

Probing gluon number density with electron-dijet correlations at EIC



NCN



The Henryk Niewodniczański
Institute of Nuclear Physics
Polish Academy of Sciences

Krzysztof Kutak



Based on:

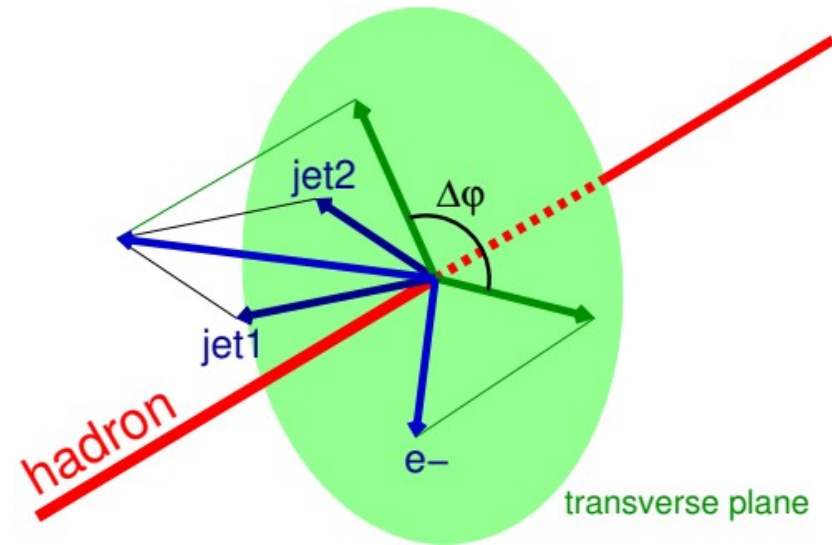
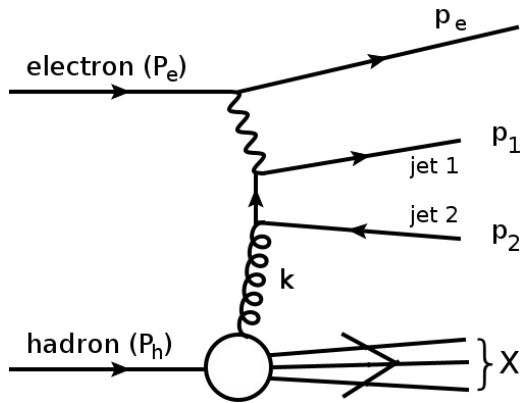
EPJC 81, 741 (2021)

P. Kotko, S. Sapeta, A. van Hameren, E. Zarow

arxiv 2110.06156

Hentschinski, Kutak

The di-jet in DIS



$$k^\mu = xP_h^\mu + k_T^\mu$$

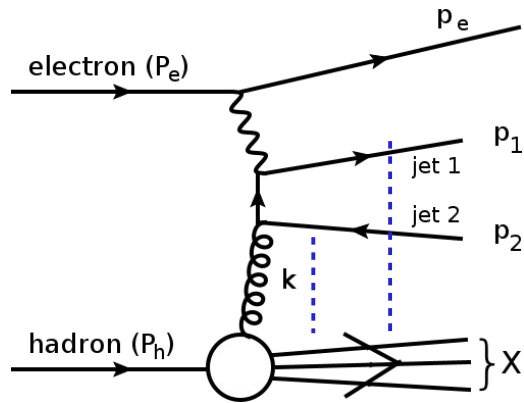
iTMD = small x Improved Transverse Momentum Dependent factorization

- counts for saturation
- correct gauge structure i.e. uses gauge links to define TMD's
- takes into account kinematical effects – the whole phase space is available at LO
- is implemented in MC event generator KaTie by van Hameren , LxJet by Kotko
- valid in region $p_T > Q_s$, k_T can be any. p_T is hard final state momentum, k_T is imbalance

P. Kotko K. Kutak , C. Marquet , E. Petreska , S. Sapeta, A. van Hameren,
JHEP 1509 (2015) 106

A. van Hameren, P. Kotko, K. Kutak, C. Marquet, E. Petreska
JHEP 12 (2016) 034

The di-jet in DIS



The same gauge link structure and as in TMD's

F. Dominguez, B. Xiao, F. Yuan

Phys.Rev.Lett. 106 (2011) 022301

F. Dominguez, C. Marquet, B. Xiao, F. Yuan

Phys.Rev. D83 (2011) 105005

$$k^\mu = xP_h^\mu + k_T^\mu$$

iTMD = small x Improved Transverse Momentum Dependent factorization

- counts for saturation
- correct gauge structure i.e. uses gauge links to define TMD's
- takes into account kinematical effects – the whole phase space is available at LO
- is implemented in MC event generator KaTie by van Hameren , LxJet by Kotko
- valid in region $p_T > Q_s$, k_T can be any. p_T is hard final state momentum, k_T is imbalance

P. Kotko, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren,
JHEP 1509 (2015) 106

A. van Hameren, P. Kotko, K. Kutak, C. Marquet, E. Petreska
JHEP 12 (2016) 034

T. Altinoluk, R. Boussarie, P. Kotko JHEP 1905 (2019) 156

Derivation within CGC T. Altinoluk, R. Boussarie, JHEP10(2019)208

Role of transverse gauge links

Boussarie, Mehtar-Tani Phys. Rev. D 103, 094012 (2021)

Set of basic TMD's for 2, 3 and 4 jets (unpolarized)

Bury, Kotko, Kutak '18

$$\mathcal{F}_{gg}^{(1)}(x, k_T) = \text{F.T.} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(2)}(x, k_T) = \text{F.T.} \left\langle \frac{\text{Tr} [\mathcal{U}^{[\square]}]}{N_c} \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(3)}(x, k_T) = \text{F.T.} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[\square]} \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(1)}(x, k_T) = \text{F.T.} \left\langle \frac{\text{Tr} [\mathcal{U}^{[\square]\dagger}]}{N_c} \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(2)}(x, k_T) = \text{F.T.} \frac{1}{N_c} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[\square]\dagger} \right] \text{Tr} \left[\hat{F}^{i+}(0) \mathcal{U}^{[\square]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(3)}(x, k_T) = \text{F.T.} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(4)}(x, k_T) = \text{F.T.} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[-]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(5)}(x, k_T) = \text{F.T.} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[\square]} \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(6)}(x, k_T) = \text{F.T.} \left\langle \frac{\text{Tr} [\mathcal{U}^{[\square]}]}{N_c} \frac{\text{Tr} [\mathcal{U}^{[\square]\dagger}]}{N_c} \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(7)}(x, k_T) = \text{F.T.} \left\langle \frac{\text{Tr} [\mathcal{U}^{[\square]}]}{N_c} \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

Set of basic TMD's for 2, 3 and 4 jets

Bury, Kotko, Kutak '18

$$\mathcal{F}_{qg}^{(1)}(x, k_T) = \text{F.T.} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

dipole gluon

$$\mathcal{F}_{qg}^{(2)}(x, k_T) = \text{F.T.} \left\langle \frac{\text{Tr} [\mathcal{U}^{[\square]}]}{N_c} \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{qg}^{(3)}(x, k_T) = \text{F.T.} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[\square]} \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(1)}(x, k_T) = \text{F.T.} \left\langle \frac{\text{Tr} [\mathcal{U}^{[\square]\dagger}]}{N_c} \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(2)}(x, k_T) = \text{F.T.} \frac{1}{N_c} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[\square]\dagger} \right] \text{Tr} \left[\hat{F}^{i+}(0) \mathcal{U}^{[\square]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(3)}(x, k_T) = \text{F.T.} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

WW gluon

$$\mathcal{F}_{gg}^{(4)}(x, k_T) = \text{F.T.} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[-]} \right] \right\rangle$$

same direction pointing Wilson lines

$$\mathcal{F}_{gg}^{(5)}(x, k_T) = \text{F.T.} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[\square]} \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(6)}(x, k_T) = \text{F.T.} \left\langle \frac{\text{Tr} [\mathcal{U}^{[\square]}]}{N_c} \frac{\text{Tr} [\mathcal{U}^{[\square]\dagger}]}{N_c} \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(7)}(x, k_T) = \text{F.T.} \left\langle \frac{\text{Tr} [\mathcal{U}^{[\square]}]}{N_c} \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

Set of basic TMD's for 2, 3 and 4 jets

$$\mathcal{F}_{qg}^{(1)}(x, k_T) = \text{F.T.} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{qg}^{(2)}(x, k_T) = \text{F.T.} \left\langle \frac{\text{Tr} [\mathcal{U}^{[\square]}]}{N_c} \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{qg}^{(3)}(x, k_T) = \text{F.T.} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[\square]} \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(1)}(x, k_T) = \text{F.T.} \left\langle \frac{\text{Tr} [\mathcal{U}^{[\square]\dagger}]}{N_c} \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(2)}(x, k_T) = \text{F.T.} \frac{1}{N_c} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[\square]\dagger} \right] \text{Tr} \left[\hat{F}^{i+}(0) \mathcal{U}^{[\square]} \right] \right\rangle$$

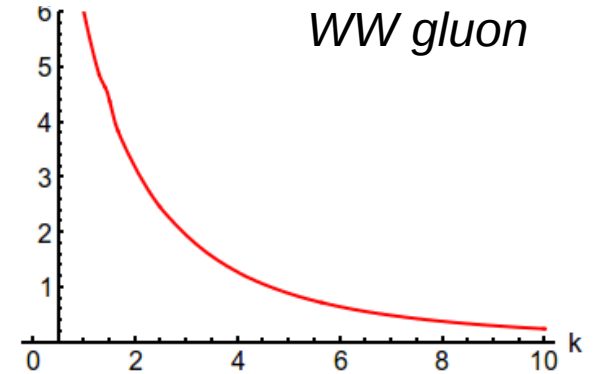
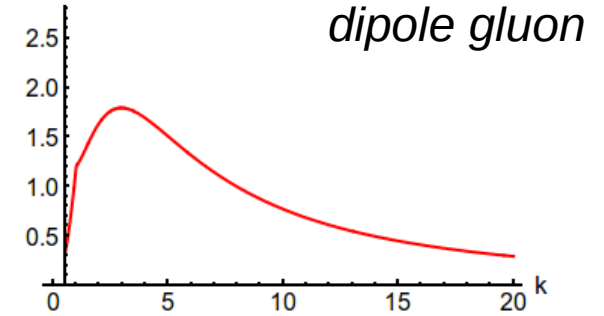
$$\mathcal{F}_{gg}^{(3)}(x, k_T) = \text{F.T.} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(4)}(x, k_T) = \text{F.T.} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[-]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[-]} \right] \right\rangle$$

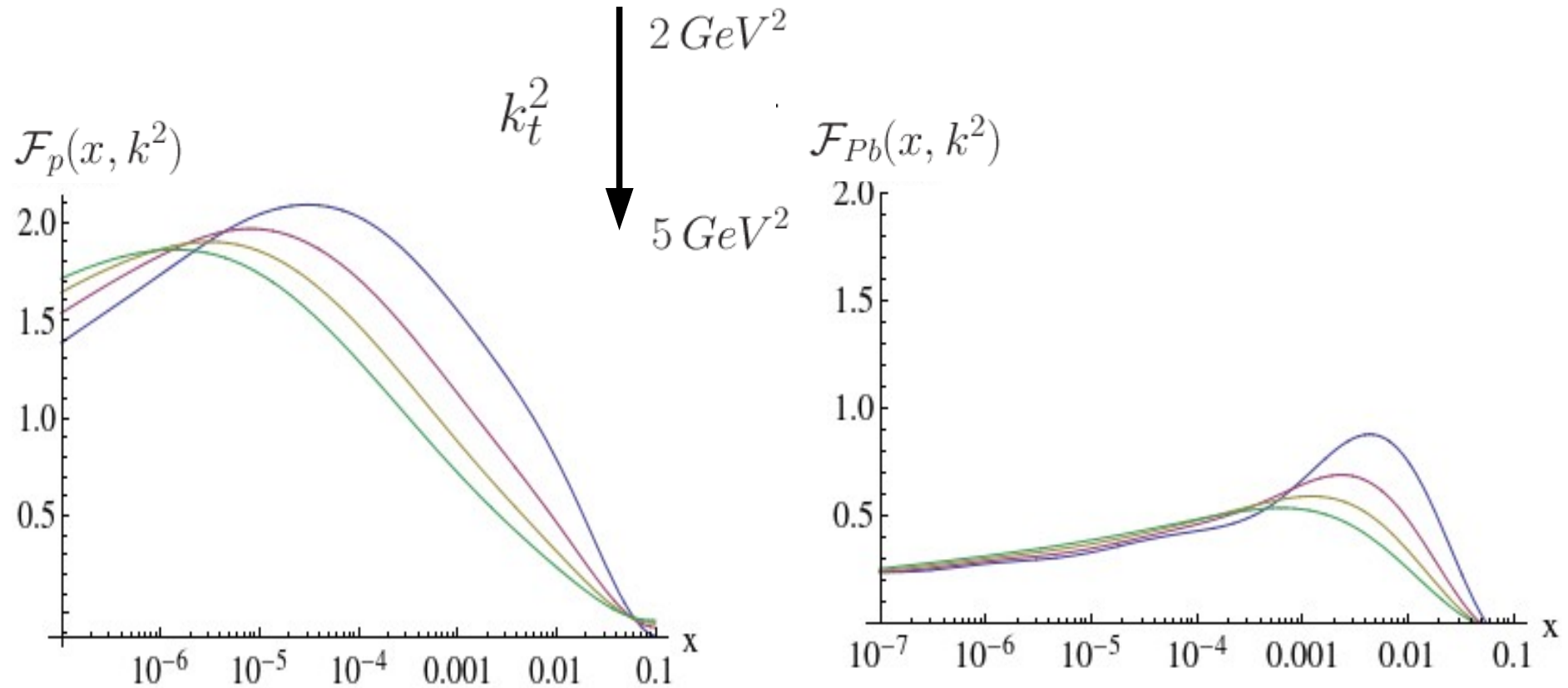
$$\mathcal{F}_{gg}^{(5)}(x, k_T) = \text{F.T.} \left\langle \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[\square]} \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(6)}(x, k_T) = \text{F.T.} \left\langle \frac{\text{Tr} [\mathcal{U}^{[\square]}]}{N_c} \frac{\text{Tr} [\mathcal{U}^{[\square]\dagger}]}{N_c} \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{gg}^{(7)}(x, k_T) = \text{F.T.} \left\langle \frac{\text{Tr} [\mathcal{U}^{[\square]}]}{N_c} \text{Tr} \left[\hat{F}^{i+}(\xi) \mathcal{U}^{[\square]\dagger} \mathcal{U}^{[+]\dagger} \hat{F}^{i+}(0) \mathcal{U}^{[+]} \right] \right\rangle$$



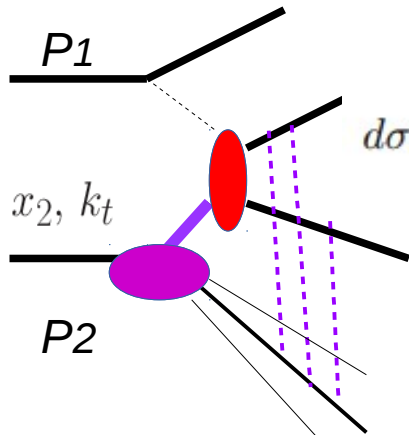
Glue from BK in p vs. glue in Pb – dipole gluon



Maximum signalize emergence of saturation scale

ITMD with Sudakov for dijets in DIS

P. Kotko, K.K. S. Sapeta, A. van Hameren, E. Zarow, EPJC



$$d\sigma_{eh \rightarrow e' + 2j + X} = \int \frac{dx}{x} \frac{d^2 k_T}{\pi} \mathcal{F}_{gg}^{(3)}(x, k_T, \mu) \frac{1}{4x P_e \cdot P_h} d\Phi(P_e, k; p_e, p_1, p_2) |\overline{M}_{eg^* \rightarrow e' + 2j}|^2$$

This process allows to probe the Weizsacker-Williams TMD

The Weizsacker-Williams TMD with Sudakov resummation to account for soft emissions

$$S_{\text{Sud}}^{g \rightarrow q\bar{q}}(\mu, b_T) = \frac{\alpha_s N_c}{4\pi} \ln^2 \frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}$$

A. Mueller, B-W. Xiao, F. Yuan, 2013

Related studies for dijet/dihadron at EIC

Back-to-back regime using MV model + Sudakov
L. Zheng, E.C. Aschenauer, J.H. Lee, B-W. Xiao, 2014

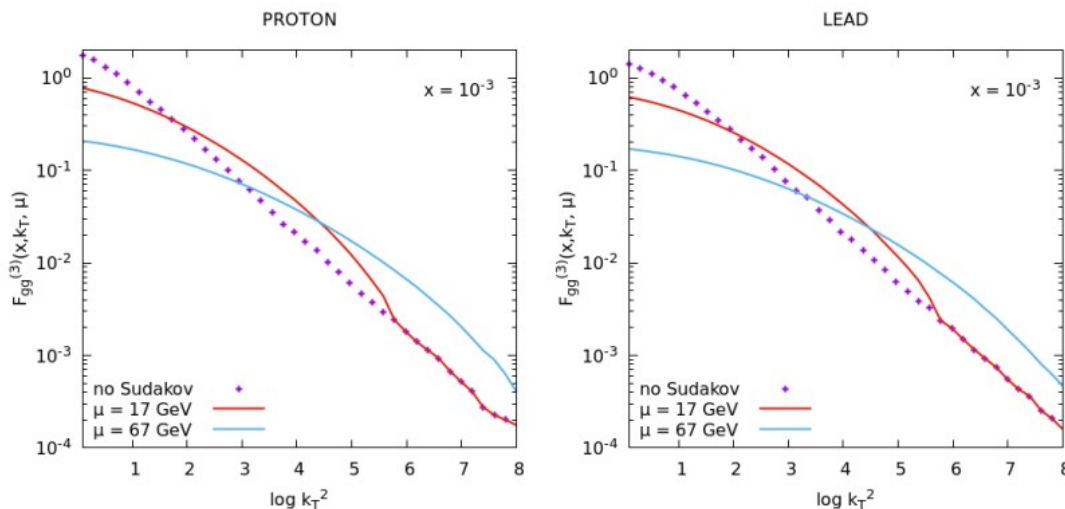
Full CGC calculations (no Sudakov)
A. Dumitru, V. Skokov, 2018

H. Mantysaari, N. Mueller, F. Salazar, B. Schenke, 2019
F. Salazar, B. Schenke, 2020

R. Boussarie, H. Mäntysaari, F. Salazar, B. Schenke
JHEP 09 (2021) 178

Y. Zhao et al. arXiv:2105.08818

Marquet, Taels, Altinoluk, JHEP 06 (2021) 085



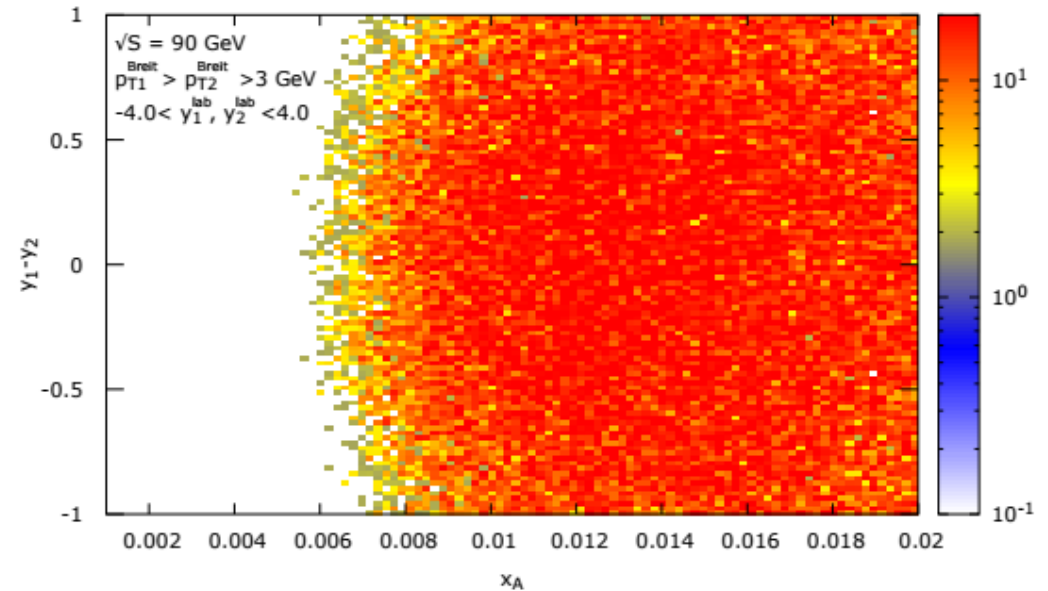
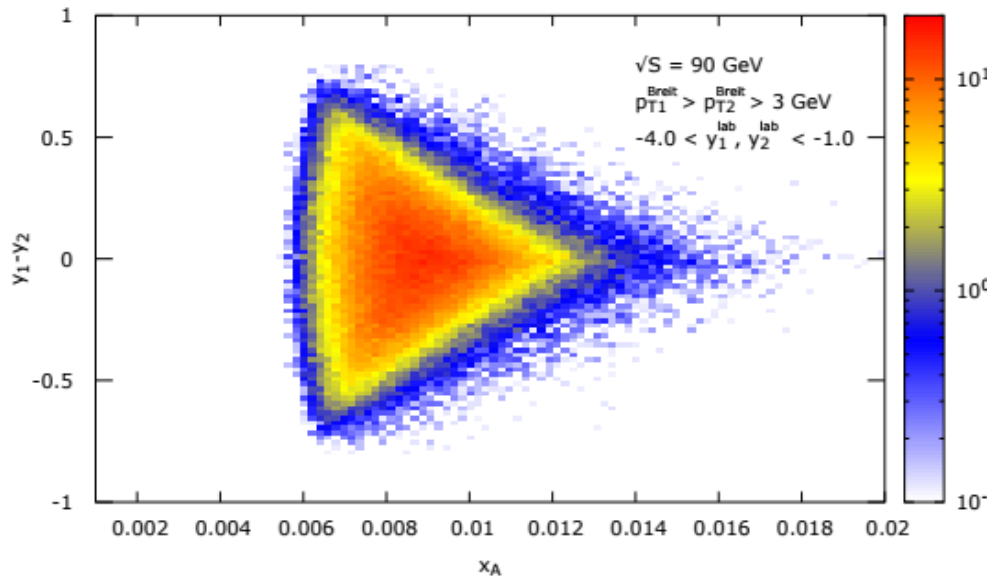
We do not take into account linearly polarised gluons

Low x region

P. Kotko, K.K. S. Sapeta, A. van Hameren, E. Zarow, '20 EPJC

For clarity:

$$k = P x_A + k_T \quad x_{Bj} = Q^2 / (P \cdot q)$$



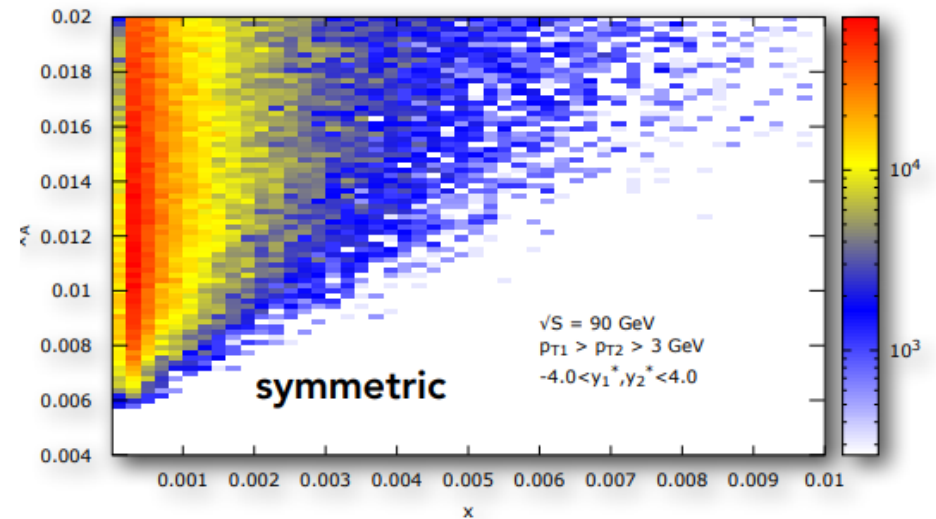
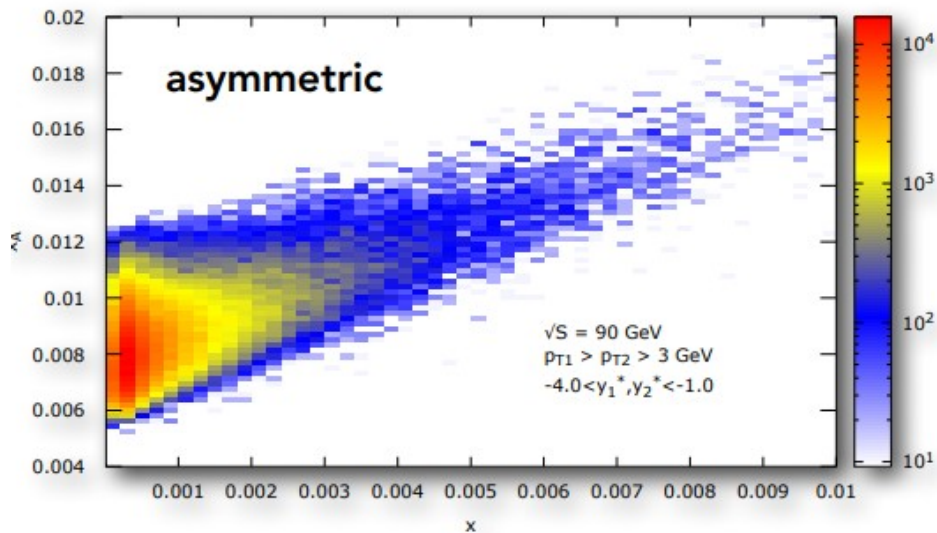
The asymmetric rapidity cuts guarantee good focusing of the cross section around smaller values of x required by the formalism.

x_{Bj} VS x_A

P. Kotko, KK. S. Sapeta, A. van Hameren, E. Zarow, '20 EPJC

For clarity:

$$k = P x_A + k_T \quad x_{Bj} = Q^2 / (P \cdot q)$$

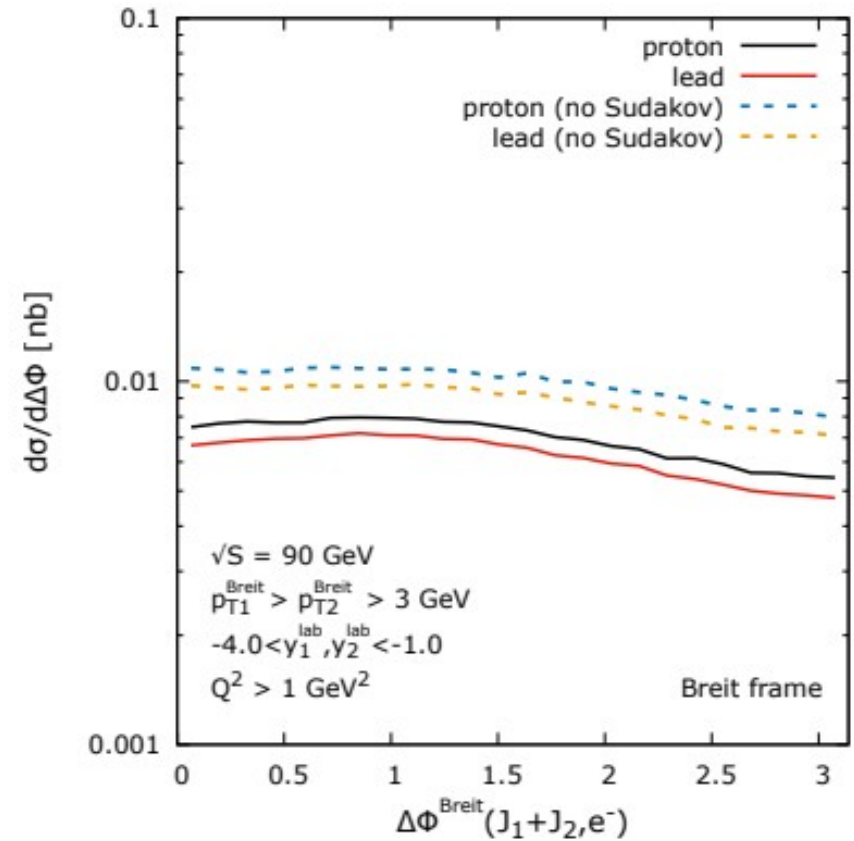
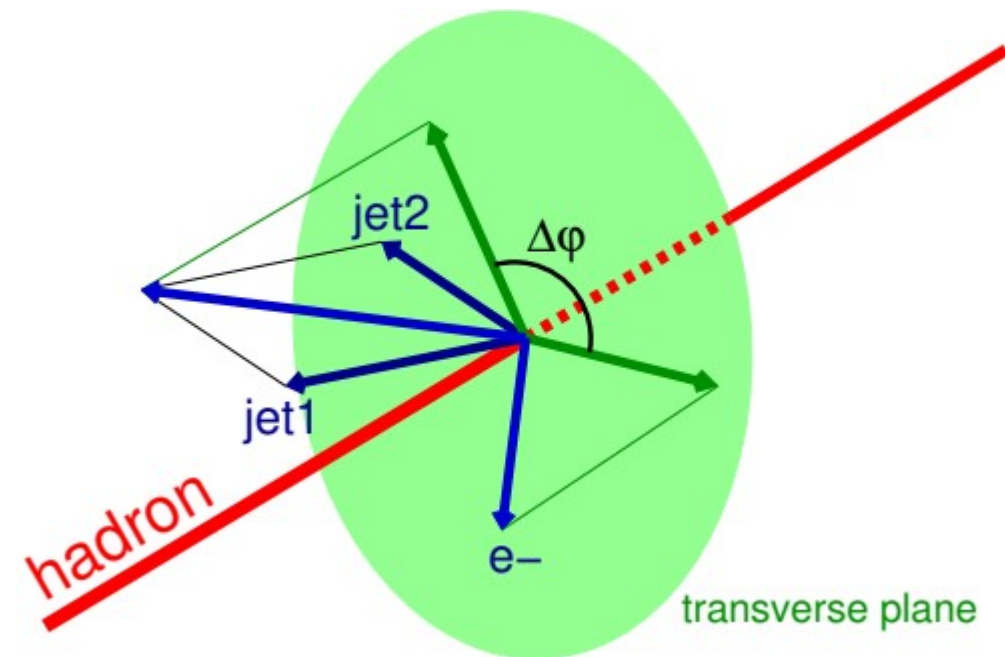


For asymmetric window $x_{Bj} \sim 10^{-4}$, $x_A \sim 5 \cdot 10^{-3}$

From P. Kotko DIS 2021

Azimuthal correlations EIC kinematics

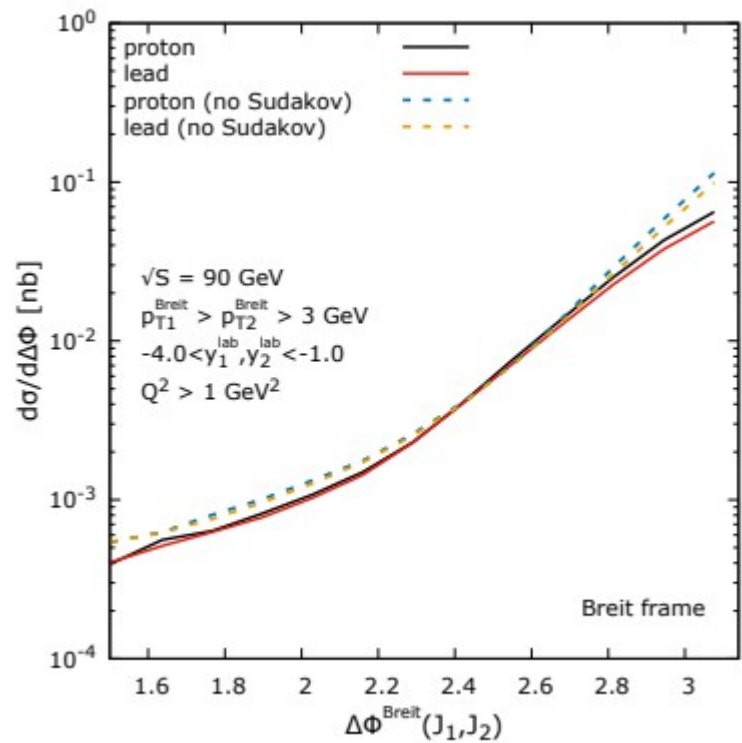
angle between di-jet and electron



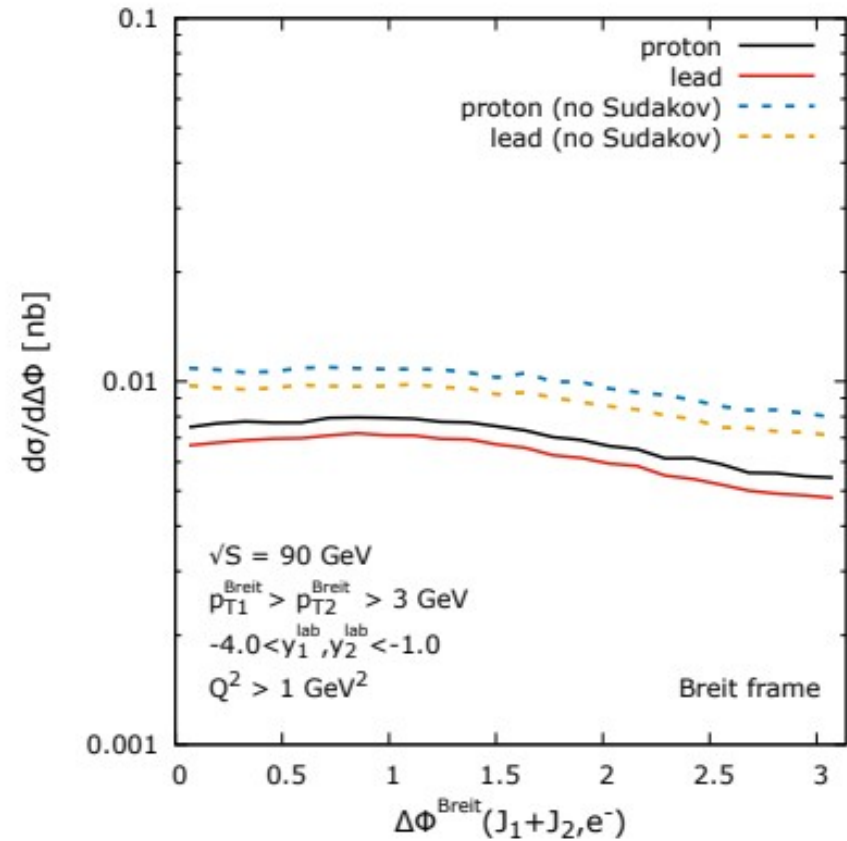
Large Sudakov effects.

Azimuthal correlations EIC kinematics

angle between dijets

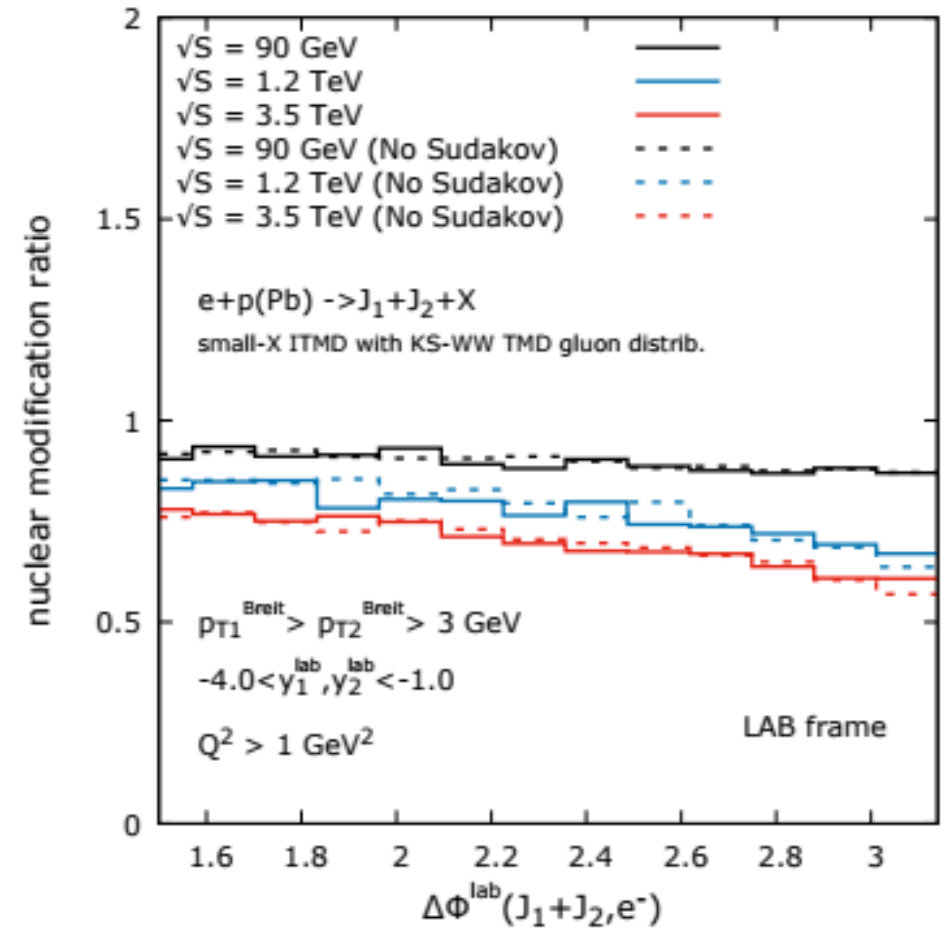
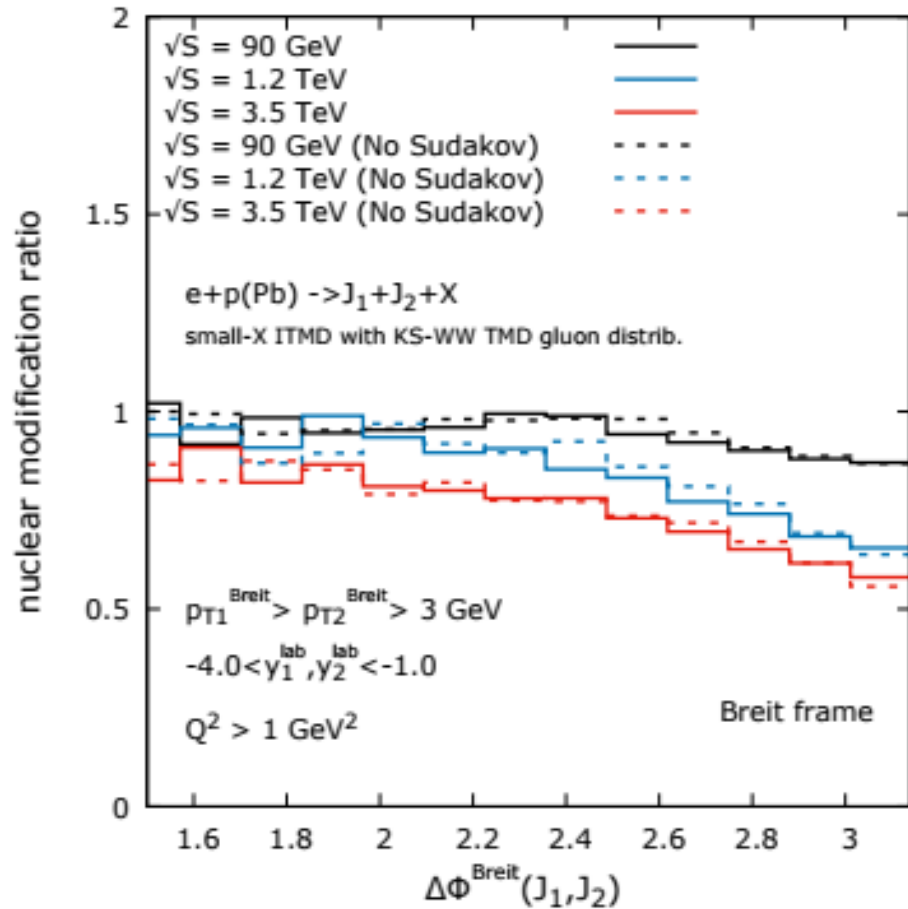


angle between di-jet and electron



Large Sudakov effects.

Energy dependence of nuclear modification factor



Evidently the larger energy the more visible saturation effects are.
Sudakov effects in R_{pA} cancel to large extent.

Monte Carlo tools for k_T dependent initial states

KaTie (A. van Hameren)

Comput.Phys.Commun. 224 (2018) 371-380

- Monte Carlo program for tree-level calculations e-p, p-p, p-A, A-A
- any process within the Standard Model (High Energy Factorization)
- *dijets, trijets using iTMD*
- initial-state partons on-shell or off-shell dependent on factorization framework
- employs numerical Dyson-Schwinger recursion to calculate helicity amplitudes
- automatic phase space optimization
- It can be used together with CASCADE to account for initial and final state showers for example using PB TMD

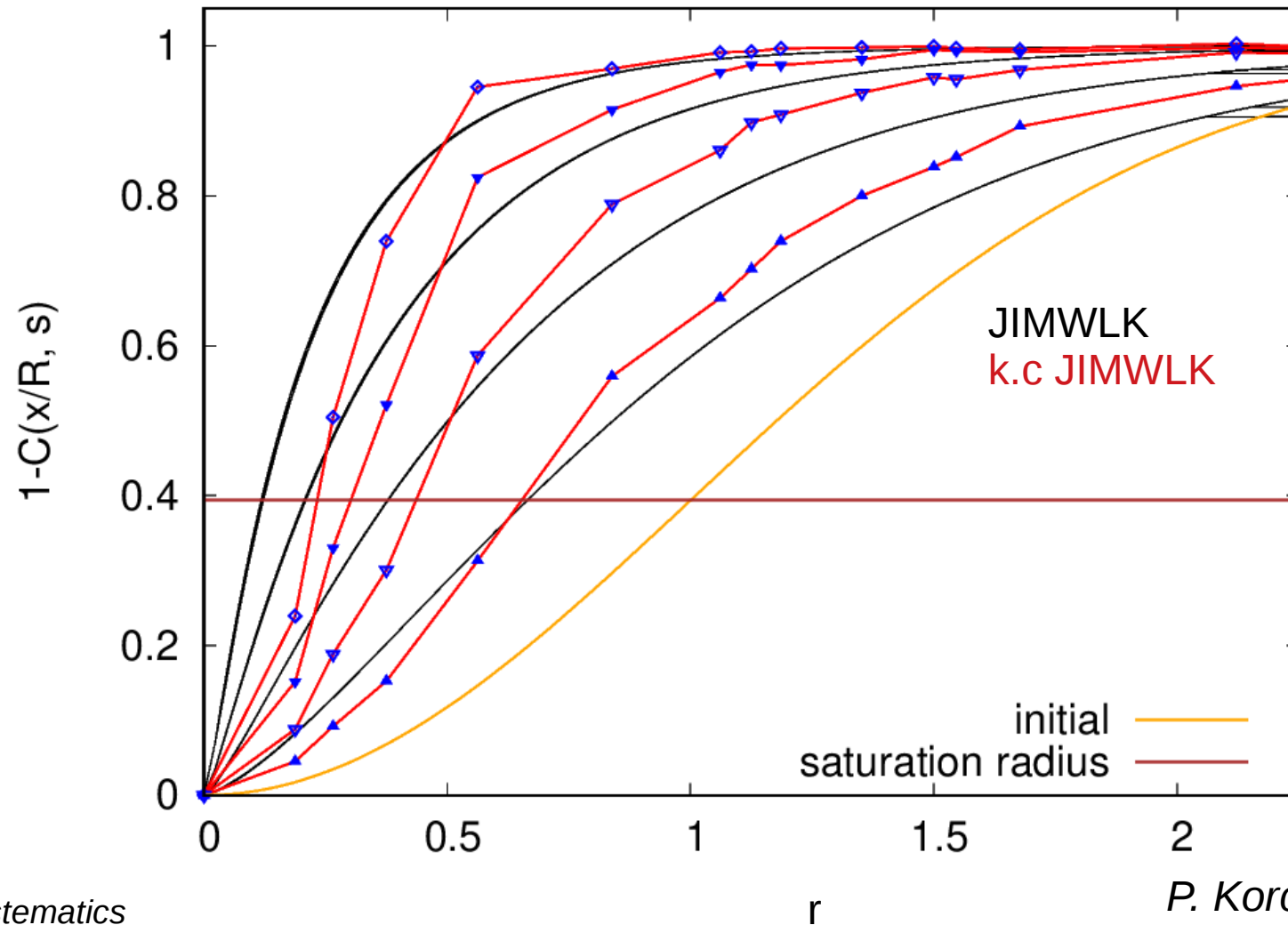
H. Van Haevermaet, A. Van Hameren, P. Kotko, K. Kutak, P. Van Mechelen
Eur.Phys.J.C 80 (2020) 7, 610

Ongoing work on developments at NLO accuracy

JIMWLK with kinematical-constraint

Proposal by Hatta, Iancu '16

See discussion by A. Stasto on Wednesday



JIMWLK systematics

Cali, Cichy, Korcyl, Kotko, Kutak, Marquet '20

New algorithm for MV initial conditions – results do not suffer from volume dependence Korcyl '21

P. Korcyl, preliminary

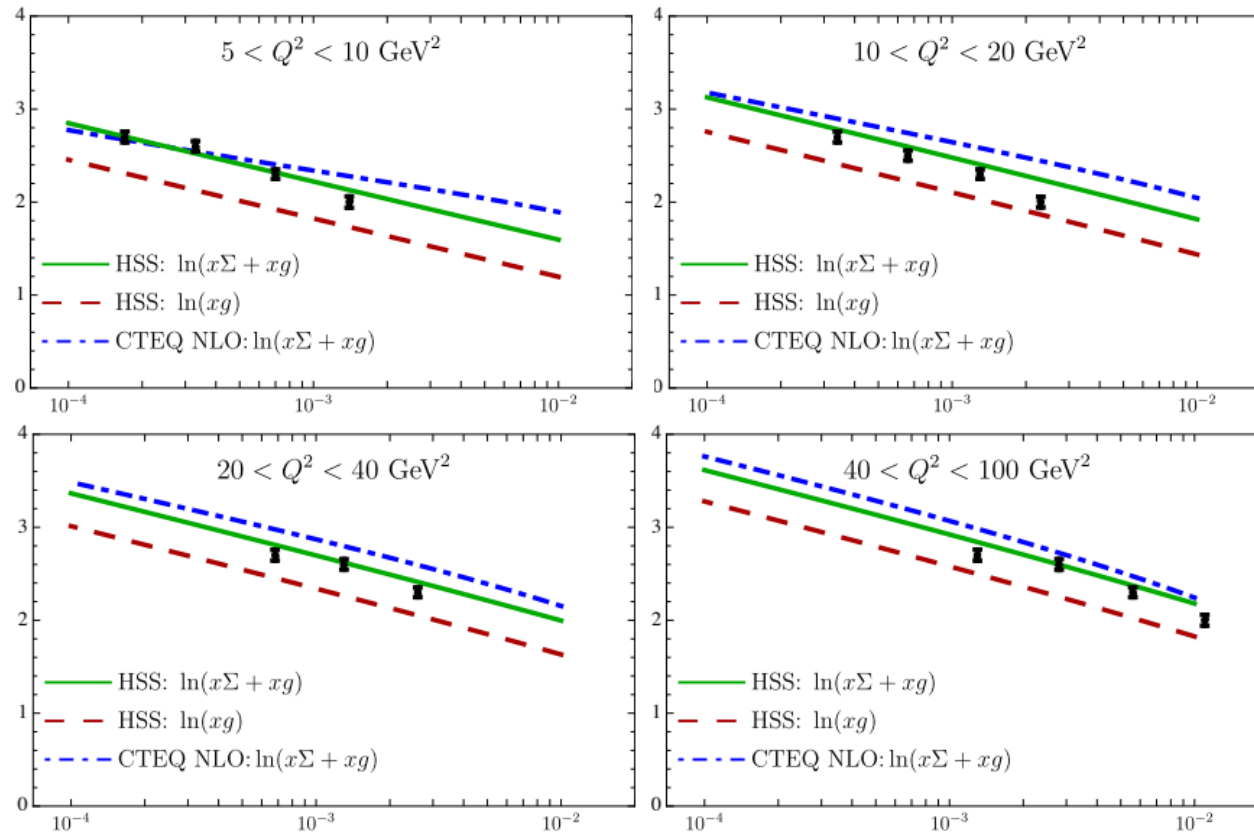
Entropy and BFKL

2110.06156 Hentschinski, Kutak

See also talk by
D. Kharzeev on Thursday

Motivated by
Kharzeev, Levin '17
Tu, Kharzeev, Ullrich '20
Kharzeev, Levin '21

HSS gluon density used
i.e. NLO BFKL +
kinematical
Improvements
Hentschinski, Sabio-Vera, Salas.
Phys.Rev.D 87 (2013) 7, 076005
Phys.Rev.Lett. 110 (2013) 4, 041601



$$S(x, Q^2) = \ln \left\langle n \left(\ln \frac{1}{x}, Q \right) \right\rangle$$

$$xg(x, Q) = \int_0^{Q^2} d\mathbf{k}^2 \mathcal{F}(x, \mathbf{k}^2)$$

Our proposal

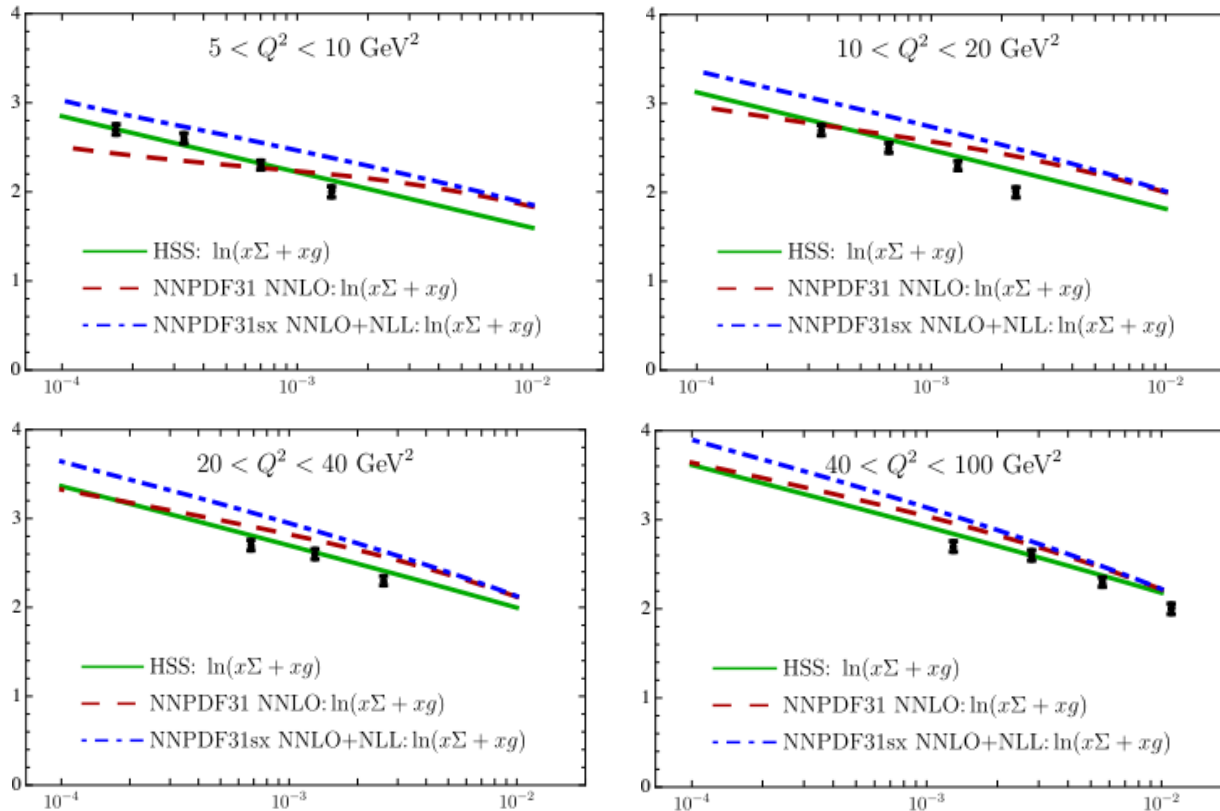
$$\left\langle n \left(\ln \frac{1}{x}, Q \right) \right\rangle = xg(x, Q) + x\Sigma(x, Q)$$

$$x\Sigma(x, Q) = P_{qg}(Q, \mathbf{k}) \otimes \mathcal{F}(x, \mathbf{k}^2)$$

Entanglement entropy and BFKL

2110.06156 Hentschinski, Kutak

Low x resummation is essential to get good description of data



HSS gluon density used
i.e. NLO BFKL +
kinematical
Improvements
[Hentschinski, Sabio-Vera, Salas. Phys.Rev.D 87 \(2013\) 7, 076005](#)
[Phys.Rev.Lett. 110 \(2013\) 4, 041601](#)

$$S(x, Q^2) = \ln \left\langle n \left(\ln \frac{1}{x}, Q \right) \right\rangle$$

$$xg(x, Q) = \int_0^{Q^2} d\mathbf{k}^2 \mathcal{F}(x, \mathbf{k}^2)$$

Our proposal

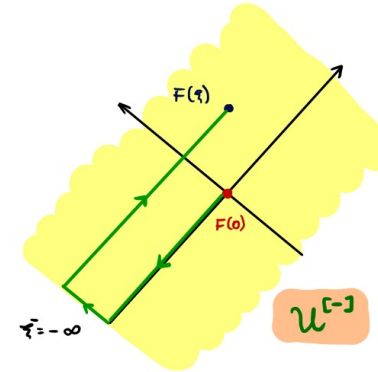
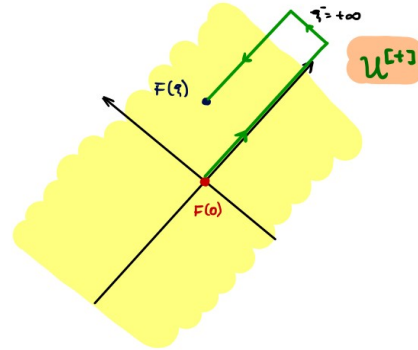
$$\left\langle n \left(\ln \frac{1}{x}, Q \right) \right\rangle = xg(x, Q) + x\Sigma(x, Q)$$

$$x\Sigma(x, Q) = P_{qg}(Q, \mathbf{k}) \otimes \mathcal{F}(x, \mathbf{k}^2)$$

Back up

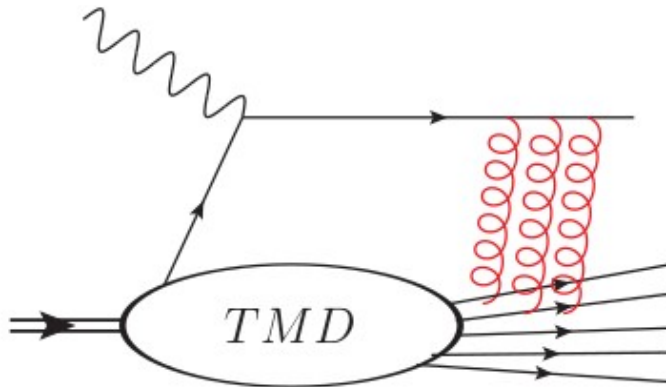
Definition of TMD – gauge links

Two basic structures arise:



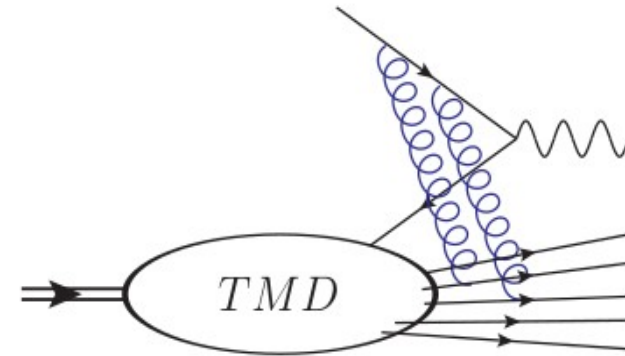
from P. Kotko, Bialasówka 2019

Semi Inclusive DIS



final state interactions

Drell-Yan



initial state interactions

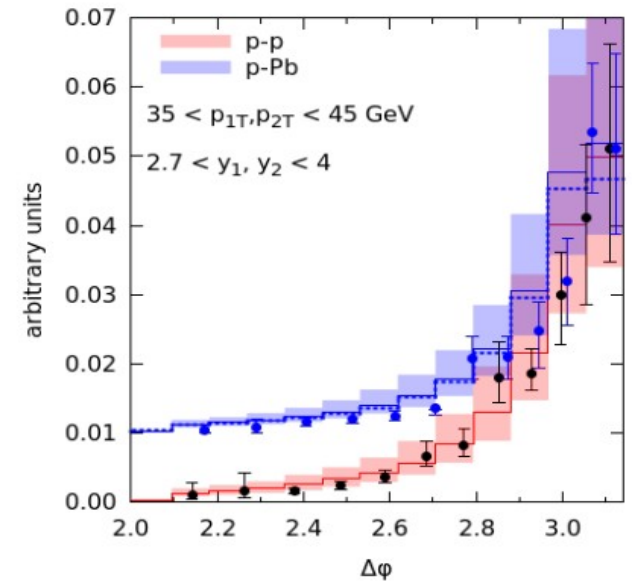
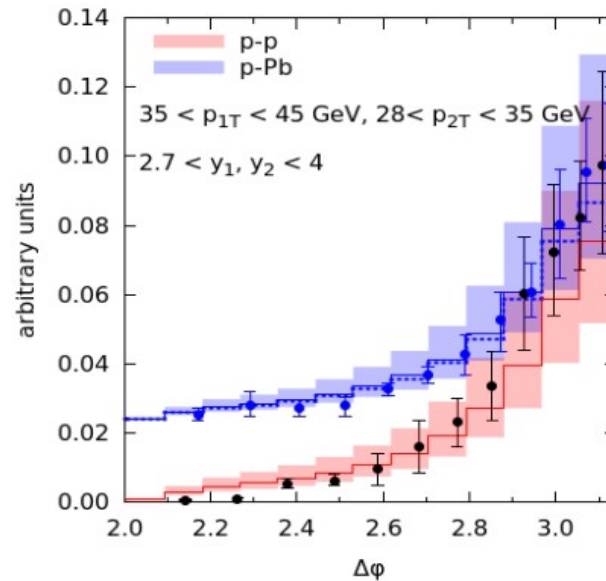
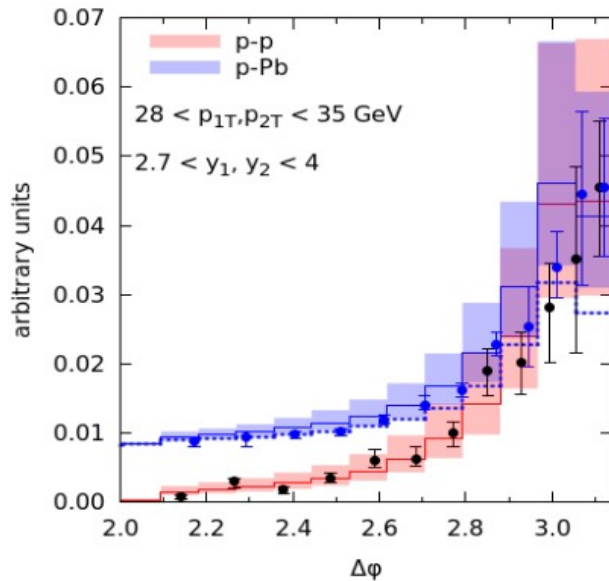
from R. Boussarie
Initial Stages 2019

$$\Phi_q^{[+]}(x, p_T) = \int \frac{d(\xi \cdot P) d^2 \xi_T}{(2\pi)^3} e^{ip \cdot \xi} \langle H | \bar{\psi}(0) \mathcal{U}^{[+]} \psi(\xi) | H \rangle$$

C.J. Bomhof, P.J. Mulders, F. Pijlman
Eur.Phys.J. C47 (2006) 147-162

$$\mathcal{U}^{[\pm]} = U_{[(0^-, \mathbf{0}_T); (\pm\infty^-, \mathbf{0}_T)]}^n U_{[(\pm\infty^-, \mathbf{0}_T); (\pm\infty^-, \infty_T)]}^T U_{[(\pm\infty^-, \infty_T); (\pm\infty^-, \xi_T)]}^T U_{[(\pm\infty^-, \xi_T); (\xi^-, \xi_T)]}^n$$

The *i*TMD factorization for di-jets in p - p and p -Pb



A. Hameren, P. Kotko, K. Kutak, S. Sapeta
*Phys.Lett. B*795 (2019) 511-515

To describe data we had to account both for Sudakov effects and for saturation.

Formalism implemented in Monte Carlo programs KaTie by A. van Hameren and LxJet by P. Kotko

ITMD from Color Glass Condensate

T. Altinoluk, R. Boussarie, P. Kotko JHEP 1905 (2019) 156

T. Altinoluk, R. Boussarie, JHEP10(2019)208

$$|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$$

TMD regime

$$+ (k_t/P_t)^n$$

$$|p_{1t}|, |p_{2t}|, |k_t| \gg Q_s$$

BFKL regime

$$+ (Q_s/k_t)^n$$

ITMD

$$|p_{1t}|, |p_{2t}| \gg Q_s$$

$$+ (Q_s/P_t)^n$$

CGC

Altinoluk, Boussarie (2019)

From Cyrille Marquet

Backup

Background

$\sim 0 p_T$ in Breit frame

$$2x_{\text{Bj}}\vec{p} + \vec{q} = 0$$

