Fragmentation Functions

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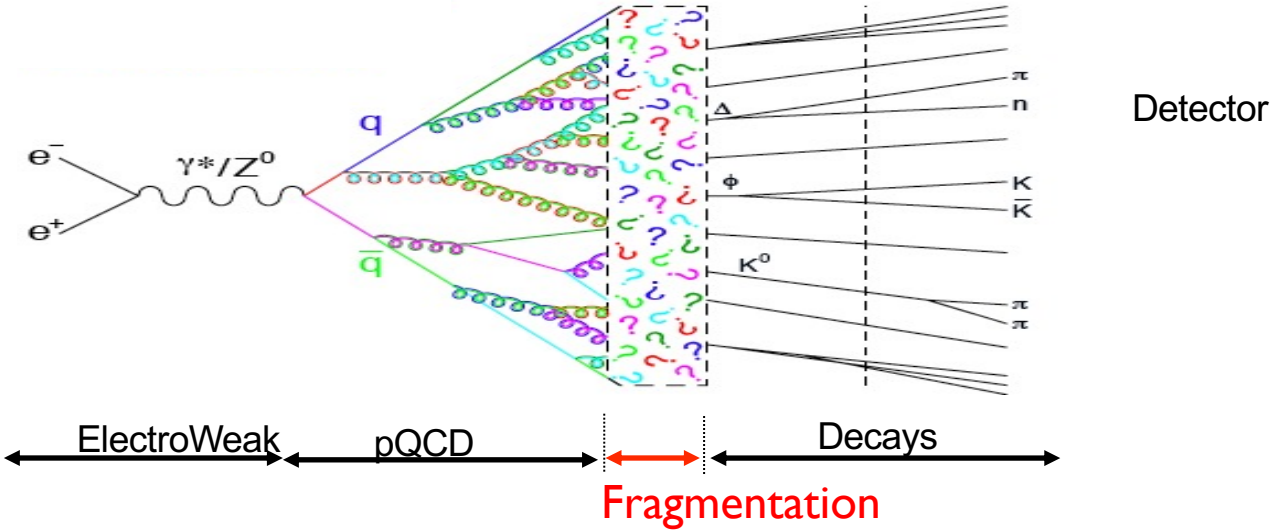
Factorized QCD: Hadronization described by Fragmentation Functions

- **Field, Feynman (1977):** Fragmentation functions encode the information on how partons produced in hard-scattering processes are turned into an observed colorless hadronic bound final-state [PRD 15 (1977) 2590]

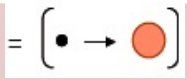


- Complementary to the study of nucleon structure (PDFs)
- Cannot be computed on the lattice
- Needed to extract nucleon structure from semi-inclusive scattering
- Factorization theorems → FFs universal objects

FFs can be organized similar to PDFs (here only leading twist)

- LO/LT: probability of finding hadron with momentum

Observables:
z: fractional energy of the quark carried by the hadron
$p_{h,T}$: transverse momentum of the hadron wrt the quark direction: **TMD FFs**

Parton polarization → **Hadron Polarization ↓**	**Spin averaged**	**longitudinal**	**transverse**
spin averaged	$D_1^{h/q}(z, p_T)$ = 		$H_1^{\perp h/q}(z, p_T)$ = 
longitudinal			
Transverse (here Λ)			

Polarization in the final States

- Analogue →similar to PDFs encoding spin/orbit correlations
- Determining final state polarization needs self analyzing decay (Λ)
- Gluon FFs similar but with circular/linear polarization (not as relevant for e+e-)

Parton polarization → / Hadron Polarization ↓	Spin averaged	longitudinal	transverse
spin averaged	$D_1^{h/q}(z, p_T)$ =		$H_1^{\perp h/q}(z, p_T)$ =
longitudinal		$G_1^{\Lambda/q}(z, p_T)$ =	$H_{1L}^{h/q}(z, p_T)$
Transverse (here Λ)	$D_{1T}^{\perp \Lambda/q}(z, p_T)$ =	$G_{1T}^{h/q}(z, p_T)$=	$H_1^{\Lambda/q}(z, p_T)$ = $H_{1T}^{\perp \Lambda/q}(z, p_T)$ =

- Encode Spin-Orbit correlations in hadronization
- Needed to access (spin dependent) parton structure of the nucleon
- Can probe fundamental QCD questions (e.g. $D_{1T}^{\perp} \leftrightarrow f_{1T}^{\perp}$)

Di-hadron fragmentation Functions

Additional Observable:

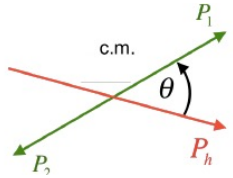
$\vec{R} = \overrightarrow{P_1} - \overrightarrow{P_2}$:

The relative momentum of the hadron pair is an additional degree of freedom:

the orientation of the two hadrons w.r.t. each other and the jet direction can be an indicator of the quark transverse spin

Parton polarization → **Hadron Polarization ↓**	**Spin averaged**	**longitudinal**	**transverse**
spin averaged	$D_1^{h/q}(z, M)$ 		$H_1^{\perp h/q}(z, p_T\mathbf{M}, \mathbf{(Ph)},\theta)$ 'Di-hadron Collins'
longitudinal			
Transverse	Type equation here.	$G_1^{\perp}(z,M,P_h,\theta)=$ T-odd, chiral-even →jet handedness QCD vaccum strucuture 	$H_1^{\sphericalangle}(z,M,(P_h),\theta)=.$ T-odd, chiral-odd Colinear 

- Relative momentum of hadrons can carry away angular momentum
  - Relative and total angular momentum →In principle endless tower of FFs



More degrees of freedom→More information about correlations in final state

Relation to MC Modeling of Hadronization

Fragmentation Functions

- Focus on more '**inclusive**' measurements → factorization holds
- Recent activity in more exclusive measurements (in particular jets)
- Needs MCEGs for experimental extraction
- Very precise extractions→Benchmark for MCEGs

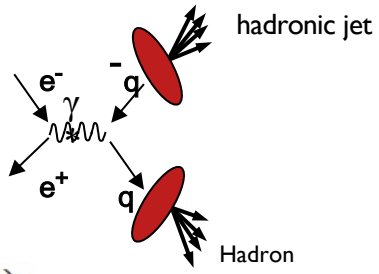
Hadronization Model in MCEG

- Exclusive final states
- "Hard" subprocesses well constrained by theory
- Measurements focusing on MCEG improvement different from measurements extracting hard physics (grooming) or FFs (more exclusive)

Access of FFs for light mesons in e^+e^- (spin averaged case)

$$\frac{1}{\sigma_{\rm tot}}\frac{d\sigma^{e^+e^-\to hX}}{dz} := \frac{1}{\sum_q e_q^2}\left(2F_1^h(z,Q^2)+F_L^h(z,Q^2)\right),$$

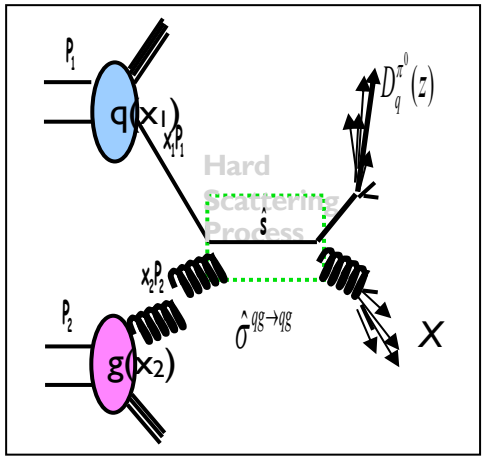
$$2F_1^h(z,Q^2) = \sum_q e_q^2\left(D_1^{h/q}(z,Q^2)+\frac{\alpha_s(Q^2)}{2\pi}\left(C_1^q\otimes D_1^{h/q}+C_1^g\otimes D_1^{h/g}\right)(z,Q^2)\right)$$

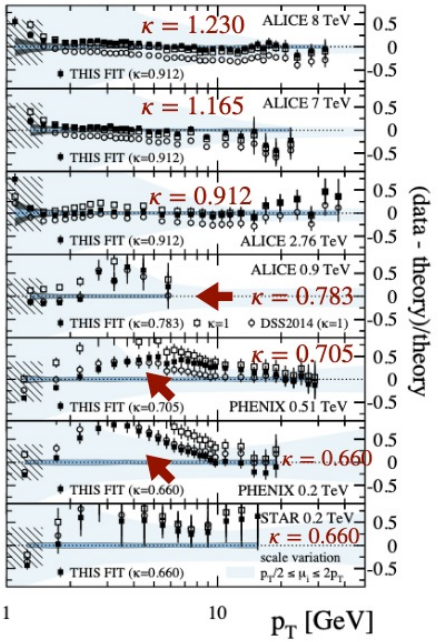


- Cleanest process→testbed for QCD calculations
- Extraction (collinear) at NNLO
- Limited access to flavor
  - (Use different couplings to γ^* and Z^0)
  - (Use polarization (SLD) and parity violating coupling)
  - **Use back-to-back correlations for different flavor combinations**
- Limited access to gluon FF
  - From evolution
  - From three jet events (but theory treatment not clear)

Flavor separated and gluon FFs from pp & $SIDIS$

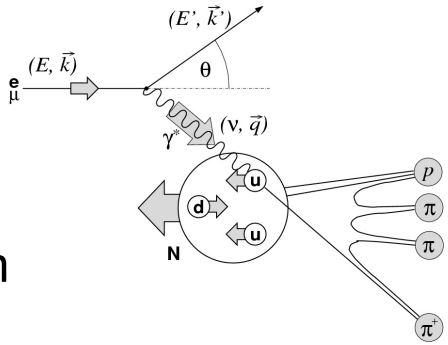
- Single Particle in pp
  - Not very clean
  - Already issues fitting (here DSS14)
  - Needed for gluon FF

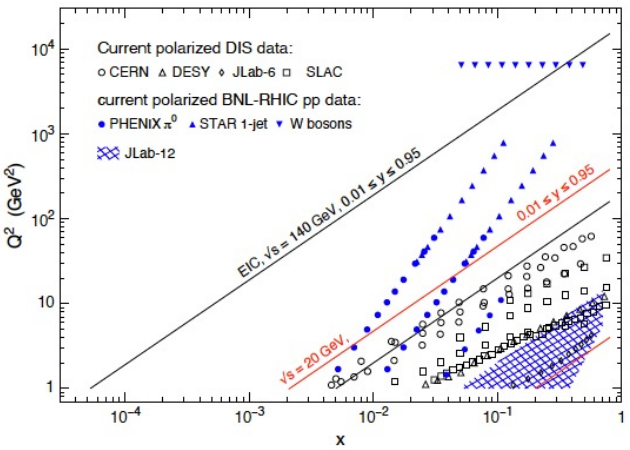




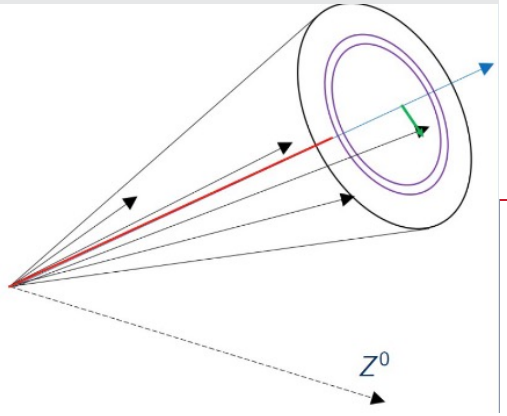
I. Borsa at INT FF workshop

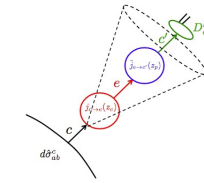
- SIDIS
  - Cleaner
  - Fixed target low scale
  - Large recent datasets from COMPASS, HERMES
  - Generally well described by theory/MC even though parts of phase space outside region of validity



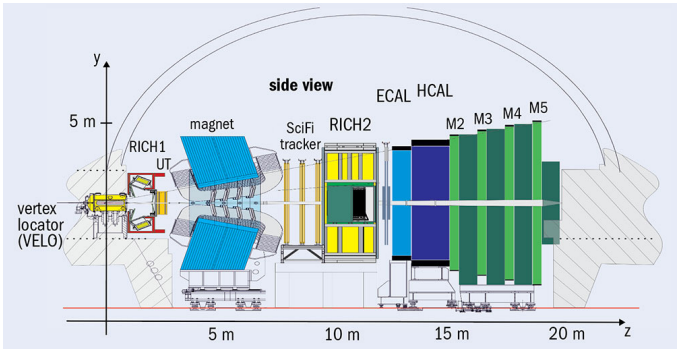


In Jet Fragmentation:



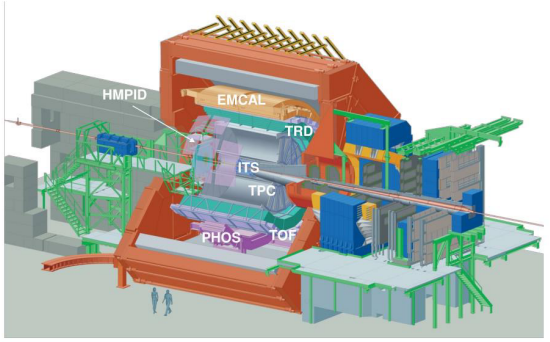


- Intense theoretical interest and development
- More exclusive final state/more complete event descriptions
  - Correlations relative to jet axis
  - Inside/outside jet axis
  - Jet tagging



- Multiple grooming techniques
  - Isolate ‘hard’ physics (e.g. Jets splitting function ↔ AP splitting function,)
  - Conversely difference between groomed/non Grooming→w/ w/o →soft contributions
  - Lund jet plane → separate soft/hadronic contributions

ALICE Detector

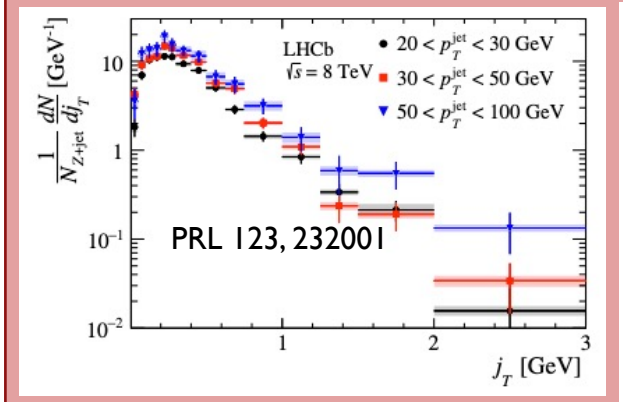


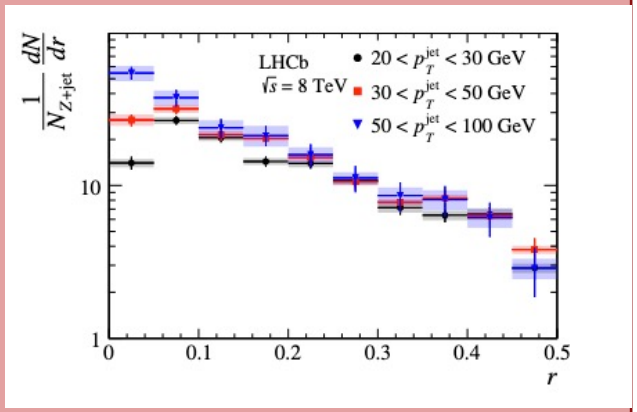
- In particular ALICE, LHCb
  - Quark enhancement in jet+Z^0

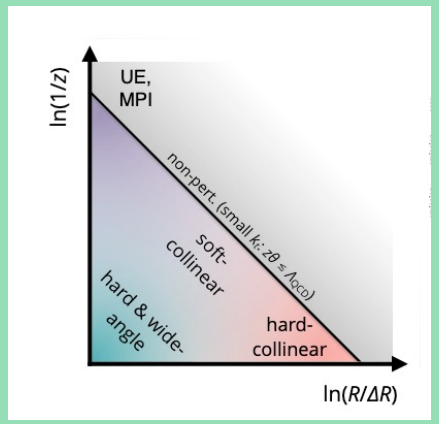
- Recent re-analysis of LEP(ALPEH) data (108.04877 [hep-ex])
- New effort at Belle II

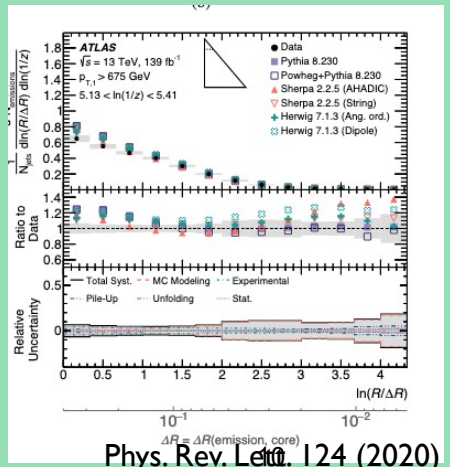
Examples of Jet Tomography at LHC

- Jet 'tomography'
- Measurement of the Lund plane
- Open HF
- J/Ψ
- Many jet observables



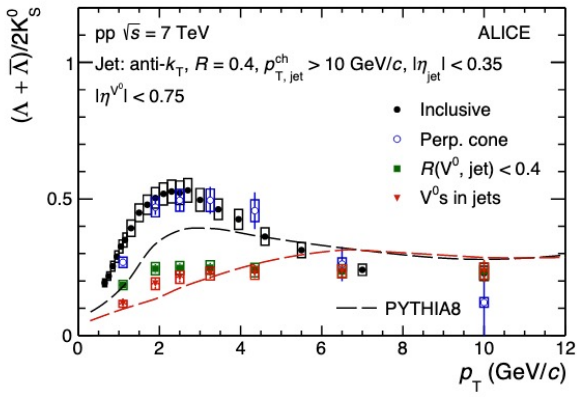






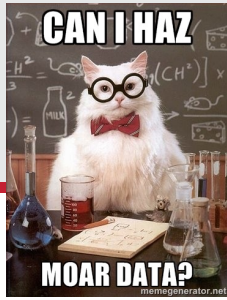
Phys. Rev. Lett. 124 (2020) 222002

Strange baryon/meson ration

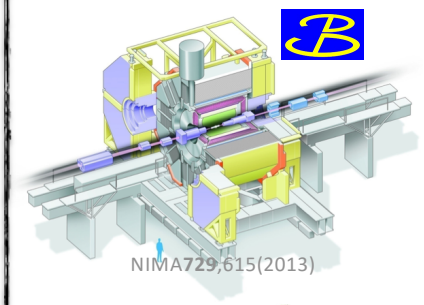


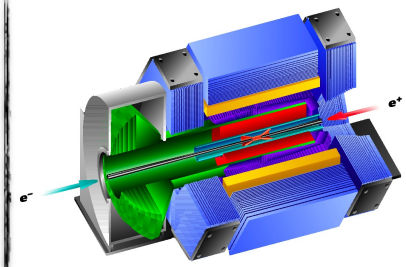
arXiv:2105.04890

Going back to e^+e^-: Role of b-factories



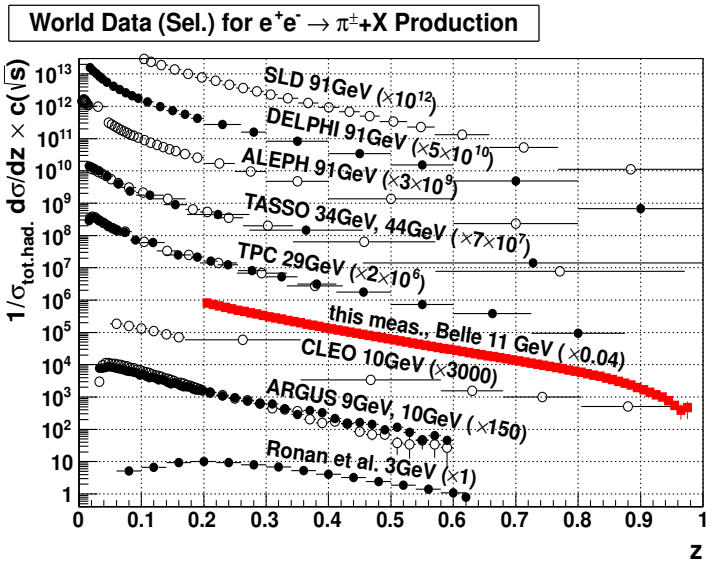
•Asymmetric-energy e^+e^- collider
• $\sqrt{s} \sim 10.6$ GeV ($\Upsilon(4S)$)
• $\beta\gamma=0.425$
• $L \sim 1$ ab^{-1}

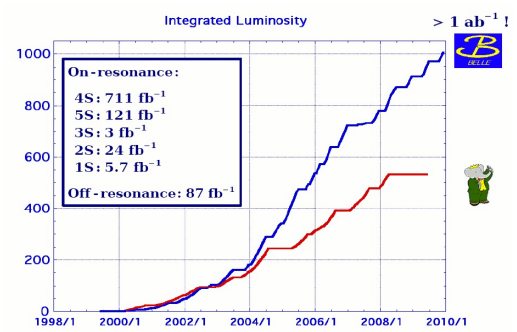




•Asymmetric-energy e^+e^- collider
• $\sqrt{s} \sim 10.6$ GeV ($\Upsilon(4S)$)
• $\beta\gamma=0.65$
• $L \sim 500$ fb^{-1}

- Dominated by B factories
- Limited lever arm in $\sqrt{s}$ in particular at high z
- Precision data includes charged single hadrons π, K Λ, charmed baryons…
- Well described at NNLO (e.g. DSS, NNFF)

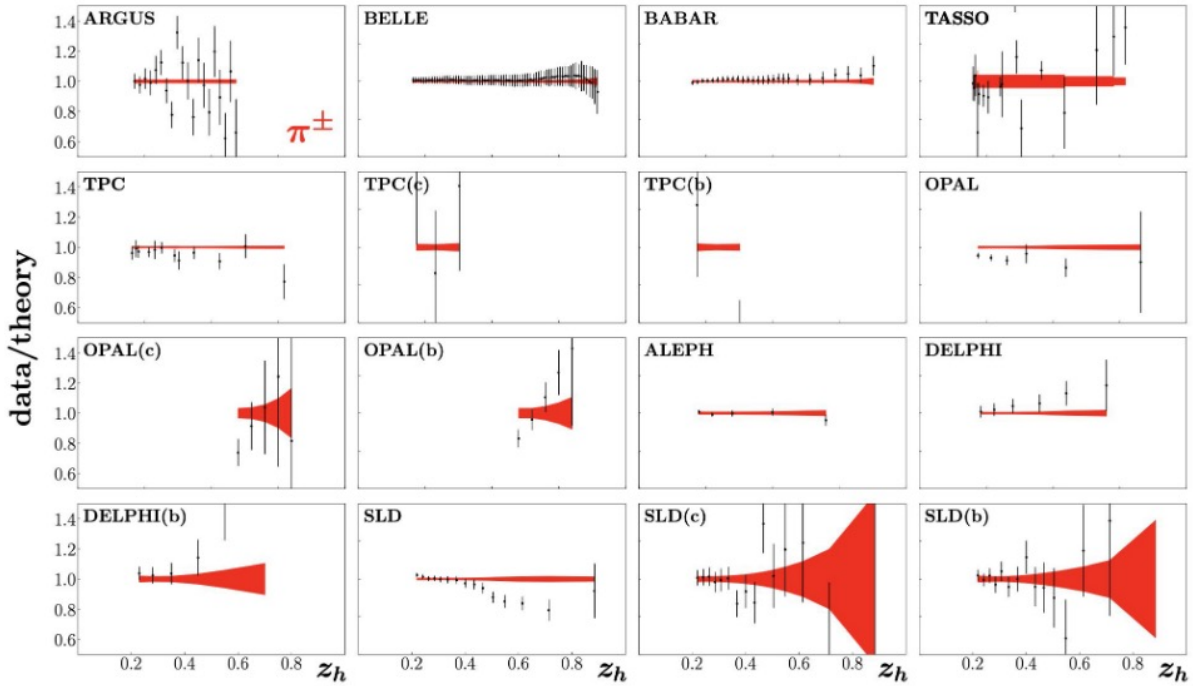


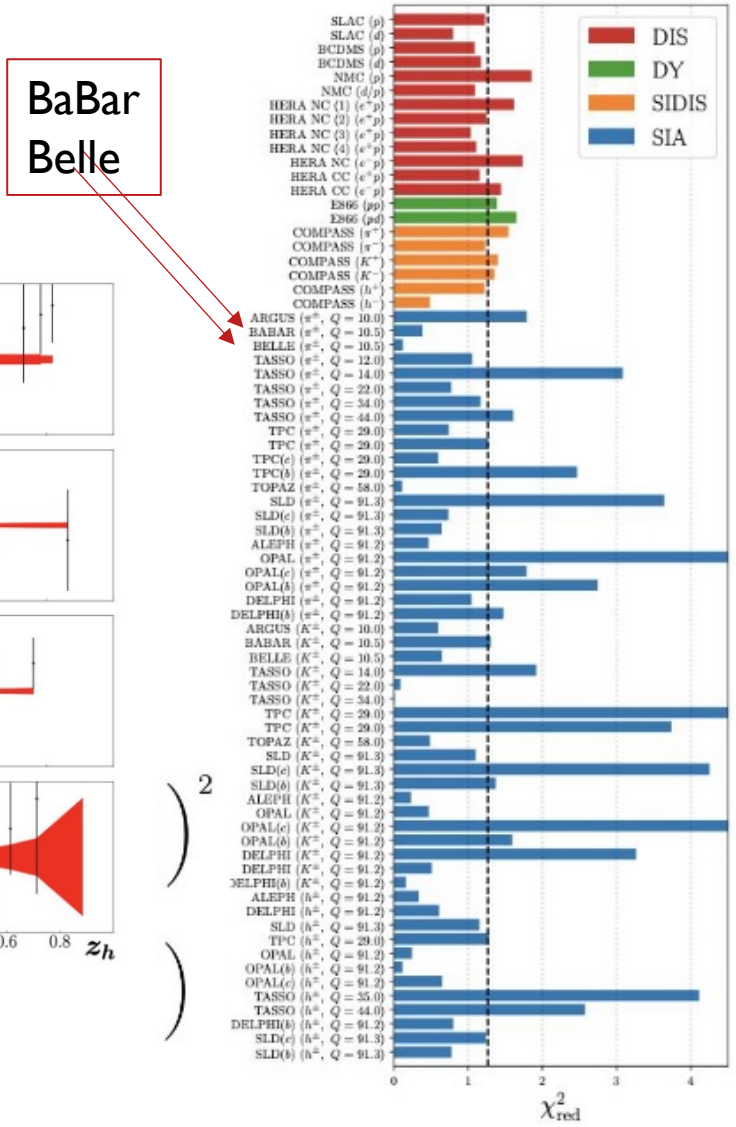


Phys.Rev.Lett. 111 (2013) 062002 (Belle)
Phys.Rev. D88 (2013) 032011 (BaBar)

Example of state of theory fitting single hadron data

- Here JAM 2021
- NLO extraction

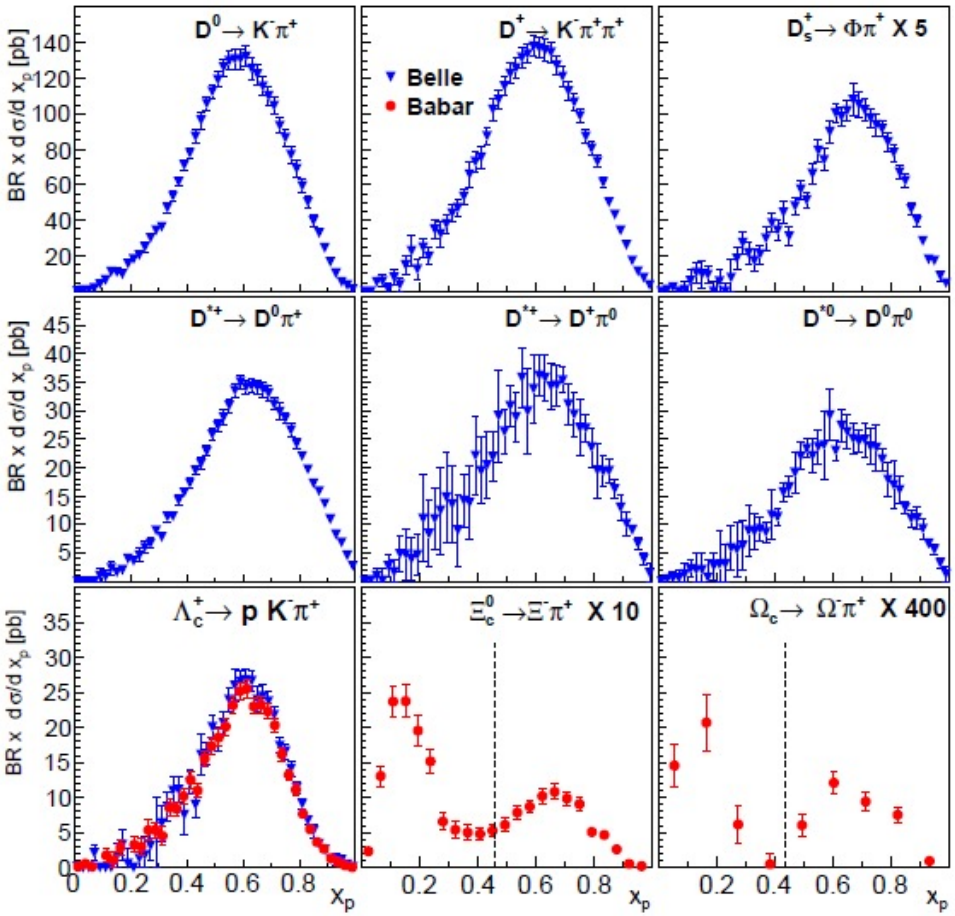




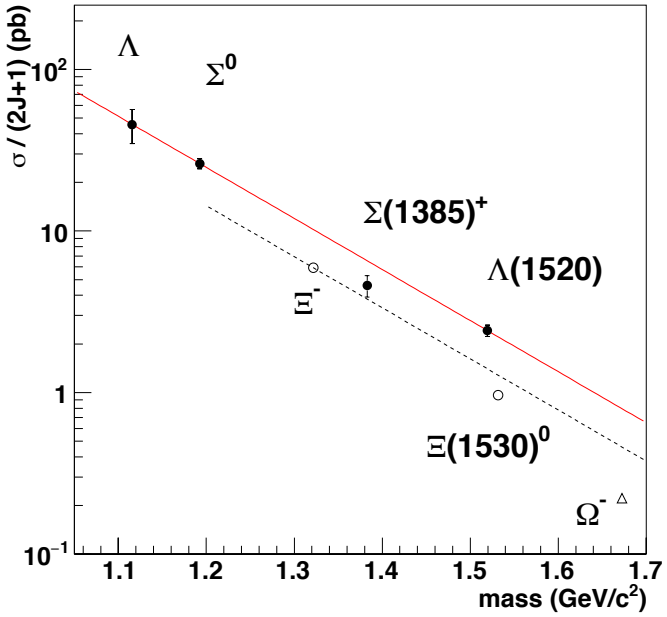
Further results from Belle

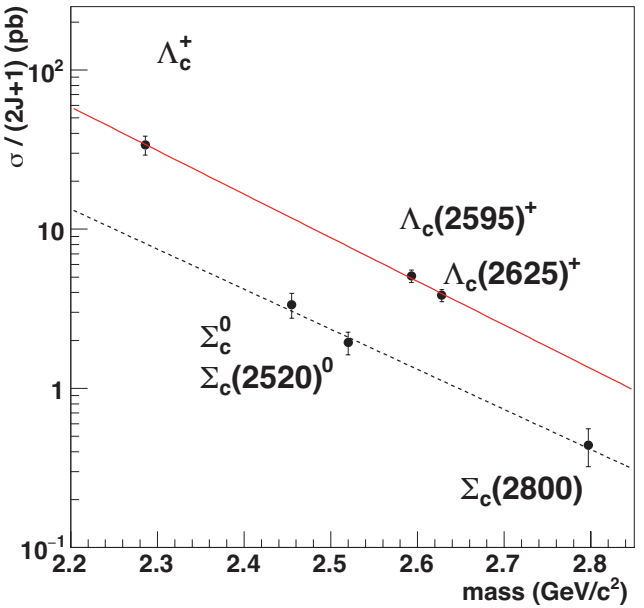
- Single hadron differential cross sections for charged pions, kaons, and protons vs z
- Single hadron differential cross sections for Λ, Σ, Ξ, Ω, Λ_c, Σ_c, Ξ_c, Ω_c (etc) vs x_p
- Heavier particles generally plotted vs normalized momentum
- Unlike light hadrons charmed hadrons contain large fraction of charm quark momentum

PRL.95, 142003 (2005)(Babar)
PRD73, 032002 (2006) (Belle)
PRD75, 012003 (2007)(Babar)
PRL 99, 062001 (2007)(Babar)



Mass Dependence of σ →test hadronization model



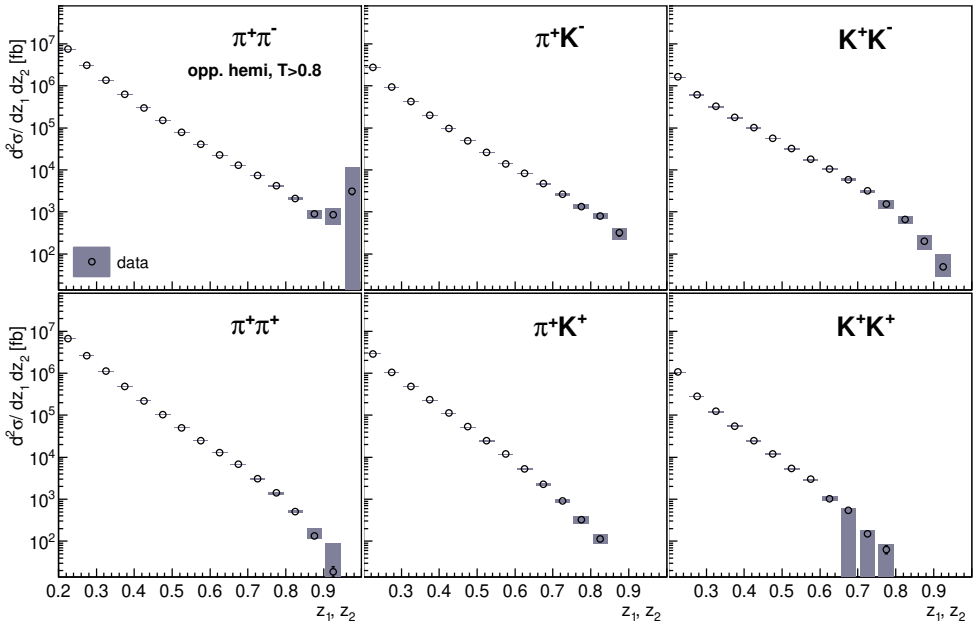


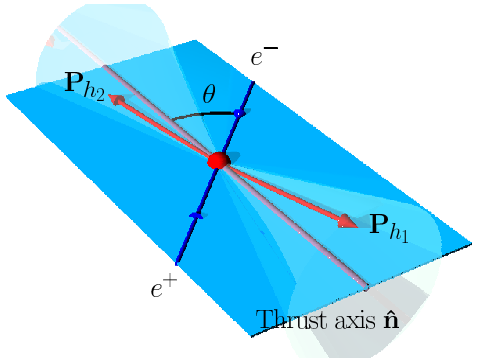
- Found consistent with di-quark model

PRD 97, 072005 (2018)

More complex final states, di-hadrons Back-to-back

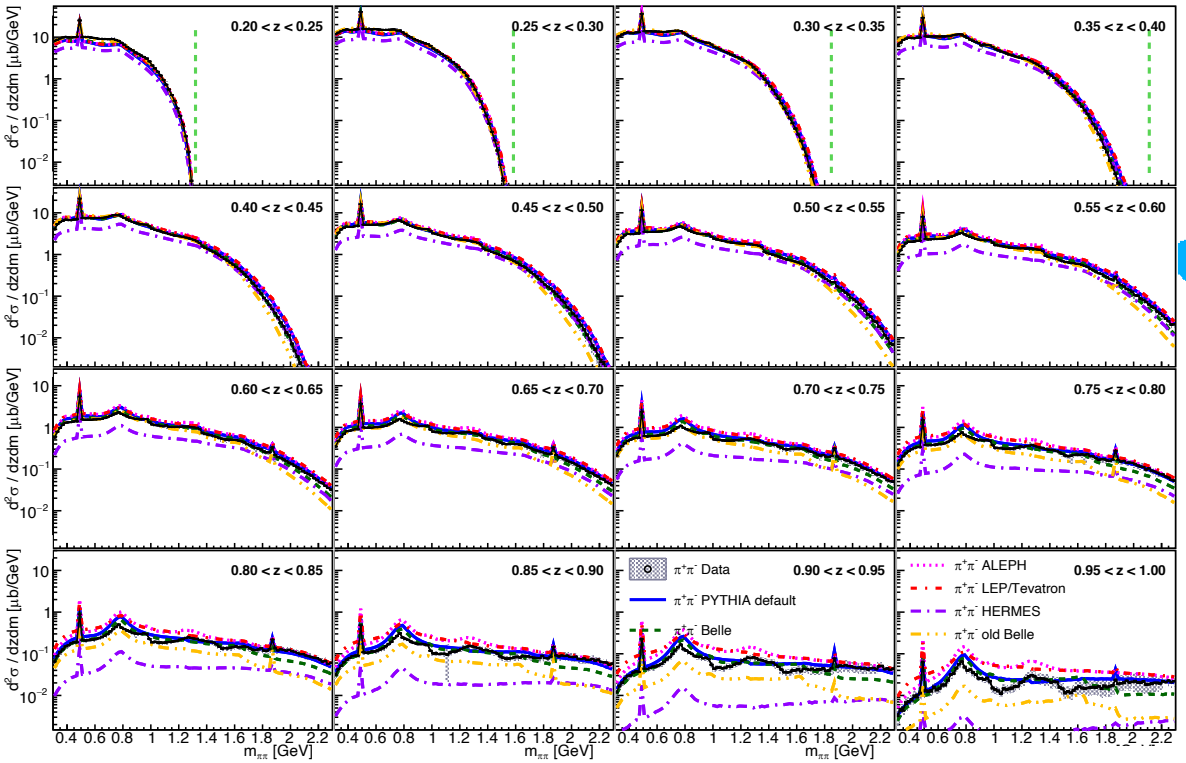
- Hadron pairs can be
  - Back-to-back (here)
  - Same hemisphere
  - ‘any-hemisphere’ (a la de Florian & Vanni (Phys Lett B 578 (2004) 139)
  - Same-hemisphere→di-hadrons
  - Systematics MC driven:
    - acceptance/smearing/non $q\bar{q}$ contribution, ISR corrections
    - Optional: charm, weak decays



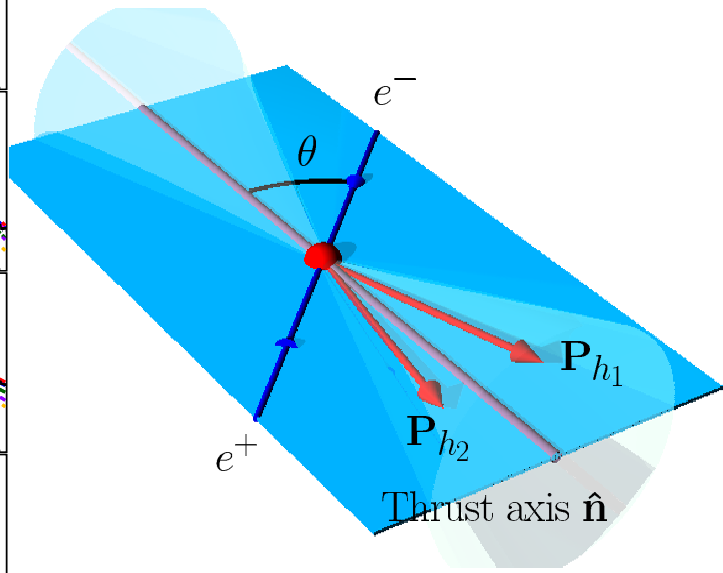


$$T = \max \frac{\sum_h |P_h^{CMS} \cdot \hat{n}|}{\sum_h |P_h^{CMS}|}$$

More complex final states, di-hadrons M_{hh} distribution

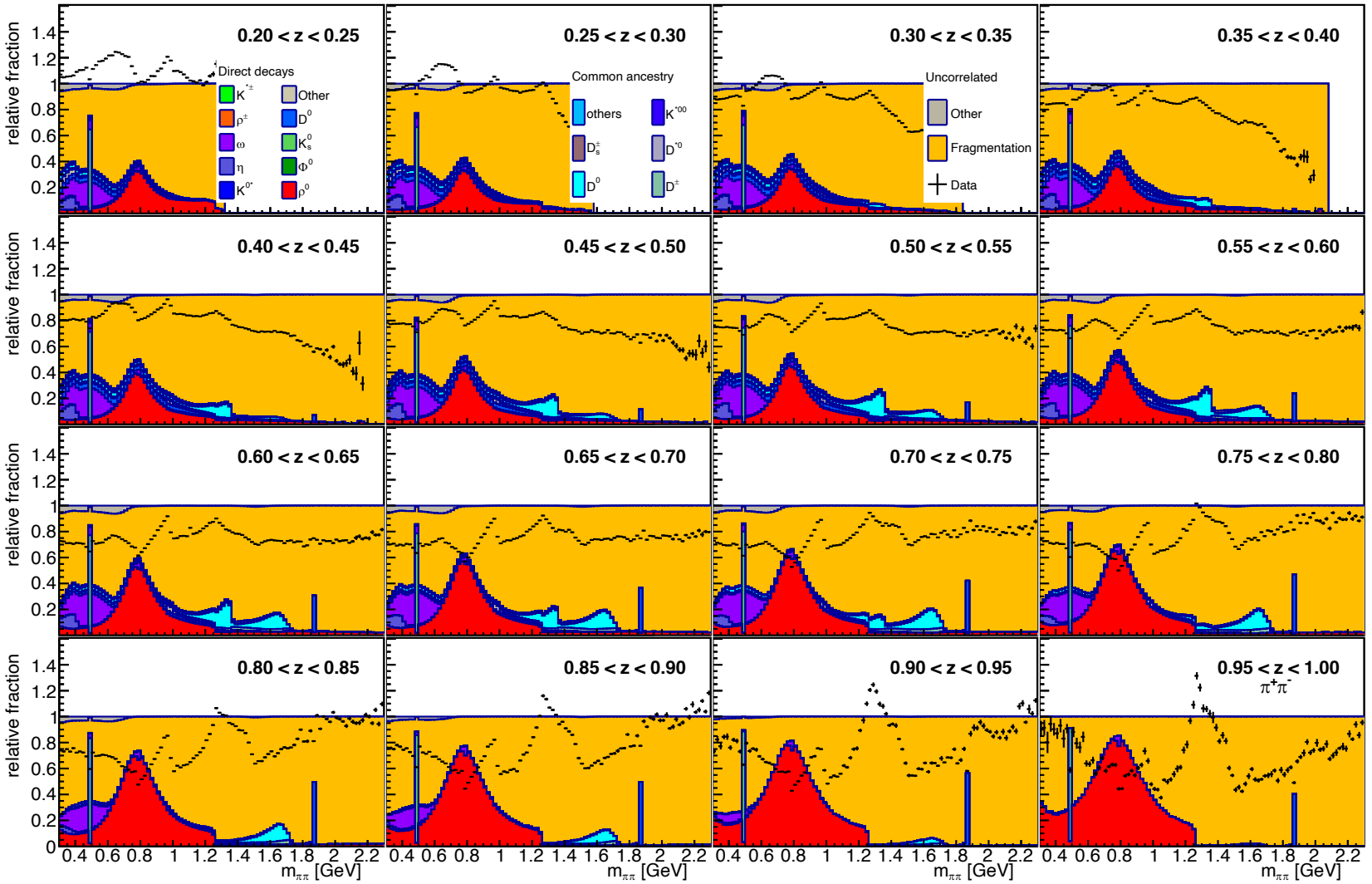


m [GeV]



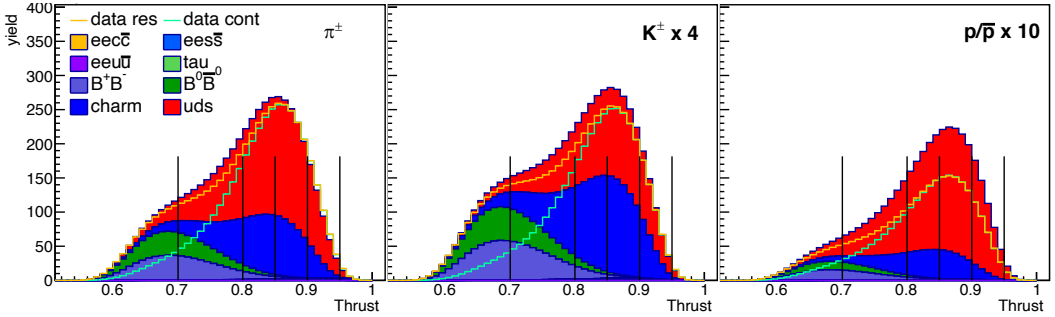
(Phys.Rev. D96 (2017) no.3, 032005)

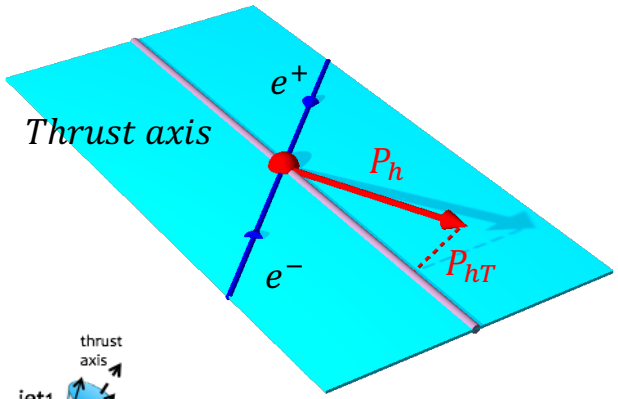
MC Origin


0.20 < z < 0.25
0.25 < z < 0.30
0.30 < z < 0.35
0.35 < z < 0.40
0.40 < z < 0.45
0.45 < z < 0.50
0.50 < z < 0.55
0.55 < z < 0.60
0.60 < z < 0.65
0.65 < z < 0.70
0.70 < z < 0.75
0.75 < z < 0.80
0.80 < z < 0.85
0.85 < z < 0.90
0.90 < z < 0.95
0.95 < z < 1.00
π+π-
Direct decays
Other
Common ancestry
others
Uncorrelated
Other
Fragmentation
Data
relative fraction
m_ππ [GeV]

P_T dependence of single hadrons

- Quasi inclusive hadron production gives access to transverse momentum in fragmentation
- **Transverse momentum measured with respect to thrust axis**
- Analysis performed differentially in bins of $z, P_{hT}, Thrust$ $(18\times20\times6)$

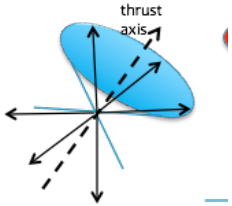




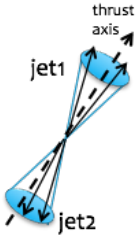
$$T = \sum_i \frac{|\mathbf{P}\cdot\hat{\mathbf{n}}|}{|P|}$$

thrust axis $\equiv \hat{\mathbf{n}}$

$0.5 \leq T \leq 1$



T~0.5

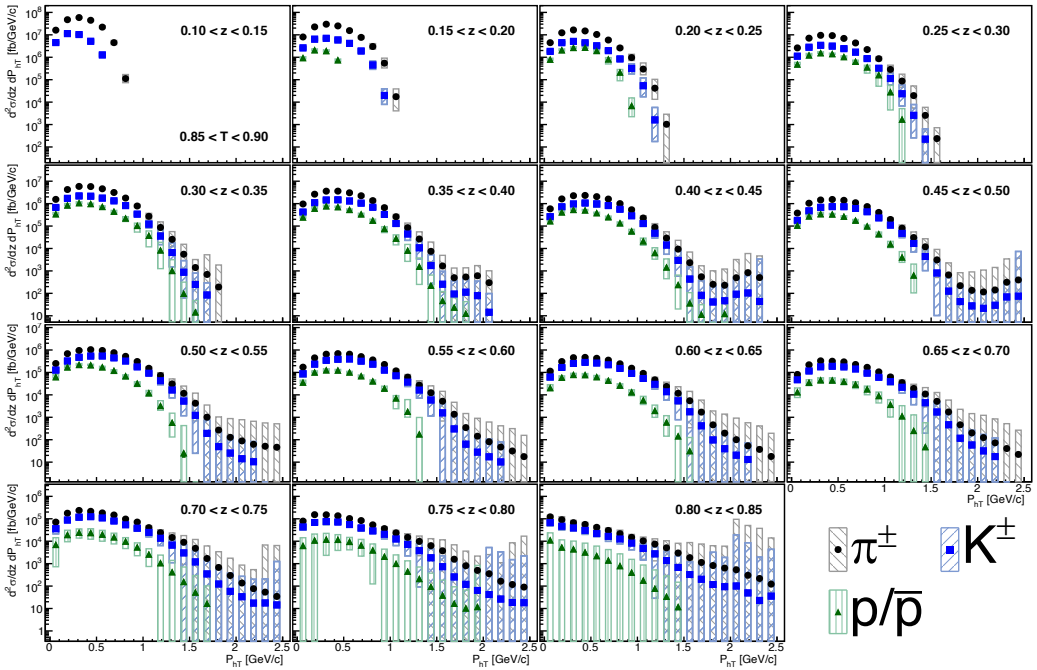


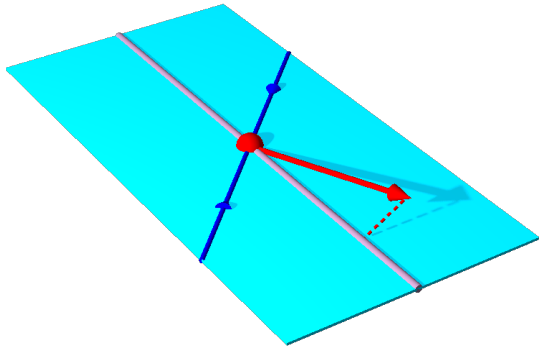
T~1

Phys.Rev. D99 (2019) no.11, 112006

Transverse momentum distributions

- 0.85< Thrust T < 0.9
  - Transverse momenta mostly Gaussian
  - Possible deviations for large P_{hT} tails, but also large uncertainties

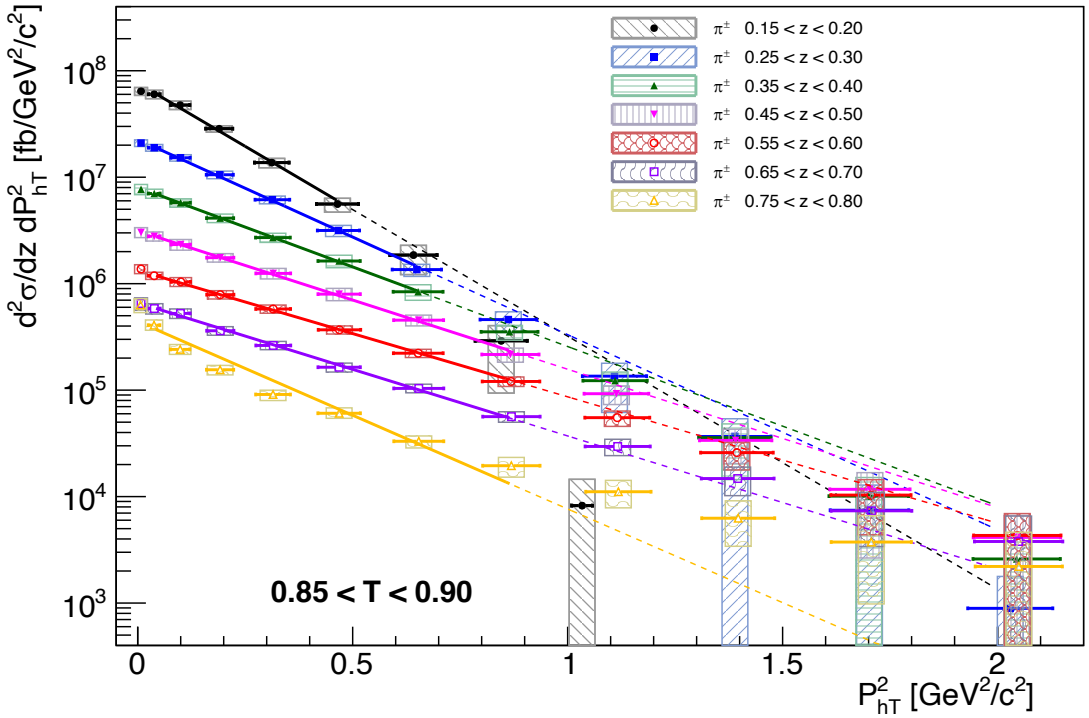




Phys.Rev.D 99 (2019) 11, 112006

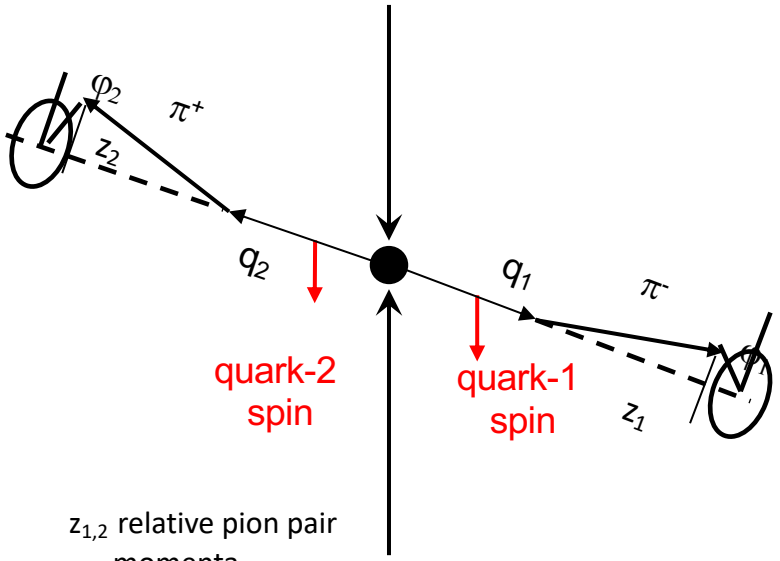
Transverse Momentum: Gaussian widths

- 0.85 < T < 0.9
  - Fit Gauss to low P_{hT} data
  - Mostly well described with possible exception at high z
  - Deviation from Gauss at large P_{hT}
  - Clear increase in width with z for low values of z



Transverse Polarization Dependent FFs IN e^+e^-

- Access spin dependence and p_T dependence (convolution or in jet) without PDF complication
- Made possible by B-factory luminosities

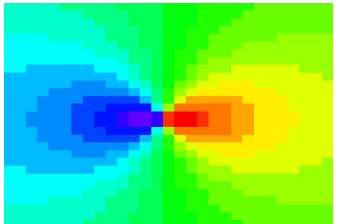


$$\text{Cross-section } e^+e^- \to (h_1 h_2)\left(\overline{h_1}\,\overline{h_2}\right) + X$$
$$\propto D_1^{\perp}\,\overline{D_1^{\perp}} + H_1^{\perp}\,\overline{H_1^{\perp}}\cos(\phi_1 + \phi_2)$$

•First non-zero independent measurement of the Collins effect for pion pairs in e^+e^- annihilation by Belle Collaboration @ $\sqrt{s} \sim 10.6$ GeV (PRL 111,062002(2008), PRD 88,032011(2013)) leads to first extraction of transversity (Phys.Rev. D75 (2007) 054032) from SIDIS and e+e-

- Confirmed by BaBar @ $\sqrt{s} \sim 10.6$ GeV (PRD 90,052003 (2014); PRD 92,111101(R)(2015) for KK and Kπ)
- Measured at BESIII @ $\sqrt{s} = 3.65$ GeV (PRL 116,42001(2016))

• Di-hadron FFs measured back-to-back:*Phys.Rev.Lett.* 107 (2011) 072004, back-to-back jets: 1505.08020 [hep-ex]



New Pt dependence from Belle

- Trend consistent with BaBar
- Direct comparison difficult due to different correction schemes (thrust vs $q\bar{q}$ −axis)

Unlike/**L**ikesign

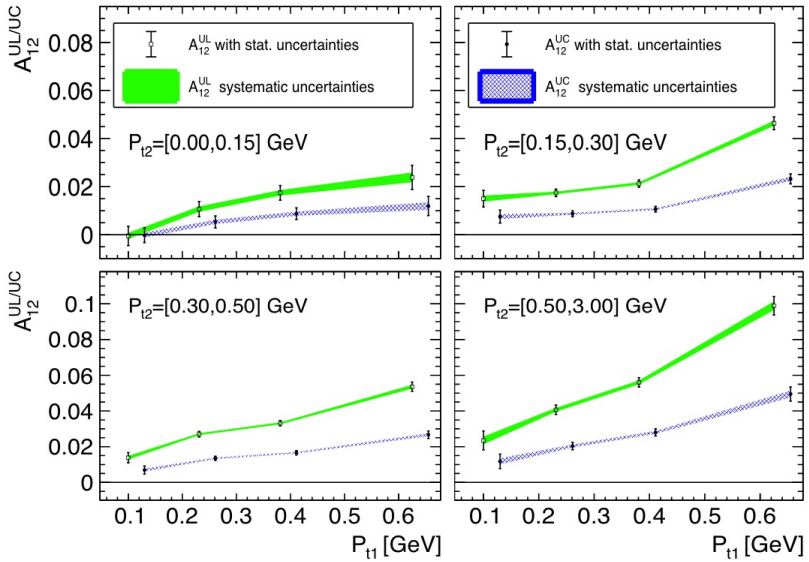
Ratios to cancel acceptance effects

Unlike:

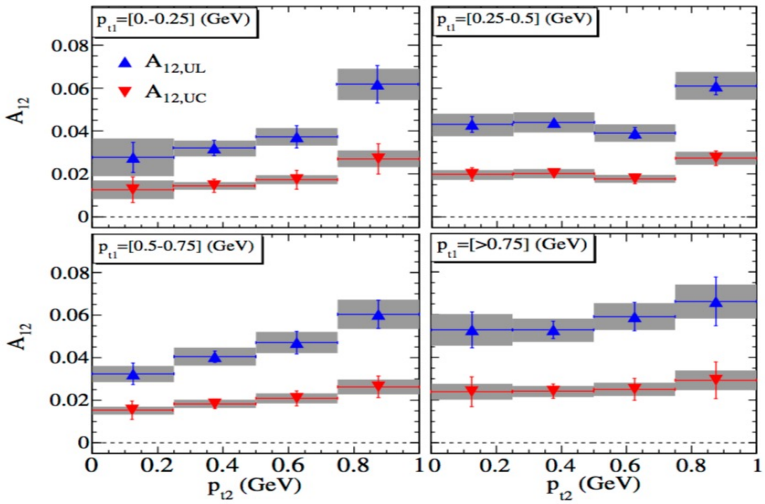
fav*fav+dis*dis

Like:

fav*dis



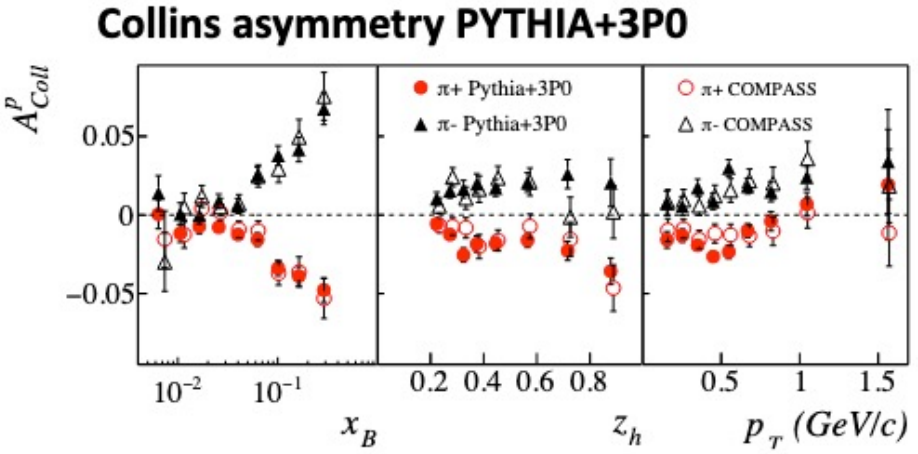
Belle



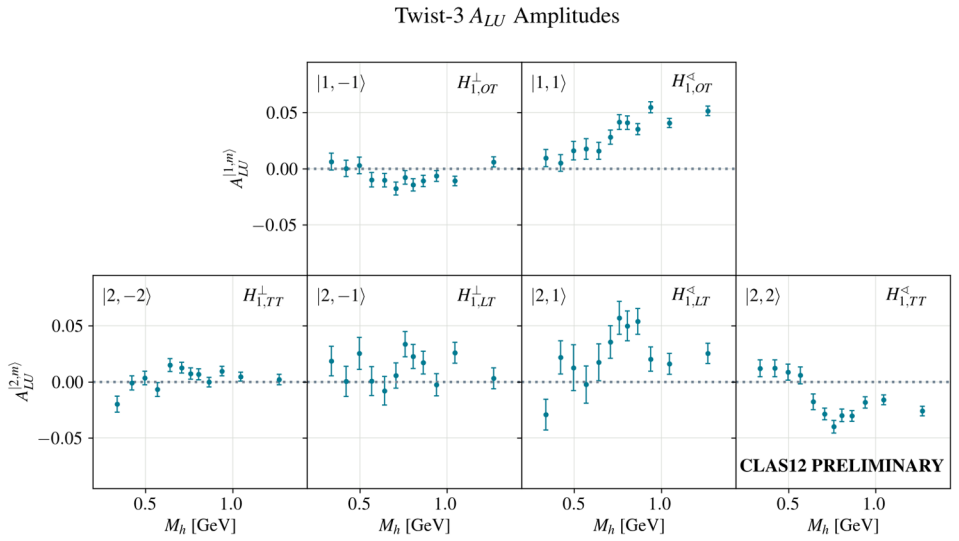
BaBar

Models for spin dependent fragmentation

- High importance for the EIC
- Recent implementation in Pythia
- Comparison to TMD Di-hadron FFs full angular dependence (ϕ_h, ϕ_R, θ) would provide comprehensive test



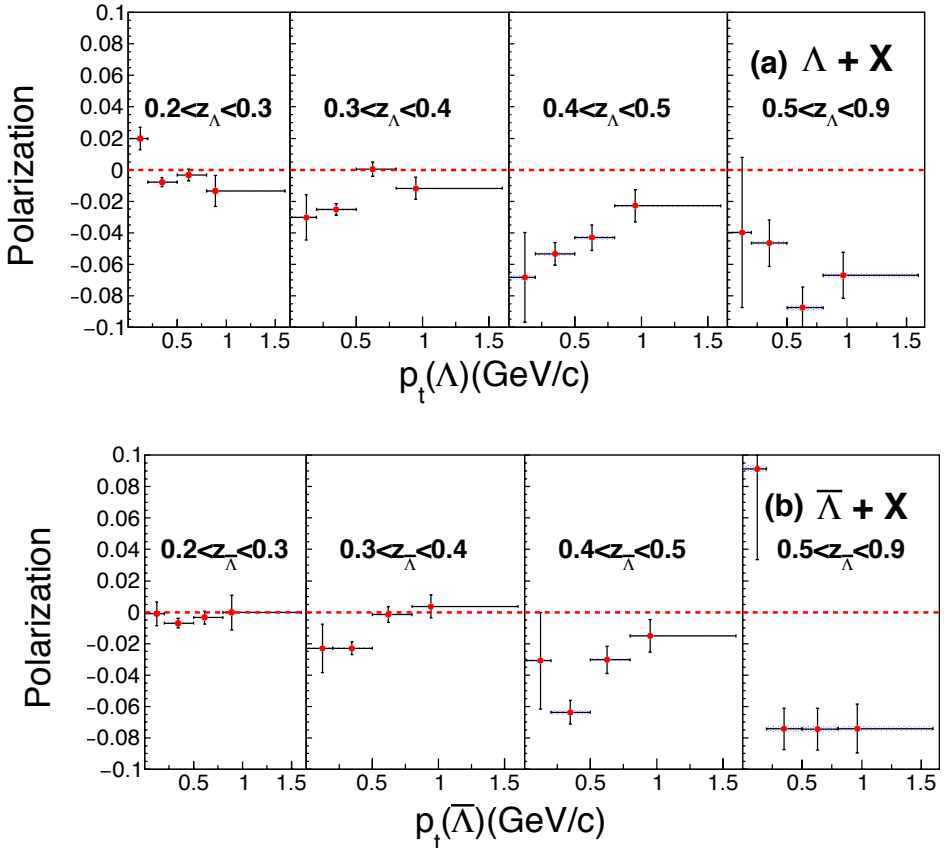
Albi KERBIZI, WorkshopTMD Studies: from Jlab to EIC



Chris Dilks at Spin 2021

Polarizing Λ Fragmentation z_Λ, p_T Dependence of observed Λ polarization

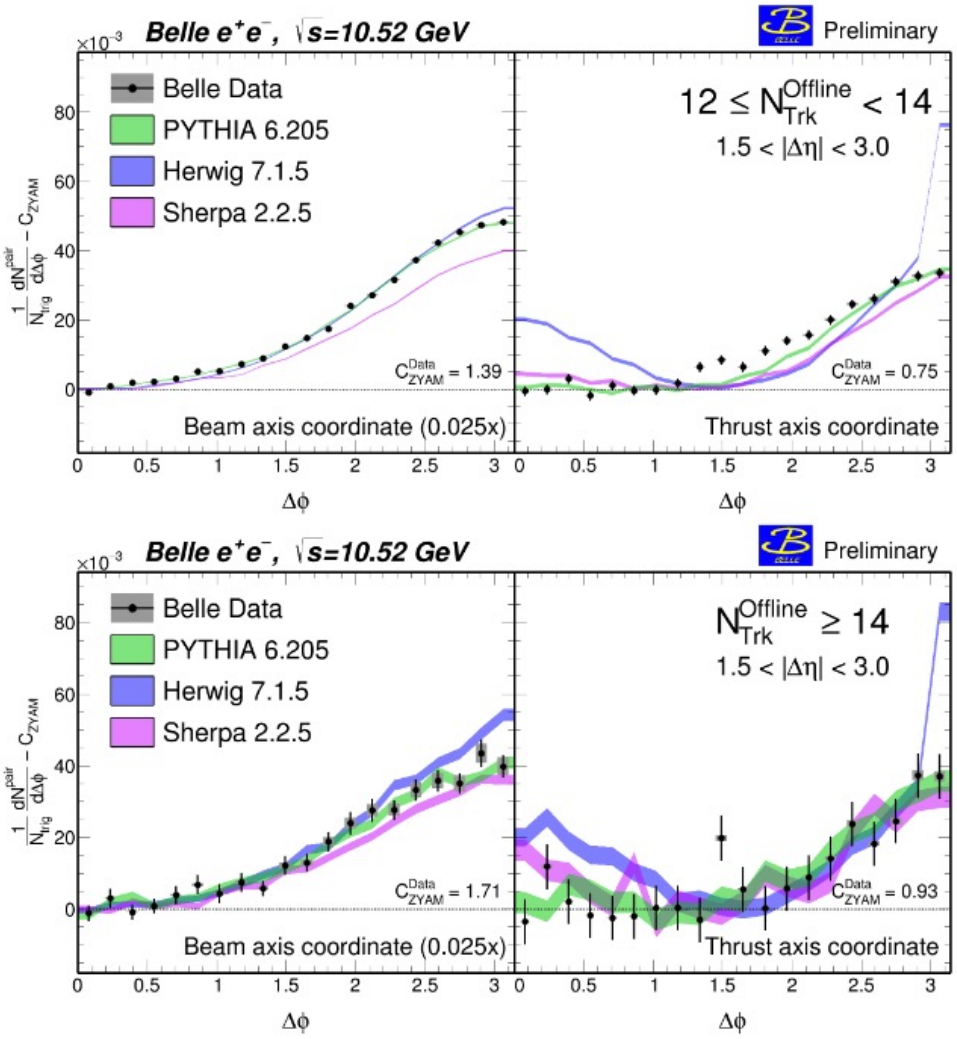
- Polarization rises with p_t in the lowest z_Λ and highest z_Λ bin. But the dependence reverses around 1 GeV in the intermediate z_Λ bins→**Unexpected!**
- Results are consistent between Λ and $(\overline{\Lambda})$
- Additional results
  - Production associated with $\pi^\pm/K^\pm$
  - Corrected for feed-down
- Comparison with SIDIS data from EIC will provide insight into gauge structure of QCD (see 2108.05383 [hep-ph])



Phys.Rev.Lett. 122 (2019) 4, 042001

“Ridge” Correlations

- Example of QCD correlation analysis ‘off the beaten path’
- Makes use of clean e^+e^- environment

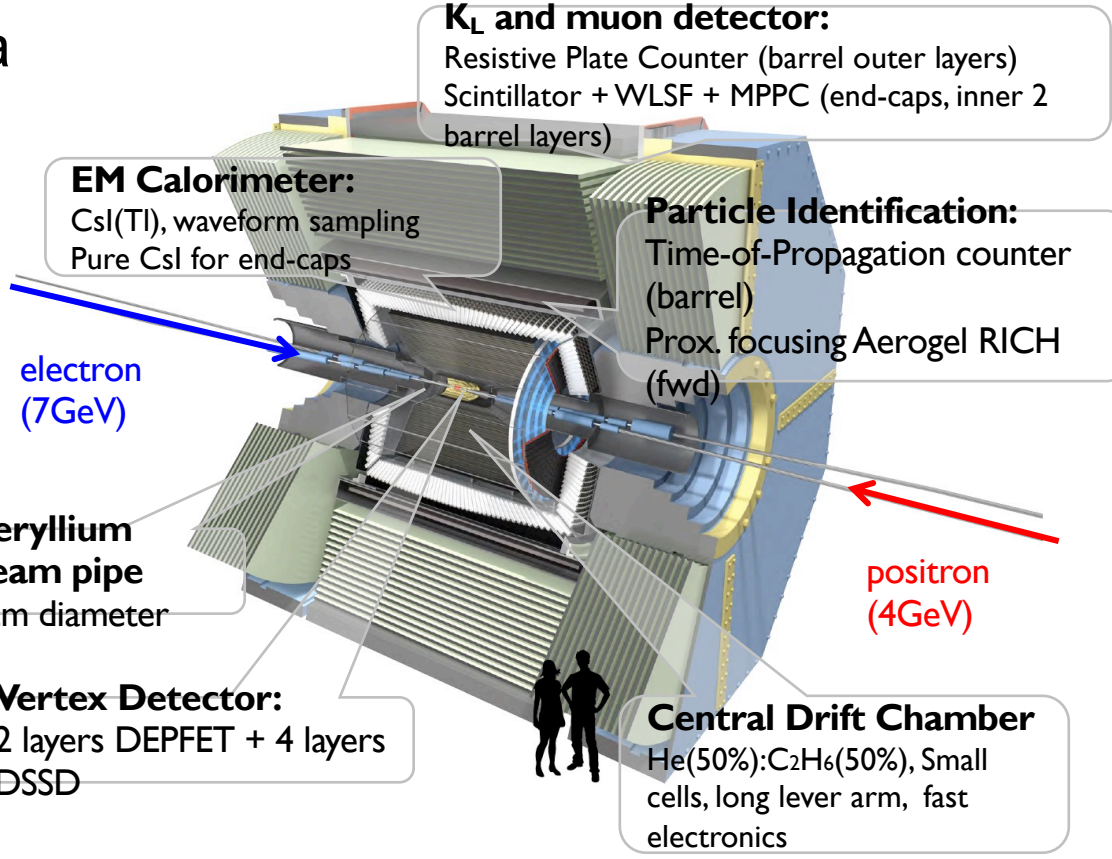


hep-ex:2008.04187

Future Plans

- Belle II currently taking data
  - Belle dataset in 2022
  - 40x lumi goal
  - 50x integrated lumi

- Drafting of Snowmass whitepaper:
  - Contributors welcome



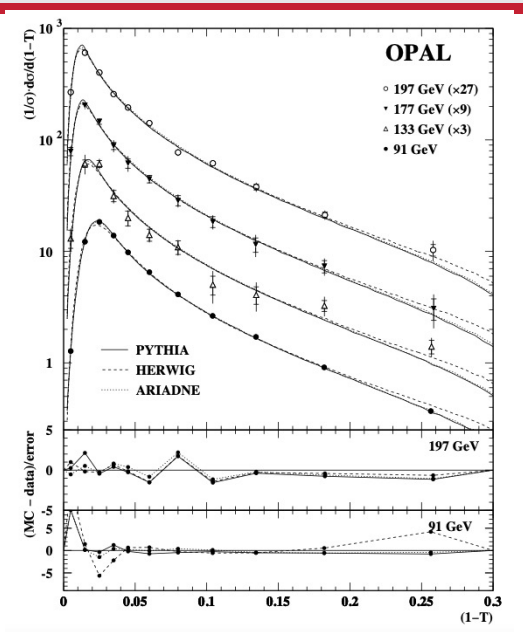
Snowmass 2021 Letter of Interest:
QCD and Hadronization Studies at Belle II

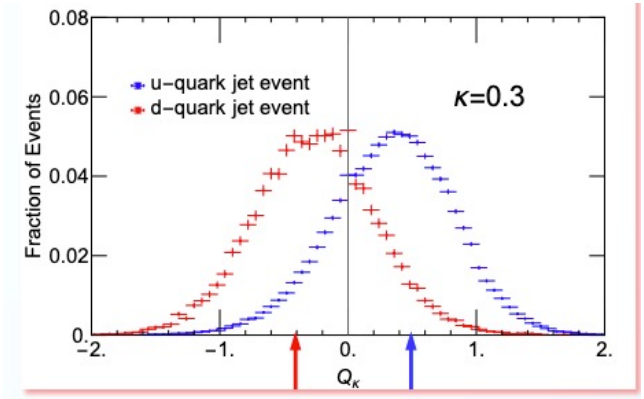
on behalf of the U.S. Belle II Collaboration

Future measurement to study hadronization in e^+e^-

- Back-to-back hadrons to explore k_T spectrum
- Event shapes
  - Rich topic at LEP
  - LEP did flavor tagged/$q/\bar{q}$
  →can this be done with jet charge?
  - Energy-Energy Correlations

- Jet topic still to be explored further in e^+e^-
  - Reanalysis of ALPEH data (MITHIG-MOD-NOTE-21-001)
  - Start of program at Belle
    - Initially focused on
      - q_T distributions in di-jets, jet-hadron correlations,
      - T-odd jets
      - WTA vs standard jet axis
      - …
    - See nice motivation in Elke's talk
  - Energy-Energy Correlations



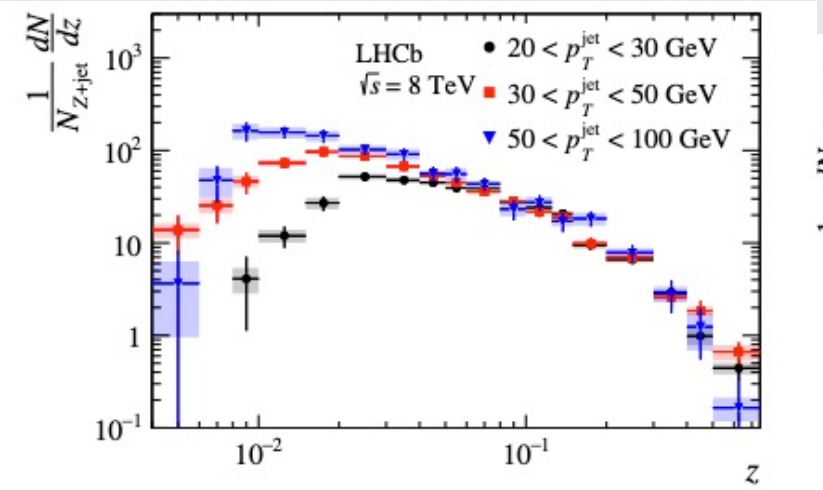


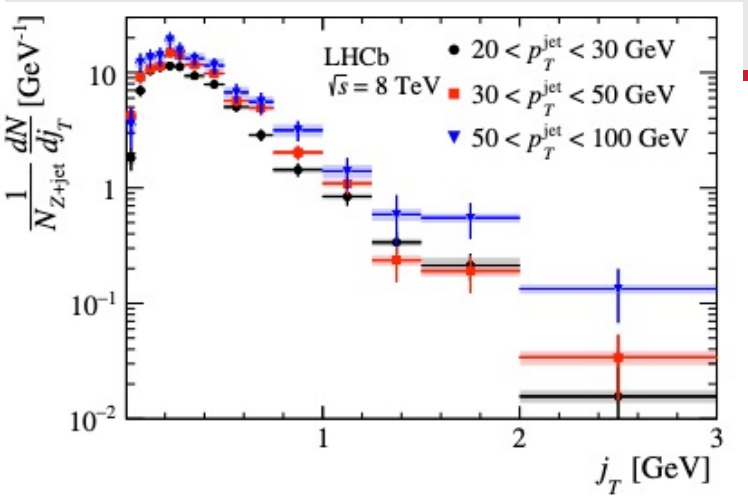
Z. Kang at INT FF worskhop 2021

Summary Outlook

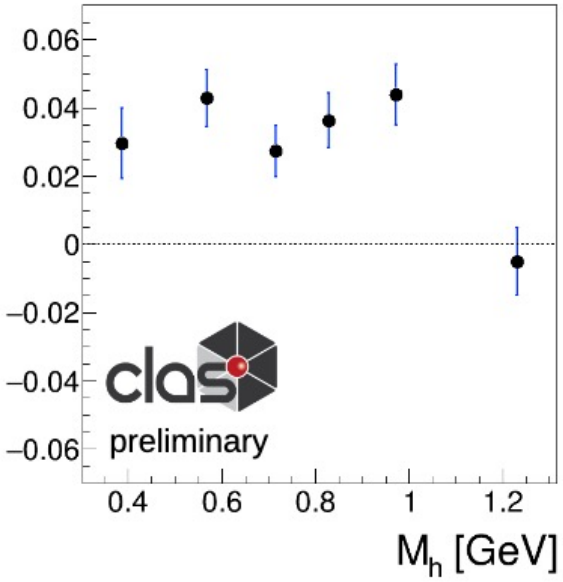
- Active programs on hadronization in $pp, SIDIS, e^+e^-$
- Precision measurements in e^+e^- needed to inform MC hadronization models
- B/Super B factories provide ample statistics for precision
- Belle program focused on polarized, TMD FFs
- Currently starting drafting program for Belle II, input welcome

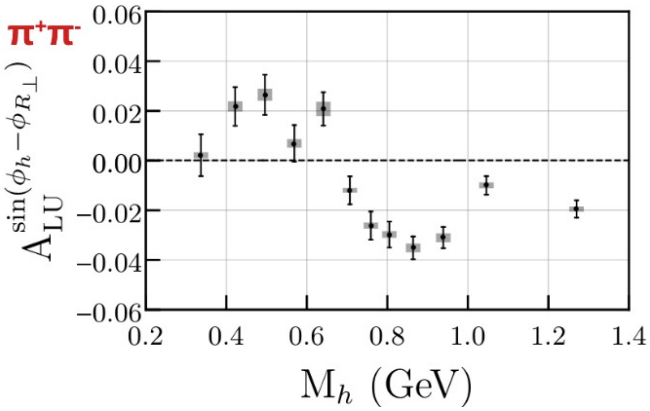
Backup


LHCb
$\sqrt{s} = 8$ TeV
$20 < p_T^{jet} < 30$ GeV
$30 < p_T^{jet} < 50$ GeV
$50 < p_T^{jet} < 100$ GeV
$\frac{1}{N_{Z+jet}} \frac{dN}{dz}$
z

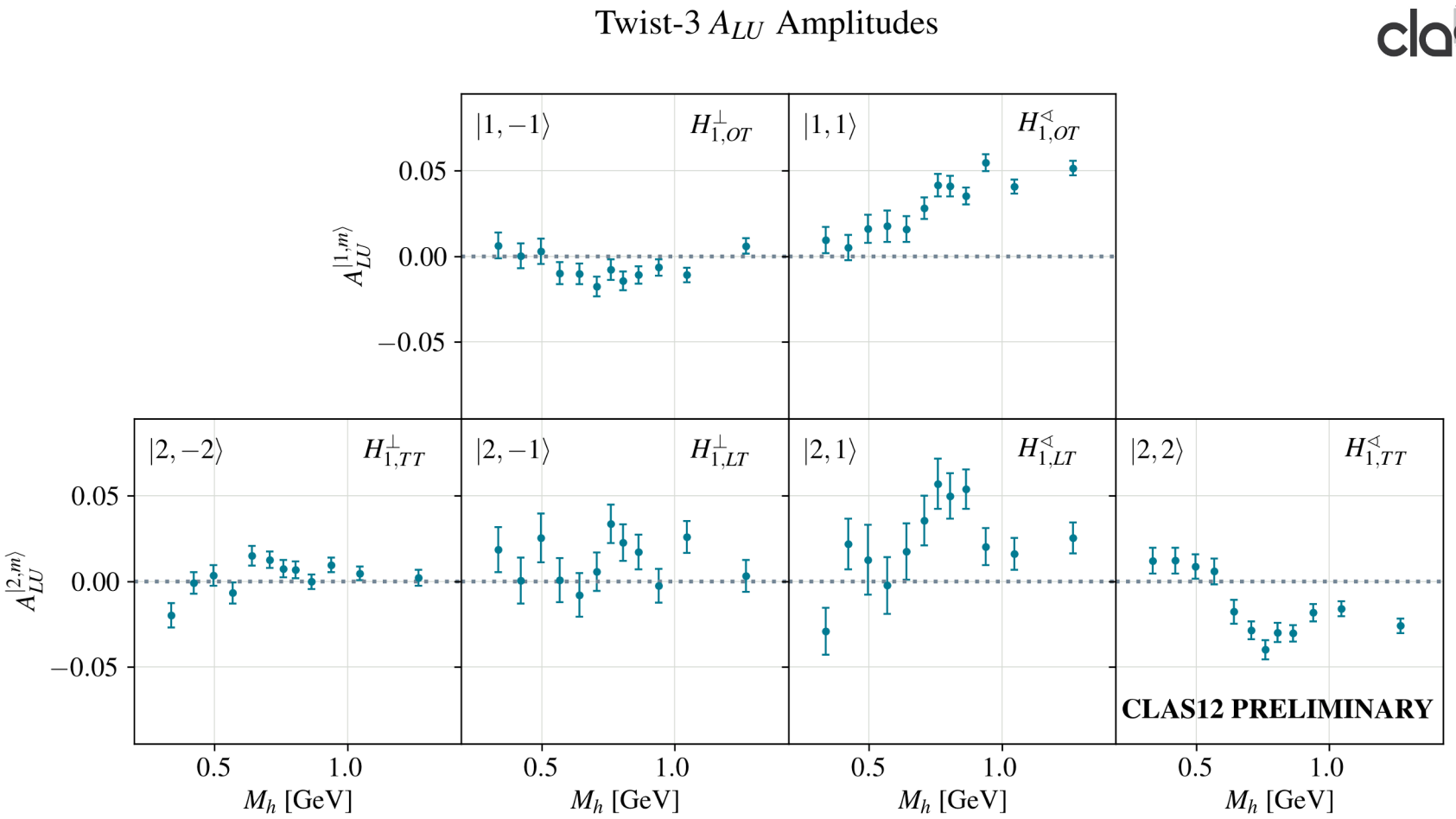

LHCb
$\sqrt{s} = 8$ TeV
$20 < p_T^{jet} < 30$ GeV
$30 < p_T^{jet} < 50$ GeV
$50 < p_T^{jet} < 100$ GeV
$\frac{1}{N_{Z+jet}} \frac{dN}{dj_T}$ [GeV^{-1}]
j_T [GeV]

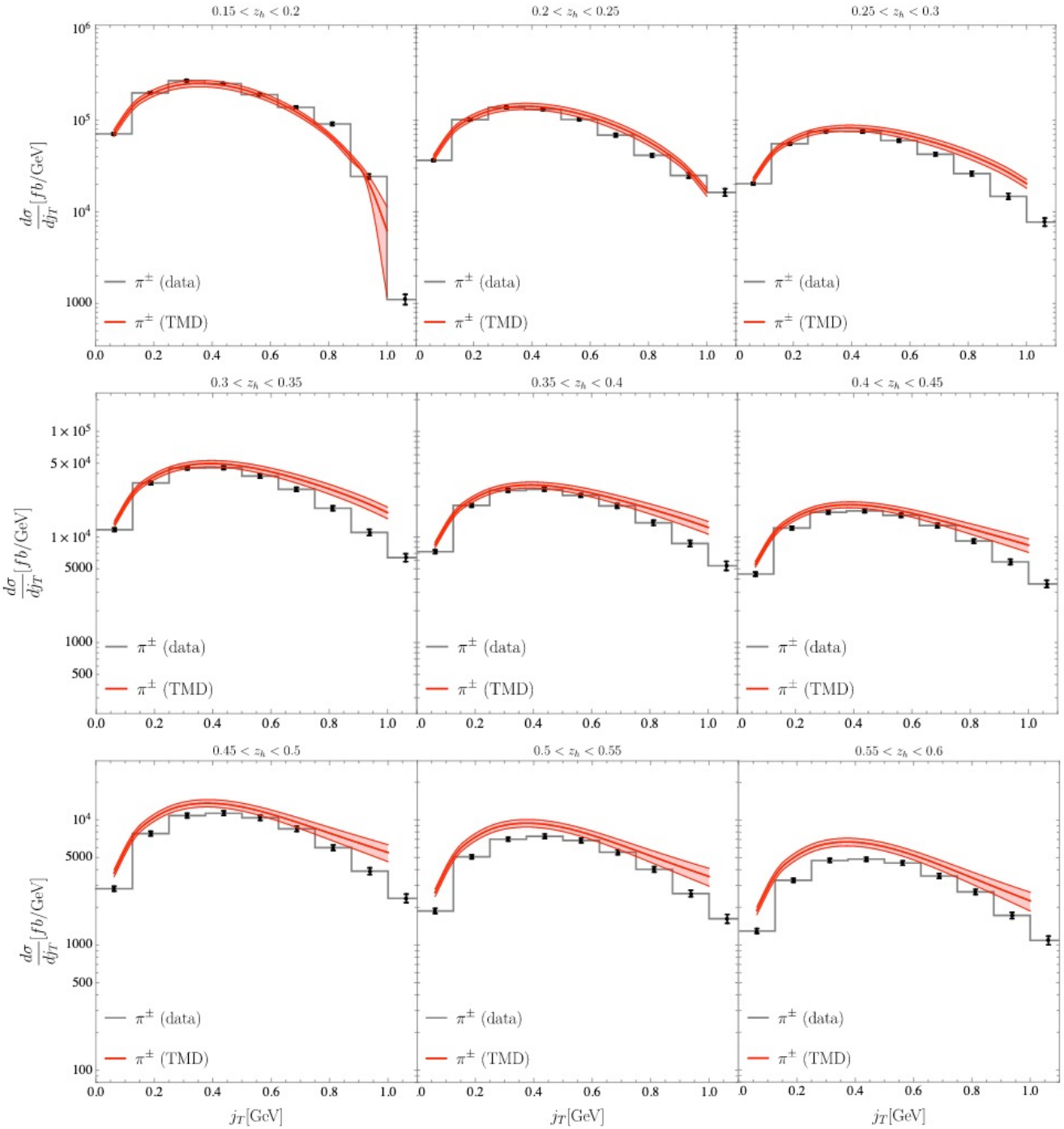
$\pi^+\pi^0$ $A_{LU}[\sin(\phi_h-\phi_R)]$ vs. M_h



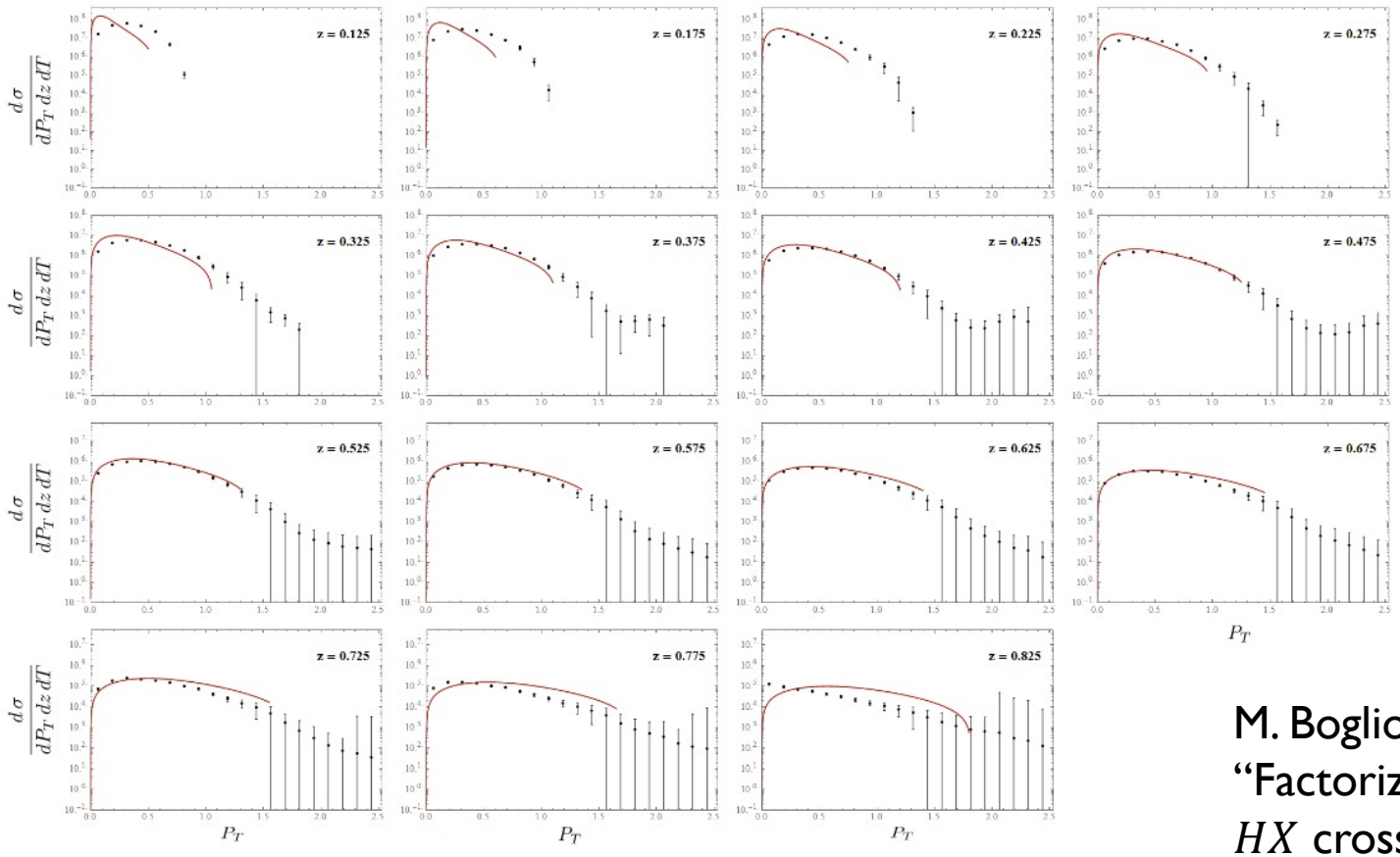


$e(x) \cdot H_1^{\sphericalangle}(z, M_h)$ vs M_h



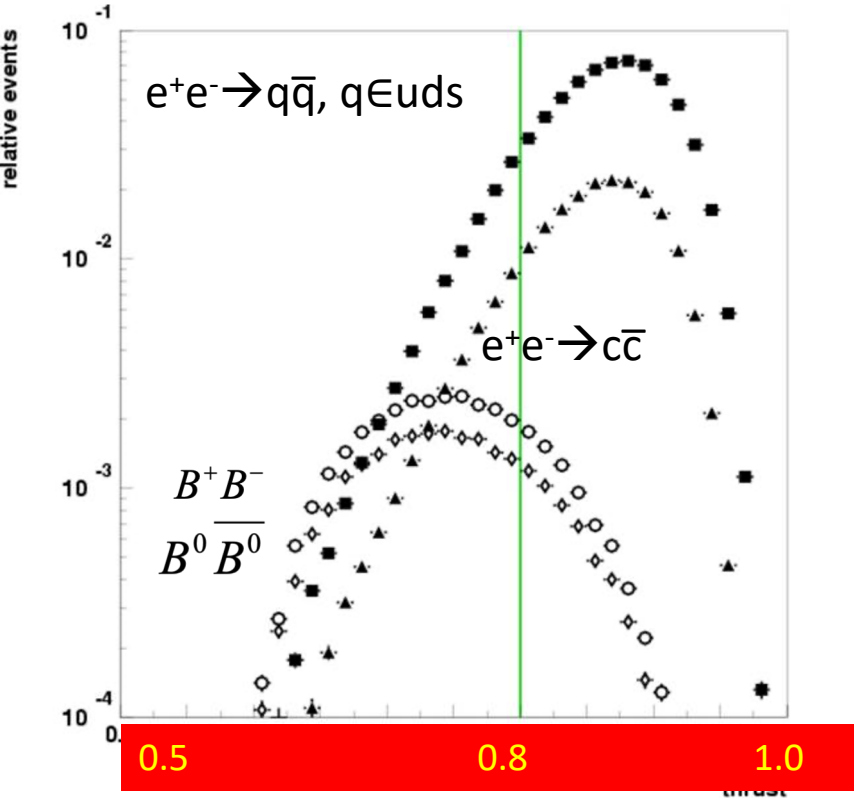


Z. Kang, D. Shao, F. Zhao, "QCD resummation on single hadron transverse momentum distribution with the thrust axis", 2007.14425 [hep-ph]

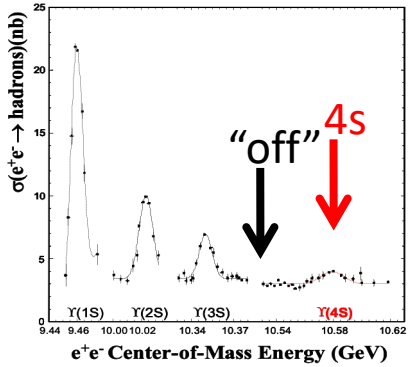


M. Boglione, A. Simonelli, "Factorization of $e^+e^- \to HX$ cross section, differential in z_h, P_T and thrust, in the 2-jet limit", 2011.07366 [hep-ph]

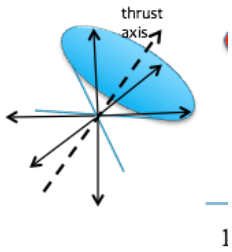
Lots of data off resonance, easy to remove resonance background



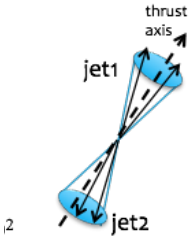
$$Thrust: T = \frac{\sum_i |p_i \cdot \hat{n}|}{\sum_i |p_i|}$$



- small B contribution (<1%) in high thrust sample
- >75% of X-section continuum under Υ (4S) resonance
- ~100 fb^{-1} ➔ ~1000 fb^{-1}



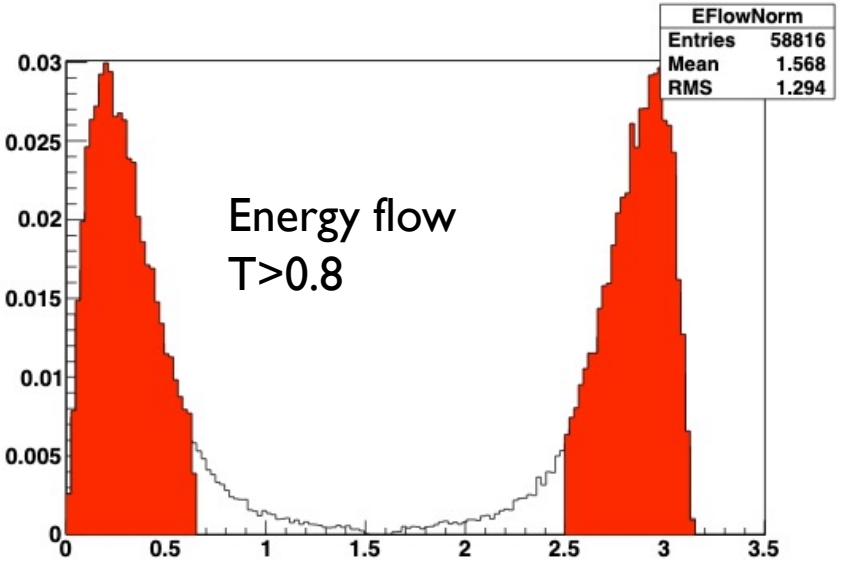
T~0.5

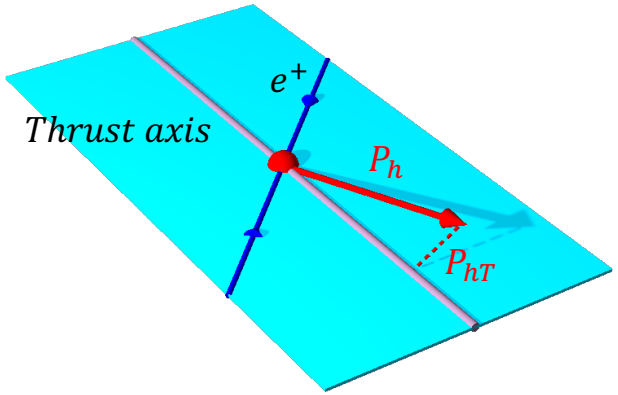


T~1

Use thrust as proxy to $q\bar{q}$ axis

- No phase space for >2 jet events
- **Transverse momentum measured with respect to thrust axis**



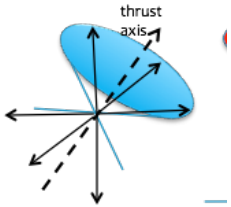


$$T = \sum_i \frac{|\mathbf{P} \cdot \hat{\mathbf{n}}|}{|P|}$$

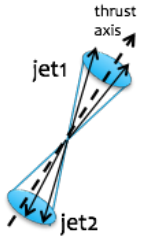
thrust axis $\equiv \hat{\mathbf{n}}$

$0.5 \leq T \leq 1$

θ_{CMS}



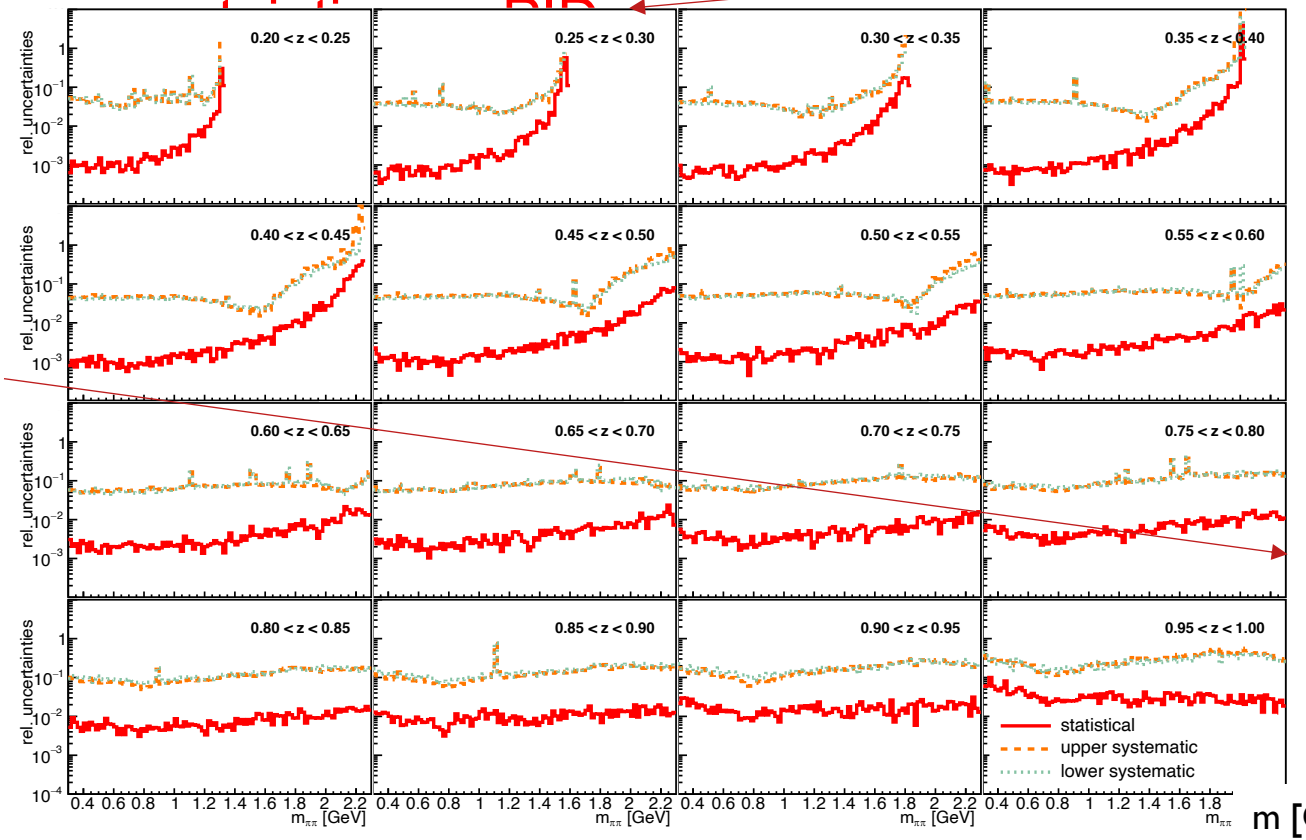
T~0.5



T~1

Results Systematics Dominated

- Low z: Dominated by PID uncertainties
  Belle II prospects: Improved PID, higher statistics to improve



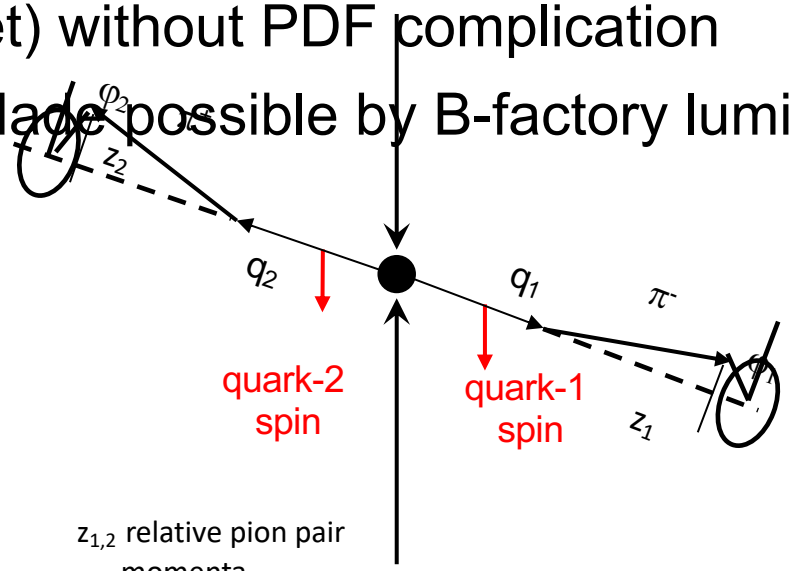
radiation with

Seidl et. al. Phys.Rev. D96 (2017) no.3, 032005

m [GeV]

Collins FFs IN e^+e^-

- Access spin dependence and p_T dependence (convolution or in jet) without PDF complication
- Made possible by B-factory lumin

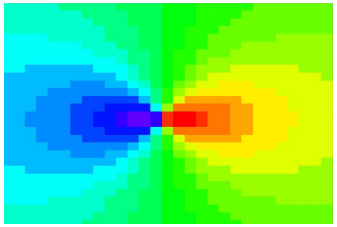


•First non-zero independent measurement of the Collins effect for pion pairs in e^+e^- annihilation by Belle Collaboration @ $\sqrt{s} \sim 10.6$ GeV (PRL 111,062002(2008), PRD 88,032011(2013)) leads to first extraction of transversity (Phys.Rev. D75 (2007) 054032) from SIDIS and e+e-

- Confirmed by BaBar @ $\sqrt{s} \sim 10.6$ GeV (PRD 90,052003 (2014); PRD 92,111101(R)(2015) for KK and Kπ)
- Measured at BESIII @ $\sqrt{s} = 3.65$ GeV (PRL 116,42001(2016))

Cross-section $e^+e^- \rightarrow (h_1 h_2)(\overline{h_1}\,\overline{h_2}) + X$

$$\propto D_1^{\perp}\,\overline{D_1^{\perp}} + H_1^{\perp}\,\overline{H_1^{\perp}}\cos(\phi_1 + \phi_2)$$



New Pt dependence from Belle

Unlike/**L**ikesign

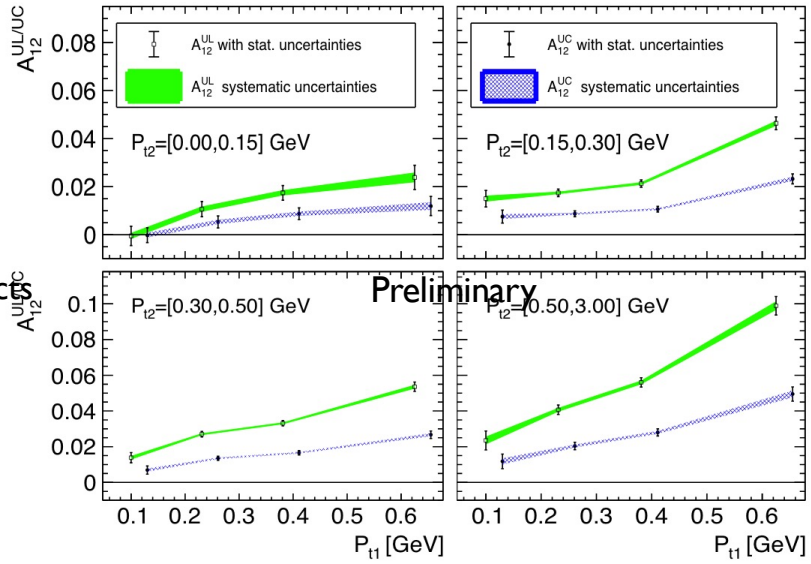
Ratios to cancel acceptance effects

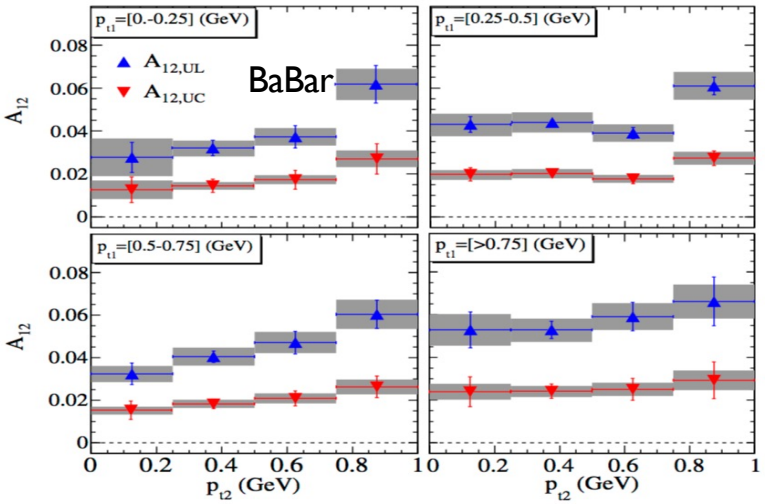
Unlike:

fav*fav+dis*dis

Like:

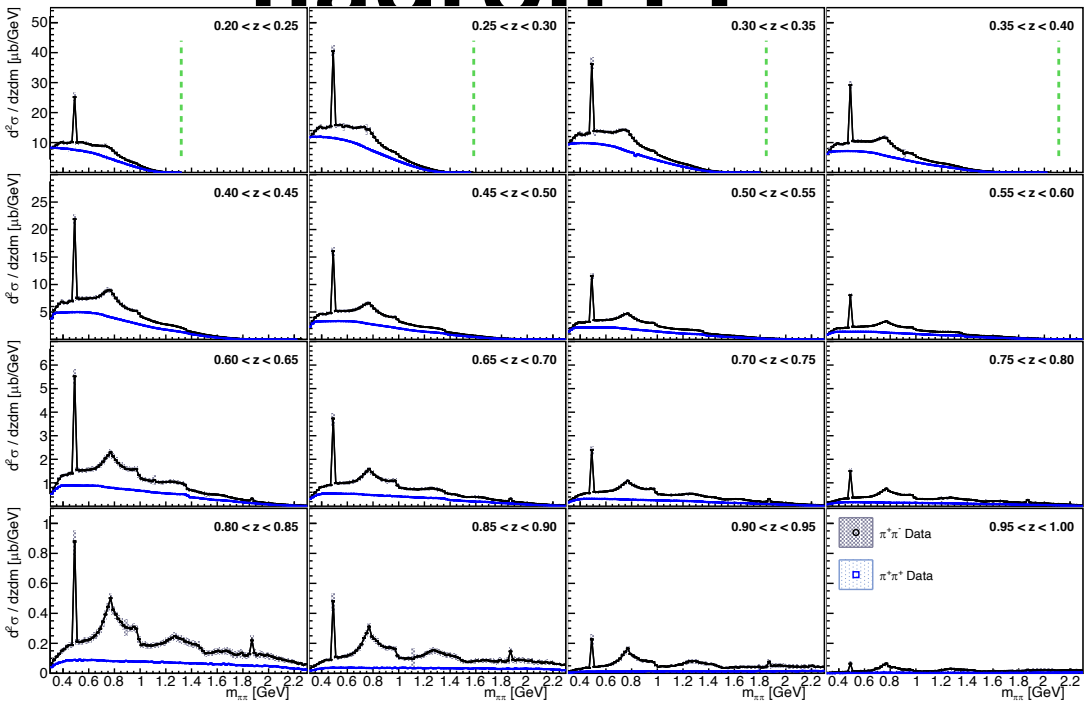
fav*dis

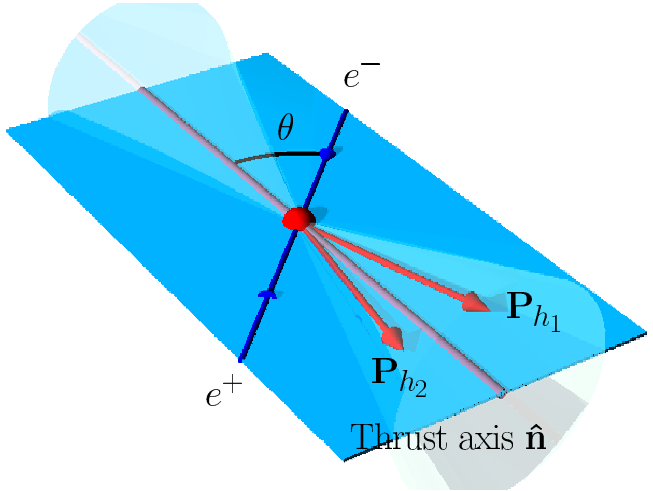




- Trend consistent with BaBar
- Direct comparison difficult due to different correction schemes (thrust vs $q\bar{q}$ −axis)

Mass dependence of di-hadrons in same hemisphere→di-hadron FF

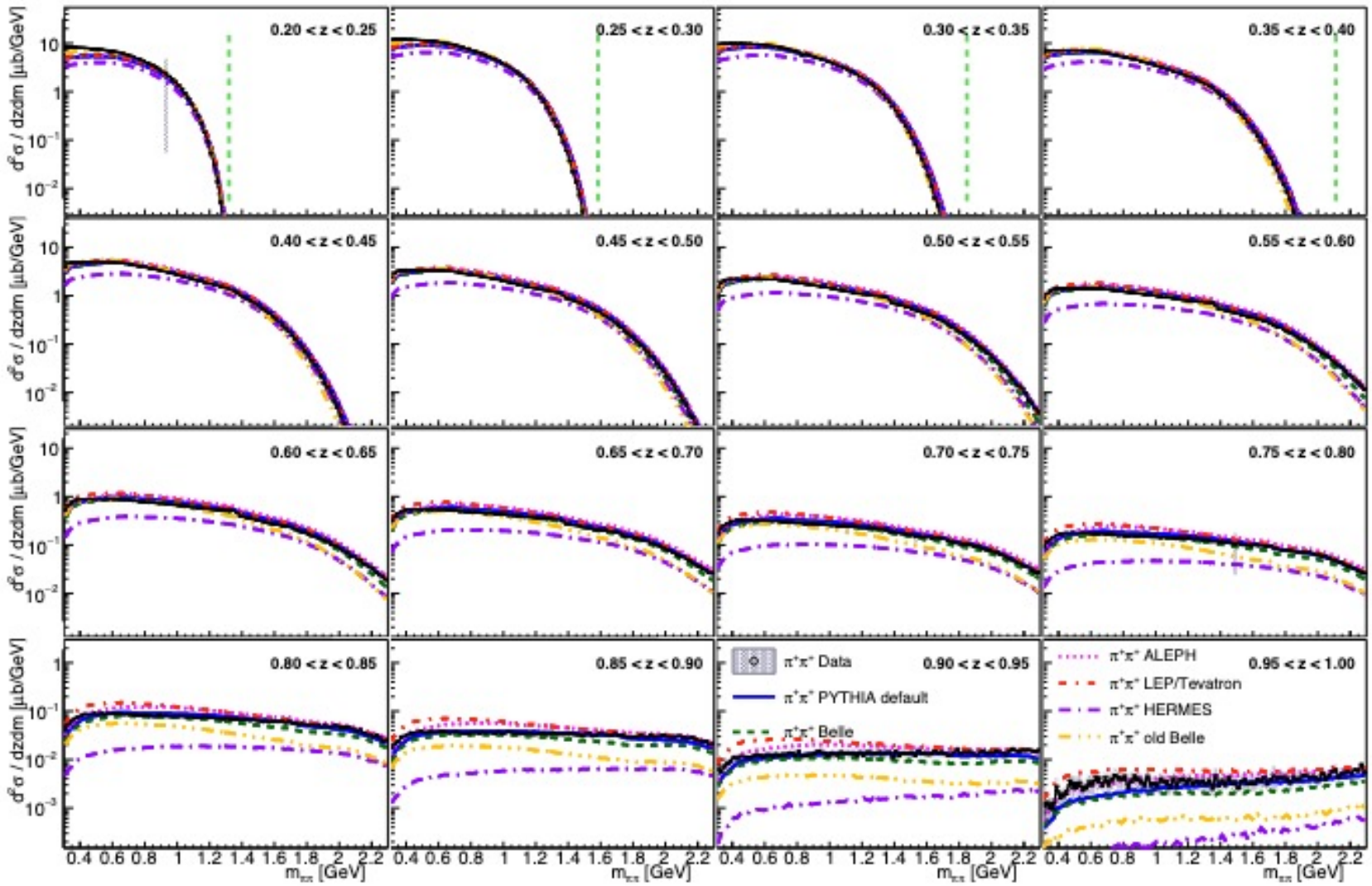




$$Q_\kappa \equiv \sum_{h \in \text{jet}} z_h^\kappa Q_h$$

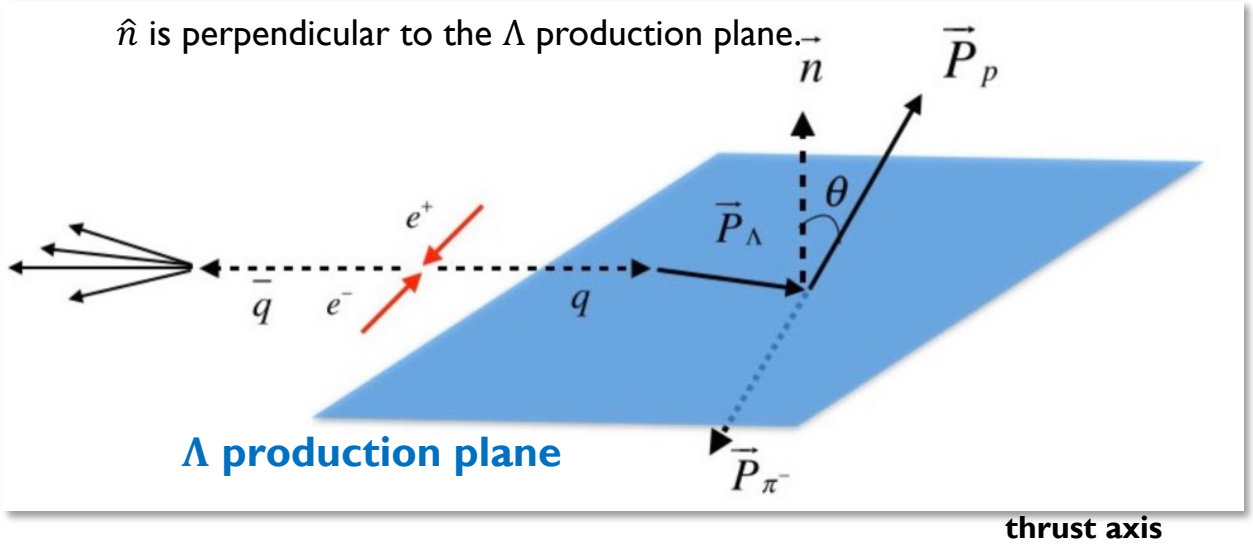
Charge of the hadr

$$z_h = \frac{p_{hT}}{\kappa = 0.3, 0.4, \ldots}$$

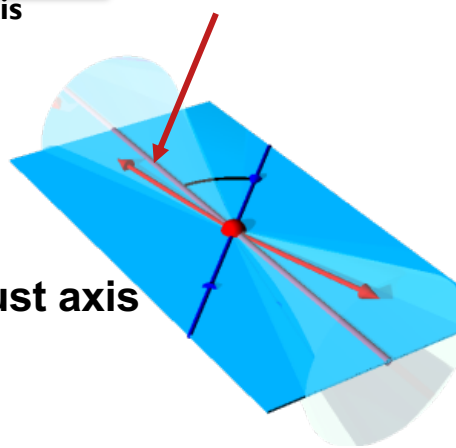

0.20 < z < 0.25
0.25 < z < 0.30
0.30 < z < 0.35
0.35 < z < 0.40
0.40 < z < 0.45
0.45 < z < 0.50
0.50 < z < 0.55
0.55 < z < 0.60
0.60 < z < 0.65
0.65 < z < 0.70
0.70 < z < 0.75
0.75 < z < 0.80
0.80 < z < 0.85
0.85 < z < 0.90
0.90 < z < 0.95
0.95 < z < 1.00
d²σ / dzdm [μb/GeV]
m_ππ [GeV]
π⁺π⁻ Data
π⁺π⁻ PYTHIA default
π⁺π⁻ Belle
π⁺π⁻ ALEPH
π⁺π⁻ LEP/Tevatron
π⁺π⁻ HERMES
π⁺π⁻ old Belle

Observables of Λ Polarization at Belle

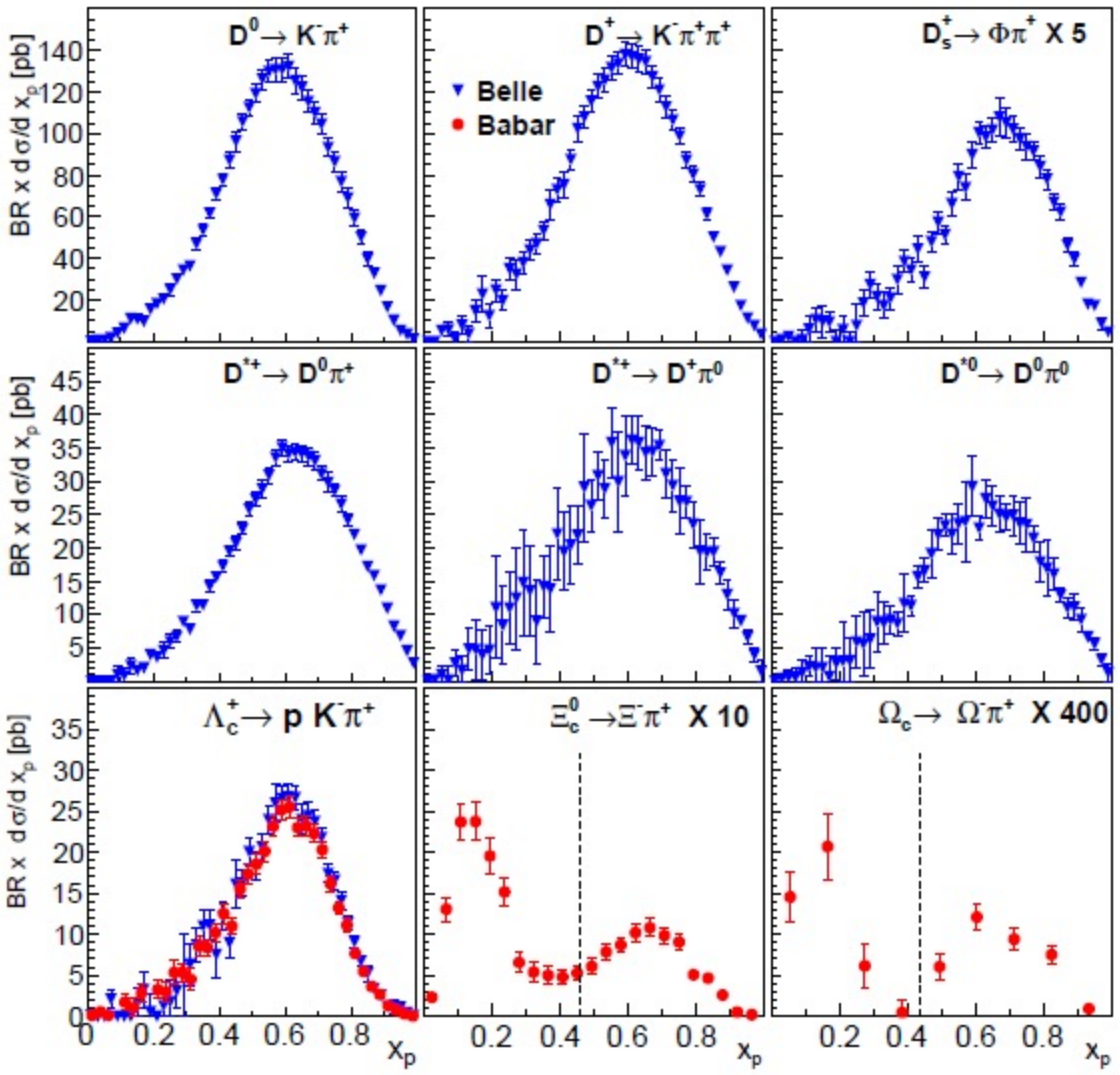
- Self-analyzing decay leads to polarization dependent distribution



- where α is the decay parameter: α_+=0.642 ± 0.013 for Λ and α_-=-0.71 ± 0.08 for $\bar{\Lambda}$ (PDG).
- The p_t is measured as the transverse momentum of Λ relative to the **thrust axis**

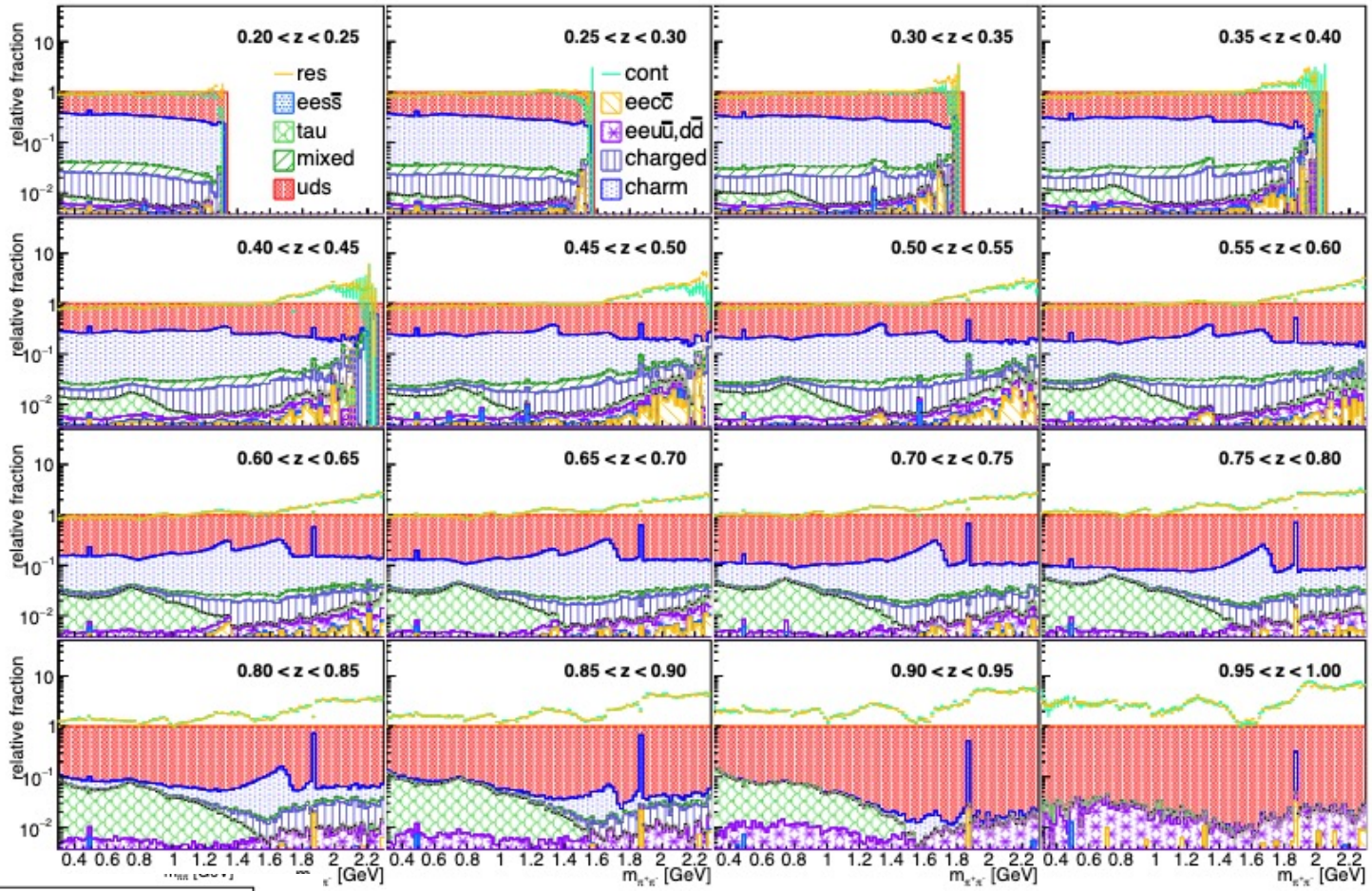


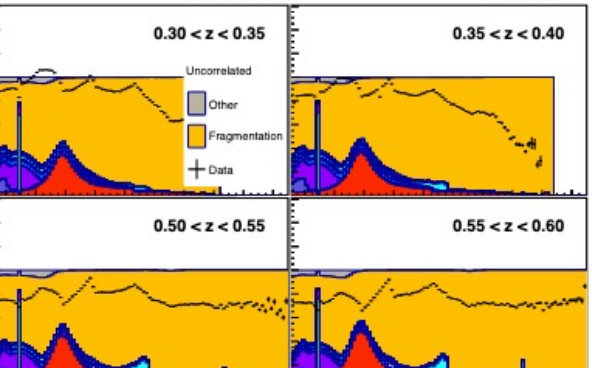
$$\frac{1}{N}\frac{dN}{d\cos\theta} = 1 + \alpha P \cos\theta$$

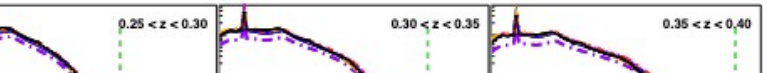


PRL.95, 142003 (2005)(Babar)
PRD73, 032002 (2006) (Belle)
PRD75, 012003 (2007)(Babar)
PRL 99, 062001 (2007)(Babar)

- Heavier particles generally plotted vs normalized momentum
- Unlike light hadrons charmed hadrons contain large fraction of charm quark momentum

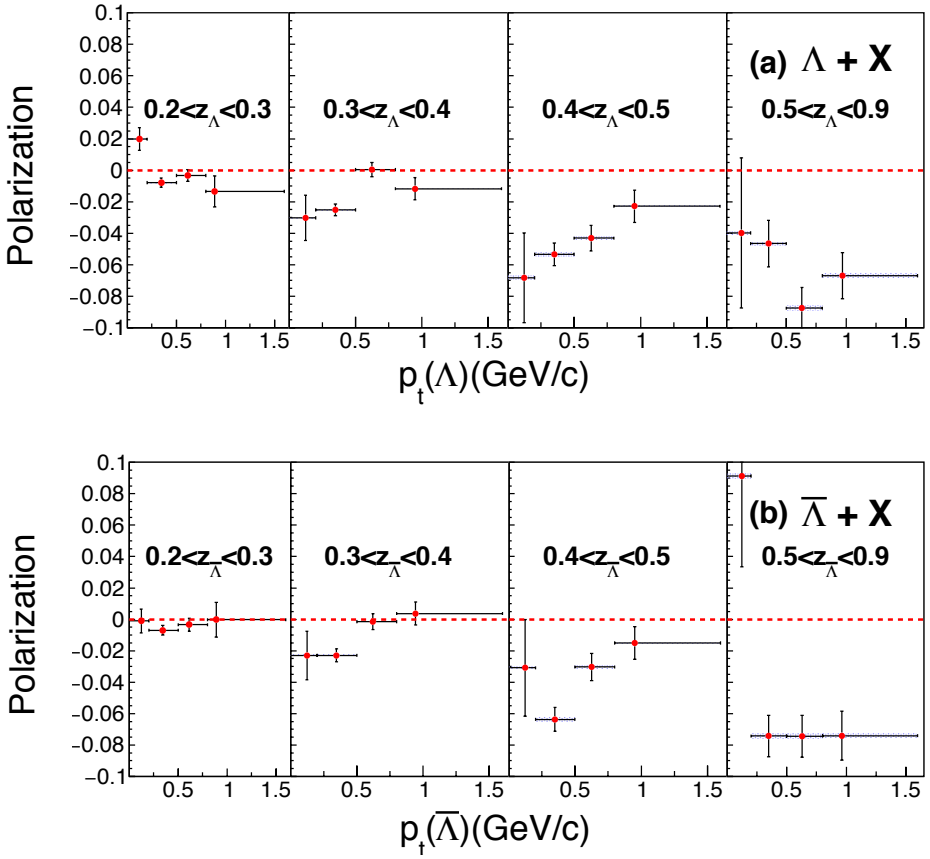

0.20 < z < 0.25
0.25 < z < 0.30
0.30 < z < 0.35
0.35 < z < 0.40
0.40 < z < 0.45
0.45 < z < 0.50
0.50 < z < 0.55
0.55 < z < 0.60
0.60 < z < 0.65
0.65 < z < 0.70
0.70 < z < 0.75
0.75 < z < 0.80
0.80 < z < 0.85
0.85 < z < 0.90
0.90 < z < 0.95
0.95 < z < 1.00
res
eess̄
tau
mixed
uds
cont
eecc̄
eeuū,dd̄
charged
charm
relative fraction
m_π⁺π⁻ [GeV]


0.30 < z < 0.35
0.35 < z < 0.40
0.50 < z < 0.55
0.55 < z < 0.60
Uncorrelated
Other
Fragmentation
Data

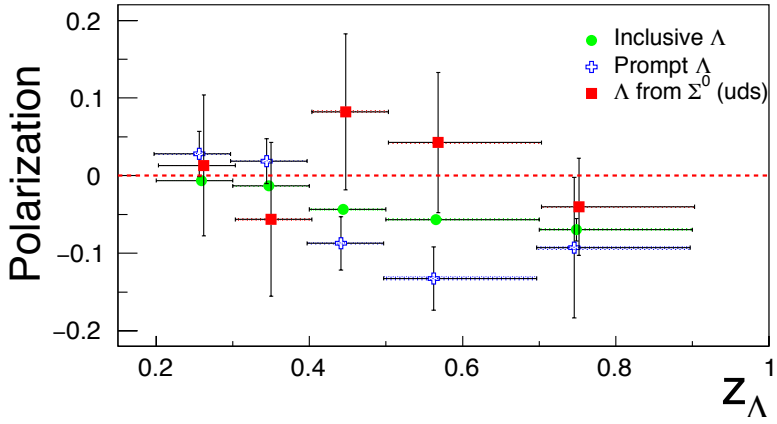

0.25 < z < 0.30
0.30 < z < 0.35
0.35 < z < 0.40

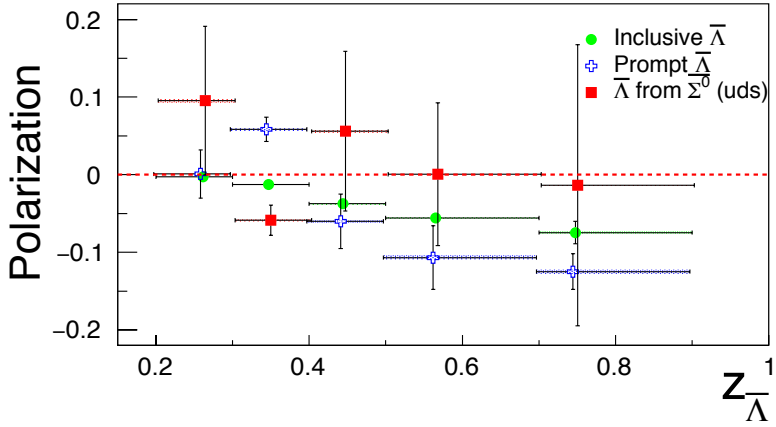
z_Λ, p_T Dependence of observed Λ polarization

- Polarization rises with p_t in the lowest z_Λ and highest z_Λ bin. But the dependence reverses around 1 GeV in the intermediate z_Λ bins→**Unexpected!**
- Results are consistent between Λ and $(\overline{\Lambda})$

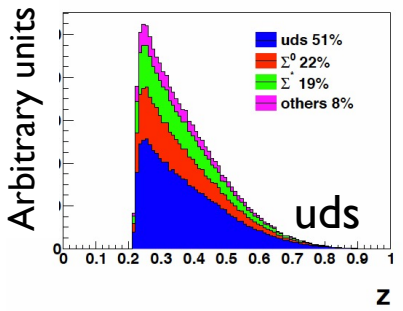


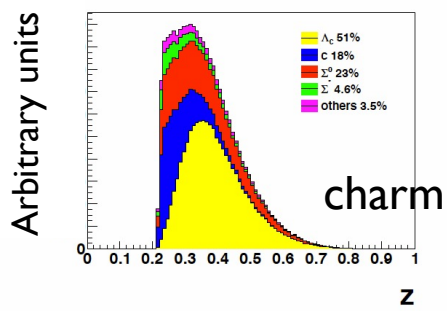
A first: Correction for feed-down and charm contribution

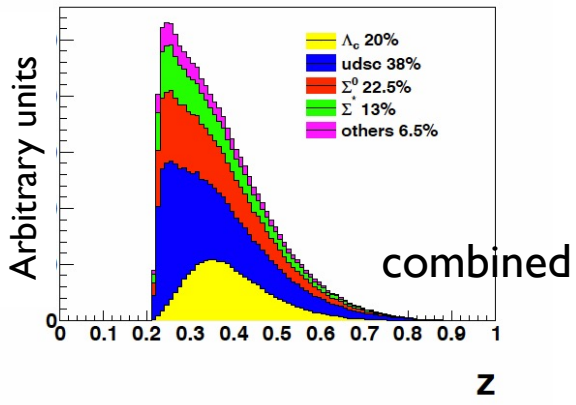




- Consider charm from D enhanced sample
- Use $\Sigma^0 \to \Lambda\gamma$, $\Xi \to \Lambda\pi$

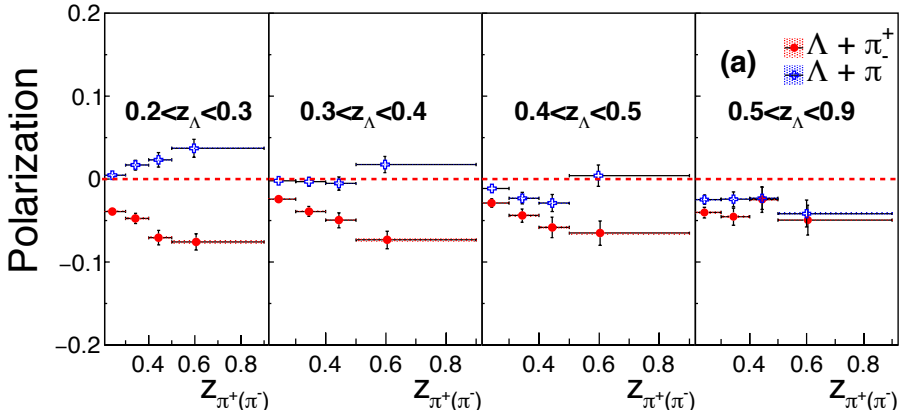


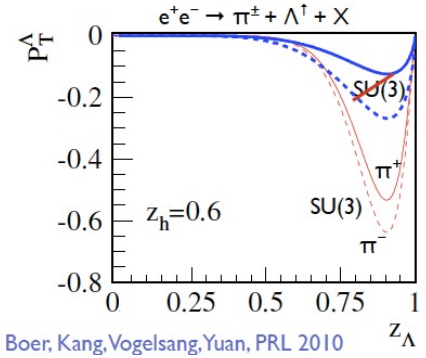




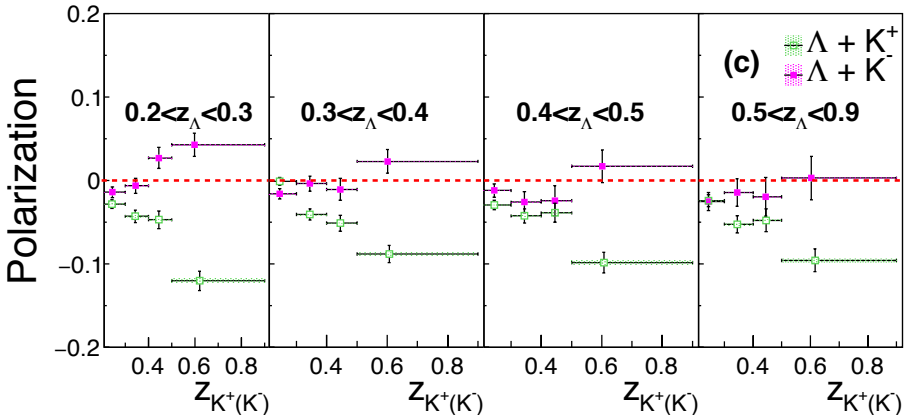
Tension with theory: ASSOCIATED PRODUCTION

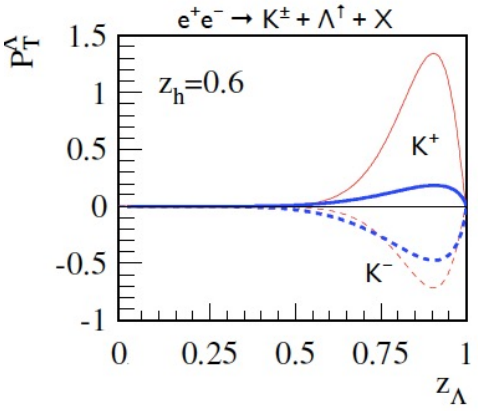
- Correlation with opposite hemisphere light meson →quark flav/charge dependence
  - Sign of asymmetry dependent on quark charge cf Sivers
- Only experimental results on T-odd, chiral even FF →**Important to understand!**





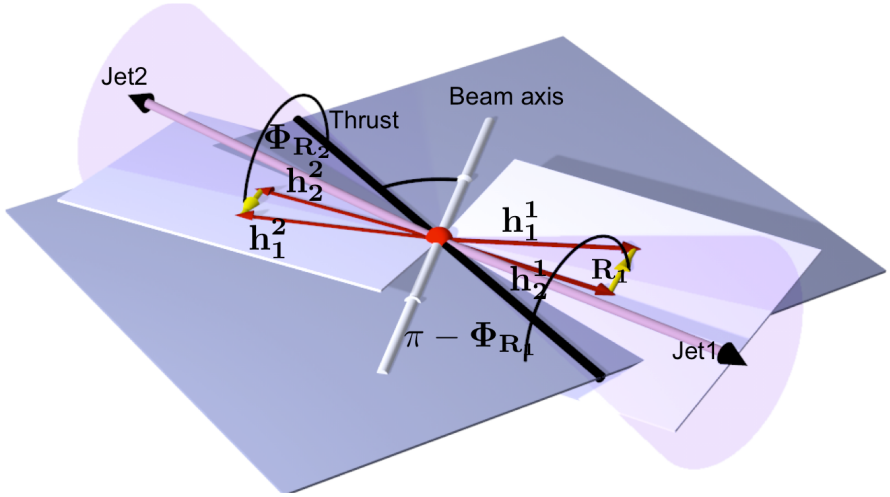
Boer, Kang, Vogelsang, Yuan, PRL 2010

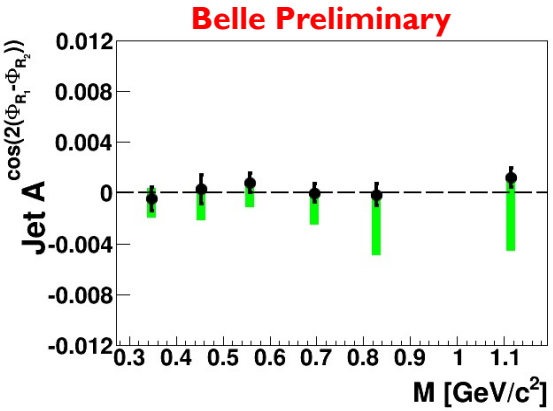


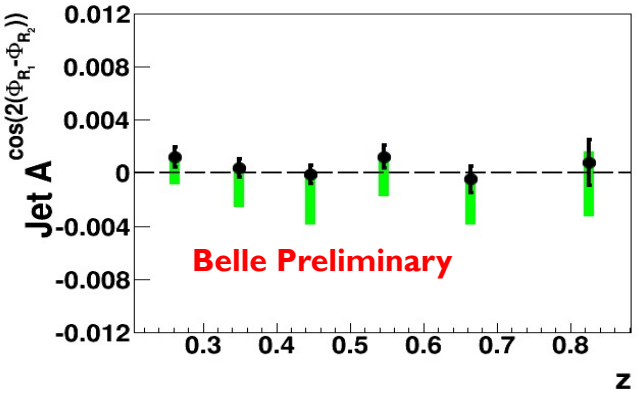


Asymmetries for Cos(2(ϕ_{R1}-ϕ_{R2})) ($G_1^{\perp}$) small

- Turned out that the asymmetry is projected to be $\equiv 0$



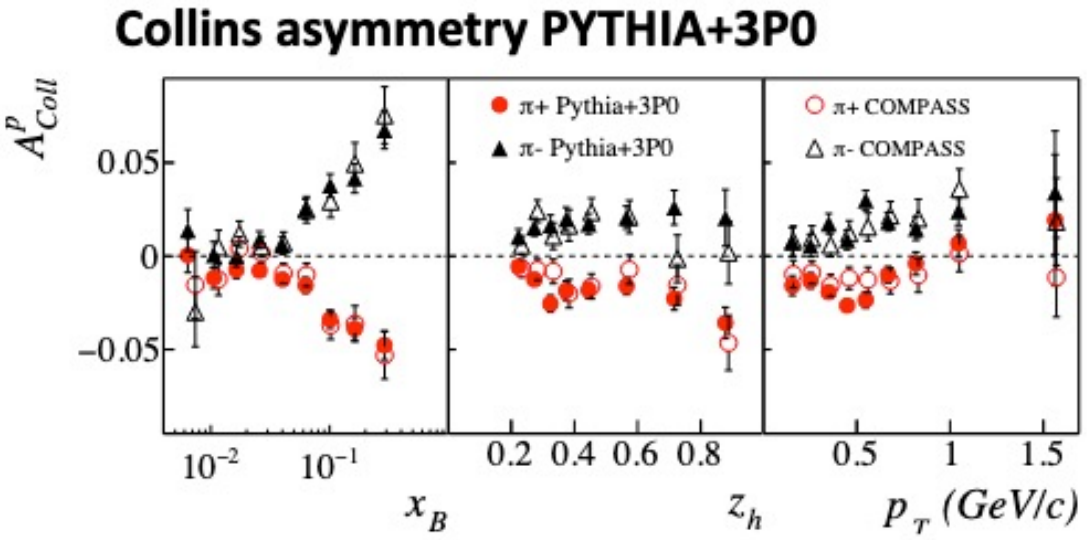




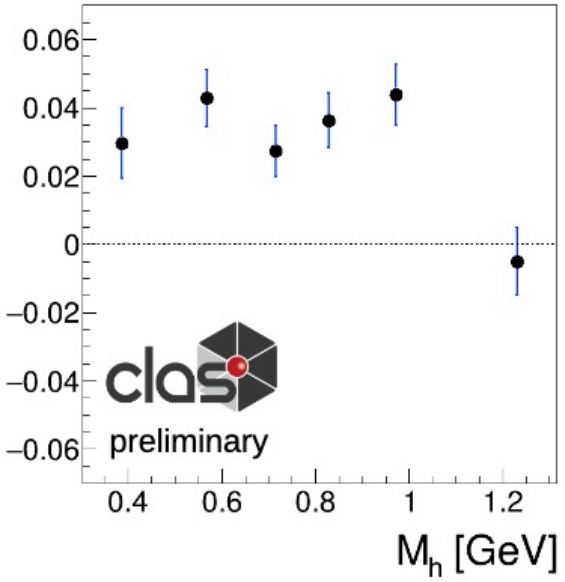
1505.08020 [hep-ex]

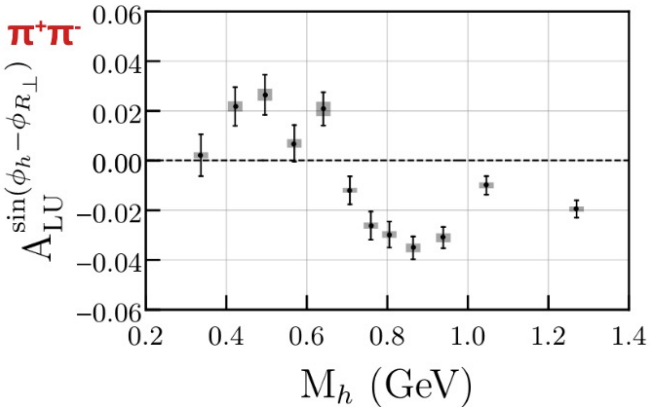
Comparing TMD Di-Hadron FFs with models

- TMD Di-hadron FFs
  - Most complex objects depending on spin orbit correlations in hadronization
- Model comparison might provide complementary insight
- Also ρ fragmentation

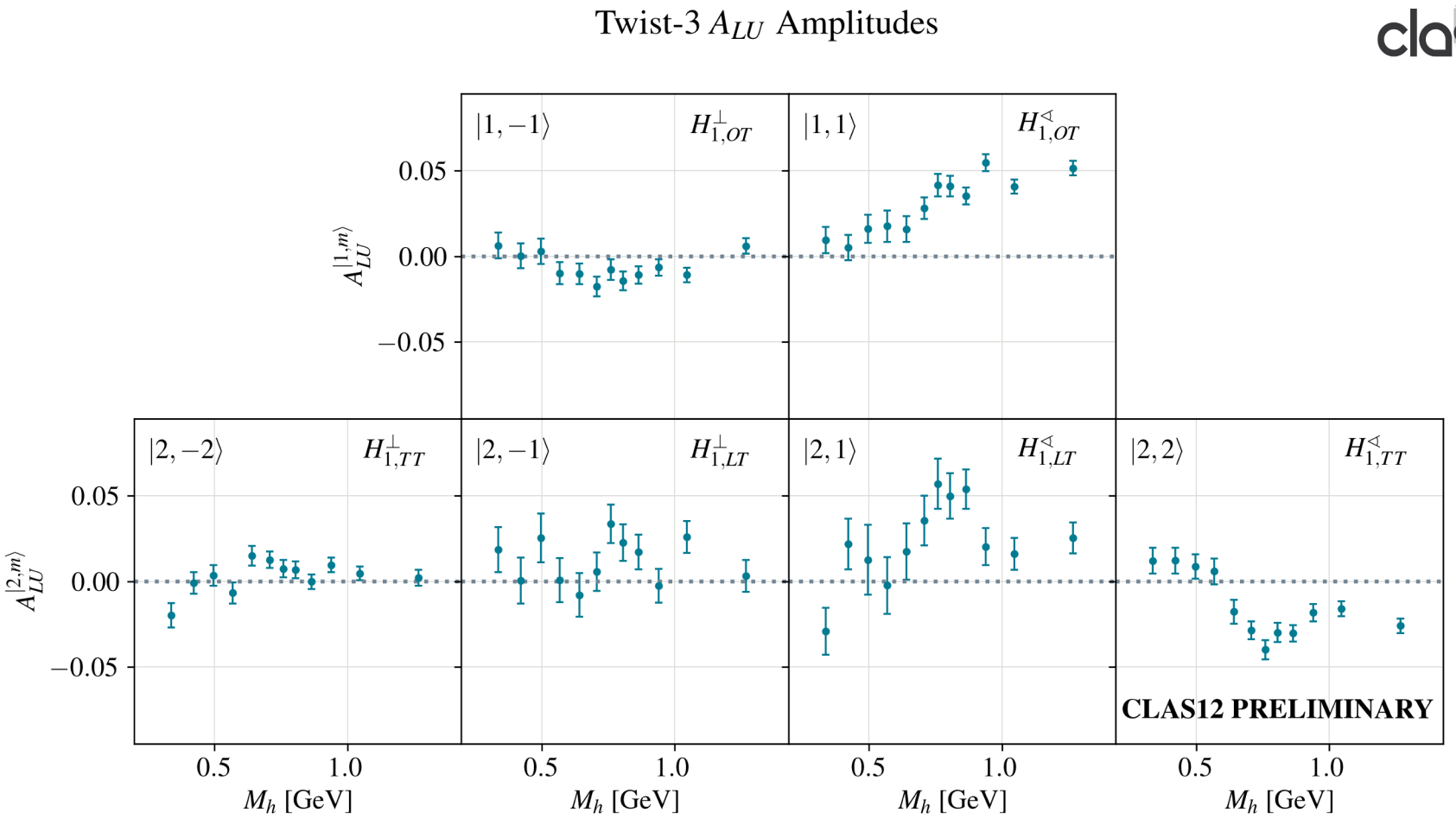


$\pi^+\pi^0$ $A_{LU}[\sin(\phi_h-\phi_R)]$ vs. M_h



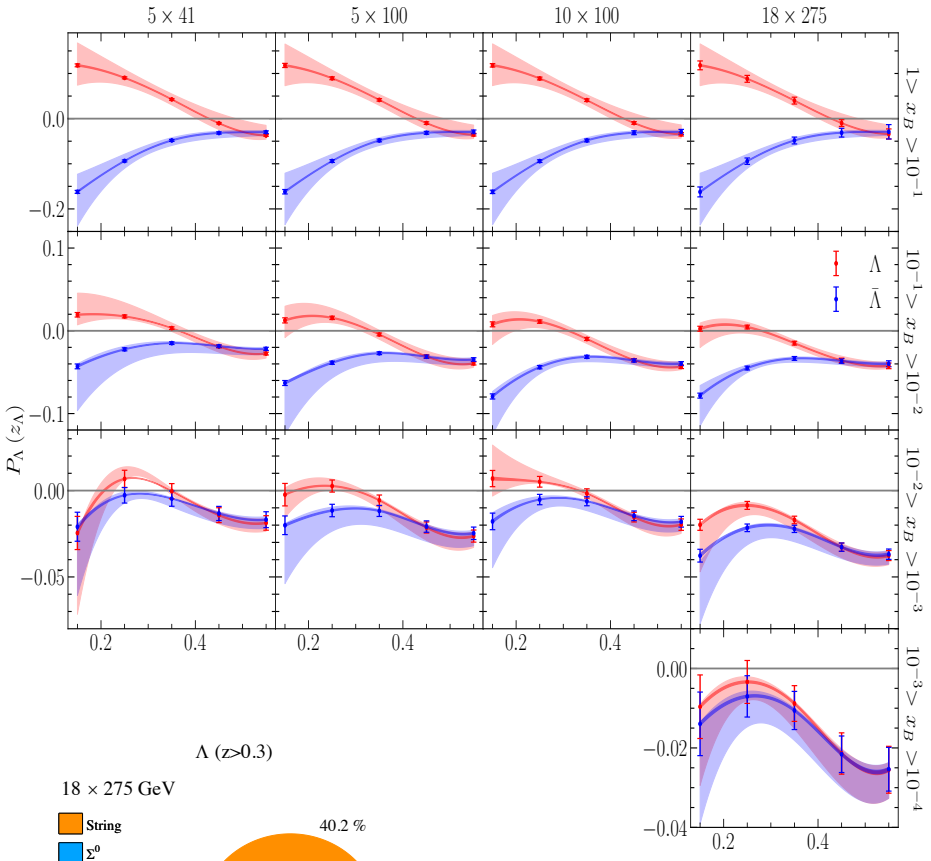


$e(x) \cdot H_1^{\sphericalangle}(z, M_h)$ vs M_h



Λ's at the EIC

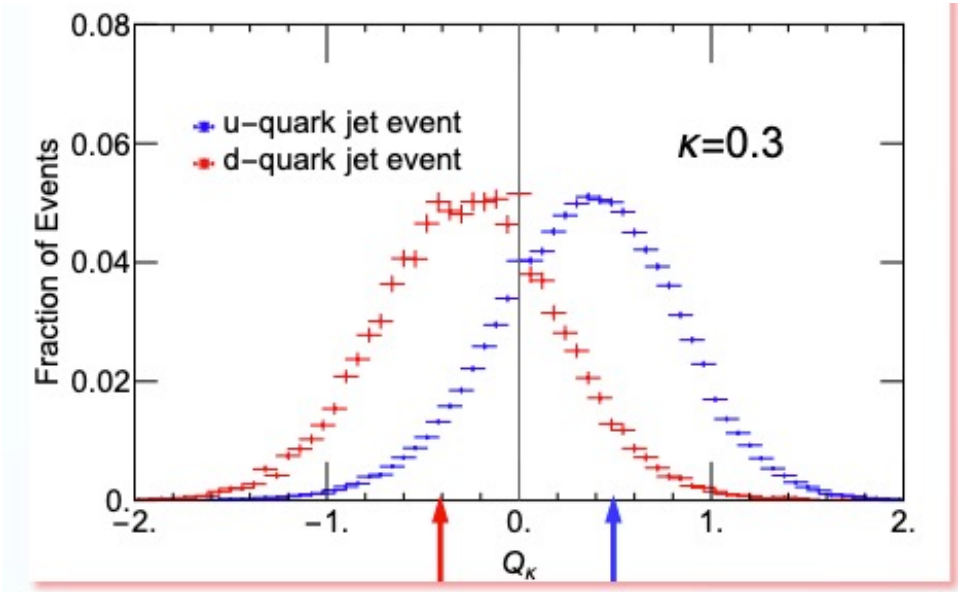
- EIC offers (finally) exciting prospect for precision Λ physics
- $\Lambda^{\uparrow}$ from polarizing FF and spin transfer (also longitudinal)
- (polarized) Λ in jets



2108.05383 [hep-ph]

Plans

- Jets
- Back to back
- …



$$Q_\kappa \equiv \sum_{h \in \text{jet}} z_h^\kappa Q_h$$

Charge of the hadron

$$z_h = \frac{p_{hT}}{p_{JT}} \qquad \kappa = 0.3, 0.4, \cdots, 1.0$$

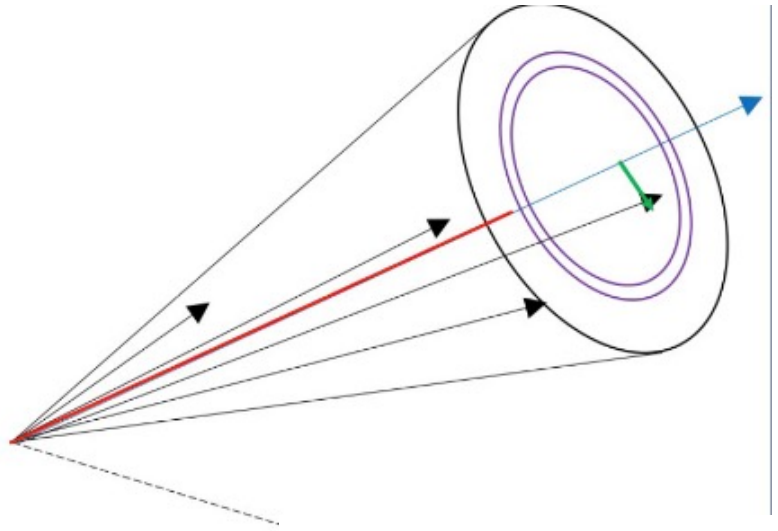
Experiment	Process	$\mathcal{L}[pb^{-1}]$	$Q^2[GeV^2]$	Final States
TPC [293, 294, 295, 296]	e^+e^-	140	29	$\pi^\pm, K^\pm, p/\bar{p}$
TASSO				
[297, 298, 299]	e^+e^-	34	34,44	$\pi^\pm, K^\pm, p/\bar{p}, K^0_S, \Lambda/\bar{\Lambda}$
[300, 301, 302, 303]				
SLD [304, 305]	e^+e^-	20	M_Z	$\pi^\pm, K^\pm, p, K^0_S, \Lambda/\bar{\Lambda}$
ALEPH [306, 307]	e^+e^-	800	M_Z	$\pi^\pm, K^\pm, p, K^0_S, \Lambda/\bar{\Lambda}$
DELPHI [308, 309, 310, 311]	e^+e^-	800	M_Z	$\pi^\pm, K^\pm, p, K^0_S, \Lambda/\bar{\Lambda}$
OPAL [312, 313, 314, 315]	e^+e^-	800	M_Z	$\pi^\pm, K^\pm, p, K^0_S, \Lambda/\bar{\Lambda}$
H1 [316, 317, 318]	$e+p$	500	27.5 (e) + 920 (p)	$h^\pm$, K^S_0
ZEUS [319, 320, 321]	$e+p$	500	27.5 (e) + 920 (p)	$h^\pm$
BELLE [322, 323]	e^+e^-	10^6	near 10.58	$\pi^\pm, K^\pm, p/\bar{p}$
BaBar [324, 325]	e^+e^-	$557 \cdot 10^3$	near 10.58	$\pi^\pm, K^\pm, \eta, p/\bar{p}$
HERMES [326, 327]	$e+p(d)$	272 (p) 329(d)	27.6 fixed target	$\pi^{\pm,0}, K^\pm$
COMPASS [328]	$\mu+p(d)$	775	160 GeV fixed target	$h^\pm$
PHENIX		16×10^{-3}	62.4	
[329, 330, 331]	pp	2.5	200	$\pi^{\pm,0}, \eta$
[332, 333, 334]		128	510	
STAR				
[335, 336, 337]	pp	8	200	$\pi^{0,\pm}, \eta, p/\bar{p}, K^0_S, \Lambda/\bar{\Lambda}$
[338, 339, 340]				
ALICE [341]	pp	6×10^{-3}	7×10^3	π^0, η
TOPAZ [342]	e^+e^-	278	52-61.4	$\pi^\pm, K^\pm, K^0_S,$
CDF [343, 344]	$p+\bar{p}$	n/a	630 (1800)	$h^\pm, K^0_S\Lambda^0$

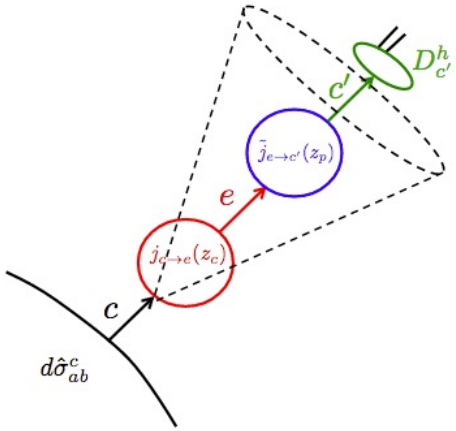
Points compared to MC generators

- FFs ’inclusive’, MCEG exclusive
- The more exclusive measurements, probably the better for MCEG
  - Event shapes
  - In jet
  - Di-hadron

In Jet

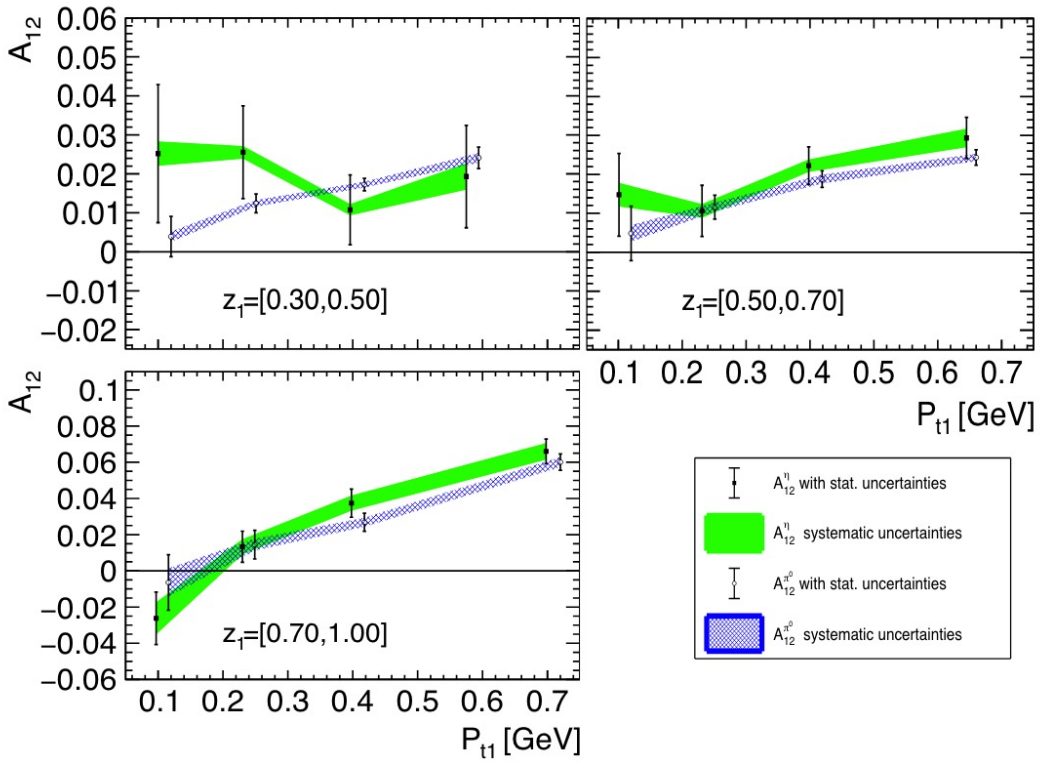
- More correlations (relative to jet axis)
- Grooming→w/ w/o →soft contributions





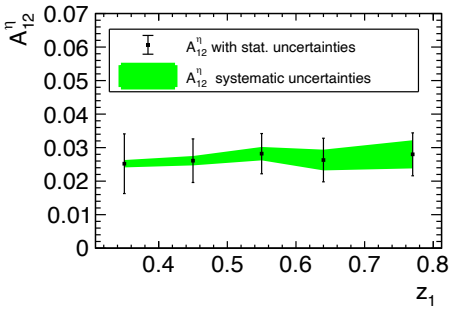
New π^0/η from Belle

- **Rise with $z_{1,2}$, similar to charged pions**



$$\mathcal{R}_{12}^{\pi^0} = \frac{R_{12}^{0\pm}}{R_{12}^{L}} = \frac{\pi^0\pi^+ + \pi^0\pi^-}{\pi^+\pi^+ + \pi^-\pi^-}$$

$$\mathcal{R}_{12}^{\eta} = \frac{R_{12}^{\eta\pm}}{R_{12}^{L}} = \frac{\eta\pi^+ + \eta\pi^-}{\pi^+\pi^+ + \pi^-\pi^-}$$

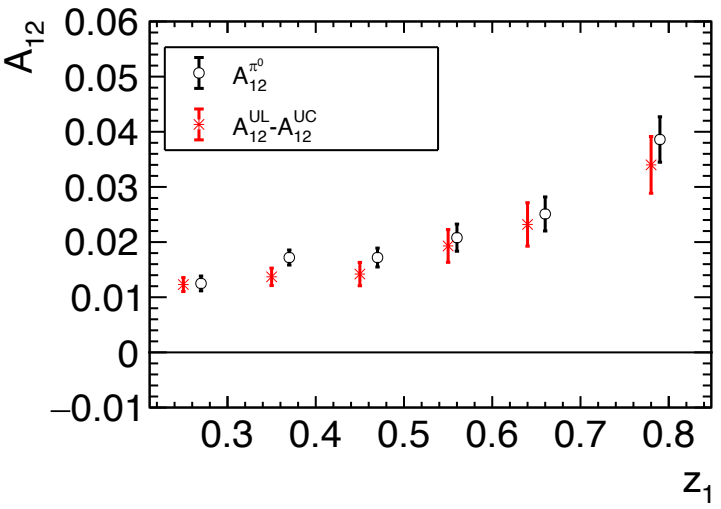


- η almost flat except large z

Consistency between Neutral and charged pions

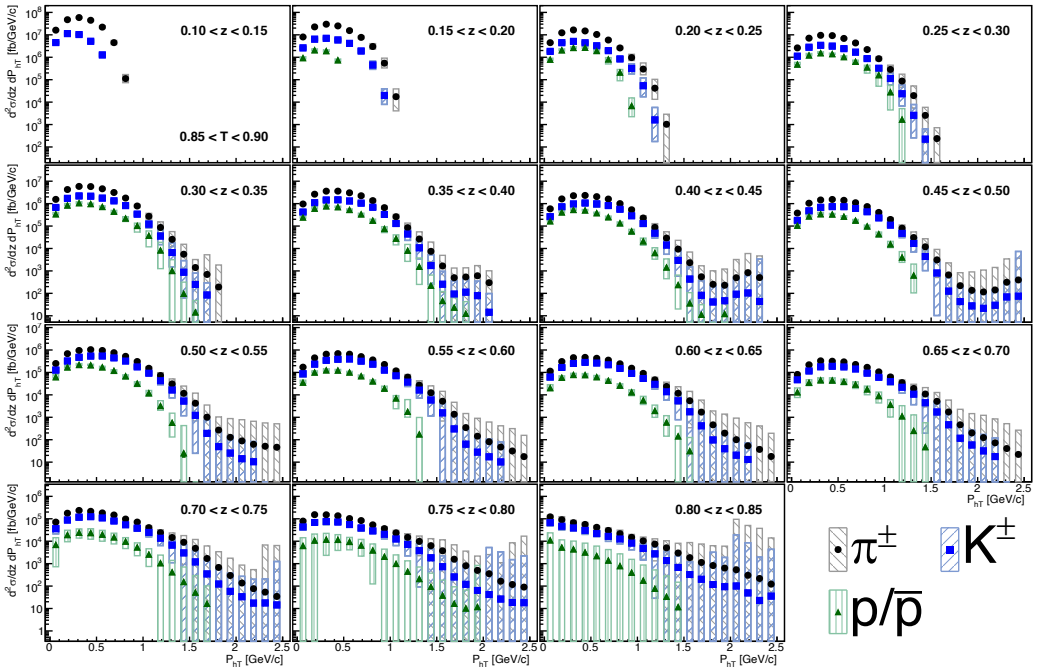
$$\mathcal{R}_{12}^{\pi^0} = \frac{R_{12}^{0\pm}}{R_{12}^{L}} \approx 1 + \cos(\phi_{12}) \frac{\sin^2(\theta)}{1+\cos^2(\theta)} \times \left\{ \frac{5(H_1^{\perp,fav} + H_1^{\perp,dis}) \otimes (H_1^{\perp,fav} + H_1^{\perp,dis}) + 4H_{1,s\to\pi}^{\perp,dis} \otimes H_{1,s\to\pi}^{\perp,dis}}{5(D_1^{fav} + D_1^{dis}) \otimes (D_1^{fav} + D_1^{dis}) + 4D_{1,s\to\pi}^{dis} \otimes D_{1,s\to\pi}^{dis}} - \frac{10H_1^{\perp,fav} \otimes H_1^{\perp,dis} + 2H_{1,s\to\pi}^{\perp,dis} H_{1,s\to\pi}^{\perp,dis}}{10D_1^{fav} \otimes D_1^{dis} + 2D_{1,s\to\pi}^{dis} \otimes D_{1,s\to\pi}^{dis}} \right\}.$$

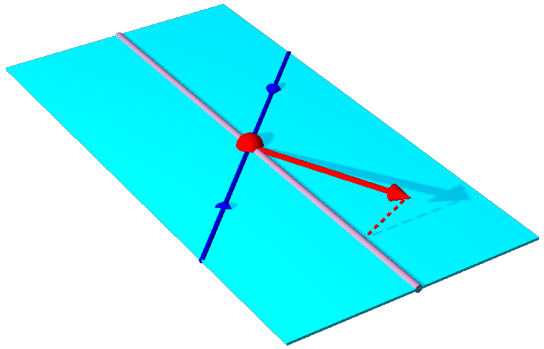
$$= A_{12}^{UL} - A_{12}^{UC} \ (Isospin)$$



Transverse momentum distributions

- 0.85< Thrust T < 0.9
  - Transverse momenta mostly Gaussian
  - Possible deviations for large P_{hT} tails, but also large uncertainties

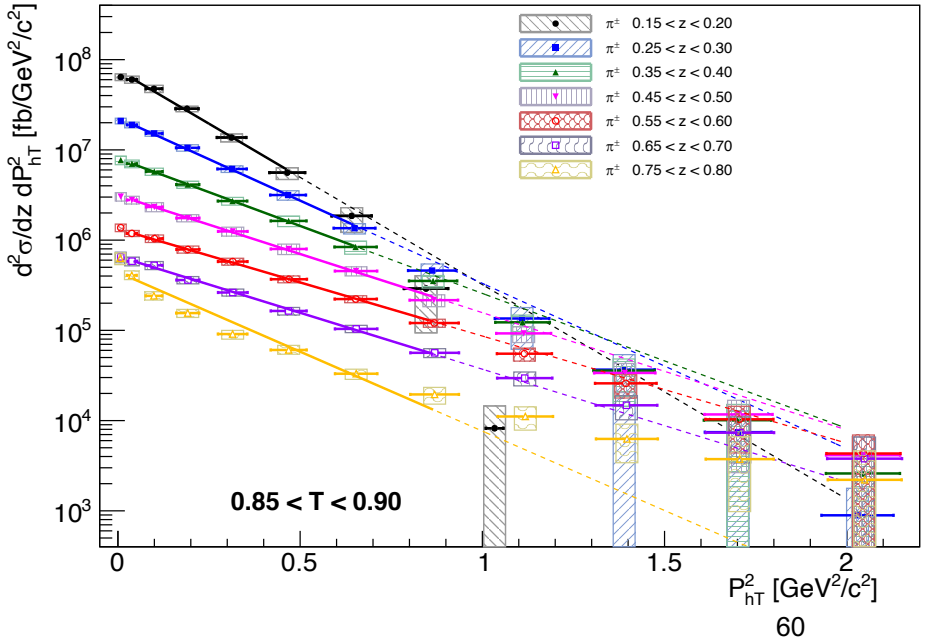




Phys.Rev.D 99 (2019) 11, 112006

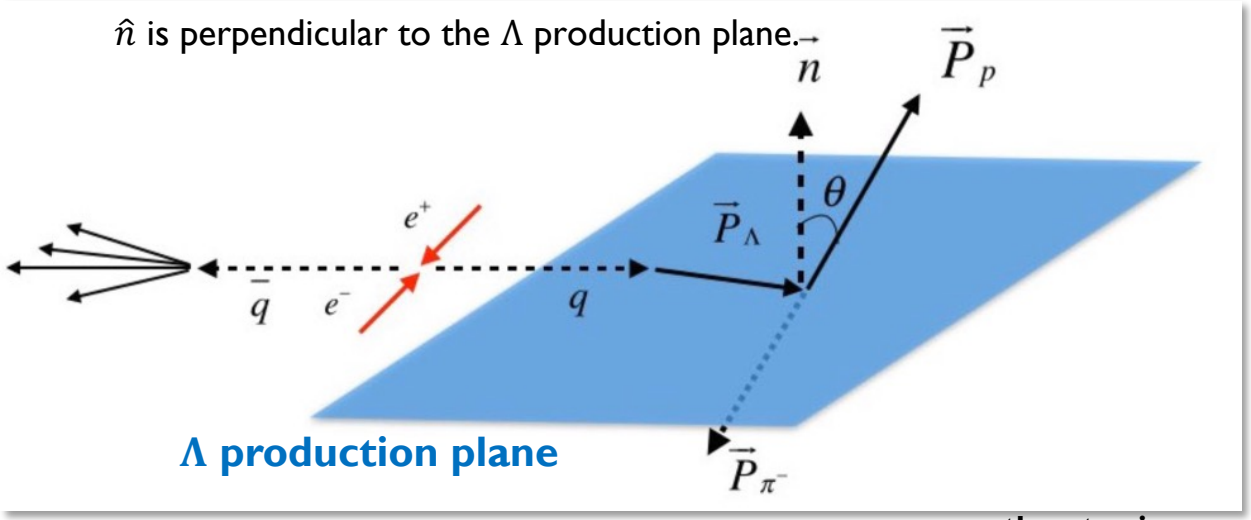
Transverse Momentum: Gaussian widths

- 0.85 < T < 0.9
  - Fit Gauss to low P_{hT} data
  - Mostly well described with possible exception at high z
  - Deviation from Gauss at large P_{hT}
  - Clear increase in width with z for low values of z

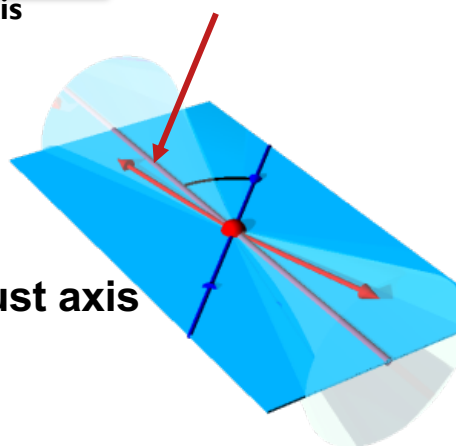


Observables of Λ Polarization at Belle

- Self-analyzing decay leads to polarization dependent distribution



thrust axis

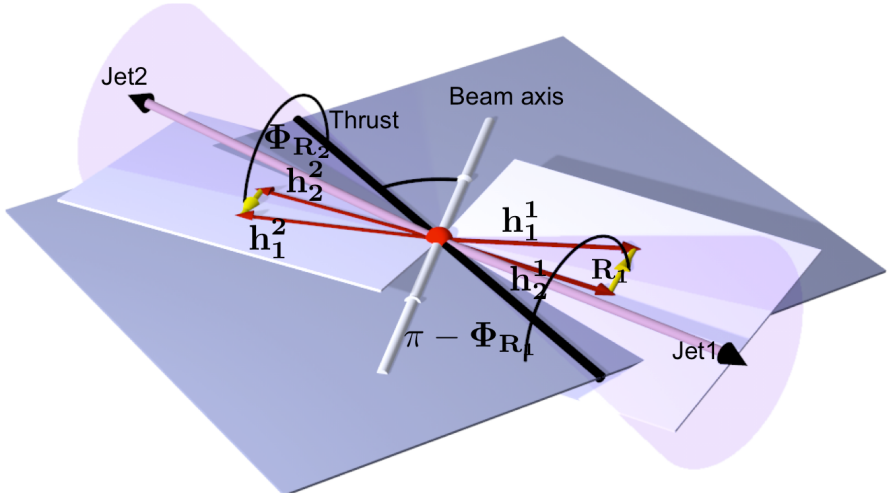


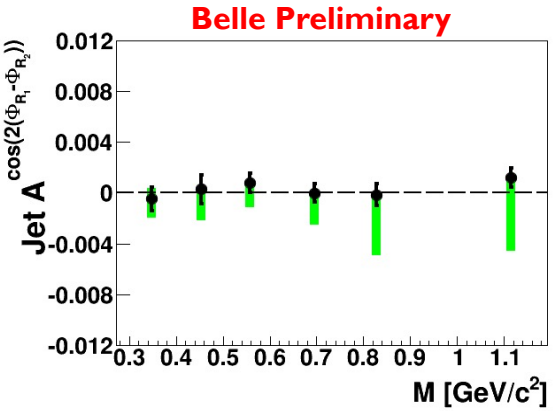
- where α is the decay parameter: α_+=0.642 ± 0.013 for Λ and α_-=-0.71 ± 0.08 for $\bar{\Lambda}$ (PDG).
- The p_t is measured as the transverse momentum of Λ relative to the **thrust axis**

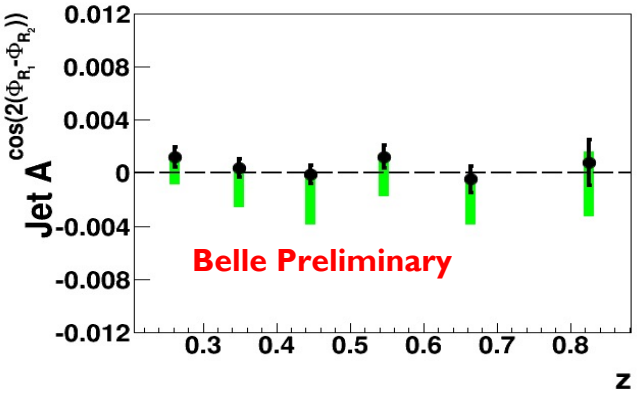
$$\frac{1}{N}\frac{dN}{d\cos\theta} = 1 + \alpha P \cos\theta$$

Asymmetries for Cos(2(ϕ_{R1}-ϕ_{R2})) ($G_1^{\perp}$) small

- Turned out that the asymmetry is projected to be $\equiv 0$







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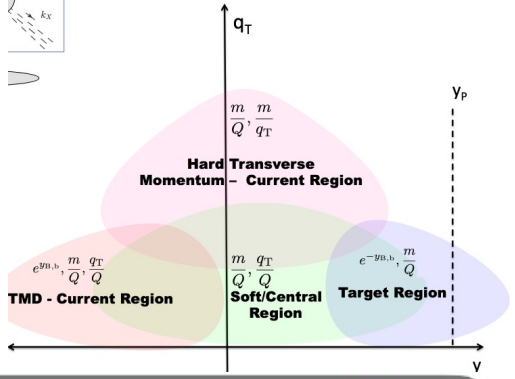
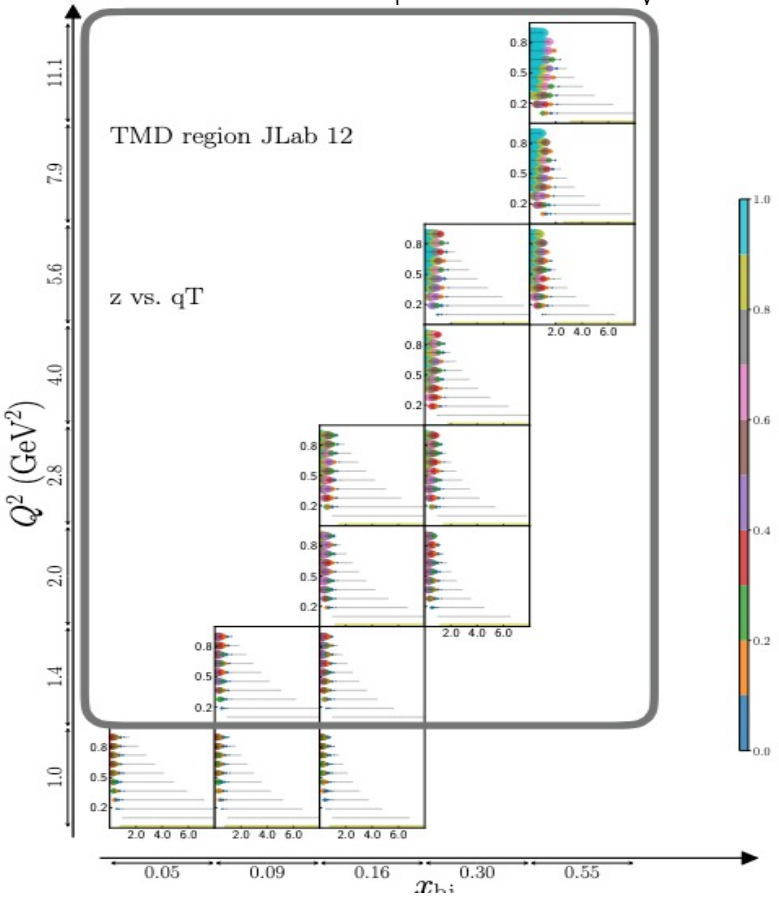
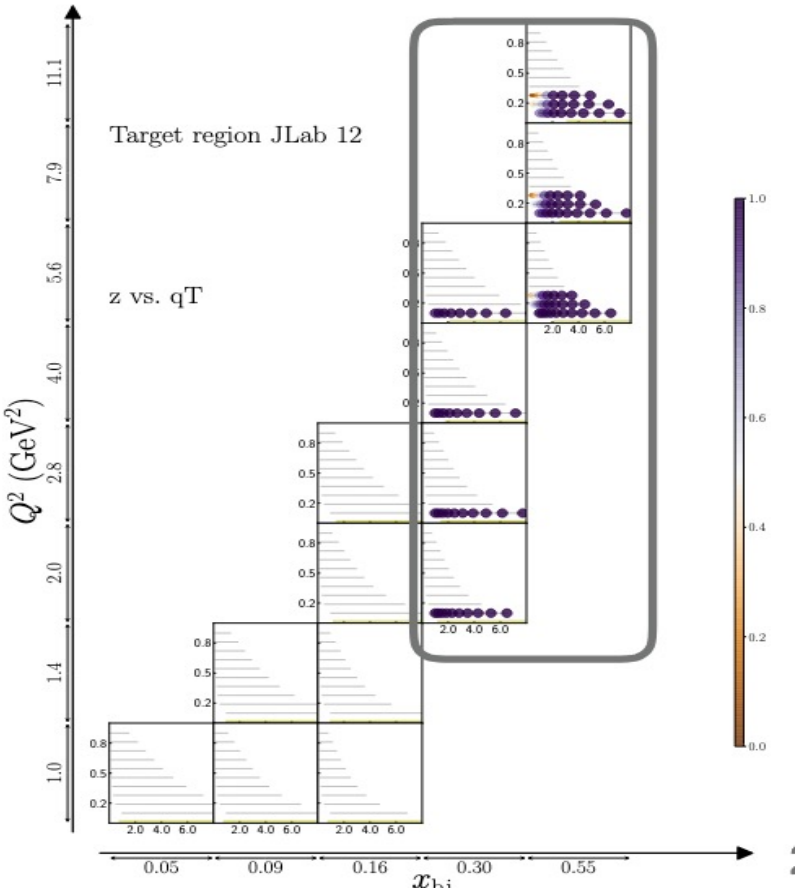
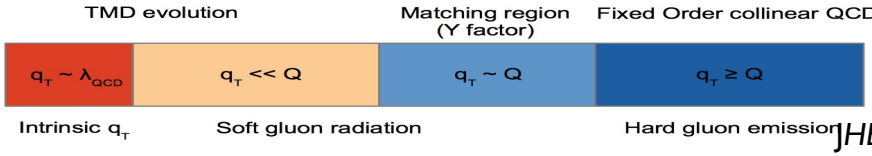


A. Prokudin et al. (YR SIDIS meeting)
See also *JHEP* 10 (2019) 122, JHEP10(2019)122

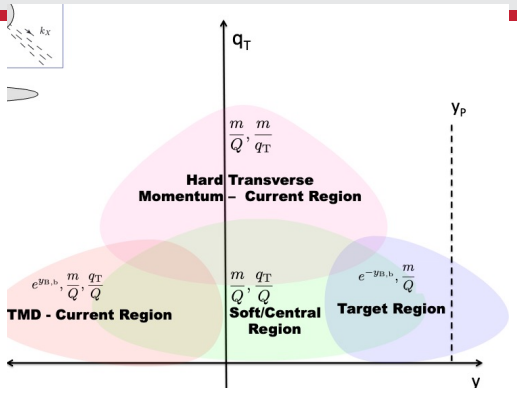


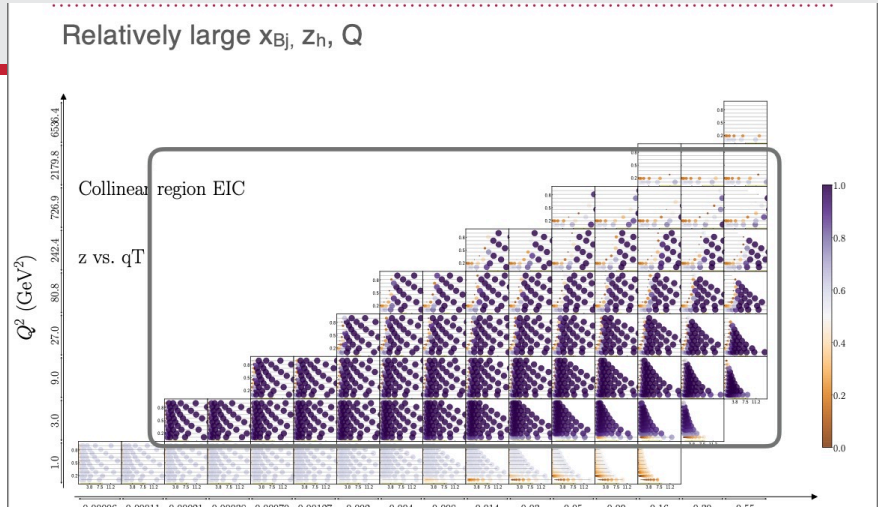


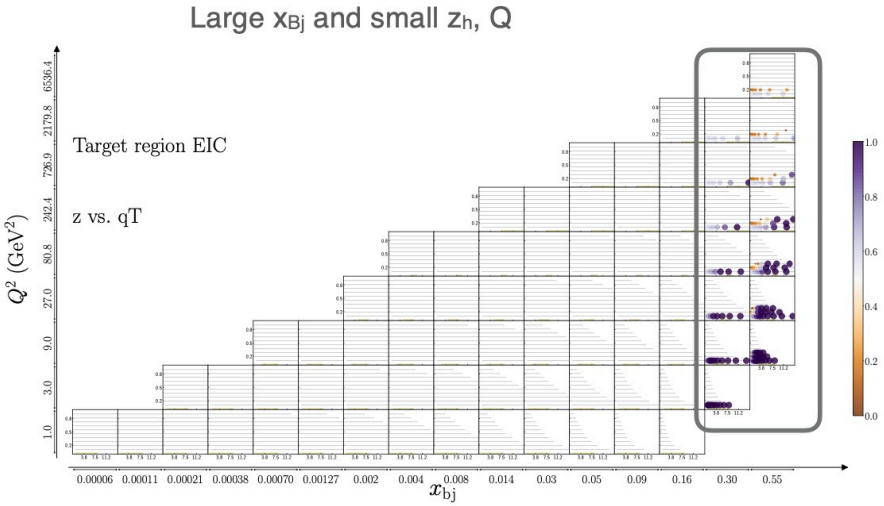


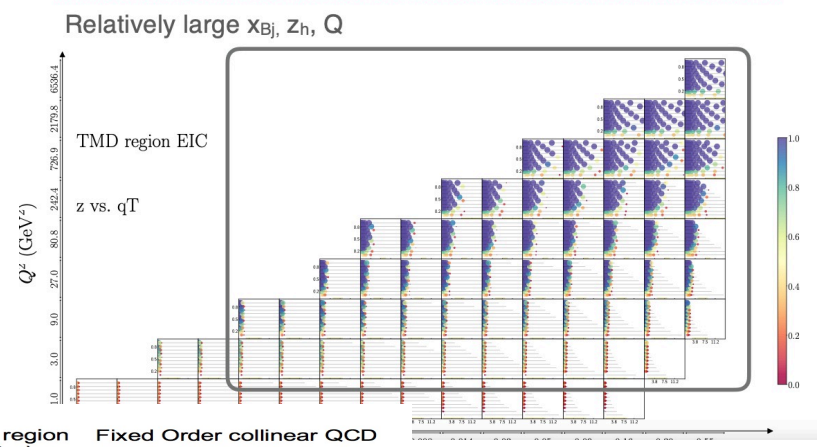
From M Boglione
PoS QCDEV2016 (2017) 026

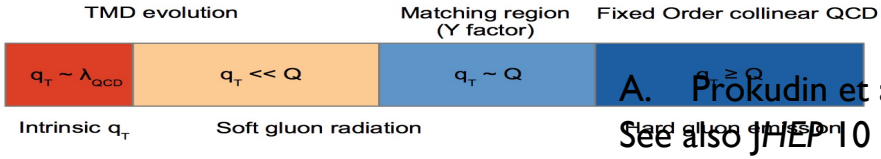
A. Prokudin et al.
JHEP 10 (2019) 122, JHEP10(2019)122









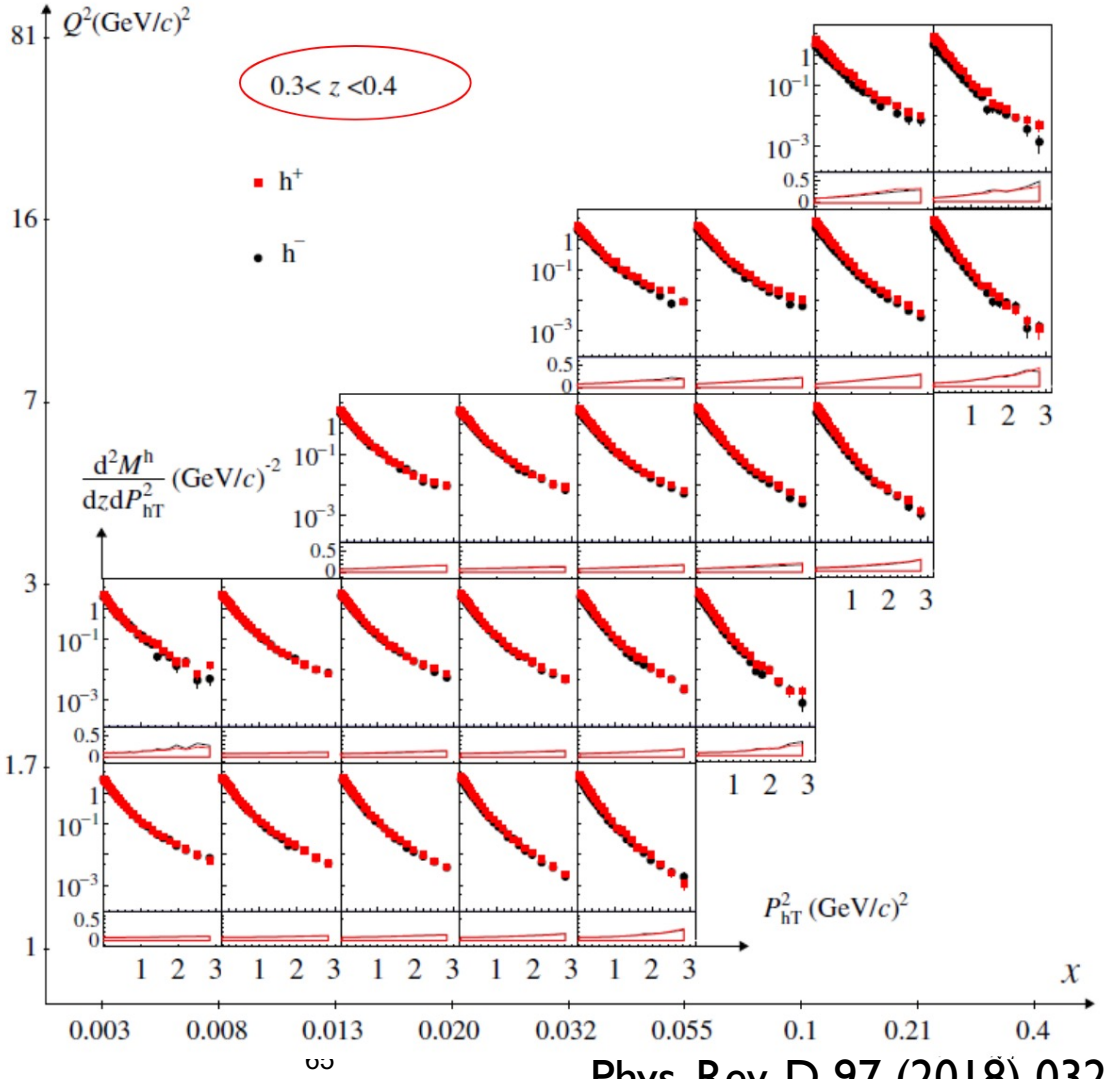


From M Boglione
PoS QCDEV2016 (2017) 026

A. Prokudin et al. (YR SIDIS meeting)
See also *JHEP* 10 (2019) 122, JHEP10(2019)122

COMPASS ^6LiD

- h^+/h^- shapes similar

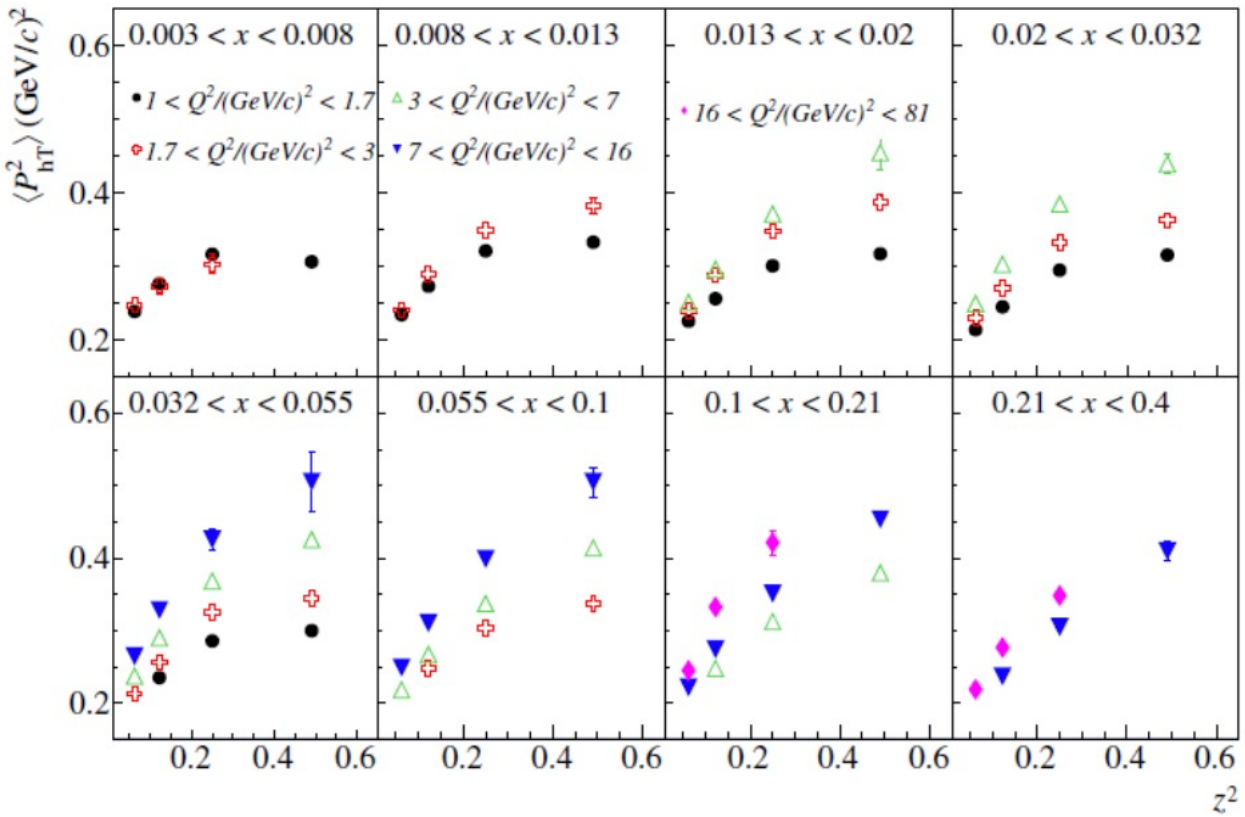


Following COMPASS plots from A. Moretti/A Martin's talks at DIS 2021 and SPIN 2021

Phys. Rev. D 97 (2018) 032006

- Strong Q^2 / W, z dependence

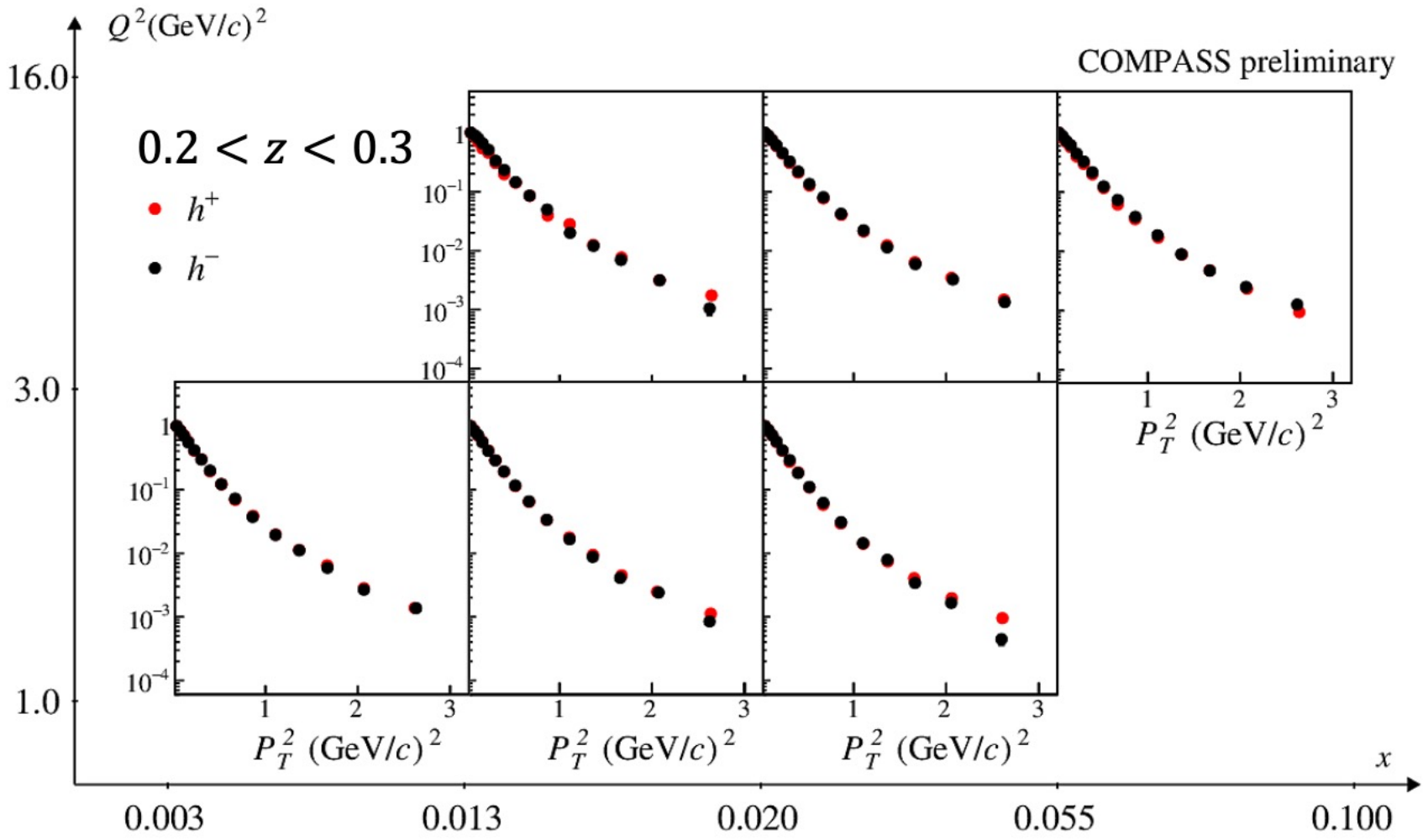
$\langle P_T^2 \rangle$ from 1 exp fit up to 0.85 GeV/c

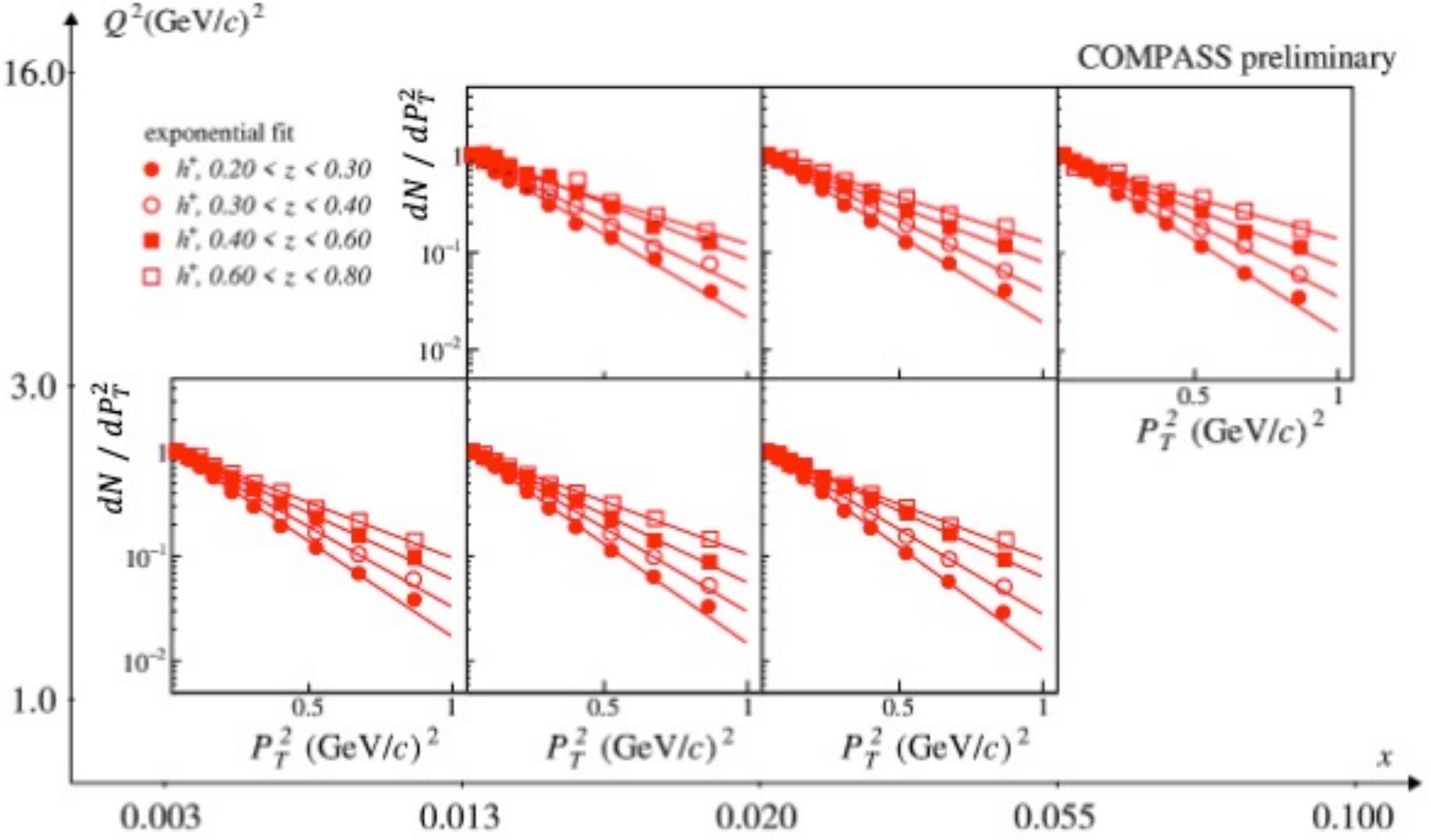


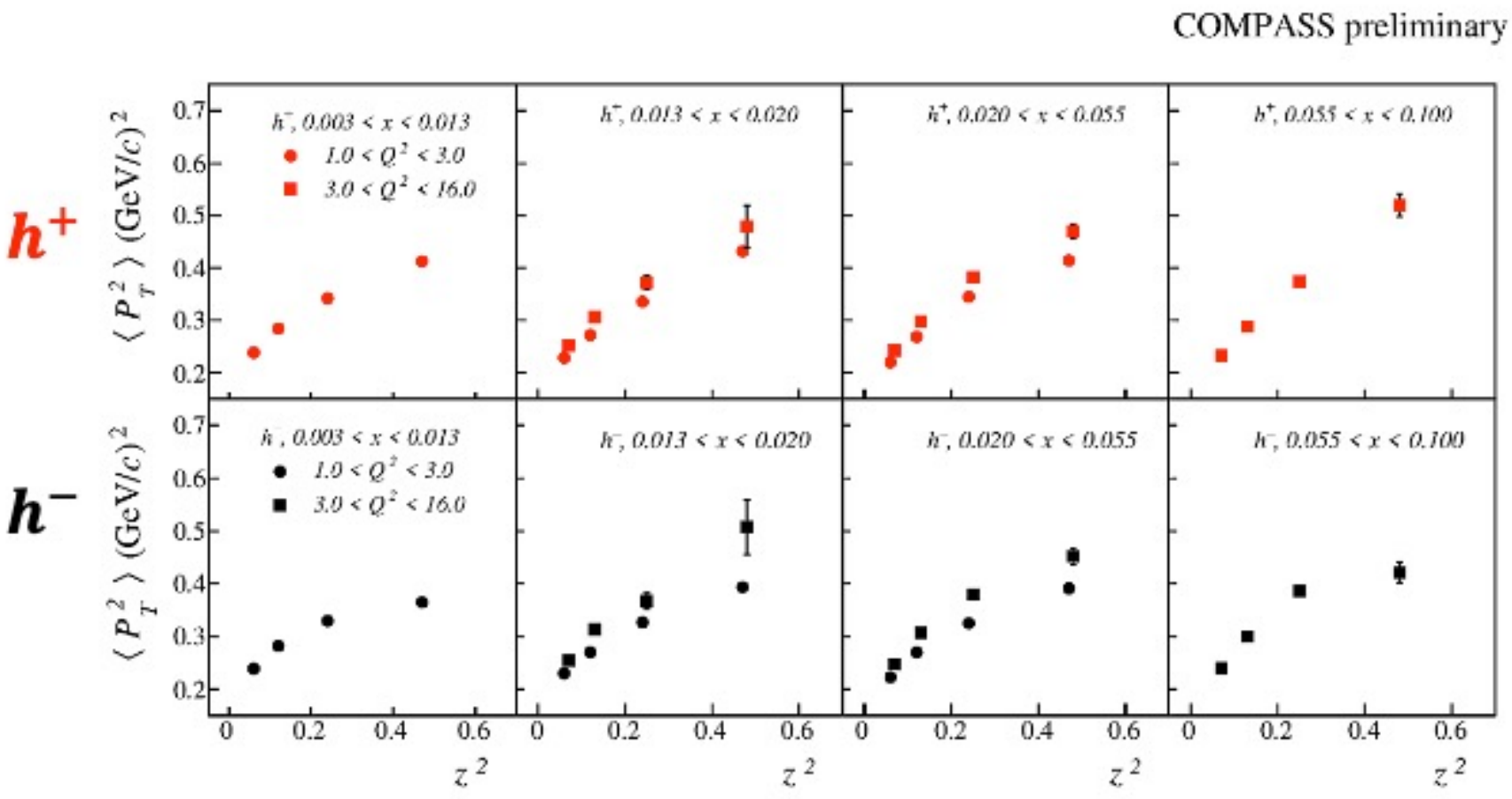
$$\vec{P}_T \simeq z\vec{k}_T + \vec{p}_\perp \Rightarrow$$

SIDIS intrinsic fragmentation

New: LH_2

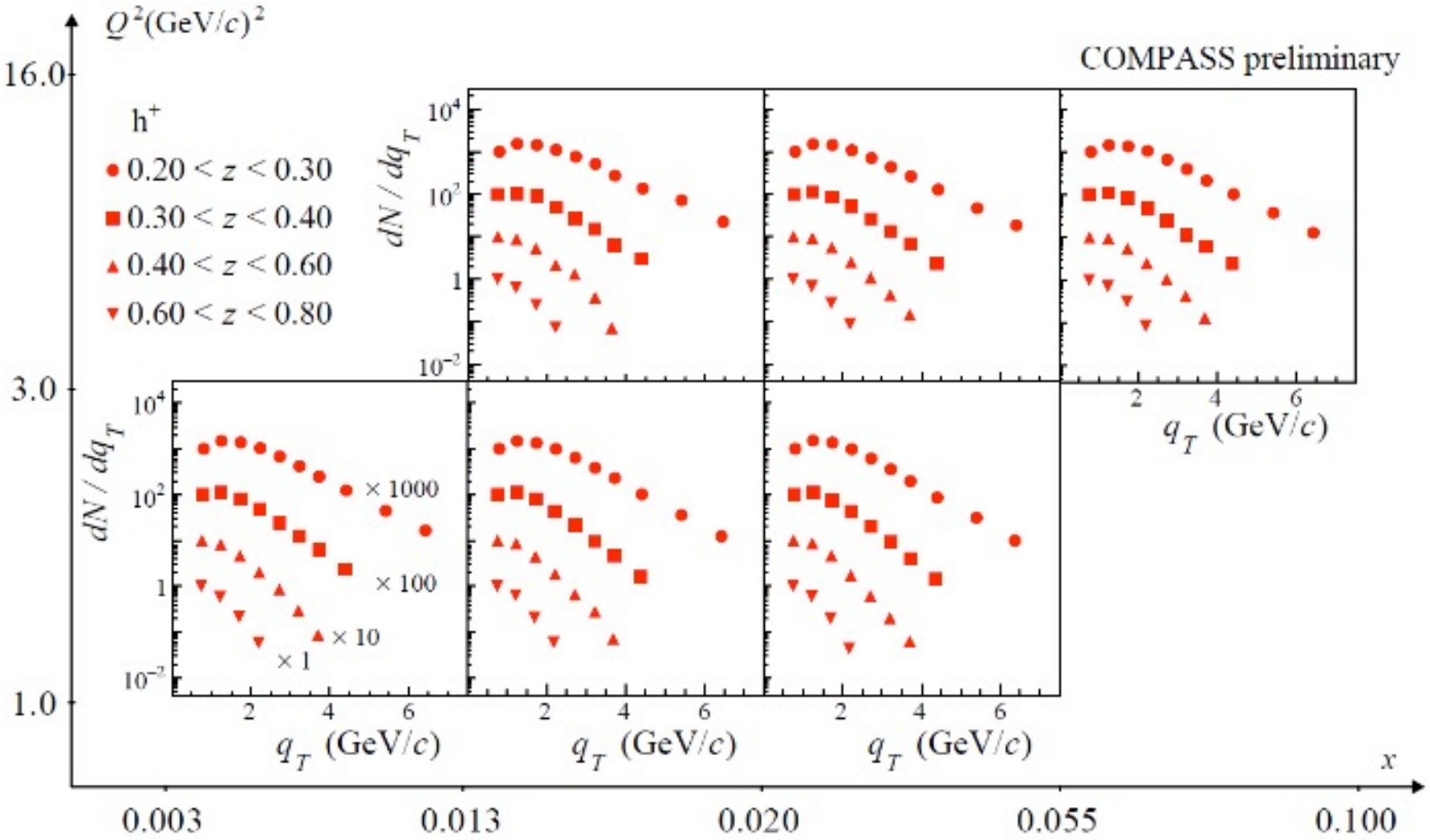

$Q^2(\mathrm{GeV}/c)^2$
16.0
3.0
1.0
COMPASS preliminary
$0.2 < z < 0.3$
h^+
h^-
$P_T^2\ (\mathrm{GeV}/c)^2$
x
0.003
0.013
0.020
0.055
0.100


Q^2(GeV/c)^2
16.0
3.0
1.0
COMPASS preliminary
exponential fit
h^+, 0.20 < z < 0.30
h^+, 0.30 < z < 0.40
h^+, 0.40 < z < 0.60
h^+, 0.60 < z < 0.80
dN / dP_T^2
10^{-1}
10^{-2}
0.5
1
P_T^2 (GeV/c)^2
x
0.003
0.013
0.020
0.055
0.100



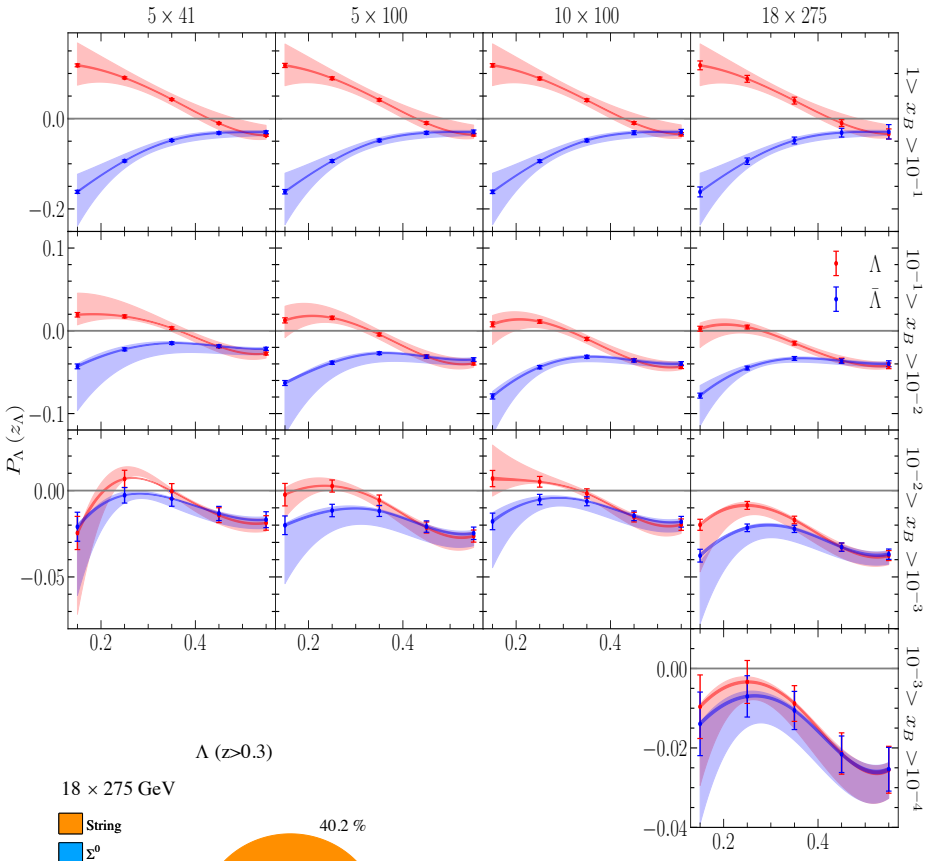
$\langle P_T^2 \rangle$ versus z^2 in the x and Q^2 bins

Deviations from the linear trend $\langle \boldsymbol{P_T^2} \rangle = \boldsymbol{z^2 \langle k_T^2 \rangle} + \boldsymbol{\langle p_\perp^2 \rangle}$


Q^2(GeV/c)^2
16.0
3.0
1.0
COMPASS preliminary
h$^+$
0.20 < z < 0.30
0.30 < z < 0.40
0.40 < z < 0.60
0.60 < z < 0.80
dN / dq_T
q_T (GeV/c)
× 1000
× 100
× 10
× 1
0.003
0.013
0.020
0.055
0.100
x

Λ's at the EIC

- EIC offers (finally) exciting prospect for precision Λ physics
- $\Lambda^{\uparrow}$ from polarizing FF and spin transfer (also longitudinal)
- (polarized) Λ in jets



2108.05383 [hep-ph]

Summary

- Rich program on TMD (FFs) at Belle (II), COMPASS, JLAB12, EIC
- Belle and COMPASS results can be used to extract intrinsic k_T
- EIC will allow precision polarized Λ program, CLAS12 can be pathway

- Honorary Mentions that couldn't be covered in this talk
  - Polarized Λ at CLAS12 and COMPASS
  - High precision, multi-dimensional measurements of A_{LU} at CLAS12
  - Rich Compass program on azimuthal asymmetries on polarized/unpolarized targets
  - Ongoing studies of the SIDIS program for the EIC
  - See overview talks at Spin 2021, DIS 2021