Pseudo and quasi gluon PDF in the BFKL approximation

Giovanni Antonio Chirilli

University of Regensburg

RBRC Workshop: Small-x Physics in the EIC Era
17 Dec, 2021

Based on arXiv: 2111.12709[hep-ph]

Motivation

- Lattice gauge theory is formulated in Euclidean space
  - direct calculation of the PDFs would be impossible for objects that are defined through the light-cone matrix element of gauge-invariant bi-local operators

- Idea Consider equal-time correlators and perform the Lattice analysis in coordinate space through the Ioffe-time distributions.

- Fourier transform in momentum space
  - quasi-PDF X. Ji (2013)
  - pseudo-PDF A. Radyushkin (2017)

- Lattice calculations provide values of the Ioffe-time distributions for a limited range of the distance separating the bi-local operators. In order to perform the Fourier transform for the quasi-PDF or the pseudo-PDF, it is then necessary to extrapolate the large-distance behavior.

Tensor decomposition

I. Balitsky, W. Morris, A. Radyuskin (2020)

Tensor decomposition over invariant amplitudes of the gluon matrix element
Tensor structures are build from P^μ, x^μ, and $g^{\mu\nu}$

$$\begin{aligned} M_{\mu\alpha;\lambda\beta} &\equiv \langle P|G_{\mu\alpha}(z)[z,0]G_{\lambda\beta}(0)|P\rangle \\ &= I_{1\mu\alpha;\lambda\beta}\mathcal{M}_{pp} + I_{2\mu\alpha;\lambda\beta}\mathcal{M}_{zz} + I_{3\mu\alpha;\lambda\beta}\mathcal{M}_{zp} \\ &\quad + I_{4\mu\alpha;\lambda\beta}\mathcal{M}_{pz} + I_{5\mu\alpha;\lambda\beta}\mathcal{M}_{ppzz} + I_{6\mu\alpha;\lambda\beta}\mathcal{M}_{gg} \end{aligned}$$

the amplitudes $\mathcal{M}$ are functions of the invariants z^2 and $z \cdot P = \varrho$ (Ioffe time)

light-cone Gluon distribution is obtained from

$$g_\perp^{\alpha\beta} M_{+\alpha;\beta_+}(z^+, P) = -2(P^-)^2 \mathcal{M}pp$$

The PDF is determined by the $\mathcal{M}_{pp}$ structure

$$M_{+i;+i} = M_{0i;0i} + M_{3i;3i} + M_{0i;3i} + M_{3i;0i}$$

$$M_{0i;i0} + M_{ji;ij} = 2p_0^2 \mathcal{M}_{pp} \stackrel{\text{high}-\text{energy}}{\longrightarrow} M_{+i;+i}$$

At high energy (Regge limit) the transverse components are suppresed and we do not distinguish between the 0-component and the 3-component

Definition of the pseudo and quasi gluon PDF

Ioffe-time distribution at high energy

$$z_\mu \, z_\nu \, \langle P|G^{ai\mu}(z)[z,0]G_i^{b\,\nu}(0)|P\rangle = 2\,\varrho^2 \mathcal{M}_{pp}(\varrho, z^2)$$

Ioffe-time $\varrho \equiv z \cdot P$ z^μ space-like vector $i = 1, 2$

Pseudo-PDF: Fourier transform with respect to P keeping its orientation fixed

$$G_{\rm p}(x_B, z^2) = \int \frac{d\varrho}{2\pi} \, e^{-i\varrho x_B} \mathcal{M}_{pp}(\varrho, z^2)$$

Quasi-PDF: Fourier transform with respect to z keeping its orientation fixed

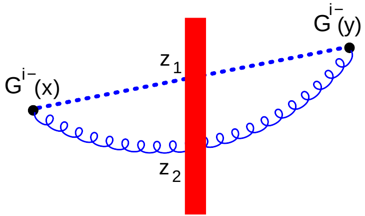
$$G_q(x_B, P_\xi) = P_\xi \int \frac{d\varsigma}{2\pi} \, e^{-i\varsigma\, P_\xi x_B} \mathcal{M}_{pp}(\varsigma P_\xi, \varsigma^2)$$

$\xi^\mu = \frac{z^\mu}{|z|}$ $P_\xi = P \cdot \xi$

High-energy operator product expansion

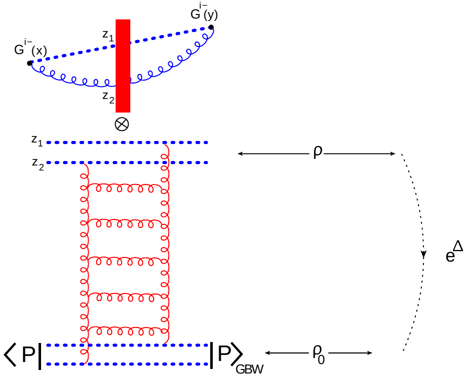
At high-energy we do not distinguish between the 0 and 3 components

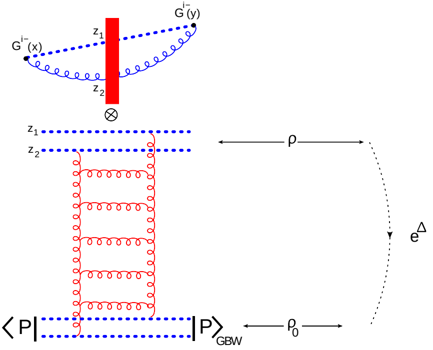
$$\langle P|G^{ai-}(z)[z,0]G_i^{b\,-}(0)|P\rangle = 2(P^-)^2\mathcal{M}_{pp}(\varrho, z^2)$$



High-energy operator product expansion formalism is formulated in coordinate space $\Rightarrow$ is suitable to reach our goal.

High-energy operator product expansion



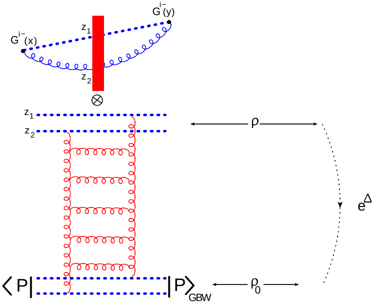


Resum $\alpha_s \ln \varrho$ with BFKL eq. $a = -\frac{2x^+y^+}{(x-y)^2 a_0}$

$$2a\frac{d}{da}\,\mathcal{V}_a(z_\perp) = \frac{\alpha_s N_c}{\pi^2}\int d^2z'\left[\frac{\mathcal{V}_a(z'_\perp)}{(z-z')^2_\perp} - \frac{(z,z')_\perp \mathcal{V}_a(z_\perp)}{z'^2_\perp(z-z')^2_\perp}\right]$$

$\frac{1}{z^2_\perp}\mathcal{U}(z_\perp) \equiv \mathcal{V}(z_\perp) \qquad \mathcal{U}(x_\perp, y_\perp) = 1 - \frac{1}{N_c}\mathrm{tr}\{U(x_\perp)U^\dagger(y_\perp)\}$

High-energy operator product expansion

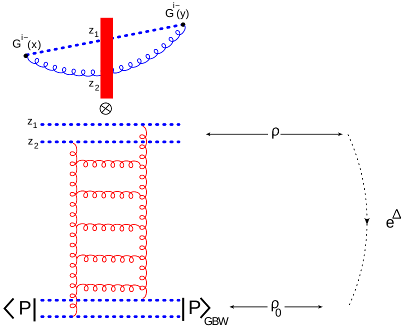


Resum $\alpha_s \ln \varrho$ with BFKL eq. $a = -\frac{2x^+y^+}{(x-y)^2 a_0}$

$$2a\frac{d}{da}\,\mathcal{V}_a(z_\perp) = \frac{\alpha_s N_c}{\pi^2}\int d^2z'\Big[\frac{\mathcal{V}_a(z'_\perp)}{(z-z')^2_\perp} - \frac{(z,z')_\perp \mathcal{V}_a(z_\perp)}{z'^2_\perp(z-z')^2_\perp}\Big]$$

solution $$\mathcal{V}^a(z_{12}) = \int \frac{d\nu}{2\pi^2}(z_{12}^2)^{-\frac{1}{2}+i\nu}\left(\frac{a}{a_0}\right)^{\frac{\aleph(\gamma)}{2}}\int d^2\omega(\omega_\perp^2)^{-\frac{1}{2}-i\nu}\mathcal{V}^{a_0}(\omega_\perp)$$

Ioffe-time distribution in the saddle-point approximation



$$\mathcal{M}_{pp}(\varrho, z^2) = \frac{3N_c^2}{128\,\varrho}\frac{Q_s\,\sigma_0}{|z|}\left(\frac{2\,\varrho^2}{z^2M_N^2} + i\epsilon\right)^{\bar{\alpha}_s 2\ln 2}\frac{e^{-\frac{\ln^2\frac{Q_s|z|}{2}}{7\zeta(3)\bar{\alpha}_s\ln\left(\frac{2\,\varrho^2}{z^2M_N^2}+i\epsilon\right)}}}{\sqrt{7\zeta(3)\bar{\alpha}_s\ln\left(\frac{2\,\varrho^2}{z^2M_N^2} + i\epsilon\right)}}$$

saturation scale Q_s , $\sigma_0 = 29.1$ mb , M_N mass of the nucleon

Leading and next-to-leading twist

Analytic continuation of local-operator $\Rightarrow$ light-ray operators

$$F^a_{p_1\xi}(x)\nabla^{j-2}F^{a\ \xi}_{p_1}(x)\Big|_{x=0} \stackrel{\text{forw.}}{=} \frac{1}{\Gamma(2-j)}\int_0^\infty dv\, v^{1-j}\, F^a_{p_1\xi}(0)[0,vp_1]^{ab}F^{b\ \xi}_{p_1}(vp_1)$$

$\omega = j-1 \to 0 \;\Leftrightarrow\; x_B \to 0$ at $\frac{\alpha_s}{\omega}\sim 1 \;\Rightarrow$ resummation: BFKL eq.

To get the leading and next-to-leading residues we need to approach the DGLAP limit $\alpha_s \ll \omega \ll 1$

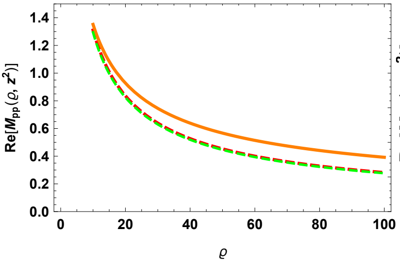
Leading and next-to-leading twist for the Ioffe-time-distribution

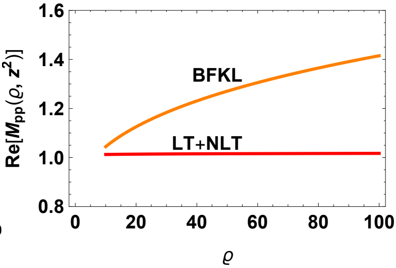
$$\mathcal{M}_{pp}(\varrho, z^2) = \frac{N_c^2}{8\pi^2\bar\alpha_s}\frac{Q_s^2\sigma_0}{\varrho}\left(\frac{4\bar\alpha_s\left|\ln\frac{Q_s|z|}{2}\right|}{\ln\left(\frac{2\,\varrho^2}{z^2M_N^2}+i\epsilon\right)}\right)^{\frac{1}{2}} I_1(\tilde t)\left(1+\frac{Q_s^2|z|^2}{5}\right) + O\left(\frac{Q_s^4|z|^4}{16}\right)$$

with

$$\tilde t = \left[4\bar\alpha_s\left|\ln\frac{Q_s|z|}{2}\right|\ln\left(\frac{2\,\varrho^2}{z^2M_N^2}+i\epsilon\right)\right]^{\frac{1}{2}}$$

Ioffe-time distribution at large-longitudinal distances





- Left panel
  - Orange curve is the BFKL resummation
  - Green-dash and red-dash are the LT and LT+NLT respectively.

- Right panel: BFKL resummation (Orange) and LT+NLT (red) both normalized to the LT.

Pseudo gluon PDF

BFKL resummation

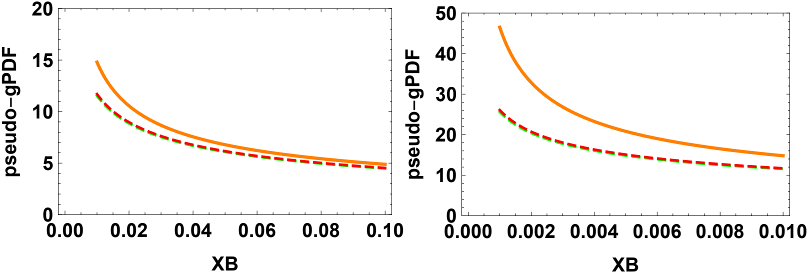
$$G_{\rm p}(x_B,z^2) = -i\frac{3N_c^2}{128\pi}\frac{Q_s\sigma_0}{|z|}\frac{\Gamma(\bar\alpha_s 4\ln 2)\sin(\frac{\pi}{2}\bar\alpha_s 4\ln 2)}{\sqrt{7\zeta(3)\bar\alpha_s\ln\left(\frac{2}{x_B^2z^2M_N^2}+i\epsilon\right)}}{\rm sign}(x_B)$$

$$\times\exp\left\{\frac{-\ln^2\frac{Q_s|z|}{2}}{28\zeta(3)\bar\alpha_s\ln\left(\frac{2}{x_B^2z^2M_N^2}+i\epsilon\right)}\right\}\left(\frac{2}{x_B^2z^2M_N^2}+i\epsilon\right)^{\bar\alpha_s 2\ln 2}$$

Leading and next-to-leading twist

$$G_{\rm p}(x_B,z^2) = \frac{N_c^2\,Q_s^2\sigma_0}{16\pi^3\bar\alpha_s}\frac{1}{2\pi i}\int_{1-i\infty}^{1+i\infty}d\omega\,\left(\frac{2}{x_B^2|z|^2M_N^2}\right)^{\frac{\omega}{2}}\left(\frac{4}{Q_s^2|z|^2}\right)^{\frac{\bar\alpha_s}{\omega}}\Gamma(\omega)$$
$$\times\left(1+\frac{Q_s^2|z|^2}{5}\right)+O\left(\frac{Q_s^4|z|^4}{16}\right)$$

Pseudo gluon PDF



Pseudo-PDF have typical behavior of gluon distribution at low-x_B.

quasi gluon PDF

BFKL resumation $\aleph(\gamma) \equiv \frac{\alpha_s N_c}{\pi}\Big(2\psi(1) - \psi(\gamma) - \psi(1-\gamma)\Big)$ $\gamma = \frac{1}{2} + i\nu$

$$G_{\rm q}(x_B, P_\xi) = i\,\frac{3N_c^2}{4\pi^4} Q_s \sigma_0 P_\xi |x_B| \int d\nu \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)^{\frac{\aleph(\gamma)}{2}} \left(\frac{Q_s^2}{4P_\xi^2 x_B^2}\right)^{i\nu} \times \frac{\gamma\,\Gamma^2(1-\gamma)\Gamma^3(1+\gamma)\Gamma(2\gamma-2)}{\Gamma(2+2\gamma)} \sinh(\pi\nu)$$

Leading + next-to-leading twist

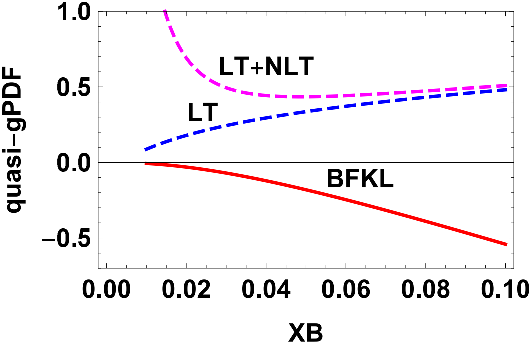
$$G_{\rm q}(x_B, P_\xi) = -\frac{N_c^2\, Q_s^2\, \sigma_0}{16\bar\alpha_s^2 \pi^3} \frac{1}{2\pi i} \int_{1-i\infty}^{1+i\infty} d\omega \left(-\frac{2P_\xi^2}{M_N^2} + i\epsilon\right)^{\frac{\omega}{2}} \left(-\frac{4P_\xi^2 x_B^2}{Q_s^2} + i\epsilon\right)^{\frac{\bar\alpha_s}{\omega}} \times \left(\omega + \frac{2\bar\alpha_s Q_s^2}{5} \frac{1}{P_\xi^2 x_B^2}\right)$$

Usual exponentiation of the BFKL pomeron intercept, which resums logarithms of x_B, is missing.

For low values of x_B and fixed values of P these corrections are enhanced rather than suppressed at this regime.

quasi gluon PDF

Here $P_\xi = 4$ GeV.



Behavior of curves will not change even for values of $P_\xi = 100$ GeV.

Quasi-PDF have rather unusual behavior at low-x_B.

The usual exponentiation of the BFKL pomeron intercept, which resums logarithms of x_B, is missing.

Conclusions

- Large-distance behavior of the gluon Ioffe-time distribution is computed
  - Ioffe-time ϱ acts as rapidity parameter.
    - $\alpha_s \ln \varrho$ resumed by BFKL eq.
  - Ioffe-time distribution is a very slowly varying function at large values of ϱ.
- Pseudo-PDF and quasi-PDF have a very different behavior at low-x_B.
  - pseudo-PDF have typical behavior of gluon distribution at low-x_B.
  - quasi-PDF have rather unusual behavior at low-x_B.
    - usual exponentiation of the BFKL pomeron intercept, which resums logarithms of x_B, is missing.
- The power corrections in the quasi-PDF do not come in as inverse powers of P but as inverse powers of $x_B P$
  - for low values of x_B and fixed values of P these corrections are enhanced rather than suppressed at this regime.