Small-$_x$ parton physics on lattice????

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BNL Small-x workshop, Dec. 15, 2021

Outline

- What is a parton?
- On lattice?
- How to go to small x?
- New ideas?
- No conclusion

What is a parton?

Partons in QFT are effective DOFs

- Partons are not just the quarks and gluons in the usual QCD lagrangian.
- Partons are a special type of IR collinear modes with momentum

$$k^\mu = \left(k^0, k^z, \vec{k}_\perp\right)$$

with $k^z \to \infty, k^0 \to \infty, k_\perp \sim \Lambda_{QCD}, k_\mu^2 \sim \Lambda_{QCD}^2$

Collins, Soper and Sterman, QCD factorization, 70'-80's

Bauer, Stewart et al, Soft-Collinear EFT, 00's

Three formalisms for partons

	States	operators	Time-signature
LFQ	P=∞	LF correlators	Minkowski
SCET	p=finite	LF correlators	Minkowski
Feynman	p=∞	Equal time	Euclidean

Partonic Effective Theories

- Hamiltonian Formulation & Light-Front Quantiziation

S. Weinberg (66')

S. J. Chang and S. K. Ma (69')

J. Kogut and D. Soper ('70) …

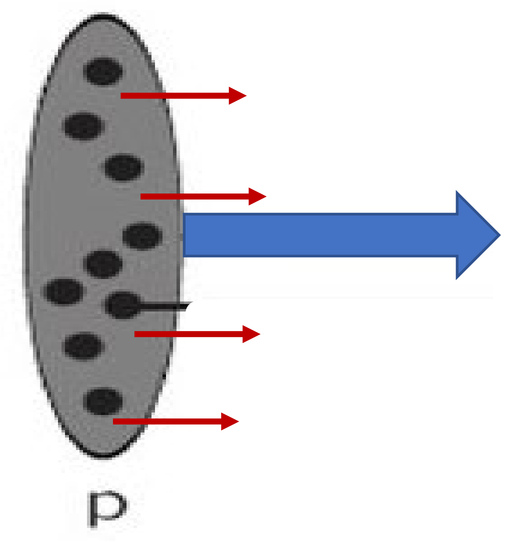
} Dirac's Light-Front Quantization

Much work has been devoted to LFQ due to the leadership of S. Brodsky and late K. Wilson. A. Mueller for PT in trees.

- Lagrangian Formulation & light-cone correlations
  - QCD factorization: light-front correlations
  - Using the soft-collinear modes in SCET can construct the bound state of the proton

Feynman formulation

- Parton distributions are the ordinary momentum distributions of constituents in a many-body system when boosted to the infinite momentum limit



Momentum distribution in NR systems

- Knock-out reactions in NR systems probes momentum distribution

$$n\left(\vec{k}\right) = \left|\psi\left(\vec{k}\right)\right|^2$$

$$\sim \int \psi^*(\vec{r})\psi(0)e^{i\vec{k}\vec{r}}d^3r$$

$$\sim \int \langle\Omega\left|\hat{\psi}^+(\vec{r})\hat{\psi}(0)\right|\Omega\rangle e^{i\vec{k}\vec{r}}d^3r$$

- Mom.dis. are related to Euclidean correlations, generally amenable for Monte Carlo simulations.

Difference between relativistic and NR systems

- NR cases, the energy transfer is small.

$$q^0 \sim \frac{1}{M} \sim 0$$

- Relativistic systems:

In DIS, if we choose a frame in which the virtual photon energy is zero

$$q^\mu = (0, 0, 0, -Q)\,,$$
$$P^\mu = \left(\frac{Q}{2x_B} + \frac{M^2 x_B}{Q}, 0, 0, \frac{Q}{2x_B}\right)\,,$$

In the Bjorken limit, $P^z \sim Q \rightarrow \infty$

Feynman's partons

- Consider the mom.dis. of constituents in a hadron

$$f(k^z, P^z) = \int d^2k_\perp \, f(k^z, k_\perp, P^z)$$

which depends on P^z because of relativity.

(H is not invariant under boost K)

- PDF is a result of the $P^z \rightarrow \infty$ limit,

$$f(k^z, P^z) \rightarrow_{p^z \rightarrow \infty} f(x) \quad with\ x = \frac{k^z}{P^z},$$

Partons from a large-P expansion of Euclidean observables

- Assuming $P^z \to \infty$ limit exists, parton physics is obtained by expansion (Feynman, 1969)

$$f(k^z, P^z) = f(x) + f_2(x)(M/P^z)^2 + ...$$

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$$f(k^z, P^z) = f(x) + f_2(x)(M/P^z)^2 + \ldots$$

- Account for subtlety of non-commuting limits of $P^z \to \infty$ & $\Lambda_{UV} \to \infty$ in QFT (Ji, 2013)

$$\tilde{f}(y, P^z) = \int Z(y/x, xP^z/\mu) f(x, \mu) dx + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{y^2(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(1-y)^2(P^z)^2}\right),$$

Going for a particular x? Weinberg EFT expansion

- Approximate $P^z = \infty$ by a finite large P^z.

  We frequently do this in QCD

  Lattice QCD & HQET

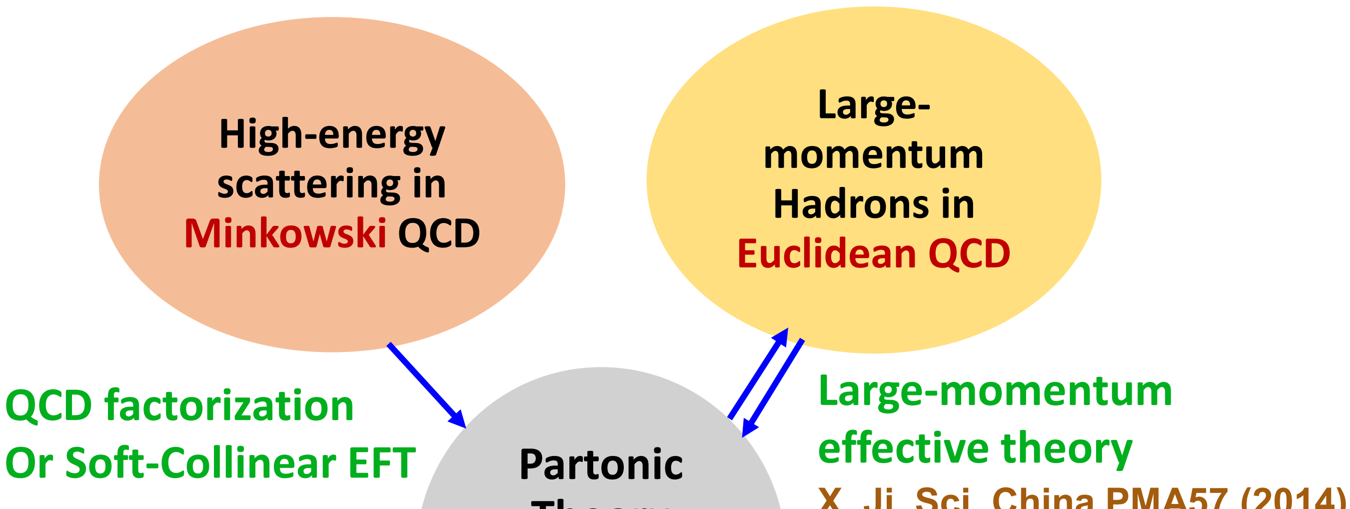
- EFT expansion for PDFs in the spirit of Weinberg,

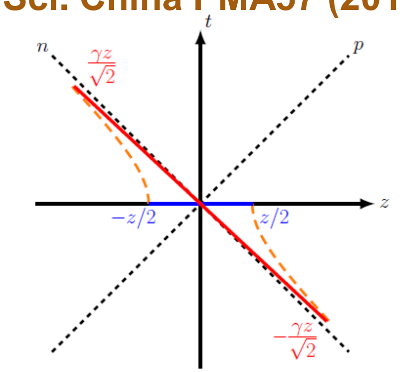
$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \tilde{C}\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) \tilde{q}(y, P^z, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right).$$

x-dependence of PDF can be calculated from QCD!

Not by fitting as in extracting PDFs from exp. data.

EFT for partons: Full Euclidean QCD





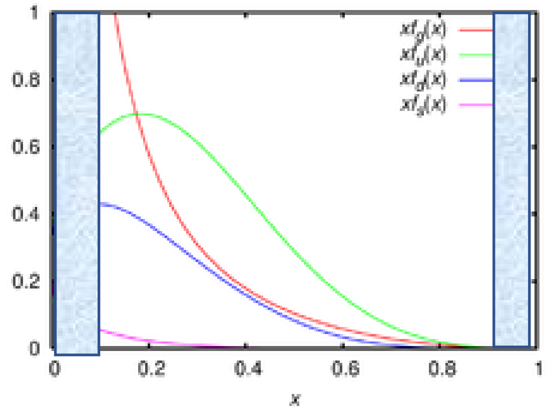
Main feature & limitation

- Large-momentum systematic expansion!
- LaMET expansion breaks done near

$$xP^z \sim \Lambda_{QCD}; \quad (1-x)P^z \sim \Lambda_{QCD}$$

where the collinear modes end. These are soft modes or zero modes.

For $P^z \sim 2-3\ GeV$, max x-range: 0.1-0.9



On lattice?

LaMET and lattice parton physics

- Creating a large momentum hadron on lattice.
- Computing space correlation functions.
- LaMET has been used to calculate extract the LF observables such as PDFs and GPDs, and TMDPDFs through lattice QCD without LFQ
- A lot of progress has been made

Ji, Liu, Liu, Zhang, Zhao, Review of Modern Physics, 93 (2021) 3

Cichy & Constantinou, Adv. High Energy Phys. 2019 (2019) 3036904

LaMET lattice QCD meetings

Martha Constantinou
- Xiangdong Ji
- Huey-Wen Lin
- Peter Petreczky
- David Richards
- Andreas Schaefer
- Yi-Bo Yang
- Yong Zhao

LaMET 2021 Online

Dec 7 – 9, 2021
Zoom
US/Eastern timezone

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Contact Dr. Yong Zhao
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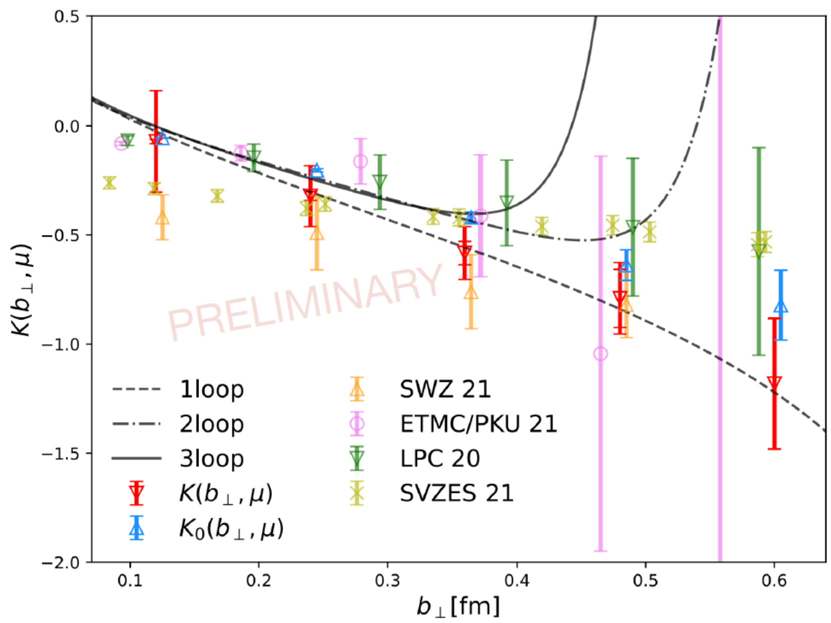
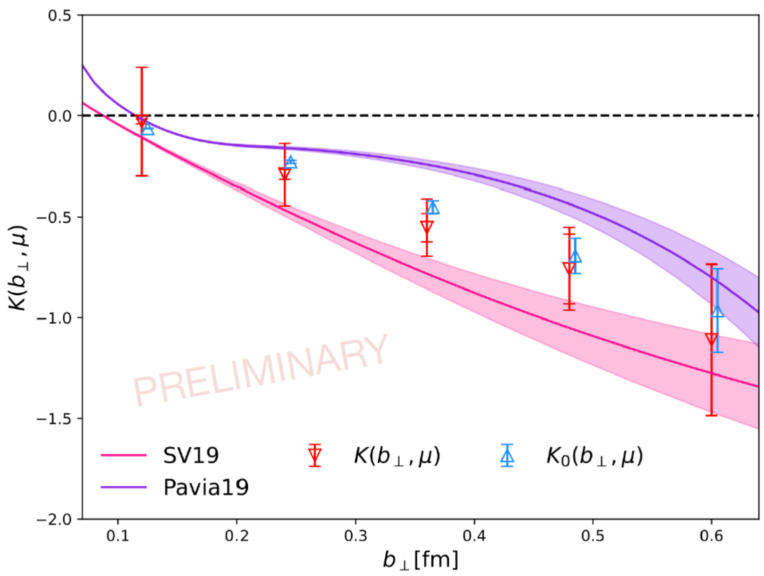
The **2021 Meeting on Lattice Parton Physics from Large-Momentum Effective Theory (LaMET 2021)** will be held online by Center for Nuclear Femtography (CNF) at SURA in Washington DC. The meeting will take place from Dec. 7-9, 2021. Registration is open now at https://indico.cern.ch/event/1082505/, and abstracts can be submitted for presentation. The deadline for abstract submission is 12:00 AM EST, Nov. 15, 2021.

Large-momentum effective theory (LaMET) is based on the field theoretical realization of Feynman parton model in which PDFs are momentum distributions of quarks and gluons in an infinite momentum hadron state. One can start from the momentum distributions in a hadron with finite but large momentum, which can be calculated in Euclidean approaches such as lattice QCD, and then expand the results systematically. The leading term is the PDFs after proper field-theoretical matching and running. Over the past years, LaMET has enabled much progress in the lattice QCD calculation of PDFs as well as GPDs and TMDs. The lattice data for LaMET calculations can also be analyzed in coordinate-space factorization approaches to get moments of PDFs or x-distributions through phenomenological parametrizations.

Rapidity evolution kernel

LPC collaboration, to be published

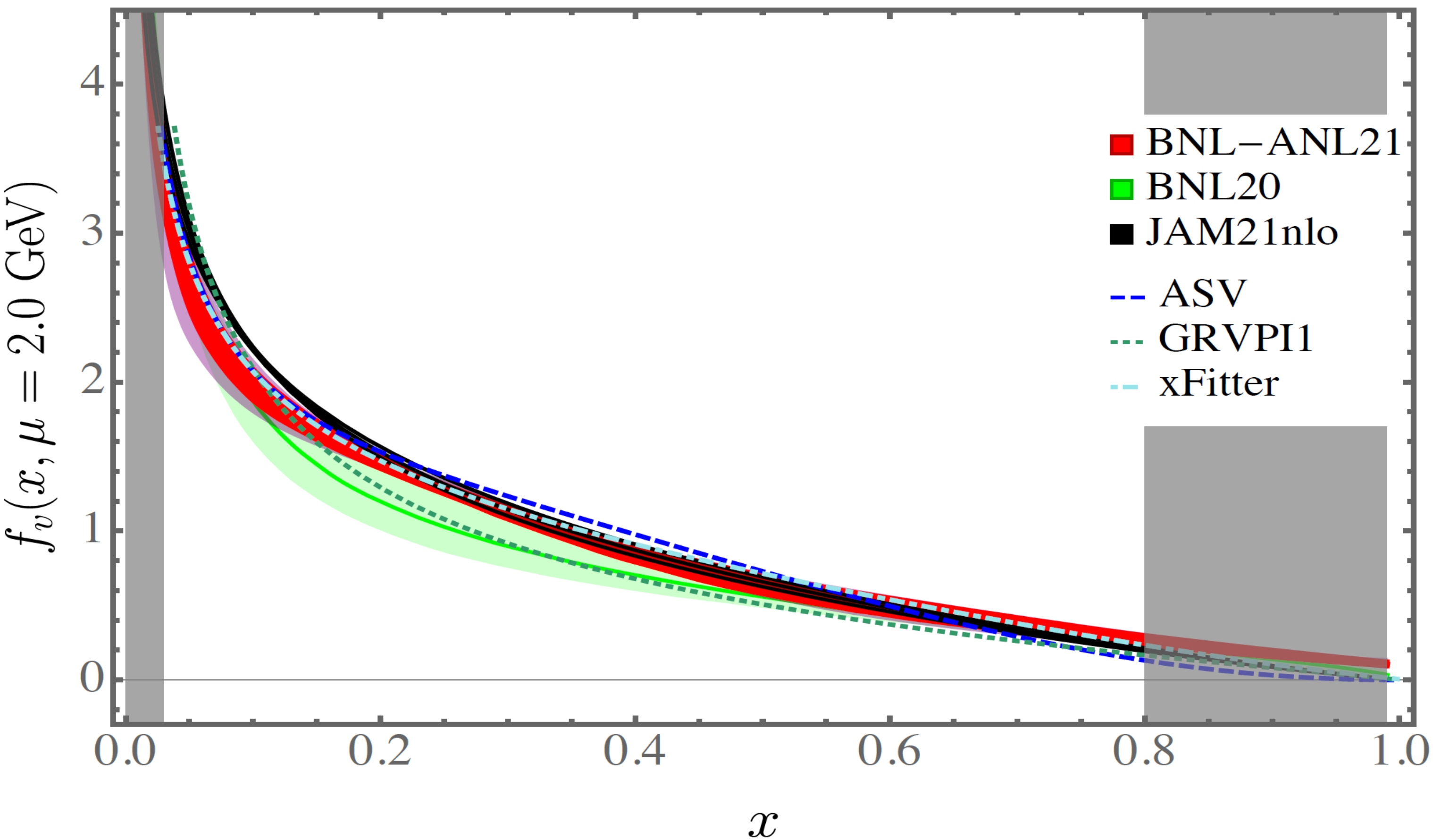




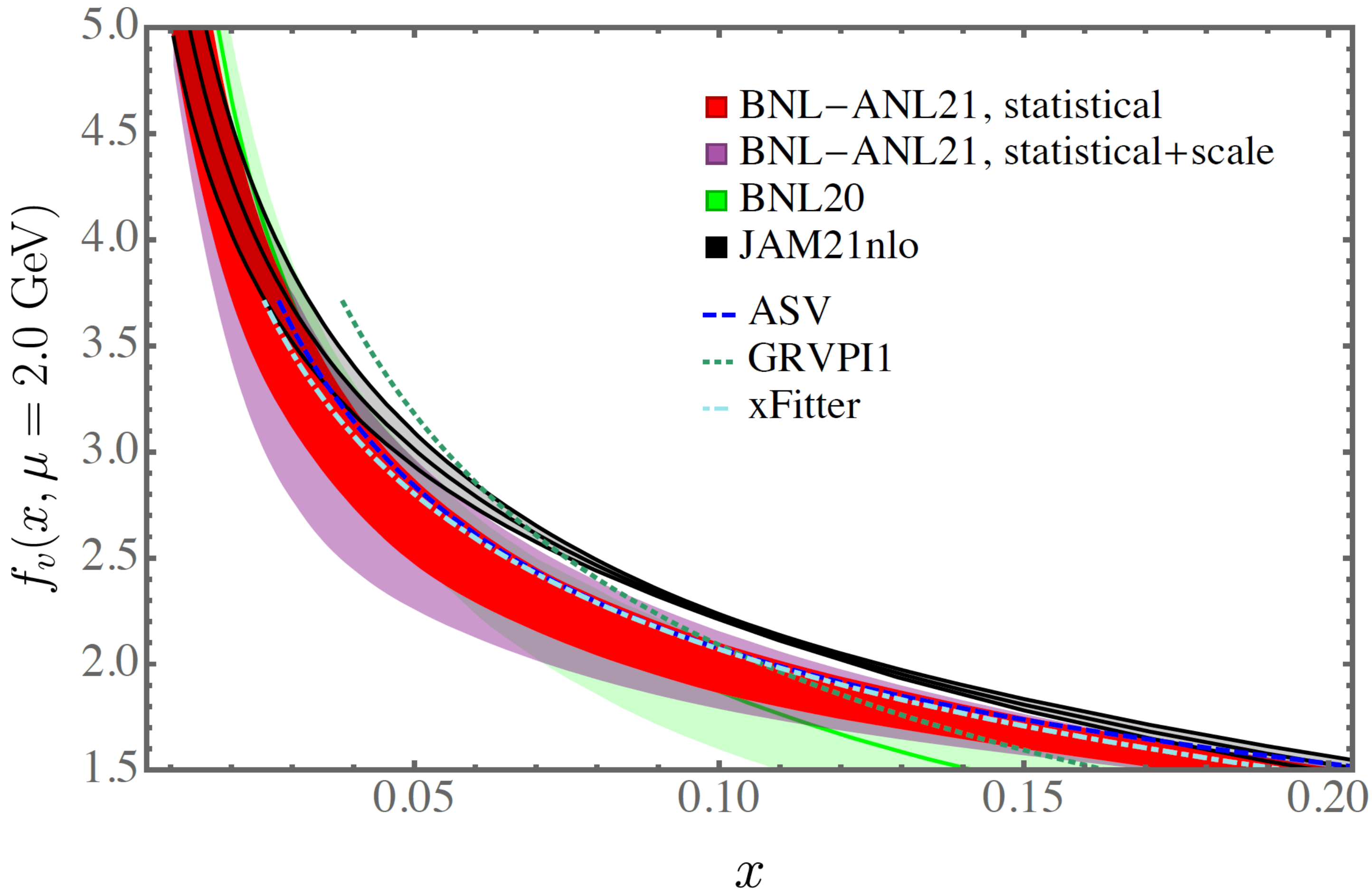
A recent pion PDF hep-lat: 2112:02208, ANL&BNL

- Lattice spacing 0.04-0.06 fm, $m_\pi = 300$ MeV

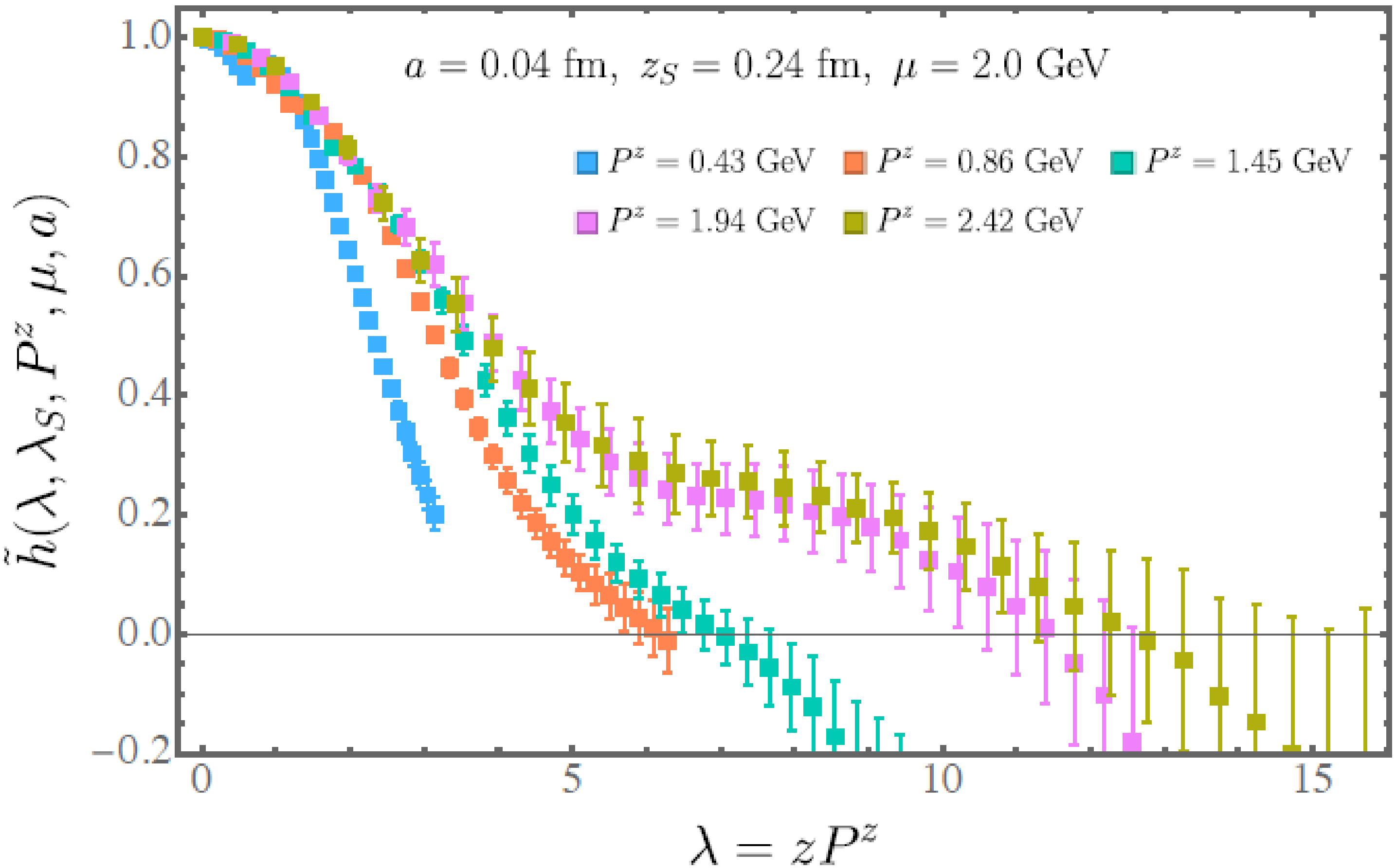
$\mathrm{P} = 2.42$ MeV



Too good to be true!

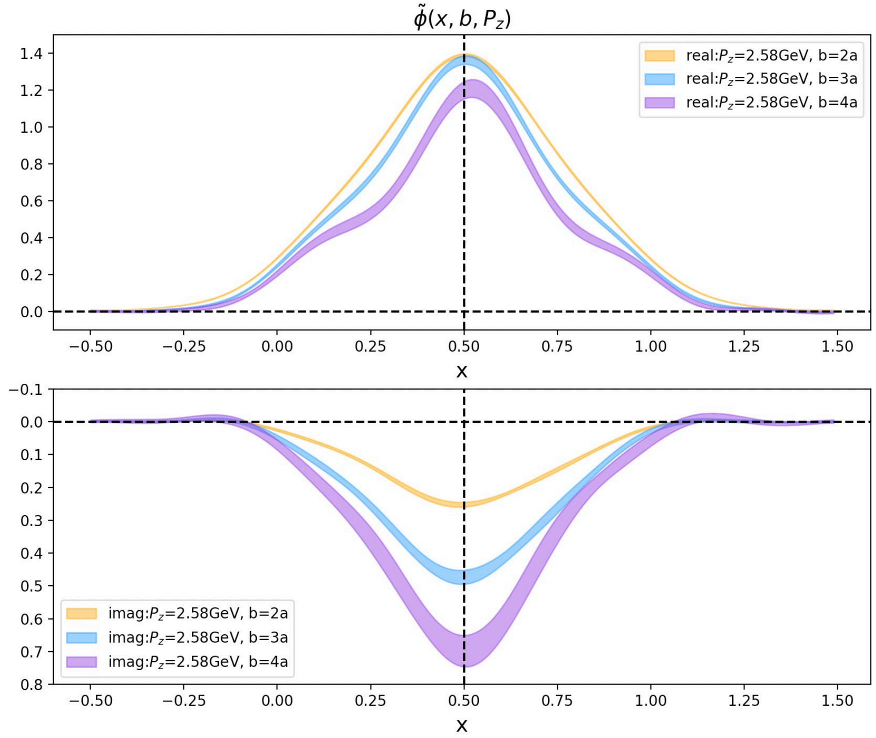


Light-cone correlation



A first example of QCD LFWF

- QCD meson LFWF (preliminary, LPC collaboration)



How to go to small x?

Small-x partons

- Small-x partons are generated from long-range correlations of quantum fields along the longitudinal direction (near critical point).

$$f(x) = \int \frac{d\lambda}{2\pi} e^{-ix\lambda} h(\lambda) \ .$$

- There are small-x pQCD evolution/resummations (BFKL, Balitsky-Kovchegov eq., JIMWLK eq.)
- However, non-perturbative approaches are important (MV model, non-pert Pomeron).

Small x: brute force

- Smallest x:

  $$x \sim \frac{\Lambda_{QCD}}{P^z}$$

  take Λ_{QCD} as 200-300 MeV

- to get x=0.01, one needs

  $P^z \sim 100\Lambda_{QCD} \sim$ 20 GeV, (filtering g.s. can be hard)

  $\gamma \sim 20$, proton long. contraction, ~ 0.05 fm

- Lattice spacing a

  Valence quark mom $k \sim 7\,\mathrm{GeV} \ll \frac{\pi}{a}$, $a \ll 0.1$

At least 3-5 points for proton, $a \sim 0.01$ fm

Small x: brute force (cont.)

- Longest correlation length λ : $\lambda \sim P^z z$

  Since the largest λ get x=0.01, is

  $$\lambda \sim \frac{1}{x} \sim 100$$

- z = 100/ P^z = 1 fm: size of the gluon cloud

  with $a \sim 0.01$ fm

- One need about 256 pts!
- Thus one needs configurations 256 pts with 0.01fm!

Nucleus

- A large nucleus can be used to accelerate parton saturations.
- Effects is linear in $A^{1/3}$
- However, the number of contractions grow like A!

  to calculate partons for a nucleus is also
challenging

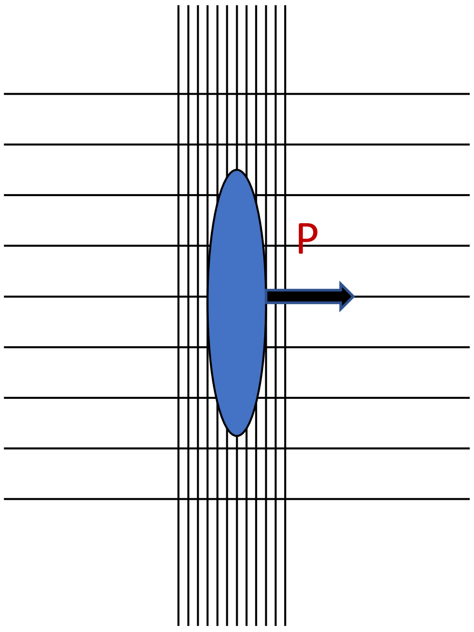
- At least, one can calculate the EMC effect for the deuteron (??)

new ideas?

- One never needs a large lattice to compute long range correlations.
- One just need small lattice spacing and a large momentum nucleon.

Asymmetric lattice

- Starting with an asymmetric lattice



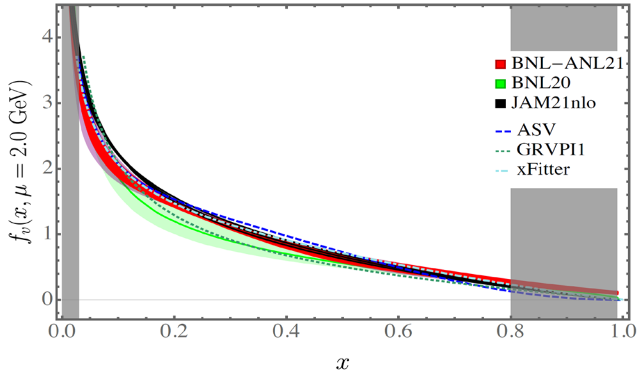
Analytically-integrate out the short-distance modes

- One can integrate out modes at small lattice spacing, using Wilson's renormalization group approach
- This generates a new extended momentum-dependent source on the lattice
- Each step of integration will generate a new source.
- Thus one gets an RGE for new sources

(Similar to JIMWLK equation?)

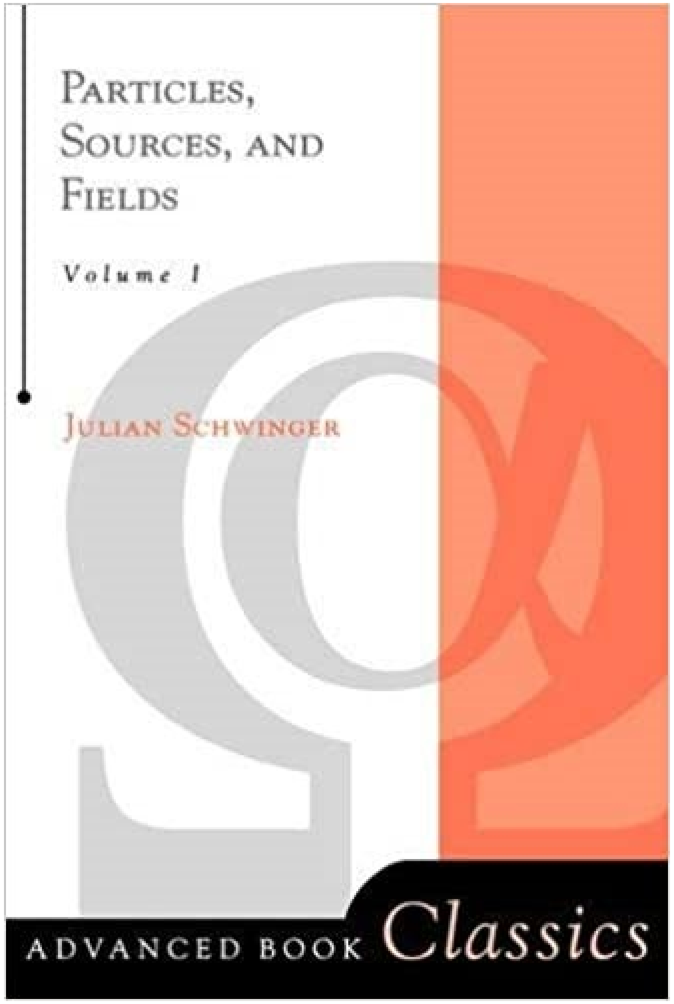
Simulating the EFT of small-x

- Once the source is has only resolution scale of order 0.03-0.05 fm, one can finally put on lattice.
- All large-x partons are in the source



- An EFT for small x.
- The end EFT can be simulated on lattice with an effective sources.

Source is important!



No conclusion

- It is just an idea (never written a small x paper!)
- It may or may not work
- Welcome collaborations!
- Thank the following people for preliminary discussions:

  U-J. Wiese, L. C. Jin, B. W. Xiao, F. Yuan

  R. Venugopalan, …