Dijet impact factor in DIS at NLO
—
some highlights.

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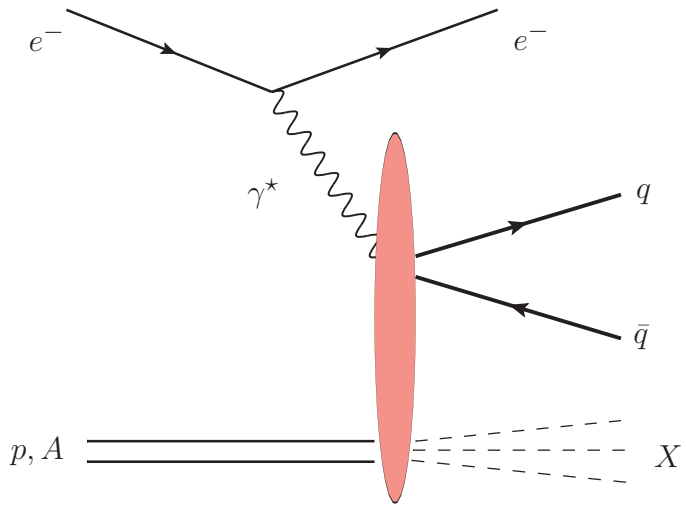
Inclusive dijet production in DIS at small x

- Inclusive dijet (or dihadron) production in DIS at small x:
  - $\Rightarrow$ probe of the saturated regime of QCD
  - $\Rightarrow$ access to the Weizsäcker-Williams gluon distribution
  - $\Rightarrow$ and to the quadrupole correlator of Wilson lines



This work

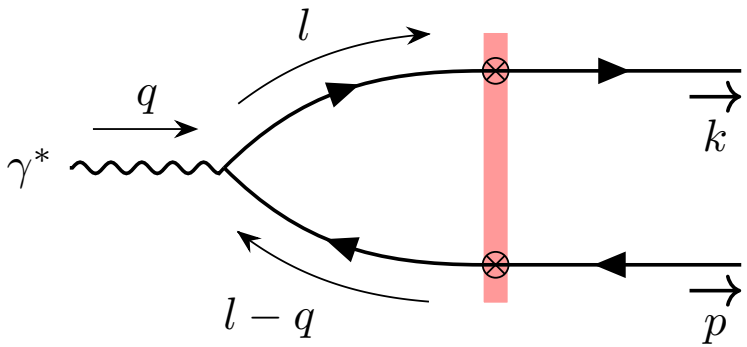
- Full NLO computation in the Regge limit $s \gg Q^2 \gg \Lambda_{\mathrm{QCD}}$, with completely general kinematics of the dijet system.
- Both longitudinal and transverse $\gamma^\star$.

Outline

- Brief overview of the computation.
- Main results.
- Prospects for the future.

Dipole picture, CGC EFT, covariant perturbation theory

- We work in the dipole picture of DIS.



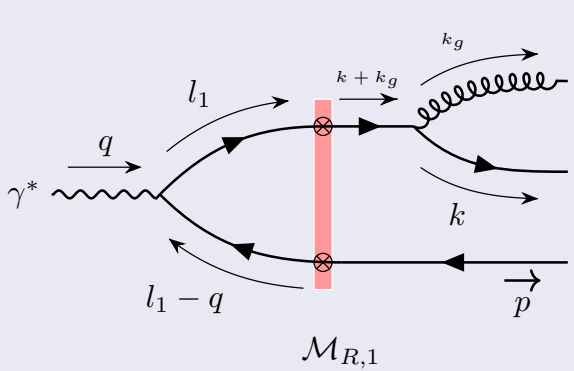
- Covariant perturbation theory.

- CGC effective vertex:
  $= (2\pi)\delta(q^- - p^-)\gamma^- \int \mathrm{d}^2\boldsymbol{x}_\perp e^{-i(\boldsymbol{q}_\perp - \boldsymbol{p}_\perp)\boldsymbol{x}_\perp} V_{ij}(\boldsymbol{x}_\perp)$

  $\Rightarrow$ multiple gluon interactions with the target resummed via path-ordered Wilson lines $V(\boldsymbol{x}_\perp)$

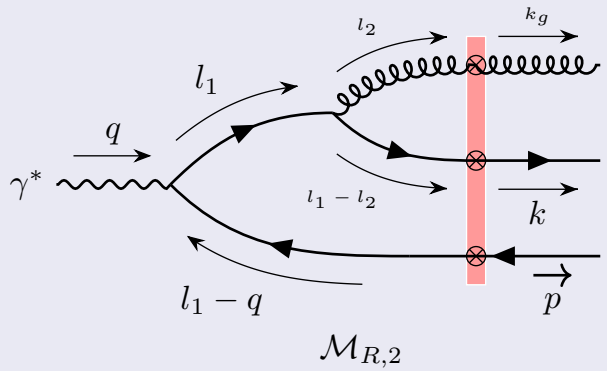
The diagrams we have computed (1/2)

Real diagrams



$\mathcal{M}_{R,1}$

$+(q \longleftrightarrow \bar{q})$



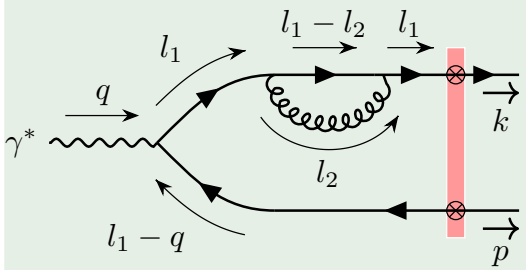
$\mathcal{M}_{R,2}$

$+(q \longleftrightarrow \bar{q})$

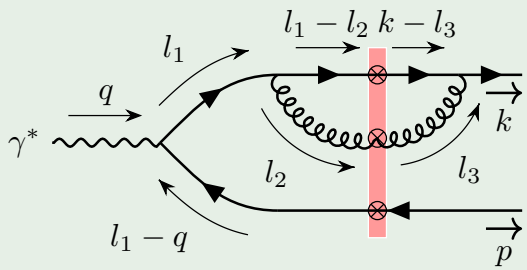
- Already computed by Ayala, Hentschinski, Jalilian-Marian, Tejeda-Yeomans, 1701.07143 using spinor helicities techniques.
- We recover their results.

The diagrams we have computed (2/2)

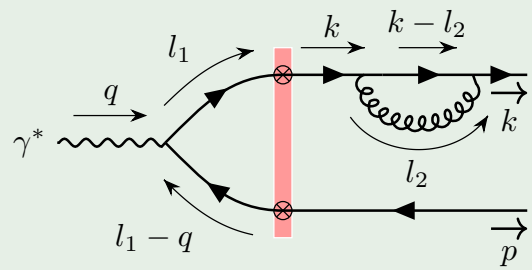
Self-energies



$+(q \longleftrightarrow \bar{q})$

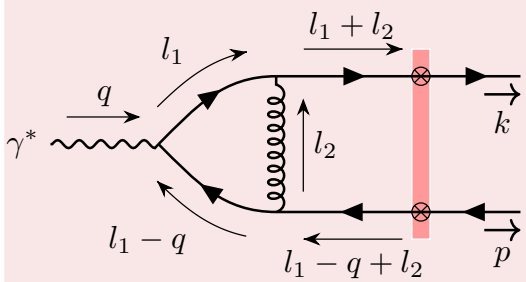


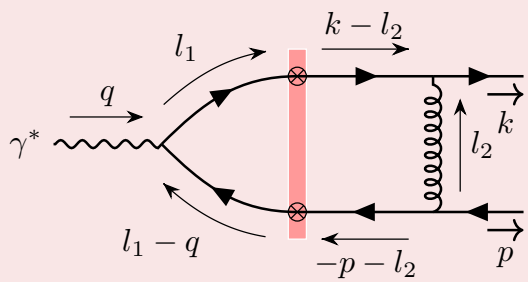
$+(q \longleftrightarrow \bar{q})$

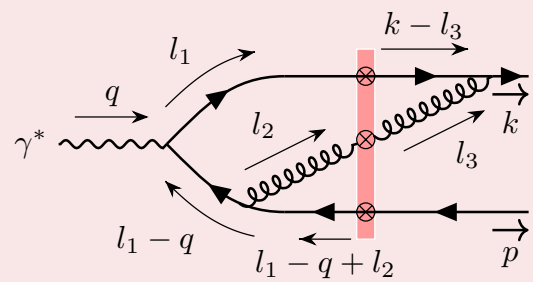


$+(q \longleftrightarrow \bar{q})$

Vertex corrections







$+(q \leftrightarrow \bar{q})$

Main results

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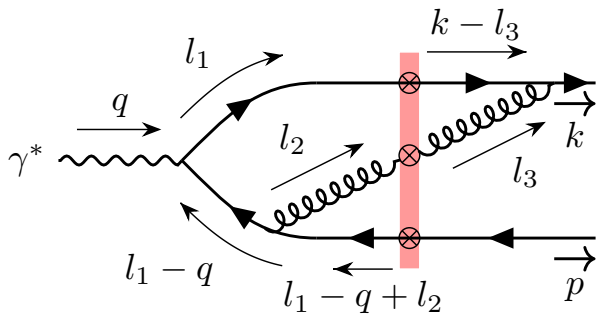
Reducing the number of integrals

- Example: the dressed vertex correction for longitudinally polarized $\gamma^\star$.



$$= \frac{ee_f q^-}{\pi} \int \mathrm{d}^2\boldsymbol{x}_\perp \mathrm{d}^2\boldsymbol{y}_\perp \mathrm{d}^2\boldsymbol{z}_\perp e^{-i\boldsymbol{k}_\perp \boldsymbol{x}_\perp - i\boldsymbol{p}_\perp \boldsymbol{y}_\perp} [t^a V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{z}_\perp) t_a V(\boldsymbol{z}_\perp) V^\dagger(\boldsymbol{y}_\perp) - t^a t_a]$$

$$\times \frac{\alpha_s}{\pi^2} 2(z_q z_{\bar{q}})^{3/2} Q \delta_{\sigma,-\bar{\sigma}} \int_0^{z_q} \frac{\mathrm{d}z_g}{z_g} e^{-iz_g \boldsymbol{k}_\perp / z_q \cdot \boldsymbol{r}_{zx}} \left(1 + \frac{z_g}{z_{\bar{q}}}\right) \left(1 - \frac{z_g}{z_q}\right) K_0\left(\Delta_{V,3} \sqrt{\boldsymbol{R}_V^2 + \omega_V \boldsymbol{r}_{zy}^2}\right)$$

$$\times \left\{ \left[1 - \frac{z_g}{2z_q} - \frac{z_g}{2(z_{\bar{q}} + z_g)}\right] \frac{\boldsymbol{r}_{zx} \cdot \boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zy}^2} + i\sigma \left[\frac{z_g}{2z_q} - \frac{z_g}{2(z_{\bar{q}} + z_g)}\right] \frac{\boldsymbol{r}_{zx} \times \boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2 \boldsymbol{r}_{zy}^2} \right\}$$

Take home message

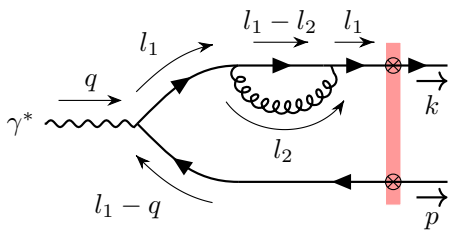
- Compact expression!
- Suitable for numerical evaluation.

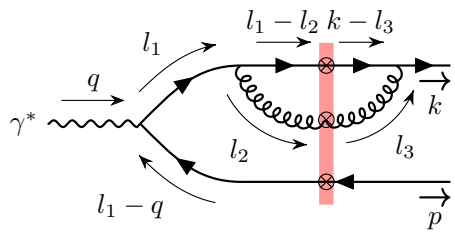
What kind of divergence do we get?

- UV (short distance) divergences
  - internal momentum goes to ∞ or $|\boldsymbol{z}_\perp - \boldsymbol{x}_\perp| \to 0$.
  - we use dim. reg. in the transverse plane to extract the UV pole of each diagram if any.
- Rapidity divergence, "slow gluon" when $k_g^- \to 0$.
- Soft divergence $k_g^\mu \to 0$.
- Collinear divergence, $z_q \boldsymbol{k}_{\perp g} - z_g \boldsymbol{k}_\perp \to 0$ or $z_{\bar{q}} \boldsymbol{k}_{\perp g} - z_g \boldsymbol{p}_\perp \to 0$.

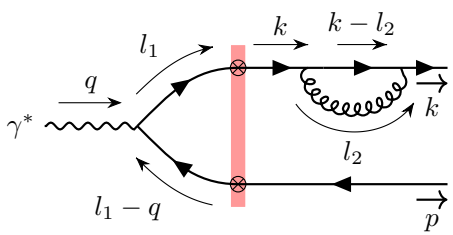
Cancellation of UV divergences

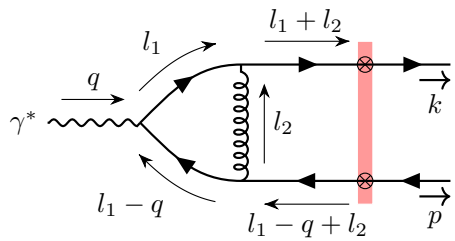
- Massless quark + universality of quark electric charge $\Rightarrow$ **no need for UV renormalization**
- UV divergence cancels between free self-energy before shock-wave and dressed self energy





- The free self-energies after SW turn UV divergence of the free vertex correction before shock-wave into IR





$+(q \leftrightarrow \bar{q})$

$\propto \left(\frac{2}{\varepsilon_{\mathrm{IR}}} - \frac{2}{\varepsilon_{\mathrm{UV}}} \right)$ $\propto \frac{2}{\varepsilon_{\mathrm{UV}}}$

Remaining $\frac{2}{\varepsilon_{\mathrm{IR}}}$ pole canceled by the real corrections.

Cancellation of rapidity divergences

- Rapidity divergence is regularized with a **longitudinal momentum cut-off** Λ^-.
- The slow gluon phase space is divided using a factorization scale k_f^-.
- We have proven:

$$\mathrm{d}\sigma_{\mathrm{NLO}}^{\gamma^\star \to q\bar{q}+X} = \alpha_s \ln\left(\frac{k_f^-}{\Lambda^-}\right) \underbrace{\mathcal{H}_{\mathrm{JIMWLK}} \otimes \mathrm{d}\sigma_{\mathrm{LO}}^{\gamma^\star \to q\bar{q}+X}}_{\text{action of JIMWLK on the LO x-section}} + \overbrace{\mathrm{finite}}^{\Lambda^- \to 0}$$

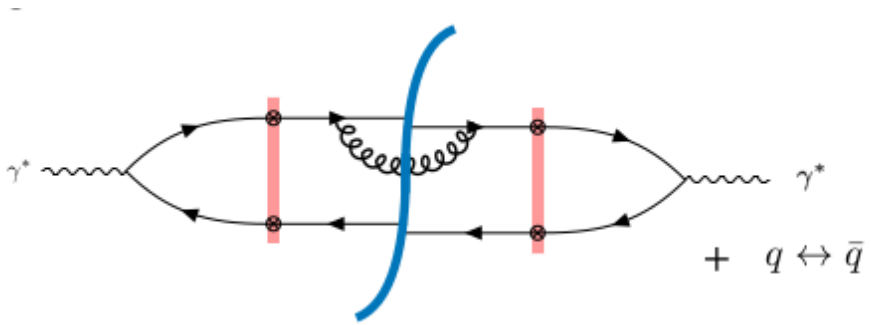
- Thus, the Λ^- dependence of the NLO impact factor is canceled by the JIMWLK evolution of the LO cross-section from Λ^- to k_f^-.

Cancellation of collinear divergences

- $1/\varepsilon$ pole from the free vertex correction before shock-wave remains.
- The pole cancels for IRC safe observable only $\Rightarrow$ jets.

$$\mathrm{d}\sigma^{\gamma^*_\lambda \to q\bar{q}+X} = \frac{\alpha_s C_F}{\pi}\left(\ln\left(\frac{k^-}{k_f^-}\right) + \ln\left(\frac{p^-}{k_f^-}\right) - \frac{3}{2}\right) \times \frac{2}{\varepsilon} \times \mathrm{d}\sigma_{\mathrm{LO}}^{\gamma^*_\lambda + A \to q\bar{q}+X}$$

$$+ \int_{k_f^-} \frac{\mathrm{d}k_g^-}{k_g^-} \frac{\mathrm{d}^{2-\varepsilon}\boldsymbol{k}_{\perp,g}}{(2\pi)^{2-\varepsilon}}\, \mathrm{d}\sigma^{\gamma^*_\lambda \to q\bar{q}g+X}\, \Theta(g \in q \text{ or } \bar{q} \text{ jet}) + \text{finite}$$



- The slow gluon phase space is $k_g^- < k_f^-$ is excluded.

Prospects for the future

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The back-to-back limit

- Focus on the regime $|P_\perp| = |z_{\bar{q}} k_\perp - z_q p_\perp| \gg |q_\perp| = |k_\perp + p_\perp|$.

- Several goals:

  - Simplify the formulas.

  - Recover the Sudakov logarithms $\sim \ln^2(P_\perp/q_\perp)$ due to soft gluon radiations.
    Mueller, Xiao, Yuan, 1308.2993

  - Clarify the resummation of these logs within the CGC effective field theory.

Apply our results to other related processes.

- Dihadron production: more relevant for the EIC?
- Inclusive production of a dijet pair in ultraperipheral nuclear collisions.
  $\Rightarrow$ photon+nucleus collision $= Q^2 \to 0$ limit of our result.
- Extend our results to massive quarks.

  see Beuf, Lappi, Paatelainen, 2103.14549 & 2112.03158

Conclusion

- **Proof of UV and IR finiteness** of the dijet cross-section within the CGC effective field theory.
- In particular: **proof of JIMWLK factorization** of the rapidity divergence for a process with non-trivial final state.
- We have obtained a **numerically tractable NLO impact factor** $\Rightarrow$ reach $\alpha_s^3 \ln(1/x)$ accuracy when combined with extant results for NLO BK-JIMWLK.
- Towards a numerical evaluation of the impact factor: **very challenging!** (9 to 11-dimensional integrals + Fourier phases...)