Spin at Small x

Dialing in on Sub-Eikonal Physics

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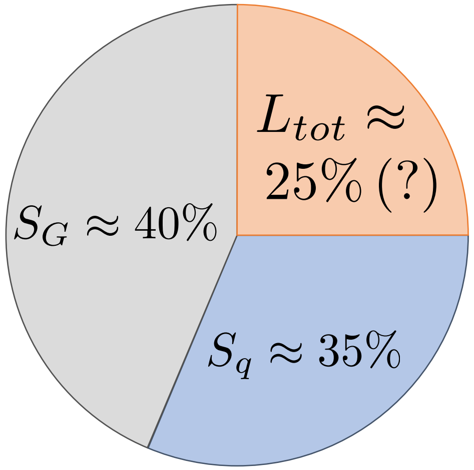
RBRC Workshop:
Small-x Physics in the EIC Era

12/16/2021

Jaffe-Manohar Spin Sum Rule:

Jaffe and Manohar, Nucl. Phys. **B337** 509 (1990)

$$\frac{1}{2} = S_q + S_G + L_q + L_G$$



- Quark Polarization:

$$S_q(Q^2) = \frac{1}{2} \sum_{f,\bar{f}} \int_0^1 dx\, \Delta q_f(x, Q^2)$$

$$\Delta q(x, Q^2) = \int \frac{dr^-}{2\pi} e^{ixp^+ r^-} \left\langle pS_L \middle| \bar{\psi}(0)\, \mathcal{U}[0,r]\, \frac{\gamma^+ \gamma^5}{2}\, \psi(r) \middle| pS_L \right\rangle$$

➢ Nonlocal generalization of the **axial vector current** $j_5^\mu = \bar{\psi}\, \gamma^\mu \gamma^5 \psi$

- Gluon Polarization:

$$S_G(Q^2) = \int_0^1 dx\, \Delta G(x, Q^2)$$

$$\Delta G(x, Q^2) = \frac{-2i}{xp^+} \int \frac{dr^-}{2\pi} e^{ixp^+ r^-} \left\langle pS_L \middle| \epsilon_T^{ij}\, \mathrm{tr}\left[F^{+i}(0)\, \mathcal{U}[0,r]\, F^{+j}(r)\, \mathcal{U}'[r,0] \right] \middle| pS_L \right\rangle$$

➢ Circular **azimuthal correlation** of gluon field strengths

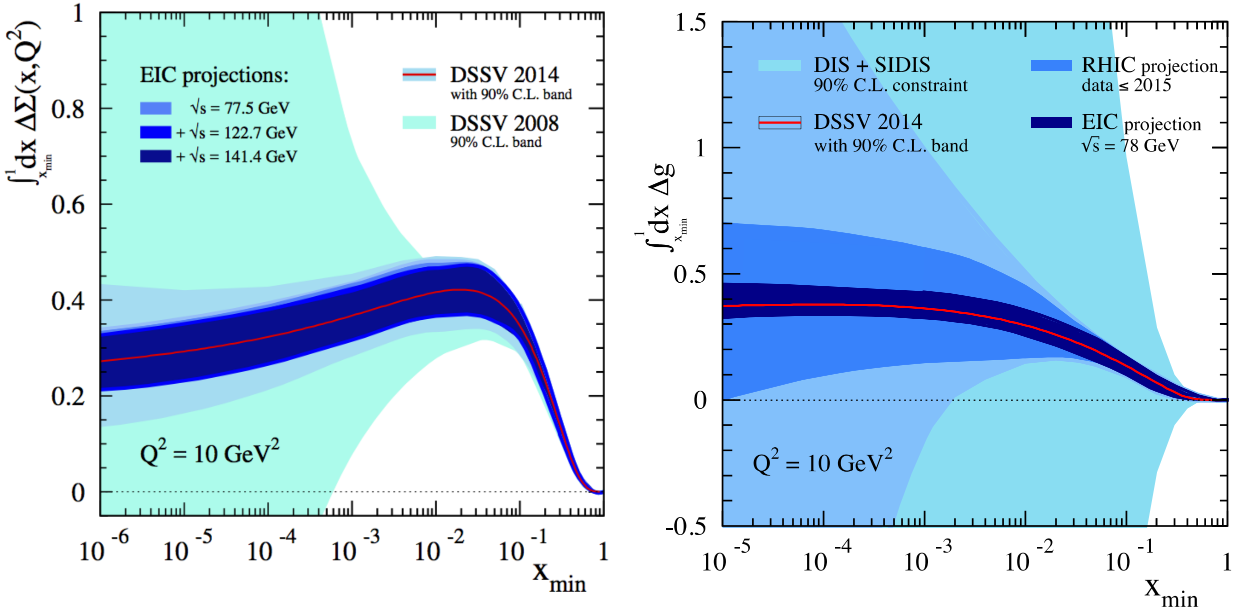
Why Should We Care About Spin at Small x?

$$\Delta q^+ \equiv q + \bar{q}$$

$$\Delta\Sigma(Q^2) = \sum_q \int_0^1 dx\, \Delta q^+(x, Q^2)$$

$$\Delta G(Q^2) = \int_0^1 dx\, \Delta g(x, Q^2)$$

Aschenauer et al., Phys. Rev. **D92** (2015) no. 9 094030

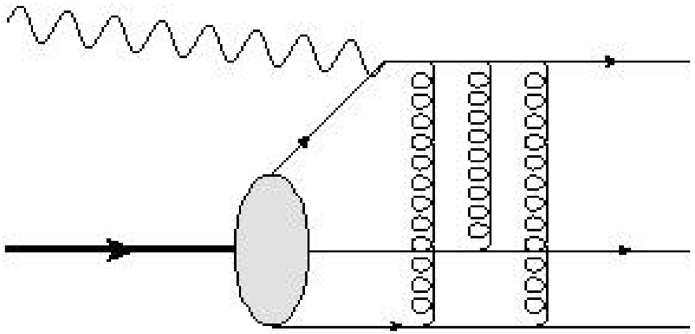


- Determination of the partonic origin of proton spin **requires extrapolation to x=0**
- Extractions based on **DGLAP** are **not predictive of the x dependence**
  - ➢ Inevitable that the **uncertainty blows up** once **data constraints run out**
  - ➢ Controlled extrapolation to x=0 **requires a theory** which **predicts spin at small x**

Spin: Beyond the Eikonal Limit

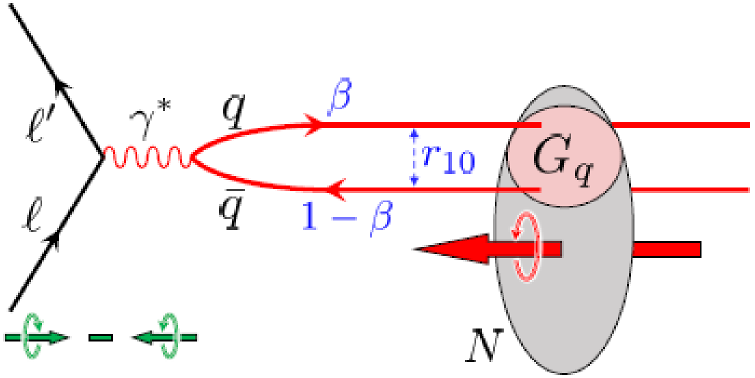
- At **large x**, DIS is dominated by a **"Knockout" process**
  - At **small x**, the leading channel is a **"dipole" process**

c.f) **Large x:** "Knockout" DIS



- **Leading-power** dipole scattering is **spin-independent**
  - Pure eikonal **Wilson lines** (gluons)

Small x: "Dipole" DIS



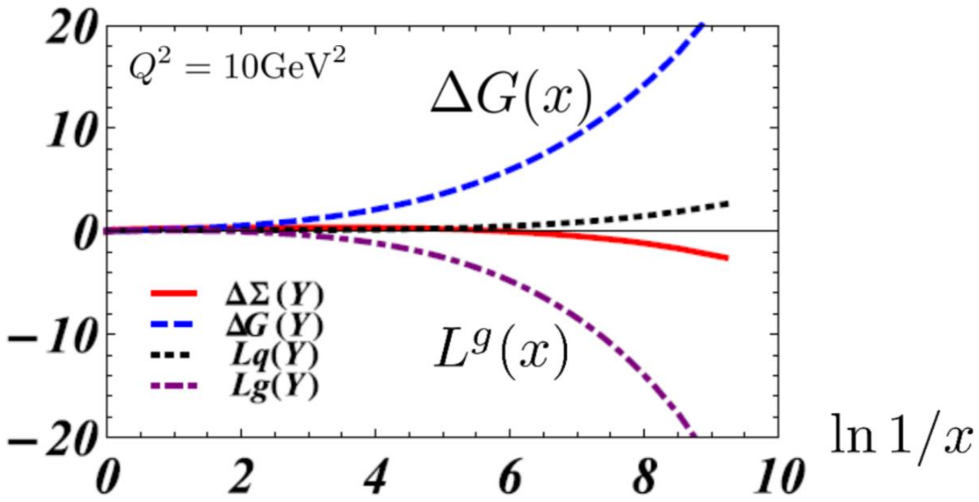
- **Spin at small x** selects on **different, sub-eikonal** dynamics
  - Spin observables are sensitive to **novel small-x physics**

First Approach: Infrared Evolution Equations

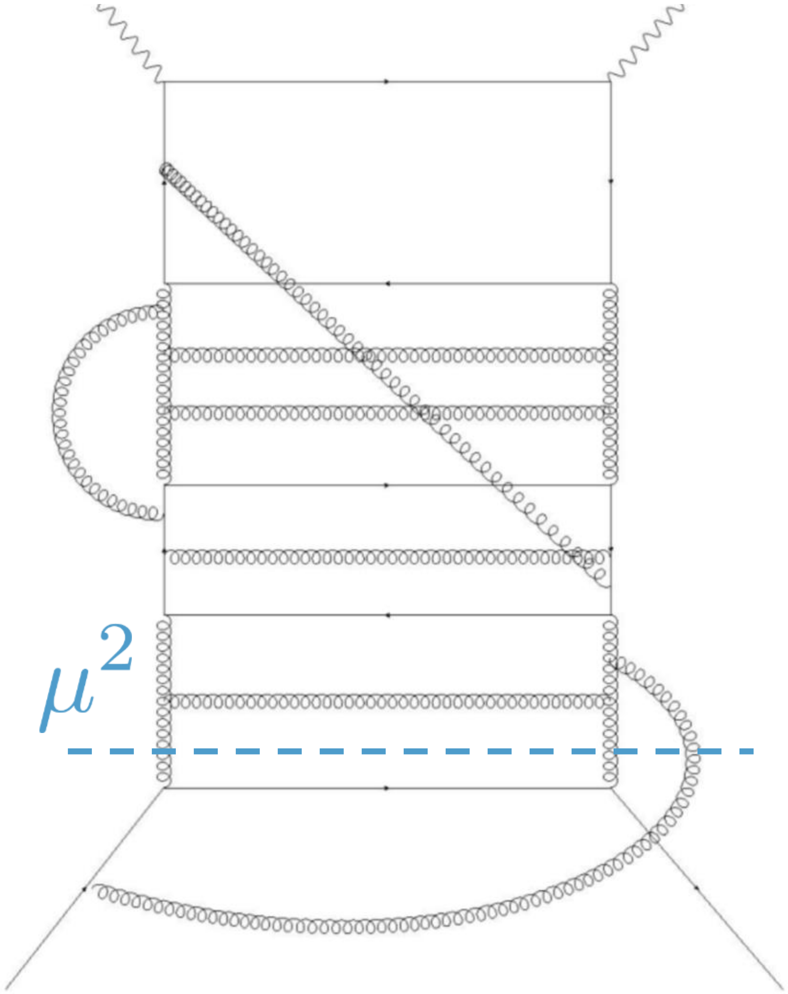
Bartels, Ermolaev, & Ryskin,
*Z. Phys. **C70** (1996), Z. Phys. **72** (1996)*

- Based on identifying where the softest momentum scale μ lives in a complex diagram
- Close relation to DGLAP
- Goal is to resum **all** double logarithms

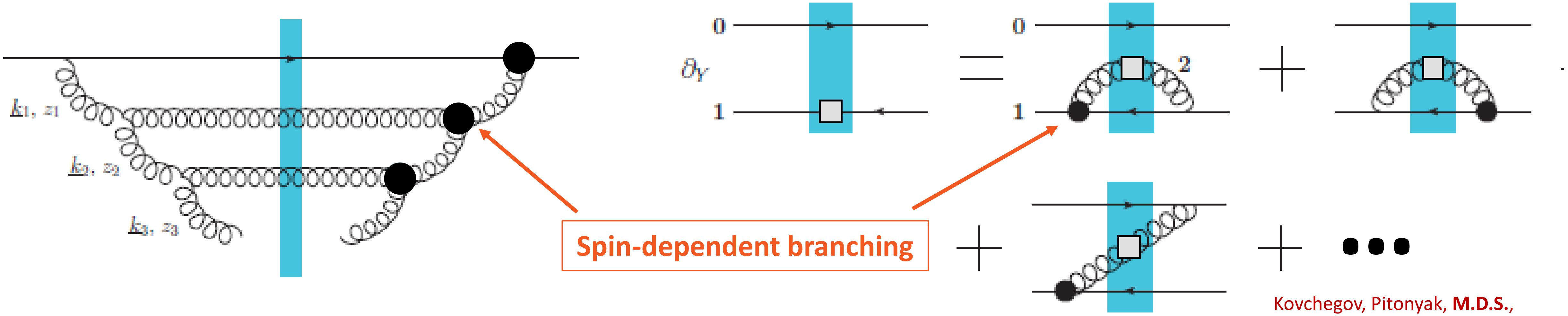
$$\alpha_s \ln^2 \frac{1}{x} \sim \mathcal{O}(1) \qquad \alpha_s \ln \frac{1}{x} \ln \frac{Q^2}{\mu^2} \sim \mathcal{O}(1)$$



*Hatta & Yao, Phys. Lett. **B798** (2019)*
*Boussarie, Hatta, & Yuan, Phys. Lett. **B797** (2019)*
*Hatta & Yang, Phys. Lett. **B781** (2018)*
*Hatta et al., Phys. Rev. **D95** (2017)*



Second Approach: Polarized Dipoles

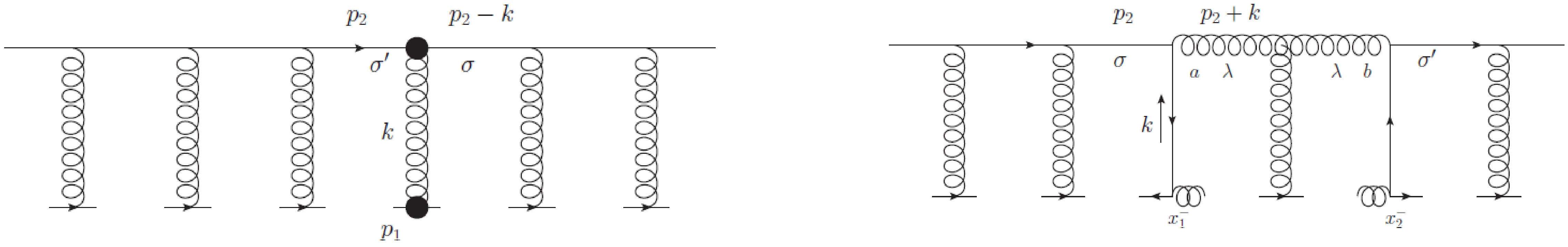


Kovchegov, Pitonyak, **M.D.S.**,
JHEP **1601** (2016), Phys. Rev. **D95** (2017)

- **Spin information** from the valence sector (large x) is **transmitted to small x** by **spin-dependent branching**

*Also G. Chirilli, JHEP **1901** (2019)*

- **Suppressed** by the **coupling** α_s but **enhanced** by the **phase space** $\ln \frac{1}{x}$
  - Resummation leads to **quantum evolution of spin at small x**
  - Analogous to **BFKL evolution** for unpolarized gluons



QCD Stern-Gerlach: $\gamma\, \vec{S} \cdot \vec{B}$

$$V_{\underline{x}}^{pol} = \frac{i g p_1^+}{s} \int\limits_{-\infty}^{\infty} dx^- \, V_{\underline{x}}[+\infty, x^-] \, F^{12}(x^-, \underline{x}) \, V_{\underline{x}}[x^-, -\infty]$$

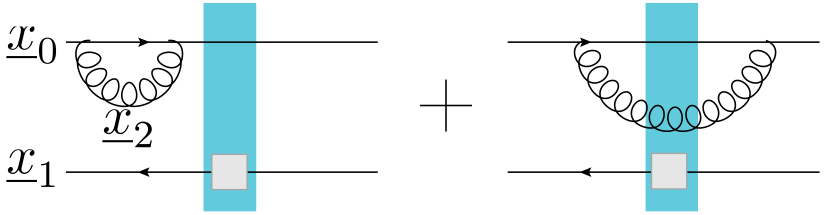
$$- \frac{g^2 p_1^+}{s} \int\limits_{-\infty}^{\infty} dx_1^- \int\limits_{x_1^-}^{\infty} dx_2^- \;\; V_{\underline{x}}[+\infty, x_2^-] \, t^b \, \psi_\beta(x_2^-, \underline{x}) \, U_{\underline{x}}^{ba}[x_2^-, x_1^-] \left[\frac{1}{2} \gamma^+ \gamma^5 \right]_{\alpha\beta} \bar{\psi}_\alpha(x_1^-, \underline{x}) \, t^a \, V_{\underline{x}}[x_1^-, -\infty].$$

Flavor-changing Wilson line

Since m=0: Quark helicity conservation

Quantum Evolution: Beyond Color Transparency

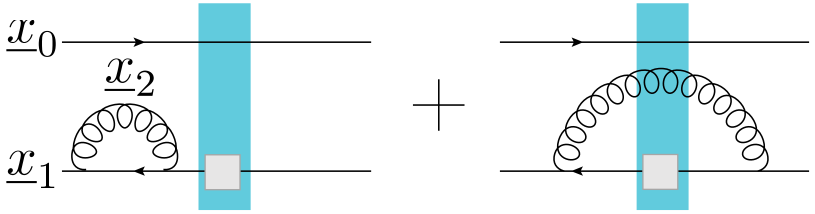
- Ladder emissions from the **unpolarized Wilson line** possess **color transparency** at short distances



$$\frac{\alpha_s N_c}{2\pi^2}\int\limits_{\Lambda^2/s}^{z}\frac{dz'}{z'}\int\frac{d^2x_2}{x_{20}^2}\times\left[\frac{1}{N_c^2}\left\langle \mathrm{tr}\left[V_2 V_1^{pol\,\dagger}\right]\,\mathrm{tr}\left[V_0 V_2^{\dagger}\right]\right\rangle_{(z's)}-\frac{1}{N_c}\left\langle \mathrm{tr}\left[V_0 V_1^{pol\,\dagger}\right]\right\rangle_{(z's)}\right]$$

Cancels when $x_2 \rightarrow x_0$

- But for ladder emissions from the **polarized Wilson line**, color transparency is **violated by spin**



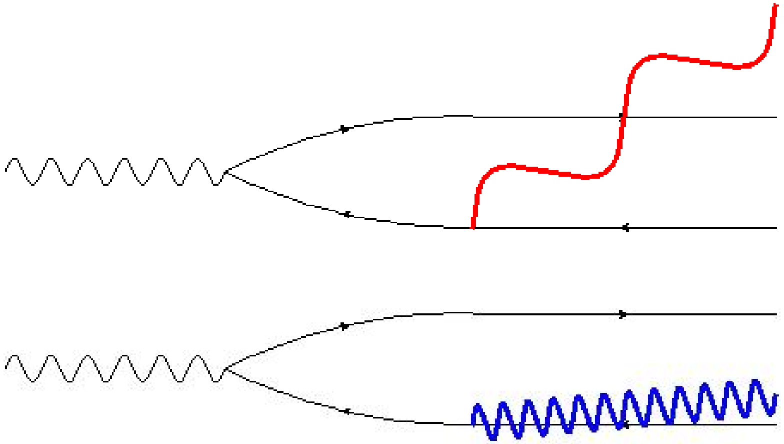
$$\frac{\alpha_s N_c}{2\pi^2}\int\limits_{\Lambda^2/s}^{z}\frac{dz'}{z'}\int\frac{d^2x_2}{x_{21}^2}\times\left[\frac{1}{N_c^2}\left\langle \mathrm{tr}\left[V_2 V_1^{pol\,\dagger}\right]\,\mathrm{tr}\left[V_0 V_2^{\dagger}\right]\right\rangle_{(z's)}-\frac{1}{N_c}\left\langle \mathrm{tr}\left[V_0 V_1^{pol\,\dagger}\right]\right\rangle_{(z's)}\right]$$

Does NOT cancel when $x_2 \rightarrow x_1$

Polarized vs. Unpolarized Small-x Evolution

Longitudinal + transverse logarithmic phase space

$$dP \sim \alpha_s \frac{dx}{x} \frac{d^2 k}{k_T^2}$$



Long wavelength/IR

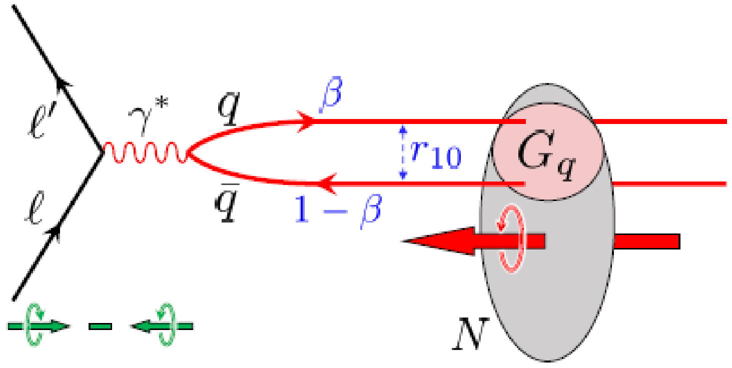
Short wavelength/UV

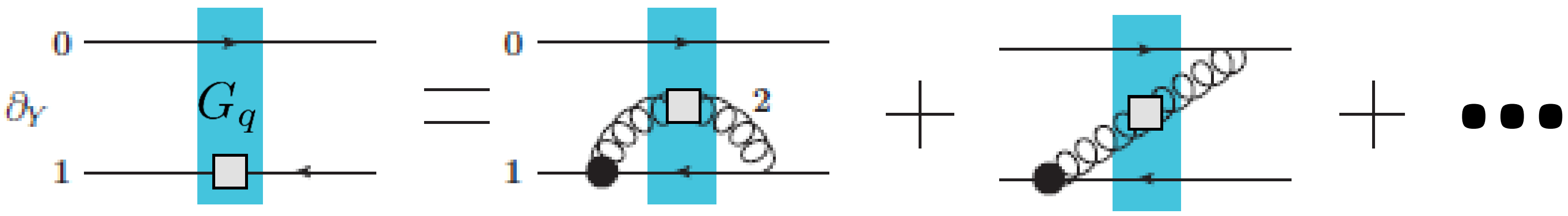
- **Unpolarized (BFKL) evolution:**
  - **Transverse logs cancel** (neutrality in the IR, transparency in the UV)
  - Only **longitudinal** phase space is logarithmic: $\alpha_s \ln \frac{1}{x} \sim O(1)$

- **Polarized (KPS) evolution:**
  - **No transparency of spin**: dominance of **UV transverse logs**
  - **Double-logarithmic** evolution: $\alpha_s \ln^2 \frac{1}{x} \sim O(1)$
  - Sensitive to **lifetime ordering** (c.f. NLO BFKL), less sensitive to saturation

The KPS Evolution Equations

Kovchegov, Pitonyak, **M.D.S.**, JHEP **1601** (2016), Phys. Rev. **D95** (2017)





$$G_q(s_{10},\eta) = G_q^{(0)}(s_{10},\eta) + \int\limits_{s_{10}}^{\eta} d\eta' \int\limits_{s_{10}}^{\eta'} ds_{21} \left[\Gamma_q(s_{10},s_{21},\eta') + 3\,G_q(s_{21},\eta')\right]$$

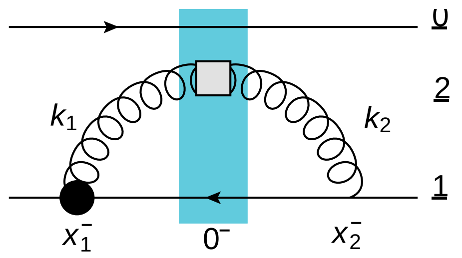
$$\eta = \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{\beta s}{\Lambda^2}, \qquad s_{10} = \sqrt{\frac{\alpha_s N_c}{2\pi}} \ln \frac{1}{r_{10}^2 \Lambda^2},$$

$$\Gamma_q(s_{10},s_{21},\eta') = G_q^{(0)}(s_{10},\eta') + \int\limits_{s_{10}}^{\eta'} d\eta'' \int\limits_{s_{32}^{\min}}^{\eta''} ds_{32} \left[\Gamma_q(s_{10},s_{32},\eta'') + 3\,G_q(s_{32},\eta'')\right]$$

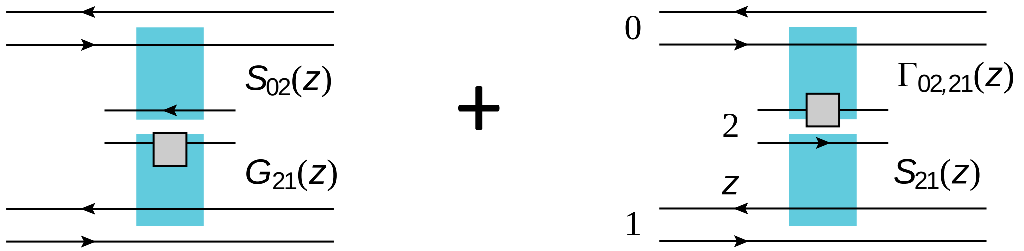
$$s_{32}^{\min} = \max\{s_{10}\,,\ s_{21} - \eta' + \eta''\}$$

- **Double-logarithmic evolution** equations written with logarithmic variables η, s_{10}
- Infinite tower of operators only **closes in certain limits (large-N_c** or large-$N_c\&N_f$)
- Auxiliary **"neighbor dipole" function Γ** necessary to enforce lifetime ordering

The Neighbor Dipole Function



Constrained by lifetime of x_{21}



Further evolution of x_{21} dipole

Evolution of x_{02} dipole is constrained by x_{21}

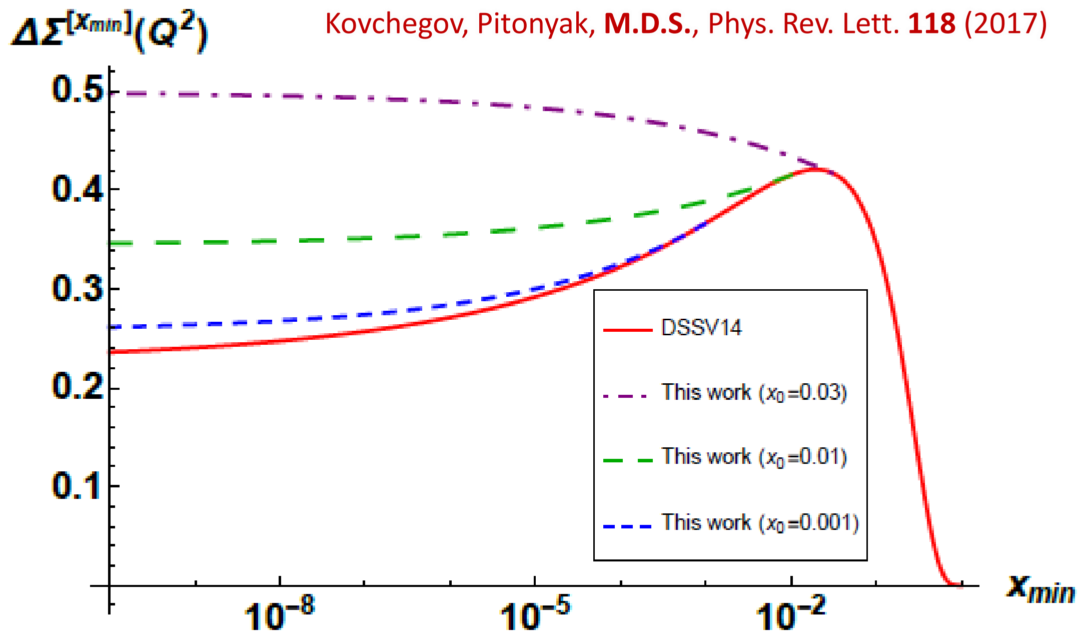
- These evolution equations describe another **operator hierarchy**
- Just like the BK equation, they do **close in the large-Nc limit** (or large Nc & Nf)
- But because **lifetime ordering** constrains the history of the polarized gluon cascade, **not all dipoles are independent**.

Kovchegov, Pitonyak, **M.D.S.**, JHEP **1601** (2016)

A Bayesian Global Analysis of Polarized DIS

- **Previous estimates:**
  - **Potentially substantial contribution** to the proton spin from small-x quarks
  - **Exploratory methods**: instantaneous transition from large-x fit (DSSV) to power-law asymptotics of KPS evolution

Kovchegov, Pitonyak, **M.D.S.**, Phys. Rev. Lett. **118** (2017)



- **This work:** D. Adamiak et al., Phys. Rev. **D104** (2021)
  - **JAM Bayesian analysis** code N. Sato et al., Phys. Rev. **D93** (2016)
  - **Global analysis** of world polarized DIS data (SLAC, EMC, SMC, COMPASS, HERMES)
  - **Proton, "neutron"** (d, ^{3}He) targets
  - **Inclusive DIS** only
    - Avoid complication of fragmentation

$$A_1 \approx \frac{g_1}{F_1}$$

$$A_{||} \propto A_1$$

Data Cuts: $N_{pts} = 122$

$$x < 0.1$$
$$Q^2 > 1.69\,\text{GeV}^2$$
$$W^2 > 4\,\text{GeV}^2$$

The Approach

1. Parameterize an **initial condition** for the polarized dipole amplitude at **large $x > x_0$**
   - Generalized form of Born approximation
   - **Frozen initial condition** at $x > x_0$

$$G_q^{(0)}(s_{10}, \eta) = a_q\, \eta + b_q\, s_{10} + c_q$$

2. **Solve KPS eqns. numerically** for $x < x_0$ to determine g_1 structure function

$$\Delta q^+(x, Q^2) = \frac{1}{\alpha_s \pi^2} \int_0^{\eta_{\max}} d\eta \int_{s_{10}^{\min}}^{\eta} ds_{10}\, G_q(s_{10}, \eta)$$

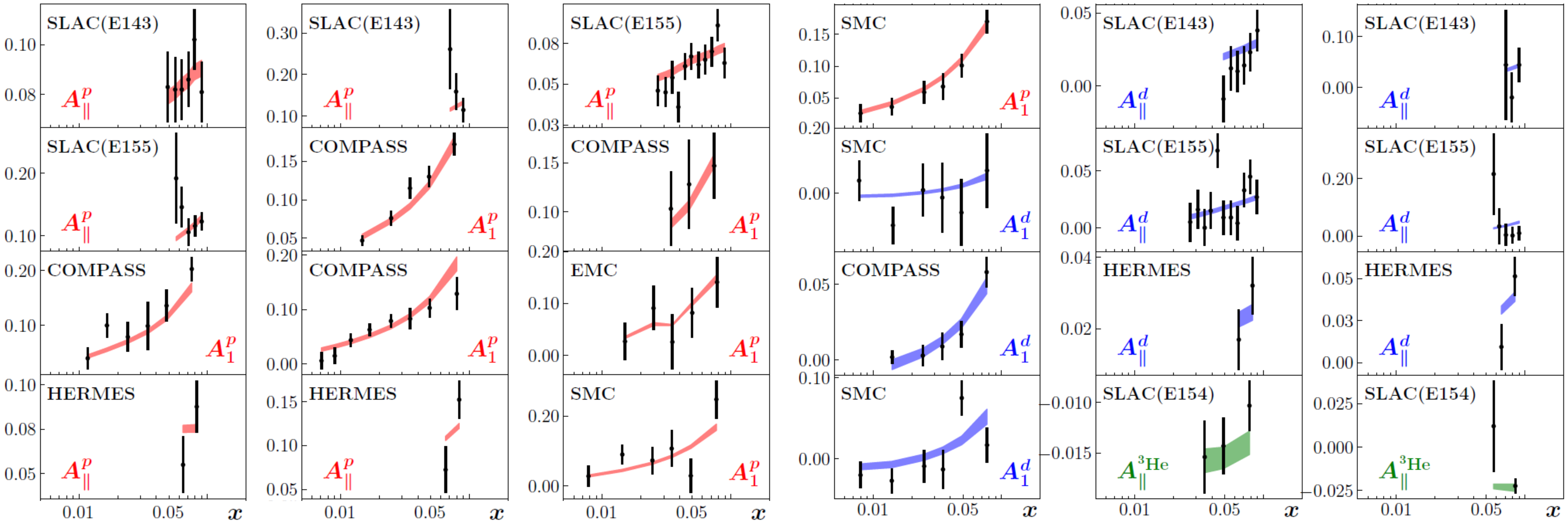
3. **Compute spin asymmetries** $A_1, A_\parallel$ and compare with experimental data
   - F_1 taken from **previous JAM fits**

$$g_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2\, \Delta q^+(x, Q^2)$$

4. **Scan parameter space** using Bayesian inference with JAM Monte Carlo framework
   - **6 parameters** scanned $(a, b, c$ for p, n$)$ **+ x_0, Λ**

$$A_1 \approx \frac{g_1}{F_1} \qquad A_\parallel \propto A_1$$

The KPS Formalism is Able to Describe World Data



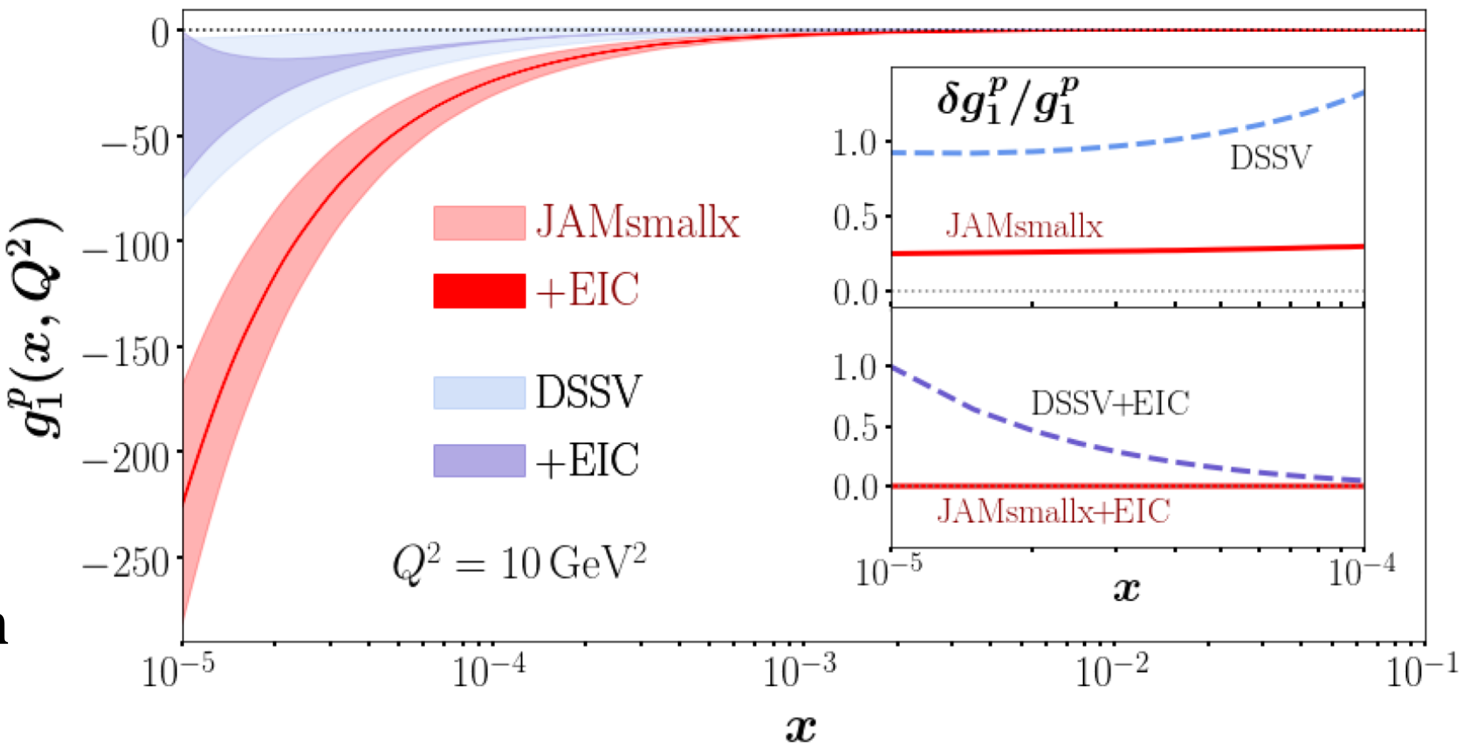
- **Nontrivial test: purely small-x theory** is able to describe the world DIS data
  - ➢ Most constraining: a **few data points below x = 0.01**

JAMsmallx: $\chi^2/N_{pts} = 1.01$

c.f. JAM16: $\chi^2/N_{pts} = 1.07$

Results: Prediction of Stronger Negative g1

- **Structure function g_1** determined down to **low x**
- **Large, negative g_1** is a robust prediction of the analysis
  - Data requires that the **dominant term** be **negative**
  - **Predictive power** at small x: controlled error in extrapolation
- The procedure **cannot describe everything**
  - Fails for **too-large x** (as expected)
  - $\frac{\chi^2}{N_{pts}} = 5.66$ for $x_0 = 0.3$



Error band: Bayesian 1σ confidence level

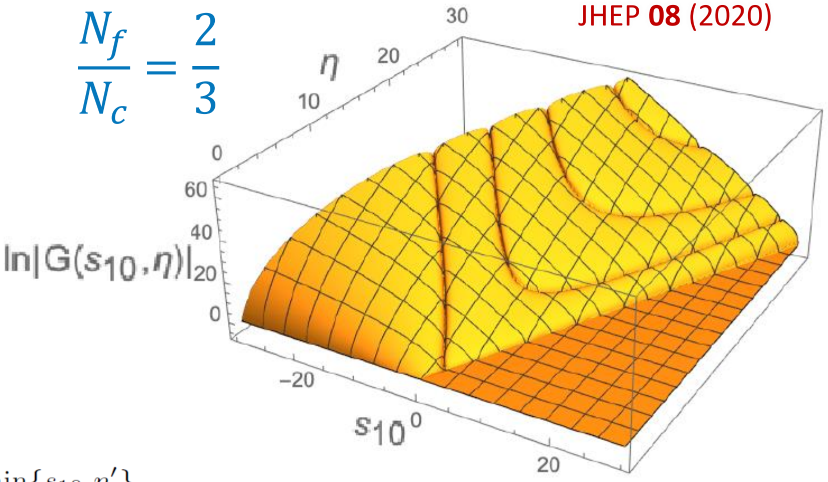
Statistical impact of **EIC data** (thin red band)

DSSV: de Florian et al., Phys. Rev. Lett. **113** (2014), Phys. Rev. **D100** (2019)

Y. V. Kovchegov, Y. Tawabutr, JHEP **08** (2020)

- KPS evolution in the **simultaneous large-N_c&N_f limit**
  - Dynamical quark loops in KPS evolution
- Numerical solution exhibits **oscillations in x**
  - Long period (many units of rapidity)

$$\frac{N_f}{N_c} = \frac{2}{3}$$



$$G(s_{10},\eta) = 1 + \int\limits_{\max\{0,s_{10}\}}^{\eta} d\eta' \int\limits_{s_{10}}^{\eta'} ds_{21} \left[\Gamma(s_{10},s_{21},\eta') + 3G(s_{21},\eta') - \frac{N_f}{N_c}\,\overline{\Gamma}(s_{10},s_{21},\eta')\right] - \frac{N_f}{N_c}\int\limits_{0}^{\eta} d\eta' \int\limits_{s_{10}+\eta'-\eta}^{\min\{s_{10},\eta'\}} ds_{21}\; Q(s_{21},\eta'),$$

$$\Gamma(s_{10},s_{21},\eta') = 1 + \int\limits_{\max\{0,s_{10}\}}^{\eta'} d\eta'' \int\limits_{\max\{s_{10},s_{21}+\eta''-\eta'\}}^{\eta''} ds_{32} \left[\Gamma(s_{10},s_{32},\eta'') + 3G(s_{32},\eta'') - \frac{N_f}{N_c}\,\overline{\Gamma}(s_{10},s_{32},\eta'')\right] - \frac{N_f}{N_c}\int\limits_{0}^{\eta'+s_{10}-s_{21}} d\eta'' \int\limits_{s_{21}+\eta''-\eta'}^{\min\{s_{10},\eta''\}} ds_{32}\; Q(s_{32},\eta''),$$

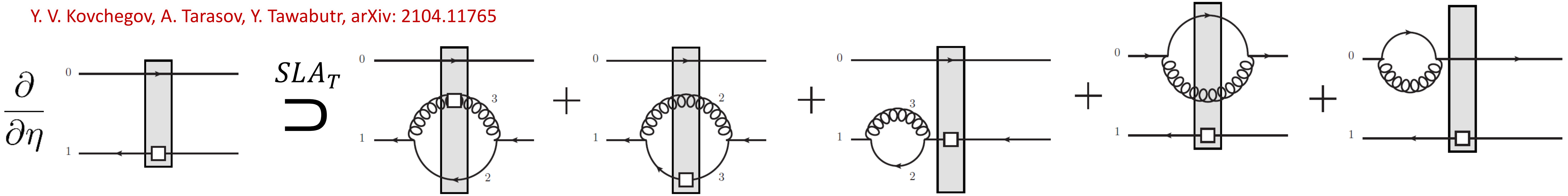
$$\Delta\Sigma(x,Q^2)\Big|_{\text{large-}N_c\&N_f} \sim \left(\frac{1}{x}\right)^{\alpha_h^q} \cos\left[\omega_q \ln\left(\frac{1}{x}\right) + \varphi_q\right]$$

$$\alpha_h^q\Big|_{\text{large-}N_c} = \frac{4}{\sqrt{3}}\sqrt{\frac{\alpha_s N_c}{2\pi}},$$

$$\omega_Q = \omega_G \approx \frac{0.22 N_f}{1 + 0.1265 N_f}$$

Other Developments: Single-Log Evolution

Y. V. Kovchegov, A. Tarasov, Y. Tawabutr, arXiv: 2104.11765



KPS (2016): $\alpha_s \ln^2 \frac{1}{x} \gtrsim 1$ $\xrightarrow{\text{resums}}$ $\Delta\Sigma \sim \left(\alpha_s \ln^2 \frac{1}{x}\right)^n$

KTT (2021): $\alpha_s \ln \frac{1}{x} \gtrsim 1$ $\xrightarrow{\text{resums}}$ $\Delta\Sigma \sim \left(\alpha_s \ln^2 \frac{1}{x}\right)^n + \left(\alpha_s \ln \frac{1}{x}\right)^{n'}$

- Calculate the **single-logarithmic corrections** to the double-logarithmic KPS eqs.
  - ➢ Exact kinematics destroy one logarithm of x
  - ➢ Also sensitive to **running coupling** corrections
  - ➢ Derived closed equations for both **large-N_c** and **large-N_c&N_f** limits

Other Developments: Role of the Axial Anomaly

- **Quark helicity** shares an intimate connection with the **axial vector current**

Jaffe and Manohar, Nucl. Phys. **B337** 509 (1990)

A. Tarasov, R. Venugopalan,
Phys. Rev. **D100** (2019), **D102** (2020), arXiv: 2109.10370

$$\Delta q(x, Q^2) = \int \frac{dr^-}{2\pi} e^{ixp^+ r^-} \left\langle pS_L \right| \bar{\psi}(0)\, \mathcal{U}[0, r]\, \frac{\gamma^+ \gamma^5}{2}\, \psi(r) \left| pS_L \right\rangle$$

- The **total quark helicity** leads to a **local matrix element** of the **axial vector current**.

$$\Delta\Sigma(Q^2) = \sum_q \int_0^1 dx\, (\Delta q + \Delta \bar{q})(x, Q^2) = \frac{1}{p^+} \sum_q \langle pS_L | \bar{\psi}(0) \frac{\gamma^+ \gamma^5}{2} \psi(0) | pS_L \rangle \sim \frac{\langle j_5^+ \rangle}{p^+}$$

- Explicitly sensitive to the **axial anomaly**
  - Different **interpretation**…?
  - Or a different **contribution to ΔΣ**…?

$$\partial_\mu j_5^\mu = \frac{\alpha_s N_f}{2\pi} \mathrm{tr} \left(F_{\mu\nu} \tilde{F}^{\mu\nu} \right)$$

Other Developments: Role of the Axial Anomaly

A. Tarasov, R. Venugopalan,
Phys. Rev. **D100** (2019), **D102** (2020), arXiv: 2109.10370

- Chiral symmetry breaking appears as a **pole** in the **forward-scattering limit**

$$\langle p' S_L | j_5^\mu | p S_L \rangle = \frac{1}{4\pi^2} \frac{\ell^\mu}{\ell^2} \int \frac{d^4 k}{(2\pi)^4} \, \mathrm{tr} F_{\alpha\beta}(k) \tilde{F}^{\alpha\beta}(-k-\ell) \qquad p' = p + \ell$$

- Subtle nuances of **pole cancellation** among the isoscalar axial vector charge and pseudoscalar charge associated with **mass generation of the η' meson**.

G. Shore, G. Veneziano, Phys Lett. **B**244 (199),
Nucl. Phys. **B381** (1992)

  - *"[The] structure function g_1 measured in polarized deeply inelastic scattering is dominated by the triangle anomaly in both Bjorken and Regge asymptotics."*
  - *"[Our result] brings into question the applicability of QCD factorization for quantities such as g_1 that are sensitive to the anomaly"*

Conclusions

- The properties of **spin at small x** force us to generalize **beyond the eikonal limit**
  - ➢ Strategic use of the **spin quantum number** to **select on sub-eikonal physics**
  - ➢ Double-logarithmic evolution, lifetime ordering, neighbor dipole amplitude…
- Significant recent progress in both **EIC phenomenology** and **fundamental theory**
  - ➢ **Viable global analyses** of world small-x polarized DIS data based on **KPS evolution**
  - ➢ **Single-log evolution** (comparable to BK) and **quark-induced oscillations**
- Important long-standing questions about **nonperturbative contributions** / **interpretation** of the **axial anomaly** in total quark spin.