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In-medium Branching Processes and their Features

MC4EIC Workshop, November 18 - 19, 2021

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Outline

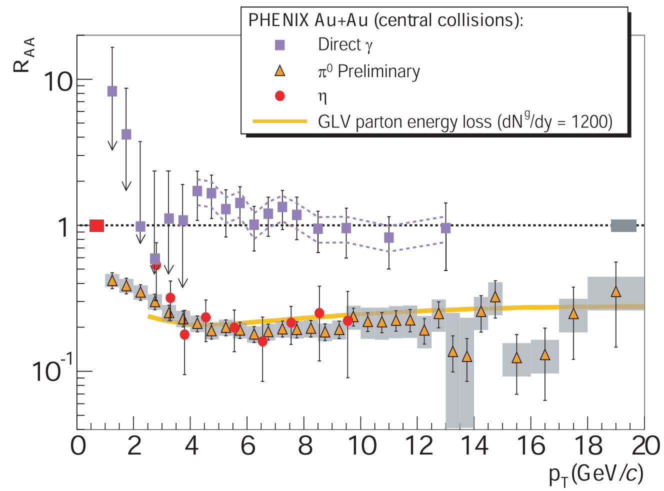
MC4EIC

- Introduction and motivation
- Why are vacuum and in-medium splitting functions different
- Calculation of all in-medium splitting functions / numerical evaluation and features
- Implementation in higher order and resumed calculations
- Existing approximate implementations in MC
- Conclusions

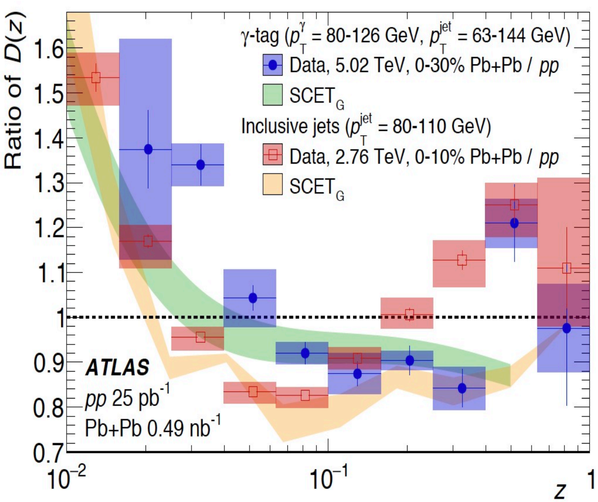
Introduction & Motivation

- In reactions with nuclei in-medium parton showers are the cornerstone of the physics of hard probes – high p_T hadrons, jets, heavy fl.
- The associated phenomena were dubbed jet quenching and established in a myriad of observables

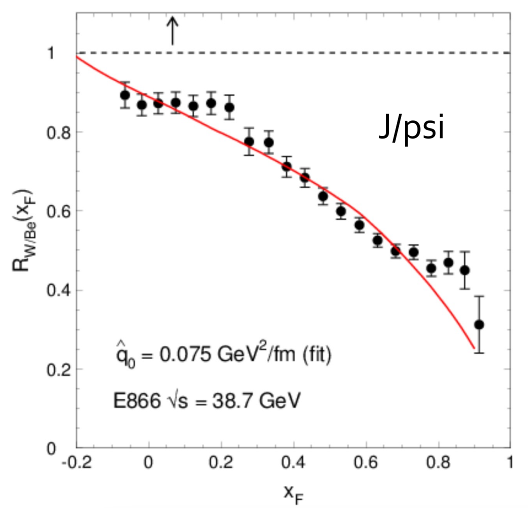
M. Gyulassy et al . (1992)



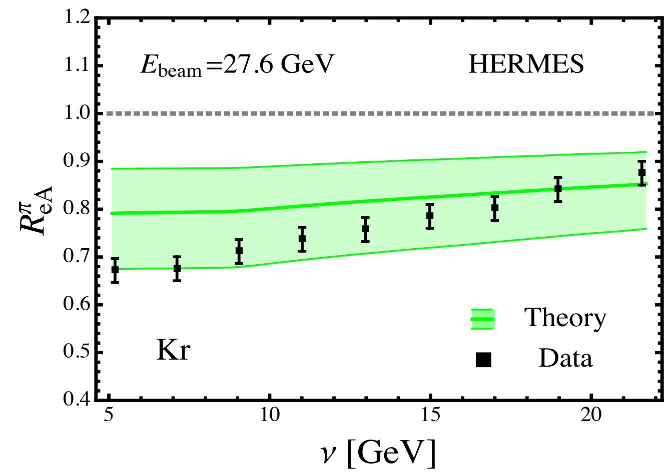
Inclusive hadron suppression, hadron correlations



Jet suppression, enhanced dijet asymmetries, jet substructure



Heavy flavor suppression, b jets,di-b jets, quarkonia



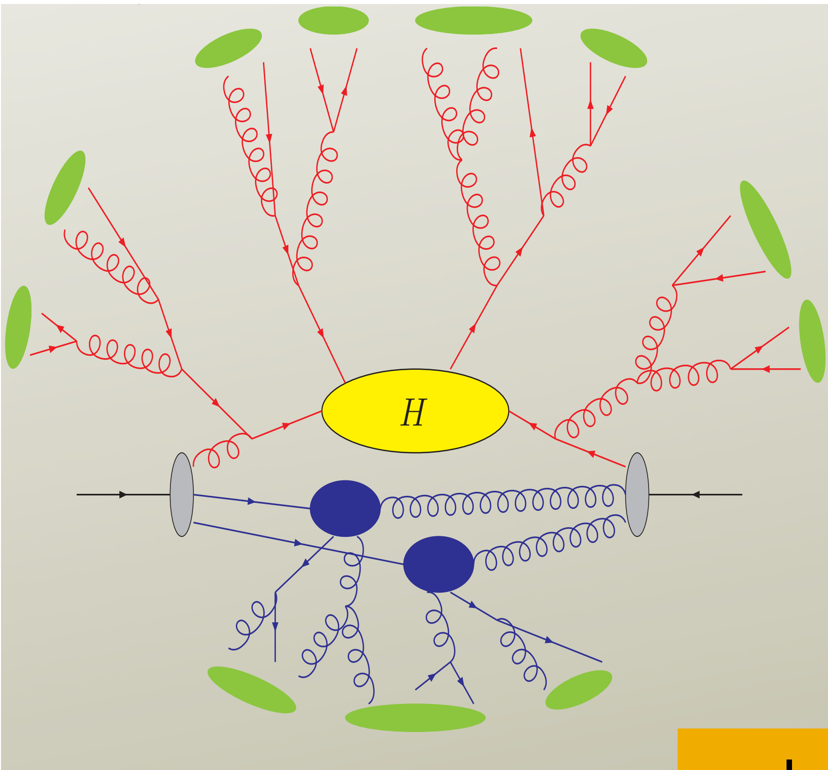
Also in cold nuclear matter

The splitting kernels

- In SCET splitting functions are related to beam (B) and jet (J) functions in SCET

W. Waalewjin. (2014)

$$
\begin{aligned}
A_{q\to qg} &= \langle J|\,T\,\bar{\chi}_n(x_0)\,e^{iS}\,|q(\boldsymbol{p})g(\boldsymbol{k})\rangle\,, \\
A_{g\to q\bar{q}} &= \langle J|\,T\,\mathcal{B}^{\lambda c}(x_0)\,e^{iS}\,|q(\boldsymbol{p})\bar{q}(\boldsymbol{k})\rangle \\
A_{g\to gg} &= \langle J|\,T\,\mathcal{B}^{\lambda c}(x_0)\,e^{iS}\,|g(\boldsymbol{p})g(\boldsymbol{k})\rangle
\end{aligned}
$$



Gribov et al. (1972)

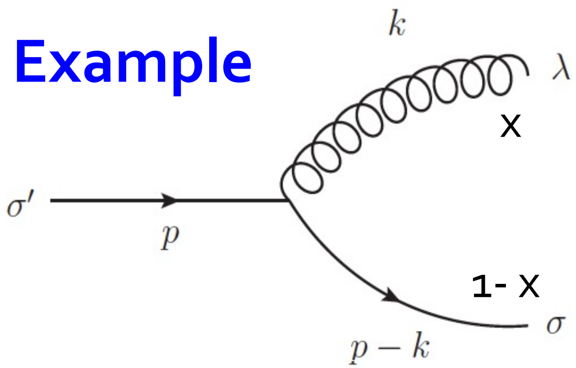
G. Altarelli et al. (1977)

Y. Dokshitzer (1977)

- In general, knowledge of branching processes is necessary for higher order and resumed calculations
- Also essential ingredient for MC event generators

Lightcone wave functions and parton branchings

Example



- The technique of lightcone wavefunctions

$$\psi(x,\underline{k}-x\underline{p}) \equiv \frac{1}{2p^+}\frac{1}{p^- - (p-k)^- - k^-}\overline{U}_\sigma(p-k)\left[-g\not{\epsilon}^*_\lambda(k)\right]U_{\sigma'}(p)$$
$$= \frac{gx(1-x)}{(\underline{k}-x\underline{p})^2_T + x^2m^2}\left\{\frac{2-x}{x\sqrt{1-x}}\left(\underline{\epsilon}^*_\lambda\cdot(\underline{k}-x\underline{p})\right)\left[\mathbb{1}\right]_{\sigma\sigma'} + \frac{\lambda}{\sqrt{1-x}}\left(\underline{\epsilon}^*_\lambda\cdot(\underline{k}-x\underline{p})\right)\left[\tau_3\right]_{\sigma\sigma'} + \frac{im\,x}{\sqrt{1-x}}\underline{\epsilon}^*_\lambda\times\left[\underline{\tau}_\perp\right]_{\sigma\sigma'}\right\}.$$

$$\langle\psi(x,\underline{\kappa})\,\psi^*(x,\underline{\kappa}')\rangle \equiv \sum_{\lambda=\pm1}\frac{1}{2}\mathrm{tr}\left[\psi(x,\underline{\kappa})\,\psi^*(x,\underline{\kappa}')\right]$$
$$= \frac{8\pi\alpha_s\,(1-x)}{\left[\kappa_T^2 + x^2m^2\right]\left[\kappa_T'^2 + x^2m^2\right]}\left[(\underline{\kappa}\cdot\underline{\kappa}')\,\left[1+(1-x)^2\right] + x^4m^2\right]$$

Useful to express in Pauli matrixes

c.f. M. Sievert et al . (2018)

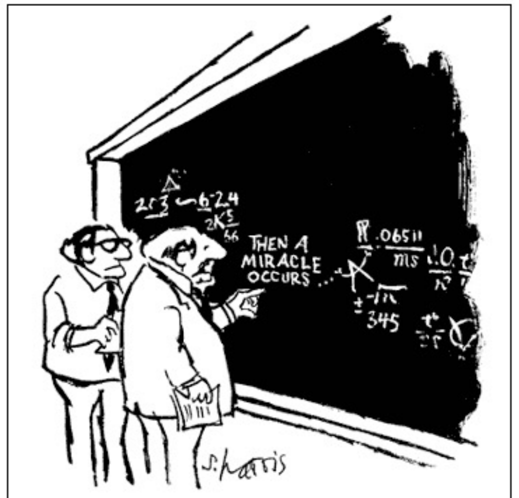
F. Ringer et al . (2016)

Branchings depending on the intrinsic momentum of the splitting $\underline{\kappa} = \underline{k - xp}.$

$$xp^+\frac{dN}{d^2k\,dx\,d^2p\,dp^+}\bigg|_{\mathcal{O}(\chi^0)} = \frac{\alpha_s C_F}{2\pi^2}\,\frac{(k-xp)^2_T\,\left[1+(1-x)^2\right] + x^4m^2}{\left[(k-xp)^2_T + x^2m^2\right]^2} \times \left(p^+\frac{dN_0}{d^2p\,dp^+}\right)$$

- Certain advantages – can provide in “one shot” both massive and massless splitting functions
- Have checked that results agree for massless and massive DGLAP

I. In-medium parton showers



"I think you should be more explicit here in step two."

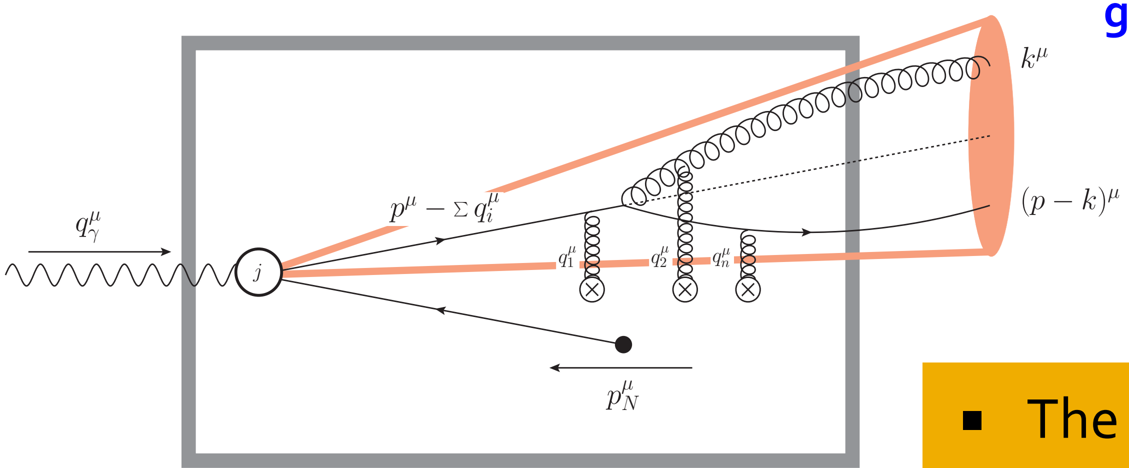
Theoretical framework

- The theoretical framework is completely general – it is applicable for both cold nuclear matter and the QGP.
- This is achieved by isolating the medium in transport parameters and universal gluon-mediated interactions

$$\mathcal{L}_{opac.} = \mathcal{L}_{QCD} + \mathcal{L}_{ext}^{qG} + \mathcal{L}_{ext}^{gG} + \mathcal{L}_{G.F.} + \cdots$$

$$v(q_T^2) \to \frac{-g_{eff}^2}{q_T^2 + \mu^2} \qquad \frac{d\sigma^{el}}{d^2q} = \frac{1}{(2\pi)^2} \frac{C_F}{2N_c} [v(q_T^2)]^2$$

In deep inelastic scattering (DIS) the lowest order processes involve prompt quark. Even at NLO the prompt gluon jet contribution is small



M. Sievert et al . (2018), (2019)

$$\frac{1}{p_N^-} \ll l_f^+ \sim \lambda^+ \sim L^+$$

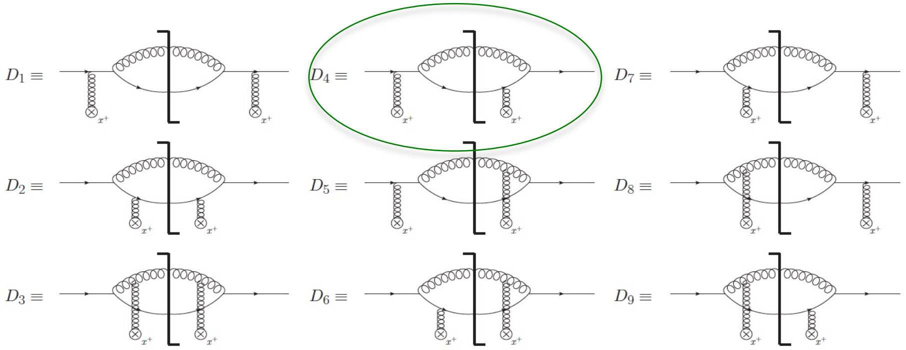
- The limit we are interested in

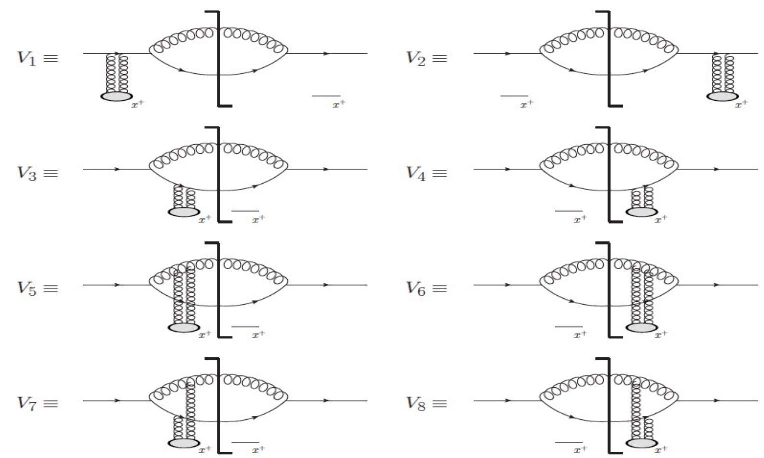
$$\mathcal{O}\left(\frac{\perp^2}{Q^2}\right)$$

Note that the leading subeikonal corrections have also been computed (not covered here)

A. Sadofyev et al . (2021)

Opacity expansion building blocks – direct and virtual terms





- Interaction in the amplitude and the conjugate amplitude (Direct). Two in the amplitude or the conjugate (Virtual)

Representative forward cut diagram. Propagators hide in wavefunction

$$D_4 = \left[\frac{-1}{2N_c C_F} \, e^{+i[\Delta E^-(\underline{k}-x\underline{p}) - \Delta E^-(\underline{k}-x\underline{p}+x\underline{q})]z^+} \right] \psi(x, \underline{k} - x\underline{p}) \left[0 - e^{-i\Delta E^-(\underline{k}-x\underline{p})z^+} \right]$$
$$\times \left[e^{+i\Delta E^-(\underline{k}-x\underline{p}+x\underline{q})z^+} - e^{+i\Delta E^-(\underline{k}-x\underline{p}+x\underline{q})x_0^+} \right] \psi^*(x, \underline{k} - x\underline{p} + x\underline{q}) \,,$$

$$p^- - k^- - (p-k)^- = \Delta E^-(\underline{k} - x\underline{p})$$

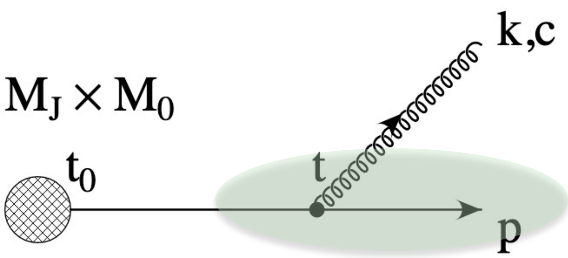
- Vitruallity changes enter the interference phases and are related to the propagators

C.f. G. Ovanesyan et al . (2011)

Z. Kang et al . (2016)

Non-local physics and coherence

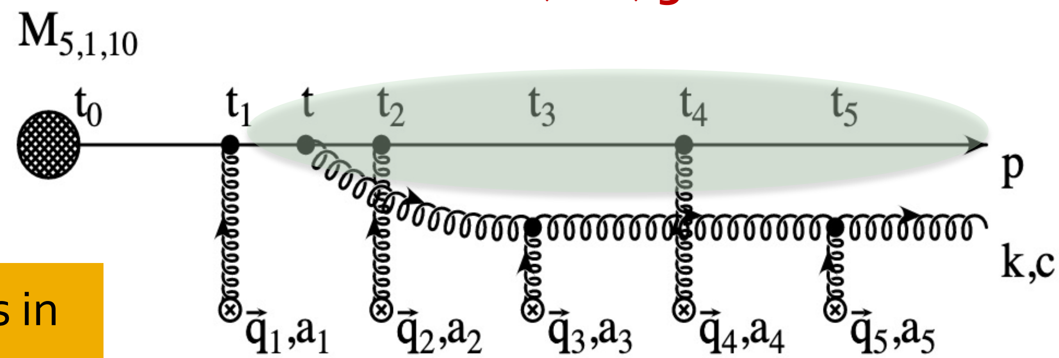
Vacuum



- Consider the formation times in the soft gluon emission limit

Medium

Let's try to evaluate the formation time of the (soft) gluon at t

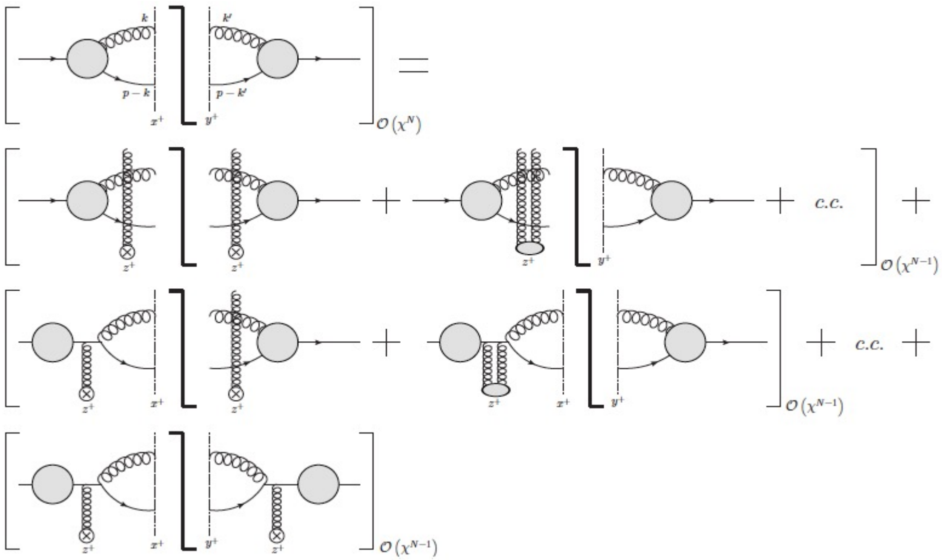


$LPM \sim L/\tau_f$ $\quad \frac{1}{\Delta E} = \tau_{form} = \frac{2\omega}{k_\perp^{\;2}} \quad \frac{1}{\Delta E} = \tau_{form} = \frac{2\omega}{(k-q_3-q_5)_\perp^{\;2}}$

- In the case of a medium we cannot guess from its final distribution. In fact future interactions can in fact affect this formation time and how the system in turn will interact. (This is a quantum coherent effect.)
- This also shows right away the difficulty of implementing LPM parton showers in time-ordered MCs

Master equation – matrix form

- Color is not entangled, homogeneous structure and multiplicative factors that can be algebraically treated
- Finally, relative to the splitting vertex we classify the as
- Initial/Initial, Initial/Final, Final/Initial and Final/Final



$$\begin{bmatrix} f^{(N)}_{F/F}(\underline{k},\underline{k}',\underline{p};\, x^+,y^+) \\ f^{(N)}_{I/F}(\underline{k}',\underline{p};\, x^+,y^+) \\ f^{(N)}_{F/I}(\underline{k},\underline{p};\, x^+,y^+) \\ f^{(N)}_{I/I}(\underline{p};\, x^+,y^+) \end{bmatrix} = \int\limits_{x_0^+}^{\min[x^+,y^+,R^+]} \frac{dz^+}{\lambda^+} \int \frac{d^2q}{\sigma_{el}} \frac{d\sigma^{el}}{d^2q} \begin{bmatrix} \mathcal{K}_1 & \mathcal{K}_2 & \mathcal{K}_3 & \mathcal{K}_4 \\ 0 & \mathcal{K}_5 & 0 & \mathcal{K}_6 \\ 0 & 0 & \mathcal{K}_7 & \mathcal{K}_8 \\ 0 & 0 & 0 & \mathcal{K}_9 \end{bmatrix} \begin{bmatrix} f^{(N-1)}_{F/F}(\underline{k},\underline{k}',\underline{p};\, x^+,y^+) \\ f^{(N-1)}_{I/F}(\underline{k}',\underline{p};\, x^+,y^+) \\ f^{(N-1)}_{F/I}(\underline{k},\underline{p};\, x^+,y^+) \\ f^{(N-1)}_{I/I}(\underline{p};\, x^+,y^+) \end{bmatrix}$$

- Upper triangular structure. Suggests specific strategy how to solve it. Calculated: initial conditions, kernels, and wrote a Mathematica code to solve it

Full in medium splitting

- Full massless and massive in-medium splitting functions now available to first order in opacity
- SCET-based effective theories also created to solve this problem

- Direct sum

$$\frac{dN(tot.)}{dxd^2k_\perp} = \frac{dN(vac.)}{dxd^2k_\perp} + \frac{dN(med.)}{dxd^2k_\perp}$$

Representative example

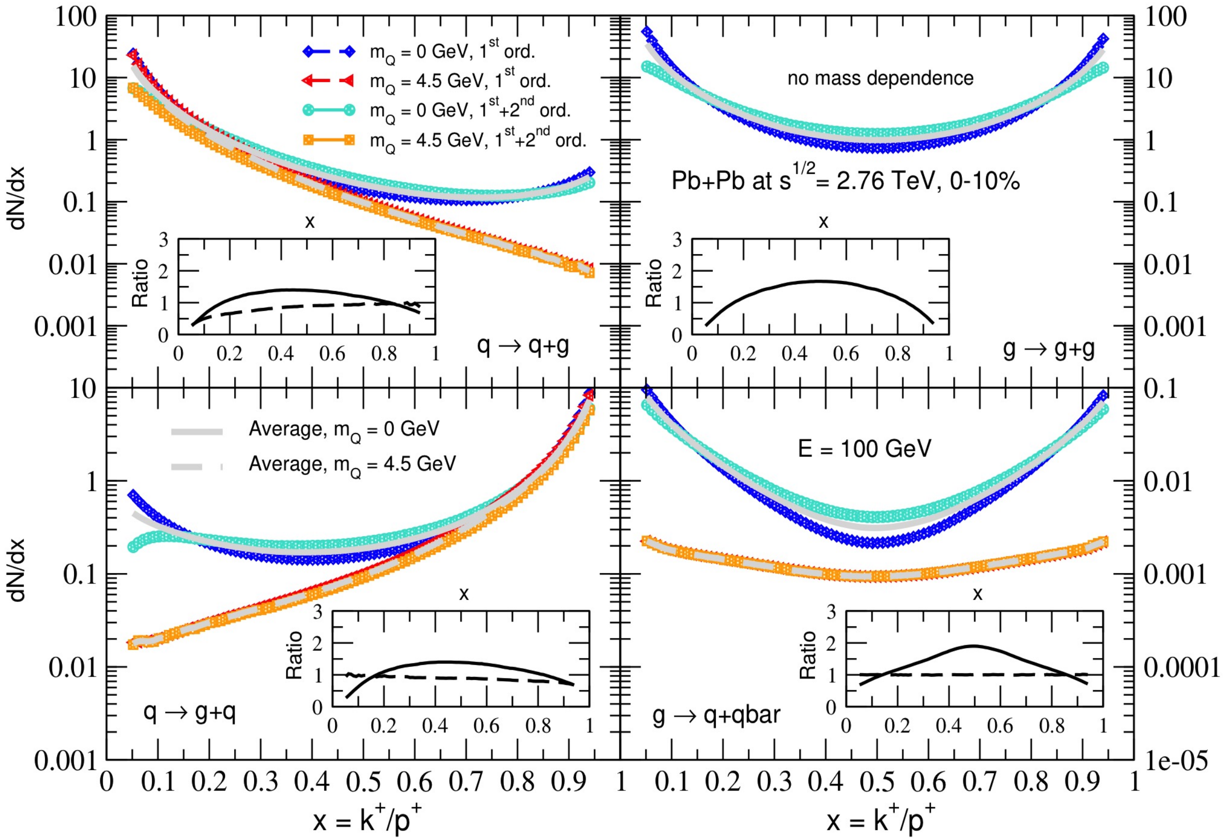
$$\left(\frac{dN^{\rm med}}{dxd^2k_\perp}\right)_{Q\to Qg} = \frac{\alpha_s}{2\pi^2}C_F\int\frac{d\Delta z}{\lambda_g(z)}\int d^2q_\perp\frac{1}{\sigma_{el}}\frac{d\sigma_{el}^{\rm med}}{d^2q_\perp}\left\{\left(\frac{1+(1-x)^2}{x}\right)\left[\frac{B_\perp}{B_\perp^2+\nu^2}\right.\right.$$
$$\times\left(\frac{B_\perp}{B_\perp^2+\nu^2}-\frac{C_\perp}{C_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z])+\frac{C_\perp}{C_\perp^2+\nu^2}\cdot\left(2\frac{C_\perp}{C_\perp^2+\nu^2}-\frac{A_\perp}{A_\perp^2+\nu^2}\right.$$
$$\left.-\frac{B_\perp}{B_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_3)\Delta z])+\frac{B_\perp}{B_\perp^2+\nu^2}\cdot\frac{C_\perp}{C_\perp^2+\nu^2}(1-\cos[(\Omega_2-\Omega_3)\Delta z])$$
$$+\frac{A_\perp}{A_\perp^2+\nu^2}\cdot\left(\frac{D_\perp}{D_\perp^2+\nu^2}-\frac{A_\perp}{A_\perp^2+\nu^2}\right)(1-\cos[\Omega_4\Delta z])-\frac{A_\perp}{A_\perp^2+\nu^2}\cdot\frac{D_\perp}{D_\perp^2+\nu^2}(1-\cos[\Omega_5\Delta z])$$
$$\left.+\frac{1}{N_c^2}\frac{B_\perp}{B_\perp^2+\nu^2}\cdot\left(\frac{A_\perp}{A_\perp^2+\nu^2}-\frac{B_\perp}{B_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z])\right]$$
$$\left.+x^3m^2\left[\frac{1}{B_\perp^2+\nu^2}\cdot\left(\frac{1}{B_\perp^2+\nu^2}-\frac{1}{C_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z])+\ldots\right]\right\}$$

- Factorize form the hard part
- Gauge-invariant
- Depend on the properties of the medium
- Can be expressed as corrections to Altarelli-Parisi

Done of course for all splitting functions

Differential branching spectra

In-medium parton showers are softer than the ones in the vacuum. There is even more soft gluon emission – medium induced scaling violations, enhancement of soft branching



Effects of opacity

- Reduction of small-x and large-x probabilities (assymptotics modulated by thermal mass)
- Enhancement of democratic branching (x~0.5)

Jet substructure – splitting functions

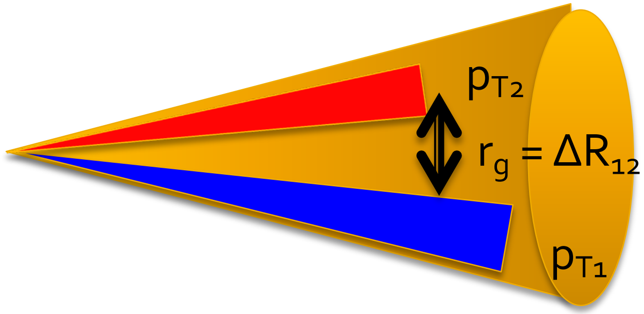
A. Larkoski et al. (2015)

$$\frac{dN_j^{\mathrm{vac,MLL}}}{dz_g d\theta_g} = \sum_i \left(\frac{dN^{\mathrm{vac}}}{dz_g d\theta_g}\right)_{j\to i\bar{i}} \underbrace{\exp\left[-\int_{\theta_g}^{1} d\theta \int_{z_{\mathrm{cut}}}^{1/2} dz \sum_i \left(\frac{dN^{\mathrm{vac}}}{dz d\theta}\right)_{j\to i\bar{i}}\right]}_{\text{Sudakov Factor}}$$

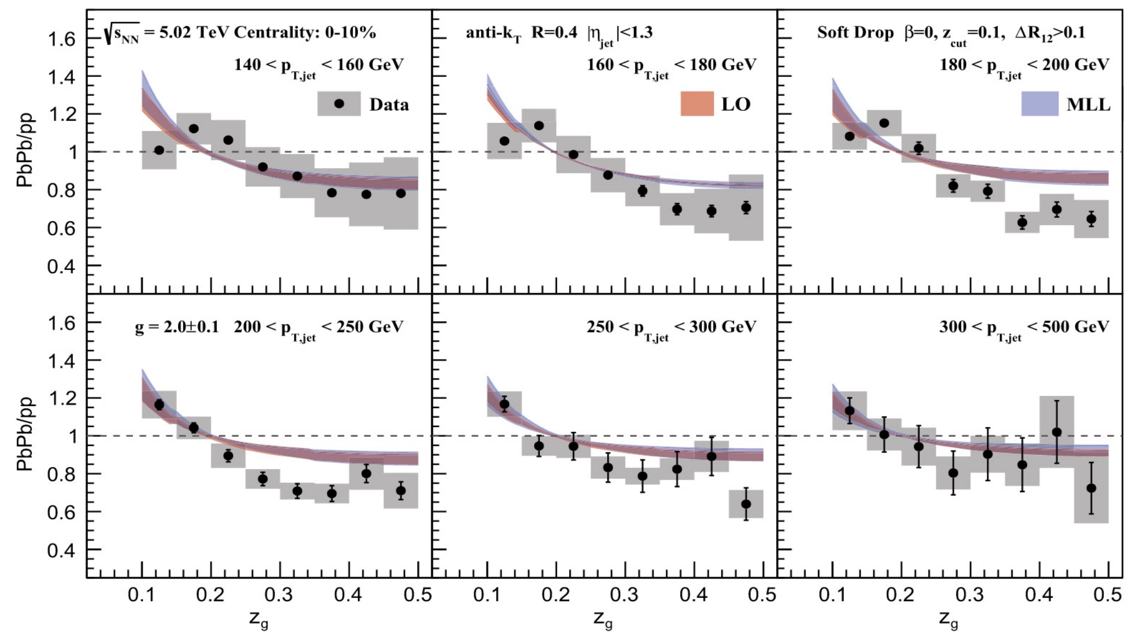
In-medium splitting functions can be measured directly through observables

Soft dropped momentum sharing distributions

$$z_g = \frac{\min(p_{T1}, p_{T2})}{p_{T1} + p_{T2}} > z_{\mathrm{cut}} \left(\frac{\Delta R_{12}}{R_0}\right)^{\beta}$$



H. Li et al. (2018)

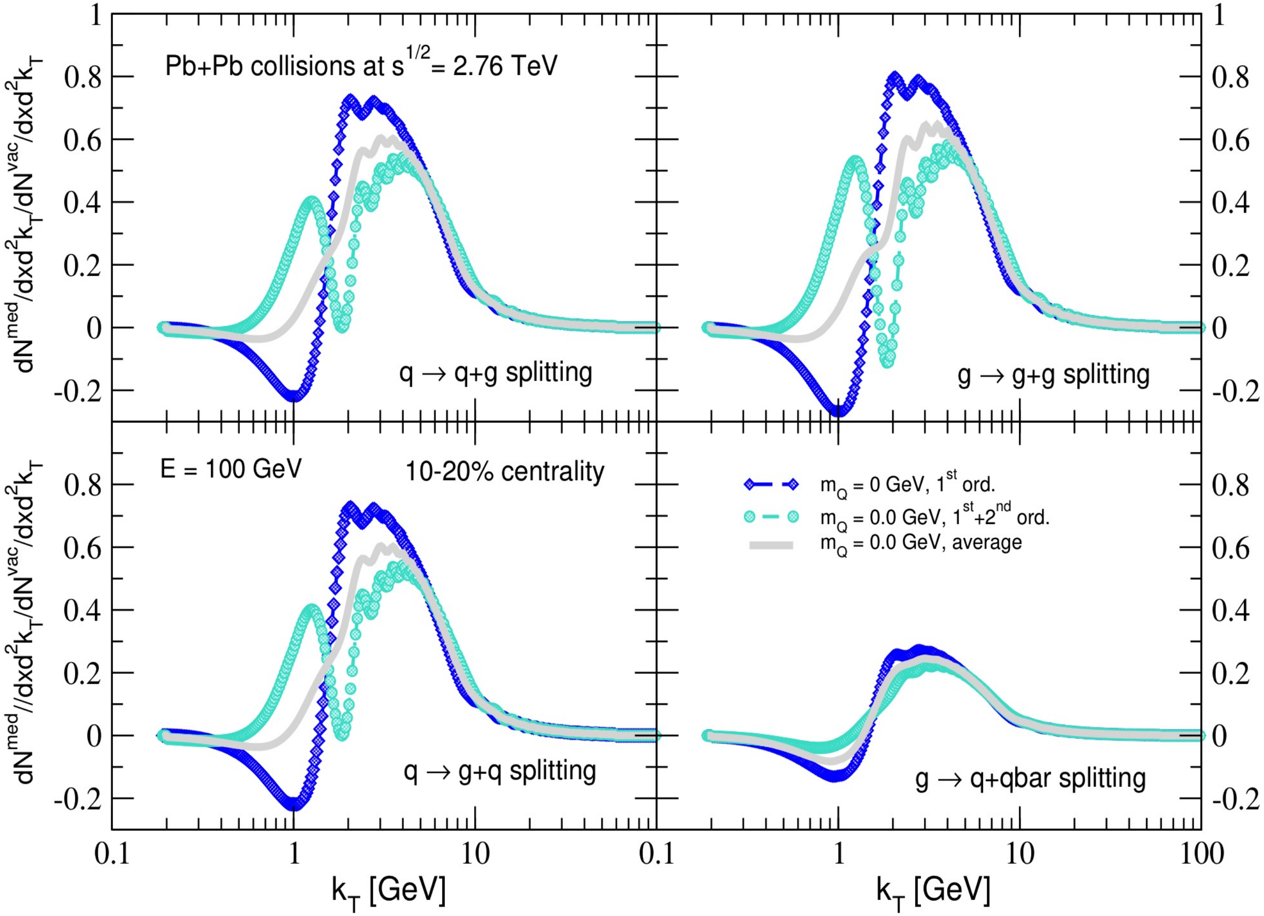


Directly proportional to the splitting functions, + resummation for small angles

The softer in-medium branching is directly observed!

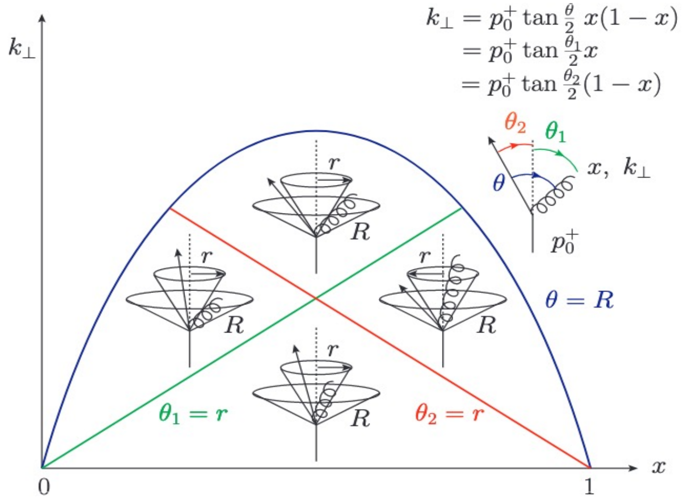
Differential branching spectra

In-medium parton showers are broader than the ones in the vacuum. There is even more large – angle gluon emission. The effect of heavy quark masses ("dead cone" effect) is also enhanced.



- Broder angular enhancement region
- Oscillating series – the average of 1^{st} and $1^{st+}2^{nd}$ order- candidate for pheno.

Jet substructure – jet shape



$$\Psi_\omega(r) = \frac{J_{\omega,E_r}(\mu)}{J_{\omega,E_R}(\mu)}$$

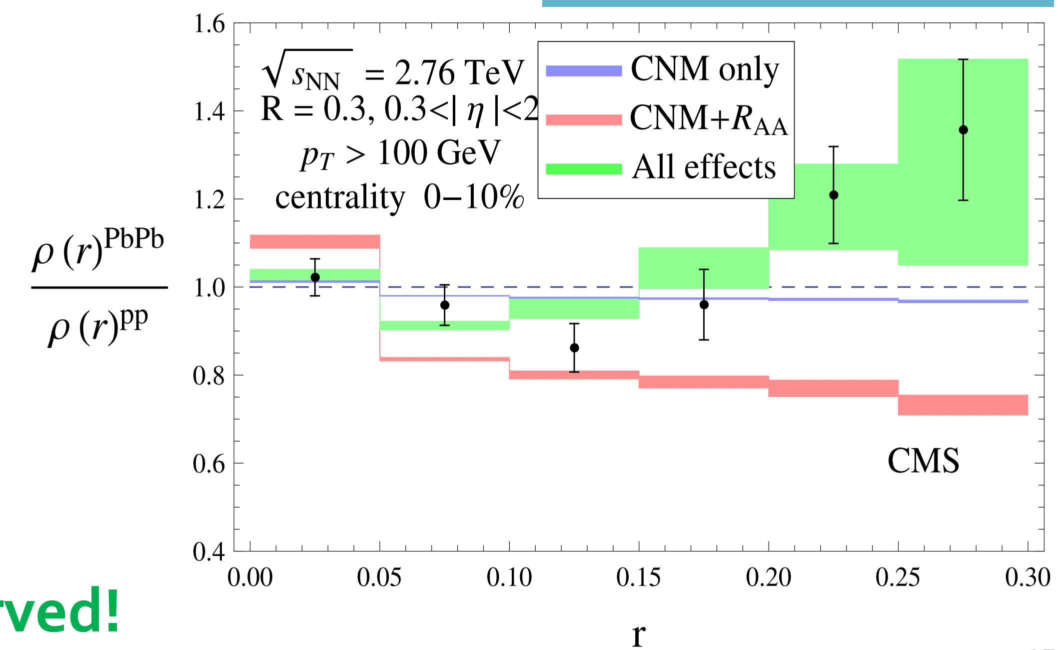
Integral jet shape

$$\Psi_J(r) = \frac{\sum_{i,\ d_{i\hat{n}}<r} E_T^i}{\sum_{i,\ d_{i\hat{n}}<R} E_T^i}\ .$$

Differential Jet shape

$$\frac{\Delta\Psi(r)}{\Delta r} = \frac{1}{N_J}\sum_{J=1}^{N_J} \frac{\Psi_J^{\rm track}(r+\delta r/2) - \Psi_J^{\rm track}(r-\delta r/2)}{\delta r}$$

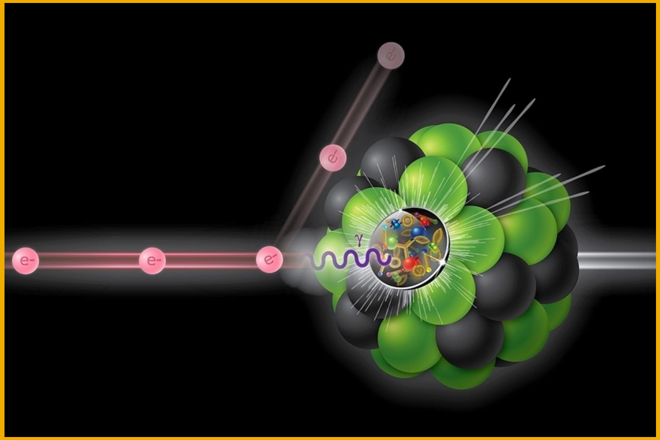
Y. Chien et al. (2015)



Expressed from jet functions, themselves computed from integrals of the splitting functions

The broader in-medium branching is directly observed!

II. EIC examples



I. Hadron production in eA

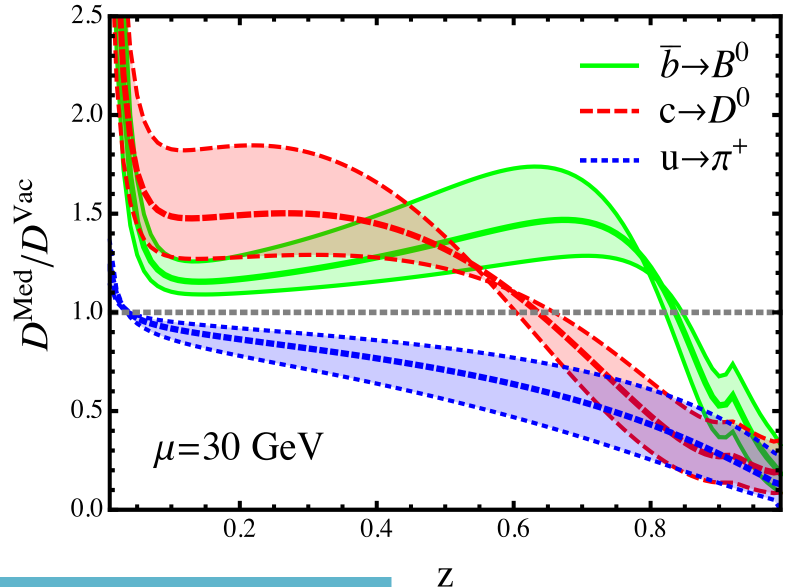
In-medium splitting functions provide correction to vacuum showers and correspondingly modification to DGLAP evolution for FFs

Integrate out the space-time information. All applications in momentum space

$$\frac{\mathrm{d}D_q(z,Q)}{\mathrm{d}\ln Q} = \frac{\alpha_s(Q^2)}{\pi}\int_z^1 \frac{\mathrm{d}z'}{z'}\left\{P_{q\to qg}(z',Q)D_q\left(\frac{z}{z'},Q\right) + P_{q\to gq}(z',Q)D_g\left(\frac{z}{z'},Q\right)\right\},$$

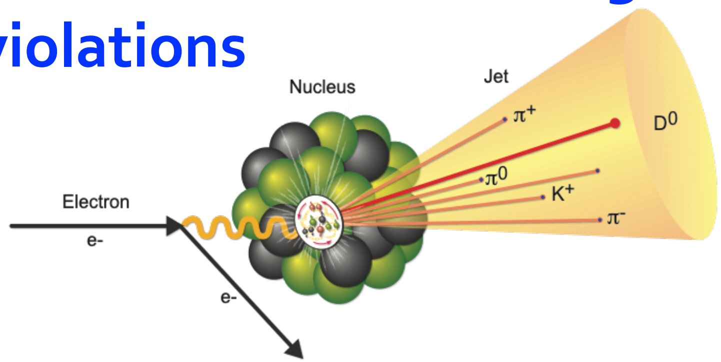
$$\frac{\mathrm{d}D_{\bar q}(z,Q)}{\mathrm{d}\ln Q} = \frac{\alpha_s(Q^2)}{\pi}\int_z^1 \frac{\mathrm{d}z'}{z'}\left\{P_{q\to qg}(z',Q)D_{\bar q}\left(\frac{z}{z'},Q\right) + P_{q\to gq}(z',Q)D_g\left(\frac{z}{z'},Q\right)\right\},$$

$$\frac{\mathrm{d}D_g(z,Q)}{\mathrm{d}\ln Q} = \frac{\alpha_s(Q^2)}{\pi}\int_z^1 \frac{\mathrm{d}z'}{z'}\left\{P_{g\to gg}(z',Q)D_g\left(\frac{z}{z'},Q\right)\right.$$
$$\left. + P_{g\to q\bar q}(z',Q)\left(D_q\left(\frac{z}{z'},Q\right) + f_{\bar q}\left(\frac{z}{z'},Q\right)\right)\right\}.$$



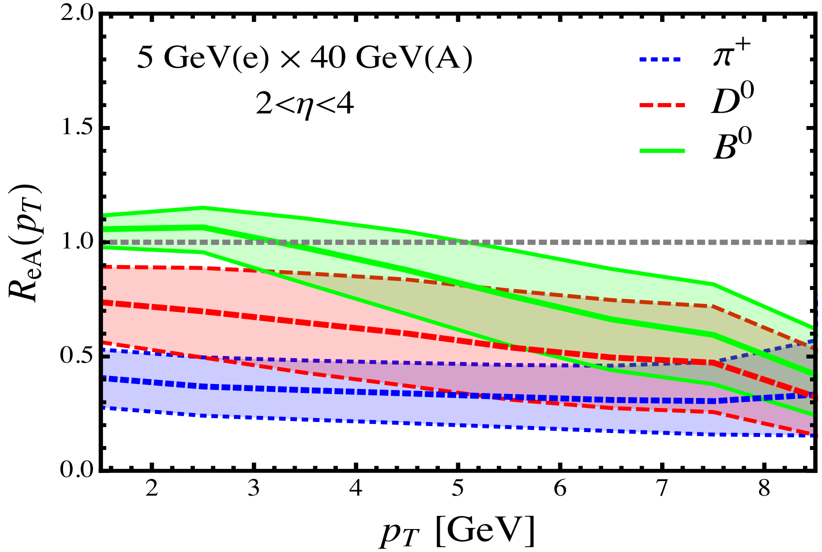
Z. Liu et al. (2020)

Medium induced scaling violations



- Always enhancement at small z but for pions (light hadrons) at very small values – mostly suppression
- Very pronounced differences between light and heavy flavor fragmentation

Light and heavy flavor suppression at the EIC



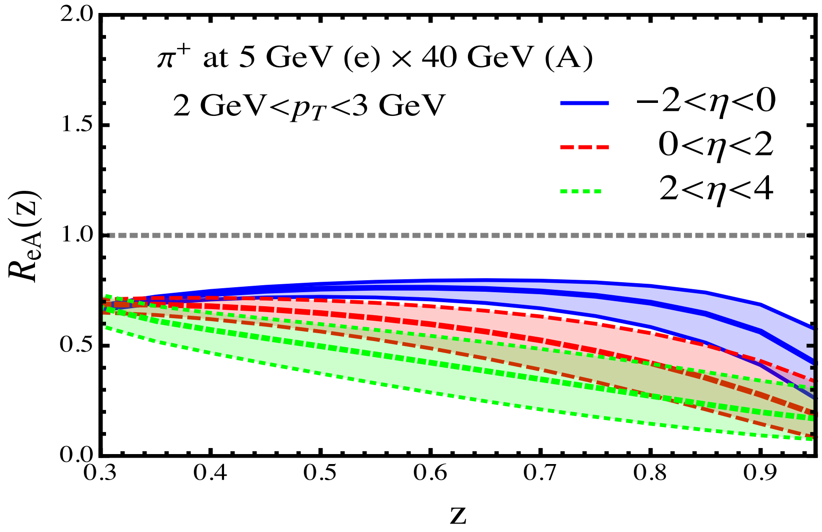
$$R^h_{eA}(p_T,\eta,z) = \frac{\left.\frac{N^h(p_T,\eta,z)}{N^{\rm inc}(p_T,\eta)}\right|_{\rm e+Au}}{\left.\frac{N^h(p_T,\eta,z)}{N^{\rm inc}(p_T,\eta)}\right|_{\rm e+p}}$$

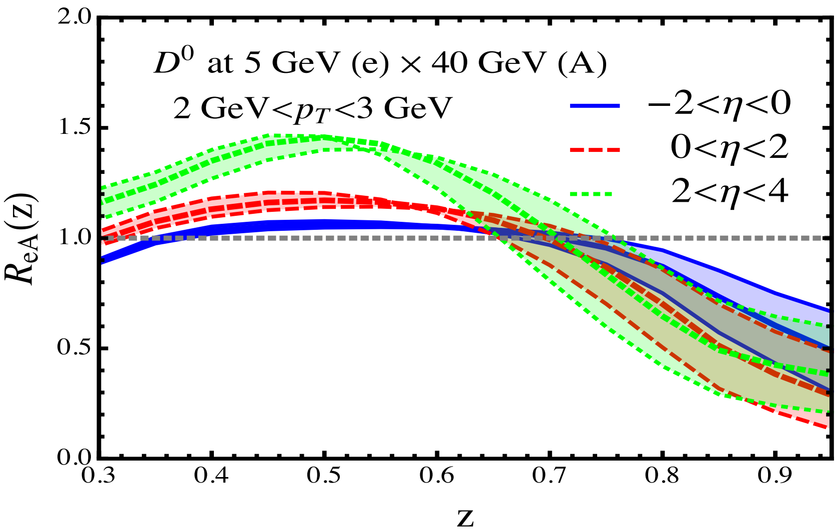
Effects are the largest at forward rapidities (p/A going)

Light pions show the largest nuclear suppression at the EIC. However to differentiate models of hadronization heavy flavor mesons are necessary

Z. Liu et al . (2020)

Differential



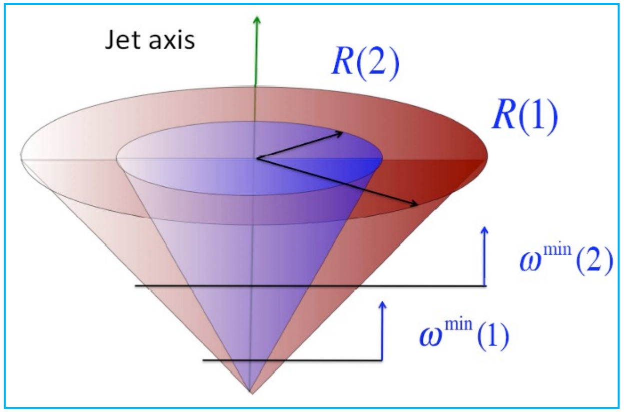


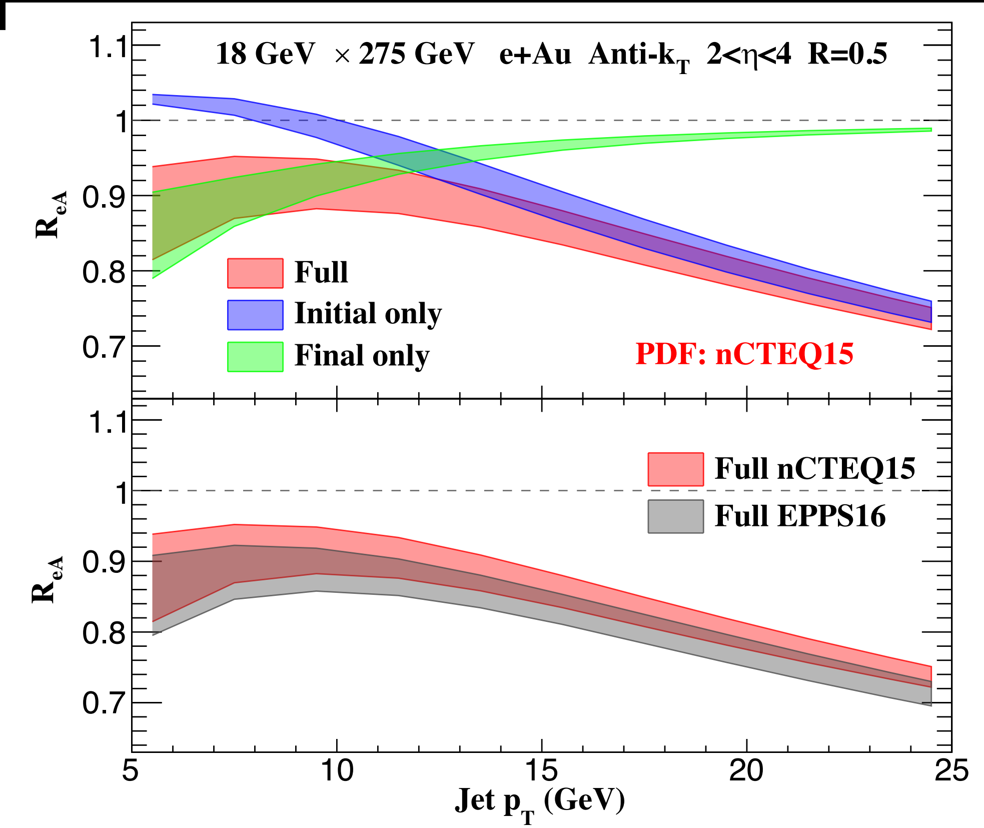
II. Jet results at the EIC

- The physics of reconstructed jet modification

H. Li et al. (2020)

$$R_{\mathrm{eA}}(R) = \frac{1}{A} \frac{\int_{\eta 1}^{\eta 2} d\sigma/d\eta dp_T \big|_{e+A}}{\int_{\eta 1}^{\eta 2} d\sigma/d\eta dp_T \big|_{e+p}}$$





Two types of nuclear effect play a role

- **Initial-state effects parametrized in nuclear parton distribution functions or nPDFs**
- **Final-state effects from the interaction of the jet and the nuclear medium – in-medium parton showers and jet energy loss**

- Net modification 20-30% even at the highest CM energy
- E-loss has larger role at lower p_T. The EMC effect at larger p_T

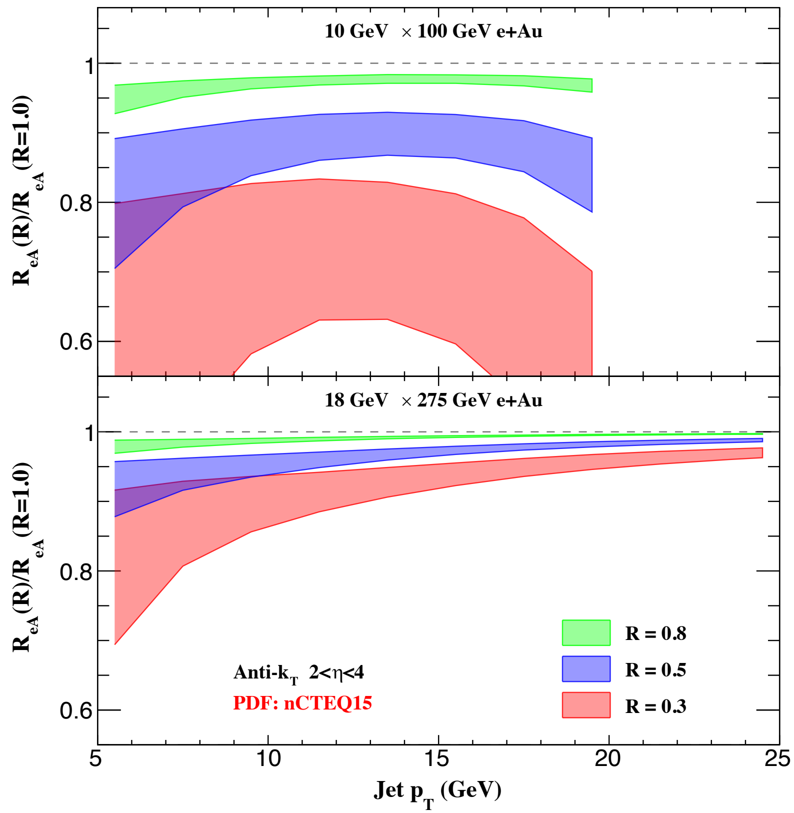
Separating initial-state from final-state effects at EIC

A key question – will benefit both nPDF extraction and understanding hadronization / nuclear matter transport properties - how to separate initial-state and final-state effects?

Define the ratio of modifications for 2 radii (it is a double ratio)

$$R_R = R_{eA}(R) / R_{eA}(R = 1)$$

- Jet energy loss effects are larger at smaller center of mass energies (electron-nuclear beam combinations)
- Effects can be almost a factor of 2 for small radii. Remarkable as it approaches magnitudes observed in heavy ion collisions (QGP)

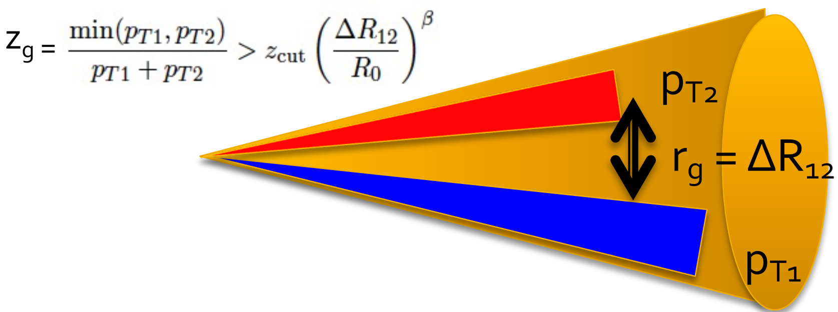


H. Li et al. (2020)

Initial-state effects are successfully eliminated

III. Heavy flavor jets substructure in DIS

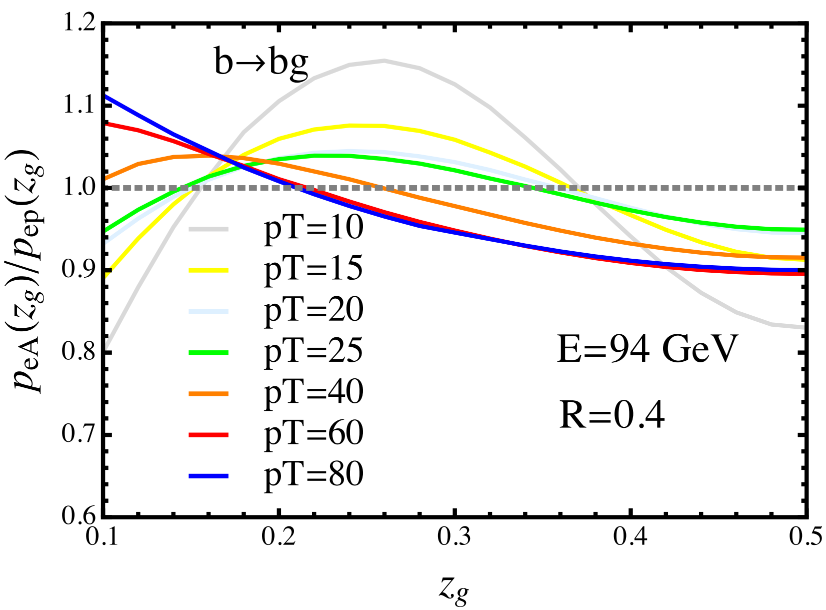
Z. Liu et al. (2021)

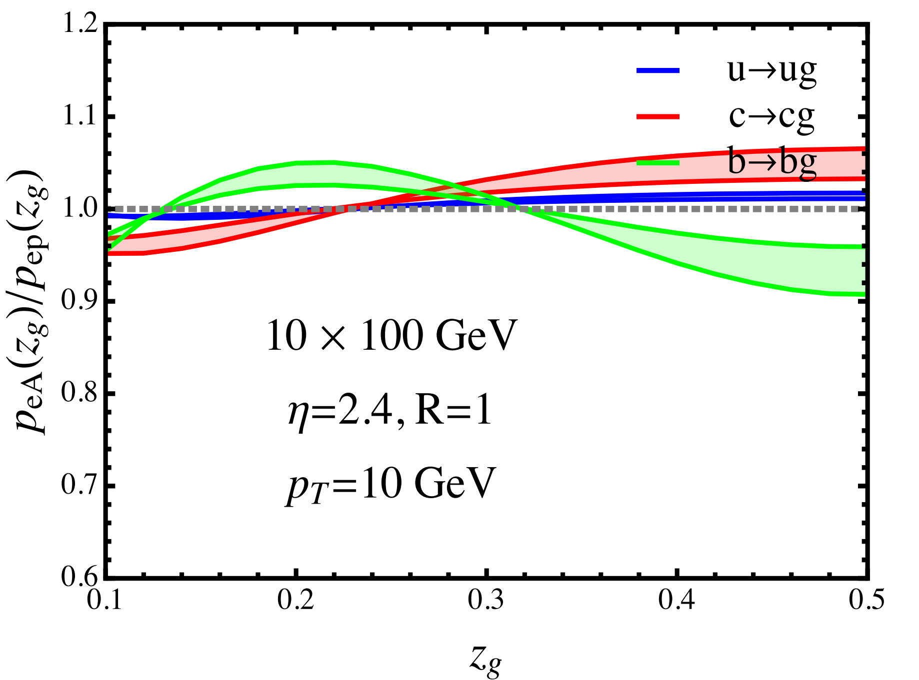


Related to the modification of jet cross sections is the modification of jet substructure. Example - Soft dropped momentum sharing distributions

- Modification of both c-jets and b-jets substructure in eA is relatively small
- It is dominated by limited phase space

Illustrative study: Kinematically not possible in DIS but illustrates very well the difference with HIC





III. Comments on MC implementation

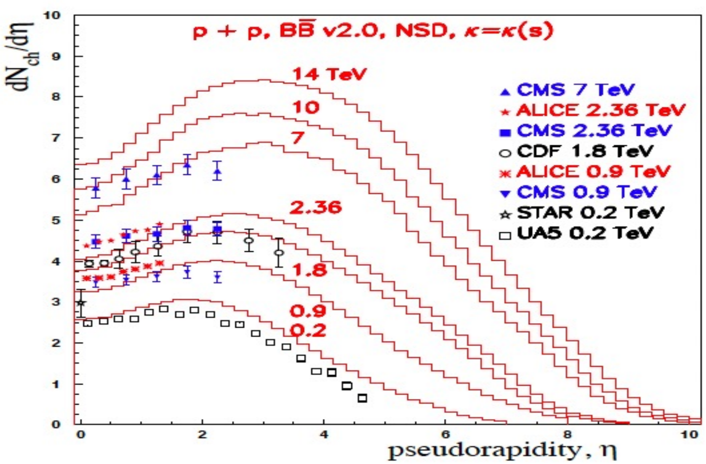
Disclaimer – I am not an expert. Selected results taken from literature, further developments might exist

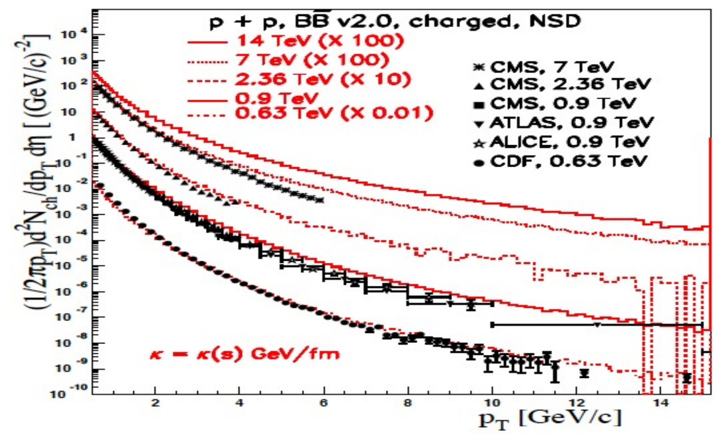
The Original Heavy Ion MC - HIJING

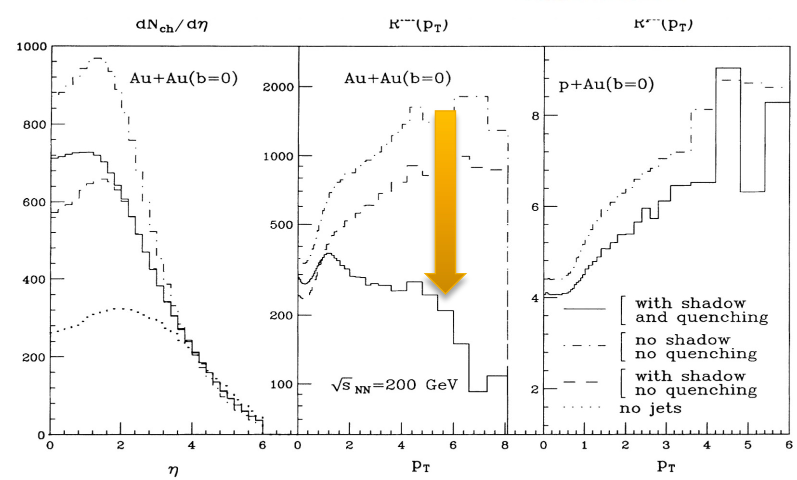
IMHO the physics I talked about has not been rigorously incorporated

The original HIJING was developed 30 years ago. It was used to develop the HI programs at RHIC and the LHC

X.N. Wang et al. (1991)







- Energy loss implemented as 1 GeV/fm (motivated by the string tension)
- Simply a guess

There are new developments such as HIJING ++

PYQUEN and JEWEL

PYQUEN

All PYTHIA based

Assumptions for angular gluon distribution

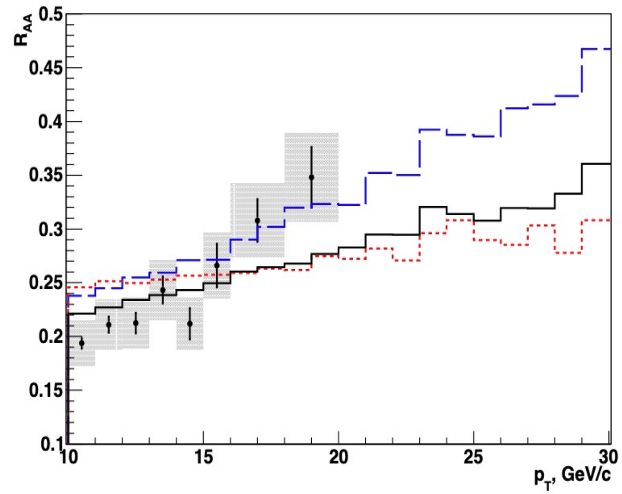
Gaussian $$\frac{dN^g}{d\theta} \propto \sin\theta \exp\left(-\frac{(\theta-\theta_0)^2}{2\theta_0^2}\right)$$

Wide $$\frac{dN^g}{d\theta} \propto 1/\theta$$

Extra wide $$\frac{dN^g}{d\theta} \propto 1/\sqrt{\theta}$$

- Collisional energy loss
- Soft gluon emission intensity

I. Lokhtin et al. (2006)



JEWEL

$$\sigma^{\mathrm{elas}} = \int_0^{|t_{\max}|} d|t| \frac{\pi\alpha_s^2(|t|+\mu_D^2)}{s^2} C_R \frac{s^2+(s-|t|)^2}{(|t|+\mu_D^2)^2}$$

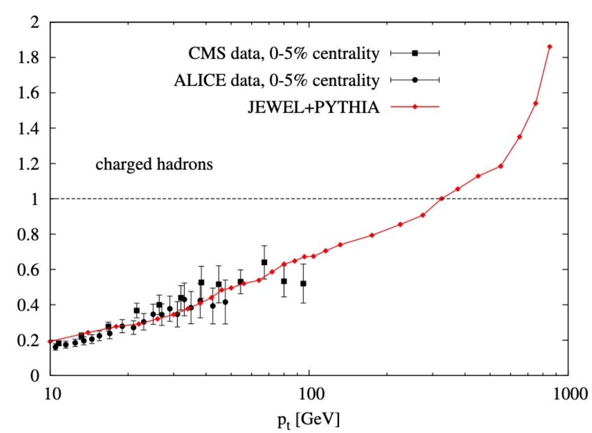
$$\hat{P}_{a\to bc}(z) \longrightarrow (1+f_{\mathrm{med}})\,\hat{P}_{a\to bc}(z)$$

Bertsch-Gunion

$$\frac{\mathrm{d}\sigma^{(\mathrm{GB})}}{\mathrm{d}\mathbf{k}_\perp \mathrm{d}\mathbf{q}_\perp} \propto \frac{\mathbf{q}_\perp^2}{\mathbf{k}_\perp^2(\mathbf{k}_\perp-\mathbf{q}_\perp)^2}$$

- Collisional energy loss implemented
- Radiative processes – just numerical enhancement of the vacuum shower
- Later version include Bertsch-Gunion radiation and formation time prescription to suppress radiation

K. Zapp et al. (2008)



Challenged by more differential observables

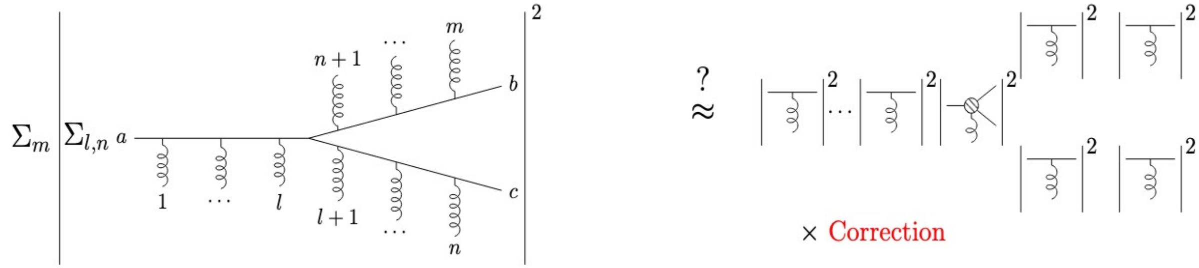
Other implementations

LIDO model

Not a general purpose model

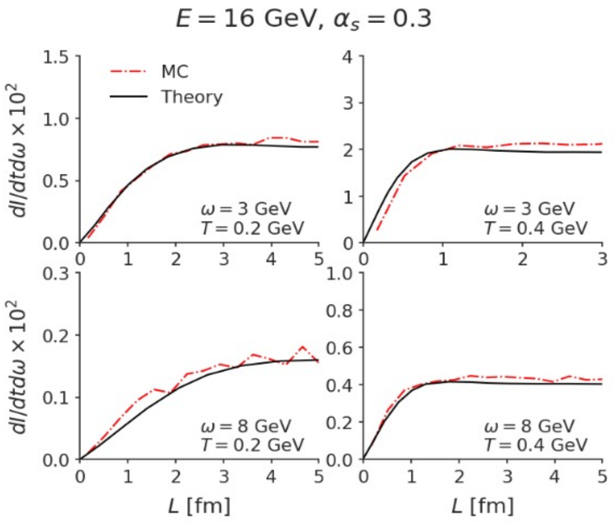
W. Ke et al. (2008)

- The same idea of reducing the incoherent radiation
- Improvement in determining the suppression factor



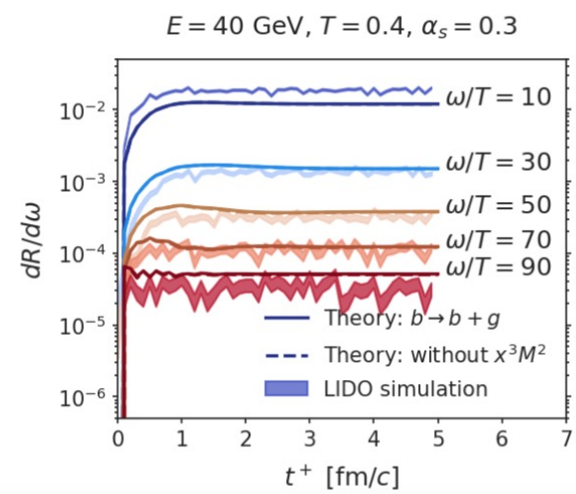
Let $N = \tau_f/\lambda$. From analysis[1] of the AMY equation[2] for the single-gluon emission rate:

Semi-classical rate ($N < 1$)	Leading-ln N ($N \gg 1$)	NLL
$\frac{dR^{\mathrm{incoh}}}{d\omega}$	$\propto \frac{dR^{\mathrm{incoh}}}{d\omega}\frac{1}{N}$	$\propto \frac{dR^{\mathrm{incoh}}}{d\omega}\frac{1}{N'}$ improved N'



- **Can reproduce radiative spectrum in the large number of scatterings limit**
- **Generalized to heavy quarks**

- Dead-cone: $1/p_\perp^2 \to (p_\perp^2 + x^2M^2)^{-1}$



Conclusions

- In the past 30 years reactions with nuclei have produced spectacular results. The key to their interpretation is in-medium parton showers
- In-medium splitting functions have been derived using different methods. In-medium parton showers are softer and broader than the ones in the vacuum. Experimentally verified. Transverse and longitudinal degrees of freedom do not factorize
- In-medium splitting functions can be calculated and tabulated (integrating out the space-time information). Implemented in higher order and resumed calculations. Very significant effects on hadrons, jets and jet substructure at the EIC
- Monte Carlos that incorporate this physics properly do not exist. The problem is the coherent nature of the emission. Various approximations and prescriptions how to mimic LPM effect proposed. The detailed shower characteristics
- For serious implementation of in-medium showers in MC generators significant effort is needed (LUTs combined with modelling coherence)

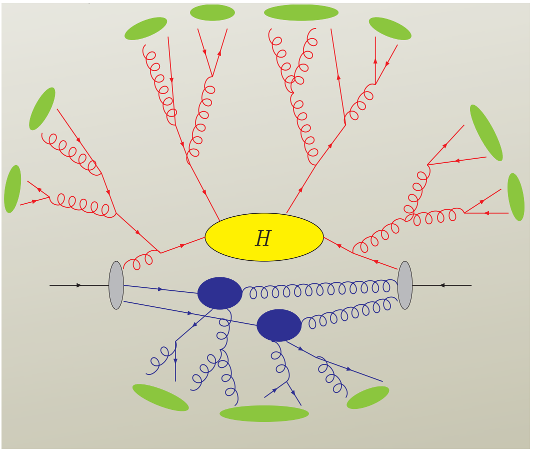
Full in medium splitting

- Full massless and massive in-medium splitting functions now available to first order in opacity

G. Ovanesyan et al . (2011)

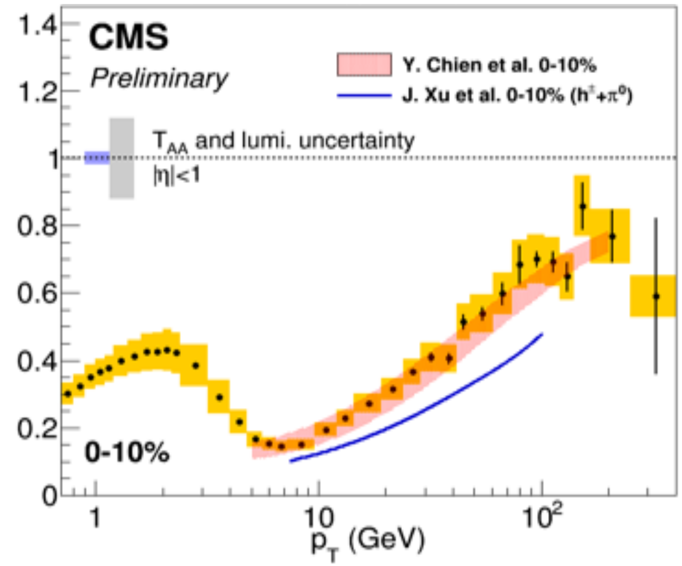
- SCET-based effective theories created to solve this problem

F. Ringer et al . (2016)



Representative example

$$\begin{aligned}
\left(\frac{dN^{\rm med}}{dx\,d^2\boldsymbol{k}_\perp}\right)_{Q\to Qg} =\; & \frac{\alpha_s}{2\pi^2}C_F\int\frac{d\Delta z}{\lambda_g(z)}\int d^2\boldsymbol{q}_\perp\frac{1}{\sigma_{el}}\frac{d\sigma_{el}^{\rm med}}{d^2\boldsymbol{q}_\perp}\Bigg\{\left(\frac{1+(1-x)^2}{x}\right)\Bigg[\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2}\\
& \times\left(\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2}-\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z])+\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2+\nu^2}\cdot\Bigg(2\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2+\nu^2}-\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2}\\
& -\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2}\Bigg)(1-\cos[(\Omega_1-\Omega_3)\Delta z])+\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2}\cdot\frac{\boldsymbol{C}_\perp}{\boldsymbol{C}_\perp^2+\nu^2}(1-\cos[(\Omega_2-\Omega_3)\Delta z])\\
& +\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2}\cdot\left(\frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2+\nu^2}-\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2}\right)(1-\cos[\Omega_4\Delta z])-\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2}\cdot\frac{\boldsymbol{D}_\perp}{\boldsymbol{D}_\perp^2+\nu^2}(1-\cos[\Omega_5\Delta z])\\
& +\frac{1}{N_c^2}\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2}\cdot\left(\frac{\boldsymbol{A}_\perp}{\boldsymbol{A}_\perp^2+\nu^2}-\frac{\boldsymbol{B}_\perp}{\boldsymbol{B}_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z])\Bigg]\\
& +x^3m^2\left[\frac{1}{\boldsymbol{B}_\perp^2+\nu^2}\cdot\left(\frac{1}{\boldsymbol{B}_\perp^2+\nu^2}-\frac{1}{\boldsymbol{C}_\perp^2+\nu^2}\right)(1-\cos[(\Omega_1-\Omega_2)\Delta z])+\ldots\right]\Bigg\}
\end{aligned}$$



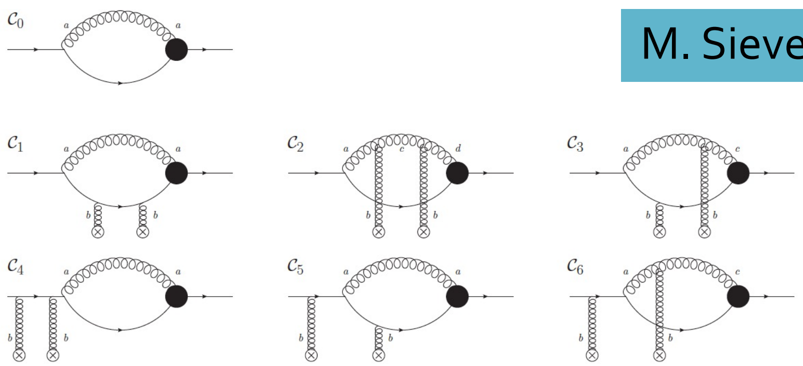
Z. Kang et al . (2015)

- For the first time we were able to do is higher order and resummed calculations

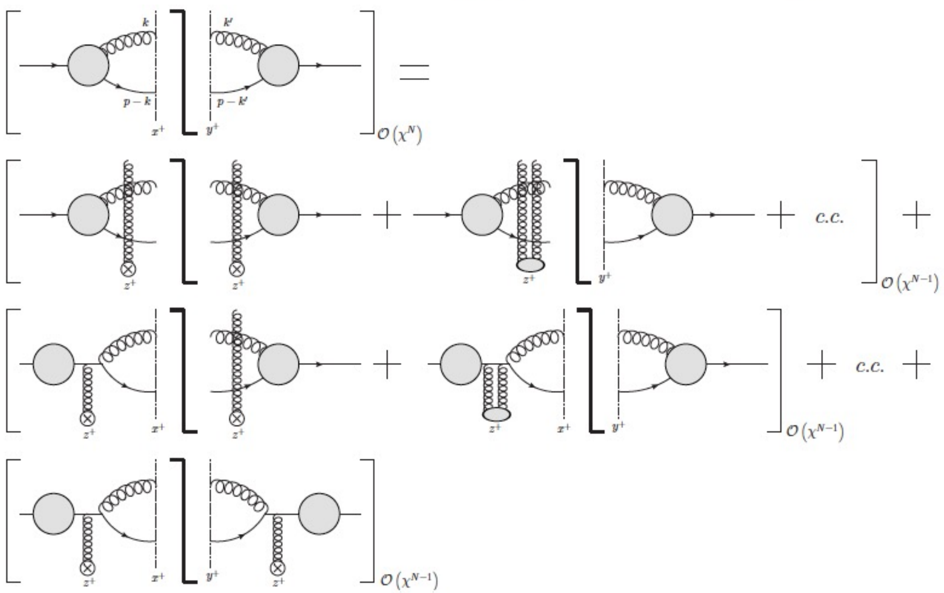
Parton branching to any order in opacity

- Treating color (one complication in QCD).

M. Sievert et al . (2018)



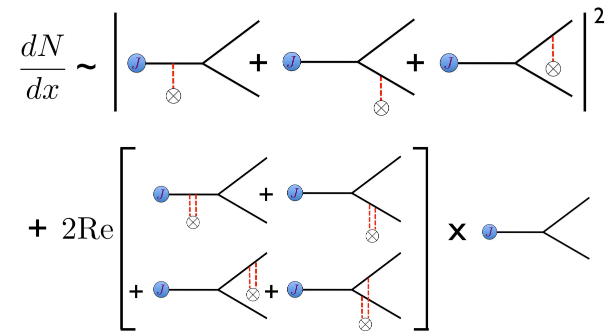
$$\mathcal{C}_1 = \frac{1}{N_c C_F} \mathrm{tr}[t^b t^b\, t^a M^a] = \mathcal{C}_0\,,$$
$$\mathcal{C}_2 = \frac{1}{N_c C_F} f^{acb} f^{cdb} \mathrm{tr}[t^a M^d] = -\frac{N_c}{C_F}\,\mathcal{C}_0\,,$$
$$\mathcal{C}_3 = \frac{1}{N_c C_F} f^{acb} \mathrm{tr}[t^b t^a M^c] = \frac{i N_c}{2C_F}\,\mathcal{C}_0\,,$$
$$\mathcal{C}_4 = \frac{1}{N_c C_F} \mathrm{tr}[t^a\, t^b t^b M^a] = \mathcal{C}_0\,,$$
$$\mathcal{C}_5 = \frac{1}{N_c C_F} \mathrm{tr}[t^b t^a t^b M^a] = \frac{-1}{2N_c C_F}\,\mathcal{C}_0\,,$$
$$\mathcal{C}_6 = \frac{1}{N_c C_F} f^{acb} \mathrm{tr}[t^a t^b M^c] = \frac{-i N_c}{2C_F}\,\mathcal{C}_0\,.$$



- Color is not entangled, homogeneous structure and multiplicative factors that can be algebraically treated
- Finally, relative to the splitting vertex we classify the as
- Initial/Initial, Initial/Final, Final/Initial and Final/Final

Generalizing the result to all in-medium splittings

- Note – all splittings have the same topology.
  Same - structure, interference phases, propagators
  Different - mass dependence, wavefunctions, color (which also affects transport coefficients)



$$\langle \psi(x,\underline{\kappa})\, \psi^*(x,\underline{\kappa}') \rangle = \frac{8\pi\alpha_s\, f(x)}{[\kappa_T^2+\nu^2 m^2]\,[\kappa_T'^2+\nu^2 m^2]} \left[g(x)\, (\underline{\kappa}\cdot\underline{\kappa}') + \nu^4 m^2 \right] \quad \Delta E^-(\underline{\kappa}) = -\frac{\kappa_T^2+\nu^2 m^2}{2x(1-x)p^+}$$

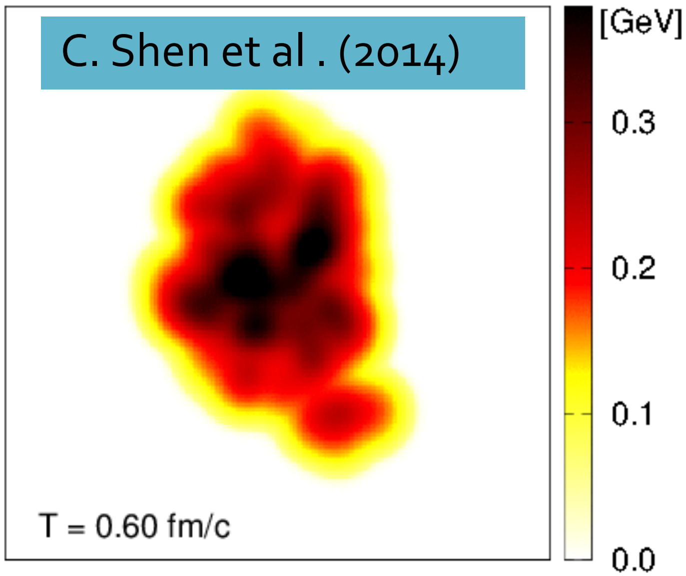
- Master table that gives all ingredients

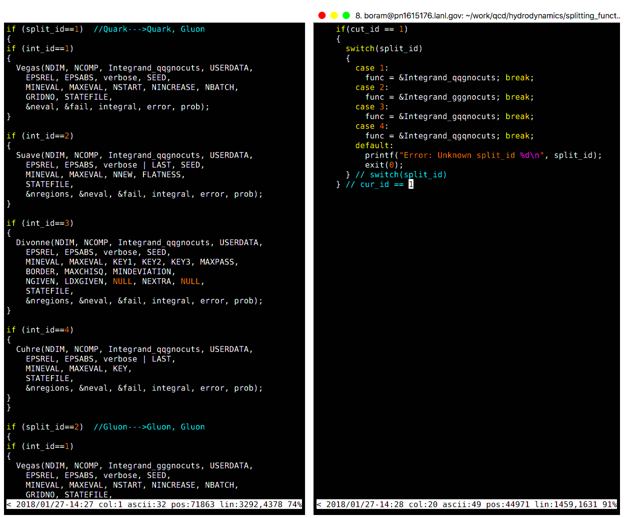
	d_1	d_2	d_3	d_4	d_5	d_6	v_1	v_2	v_3	v_4	λ_R^+	$\mathcal{C}_0$	ν	$f(x)$	$g(x)$
G/q	1	1	$\frac{N_c}{C_F}$	$\frac{-1}{2N_c C_F}$	$\frac{N_c}{2C_F}$	$\frac{-N_c}{2C_F}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{-N_c}{2C_F}$	$\frac{N_c}{2C_F}$	λ_q^+	C_F	x	$1-x$	$1+(1-x)^2$
q/q	1	1	$\frac{N_c}{C_F}$	$\frac{-1}{2N_c C_F}$	$\frac{N_c}{2C_F}$	$\frac{-N_c}{2C_F}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{-N_c}{2C_F}$	$\frac{N_c}{2C_F}$	λ_q^+	C_F	$1-x$	x	$1+x^2$
q/G	1	$\frac{C_F}{N_c}$	$\frac{C_F}{N_c}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2N_c^2}$	$-\frac{1}{2}$	$-\frac{C_F}{2N_c}$	$\frac{-N_c}{2C_F}$	$\frac{-1}{2N_c^2}$	λ_G^+	$\frac{1}{2}$	1	$x(1-x)$	$x^2+(1-x)^2$
G/G	1	1	1	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	λ_G^+	N_c	0	$1+x^4+(1-x)^4$	1

We have now solved the problem for all splitting functions

M. Sievert et al . (2019)

Improvements in physics & code





Refactoring

- Code is **restructured** (in C++) and shortened (**24K → 8K lines**). 20x speed improvement

Effective incorporation of simulated QGP medium

- Reduced overhead for calling QGP medium grid function. 2x speed improvement

Efficient on-node parallelization

- New parallelization shows much better scaling 10x speed improvement

Overall improvement:

18 days → 1 hour