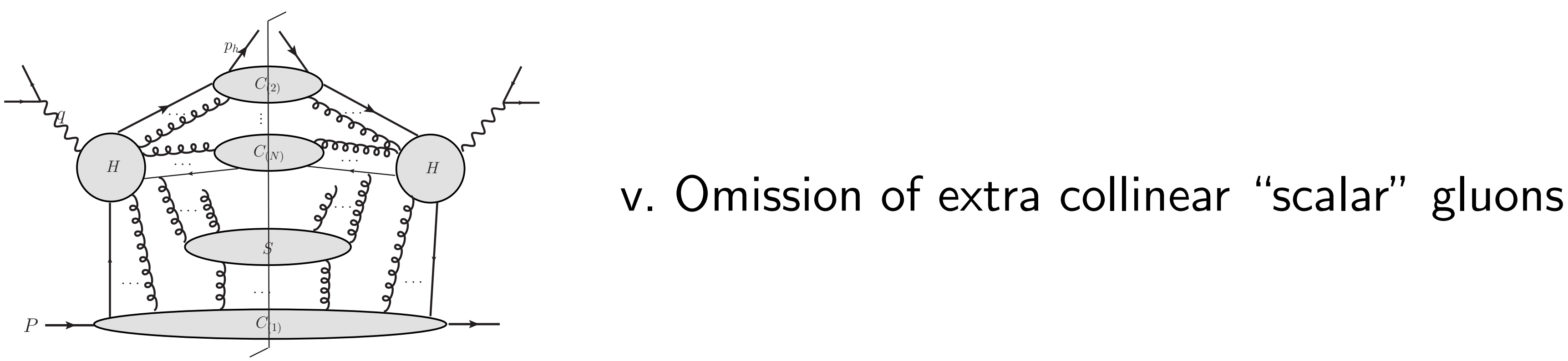
- In factorization, kinematic approximations shift final state, valid to leading power in Q for inclusive cross sections.
  Momentum conservation violated, but in unobserved part X of final state.
  MCEG must conserve momentum.

- In N*LO TMD and collinear factorization, there are subtractions constructed to correctly stop double counting.
  Different terms may make different and large changes to unobserved final state.
  Care here!

- MCEG gives probabilities, i.e., squared amplitudes, not QM amplitudes.
  Line kinematics in amplitude and complex conjugate must match.
  OK in parton model, and in LLA (parton quasiparticles in space-time)



- Concept of a line with momentum is effectively gauge dependent, as in



- *Perhaps the above suggests where to go next.*