Entropy and multiplicity distributions in DIS

CGC perspective

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Outline

- CGC hadron wavefunction
- Density matrix for small x gluons and relation to observables
- Entanglement entropy of small x gluons and its properties

Disclaimer

- In this talk, CGC as a *model* of hadron wavefunction. Computable! Genuine non-perturbative effects are missing.

- Results are for small x components; thus most appropriate for mid-rapidity observables in DIS, UPC, p-p/A.

- Goal is to explore if concepts and methods of quantum information theory could give new insights to/simpler computational approach to high energy collisions.

Hadron wavefunction at high energy

- Large fraction of the longitudinal momentum is carried by the valence d.o.f.
- Radiated gluons have significantly lower longitudinal momentum and relatively shorter lifetime
- This leads to natural separation:

$$|\psi\rangle = |s\rangle \otimes |v\rangle$$

$|v\rangle =$ the state vector characterizing the valence d.o.f.;
$|s\rangle =$ the vacuum of the soft fields

Valence degrees of freedom

- For a large nucleus or proton at high energy $|v\rangle$ can be approximated by McLerran-Venugopalan model

$$\langle\rho|v\rangle\langle v|\rho\rangle = \mathcal{N}\, e^{-\int_{\underline{k}} \frac{1}{2\mu^2}\rho_a(\underline{k})\rho_a^*(\underline{k})}$$

ρ is the color charge density of the valence d.o.f.

- The phase of $\langle\rho|v\rangle$ is not known; not required for the purpose of this talk
- Possible to go beyond this approximation;

  e.g. use dipole wavefunction or LC model of proton

Adrian Dumitru's talk

Soft fields

- QCD Hamiltonian can be diagonalized at leading perturbative order
- Soft gluon vacuum is a coherent state

$$|s\rangle = \mathcal{C}|0\rangle; \quad \mathcal{C} = \exp\left\{2i\mathrm{tr}\int_{\underline{k}} b^i(\underline{k})\phi_i(\underline{k})\right\}; \quad \phi_i(\underline{k}) \equiv a_i^+(\underline{k}) + a_i(-\underline{k})$$

where b^i is Weizsäcker-Williams field – solution of static Yang-Mills equation

$$\partial_i b^i = g\rho; \qquad b_i = \frac{1}{ig} V\partial_i V^+$$

At leading order in color charge density:

$$b_a^i(\underline{k}) = g\rho_a(\underline{k})\frac{i\underline{k}_i}{k^2} + \mathcal{O}(\rho^2)$$

- Can be systematically improved: $|s\rangle = \mathcal{CB}|0\rangle$, $\mathcal{B} \propto \exp(\phi B^{-1}\phi)$; reproduces JIMWLK

A. Kovner, M. Lublinsky, et al, 2007-2009

(Reduced) density matrix

- Hadron density matrix:

$$\hat{\rho} = |v\rangle \otimes |s\rangle\langle s| \otimes \langle v|$$

with the property $\hat{\rho}^{n>1} = \hat{\rho}$. That is the density matrix is pure.

- It is natural to consider soft sector and integrate out valence degrees of freedom. Phenomenological motivation: measurements at mid rapidity reflect the properties of soft gluons.
- Reduced density matrix for small-x d.o.f.:

$$\hat{\rho}_r = \mathrm{Tr}_\rho \hat{\rho} \equiv \int D\rho \, \langle \rho | \hat{\rho} | \rho \rangle = \int D\rho \, \langle \rho | v \rangle \, |s\rangle\langle s| \, \langle v | \rho \rangle$$

(Reduced) density matrix

- This set-up is not the same as in the talk by Dima Kharzeev.
  Here: the entanglement between small-x and valence degrees of freedom.
- Still might be a meaningful proxy for parton model?!
  Common element is the natural bi-partitioning of the d.o.f. in the underlying wave funct.
- Expected properties

  Due to translational invariance: diagonality in momentum space
  Due to reality of b_i: coupling between positive and negative modes w/ same $|k|$
  Due to integration of d.o.f.: mixed state

Reduced density matrix

Calculation is well-defined; rather technical for a short talk

For simplicity, consider the dilute approximation for WW field first

- Calculations are easier in coherent field basis
  To mimic parton model we used number basis representation
- The density matrix elements

$$\langle n_c(\underline{q}), m_c(-\underline{q})|\hat{\rho}_r(\underline{q})|\alpha_c(\underline{q}), \beta_c(-\underline{q})\rangle = (1-R)\frac{(n+\beta)!}{\sqrt{n!m!\alpha!\beta!}}\left(\frac{R}{2}\right)^{n+\beta}\delta_{(n-m),(\alpha-\beta)}$$

with

$$R = \left(1 + \frac{1}{2}\frac{q^2}{g^2\mu^2}\right)^{-1} \qquad Q_s^2 = \alpha_s N_c g^2 \mu^2$$

Haowu Duan et al, 2001.01726

What is it good for?

Simplest observables:

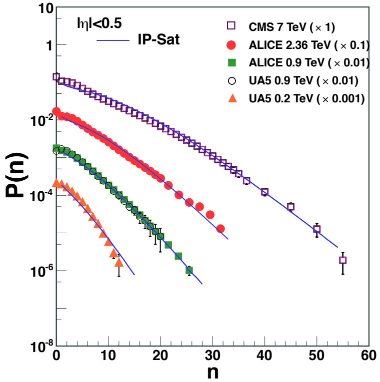
- particle number

$$\langle a^+(k)a(k)\rangle = \mathrm{Tr}[a^+(k)a(k)\hat{\rho}_r]$$

- and fluctuations

$$\langle (a^+(k)a(k))^n\rangle$$

Glittering glasma (0905.3234) is due to nontrivial $\hat{\rho}_r$; negative binomial distribution of gluons is encoded in the reduced density matrix.



P. Tribedy and R. Venugopalan, 1112.2445

These observables only probe diagonal components; off-diagonal components of $\hat{\rho}_r$ are irrelevant for the purpose of describing this limmitted set of data.

Density matrix of "ignorance"

- Ignoring/Setting off-diagonals components to zero $\rightsquigarrow$ well-defined density matrix: positive-definite and normalized
- Identical particle number fluctuations as obtained in full reduced density matrix
  at least for the initial state; wait for more details
- Associated loss of information can be characterized by entropy: entr. of ignorance > 0
  $S(\hat{\rho}_r) \leq S(\hat{\rho}_i)$
- By construction, this entropy is basis-dependent
  For phenomenology of high-energy collisions, preferred basis = number of particles
- Is $S(\hat{\rho}_r) = S(\hat{\rho}_i)$ in CGC?

Haowu Duan et. al., 2001.01726

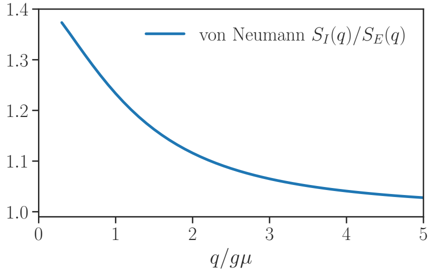
Entropy of ignorance

- $S(\hat{\rho}_r) = S(\hat{\rho}_i)$ only if there are no off-diagonal elements in reduced density matrix

$$\langle n_c(\underline{q}), m_c(-\underline{q}) | \hat{\rho}_r(\underline{q}) | \alpha_c(\underline{q}), \beta_c(-\underline{q}) \rangle = (1-R) \frac{(n+\beta)!}{\sqrt{n!m!\alpha!\beta!}} \left(\frac{R}{2}\right)^{n+\beta} \delta_{(n-m),(\alpha-\beta)}$$

- For illustration $\langle 2n | \hat{\rho}_r | 0 \rangle$:

$$\langle n_c(\underline{q}), n_c(-\underline{q}) | \hat{\rho}_r(\underline{q}) | 0, 0 \rangle = (1-R) \left(\frac{R}{2}\right)^{n} \neq 0$$

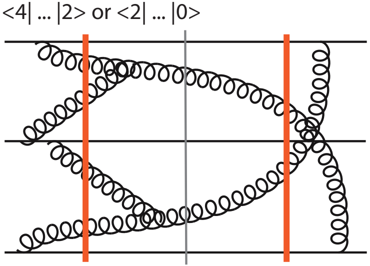


The difference between entropies $S(\hat{\rho}_r) = S(\hat{\rho}_i)$ is due to off-diagonal elements.
Do off-diagonal elements contribute to observables?

Haowu Duan et al, 2001.01726

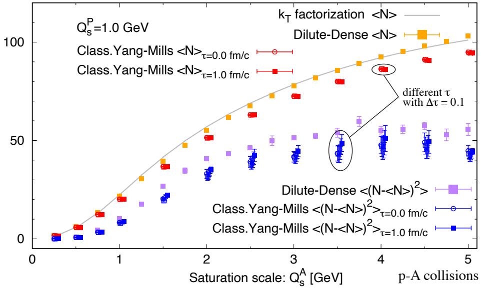
Off-diagonal elements in observables

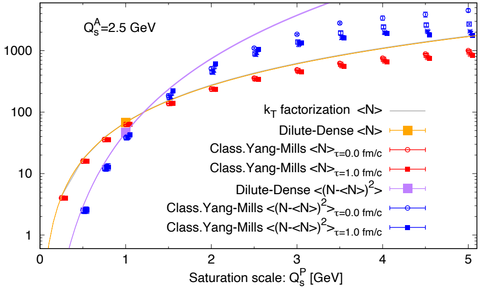
- Single transverse spin asymmetry
  J.-W. Qiu and G. Sterman, Phys.Rev.Lett. 67 (1991) 2264
- Odd azimuthal anisotropy of two gluon production in CGC includes contributions from off-diagonal elements in density matrix of dilute projectile
  Do similar higher-order density corrections important for final state gluon fluctuations?



Yu. Kovchegov, V. S., 1802.08166

Off-diagonal elements in observables: mid-rapidity p-A





S. Schlichting, V. S., 1910.12496

In dilute-dense collisions, final state interactions do not change gluon number fluctuations

Thus off-diagonal components of CGC reduced density do not play a significant role in defining gluon number fluctuations/gluon entropy.

Entanglement entropy

- Entanglement entropy = Von Neumann entropy of reduced matrix $S_E = -\mathrm{Tr}\,\hat{\rho}_r \ln \hat{\rho}_r$

$$S_E = \frac{1}{2}(N_c^2-1)S_\perp \int \frac{d^2q}{(2\pi)^2}\left[\ln\left(\frac{g^2\mu^2}{q^2}\right) + \sqrt{1+4\frac{g^2\mu^2}{q^2}}\,\ln\left(1+\frac{q^2}{2g^2\mu^2}+\frac{q^2}{2g^2\mu^2}\sqrt{1+4\frac{g^2\mu^2}{q^2}}\right)\right]$$

A. Kovner, M. Lublinsky, 1506.05394

- Including saturation effects ($b_i \propto V\partial_i V^\dagger$)

$$S^E = \frac{N_c^2-1}{2}\sum_{\nu=\pm}\int_{\underline{k}}\left[\ln \tilde{M}_\nu(\underline{k}) + \sqrt{1+4\tilde{M}_\nu(\underline{k})}\,\ln\left(1+\frac{1}{2\tilde{M}_\nu(\underline{k})}+\frac{\sqrt{1+4\tilde{M}_\nu(\underline{k})}}{2\tilde{M}_\nu(\underline{k})}\right)\right]$$

Two polarizations with eigenvalues defined by the WW gluon distribution functions

$$\tilde{M}_\pm = \frac{(2\pi)^3}{2S_\perp(N_c^2-1)}\,\frac{xG_g^{(1)} \pm xh_g^{(1)}}{2}$$

H. Duan, A. Kovner, V.S., 2111.06475

Entanglement entropy: Boltzmann form

- Amusingly entanglement entropy can be written in Boltzmann form

$$S_E = (N_c^2 - 1) S_\perp \sum_{i=\pm} \int \frac{d^2 q}{(2\pi)^2} \left[(1 + f_i) \ln(1 + f_i) - f_i \ln f_i \right], f_\pm = \frac{1}{\exp(\beta\omega_\pm) - 1}$$

with

$$\beta\omega_\pm = 2 \ln \left(\frac{1}{2\sqrt{M_\pm(k)}} + \sqrt{1 + \frac{1}{4M_\pm(k)}} \right)$$

- This suggests that reduced density matrix can be diagonalized to $\langle N | \hat{\rho}_r | M \rangle = \mathrm{diag}(\lambda^0, \lambda^1, \lambda^2, ...)$

For general Gaussian density matrices,
J. Berges, S. Floerchinger, and R. Venugopalan, 1712.09362

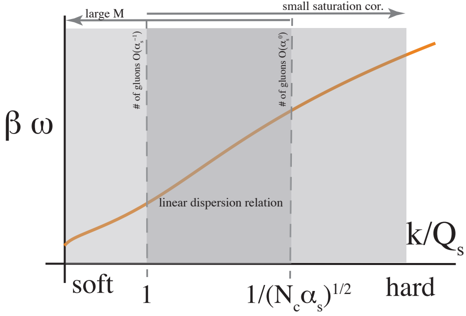
$$\beta\omega_{\pm} = 2\ln\left(\frac{1}{2\sqrt{M_{\pm}(k)}} + \sqrt{1 + \frac{1}{4M_{\pm}(k)}}\right)$$

Large eigenvalues $M \gg 1$ (# of gluons $\sim \mathcal{O}(\alpha_s^0)$)

$$\beta\omega \approx \frac{1}{\sqrt{M(k)}}$$

Small eigenvalues

$$\beta\omega \approx -\ln M(k)$$



In the dilute limit $k \gg Q_s$, $M(k) = \frac{1}{N_c\alpha_s}\frac{Q_s^2}{k^2}$ and $M(k) \gg 1$ for $Q_s < k < Q_s/\sqrt{N_c\alpha_s}$.
In this regime, $\beta\omega \approx \frac{k}{T_{\text{eff}}}$ with $T_{\text{eff}} = \frac{Q_s}{\sqrt{N_c\alpha_s}}$.

Soft momenta ($k < Q_s$), including saturation effects, $\beta\omega \approx 1/\sqrt{\ln(Q_s/k)}$.

Hard momenta ($k > Q_s/\sqrt{N_c\alpha_s}$), $\beta\omega \approx \ln(k/Q_s)$.

Quasi-particle dispersion relation II

- Quasiparticles have Boltzmann-like density matrix

- In semihard momentum range, $Q_s < k < Q_s/\sqrt{N_c\alpha_s}$,
their spectrum corresponds to massless particles
in a heat-bath of temperature $T_{\text{eff}} = \frac{Q_s}{\sqrt{N_c\alpha_s}}$

see also, CGC-Black Hole correspondence by Gia Dvali and Raju Venugopalan, 2106.11989

- Explicit unitary transformation to this quasiparticle basis was constructed for creation-annihilation operators

$$c_\pm(k) = \cosh(B_\pm)\, a_\pm(k) + \sinh(B_\pm)\, a^\dagger_\pm(-k)$$

with $B_\pm = \ln 2\sqrt{\alpha_\pm} = \frac{1}{4}\ln\left(1 + 4\tilde{M}_\pm\right)$. For large $\tilde{M}_\pm$, $c_\pm(k) = a_\pm(k) + a^\dagger_\pm(-k)$.

- Are there any phenomenological implications?

Outlook

To make this discussion phenomenologically relevant, need to consider

- Evolution

  Linblad evolution by Nestro Armesto et al, 1901.08080

- Quantitative effects of scattering
- Fragmentation
- Soft emissions and decoherence

  G. Semenoff et al "unobserved soft photons decohere nearly all outgoing momentum superpositions of charged particles" (see e.g. 1706.03782)

  Associated entropy? For entropy if a jet: Duff Neil and Wouter Waalewijn, 1811.01021

Conclusions

- CGC density matrix has of diagonal components in the number basis representation

- Diagonal components appear to be fully responsible for gluon number distributions; this conclusion is not affected by finite state interaction in glasma

- Off-diagonal components contribute to some observables, including v_3.
  Origin of v_3 in UPC?


BACKUP

Example: two fermion model I

- Two fermions, A and B in **pure** state

$$|\phi_{AB}\rangle = \frac{\sqrt{2}}{2}|0_A\rangle \otimes |0_B\rangle + \frac{1}{2}|1_A\rangle \otimes (|0_B\rangle + |1_B\rangle)$$

- Reduced density matrix for subsystem A and B are

$$\rho_A = \frac{1}{2}\begin{pmatrix} 1 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 1 \end{pmatrix} \quad \rho_B = \frac{1}{4}\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$$

Entanglement entropies for A and its complement are identical

$$S_E(\rho_A) = S_E(\rho_B) = \frac{3}{2}\ln 2 + \frac{1}{\sqrt{2}}\operatorname{acoth}\sqrt{2} \approx 0.416496$$

Example: two fermion model II

- Ignorance entropy depends on set of defining operators $\{O_i\}$
- First: $\{O_i\}$ as all operators diagonal in particle number basis. To calculate S_I: discard off-diagonal matrix elements in number basis $\rho_{AB} = \mathrm{diag}\,\{1/2,\, 1/4,\, 0,\, 1/4\}$

$$S_I(\rho_{AB}) = -\sum_i p_i \ln p_i = \frac{3}{2} \ln 2 \approx 1.03972$$

- Entropy of ignorance for reduced density matrix ρ_A: measurable quantities are operators diagonal in Fock space of fermion A. Drop off-diagonal matrix elements of ρ_A: $\rho_A^I = \mathrm{diag}\,\{1/2,\, 1/2\}$

$$S_I(\rho_A) = \ln 2 \approx 0.693147$$

- Similarly, $\rho_B^I = \mathrm{diag}\,\{3/4,\, 1/4\}$, and corresponding entropy of ignorance is

$$S_I(\rho_B) = 2 \ln 2 - \frac{3}{4} \ln 3 \approx 0.56233$$

Example: two fermion model III

- Entanglement: $S_E(\rho_A) = S_E(\rho_B)$
- Ignorance: $S_I(\rho_A) \neq S_I(\rho_B)$.
- Entanglement: $S_E(\rho_{AB}) = 0$
- Ignorance: $S_I(\rho_{AB}) \neq 0$