Wigner distributions at small-x

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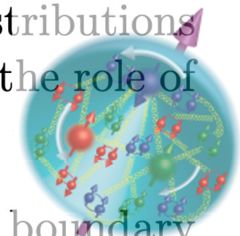
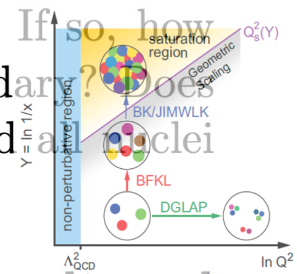
Big questions for EIC

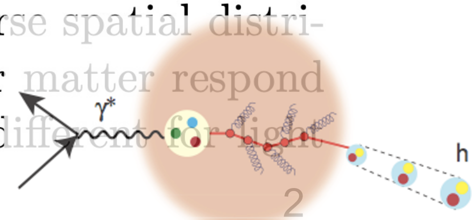
- **How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?** How are these quark and gluon distributions correlated with overall nucleon properties, such as spin direction? What is the role of the orbital motion of sea quarks and gluons in building the nucleon spin?



- **Where does the saturation of gluon densities set in?** Is there a simple boundary that separates this region from that of more dilute quark-gluon matter? If so, how do the distributions of quarks and gluons change as one crosses the boundary? Does this saturation produce matter of universal properties in the nucleon and all nuclei viewed at nearly the speed of light?



- **How does the nuclear environment affect the distribution of quarks and gluons and their interactions in nuclei?** How does the transverse spatial distribution of gluons compare to that in the nucleon? How does nuclear matter respond to a fast moving color charge passing through it? Is this response different for light and heavy quarks?

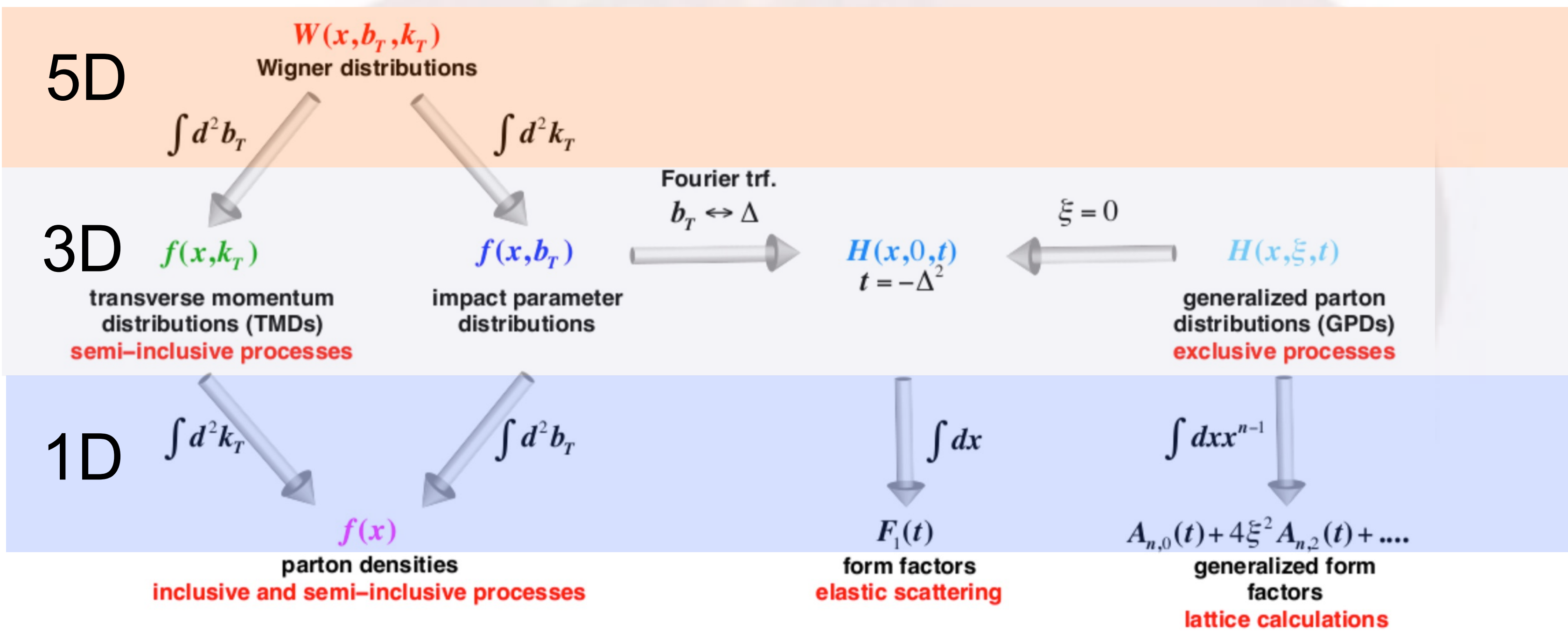


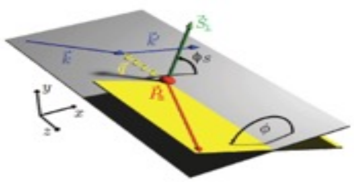
New ways to look at partons

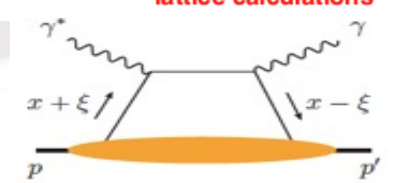
- We not only need to know that partons have long. momentum, but must have transverse degrees of freedom as well
- Partons in transverse coordinate space
  - Generalized parton distributions (GPDs)
- Partons in transverse momentum space
  - Transverse-momentum distributions (TMDs)
- Both? Wigner distributions!

Unified view of the Nucleon

❑ Wigner distributions (Belitsky, Ji, Yuan)





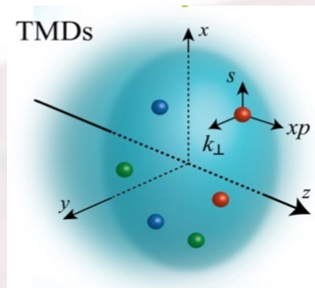


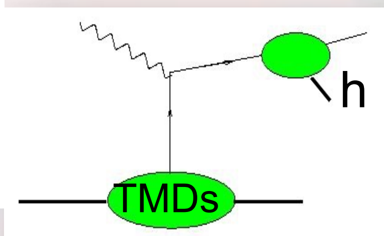
What can we learn

- 3D Imaging of partons inside the nucleon (non-trivial correlations)
  - Try to answer more detailed questions as Rutherford was doing more than 100 years ago
- QCD dynamics involved in these processes
  - Transverse momentum distributions: universality, factorization, evolutions,…
  - Small-x: BFKL vs Sudakov?

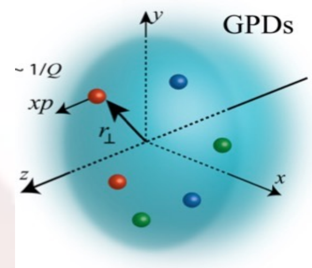
Zoo of TMDs & GPDs

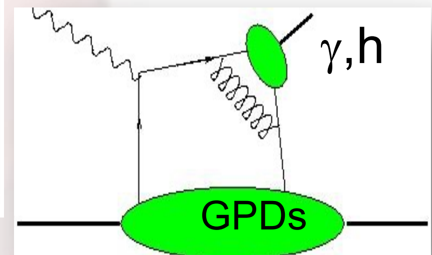
	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_1, h_{1T}^{\perp}$





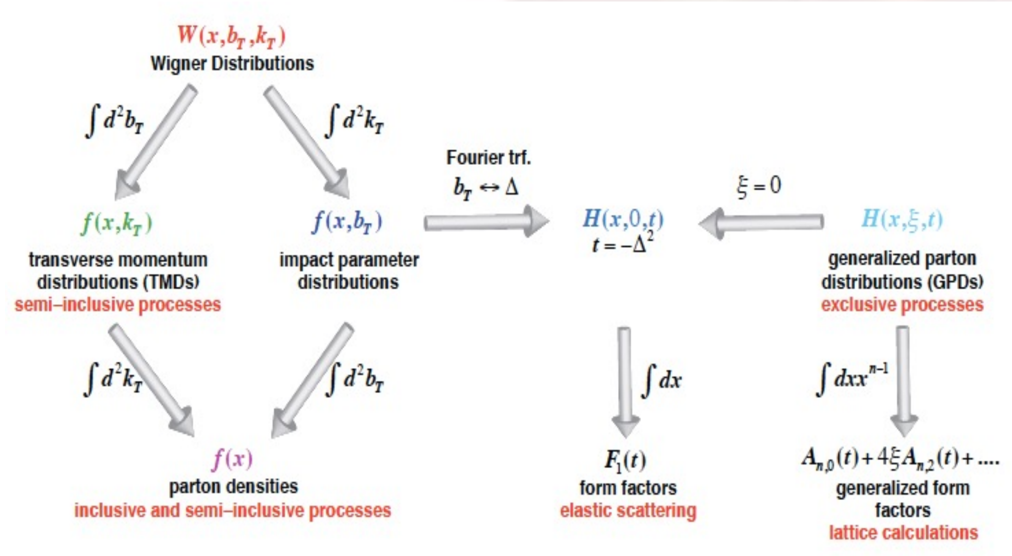
	U	L	T
U	H		$\mathcal{E}_T$
L		$\widetilde{H}$	
T	E		$H_T, \widetilde{H}_T$





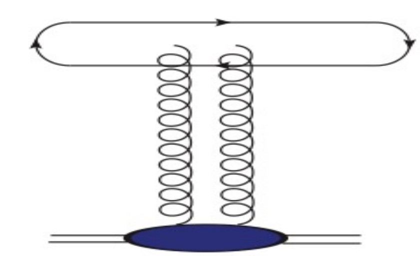
- NOT directly accessible
- Their extractions require measurements of x-sections and asymmetries in **a large kinematic domain of** $\mathbf{x_B}$**, t,** $\mathbf{Q^2}$ (GPD) and $\mathbf{x_B}$**,** $\mathbf{P_T}$**,** $\mathbf{Q^2}$ **, z** (TMD)

Small-x Approximation



Small-x

$$\frac{1}{N_c}\left\langle \mathrm{Tr}\left[U\left(R_\perp\right)U^\dagger\left(R'_\perp\right)\right]\right\rangle_x$$



Hatta-Xiao-Yuan,1601.01585
earlier: Mueller, NPB 1999

TMDs at small-x

- Consistency between the collinear TMD definitions and the CGC calculations have been established
  - Dominguez-Marquet-Xiao-Yuan 2011
- They are the most studied subjects in small-x phenomenology: inclusive, semi-inclusive processes
- DIS dijet can probe the WW gluon distribution
- Talks by Bowen, Matt, Yacine, …

Comments

- We don't lose the sensitivity to the saturation physics even with Large Q
- We gain the direct probe for the transverse momentum dependence of partons
- Beyond the leading order, additional dynamics involved
  - Soft gluon resummation

Among recent developments

- Spin-dependent TMD gluon at small-x
  - Related to the spin-dependent odderon, Boer-Echevarria-Mulders-Zhou, PRL 2016
  - Gluon/quark helicity distributions, Kovchegov-Pitonyak-Sievert, 2016, 2017, 2018, and other spin-dependent distributions
- Subleading power corrections in the TMD gluon/quark distributions
  - Balitsky-Tarasov, 2017, 2018
- Sudakov resummation for small-x TMDs
  - Mueller-Xiao-Yuan, PRL110, 082301 (2013); Xiao-Yuan-Zhou, NPB921, 104 (2017); Zhou 2018
  - Balitsky-Tarasov, JHEP1510,017 (2015)

WW-gluon distribution with TMD resummation

$$xG^{(1)}(x,k_\perp,\zeta_c=\mu_F=Q)$$

→ Hard scale entering TMD Factorization, e.g., Higgs

$$-\frac{2}{\alpha_S}\int\frac{d^2x_\perp d^2y_\perp}{(2\pi)^4}e^{ik_\perp\cdot r_\perp}\mathcal{H}^{WW}(\alpha_s(Q))e^{-S_{sud}(Q^2,r_\perp^2)}$$
$$\times\mathcal{F}^{WW}_{Y=\ln 1/x}(x_\perp,y_\perp)\ ,$$

Small-x evolution

Pert. corrections

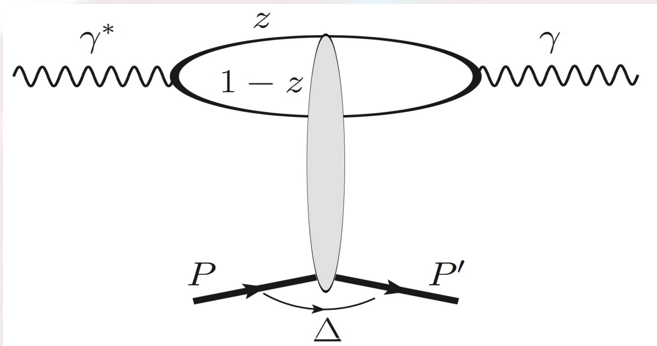
Sudakov resum.

Prediction Power!!

GPDs at small-x

- Can be studied through exclusive processes: deeply virtual Compton scattering (DVCS), deeply virtual meson production, and various diffractive processes
- In the following, take the example of DVCS
  - Heikki's talk

DVCS and GPDs at small-x



$$\frac{1}{P^+}\int\frac{d\zeta^-}{2\pi}e^{ixP^+\zeta^-}\langle p'|F^{+i}(-\zeta/2)F^{+j}(\zeta/2)|p\rangle$$
$$=\frac{\delta^{ij}}{2}xH_g(x,\Delta_\perp)+\frac{xE_{Tg}(x,\Delta_\perp)}{2M^2}\left(\Delta_\perp^i\Delta_\perp^j-\frac{\delta^{ij}\Delta_\perp^2}{2}\right)+$$

Hoodbhoy-Ji 98
Diehl 01

- Two GPDs at the leading small-x approximation

DVCS: Collinear factorization

$$T^{\mu\nu} = i\int d^4z e^{-iq\cdot z}\langle P'|j^\mu(z/2)j^\nu(-z/2)|P\rangle \equiv g_\perp^{\mu\nu}T_0 + h_\perp^{\mu\nu}T_2$$

$$T_0 = -\sum_q e_q^2 \int dx\, \alpha(x) H_q(x,\xi,\Delta_\perp^2)\ ,$$

$$h_\perp^{\mu\nu} = \frac{2\Delta_\perp^\mu \Delta_\perp^\nu}{\Delta_\perp^2} - g_\perp^{\mu\nu}$$

$$T_2 = \sum_q e_q^2 \frac{\alpha_s}{4\pi}\frac{\Delta_\perp^2}{4M^2}\int dx\, \alpha(x) E_{Tg}(x,\xi,\Delta_\perp^2)$$

$$\alpha(x) = \frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi-i\epsilon}$$

- Imaginary part at xi=x

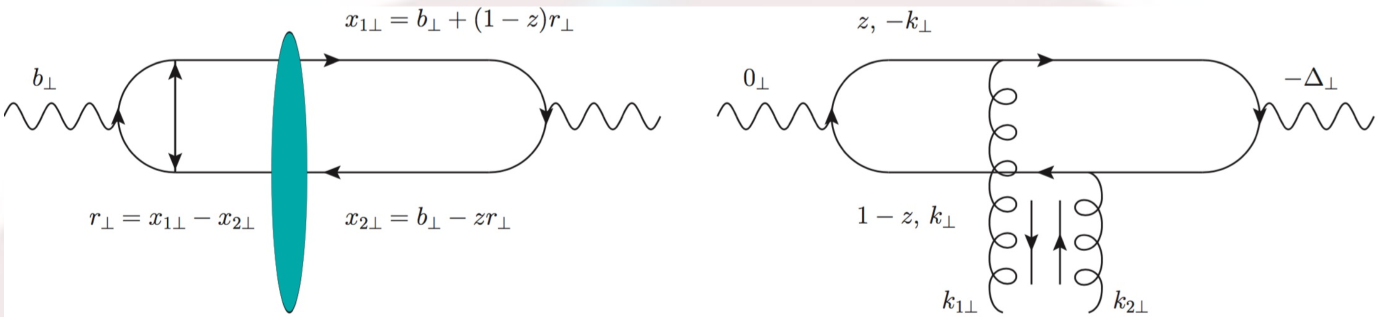
Hoodbhoy-Ji 98

$$\mathrm{Im}\, T_0 = \frac{\pi}{\xi}\sum_q e_q^2\left[\xi H_q(\xi,\xi,\Delta_\perp^2) + \xi H_{\bar q}(\xi,\xi,\Delta_\perp^2)\right]$$

$$\mathrm{Im}\, T_2 = -\frac{\pi}{\xi}\frac{\alpha_s}{2\pi}\frac{\Delta_\perp^2}{4M^2}\sum_q e_q^2 \xi E_{Tg}(\xi,\xi,\Delta_\perp^2)\ ,$$

Vanishes at LO

Dipole formalism



$$F_x(q_\perp, \Delta_\perp) = \int \frac{d^2 r_\perp d^2 b_\perp}{(2\pi)^4} e^{ib_\perp \cdot \Delta_\perp + ir_\perp \cdot q_\perp} S_x\left(b_\perp + \frac{r_\perp}{2}, b_\perp - \frac{r_\perp}{2}\right)$$

- Elliptic gluon distribution (Hatta-Xiao-Yuan 16)

$$F_x(q_\perp, \Delta_\perp) = F_0(|q_\perp|, |\Delta_\perp|) + 2\cos 2(\phi_{q_\perp} - \phi_{\Delta_\perp}) F_\epsilon(|q_\perp|, |\Delta_\perp|)$$

Gluon GPDs and dipole amplitudes

$$xH_g(x,\Delta_\perp) = \frac{2N_c}{\alpha_s}\int d^2q_\perp q_\perp^2 F_0\,,$$

$$xE_{Tg}(x,\Delta_\perp) = \frac{4N_cM^2}{\alpha_s\Delta_\perp^2}\int d^2q_\perp q_\perp^2 F_\epsilon$$

→ Elliptic gluon distribution

Hatta-Xiao-Yuan 1703.02085

Quark/GPD quark at small-x

- DGLAP splitting dominated by gluon distribution/GPD gluon

$$xq(x) = \frac{\alpha_s}{2\pi}\frac{1}{2}\int_x^1 d\zeta\left(\zeta^2+(1-\zeta)^2\right)x'G(x')\int\frac{dk_\perp^2}{k_\perp^2}$$

$$\approx xG(x)\frac{\alpha_s}{2\pi}\frac{1}{2}\cdot\frac{2}{3}\int\frac{dk_\perp^2}{k_\perp^2}$$

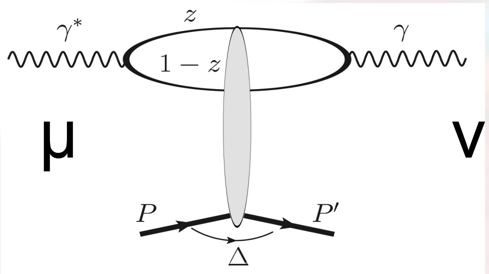
$$xH_q(x,\xi,\Delta_\perp^2) = \frac{\alpha_s}{2\pi}\frac{1}{2}\int_x^1 d\zeta\frac{\zeta^2+(1-\zeta)^2-\frac{\xi^2}{x^2}\zeta^2}{(1-\frac{\xi^2}{x^2}\zeta^2)^2}x'H_g(x',\xi,\Delta_\perp^2)\int\frac{dk_\perp^2}{k_\perp^2}$$

Ji 97, Radyushkin 97

GPD quark distribution

$$\approx \xi H_g(\xi,\xi)\frac{\alpha_s}{2\pi}\frac{1}{2}\cdot 1\int\frac{dk_\perp^2}{k_\perp^2}$$

DVCS: Helicity-conserved Amp.



$$g_\perp^{\mu\nu}\mathcal{A}_0(\Delta_\perp)+h_\perp^{\mu\nu}\mathcal{A}_2(\Delta_\perp)$$

$$\int dz d^2q_\perp d^2k_\perp \frac{(z^2+(1-z)^2)k_\perp\cdot(k_\perp+q_\perp)}{(k_\perp+q_\perp)^2(k_\perp^2+\epsilon_q^2)}F_x(q_\perp,\Delta_\perp)$$

$$\epsilon_q^2=z(1-z)Q^2$$

- Dominant contributions from z~1 or 0,

$$\int\frac{d^2k'_\perp}{(2\pi)^2}\frac{1}{k'^2_\perp}\int d^2q_\perp q_\perp^2 F_x(q_\perp,\Delta_\perp) \Rightarrow \int\frac{d^2k'_\perp}{(2\pi)^2}\frac{1}{k'^2_\perp}xH_g(x)$$

Hatta-Xiao-Yuan 1703.02085

Helicity-flip amplitude

$$\int dz d^2q_\perp d^2q_{1\perp} \frac{z(1-z)\left[2q_{1\perp}\cdot\Delta_\perp k_\perp\cdot\Delta_\perp - q_{1\perp}\cdot k_\perp\Delta_\perp^2\right]}{q_{1\perp}^2(k_\perp^2+\epsilon_q^2)\Delta_\perp^2} F_x(q_\perp,\Delta_\perp)$$

- In the DVCS limit, Q>>Δ, obtain the same result as coll. factorization

$$\mathcal{A}_2 = -\sum_q \frac{e_q^2 N_c}{Q^2}\int d^2q_\perp q_\perp^2 F_\epsilon(q_\perp,\Delta_\perp)$$

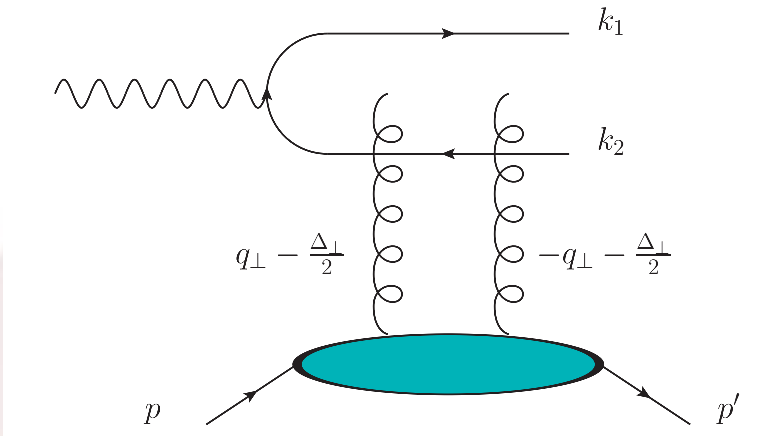
$$= -\frac{e_q^2\alpha_s\Delta_\perp^2}{4Q^2M^2}E_{Tg}(x,\Delta_\perp)$$

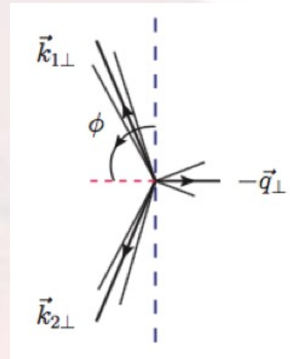
Hatta-Xiao-Yuan 1703.02085

- For DVCS at small-x, again, there exists consistency between the dipole formalism and the collinear GPD framework
  - GPD gluon distribution is directly related to the dipole amplitude, and the GPD quark distribution can be computed
- The cos(2phi) asymmetry in DVCS will provide information on the elliptic gluon distribution at small-x

Directly measure the gluon Wigner distribution?

Hatta-Xiao-Yuan,1601.01585





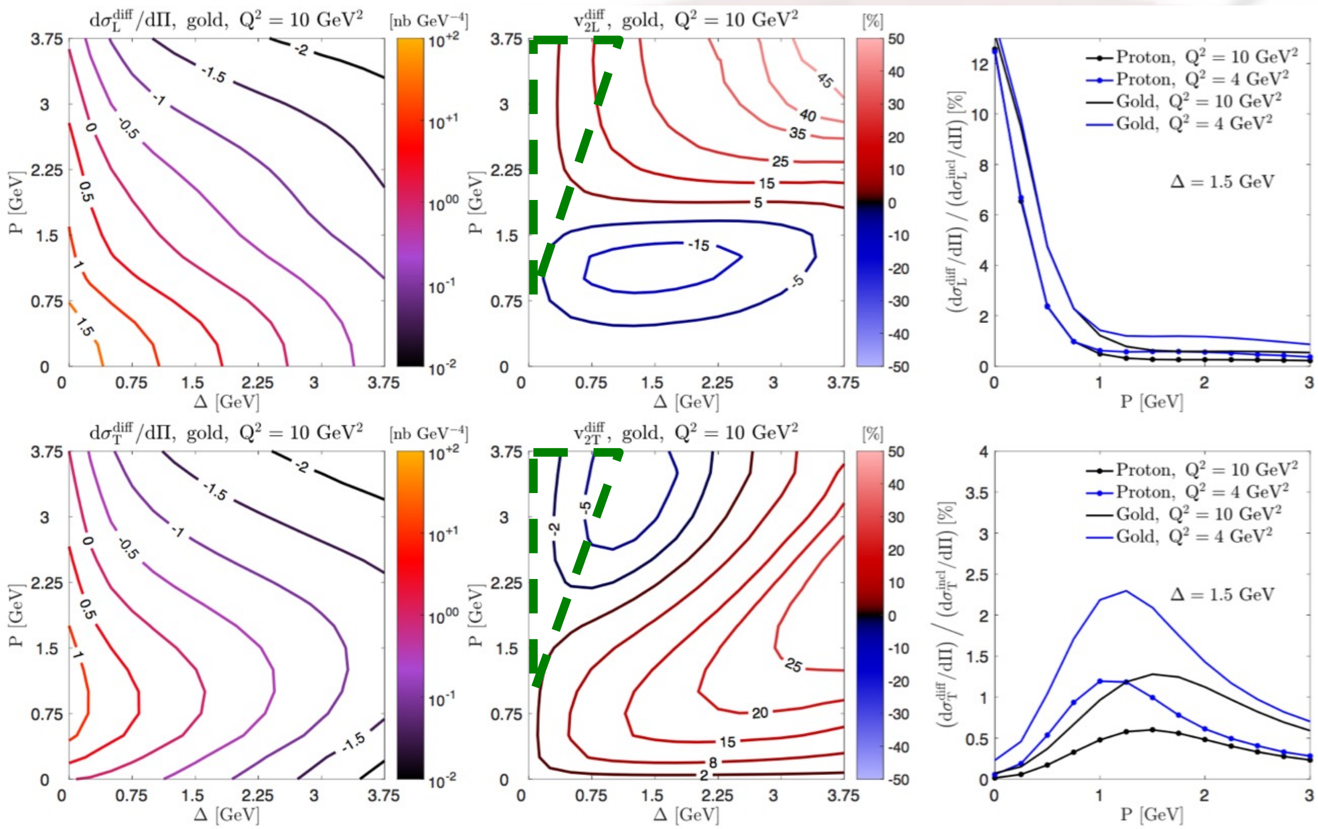
cos(2ϕ) anisotropy

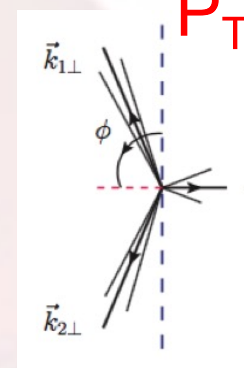
- In the Breit frame, by measuring the recoil of final state proton, one can access Δ_T. By measuring jets momenta, one can approximately access q_T.
- The diffractive dijet cross section is proportional to the square of the Wigner distribution → nucleon/nucleus tomography

$$x\mathcal{W}_g^T(x, |\vec{q}_\perp|, |\vec{b}_\perp|) + 2\cos(2\phi)x\mathcal{W}_g^\epsilon(x, |\vec{q}_\perp|, |\vec{b}_\perp|)$$

Anisotropy ~ few %

This has generated a lot of interests…





CGC calculations: Mäntysaari-Mueller-Salazar-Schenke, 1912.05586, 1902.05087; Mäntysaari-Roy-Salazar-Schenke 2011.02464

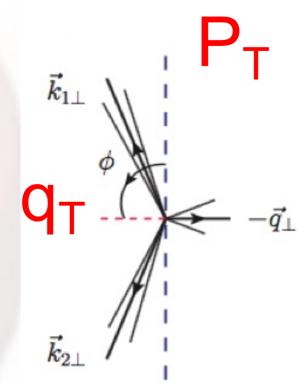
However, life is more complicated, ☹

- Gluon radiation adds additional complications
- In particular, because final state jet carries color, soft gluon contribution will modify the intuitive and simple picture from the leading order analysis
- On the other hand, gluon radiation may offer a unique opportunity to study different perspective of gluon saturation, Iancu-Mueller-Triantafyllopoulos, 2112.06353
  - Edmond's talk

Soft gluon radiations can generate an azimuthal asymmetry

Catani-Grazzini-Sargsyan 2017

- Azimuthal angular asymmetries arise from soft gluon radiations
  - ϕ is defined as angle between total and different transverse momenta of the two final state particles
- Infrared safe but divergent
  - <cos(ϕ)>, <cos(2ϕ)>, … divergent, $\sim 1/q_T^2$
  - Examples discussed include Vj, top quark pair production

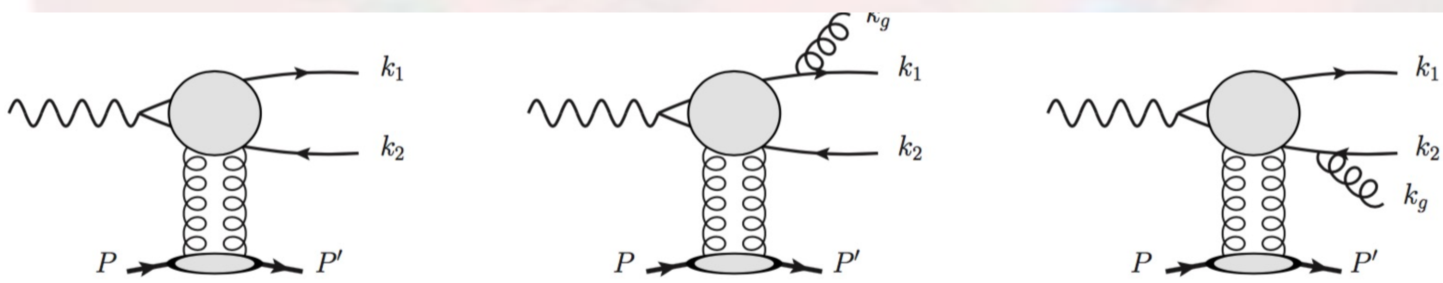


Diffractive dijet production

- Gluon radiation tends to be aligned with the jet direction

$$S_J(q_\perp) = \delta(q_\perp) + \frac{\alpha_s}{2\pi^2}\int dy_g \left(\frac{k_1\cdot k_2}{k_1\cdot k_g k_2\cdot k_g}\right)_{\vec{q}_\perp=-\vec{k}_{g\perp}}$$

$$S_{J0}(|q_\perp|) + 2\cos(2\phi)S_{J2}(|q_\perp|) + \cdots$$



Hatta-Xiao-Yuan-Zhou, 2010.10774, 2106.05307
anisotropy was neglected in an earlier paper:
Hatta-Mueller-Ueda-Yuan, 1907.09491

Leading power contributions, explicit result at α_s

$$S_J(q_\perp) = S_{J0}(|q_\perp|) + 2\cos(2\phi) S_{J2}(|q_\perp|)$$

$$S_{J0}(q_\perp) = \delta(q_\perp) + \frac{\alpha_0}{\pi}\frac{1}{q_\perp^2}\,, \quad S_{J2}(q_\perp) = \frac{\alpha_2}{\pi}\frac{1}{q_\perp^2}\,,$$

where

$$\alpha_0 = \frac{\alpha_s C_F}{2\pi} 2\ln\frac{a_0}{R^2}\,, \quad \alpha_2 = \frac{\alpha_s C_F}{2\pi} 2\ln\frac{a_2}{R^2}\,.$$

a_0, a_2 are order 1 constants, so,

in the small-R limit, <cos(2ϕ)> goes to 1

Additional gluon radiation contributions,

- In the momentum space, it will be a convolution
  - $q_T = k_{g1} + k_{g2} + \ldots$
  - Dominant contributions will be ϕ-independent
- It is convenient to perform resummation in Fourier-b space

$$\widetilde{S}_J(b_\perp) = \int d^2q_\perp e^{iq_\perp \cdot b_\perp} S_J(q_\perp)$$
$$= \widetilde{S}_{J0}(|b_\perp|) - 2\cos(2\phi_b)\widetilde{S}_{J2}(|b_\perp|) + \cdots$$

$$\widetilde{S}_{J0}(b_\perp) = 1 + \alpha_0 \ln(\mu_b^2/P_\perp^2)\ , \quad \widetilde{S}_{J2}(b_\perp) = \alpha_2$$

All order resummation, in Fourier-b space

$$\widetilde{S}_{J0}(b_\perp) = e^{-\Gamma_0(b_\perp)}\ , \quad \widetilde{S}_{J2}(b_\perp) = \alpha_2 e^{-\Gamma_0(b_\perp)} \quad \Gamma_0(b_\perp) = \int_{\mu_b^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2}\alpha_0$$

EIC Kinematics:

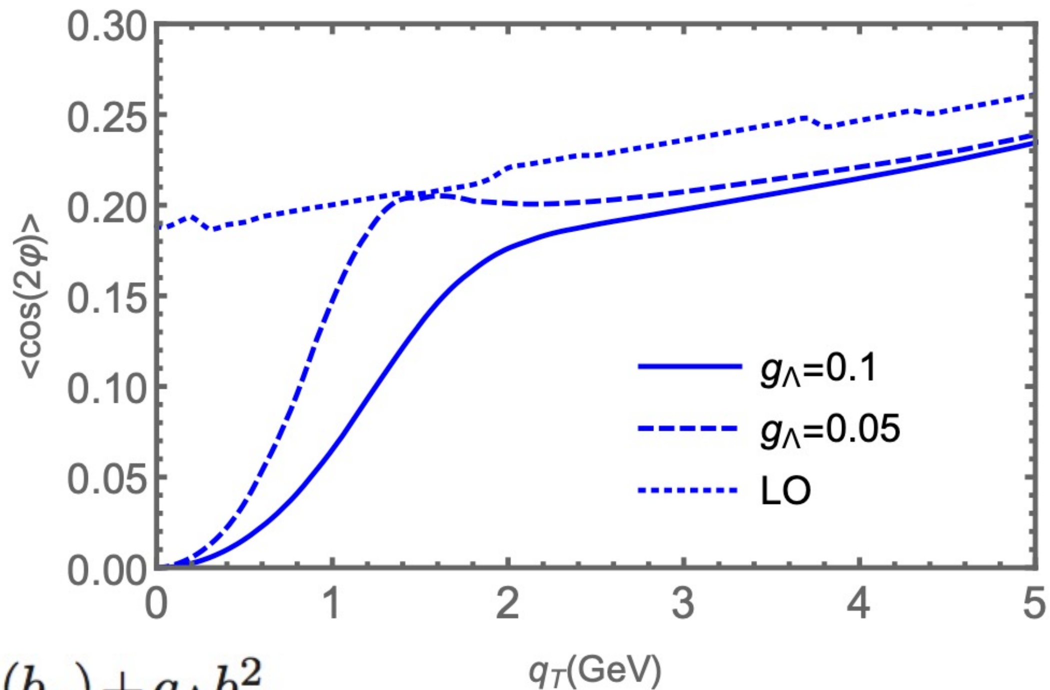
P_T~15GeV

R=0.4

y_1=y_2

Non-pert. input:

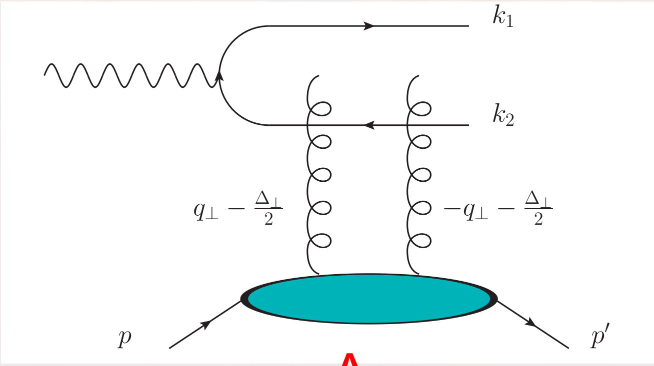
$$\Gamma_0(b_\perp) \Longrightarrow \Gamma_0(b_*) + g_\Lambda b_\perp^2$$



$\Longrightarrow \alpha_2/\alpha_0 \approx 0.14$

Comments

- To avoid the soft gluon radiation contribution, we need to reconstruct nucleon/nucleus recoil momentum to study the tomography



Δ_T

Conclusion

- Small-x physics provides a unique opportunity to explore nucleon tomography through parton Wigner distributions
  - Unified description with dipole amplitude starts to emerge
- Further developments are needed to explore the full potential of the future electron-ion collider
  - More processes to probe the dipole amplitudes, including its spin-dependence
  - Precision toward next-to-leading order computations!