From Fermi's interaction to SCET Gravity - introduction to EFT

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Gravitational soft theorem from emergent soft gauge symmetries

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AND

Soft-collinear Gravity Beyond Leading Power

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In preparation

Before we jump to SCET GR let us discuss EFTs

- Every QFT is EFT
- Typically, when we think of EFT, we mean some simplified version of the full theory
- Basic idea: remove (integrate out) complicated short distance physics and focus only on scales relevant for a given problem (long-distance contributions)
- Landscape of EFTs is too large to describe in one talk - we will only focus on few simple EFTs with perturbative matching

This is talk about formalism, but I hope to shed a light on the physics behind this formalism

I will (re)introduce following concepts

	Basic EFTs	Modern EFTs
Heavy/energetic states	Not dynamical DOF	Can be dynamical DOF
Power-counting	Mass dimension	Arbitrary parameter
Locality	Theory is local with higher derivative terms	Theory can be non-local
Multipole expansion	Not needed - they already have homogenous power counting	Essential to achieve homogenous power counting
Gauge symmetry	Same as in the full theory	Each mode has its own

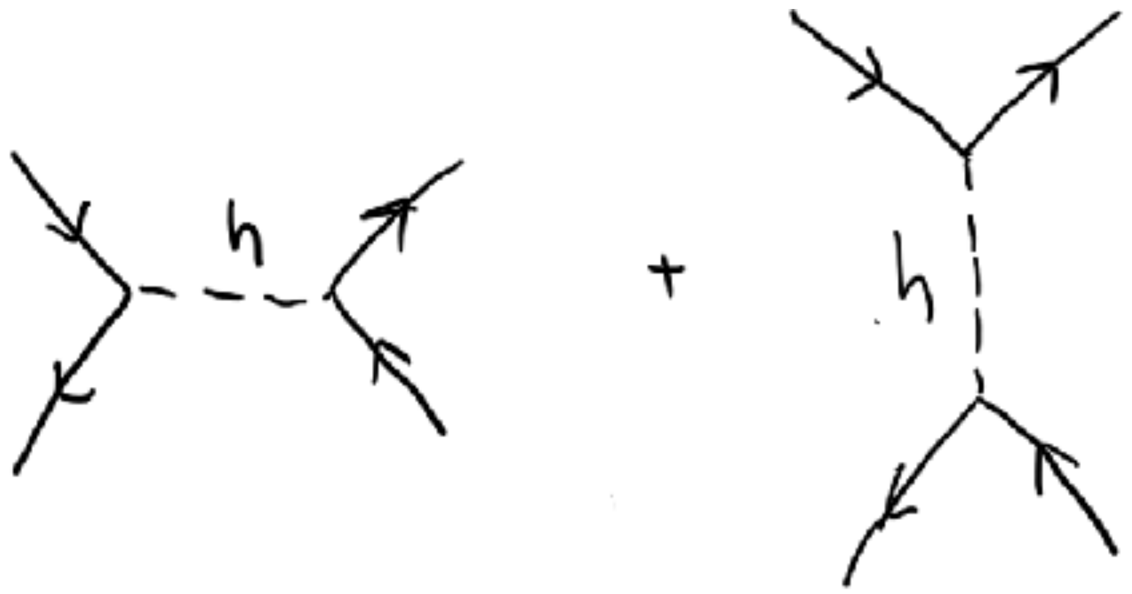
A tale of two scalars

Consider heavy and light scalar fields interacting with each other

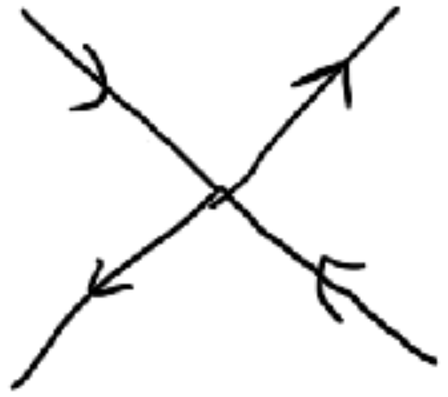
$$\mathscr{L} = \partial^\mu \phi^\dagger \partial_\mu \phi - m^2 \phi^\dagger \phi + \frac{1}{2}\left(\partial^\mu h \partial_\mu h - M^2 h^2\right) - g h \phi^\dagger \phi$$

If we are interested only in physics at scales below M, then we can use EFT

$$\mathscr{L}_{\text{eff}} = \partial^\mu \phi^\dagger \partial_\mu \phi - m^2 \phi^\dagger \phi - \frac{\lambda}{4}(\phi^\dagger \phi)^2$$



$$(-ig)^2\left(\frac{i}{s-M^2} + \frac{i}{t-M^2}\right) \rightarrow i\frac{2g^2}{M^2}$$



$$-i\lambda$$

On-shell matching gives

$$\lambda = -\frac{2g^2}{M^2}$$

This is the first term in the expansion of

$$\mathscr{L}_{\text{eff}} = \partial^\mu \phi^\dagger \partial_\mu \phi - m^2 \phi^\dagger \phi + \frac{g^2}{2}\phi^\dagger \phi \frac{1}{\Box + M^2}\phi^\dagger \phi$$

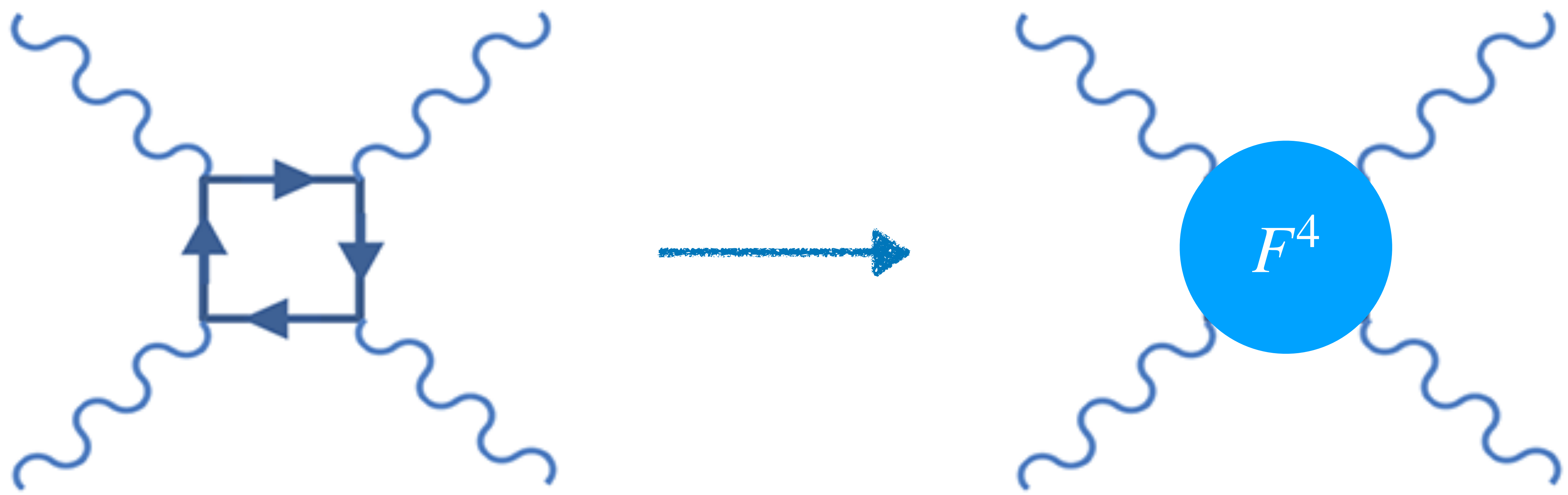
Basic EFTs

Previous example was what most people associate with EFT:

There are some heavy particles which are too heavy to be produced on-shell so we integrate them out and obtain series of local interactions

Typical examples:

- Fermi's interaction
- SMEFT
- Weak EFT
- …



Power-counting = mass dimension

But heavy particles decay ...

We want to describe for example heavy quark which decays into something light

Examples

HQET $M \gg \Lambda_{\rm QCD}$

(Expansion in $\Lambda_{\rm QCD}/M$)

NRQED/QCD $M \gg vM$

(Expansion in v - velocity)

We need to introduce modes

Hard $k \sim M$

Soft $k \sim \Lambda$

Hard $k \sim M$

Potential $k_0 \sim v^2 M, \overrightarrow{k} \sim vM$

This time we integrate-out only the hard modes of the heavy field while we keep the soft/potential mode in the theory

HQET

Consider a quark Lagrangian with mass $M \gg \Lambda_{\text{QCD}}$

$$\mathscr{L} = \overline{Q}\left(i\gamma^\mu D_\mu - M\right)Q$$

Mode decomposition is equivalent to isolating large momentum $p = Mv + k$, **with** $v^2 = 1$

Projectors allow to isolate heavy and light modes

On the operatorial level, this amounts to redefinition $Q(x) = e^{-iMvx}\left(Q_v(x) + B_v(x)\right)$, **with** $\frac{1+\not{v}}{2}Q_v = Q_v$ **and** $\frac{1-\not{v}}{2}B_v = B_v$

$$\mathscr{L} = \overline{Q}\left(i\gamma^\mu D_\mu - M\right)Q = \left(\overline{Q}_v + \overline{B}_v\right)\left(i\gamma^\mu D_\mu - (1-\not{v})M\right)\left(Q_v + B_v\right)$$

Using projection properties and introducing $D_T^\mu = D^\mu - v^\mu\, v \cdot D$

$$\mathscr{L} = \underbrace{\overline{Q}_v ivD Q_v}_{\text{Light}} - \underbrace{\overline{B}_v\left(ivD + 2M\right)B_v}_{\text{Heavy}} + \overline{Q}_v i\not{D}_T B_v + \overline{B}_v i\not{D}_T Q_v$$

This is still the QCD Lagrangian just written in a funny way

HQET

$$\mathscr{L} = \bar{Q}_v ivDQ_v - \bar{B}_v\left(ivD + 2M\right)B_v + \bar{Q}_v i\not{D}_T B_v + \bar{B}_v i\not{D}_T Q_v$$

We integrate out the heavy DOF

Use EOM (or field redefinition) $B_v = \frac{\not{D}_T Q_v}{iv \cdot D + 2M}$

This is still the expanded QCD Lagrangian = HQET

$$\mathscr{L} = \bar{Q}_v\left(iv \cdot D + i\not{D}_T \frac{1}{iv \cdot D + 2M} i\not{D}_T\right)Q_v = \bar{Q}_v\left(iv \cdot D + \frac{1}{2M} i\not{D}_T i\not{D}_T + \ldots\right)Q_v$$

This is still the QCD Lagrangian just written in a funny way

Soft $iv \cdot D \sim \Lambda \ll M$

Power-correction (mass suppressed)

NRQED works the same, but counting is different $iv \cdot D \sim v^2 M$ and $D_T \sim vM$

$$\mathscr{L}^{(0)}_{\mathrm{NRQCD}} = \mathscr{L} = \bar{Q}_v\left(iD_0 + \frac{1}{2M} i\vec{\not{D}}\, i\vec{\not{D}}\right)Q_v$$

First time we encounter theory where counting is not related to the mass dimension

When NRQED is not enough ...

NRQED does not have homogenous power-counting for threshold problems

We need more modes and we need them to interact with each other

		Photon $k^2 = 0$	**Fermion** $k_0 - \frac{\vec{k}^2}{2M} = 0$
QED	**Hard,** $k \sim M$	✗	✗
NRQED	**Soft,** $k \sim vM$	✗ / ✓	✗
PNRQED	**Potential,** $k_0 \sim v^2M,\ \vec{k} \sim vM$	✗	✓
	Ultra-soft, $k \sim v^2M$	✓	

Only on-shell modes can be present in the EFT

We keep fermion modes with $\vec{k} \sim vM$ but integrate out the same photons modes

Non-localities in modern EFT

Our first example

$$\mathscr{L}_{\text{eff}} = \partial^\mu \phi^\dagger \partial_\mu \phi - m^2 \phi^\dagger \phi + \frac{g^2}{2} \phi^\dagger \phi \frac{1}{\Box + M^2} \phi^\dagger \phi$$

When all momenta $\ll M$, we can expand the denominator and obtain local EFT

But if we allow momenta to be of the order of M, the theory becomes non-local

The same happens in pNRQED/QCD and SCET when we integrate modes which have components of momenta of the same order as modes which we need in the low energy EFT

PNRQED

1) Start from NRQED

$$\mathscr{L}^{(0)}_{\rm NRQCD} = \mathscr{L} = Q_v^\dagger \left(iD_0 + \frac{1}{2M} i\vec{D\!\!\!/}\, i\vec{D\!\!\!/} \right) Q_v$$

2) Insert field decomposition

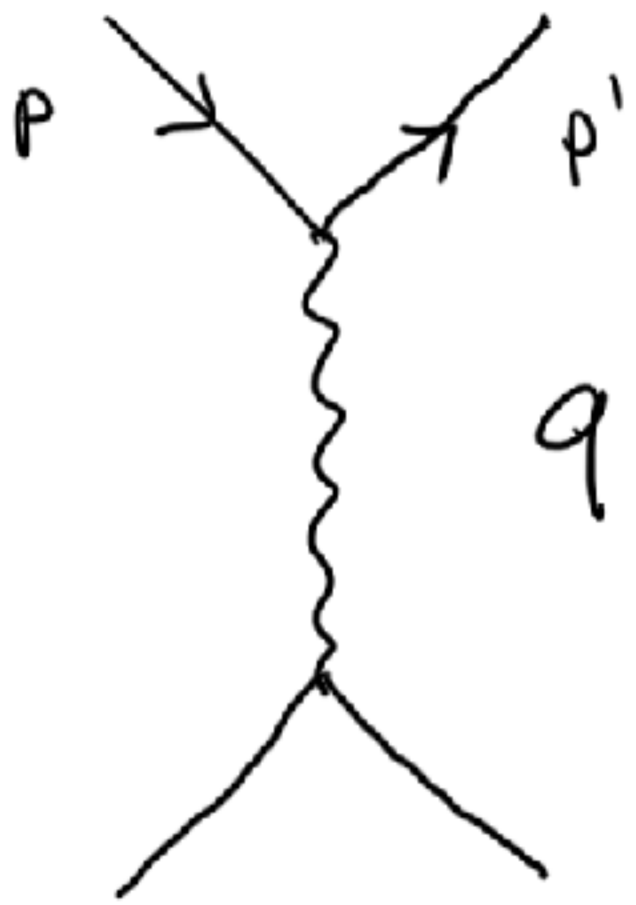
$$A^\mu = A^\mu_s + A^\mu_{us} + A^\mu_p \qquad Q_v = \chi$$

3) Integrate-out soft and potential photons and soft fermions

$$\mathscr{L}_{\rm PNQED} = \chi^\dagger(x)\left(iD_0^{us} + \frac{\vec{D}^2_{us}}{2M} \right)\chi(x) + \int d^3r \chi^\dagger \chi(t, \vec{x})\ V(r)\ \chi^\dagger \chi(t, \vec{x} + \vec{r})$$

Example: integrating out potential gluon

NRQED side



$$q = p' - p \quad (ie)^2 \frac{-i}{q^2} = i\frac{e^2}{\vec{q}^{\,2}}$$

PNRQED side

$$\int d^4x \left\langle p', k' \middle| \int d^3r \chi^\dagger \chi(t, \vec{x})\ V(r)\ \chi^\dagger \chi(t, \vec{x} + \vec{r}) \middle| p, k \right\rangle$$

$$= (2\pi)^4 \delta^{(4)}(p + k - p' - k') \int d^3r e^{-i\vec{q}\vec{r}} V(r)$$

Result

$$V(r) = \frac{e^2}{r}$$

Coulomb potential is a matching coefficient of NRQED to PNRQED

Power-counting

$$\mathscr{L}_{\text{PNQED}} = \chi^\dagger(x)\left(iD_0^{us} + \frac{\vec{D}_{us}^2}{2M}\right)\chi(x) + \int d^3r \chi^\dagger\chi(t,\vec{x})\; V(r)\, \chi^\dagger\chi(t,\vec{x}+\vec{r})$$

Potential momentum

$$k_0 \sim v^2 M \to x_0 \sim 1/v^2$$

$$\vec{k} \sim vM \to \vec{x} \sim 1/v$$

$$d^4x \sim v^{-5}$$

$$\int d^4x \chi^\dagger(x)\left(iD_0^{us} + \frac{\vec{D}_{us}^2}{2M}\right)\chi(x) \sim v^0$$

Use propagator to determine field counting
(since the mass dimension does not count anymore)

$$\left\langle 0 \left| \chi(0)\chi^\dagger(x) \right| 0 \right\rangle = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^0 - \frac{\vec{p}^2}{2M}} e^{ipx}$$

$\int \frac{d^4p}{(2\pi)^4}$	$\frac{i}{p^0 - \frac{\vec{p}^2}{2M}}$
v^5	$1/v^2$

$\to$ $\chi \sim v^{3/2}$

Now, we can count the potential

$$\int d^4x \int d^3r \chi^\dagger\chi(t,\vec{x})\; V(r)\, \chi^\dagger\chi(t,\vec{x}+\vec{r}) \sim v^{-5} \times v^{-3} \times v^6 \times V(r) \sim v^{-2} V(r)$$

For $V(r) \sim e^2/r \sim e^2 v$

In bound state/threshold we have $e^2 \sim v$ — the Coulomb potential is LO effect

Multipole expansion

Leading ultra-soft — potential interaction is $\chi^\dagger e A_0 \chi$

The Feynman rule is $e(2\pi)^4\delta^{(4)}(p-p'-k)$

But p and p' have potential scaling, hence $\vec{p}, \vec{p} \gg \vec{k}$

We need to multipole expand the ultra-soft fields to achieve homogenous counting

$$\chi^\dagger(x)eA_0(x)\chi(x) = \chi^\dagger(x)eA_0(t,\vec{0})\chi(x) + \chi^\dagger(x)e\,\vec{x}\cdot\vec{\nabla}A_0(t,\vec{0})\chi(x) + \ldots$$

leading term $\sim v^5$

sub-leading term $\sim v^6$

Ultra-soft field fluctuate at distances $\sim 1/v^2$ and cannot resolve short distance fluctuations in the spatial direction $\sim 1/v$

Taking into account terms from the spatial derivative (and non-abelian potential for QCD) we can derive the Lagrangian in manifestly gauge invariant form

$$\mathscr{L}_{\text{PNQED}} = \chi^\dagger(x)\left(iD_0^{us} + \frac{\vec{\partial}^2}{2M} - e\,\vec{x}\cdot\vec{E}(t,0)\right)\chi(x) + \int d^3r\chi^\dagger\chi(t,\vec{x})\;V(r)\;\chi^\dagger\chi(t,\vec{x}+\vec{r})$$

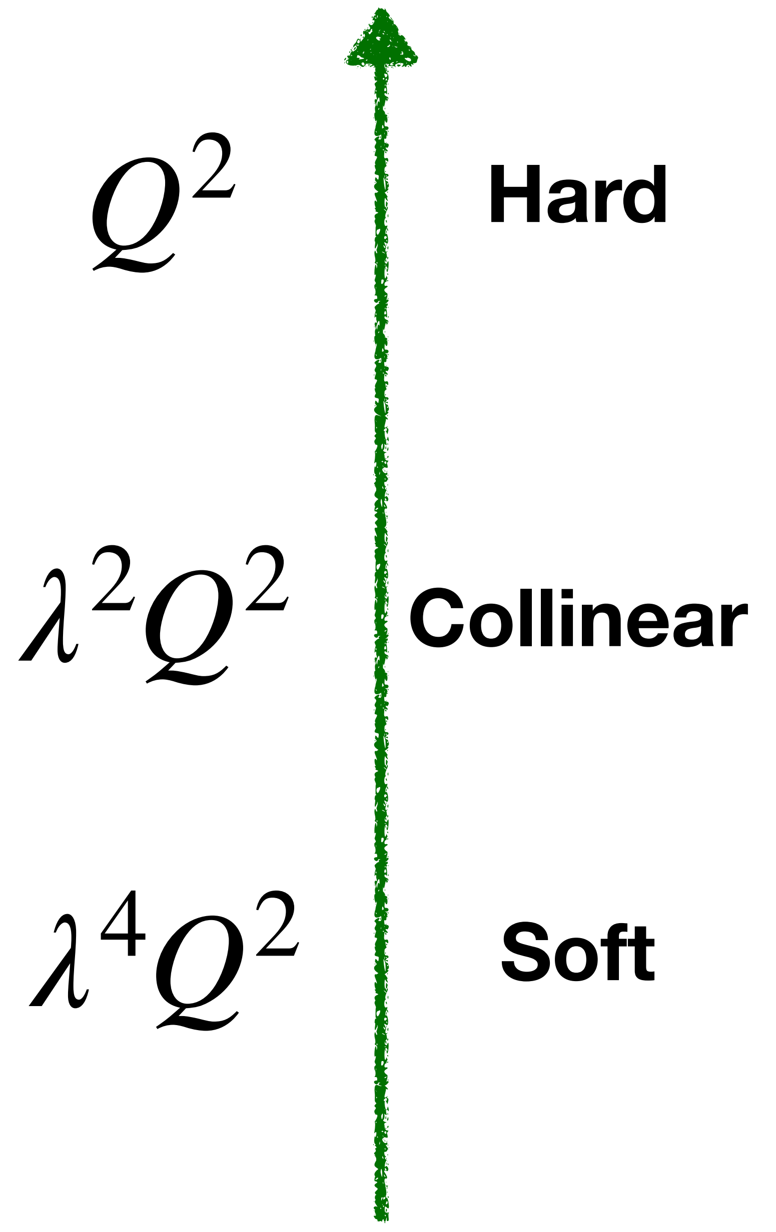
Multipole expansion and gauge symmetry are tightly connected: more on this in SCET example

(scalar) SCET

We want to describe massless energetic particles

First: light cone parametrization $n_+ n_- = 2\,, \quad n_\pm^2 = 0$

$$p^\mu = n_+ p \frac{n_-^\mu}{2} + n_- p \frac{n_+^\mu}{2} + p_\perp^\mu$$



Collinear counting

Analog of the potential mode in PNQED

$$(n_+p, p_\perp, n_-p) \sim (1, \lambda, \lambda^2) Q$$

$$n_+\partial\phi_c \sim \phi_c\,, \quad \partial_\perp\phi_c \sim \lambda\phi_c\,, \quad n_-\partial\phi_c \sim \lambda^2\phi_c$$

Soft counting

Analog of the ultra-soft mode in PNQED

$$(n_+p, p_\perp, n_-p) \sim (\lambda^2, \lambda^2, \lambda^2) Q$$

$$\partial_\mu\phi_s \sim \lambda^2\phi_s$$

SCET power-counting

Use the trick from PNRQED - field counting from propagator

$$\left\langle 0 \left| T\phi(x)\phi(y) \right| 0 \right\rangle = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2 + i\varepsilon} e^{-ip(x-y)}$$

For collinear momentum

$$d^4p = \frac{1}{2} dn_+p dn_-p d^2p_\perp \sim \lambda^0 \times \lambda^2 \times \lambda^2 = \lambda^4$$

$$p^2 = n_+p n_-p + p_\perp^2 \sim \lambda^2$$

Which implies $\phi_c \sim \lambda$

True also for fermions — independent of mass dimension

For soft momentum

$$d^4p \sim \lambda^8$$

$$p^2 \sim \lambda^4$$

Which implies $\phi_s \sim \lambda^2$

Soft fermion scale as λ^3

Gauge symmetry

Start from field decomposition, just like in PNRQED

$$\phi = \phi_c + WZ^\dagger \phi_s$$

$$A_\mu(x) = A_{c\mu}(x) + A_{s\mu}(x)$$

In SCET we need to be more careful with gauge symmetry

$$WZ^\dagger = P\exp\left[ig\int_{-\infty}^{0} dsn_+ A(x+sn_+)\right] \overline{P}\exp\left[-ig\int_{-\infty}^{0} dsn_+ A_s(x+sn_+)\right]$$

$$\begin{aligned}
\text{collinear:}\quad & A_c \to U_c A_c U_c^\dagger + \frac{i}{g} U\left[D_s\,, U^\dagger\right]\,, && \phi_c \to U_c\phi_c\,, \\
& A_s \to A_s\,, && \phi_s \to \phi_s\,, \\
\text{soft:}\quad & A_c \to U_s A_c U_s^\dagger\,, && \phi_c \to U_s\phi_c\,, \\
& A_s \to U_s A_s U_s^\dagger + \frac{i}{g} U_s\left[\partial\,, U_s^\dagger\right]\,. && \phi_s \to U_s\phi_s\,.
\end{aligned}$$

Collinear fields transform under collinear and soft gauge transformation

Soft fields can be treated as a background field and they do not transform under the collinear gauge transformation

Multipole expansion

Collinear

$$(n_+p, p_\perp, n_-p) \sim (1, \lambda, \lambda^2)Q$$

$$(n_+x, x_\perp, n_-x) \sim (1/\lambda^2, 1/\lambda, 1)1/Q$$

Soft

$$(n_+p, p_\perp, n_-p) \sim (\lambda^2, \lambda^2, \lambda^2)Q$$

$$(n_+x, x_\perp, n_-x) \sim (1/\lambda^2, 1/\lambda^2, 1/\lambda^2)1/Q$$

Soft field cannot resolve short-wavelength fluctuations along n_-x and $x_\perp$

In NRQED, ultra-soft photons could not resolve potential flections along the spatial direction

Introduce $x_- = n_+ \cdot x \dfrac{n_-}{2}$ **then**

$$\phi_s(x) = \phi_s(x_-) + x_\perp \partial \phi_s(x_-) + \frac{1}{2} n_- x n_+ \partial \phi_s(x_-) + \frac{1}{2} x_{\perp\alpha} x_\perp^\beta \partial_\alpha \partial_\beta \phi_s(x_-) + \mathcal{O}(\lambda^3 \phi_s)\,,$$

Relevant for factorization theorems

QED: ultra-soft photon couples only to the total charge of a bound state
For neutral atoms - dipole interaction is the leading one

QCD: soft gluon couples only to the total color charge of a jet
For colorless jets - dipole interaction is the leading one

Multipole expansion and gauge symmetry

$$\text{collinear:}\quad \hat{A}_c \to U_c \hat{A}_c U_c^\dagger + \frac{i}{g} U_c \left[\hat{D}_s(x_-)\,, U_c^\dagger\right]\,,\qquad \hat{\phi}_c \to U_c \hat{\phi}_c\,,$$

$$\text{soft:}\quad A_c \to U_s(x_-) \hat{A}_c U_s^\dagger(x_-)\,,\qquad \hat{\phi}_c \to U_s(x_-)\hat{\phi}_c\,,$$

We need to enforce that gauge symmetry is consistent with multipole expansion

$$\hat{D}_{s\mu} = \partial_\mu - ig\frac{n_{+\mu}}{2} n_- A_s(x_-)$$

Derivation involves parallel transport in gauge space from x to x_-

Only n_- component is a soft-covariant derivative, rest are ordinary derivatives

$$\mathcal{L}^{(0)} = \frac{1}{2}\left[n_+ D_c \hat{\phi}_c\right] n_- D\hat{\phi}_c + \frac{1}{2}\left[n_- D\hat{\phi}\right] n_+ D_c \hat{\phi}_c + \left[D_{c\mu_\perp}\hat{\phi}_c\right] D_c^{\mu_\perp}\hat{\phi}_c\,,$$

$$\mathcal{L}_\chi^{(1)} = \frac{1}{2} x_\perp^\mu n_-^\nu g F_{\mu\nu}^a n_+ j^a\,,$$

$$\mathcal{L}_\chi^{(2)} = \frac{1}{4} n_- x n_+^\mu n_-^\nu g F_{\mu\nu}^a n_+ j^a + \frac{1}{4} x_\perp^\mu x_{\perp\rho} n_-^\nu \operatorname{tr}\left(\left[D_s^\rho\,, gF_{\mu\nu}\right] t^a\right) n_+ j^a + \frac{1}{2} x_\perp^\mu g F_{\mu\nu_\perp}^a j^{a\nu_\perp}$$

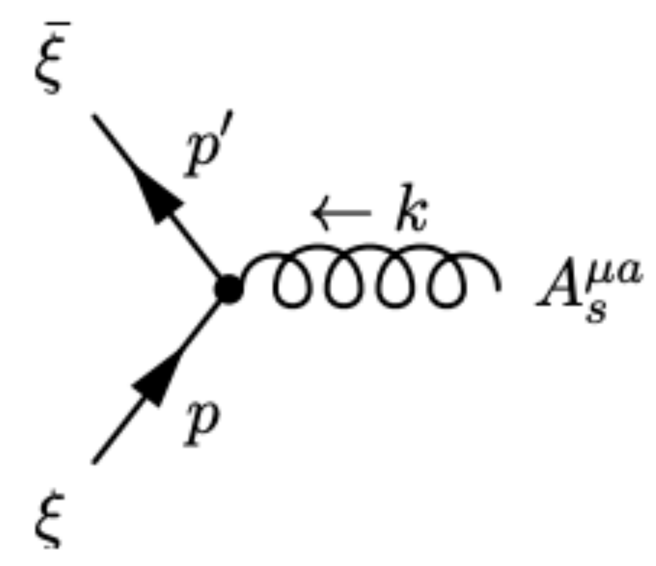
For PNRQED only D_t involved coupling to ultra-soft photons

The Lagrangian in a Gauge-Invariant form $x_\mu F^{\mu\nu} \leftrightarrow \vec{x}\cdot\vec{E}$

The derivation is actually not trivial - it involves a lot field redefinitions with special Wilson lines to ensure gauge covariance, but it can be performed to all orders and powers — if you are interested in the details ask me afterwards

How do the Feynman rules look like?

Example: single soft gluon emission in spin 1/2 QCD



$$ig_s t^a \begin{cases} \frac{\not{n}_+}{2} n_{-\mu} & \mathcal{O}(\lambda^0) \\ \frac{\not{n}_+}{2} X_\perp^\rho n_-^\nu (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\ S^{\rho\nu}(k,p,p') \frac{\not{n}_+}{2} (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}$$

where

$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2}\left[(n_- X) n_+^\rho n_-^\nu + (k X_\perp) X_\perp^\rho n_-^\nu + X_\perp^\rho \left(\frac{\not{p}'_\perp}{n_+ p'} \gamma_\perp^\nu + \gamma_\perp^\nu \frac{\not{p}_\perp}{n_+ p}\right)\right]$$

x-dependent terms in the Lagrangian violate translational invariance

Dependence on x is always polynomial so we get derivatives of momentum conservation deltas

$$X^\mu \equiv \partial^\mu \left[(2\pi)^4 \delta^{(4)}\left(\sum p_{in} - \sum p_{out}\right)\right],$$

$$X^\mu X^\nu \equiv \partial^\mu \partial^\nu \left[(2\pi)^4 \delta^{(4)}\left(\sum p_{in} - \sum p_{out}\right)\right]$$

which is what you expect if remember the reason behind multipole expansion

In PNRQED we said, $\delta^{(4)}(p - p' - k) = \delta^{(4)}(p - p' - k_0) + \dots$

In practice, PNRQED computations are often performed in position space (it is almost QM), but in SCET we typically work in momentum space

Non-locality

In addition to the Lagrangian which describes interactions within a single collinear sector with the soft background we need also the so-called sources, currents or N-jet operators

$$\mathcal{J} = \int [dt]_N \, C(t_{i_1}, t_{i_2}, \ldots) J_s(0) \prod_{i=1}^{N} J_i(t_{i_1}, t_{i_2}, \ldots) \, ,$$

In PNRQED, non-locality appeared after integrating out potential photons which had the same spatial momentum scaling as potential fermions retained in the EFT

In SCET, we integrate out the hard scale but large momentum components are of the order of the hard scale hence sources become non-local along the collinear directions

In, general, we have N-collinear directions

Each collinear direction must be separately collinear gauge invariant

$$J_i(x) \stackrel{\text{col.}}{\longrightarrow} J_i(x) \, , \quad J_i(x) \stackrel{\text{soft}}{\longrightarrow} U_s(x_{i-}) J_i(x) \, ,$$

$$\chi_{c_i} = W_i^\dagger \phi_{c_i} \, , \quad \mathcal{A}^{\mu}_{i\perp_i} = W_i^\dagger \left[i D^{\mu}_{\perp_i} W_i \right]$$

Total operator must be soft gauge invariant

Multipole expansion with respect to other directions sets soft field to zero

Thus soft gauge invariance is just a requirement of color-neutrality

W_i **is a collinear Wilson line introduced to make the collinear fields collinear gauge invariant**

$$J_i^{A0}(t_i) \in \left\{ \chi_{c_i}(t_i n_{i+}) \, , \chi^\dagger_{c_i}(t_i n_{i+}) \, , \mathcal{A}_{i\perp_i}(t_i n_{i+}) \right\}$$

$$\phi_s(x) \sim \lambda^2 \, , \quad F_{\mu\nu} \sim \lambda^4 \, , \quad i D_s^{\mu} \phi_s(x) \sim \lambda^4$$

Currents are rarely discussed in PNRQED, they appear if we are concerned with production or decay of non-relativistic states

Operator basis

$$J_i(x) \xrightarrow{\text{col.}} J_i(x)\,, \quad J_i(x) \xrightarrow{\text{soft}} U_s(x_{i-})J_i(x)\,,$$

Collinear gage invariant building blocks

$$\chi_{c_i} = W_i^\dagger \phi_{c_i}\,, \quad \mathscr{A}^\mu_{i\perp_i} = W_i^\dagger \left[iD^\mu_{\perp_i} W_i\right]$$

Each costs one power of λ

W_i is a collinear Wilson line introduced to make the collinear fields collinear gauge invariant

Leading power currents

$$J_i^{A0}(t_i) \in \left\{\chi_{c_i}(t_i n_{i+})\,, \chi^\dagger_{c_i}(t_i n_{i+})\,, \mathscr{A}_{i\perp_i}(t_i n_{i+})\right\}$$

At subleading power we can have, for example $\partial^\mu_\perp \chi_{c_i}$ - note this is ordinary derivative thanks to multipole expansion

Next, we can have $n\_D_s\chi_{c_i}$ but this term can be eliminated using EOM

Explicit soft building blocs

$$\phi_s(x) \sim \lambda^2\,, \quad F_{\mu\nu} \sim \lambda^4\,, \quad iD^\mu_s \phi_s(x) \sim \lambda^4$$

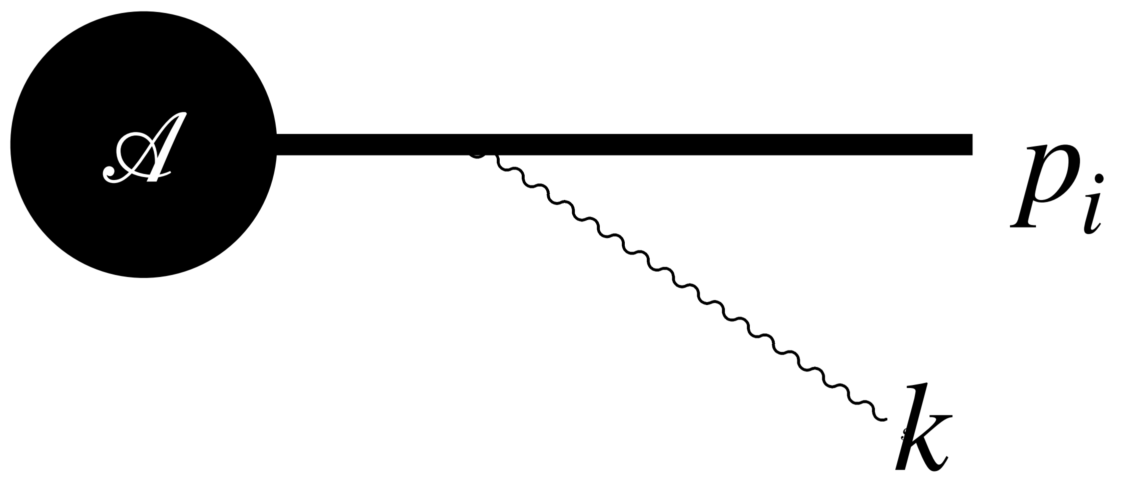
No gluon soft building blocks up to λ^4

Example: Soft theorem in QCD

LBK theorem

Low, 1958
Burnett, Kroll, 1968

$$\mathscr{A}_{\rm rad} = -\, g_s \sum_{i=1}^{n} t_i^a \overline{u}(p_i) \left(\frac{p_i \cdot \varepsilon^a(k)}{p_i \cdot k} + \frac{k_\nu \varepsilon^a_\mu(k) J_i^{\mu\nu}}{p_i \cdot k} \right) \mathscr{A}$$

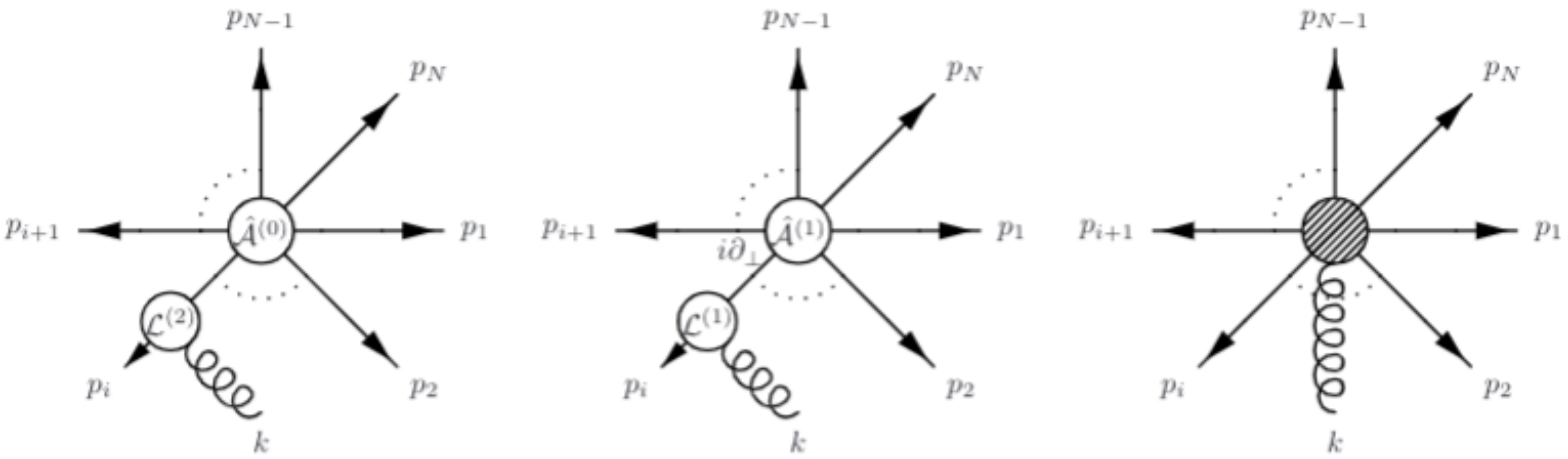


Radiative amplitude for soft gluon/photon

Is related in a universal way, up to order $(k)^0$

To the non-radiative amplitude

$$J_i^{\mu\nu} = L_i^{\mu\nu} + \Sigma_i^{\mu\nu} = p_i^\mu \frac{\partial}{\partial p_{i\nu}} - p_i^\nu \frac{\partial}{\partial p_{i\mu}} + \Sigma_i^{\mu\nu}$$



SCET derivation: First two terms come from time-ordered product of the Lagrangian and the operator representing non-radiative amplitude

At $\mathcal{O}(\lambda^4)$ we can write for the first time soft gluon building block $F_s^{\mu\nu} \sim \lambda^4$ so this term is no longer universal

Perturbative Gravity

Gravity is an EFT

$$S_{\text{grav,EFT}} = \int d^4x\sqrt{-g}\left(\Lambda + \frac{2}{\kappa^2}R + c_1R^2 + c_2R_{\mu\nu}R^{\mu\nu} + \dots\right)$$

We will focus on a scalar coupled to gravity

$$S_\varphi = \int d^4x\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\left[\partial_\mu\varphi\right]\partial_\nu\varphi - \frac{\lambda_\varphi}{4!}\varphi^4\right)$$

Gauge symmetry is the diffeomorphisms group

$$x^\mu \to y^\mu(x)$$

More convenient to work with local translations

$$x^\mu \to y^\mu(x) = x^\mu + \varepsilon^\mu(x)\,,$$

We work in Minkowski background metric

$$S^{(0)}_{\text{EH}} = \int d^4x\left[\partial_\alpha h_{\mu\nu}\partial^\alpha h^{\mu\nu} - \partial_\alpha h\partial^\alpha h - 2\partial_\mu h^{\mu\nu}(\partial_\alpha h^\alpha_\nu - \partial_\nu h)\right]$$

$$\mathcal{L}^{(0)} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{\lambda_\varphi}{4!}\varphi^4$$

$$\mathcal{L}^{(1)} = -\frac{1}{2}h_{\mu\nu}\left(\partial^\mu\varphi\,\partial^\nu\varphi - \eta^{\mu\nu}\frac{1}{2}\partial_\alpha\varphi\,\partial^\alpha\varphi\right) - \frac{1}{2}h\frac{\lambda_\varphi}{4!}\varphi^4$$

Massless tensor coupled to matter

Soft-Collinear Gravity

As always, start from mode decomposition

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + s_{\mu\nu} \equiv g_{s\mu\nu} + h_{\mu\nu} \,,$$

Dangerous components

$$h_{++} \sim \lambda^{-1} \,, \; h_{+\mu_\perp} \sim 1 \,, \; h_{+-} \sim \lambda \,, \; h_{\mu_\perp \nu_\perp} \sim \lambda \,, \; h_{\mu_\perp -} \sim \lambda^2 \,, \; h_{--} \sim \lambda^3$$

Wilson lines (parallel transport) save the day!

After some lengthy derivation, which involves a lot of field redefinitions and Wilson lines, we find gravitation covariant derivative

$$n_- D_s \varphi \equiv \left(\partial_- - \frac{1}{2} s_{-\mu} \partial^\mu + \frac{1}{8} s_{+-} s_{--} \partial_+ + \frac{1}{16} s_{-\alpha_\perp} s_-^{\alpha_\perp} \partial_+ - \frac{1}{4} \left[\Omega_-\right]_{\mu\nu} J^{\mu\nu} \right) \varphi + \mathcal{O}(\lambda^3) \,.$$

Coupling to momentum

GR non-linear terms, not interesting for us now

Coupling to angular momentum

Gravity is a gauge theory with a charges equal to momentum and angular momentum!

with spin-connection

$$\Omega_{-\alpha\beta} = -\frac{1}{2} \Big([\partial_\alpha s_{\beta -}] - [\partial_\beta s_{\alpha -}] \Big) + \mathcal{O}(s^2)$$

The Lagrangian and operator basis

$$\mathcal{L}^{(0)}_{D_s} = \frac{1}{2}\partial_+\varphi D_-\varphi + \frac{1}{2}\partial_{\alpha_\perp}\varphi\partial^{\alpha_\perp}\varphi\,,$$

$$\mathcal{L}^{(1)}_{D_s} = -\frac{1}{2}\mathfrak{h}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{4}\mathfrak{h}^{\alpha_\perp}{}_{\alpha_\perp}\left(\partial_+\varphi D_-\varphi + \partial_{\alpha_\perp}\varphi\partial^{\alpha_\perp}\varphi\right)\,,$$

$$\mathcal{L}^{(2)}_{D_s} = -\frac{1}{8}x^\alpha_\perp x^\beta_\perp R^s_{\alpha-\beta-}(\partial_+\varphi)^2 + \frac{1}{2}\mathfrak{h}^{\mu\alpha}\mathfrak{h}^\nu_\alpha\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{16}\left((\mathfrak{h}^{\alpha_\perp}_{\alpha_\perp})^2 - 2\mathfrak{h}^{\alpha\beta}\mathfrak{h}_{\alpha\beta}\right)\left(\partial_+\varphi D_-\varphi + \partial_{\mu_\perp}\varphi\partial^{\mu_\perp}\varphi\right)\,,$$

Like in QCD, we need collinear Wilson lines to make the field collinear gauge invariant

$$\chi_c = \left[W_c^{-1}\varphi_c\right]\,,\quad \mathfrak{h}_{\mu\nu} = W_\mu{}^\alpha W_\nu{}^\beta\left[W_c^{-1}h_{\alpha\beta}\right] + \left(W_\mu{}^\alpha W_\nu{}^\beta \hat{g}_{s\alpha\beta} - \hat{g}_{s\mu\nu}\right)$$

W_c: gravitational collinear Wilson line

Collinear gauge invariant graviton: dangerous components are absent

Universal interaction

NRQED: $\vec{x}\cdot\vec{E} = x^\alpha_T v^\beta F^{us}_{\alpha\beta}$

SCET: $x^\alpha_\perp n^\beta_- F^s_{\alpha\beta}$

SCET GR: $x^\alpha_\perp x^{\alpha'}_\perp n^\beta_- n^{\beta'}_- R^s_{\alpha\beta\alpha'\beta'}$

Analog of the field-strength tensor: Riemann tensor

Note that the Riemann tensor contains two derivatives $R \sim \lambda^6$

In GR, it is actually a quadrupole rather than dipole — not unexpected compare EM vs GR waves

Example: soft theorem in GR

Turns out, gravitational gauge symmetry is a stronger constraint than YM and even the sub-subleading terms are universal

$$\mathscr{A}_{\text{rad}} = -\frac{\kappa}{2}\sum_i \bar{u}(p_i)\left(\frac{\varepsilon_{\mu\nu}(k)p_i^\mu p_i^\nu}{p_i\cdot k} + \frac{\varepsilon_{\mu\nu}(k)p_i^\mu k_\rho J_i^{\nu\rho}}{p_i\cdot k} + \frac{1}{2}\frac{\varepsilon_{\mu\nu}(k)k_\rho k_\sigma J_i^{\rho\mu}J_i^{\sigma\nu}}{p_i\cdot k}\right)\mathscr{A}$$

Like in the gauge theory case the soft theorem comes from the time-ordered product

First available graviton building block is given by a Riemann tensor $R_s^{\mu\nu\rho\sigma} \sim \lambda^6$

$$\mathscr{A}_{\text{rad}} = -g_s\sum_{i=1}^{n} t_i^a\bar{u}(p_i)\left(\frac{p_i\cdot\varepsilon^a(k)}{p_i\cdot k} + \frac{k_\nu\varepsilon_\mu^a(k)J_i^{\mu\nu}}{p_i\cdot k}\right)\mathscr{A}$$

Check gauge invariance of each term!

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These terms come for subleading Lagrangian: the "$\vec{x}\cdot\vec{E}$" dipole terms

$$\mathscr{A}_{\text{rad}} = -g_s\sum_{i=1}^{n} t_i^a\bar{u}(p_i)\left(\frac{p_i\cdot\varepsilon^a(k)}{p_i\cdot k} + \frac{k_\nu\varepsilon_\mu^a(k)J_i^{\mu\nu}}{p_i\cdot k}\right)\mathscr{A}$$

Check gauge invariance of each term!

That's it?

Besides re-deriving known theorems or rewriting GR in with fancy covariant derivative why bother with SCET power corrections?

- Energetic particles which couple to soft and collinear radiation are ubiquitous in HEP
- Such radiation leads to large logarithmic correction — typically in QCD, but also in QED and even in weak interactions
- EFT allows for scale factorization and resummation of these logs using RG methods
- Many of these corrections are universal — soft theorem are the simplest examples of such universality, but it becomes manifest in the EFT formulation
- Subleading terms are phenomenologically relevant for the precision collider physics
- Some processes start at subleading power

Further improvement of precision for resummation at the LHC is very difficult without EFT approach — systematic study of power corrections started only few years ago

Take home messages

- Modern EFT can handle a lot more than just integrating out heavy particles
- Homogenous power-counting is build in into modes EFT and allows for a systematic expansion
- The price to pay is a very complicated QFT, typically non-local, with x-dependent Lagrangian
- Gauge symmetry plays a central role in organizing the expansion (huge advantage over diagrammatic methods!)
- Once you pay the overheads, many things come out almost for free like the soft theorems
- Surprisingly, SCET reveals hidden structure of the gauge symmetry in GR