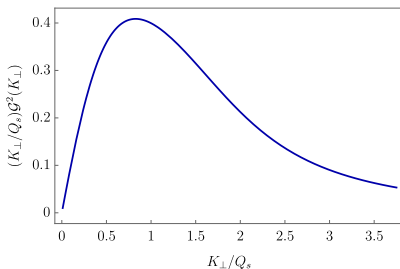
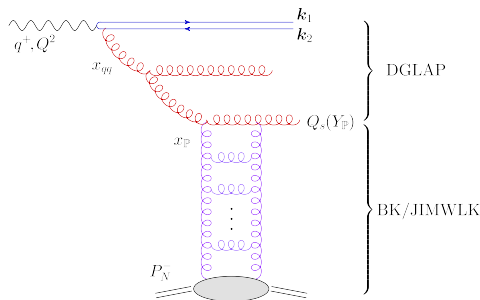


Probing gluon saturation via diffractive jet production at the EIC

Edmond Iancu

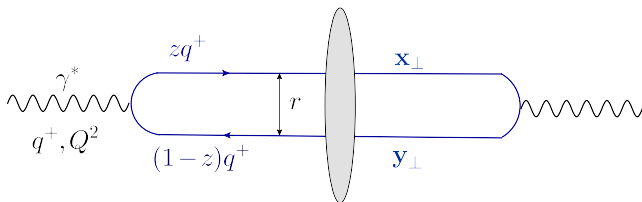
IPhT, Université Paris-Saclay

with A.H. Mueller and D.N. Triantafyllopoulos, arXiv:2112.06353



How to measure gluon saturation at the EIC ?

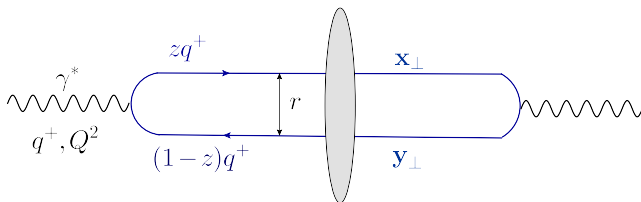
- Total $\gamma^* A$ cross-section at **small** $x_{\text{Bj}} \equiv \frac{Q^2}{2P_N \cdot q}$ and **not too high** Q^2



- Excellent saturation/CGC fits at HERA: $x_{\text{Bj}} \leq 10^{-2}$, $Q^2 \leq 50 \text{ GeV}^2$
 - the pioneers: Golec-Biernat and Wüsthoff, 1999
 - Ducloué, E.I., Soyez and Triantafyllopoulos, 2019
 - Beuf, Hänninen, Lappi, Mäntysaari, 2020 ...
(*BK/JIMWLK, NLO effects, resummations*) [talks by Anna, Tuomas]
- However, gluon saturation only **marginally** probed: $Q_s^2 \sim 1 \text{ GeV}^2$
 - limited region in phase-space, non-perturbative contamination

How to measure gluon saturation at the EIC ?

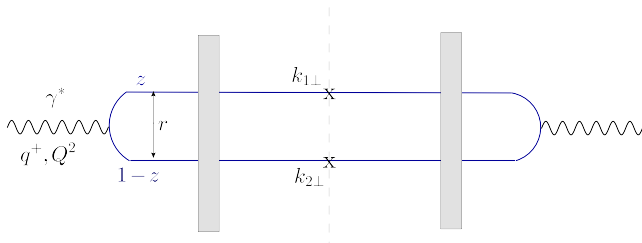
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(*BK/JIMWLK, NLO effects, resummations*) [talks by Anna, Tuomas]
- Can one measure saturation in **hard DIS** ? ($Q^2 \gg Q_s^2$)
- Consider more exclusive processes: **correlations in particle production**

Inclusive dijets in the correlation limit

- The $q\bar{q}$ fluctuation is put on shell by **inelastic scattering** off the nucleus

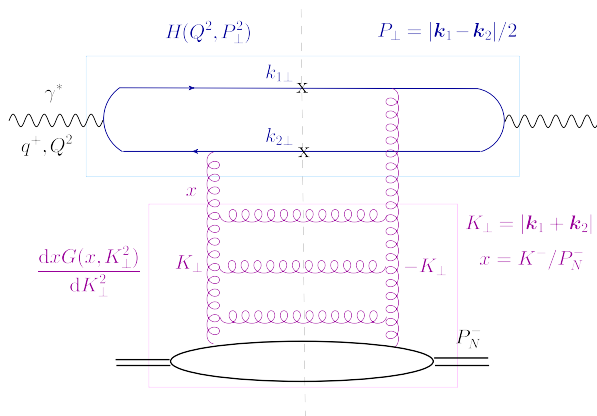


- Correlation limit** : 2 hard jets which are nearly back-to-back

$$k_{1\perp} \simeq k_{2\perp} \sim Q \gg K_{\perp} \equiv |\mathbf{k}_{1\perp} + \mathbf{k}_{2\perp}| \sim Q_s$$

- scattering is weak, by color transparency: small $q\bar{q}$ dipole: $r \sim 1/k_{i\perp}$
- momentum imbalance fixed by scattering off the saturated gluons
- Measure saturation from azimuthal correlations ! (*Marquet, 2007*)
- TMD factorisation**: hard impact factor \times unintegrated gluon distribution (*Dominguez, Marquet, Xiao and Yuan, 2011*) [talk by Feng]

TMD factorisation for inclusive dijets

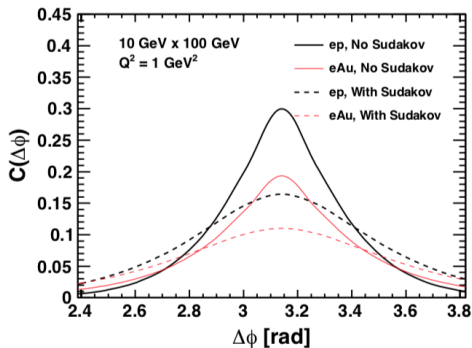


- For $Q^2 \gg Q_s^2$, one typically has $Q_s^2 \ll K_\perp^2 \ll Q^2$: **saturation still marginal**
 - the UGD decreases only like $1/K_\perp^2$ when $K_\perp \gg Q_s$
- **Sudakov effect** $\sim \alpha_s \ln^2(Q^2/Q_s^2)$ (Mueller, Xiao and Yuan, 2013)
 - strong additional broadening due to final-state radiation

Azimuthal correlations in dijets

(Zheng, Aschenauer, Lee, and Xiao, arXiv:1403.2413)

- Large competing effect due to **final-state radiation** ($Q \gg Q_s$): “Sudakov”



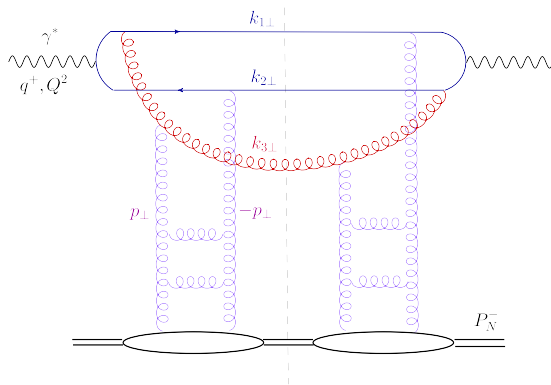
Metz, Zhou, 1105.1991; Dumitru, Lappi, Skokov, 1508.04438

Marquet, Petreska, Roiesnel, 1608.02577;

Dumitru, Skokov, Ullrich, 1809.02615; Mäntysaari et al, arXiv:1912.05586 ...

2+1 jets in hard DIS diffraction

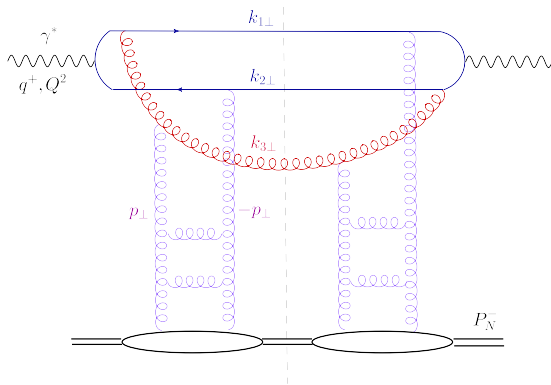
- 2 hard quark-antiquark jets + 1 softer gluon jet: $k_{1\perp}, k_{2\perp} \sim Q \gg k_{3\perp}$



- Coherent diffraction:** elastic scattering, the target is not broken
 - colorless exchange (2-gluon ladder): “Pomeron”
 - rapidity gap $Y_{\mathbb{P}}$ between the 3 jets and the nucleus
 - the momentum transfer is negligible: $|k_1 + k_2 + k_3| \sim \Lambda$

2+1 jets in hard DIS diffraction

- Hard dijet **imbalance** controlled by the gluon jet: $K_{\perp} \equiv |\mathbf{k}_1 + \mathbf{k}_2| \simeq k_{3\perp}$

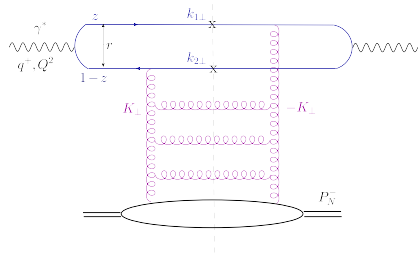
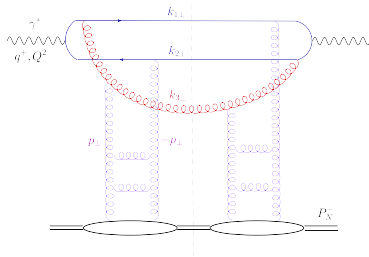


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Why is this interesting ?

- Elastic scattering is **more sensitive to gluon saturation** [talk by Heikki]

$$\sigma_{el} \propto |\mathcal{A}_{el}|^2 \longleftrightarrow \sigma_{tot} \propto 2\text{Im} \mathcal{A}_{el}$$

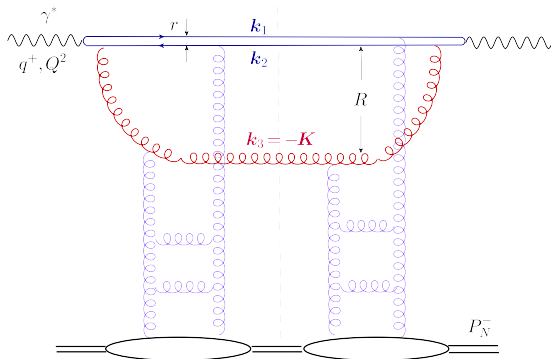


- At weak scattering $|\mathcal{A}_{el}| \ll 1$, elastic scattering is strongly suppressed
- Diffraction is controlled by strong scattering $|\mathcal{A}_{el}| \sim 1$: **Black Disk Limit**
- 2+1 jets**: elastic scattering becomes strong when $k_{3\perp} \sim Q_s(Y_{\mathbb{P}})$
 - the gluon jet redistributes colour over a large transverse size $\sim 1/k_{3\perp}$

The gluon dipole

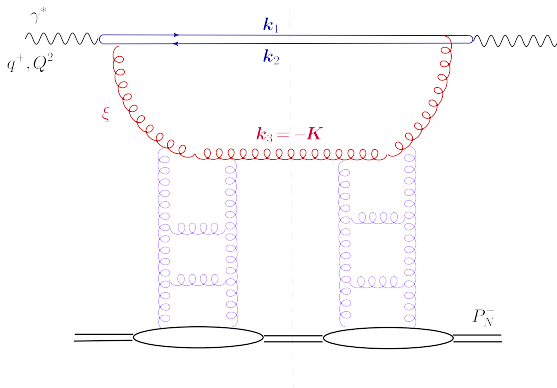
- The typical “hard dijet” events truly are 2+1 jets with $k_{3\perp} \sim Q_s(Y_{\mathbb{P}})$
 - dominate over exclusive dijets when $Q^2 \gg Q_s^2$
- A simpler, effective, picture for the colour flow (*Wüsthoff, 97*)

$R \sim 1/k_{3\perp} \sim 1/Q_s \gg r \sim 1/P_{\perp} \sim 1/Q \Rightarrow$ effective gg dipole



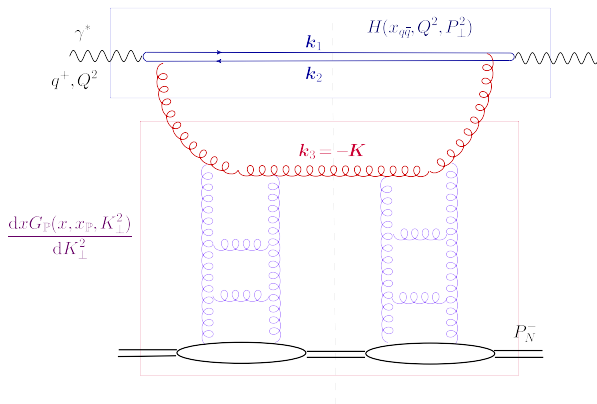
- 2+1 diffractive jets: an opportunity to measure the gluon dipole

TMD-like factorisation for diffractive dijets



- Gluon formation time $\tau_3 = \frac{2k_3^+}{k_{3\perp}^2} \lesssim$ photon coherence time $\tau_q = \frac{2q^+}{Q^2}$
- Soft gluon: $\xi \equiv \frac{k_3^+}{q^+} \lesssim \frac{k_{3\perp}^2}{Q^2} \ll 1 \implies$ eikonal approx \implies factorisation

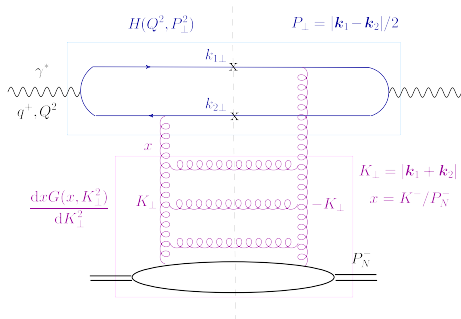
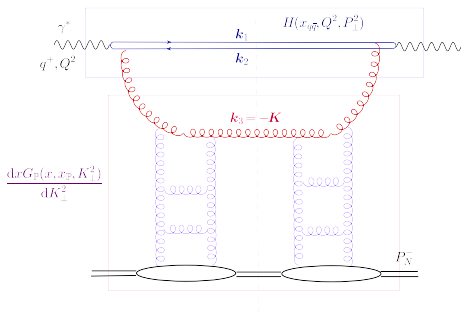
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- Trijet cross-section = Hard factor \times UGD of the Pomeron

The hard impact factor

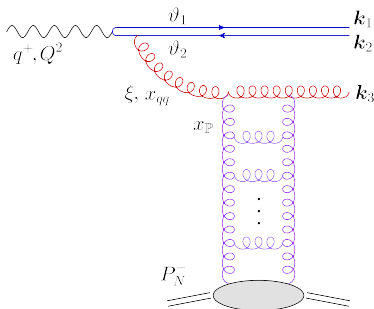
- Exactly the same for diffractive and inclusive dijets in the correlation limit



- the decay of the virtual photon $\gamma^* \rightarrow q\bar{q}$
- the scattering between a small $q\bar{q}$ pair and a gluon from the target
- What is different though is the actual **source** of this gluon
 - Pomeron (evolved to $x_{\mathbb{P}}$) vs. Hadronic target (evolved to $x_{q\bar{q}}$)

Rapidity scales

- $x_{\mathbb{P}}$: fraction of target longitudinal momentum P_N^- carried by the Pomeron
- $x_{q\bar{q}} \sim x_{B_j}$: the corresponding quantity for the exchanged gluon
 - the exchanged gluon has $k^+ = \xi q^+$ and $k^- = x_{q\bar{q}} P_N^-$
- $x \equiv x_{q\bar{q}}/x_{\mathbb{P}}$: splitting fraction of the exchanged gluon w.r.t. the Pomeron



$$\left\{ \Delta Y = \ln \frac{x_{\text{P}}}{x_{\text{qg}}} \right.$$

$$\left\{ Y_{\mathbb{P}} = \ln \frac{1}{x_{\mathbb{P}}} \right.$$

$$x \equiv \frac{x_{q\bar{q}}}{x_{\mathbb{P}}} = \frac{Q^2 + M_{q\bar{q}}^2}{Q^2 + M_{q\bar{q}q}^2}$$

$$M_{q\bar{q}}^2 = \frac{P_{\perp}^2}{\vartheta_1 \vartheta_2} \sim Q^2$$

$$M_{q\bar{q}g}^2 = M_{q\bar{q}}^2 + \frac{k_{3\perp}^2}{\xi}$$

- The eikonal approximation applies when $M_{q\bar{q}q}^2 \gg M_{q\bar{q}}^2$, or $x \ll 1$

The Pomeron unintegrated gluon distribution (1)

$$\frac{dx G_{\mathbb{P}}(x, x_{\mathbb{P}}, K_{\perp}^2)}{d^2\mathbf{K} d^2\mathbf{b}} = \frac{N_c^2 - 1}{8\pi^4} [\mathcal{G}(K_{\perp}, Y_{\mathbb{P}})]^2$$

- $\mathcal{G}(K_{\perp}, Y_{\mathbb{P}})$: a Fourier transform of the gluon dipole scattering amplitude

$$\mathcal{G}(K_{\perp}, Y_{\mathbb{P}}) = 2 \int_0^{\infty} \frac{dR}{R} J_2(K_{\perp} R) \mathcal{T}_g(R, Y_{\mathbb{P}})$$

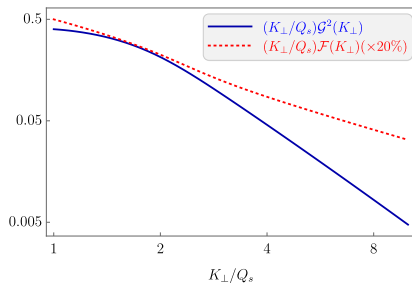
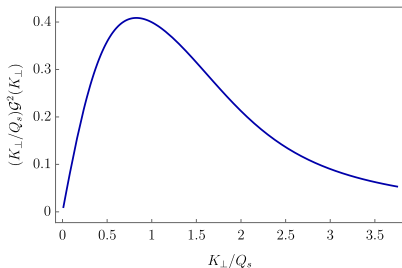
- integral controlled by dipole sizes $R \sim 1/K_{\perp}$

$$\mathcal{T}_g(R) \simeq \begin{cases} r^2 Q_s^2, & R \ll 1/Q_s \\ 1, & R \gg 1/Q_s \end{cases} \quad \Rightarrow \quad \mathcal{G}(K_{\perp}) \simeq \begin{cases} Q_s^2/K_{\perp}^2, & K_{\perp} \gg Q_s \\ 1, & K_{\perp} \ll Q_s \end{cases}$$

- saturation scale is harder for a gluon dipole: $Q_s^2(gg) = (9/4)Q_s^2(q\bar{q})$
- Similar to the Weizsäcker-Williams gluon TMD from inclusive dijets ...
except that the diffractive cross-section involves the square of $\mathcal{G}(K_{\perp})$

The Pomeron unintegrated gluon distribution (2)

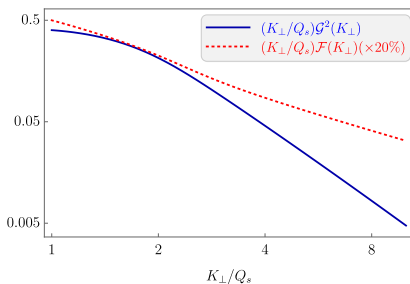
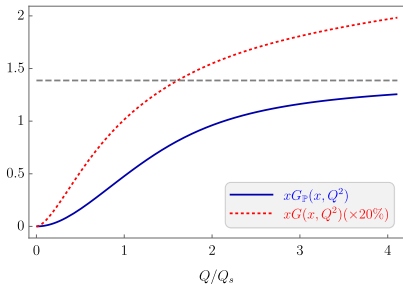
- At large $K_{\perp} \gg Q_s$, $\mathcal{G}^2(K_{\perp}) \propto 1/K_{\perp}^4$ decreases very fast.
- the bulk of the diffractive cross-section lies at momenta $K_{\perp} \lesssim Q_s(Y_{\mathbb{P}})$



- Diffractive trijets are more sensitive to saturation than inclusive dijets
 - no need to explicitly measure (or even see) the third, softer, jet
 - study the distribution of the hard dijets in their momentum imbalance K_{\perp} , for various values of the rapidity gap $\Rightarrow Q_s(Y_{\mathbb{P}})$

The Pomeron unintegrated gluon distribution (2)

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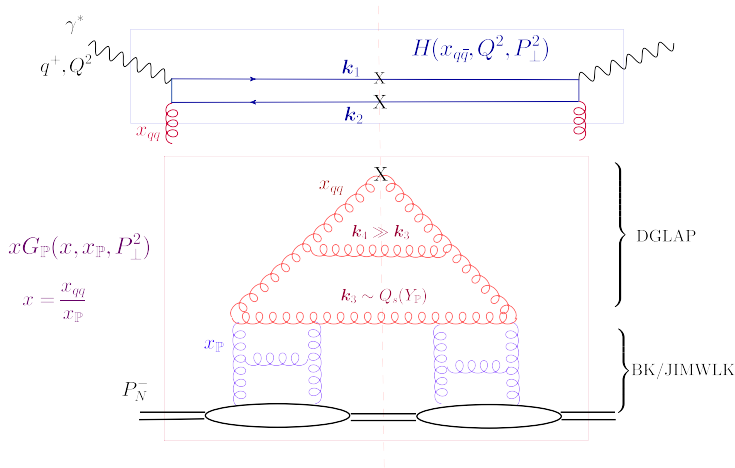


- This sensitivity persists after integrating out the K_\perp -distribution.
- the integrated gluon distribution of the Pomeron $xG_\mathbb{P}(x, x_\mathbb{P}, Q^2)$

$$xG_\mathbb{P}(x, x_\mathbb{P}, Q^2) \propto Q_s^2(Y_\mathbb{P}) \quad \text{vs.} \quad xG(x, Q^2) \propto Q_0^2 \ln \frac{Q^2}{Q_0^2}$$

Collinear factorisation for diffractive dijets

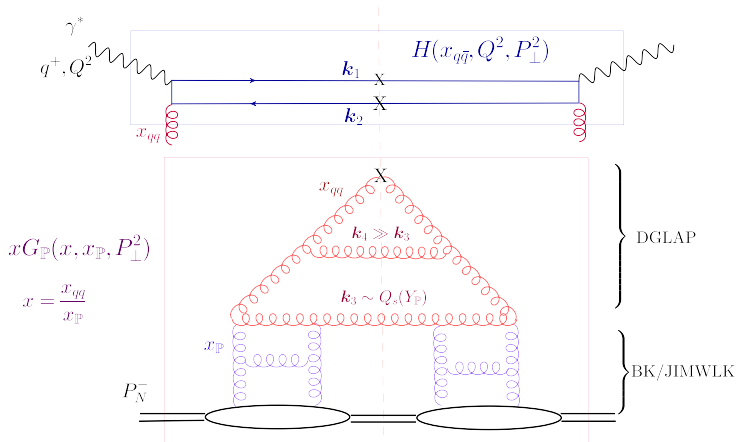
- Measure the dijet distribution **integrated over the imbalance K_\perp**



$$\frac{d\sigma_D^{\gamma^* A \rightarrow q\bar{q} A' X}}{dy_1 dy_2 d^2 P dY_P} = H(x_{q\bar{q}}, Q^2, P_\perp^2) xG_P(x, x_P, P_\perp^2)$$

Collinear factorisation for diffractive dijets

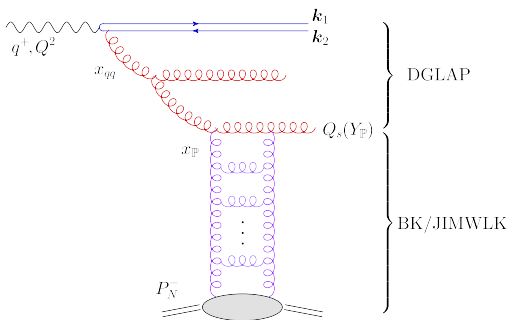
- Measure the dijet distribution **integrated over the imbalance K_\perp**



- eliminates the Sudakov effect $\sim \alpha_s \ln^2 \frac{P_\perp^2}{K_\perp^2}$ (final state radiation)
- Sudakov double logs replaced by the **DGLAP** single logs $\sim \alpha_s \ln \frac{P_\perp^2}{K_\perp^2}$

Collinear factorisation for diffractive dijets

- Gluon saturation provides the **initial condition** for the DGLAP evolution



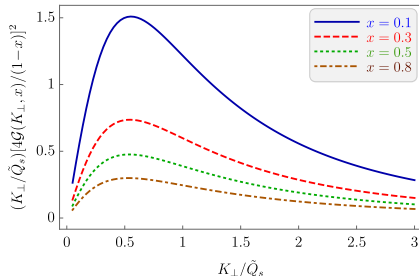
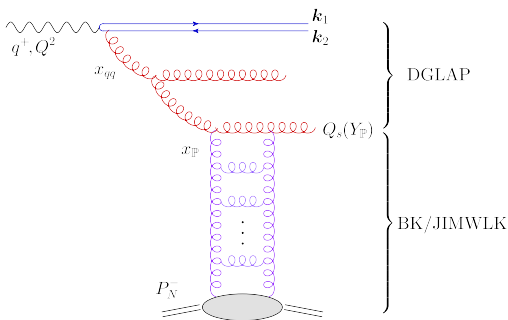
- $xG_{\mathbb{P}}(x, x_{\mathbb{P}}, Q^2)$ for $Q^2 \sim Q_s^2(Y_{\mathbb{P}})$ and for any $x \leq 1$
- $x \equiv x_{q\bar{q}}/x_{\mathbb{P}}$: splitting fraction w.r.t. the Pomeron
- $x \sim 1$ is beyond eikonal approx.
- exploit black disk limit

$$xG_{\mathbb{P}}(x, x_{\mathbb{P}}, Q^2) = S_{\perp}(1-x)^2 \frac{N_c^2 - 1}{(2\pi)^3} \kappa Q_s^2(Y_{\mathbb{P}})$$

- A unique situation where DGLAP and BK/JIMWLK can be interconnected
- Diffraction matches the two evolutions at a **fixed transverse scale** $Q_s^2(Y_{\mathbb{P}})$

Collinear factorisation for diffractive dijets

- Gluon saturation provides the **initial condition** for the DGLAP evolution



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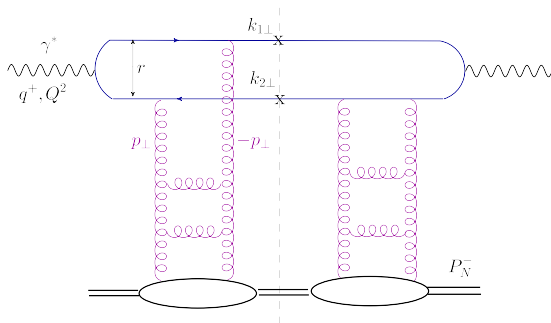
Conclusions

- A new, **hard**, process to study gluon saturation via **final-state correlations**
 - yet another avatar of “dijet production in the correlation limit”
- Especially sensitive to saturation since relying on **elastic scattering**
- Additional, comparatively soft, **radiation** plays an essential role:
 - use the 3rd jet to “open up the colour space” (*Bowen, via email*)
 - large-size (R), effective, **gluon-gluon dipole**
 - use elastic scattering as a filter to select $R \sim 1/Q_s$
- The actual measurement of the **gg dipole** would be a bonus !
- Saturation remains important if the dijet correlation is **not measured**
 - initial condition for DGLAP evolution emerging from first principles
- Similar processes might be interesting in **ultraperipheral pA and AA**

THANK YOU & STAY HEALTHY !

Back-up: Exclusive hard dijet production

- Elastic scattering off a small quark-antiquark dipole: $r \sim 1/P_\perp \ll 1/Q_s(Y)$
 - scattering is weak by colour transparency: $\mathcal{T}(r, Y) \simeq r^2 Q_s^2(Y) \ll 1$
 - cross-section: $\mathcal{T}^2(r, Y) \propto r^4$: strongly suppressed at high P_\perp^2



$$\sigma_{\text{el}} \propto \frac{1}{P_\perp^6} \left[\frac{\alpha_s x G(x, P_\perp^2)}{S_\perp} \right]^2 \longleftrightarrow \sigma_{\text{diff}} \propto \frac{\alpha_s}{P_\perp^4} Q_s^2(Y_{\mathbb{P}})$$